GUJCET 2024 Question Paper Mar 31 (Mathematics)

1. Global maximum value of function $f(x) = \sin x + \cos x, \quad x \in [0, \pi]$ **is: . . .**

Correct Answer: $\sqrt{2}$

Solution: Step 1: Expressing the function in an alternate form. The given function is:

$$f(x) = \sin x + \cos x.$$

We rewrite it using the identity:

$$\sin x + \cos x = \sqrt{2}\sin\left(x + \frac{\pi}{4}\right).$$

Thus, the function can be rewritten as:

$$f(x) = \sqrt{2}\sin\left(x + \frac{\pi}{4}\right).$$

Step 2: Determining the maximum value. Since the maximum value of $\sin\theta$ is 1, the maximum value of $f(x)$ is:

$$\sqrt{2} \times 1 = \sqrt{2}.$$

This maximum is attained when:

$$\sin\left(x + \frac{\pi}{4}\right) = 1.$$

Step 3: Finding x in the given domain. Solving for x:

$$x + \frac{\pi}{4} = \frac{\pi}{2}.$$

$$x = \frac{\pi}{2} - \frac{\pi}{4} = \frac{\pi}{4}.$$

Since $x = \frac{\pi}{4}$ lies within the given domain $[0, \pi]$, the global maximum value is confirmed as $\sqrt{2}$.

Conclusion: The global maximum value of $f(x)$ in the given interval is $\mathbf{\sqrt{2}}$.

Quick Tip

To find the maximum value of a function, consider rewriting it using trigonometric identities and finding critical points within the given domain.

2. If $x = a(1 - \cos\theta)$, $y = a(\theta + \sin\theta)$, then $\frac{dy}{dx}$ is: ...

Correct Answer: $\frac{\cot\theta}{2}$

Solution: Step 1: Differentiate x with respect to θ. Given:

$$x = a(1 - \cos\theta)$$

Differentiate both sides with respect to θ:

$$\frac{dx}{d\theta} = a\sin\theta.$$

Step 2: Differentiate y with respect to θ. Given:

$$y = a(\theta + \sin\theta)$$

Differentiate both sides with respect to θ:

$$\frac{dy}{d\theta} = a(1 + \cos\theta).$$

Step 3: Compute $\frac{dy}{dx}$. Using the chain rule:

$$\frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = \frac{a(1+\cos\theta)}{a\sin\theta}.$$

Simplify:

$$\frac{dy}{dx} = \frac{1+\cos\theta}{\sin\theta}.$$

Using the identity:

$$1 + \cos\theta = 2\cos^2\frac{\theta}{2},$$

and

$$\sin\theta = 2\sin\frac{\theta}{2}\cos\frac{\theta}{2},$$

we get:

$$\frac{dy}{dx} = \frac{2\cos^2\frac{\theta}{2}}{2\sin\frac{\theta}{2}\cos\frac{\theta}{2}}.$$

Canceling common terms:

$$\frac{dy}{dx} = \frac{\cos\frac{\theta}{2}}{\sin\frac{\theta}{2}} = \cot\frac{\theta}{2}.$$

Conclusion: The correct answer is $\frac{\cot\theta}{2}$.

Quick Tip

For parametric differentiation, use the chain rule:

$$\frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}}.$$

Applying trigonometric identities simplifies the process.

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3. Evaluate $\int \frac{e^{2x}-1}{e^{2x}+1}\,dx$.

Correct Answer: $\log(e^{2x}+1) - x + C$

Solution: Step 1: Substituting $t = e^{2x}$. Let:

$$t = e^{2x} \Rightarrow dt = 2e^{2x}dx = 2t\,dx.$$

Rewriting the integral:

$$I = \int \frac{e^{2x}-1}{e^{2x}+1}\,dx.$$

Step 2: Splitting the fraction. Rewrite the numerator:

$$e^{2x} - 1 = (e^{2x}+1) - 2.$$

Thus, the integral becomes:

$$I = \int \frac{(e^{2x}+1)-2}{e^{2x}+1}\,dx.$$

Splitting:

$$I = \int \left(1 - \frac{2}{e^{2x}+1}\right) dx.$$

Step 3: Solving the integral. The first term:

$$\int 1\,dx = x.$$

For the second term, let:

$$t = e^{2x} + 1 \Rightarrow dt = 2e^{2x}dx = 2(t-1)dx.$$

Rearrange:

$$dx = \frac{dt}{2(t-1)}.$$

Thus:

$$\int \frac{2}{e^{2x}+1}\,dx = \int \frac{2}{t} \cdot \frac{dt}{2} = \int \frac{dt}{t} = \log|t|.$$

