GATE 2025 CE 16/2/25 Shift-2 Question Paper with Solution

Time Allowed :3 Hours	**Maximum Marks :**100	**Total Questions :**65

General Instructions

Read the following instructions very carefully and strictly follow them:

1. The GATE Exam will be structured with a total of 100 marks.
2. The exam mode is Online CBT (Computer Based Test)
3. The total duration of Exam is 3 Hours.
4. It will include 65 questions , divided in 2 sections.
5. Section 1 : General Aptitude.
6. Section 2 : Technical.
7. The marking scheme is as such : 1 and 2 marks Questions. Each correct answer will carry marks as specified in the question paper. Incorrect answers may carry negative marks, as indicated in the question paper.
8. Question Types: The exam will include a mix of Multiple Choice Questions (MCQs), Multiple Select Questions (MSQs), and Numerical Answer Type (NAT). questions.

GENERAL APTITUDE

Question 1-5 carry one mark each

1. Even though I had planned to go skiing with my friends, I had to ..... at the last moment because of an injury.

(A) back up

(B) back of

(C) back on

(D) back out

Correct Answer: (D) back out

Solution:

The expression ”back out” means to withdraw from a commitment, plan, or agreement. In the sentence, the speaker had initially made plans to go skiing but could not follow through because of an injury. Therefore, ”back out” is the correct phrasal verb to describe this withdrawal.

Other options are incorrect:

- ”back up” means to support or reverse a vehicle.
- ”back of” is not a valid phrasal verb.
- ”back on” doesn’t fit the sentence grammatically or idiomatically.

Thus, the most appropriate choice is ”back out”.

Quick Tip

Phrasal verbs often change the meaning of the root verb entirely—always learn them in context.

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2. The President, along with the Council of Ministers, ... to visit India next week.

(A) wish

(B) wishes

(C) will wish

(D) is wishing

Correct Answer: (B) wishes

Solution:

The subject of the sentence is ”The President”, which is singular. The phrase ”along with the Council of Ministers” is a modifying phrase and does not affect the subject-verb agreement. Therefore, the verb should also be singular.

- ”wishes” is the singular form of the verb and agrees with the singular subject ”The President”.
- ”wish” is plural and would be incorrect here.
- ”will wish” changes the tense unnecessarily.
- ”is wishing” is awkward and not appropriate in this context.

Hence, the correct form is: ”The President, along with the Council of Ministers, wishes to visit India next week.”

Quick Tip

Ignore interrupting phrases like ”along with”, ”as well as”, etc., when determining subject-verb agreement.

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3. An electricity utility company charges 7 per kWh. If a 40-watt desk light is left on for 10 hours each night for 180 days, what would be the cost of energy consumption? If the desk light is on for 2 more hours each night for the 180 days, what would be the percentage-increase in the cost of energy consumption?

(A) 604.8; 10%

(B) 504; 20%

(C) 604.8; 12%

(D) 720; 15%

Correct Answer: (B) 504; 20%

Solution:

First, convert the power rating to kilowatts:

$$40 \text{ W} = \frac{40}{1000} = 0.04 \text{ kW}$$

Case 1: Desk light used for 10 hours per day

$$\text{Energy} = 0.04 \times 10 \times 180 = 72 \text{ kWh}$$

$$\text{Cost} = 72 \times 7 = 504$$

Case 2: Desk light used for 12 hours per day

$$\text{Energy} = 0.04 \times 12 \times 180 = 86.4 \text{ kWh}$$

$$\text{Cost} = 86.4 \times 7 = 604.8$$

Percentage Increase:

$$\frac{604.8 - 504}{504} \times 100 = \frac{100.8}{504} \times 100 \approx 20\%$$

Therefore, the percentage increase in cost is 20% and the original cost is 504.

Quick Tip

Always convert watts to kilowatts and use the formula: Energy = Power × Time × Days. Then multiply by rate to calculate cost.

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4. In the context of the given figure, which one of the following options correctly represents the entries in the blocks labelled (i), (ii), (iii), and (iv), respectively?

N	U	F	(i)
21	14	9	6
H	L	(ii)	O
12	(iv)	15	(iii)

(A) Q, M, 12 and 8

(B) K, L, 10 and 14

(C) I, J, 10 and 8

(D) L, K, 12 and 8

Correct Answer: (D) L, K, 12 and 8

Solution:

We analyze the logic based on letter-number patterns. Let's look at the structure row-wise:

Top Row: N (21), U (14), F (9), (i) = ?

Bottom Row: H (12), L (?), O (15), (?) = ?

Now map letters to positions in the alphabet:

$$\text{N} = 14, \quad \text{U} = 21, \quad \text{F} = 6, \quad \text{H} = 8, \quad \text{L} = 12, \quad \text{O} = 15$$

Compare these with the numbers in the second row:

$$21 \leftrightarrow N = 14 \Rightarrow 14 + 7 = 21$$

$14 \leftrightarrow U = 21 \Rightarrow 21 - 7 = 14$

$9 \leftrightarrow F = 6 \Rightarrow 6 + 3 = 9$

$6 \leftrightarrow (i) = ? \Rightarrow$ To balance pattern $\Rightarrow (i) = L(12) \Rightarrow 12 - 6 = 6$ ✓

Now the second row:

$$H = 8, \quad 12 \Rightarrow 8 + 4 = 12$$

(iv) = 8

L = 12, (ii) = ? $\Rightarrow 12 + 3 = 15(ii) = K(11)$

(iii) = 12 O = 15 15 - 3 = 12

So correct set: (i) = L, (ii) = K, (iii) = 12, (iv) = 8

Thus, the correct option is (D).

Quick Tip

When solving letter-number reasoning grids, convert letters to their alphabet positions (A=1 to Z=26), then analyze the mathematical pattern row-wise or column-wise.

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5. A bag contains Violet (V), Yellow (Y), Red (R), and Green (G) balls. On counting them, the following results are obtained:

(i) The sum of Yellow balls and twice the number of Violet balls is 50.

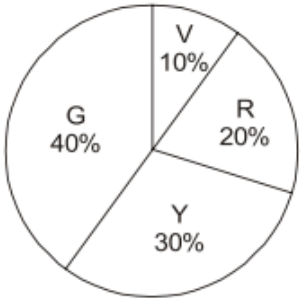
(ii) The sum of Violet and Green balls is 50.

(iii) The sum of Yellow and Red balls is 50.

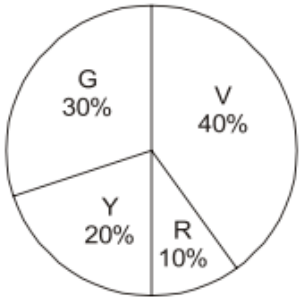
(iv) The sum of Violet and twice the number of Red balls is 50.

Which one of the following Pie charts correctly represents the balls in the bag?

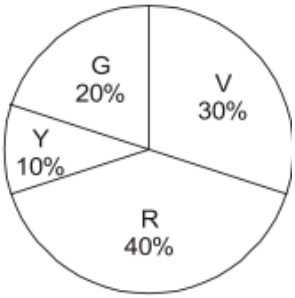
(A)



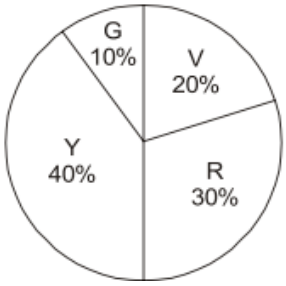
(B)



(C)



(D)



Correct Answer: (A) V: 10%, Y: 30%, R: 20%, G: 40%

Solution:

Let the total number of balls be 100 (since percentages are given). So, the actual number of each type of ball in option (A) is:

$$V = 10, \quad Y = 30, \quad R = 20, \quad G = 40$$

Now verify the conditions:

(i) $Y + 2V = 30 + 2 \times 10 = 30 + 20 = 50$

(ii) $V + G = 10 + 40 = 50$

(iii) $Y + R = 30 + 20 = 50$

(iv) $V + 2R = 10 + 2 \times 20 = 10 + 40 = 50$

All conditions are satisfied only in option (A). Other options do not verify all four conditions simultaneously.

Quick Tip

Assume the total is 100 when pie chart percentages are given. Translate each condition into equations and verify using actual values from the options.

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6. "His life was divided between the books, his friends, and long walks. A solitary man, he worked at all hours without much method, and probably courted his fatal illness in this way. To his own name there is not much to show; but such was his liberality that he was continually helping others, and fruits of his erudition are widely scattered, and have gone to increase many a comparative stranger's reputation."

(From E.V. Lucas's "A Funeral")

Based only on the information provided in the above passage, which one of the following statements is true?

(A) The solitary man described in the passage is dead.

(B) Strangers helped create a grand reputation for the solitary man described in the passage.

(C) The solitary man described in the passage found joy in scattering fruits.

(D) The solitary man worked in a court where he fell ill.

Correct Answer: (A) The solitary man described in the passage is dead.

Solution:

The title of the passage, "A Funeral," and the use of past tense verbs such as "was divided," "worked," and "courted" indicate that the person being discussed is no longer alive. The statement "he probably courted his fatal illness" also supports this inference, implying he ultimately succumbed to that illness. The passage is reflective and eulogistic in nature, pointing toward the man's death.

The other options include unsupported claims. For instance, there is no mention of the man working in a court or finding joy in scattering fruits. The "fruits of his erudition" refers

metaphorically to the impact of his knowledge, not literal joy or fruit scattering.

Quick Tip

Pay attention to past tense usage and the title or source of a passage—it often provides key contextual clues for inference-based questions.

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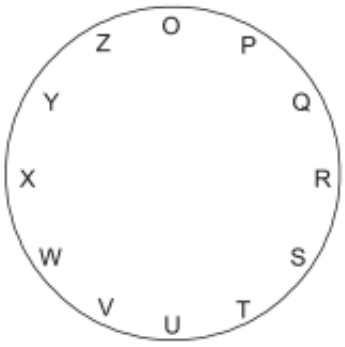
7. For the clock shown in the figure, if

O* = O Q S Z P R T, and

X* = X Z P W Y O Q,

then which one among the given options is most appropriate for P*?



(A) P U W R T V X

(B) P R T O Q S U

(C) P T V Q S U W

(D) P S U P R T V

Correct Answer: (B) P R T O Q S U

Solution:

We are given two sequences of letters representing paths around a circular clock-like figure. Each sequence starts from a reference letter and continues in a specific order:

- O* = O Q S Z P R T - X* = X Z P W Y O Q

These sequences follow a clockwise path on the circle. To find P*, we need to start from P and trace a similar clockwise pattern.

Looking at the clock diagram, starting from P and moving clockwise gives the sequence:

$$P \to R \to T \to O \to Q \to S \to U$$

This matches option (B).