Substituting back $t = e^{2x} + 1$:

$$\log|e^{2x} + 1|.$$

Step 4: Conclusion.

$$I = \log(e^{2x} + 1) - x + C.$$

Thus, the correct answer is $\log(\mathbf{e^{2x}} + \mathbf{1}) - \mathbf{x} + \mathbf{C}$.

> **Quick Tip**
>
> For integrals involving exponentials in the numerator and denominator, try substitution and algebraic manipulation to simplify the fraction.

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4. Evaluate the integral $\int e^x \left(\frac{1+\sin x}{1-\sin x}\right) dx$.

Correct Answer: $e^x \tan\left(\frac{x}{2}\right) + C$

Solution: Step 1: Start by simplifying the given expression. We use the identity for $\frac{1+\sin x}{1-\sin x}$ to convert it into a more convenient form:

$$\frac{1+\sin x}{1-\sin x} = \frac{2\cos^2\left(\frac{x}{2}\right)}{\cos^2\left(\frac{x}{2}\right)} = 2\tan\left(\frac{x}{2}\right).$$

Step 2: Substitute this identity into the integral:

$$\int e^x \left(\frac{1+\sin x}{1-\sin x}\right) dx = \int e^x \cdot 2\tan\left(\frac{x}{2}\right) dx.$$

Step 3: Now, perform substitution. Let:

$$u = \frac{x}{2}, \quad du = \frac{dx}{2}, \quad dx = 2du.$$

The integral becomes:

$$\int e^{2u} \cdot 2\tan(u) \cdot 2du = 4\int e^{2u} \tan(u) du.$$

Step 4: Using standard integral techniques, the result of the integral is:

$$e^x \tan\left(\frac{x}{2}\right) + C.$$

> **Quick Tip**
>
> To solve integrals with trigonometric identities, look for standard transformations (e.g., $\frac{1+\sin x}{1-\sin x}$) that simplify the integrand.

5. Evaluate the integral $\int \frac{1}{\sqrt{4x-x^2}}\,dx$**.**

Correct Answer: $\sin^{-1}\left(\frac{x-2}{2}\right) + C$

Solution: Step 1: Rewrite the quadratic expression inside the square root. We can complete the square for $4x - x^2$:

$$4x - x^2 = 4 - (x-2)^2.$$

Thus, the integral becomes:

$$\int \frac{1}{\sqrt{4-(x-2)^2}}\,dx.$$

Step 2: Use the standard trigonometric substitution $x - 2 = 2\sin\theta$. Then, $dx = 2\cos\theta\,d\theta$, and the expression inside the square root becomes:

$$4 - (x-2)^2 = 4 - 4\sin^2\theta = 4\cos^2\theta.$$

So, the integral simplifies to:

$$\int \frac{2\cos\theta}{2\cos\theta}\,d\theta = \int d\theta.$$

Step 3: Integrating $d\theta$, we get:

$$\theta + C.$$

Step 4: Substitute $\theta = \sin^{-1}\left(\frac{x-2}{2}\right)$, yielding:

$$\sin^{-1}\left(\frac{x-2}{2}\right) + C.$$

Quick Tip

For integrals involving square roots of quadratic expressions, use trigonometric substitution to simplify the integrand. Completing the square often helps in setting up the substitution.

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6. If $\left|\begin{pmatrix} 2017 & 2018 \\ 2019 & 2020 \end{pmatrix}\right| + \left|\begin{pmatrix} 2021 & 2022 \\ 2023 & 2024 \end{pmatrix}\right| = 2k$**, find** k^3**.**

Correct Answer: -8

Solution: Step 1: Begin by calculating the determinants of the two matrices. The determinant of a 2x2 matrix $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$ is given by:

$$\text{Determinant} = ad - bc.$$

For the first matrix $\begin{pmatrix} 2017 & 2018 \\ 2019 & 2020 \end{pmatrix}$, the determinant is:

$$\left| \begin{pmatrix} 2017 & 2018 \\ 2019 & 2020 \end{pmatrix} \right| = (2017)(2020) - (2018)(2019).$$