To verify, count each step from P in the figure: P $\rightarrow$ R $\rightarrow$ T $\rightarrow$ O $\rightarrow$ Q $\rightarrow$ S $\rightarrow$ U – all in clockwise direction, and all letters are unique with no repetitions, matching the style of the given sequences.

Quick Tip

For circular reasoning questions, sketch or trace the path visually on the diagram and ensure you're moving in a consistent direction (clockwise or counter-clockwise).

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8. Consider a five-digit number PQRST that has distinct digits P, Q, R, S, and T, and satisfies the following conditions:

1. $P < Q$
2. $S > P > T$
3. $R < T$

If integers 1 through 5 are used to construct such a number, the value of P is:

(A) 1

(B) 2

(C) 3

(D) 4

Correct Answer: (C) 3

Solution:

We are given the constraints:

$$P < Q, \quad S > P > T, \quad R < T$$

We need to assign the digits 1 through 5 (each used only once) to P, Q, R, S, and T in a way that satisfies all the above conditions.

Let's try to assign values that satisfy these relations step-by-step:

From $S > P > T$, we can choose:

$$S = 5, \quad P = 3, \quad T = 2$$

This satisfies $S > P > T$. Now for $P < Q$, if $P = 3$, then Q must be greater than 3, so we can take:

$$Q = 4$$

That leaves only 1 unused, which can go to:

$$R = 1$$

Now check if all conditions are satisfied: - $P = 3 < Q = 4$ - $S = 5 > P = 3 > T = 2$ - $R = 1 < T = 2$

All conditions are satisfied.

Thus, the value of P is 3.

Quick Tip

When solving such logic puzzles with digit constraints, list available digits and test possible combinations systematically to satisfy all inequalities.

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9. A business person buys potatoes of two different varieties P and Q, mixes them in a certain ratio and sells them at 192 per kg.

The cost of the variety P is 800 for 5 kg.

The cost of the variety Q is 800 for 4 kg.

If the person gets 8% profit, what is the P : Q ratio (by weight)?

(A) 5 : 4

(B) 3 : 4

(C) 3 : 2

(D) 1 : 1

Correct Answer: (A) 5 : 4

Solution:

Given:

$$\text{Cost of 5 kg of variety P} = 800 \Rightarrow \text{Cost per kg} = 160$$

Cost of 4 kg of variety Q = 800 $\Rightarrow$ Cost per kg $= 200$

Let the seller mix 5 kg of P and 4 kg of Q (to match the quantity from the cost data).

$$\text{Total cost} = 800 + 800 = 1600$$

Total weight = 5 + 4 = 9 kg

Selling price per kg = 192 $\Rightarrow$ Total selling price $= 9 \times 192 = 1728$

$$\text{Profit} = 1728 - 1600 = 128$$

Profit % = $\frac{128}{1600} \times 100 = 8\%$

Thus, the assumed mixture gives exactly 8% profit, which matches the condition. Therefore, the weight ratio P : Q is:

$$5 : 4$$

Quick Tip

In mixture problems involving profit, use assumed weights based on cost data to match the required profit percentage. Compare cost price and selling price for total quantity.

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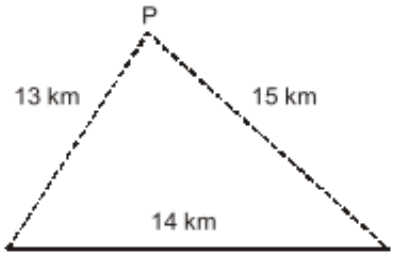
10. Three villages P, Q, and R are located in such a way that the distance PQ = 13 km, QR = 14 km, and RP = 15 km, as shown in the figure. A straight road joins Q and R. It is proposed to connect P to this road QR by constructing another road. What is the minimum possible length (in km) of this connecting road?

Note: The figure shown is representative.



(A) 10.5

(B) 11.0

(C) 12.0

(D) 12.5

Correct Answer: (C) 12.0

Solution:

Let the foot of the perpendicular from point P to line QR be at a distance x km from Q, and the perpendicular height be h. We can now apply the Pythagorean theorem to two right-angled triangles:

$$h^2 + x^2 = 13^2 = 169 \quad \text{(i)}$$

$$h^2 + (14 - x)^2 = 15^2 = 225 \quad \text{(ii)}$$

Now subtract equation (i) from (ii):

$$[h^2 + (14 - x)^2] - [h^2 + x^2] = 225 - 169$$

$$(14 - x)^2 - x^2 = 56$$

$$196 - 28x = 56 \Rightarrow 28x = 140 \Rightarrow x = 5$$

Substitute $x = 5$ in equation (i):

$$h^2 + 25 = 169 \Rightarrow h^2 = 144 \Rightarrow h = \sqrt{144} = 12$$

Therefore, the minimum possible length of the connecting road is $\boxed{12 \text{ km}}$.

Quick Tip

To find the shortest distance from a point to a line segment, drop a perpendicular and apply the Pythagorean theorem to form solvable right triangles.

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11. For the matrix [A] given below, the transpose is ____.

$$A = \begin{bmatrix} 2 & 3 & 4 \\ 1 & 4 & 5 \\ 4 & 3 & 2 \end{bmatrix}$$

(A) $\begin{bmatrix} 2 & 1 & 4 \\ 3 & 4 & 3 \\ 4 & 5 & 2 \end{bmatrix}$

(B) $\begin{bmatrix} 4 & 3 & 2 \\ 5 & 4 & 1 \\ 2 & 3 & 4 \end{bmatrix}$

(C) $\begin{bmatrix} 4 & 2 & 3 \\ 5 & 1 & 4 \\ 2 & 4 & 3 \end{bmatrix}$

(D) $\begin{bmatrix} 2 & 3 & 4 \\ 1 & 4 & 5 \\ 4 & 3 & 2 \end{bmatrix}$

Correct Answer: (A) $\begin{bmatrix} 2 & 1 & 4 \\ 3 & 4 & 3 \\ 4 & 5 & 2 \end{bmatrix}$

Solution:

The transpose of a matrix is obtained by interchanging its rows and columns. That is, the element at position (i, j) in the original matrix becomes the element at position (j, i) in the transposed matrix.

Given:

$$A = \begin{bmatrix} 2 & 3 & 4 \\ 1 & 4 & 5 \\ 4 & 3 & 2 \end{bmatrix}$$

Now taking the transpose:

- First row $(2\ 3\ 4)$ becomes first column - Second row $(1\ 4\ 5)$ becomes second column - Third row $(4\ 3\ 2)$ becomes third column

So,

$$A^T = \begin{bmatrix} 2 & 1 & 4 \\ 3 & 4 & 3 \\ 4 & 5 & 2 \end{bmatrix}$$

Hence, the correct option is (A).

Quick Tip

To transpose a matrix, flip it over its main diagonal. This is useful in many areas of linear algebra, especially in symmetric and orthogonal matrices.

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12. Integration of $\ln(x)$ with x, i.e. $\int \ln(x)dx =$ ____

(A) $x \cdot \ln(x) - x +$ Constant

(B) $x - \ln(x) +$ Constant

(C) $x \cdot \ln(x) + x +$ Constant

(D) $\ln(x) - x +$ Constant

Correct Answer: (A) $x \cdot \ln(x) - x +$ Constant

Solution:

To integrate $\int \ln(x)dx$, we use the method of integration by parts.

Recall the formula:

$$\int u\,dv = uv - \int v\,du$$

Choose:

$$u = \ln(x) \Rightarrow du = \frac{1}{x}dx, \quad dv = dx \Rightarrow v = x$$

Now apply the integration by parts formula:

$$\int \ln(x)\,dx = x \cdot \ln(x) - \int x \cdot \frac{1}{x}\,dx$$

$$= x \cdot \ln(x) - \int 1\,dx = x \cdot \ln(x) - x + C$$

Therefore,

$$\int \ln(x)\,dx = x \cdot \ln(x) - x + \text{Constant}$$

This matches option (A).

Quick Tip

When integrating logarithmic functions like $\ln(x)$, consider integration by parts with $u = \ln(x)$. This technique often simplifies otherwise difficult integrals.

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13. Consider the following statements (P) and (Q):

(P): Fly ash and ground granulated blast furnace slag can be used as mineral admixtures in concrete.

(Q): As per IS 456:2000, the minimum moist curing period becomes higher when a mineral admixture is added to concrete.

Identify the CORRECT option from choices given below.

(A) Both (P) and (Q) are TRUE.

(B) (P) is TRUE and (Q) is FALSE.

(C) (P) is FALSE and (Q) is TRUE.

(D) Both (P) and (Q) are FALSE.

Correct Answer: (A) Both (P) and (Q) are TRUE.

Solution:

Statement (P): Fly ash and ground granulated blast furnace slag are pozzolanic materials and are classified as mineral admixtures. According to IS standards, they are commonly used to enhance concrete properties and are considered artificial mineral admixtures. Hence, statement P is correct.

Statement (Q): IS 456:2000 specifies that when mineral admixtures are used in concrete, the rate of hydration tends to slow down. This requires a longer moist curing period to ensure sufficient strength development.

- Ordinary Portland Cement (OPC) usually requires 7 to 10 days of curing.
- With mineral admixtures, a minimum of 10 days of moist curing is recommended.

Thus, both statements (P) and (Q) are correct, making option (A) the right choice.

Quick Tip

Pozzolanic materials like fly ash and slag reduce the rate of hydration, thus increasing the required curing time to achieve optimal strength in concrete.

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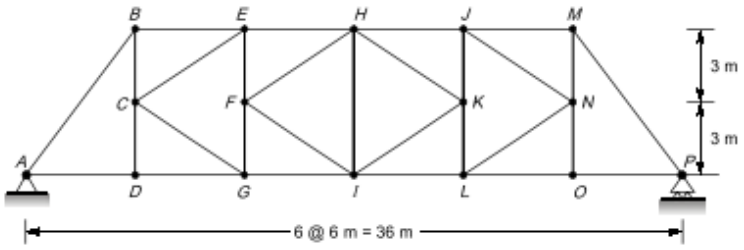
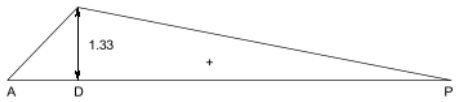
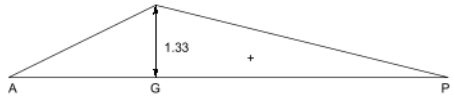
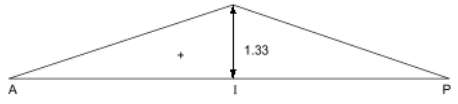
14. Consider the pin-jointed truss shown in the figure. Influence line is drawn for the axial force in the member G-I, when a unit load travels on the bottom chord of the truss. Identify the CORRECT influence line from the following options:

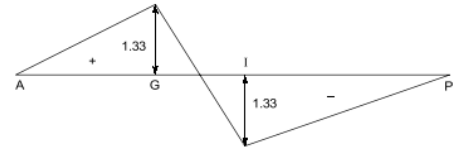
Note: Positive value corresponds to tension and negative value corresponds to compression in the member.