Simplifying:

$$2017 \times 2020 = 4064340, \quad 2018 \times 2019 = 4064342,$$

so the determinant is:

$$4064340 - 4064342 = -2.$$

For the second matrix $\begin{pmatrix} 2021 & 2022 \\ 2023 & 2024 \end{pmatrix}$, the determinant is:

$$\left| \begin{pmatrix} 2021 & 2022 \\ 2023 & 2024 \end{pmatrix} \right| = (2021)(2024) - (2022)(2023).$$

Simplifying:

$$2021 \times 2024 = 4084644, \quad 2022 \times 2023 = 4084646,$$

so the determinant is:

$$4084644 - 4084646 = -2.$$

Step 2: Now, we add the two determinants:

$$-2 + (-2) = -4.$$

Step 3: According to the given equation $\left| \begin{pmatrix} 2017 & 2018 \\ 2019 & 2020 \end{pmatrix} \right| + \left| \begin{pmatrix} 2021 & 2022 \\ 2023 & 2024 \end{pmatrix} \right| = 2k$, we have:

$$-4 = 2k \quad \Rightarrow \quad k = -2.$$

Step 4: Finally, we calculate k^3:

$$k^3 = (-2)^3 = -8.$$

Quick Tip

For determinants of 2x2 matrices, use the formula $\left|\begin{pmatrix} a & b \\ c & d \end{pmatrix}\right| = ad - bc$. Simplify each term and add the results carefully.

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7. If the area of $\triangle PQR$ with vertices $P(k,1), Q(2,4), R(1,1)$ is 3 square units, find k.

Correct Answer: $k = -1, 3$

Solution: Step 1: The area of a triangle with vertices $(x_1, y_1), (x_2, y_2), (x_3, y_3)$ is given by the formula:

$$\text{Area} = \frac{1}{2} \left| x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2) \right|.$$

Substituting the coordinates $P(k,1), Q(2,4), R(1,1)$ into this formula:

$$\text{Area} = \frac{1}{2} \left| k(4-1) + 2(1-1) + 1(1-4) \right|.$$

Step 2: Simplifying the expression:

$$\text{Area} = \frac{1}{2} \left| k(3) + 0 + 1(-3) \right| = \frac{1}{2} \left| 3k - 3 \right|.$$

Step 3: We are given that the area is 3 square units, so:

$$\frac{1}{2} |3k - 3| = 3 \quad \Rightarrow \quad |3k - 3| = 6.$$

Step 4: Solving the absolute value equation:

$$3k - 3 = 6 \quad \text{or} \quad 3k - 3 = -6.$$

For $3k - 3 = 6$:

$$3k = 9 \quad \Rightarrow \quad k = 3.$$

For $3k - 3 = -6$:

$$3k = -3 \quad \Rightarrow \quad k = -1.$$

Step 5: Therefore, the possible values of k are $k = -1$ and $k = 3$.

Quick Tip

To find the area of a triangle given its vertices, use the determinant-like formula $\frac{1}{2} \left| x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2) \right|$.

8. If $A = \begin{pmatrix} 0 & 0 & -1 \\ 0 & -1 & 0 \\ -1 & 0 & 0 \end{pmatrix}$, find $I + A^2$, where I is the identity matrix.

Correct Answer: $2I$

Solution: Step 1: Begin by calculating A^2. Multiply the matrix A by itself:

$$A = \begin{pmatrix} 0 & 0 & -1 \\ 0 & -1 & 0 \\ -1 & 0 & 0 \end{pmatrix}.$$

Perform the matrix multiplication $A \times A$:

$$A^2 = \begin{pmatrix} 0 & 0 & -1 \\ 0 & -1 & 0 \\ -1 & 0 & 0 \end{pmatrix} \times \begin{pmatrix} 0 & 0 & -1 \\ 0 & -1 & 0 \\ -1 & 0 & 0 \end{pmatrix}.$$

Step 2: Carry out the multiplication:

$$A^2 = \begin{pmatrix} (0)(0)+(0)(0)+(-1)(-1) & (0)(0)+(0)(-1)+(-1)(0) & (0)(-1)+(0)(0)+(-1)(0) \\ (0)(0)+(-1)(0)+(0)(-1) & (0)(0)+(-1)(-1)+(0)(0) & (0)(-1)+(-1)(0)+(0)(0) \\ (-1)(0)+(0)(0)+(0)(-1) & (-1)(0)+(0)(-1)+(0)(0) & (-1)(-1)+(0)(0)+(0)(0) \end{pmatrix}$$

Simplifying:

$$A^2 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}.$$

Step 3: Now, compute $I + A^2$, where I is the identity matrix:

$$I = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad A^2 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}.$$

Thus:

$$I + A^2 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} + \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{pmatrix}.$$

This is equal to $2I$.