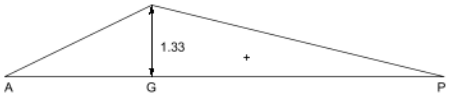


(A) 

(B) 

(C) 

(D) 

Correct Answer: (B) 

Solution:

We are to find the influence line for the axial force in member G–I when a unit load travels along the bottom chord of the truss.

From the geometry of the truss: - Total span = $6 \times 6 = 36$ m - Height of truss $h = 6$ m - Panel point spacing = 6 m - Distance from A to G = 2 panels = 12 m - Distance from G to right support = 4 panels = 24 m

Now, to find the ordinate of the influence line for member G-I under G, we use the standard formula for diagonal or bottom chord members in simple trusses:

$$\text{Ordinate} = \frac{ab}{lh}$$

Where: - $a = 12$ m, $b = 24$ m, $l = 36$ m, $h = 6$ m

Substitute into the formula:

$$\frac{12 \times 24}{36 \times 6} = \frac{288}{216} = 1.33$$

Hence, the influence line has an ordinate of $+1.33$ under joint G, which corresponds to tension in member G-I. Therefore, the correct diagram is option (B).

Quick Tip

To find influence lines for truss members, use geometry-based influence line theory and remember that bottom chord members generally carry tension under downward unit loads.

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15. The most suitable test for measuring the permeability of clayey soils in the laboratory is ____.

(A) Constant head test

(B) Pumping out test

(C) Hydrometer test

(D) Falling head test

Correct Answer: (D) Falling head test

Solution:

Clayey soils have very low permeability due to their fine-grained structure. The constant head test is generally suitable for coarse-grained soils like sand and gravel, which allow faster water flow. On the other hand, the falling head test is better suited for fine-grained soils like clay because it can more accurately measure the slow rate of flow through such soils.

In the falling head test, the rate at which the water level falls in a standpipe is monitored, allowing precise calculation of permeability even in low-permeability soils.

Hence, the falling head test is most appropriate for laboratory measurement of permeability in clayey soils.

Quick Tip

Use the falling head test for fine-grained soils (clays and silts), and the constant head test for coarse-grained soils (sands and gravels).

16. A hydraulic jump occurs in an open channel when the slope of the channel changes from ____.

(A) MILD slope to STEEP slope

(B) STEEP slope to MILD slope

(C) MILD slope to ZERO slope

(D) STEEP slope to a STEEPER slope

Correct Answer: (B) STEEP slope to MILD slope

Solution:

A hydraulic jump is formed in an open channel when the flow transitions from supercritical flow $(F_r > 1)$ to subcritical flow $(F_r < 1)$. This typically occurs when a steep slope (which supports supercritical flow) changes to a mild slope, causing the flow to decelerate and a jump to form.

This change in slope causes the kinetic energy of the flow to be suddenly dissipated, forming a jump which is visible as a sudden rise in the water surface.

Hence, the correct answer is (B).

Quick Tip

Hydraulic jumps are energy-dissipating phenomena in channels, usually formed when supercritical flow transitions to subcritical flow.

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17. The bacteria mainly responsible for crown corrosion in a sewer is ____.

(A) Methanogenic bacteria

(B) Denitrifying bacteria

(C) Sulphur reducing bacteria

(D) Pseudomonas bacteria

Correct Answer: (C) Sulphur reducing bacteria

Solution:

Sulphur reducing bacteria (SRB) are anaerobic bacteria that reduce sulphates to hydrogen sulphide gas (H_2S) in sewage systems. This hydrogen sulphide, upon reaching the crown of the sewer where oxygen is present, gets oxidized to sulphuric acid.

The sulphuric acid reacts with the concrete and leads to deterioration of the sewer crown, a process known as crown corrosion.

Hence, the correct answer is (C).

Quick Tip

Crown corrosion in sewers is due to biological activity of sulphur reducing bacteria which produce hydrogen sulphide, leading to sulphuric acid formation.

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18. The recommended minimum traffic growth rate and design period considered for structural design of flexible pavements for national highways in India as per IRC 37:2018 is ____ percentage and ____ years, respectively.

(A) 5, 20

(B) 5, 30

(C) 7, 20

(D) 7, 30

Correct Answer: (A) 5, 20

Solution:

As per IRC 37:2018 (Guidelines for the Design of Flexible Pavements), the following values are recommended for the structural design of flexible pavements for national highways:

- **Minimum traffic growth rate:** 5% - **Design period:** 20 years

These values are taken as the default unless actual traffic studies suggest otherwise.

Hence, the correct option is (A).

Quick Tip

Refer to IRC 37:2018 for standards related to pavement design parameters like design life and traffic growth.

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19. After applying the correction for elevation and temperature, the runway length is 700 m.

The corrected runway length (in m) for an effective gradient of 1% is ____ (round off to the nearest integer).

(A) 840

(B) 700

(C) 720

(D) 740

Correct Answer: (A) 840

Solution:

Given: Corrected runway length after applying elevation and temperature corrections:

$$l = 700 \text{ m}, \quad \text{Gradient} = 1\%$$

According to standards, for every 1% gradient, the runway length increases by 20%.

$$\text{Correction} = 700 \times \frac{20}{100} = 140$$

$$\text{Corrected length} = 700 + 140 = 840 \text{ m}$$

$$\text{Or simply: } 700 \times 1.2 = 840 \text{ m}$$

Hence, the final corrected length is $\boxed{840 \text{ m}}$.

Quick Tip

Remember: Runway length is increased by 20% for every 1% gradient as per ICAO or aviation design guidelines.

---

20. The point where the road alignment changes from a tangent to a curve is known as ____.

(A) Point of deflection

(B) Point of intersection

(C) Point of curve

(D) Point of tangency

Correct Answer: (C) Point of curve

Solution:

In highway geometry, the transition from a straight path (tangent) to a circular curve is marked by specific points:

- **Point of Intersection (PI):** Where two tangents meet. - **Point of Curve (PC):** Where the curve begins (tangent ends and curve starts). - **Point of Tangency (PT):** Where the curve ends and tangent resumes.

Therefore, the point where the road alignment changes from a tangent to a curve is called the **Point of Curve (PC)**.

Hence, the correct answer is (C).

Quick Tip

Remember the road alignment transition points: PI (intersection), PC (start of curve), PT (end of curve).

---

21. Consider a velocity vector, $\vec{V}$ in (x, y, z) coordinates given below. Pick one or more CORRECT statement(s) from the choices given below:

$$\vec{V} = u\hat{i} + v\hat{j}$$

(A) z-component of Curl of velocity; $\nabla \times \vec{V} = \left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right)\hat{z}$

(B) z-component of Curl of velocity; $\nabla \times \vec{V} = \left(\frac{\partial u}{\partial x} - \frac{\partial v}{\partial y}\right)\hat{z}$

(C) Divergence of velocity; $\nabla \cdot \vec{V} = \left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right)$

(D) Divergence of velocity; $\nabla \cdot \vec{V} = \left(\frac{\partial u}{\partial x} - \frac{\partial v}{\partial y}\right)$

Correct Answer: (A), (C)

Solution:

Given:

$$\vec{V} = u(x, y)\,\hat{i} + v(x, y)\,\hat{j}$$

There is no component in the z-direction, so the flow is 2D.

Divergence of velocity:

$$\nabla \cdot \vec{V} = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}$$

Hence, option (C) is correct.

Curl of velocity (vector form):

$$\nabla \times \vec{V} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ u & v & 0 \end{vmatrix}$$

Evaluating the determinant:

$$\nabla \times \vec{V} = \left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} \right) \hat{k}$$

Thus, the z-component of curl is:

$$\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} \right) \hat{z}$$

So, option (A) is also correct. Options (B) and (D) are incorrect as they show incorrect expressions for curl and divergence respectively.

Quick Tip

For 2D velocity fields, divergence is $\nabla \cdot \vec{V} = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}$, and curl's z-component is $\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}$.

---

22. Given that A and B are not null sets, which of the following statements regarding probability is/are CORRECT?

(A) $P(A \cap B) = P(A) \cdot P(B)$, if A and B are mutually exclusive.

(B) Conditional probability, $P(A \mid B) = 1$ if $B \subseteq A$.

(C) $P(A \cup B) = P(A) + P(B)$, if A and B are mutually exclusive.

(D) $P(A \cap B) = 0$, if A and B are independent.

Correct Answer: (B), (C)

Solution:

Option (A): If A and B are mutually exclusive, then:

$$P(A \cap B) = 0$$

So the expression $P(A \cap B) = P(A) \cdot P(B)$ is incorrect unless A and B are independent, not mutually exclusive. Hence, option (A) is incorrect.

Option (B): Conditional probability is defined as:

$$P(A \mid B) = \frac{P(A \cap B)}{P(B)}$$

If $B \subseteq A$, then $A \cap B = B$, so:

$$P(A \mid B) = \frac{P(B)}{P(B)} = 1$$

Thus, option (B) is correct.

Option (C): If A and B are mutually exclusive, then:

$$P(A \cap B) = 0 \Rightarrow P(A \cup B) = P(A) + P(B)$$

So, option (C) is correct.

Option (D): If A and B are independent, then:

$$P(A \cap B) = P(A) \cdot P(B)$$

This value is generally non-zero unless either P(A) or P(B) is zero. Hence, option (D) is incorrect.

Quick Tip

Mutual exclusivity implies $P(A \cap B) = 0$, while independence implies $P(A \cap B) = P(A) \cdot P(B)$. These two are not the same and should not be confused.

---

23. In the context of construction materials, which of the following statements is/are CORRECT?

(A) If the characteristic strength is defined as that value below which not more than 50% results are expected to fall, the target mean strength in mix design will be the same as the characteristic strength irrespective of the degree of quality control expected at the site.

(B) Ten percent fines value is a non-dimensional quantity.

(C) The stress-strain curve of concrete for 1-day duration of loading is associated with a smaller secant modulus of elasticity compared to the stress-strain curve of the same concrete for 10-minutes duration of loading.

(D) The increase of carbon in steel usually leads to an increase in its 0.2% proof stress.

Correct Answer: (A), (C), (D)

Solution:

Option (A): This statement is correct. Characteristic strength is defined as the value below which not more than 5% of test results are expected to fall. The target mean strength is

calculated based on the characteristic strength and the standard deviation, which depends on the quality control. The given statement is therefore a contradiction and hence CORRECT in pointing out the flaw.

Option (B): Incorrect. Ten percent fines value has units (typically in terms of load), and hence it is not a non-dimensional quantity.

Option (C): Correct. With shorter loading durations, concrete exhibits higher stiffness. A longer duration like 1-day results in a smaller secant modulus than that for a quick loading of 10 minutes.