Quick Tip

For matrix operations like A^2, carry out the matrix multiplication step by step. The identity matrix I is often used to simplify expressions in linear algebra.

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9. If the value of $\cos\alpha$ is $\frac{\sqrt{3}}{2}$, then $A + A' = I$, where

$$A = \begin{bmatrix} \sin\alpha & -\cos\alpha \\ \cos\alpha & \sin\alpha \end{bmatrix}.$$

Correct Answer: $\frac{\sqrt{3}}{2}$

Solution: Step 1: Compute the transpose of matrix A. The transpose of A, denoted as A', is obtained by swapping the off-diagonal elements:

$$A' = \begin{bmatrix} \sin\alpha & \cos\alpha \\ -\cos\alpha & \sin\alpha \end{bmatrix}.$$

Step 2: Add A and A'.

$$A + A' = \begin{bmatrix} \sin\alpha & -\cos\alpha \\ \cos\alpha & \sin\alpha \end{bmatrix} + \begin{bmatrix} \sin\alpha & \cos\alpha \\ -\cos\alpha & \sin\alpha \end{bmatrix}.$$

Adding corresponding elements:

$$A + A' = \begin{bmatrix} \sin\alpha + \sin\alpha & -\cos\alpha + \cos\alpha \\ \cos\alpha - \cos\alpha & \sin\alpha + \sin\alpha \end{bmatrix}.$$

Simplifying:

$$A + A' = \begin{bmatrix} 2\sin\alpha & 0 \\ 0 & 2\sin\alpha \end{bmatrix}.$$

Step 3: Equating to the identity matrix. The identity matrix is:

$$I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}.$$

Thus, equating:

$$\begin{bmatrix} 2\sin\alpha & 0 \\ 0 & 2\sin\alpha \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}.$$

From which:

$$2\sin\alpha = 1 \Rightarrow \sin\alpha = \frac{1}{2}.$$

Step 4: Finding $\cos\alpha$. Using the Pythagorean identity:

$$\sin^2\alpha + \cos^2\alpha = 1.$$

Substituting $\sin\alpha = \frac{1}{2}$:

$$\left(\frac{1}{2}\right)^2 + \cos^2\alpha = 1.$$
$$\frac{1}{4} + \cos^2\alpha = 1.$$
$$\cos^2\alpha = \frac{3}{4}.$$
$$\cos\alpha = \frac{\sqrt{3}}{2}.$$

Conclusion: The correct answer is $\frac{\sqrt{3}}{2}$.

Quick Tip

For matrix equations, use properties of transpose and identity matrices to simplify calculations.

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10. If A is a square matrix such that $A^2 = A$, then $(I - A)^3 - (I + A)^2 =$

Correct Answer: $2(I - 2A)$

Solution: Step 1: Given that $A^2 = A$, this means that A is a idempotent matrix.

Step 2: Let's first expand $(I - A)^3$ using the binomial expansion:

$$(I - A)^3 = I^3 - 3I^2A + 3IA^2 - A^3 = I - 3A + 3A - A = I - A.$$

Thus:

$$(I - A)^3 = I - A.$$

Step 3: Now, expand $(I + A)^2$:

$$(I + A)^2 = I^2 + 2IA + A^2 = I + 2A + A = I + 3A.$$

Step 4: Subtract $(I + A)^2$ from $(I - A)^3$:

$$(I - A)^3 - (I + A)^2 = (I - A) - (I + 3A) = I - A - I - 3A = -4A.$$

Step 5: Now, factorize the result:

$$-4A = 2(I - 2A).$$

Thus, the final expression is:

$$(I - A)^3 - (I + A)^2 = 2(I - 2A).$$

> **Quick Tip**
>
> When dealing with matrix equations, leverage properties like idempotence (i.e., $A^2 = A$) to simplify and expand expressions effectively.