Option (D): Correct. Increasing the carbon content in steel generally increases its strength characteristics including the 0.2% proof stress (yield strength).

Quick Tip

Remember: Ten percent fines value has units (it's not dimensionless), and characteristic strength considers 5% chance of failure, not 50%.

---

24. Which of the following statements is/are INCORRECT?

(A) As the depth of the ground water table from the ground surface increases, the effective stress in the soil decreases.

(B) Bulking of the moist sand is due to the capillary action in the sand.

(C) The effective stress in a liquefied soil is almost zero.

(D) The earth pressure at any point in the soil, under all conditions, is always smaller than the vertical effective stress at that point.

Correct Answer: (A), (D)

Solution:

Option (A): Incorrect. As the water table goes deeper (i.e., increases in depth from surface), the pore water pressure acting at a point in the soil decreases. Hence, the effective stress $(\sigma' = \sigma - u)$ increases, not decreases.

Option (B): Correct. Bulking of sand occurs due to moisture forming meniscus at particle contacts, leading to capillary tension and apparent volume increase.

Option (C): Correct. In liquefied soils, the pore water pressure equals the total stress,

leading to near-zero effective stress.

Option (D): Incorrect. Earth pressure can be greater, equal, or less than vertical effective stress depending on the stress state (active, passive, or at-rest). Therefore, the statement is not always true.

Quick Tip

Effective stress increases as the water table depth increases. Earth pressure can vary with stress condition; don't assume it's always less than vertical stress.

---

25. Pick one or more CORRECT statement(s) from the choices given below, in the context of upstream and downstream cut-offs provided below the concrete apron of weirs/barrages constructed across alluvial rivers.

(A) Cut-offs are provided to increase the rate of flow over the weir / barrage.

(B) Cut-offs are provided to increase the seepage length and prevent failure due to piping.

(C) The bottom level of cut-offs mainly depends on the scour depth.

(D) Cut-offs are provided to ensure occurrence of hydraulic jump within the stilling basin.

Correct Answer: (B), (C)

Solution:

Cut-offs are vertical or inclined structural members provided below the floor of weirs or barrages to prevent sub-surface flow that may lead to piping.

- **Option (B):** Correct. Cut-offs increase the seepage path length beneath the floor, thereby reducing the exit gradient and minimizing the risk of piping, which is a major cause of failure in hydraulic structures.
- **Option (C):** Correct. The depth of cut-offs is designed considering the maximum anticipated scour depth, to ensure that the foundation remains safe even if the bed material is eroded during floods.
- **Option (A):** Incorrect. Cut-offs do not influence surface flow rates; they function below the floor and impact sub-surface flow patterns.
- **Option (D):** Incorrect. Hydraulic jumps are managed using stilling basins, baffle blocks, and aprons, not cut-offs.

Quick Tip

Cut-offs protect structures against piping and undermining. Their depth is governed by scour analysis, not flow behavior above the structure.

---

26. In the context of the effect of drainage density on the run-off generation and the hydrograph at the catchment outlet, all other factors remaining the same, pick one or more CORRECT statement(s):

(A) Lower drainage density results in higher peak in flood hydrograph compared to that when the drainage density is higher.

(B) Lower drainage density results in lower peak in flood hydrograph compared to that when the drainage density is higher.

(C) Lower drainage density results in a flood hydrograph with a longer time base compared to that when the drainage density is higher.

(D) Lower drainage density results in a flood hydrograph with a shorter time base compared to that when the drainage density is higher.

Correct Answer: (B), (C)

Solution:

Drainage density is defined as the total length of streams per unit area of the watershed. It significantly influences the shape of the flood hydrograph:

- **Lower drainage density** indicates fewer channels and longer flow paths. As a result: - Runoff takes longer to reach the outlet, - Peak discharge is lower, - Time base of the hydrograph is longer.

Hence: - **Option (B)** is correct — lower drainage density leads to lower peak discharge. - **Option (C)** is correct — lower drainage density leads to longer duration of flow.

- **Option (A)** is incorrect — lower drainage density causes a lower, not higher, peak. - **Option (D)** is incorrect — it reverses the effect on time base.

Quick Tip

High drainage density results in faster, sharper hydrographs; low drainage density causes slower, flatter hydrographs with longer duration.

27. Identify the treatment technology/technologies NOT recommended for highly biodegradable organic solid wastes.

(A) Biohydrogenation

(B) Anaerobic digestion

(C) Composting

(D) Open dumping

Correct Answer: (D) Open dumping

Solution:

Biodegradable organic waste should be managed using sustainable and environment-friendly technologies:

- **Biohydrogenation (A):** Suitable for converting organic material into bio-hydrogen, an emerging green fuel.
- **Anaerobic digestion (B):** Widely used method for treating organic waste in the absence of oxygen, producing biogas as a useful by-product.
- **Composting (C):** An aerobic process of biodegradation that converts organic matter into useful compost.
- **Open dumping (D): Incorrect practice**. It leads to uncontrolled decomposition, release of methane, foul smell, contamination of soil and groundwater, and spread of disease vectors. Therefore, open dumping is not recommended for any kind of waste, especially not for highly biodegradable organic matter.

Quick Tip

Avoid open dumping — it causes pollution and health hazards. Use biological treatments like composting or digestion for organic waste.

---

28. Which of the following statements is/are INCORRECT?

(A) Bitumen having lower softening point is preferred in warm climate regions.

(B) The viscosity of bitumen influences the mixing and compaction of bituminous mix.