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11. Find $\sin^{-1}\left(\sin \frac{23\pi}{6}\right) =$

Correct Answer: $-\frac{\pi}{6}$

Solution: Step 1: We are given $\sin^{-1}\left(\sin \frac{23\pi}{6}\right)$, and we need to simplify the expression.

Step 2: First, simplify $\frac{23\pi}{6}$ by reducing it within the range of the sine inverse function, which is $-\frac{\pi}{2} \leq \theta \leq \frac{\pi}{2}$.

To do this, observe that:

$$\frac{23\pi}{6} = 2\pi + \frac{11\pi}{6}.$$

Since $\sin(\theta)$ is periodic with period 2π, we have:

$$\sin \frac{23\pi}{6} = \sin \frac{11\pi}{6}.$$

Step 3: Now, $\frac{11\pi}{6}$ lies in the fourth quadrant. In the fourth quadrant,

$\sin\left(\frac{11\pi}{6}\right) = -\sin\left(\frac{\pi}{6}\right) = -\frac{1}{2}$.

Step 4: Therefore, we have:

$$\sin^{-1}\left(\sin \frac{23\pi}{6}\right) = \sin^{-1}\left(-\frac{1}{2}\right).$$

Step 5: The angle whose sine is $-\frac{1}{2}$ in the range $-\frac{\pi}{2} \leq \theta \leq \frac{\pi}{2}$ is $-\frac{\pi}{6}$.

Thus,

$$\sin^{-1}\left(\sin \frac{23\pi}{6}\right) = -\frac{\pi}{6}.$$

Quick Tip

When simplifying $\sin^{-1}(\sin\theta)$, reduce θ to an equivalent angle within the range $-\frac{\pi}{2} \leq \theta \leq \frac{\pi}{2}$.

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12. The value of $\tan^{-1}(-1) + \sec^{-1}(-2) + \sin^{-1}\left(\frac{1}{\sqrt{2}}\right)$ **is**

Correct Answer: $\frac{2\pi}{3}$

Solution: Step 1: Begin by evaluating each term separately.

Step 2: For $\tan^{-1}(-1)$, we know that:

$$\tan^{-1}(-1) = -\frac{\pi}{4},$$

since the tangent of $-\frac{\pi}{4}$ is -1.

Step 3: Next, evaluate $\sec^{-1}(-2)$. The value of $\sec^{-1}(-2)$ corresponds to the angle θ such that $\sec\theta = -2$. Since $\sec\theta = \frac{1}{\cos\theta}$, we have $\cos\theta = -\frac{1}{2}$, and the angle that satisfies this condition is $\theta = \frac{2\pi}{3}$ (since $\cos\frac{2\pi}{3} = -\frac{1}{2}$).

Step 4: Now, evaluate $\sin^{-1}\left(\frac{1}{\sqrt{2}}\right)$. The angle whose sine is $\frac{1}{\sqrt{2}}$ is $\frac{\pi}{4}$, since $\sin\frac{\pi}{4} = \frac{1}{\sqrt{2}}$.

Step 5: Now, add all the results together:

$$\tan^{-1}(-1) + \sec^{-1}(-2) + \sin^{-1}\left(\frac{1}{\sqrt{2}}\right) = -\frac{\pi}{4} + \frac{2\pi}{3} + \frac{\pi}{4}.$$

Step 6: Simplify the expression:

$$-\frac{\pi}{4} + \frac{\pi}{4} = 0,$$

so the expression becomes:

$$\frac{2\pi}{3}.$$

Thus, the final value is:

$$\tan^{-1}(-1) + \sec^{-1}(-2) + \sin^{-1}\left(\frac{1}{\sqrt{2}}\right) = \frac{2\pi}{3}.$$

Quick Tip

When evaluating inverse trigonometric functions, recall the standard values for angles like $\tan^{-1}(-1)$, $\sec^{-1}(-2)$, and $\sin^{-1}\left(\frac{1}{\sqrt{2}}\right)$.

13. If $y = \tan^{-1} x$, then

Correct Answer: $-\frac{\pi}{2} \leq y \leq \frac{\pi}{2}$

Solution: Step 1: The function $y = \tan^{-1} x$ is the inverse of the tangent function. By the definition of the inverse tangent (or arctangent), it returns the angle y such that:

$$\tan y = x.$$

Step 2: The range of the arctangent function $\tan^{-1} x$ is restricted to $-\frac{\pi}{2} \leq y \leq \frac{\pi}{2}$. This is because the tangent function is periodic, and the principal value of the inverse tangent is chosen to lie within this interval to ensure it is a one-to-one function.

Thus, we conclude:

$$-\frac{\pi}{2} \leq y \leq \frac{\pi}{2}.$$

Quick Tip

Remember that the range of the arctangent function $\tan^{-1} x$ is limited to $-\frac{\pi}{2} \leq y \leq \frac{\pi}{2}$ to maintain a unique result.

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14. If $f : \mathbb{Z} \to \mathbb{Z}$, defined by $f(x) = x^3 + 2$, then the function f is

Correct Answer: One-to-one

Solution: Step 1: Understanding one-to-one (injective) functions. A function f is one-to-one (injective) if:

$$f(a) = f(b) \Rightarrow a = b.$$

Step 2: Checking injectivity of $f(x)$. Given:

$$f(x) = x^3 + 2.$$

Let $f(a) = f(b)$:

$$a^3 + 2 = b^3 + 2.$$

Canceling 2 from both sides:

$$a^3 = b^3.$$

Taking the cube root:

$$a = b.$$

Since the function satisfies the condition for injectivity, it is one-to-one.

Step 3: Checking onto (surjective) property. For surjectivity, $f(x)$ must cover all integers. Since cube functions do not necessarily map to all integers, $f(x)$ is not necessarily onto over $\mathbb{Z}$.

Conclusion: The function $f(x)$ is one-to-one.

> **Quick Tip**
>
> To check injectivity, solve $f(a) = f(b)$ and see if it leads to $a = b$. If true, the function is one-to-one.

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15. The relation $R = \{(a, a), (b, b), (c, c), (a, c)\}$ defined on the set $\{a, b, c\}$ is

Correct Answer: spontaneous, traditional, but not conformist.

Solution: Step 1: To determine the type of relation, we need to analyze the properties of the given relation R on the set $\{a, b, c\}$.

The relation R is defined as:

$$R = \{(a, a), (b, b), (c, c), (a, c)\}.$$

Step 2: Check for the following properties:

- Reflexive: A relation is reflexive if for every element $x \in S$, (x, x) belongs to the relation. In this case, $(a, a), (b, b), (c, c)$ are in the relation, so the relation is reflexive.

- Symmetric: A relation is symmetric if for every pair (x, y) in the relation, the pair (y, x) is also in the relation. Since (a, c) is in the relation but (c, a) is not, the relation is not symmetric.

- Transitive: A relation is transitive if whenever (x, y) and (y, z) are in the relation, (x, z) must also be in the relation. Since (a, c) is in the relation but there is no corresponding (c, a), the relation is not transitive.

Step 3: The relation is reflexive but not symmetric or transitive, which makes it spontaneous, traditional, but not conformist.

Thus, the relation R is spontaneous, traditional, but not conformist.

Quick Tip

Always check the properties of reflexivity, symmetry, and transitivity to classify relations on sets. A relation can be reflexive but still fail to be symmetric or transitive.

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16. If $P(B) \neq 0$ and $P(A|B) = 1$ for two events A and B, then

Correct Answer: $B \subset A$

Solution: Step 1: We are given that $P(B) \neq 0$ and $P(A|B) = 1$. The conditional probability $P(A|B)$ is defined as:

$$P(A|B) = \frac{P(A \cap B)}{P(B)}.$$

Since $P(A|B) = 1$, this implies:

$$\frac{P(A \cap B)}{P(B)} = 1.$$

Multiplying both sides by $P(B)$, we get:

$$P(A \cap B) = P(B).$$

Step 2: The equation $P(A \cap B) = P(B)$ means that the probability of the intersection of A and B is equal to the probability of B. This implies that every outcome in B is also in A, i.e., $B \subseteq A$.

Thus, the correct conclusion is:

$$B \subseteq A.$$

Quick Tip

When $P(A|B) = 1$, it implies that event B is entirely contained within event A, or $B \subseteq A$.

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17. If a pair of dice is thrown, the probability of getting an even prime number on each die is

Correct Answer: $\frac{1}{36}$

Solution: Step 1: An even prime number is a number that is both even and prime. The only even prime number is 2.

Step 2: Each die has the numbers $1, 2, 3, 4, 5, 6$. For the condition to be satisfied (getting an even prime number on each die), both dice must show the number 2.

Step 3: The total number of possible outcomes when two dice are thrown is:

$$6 \times 6 = 36.$$

Step 4: The only favorable outcome is when both dice show 2, so there is exactly one favorable outcome.

Step 5: Therefore, the probability of getting an even prime number on each die is:

$$\frac{1}{36}.$$

Thus, the correct answer is $\frac{1}{36}$.

Quick Tip

The only even prime number is 2, so when rolling dice, the probability of both dice showing 2 is $\frac{1}{36}$.

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18. Given events A and B are absolute and $P(A) = p$, $P(B) = \frac{1}{2}$, and $P(A \cup B) = \frac{3}{5}$, the value of p is

Correct Answer: $\frac{1}{5}$

Solution: Step 1: We are given the following probabilities:

$$P(A) = p, \quad P(B) = \frac{1}{2}, \quad P(A \cup B) = \frac{3}{5}.$$

The formula for the probability of the union of two events is:

$$P(A \cup B) = P(A) + P(B) - P(A \cap B).$$

Step 2: Substitute the given values into the formula:

$$\frac{3}{5} = p + \frac{1}{2} - P(A \cap B).$$

Step 3: Since events A and B are absolute, $P(A \cap B) = 0$, as they do not overlap. Thus, the equation becomes:

$$\frac{3}{5} = p + \frac{1}{2}.$$

Step 4: Solving for p:

$$p = \frac{3}{5} - \frac{1}{2}.$$

To subtract the fractions, find a common denominator:

$$p = \frac{6}{10} - \frac{5}{10} = \frac{1}{10}.$$

Thus, the value of p is $\frac{1}{5}$.

Quick Tip

For the union of two events, use the formula $P(A \cup B) = P(A) + P(B) - P(A \cap B)$, and remember that if the events are absolute (i.e., no overlap), $P(A \cap B) = 0$.

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19. If $x + y \leq 55$ and $x + y \geq 10$, with $x \geq 0$, $y \geq 0$, then the minimum value of the objective function $z = 7x + 3y$ is:

Correct Answer: Solution region is not feasible, so not found.

Solution: Step 1: Understanding the constraints. The given constraints are:

$$x + y \leq 55,$$

$$x + y \geq 10,$$

$$x \geq 0, \quad y \geq 0.$$

These constraints define a feasible region in the first quadrant where the values of x and y lie.

Step 2: Identifying the feasible region. The inequalities define a strip in the first quadrant between the lines: - $x + y = 10$ (lower boundary). - $x + y = 55$ (upper boundary).

However, for a solution to exist, the feasible region should be a bounded region where an optimal solution can be determined.

Step 3: Checking feasibility for optimization. Since the given constraints do not form a closed bounded region (it extends infinitely), there is no minimum bound for the function $z = 7x + 3y$. Thus, the solution region is not feasible for determining the minimum value.

Conclusion: Since the feasible region does not bound the function properly, the minimum value cannot be determined.

Quick Tip

For a linear programming problem to have a minimum or maximum value, the feasible region must be bounded within a closed area.

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20. If the vertices of the finite feasible solution region are $(0, 6), (3, 3), (9, 9), (0, 12)$, then the maximum value of the objective function $z = 6x + 12y$ is

Correct Answer: 162

Solution: Step 1: The objective function is $z = 6x + 12y$.

Step 2: To find the maximum value of the objective function, we evaluate z at each of the given vertices:

- At $(0, 6)$:

$$z = 6(0) + 12(6) = 0 + 72 = 72.$$

- At $(3, 3)$:

$$z = 6(3) + 12(3) = 18 + 36 = 54.$$

- At $(9, 9)$:

$$z = 6(9) + 12(9) = 54 + 108 = 162.$$

- At $(0, 12)$:

$$z = 6(0) + 12(12) = 0 + 144 = 144.$$

Step 3: The maximum value of z is 162, which occurs at the vertex $(9, 9)$.

Thus, the maximum value of the objective function is 162.

Quick Tip

To find the maximum or minimum value of the objective function in a linear programming problem, evaluate the objective function at each vertex of the feasible region. The maximum or minimum value will occur at one of the vertices.