(C) The air voids in the range of 3% to 5% are required to arrive at the optimum bitumen

content.

(D) The purity of bitumen can be determined using Solubility Test.

Correct Answer: (A)

Solution:

Option (A): Incorrect. In warm climate regions, bitumen with a higher softening point is desirable to prevent deformation and bleeding under high temperatures. A lower softening point would cause the pavement to fail prematurely.

Option (B): Correct. The viscosity of bitumen affects both mixing and compaction temperatures. A proper viscosity ensures good coating of aggregates and workability during paving.

Option (C): Correct. Air voids in the range of 3–5% ensure that the pavement has sufficient durability and strength while maintaining resistance to deformation.

Option (D): Correct. The solubility test determines the proportion of pure bitumen in a sample and is used to assess the quality and presence of impurities.

Quick Tip

In hot climates, always use bitumen with a high softening point to prevent surface distress.

---

29. The "order" of the following ordinary differential equation is ____.

$$\frac{d^3y}{dx^3} + \left(\frac{d^2y}{dx^2}\right)^6 + \left(\frac{dy}{dx}\right)^4 + y = 0$$

(A) (1)(6 to 6)

(B) (2)(2 to 2)

(C) (3)(3 to 3)

(D) (4)(4 to 4)

Correct Answer: (C) (3)(3 to 3)

Solution:

The order of a differential equation is determined by the highest derivative present. Here, the

highest order derivative is:

$$\frac{d^3y}{dx^3} \Rightarrow \text{Order} = 3$$

The degree is the highest power to which the highest order derivative is raised, provided the equation is polynomial in derivatives. In this case:

$$\left(\frac{d^3y}{dx^3}\right)^1 \Rightarrow \text{Degree} = 1$$

So, order = 3 and degree = 1.

Quick Tip

The order is the highest derivative present, and degree is the power of that derivative (in polynomial form).

---

30. The design shear strength of a reinforced concrete rectangular beam with a width of 250 mm and an effective depth of 500 mm, is 0.3 MPa. The torsional moment capacity of the section (in kN.m) under pure torsion, as per IS 456:2000, is ____ (round off to one decimal place).

(A) 5.86

(B) 5.70

(C) 5.60

(D) 6.00

Correct Answer: (A) 5.86

Solution:

Given: Width, $b = 250$ mm

Effective depth, $d = 500$ mm

Design shear strength, $\tau_c = 0.3$ N/mm^2

Step 1: Calculate shear force,

$$V_e = \tau_c \cdot b \cdot d = 0.3 \times 250 \times \frac{500}{1000} = 37.5 \text{ kN}$$

Step 2: Use IS 456:2000 torsion formula,

$$T_u = \frac{V_e \cdot b}{1.6 + \frac{1.67b}{d}} = \frac{37.5 \times 0.25}{1.6} = 5.86 \text{ kN.m}$$

Hence, the torsional moment capacity is $\boxed{5.86 \text{ kN.m}}$.

Quick Tip

Ensure all values are in consistent units before applying IS 456:2000 design formulas for torsion.

---

31. From a flow-net diagram drawn under a concrete dam, the following information is obtained:

(i) The head difference between upstream and downstream side of the dam is 9 m.

(ii) The total number of equipotential drops between upstream and downstream side of the dam is 10.

(iii) The length of the field nearest to the toe of the dam on the downstream side is 1 m.

If the soil below the dam is having a saturated unit weight of 21 kN/m^3 and the unit weight of water is 9.81 kN/m^3, then the factor of safety against the quick condition will be ____ (round off to two decimal places).

Correct Answer: (B) 1.27

Solution:

The factor of safety (FOS) against quick condition is given by:

$$\text{FOS} = \frac{i_c}{i_{\text{exit}}}$$

Where,

$$i_{\text{exit}} = \frac{\Delta h}{N_d \cdot L} = \frac{9}{10 \cdot 1} = 0.9$$

$$i_c = \frac{\gamma'}{\gamma_w} = \frac{21 - 9.81}{9.81} = \frac{11.19}{9.81} \approx 1.14$$

$$\text{FOS} = \frac{1.14}{0.9} \approx 1.27$$

Thus, the factor of safety against the quick condition is $\boxed{1.27}$.

Quick Tip

To assess piping safety in soil under hydraulic structures, use FOS $= i_c/i_{\text{exit}}$ with exit gradient from flow net geometry.

---

32. A 6 m thick clay stratum has drainage at both its top and bottom surface due to the presence of sand strata. The time to complete 50% consolidation is 2 years.
The coefficient of volume change (m_v) is 1.51×10^{-3} m^2/kN and the unit weight of water is 9.81 kN/m^3.
The coefficient of permeability (in m/year) is ____ (round off to three decimal places).

Correct Answer: (B) 0.013

Solution:

For double drainage (top and bottom), the drainage path length:

$$d = \frac{H}{2} = \frac{6}{2} = 3 \text{ m}$$

Time factor for 50% consolidation:

$$T_v = \frac{\pi}{4} \times (0.5)^2 = 0.196$$

The time factor is also defined as:

$$T_v = \frac{C_v \cdot t}{d^2} = \frac{k \cdot t}{m_v \cdot \gamma_w \cdot d^2}$$

Rearranging for permeability k:

$$k = \frac{T_v \cdot m_v \cdot \gamma_w \cdot d^2}{t}$$

Substitute the values:

$$k = \frac{0.196 \times 1.51 \times 10^{-3} \times 9.81 \times 3^2}{2} \approx 0.013 \text{ m/year}$$

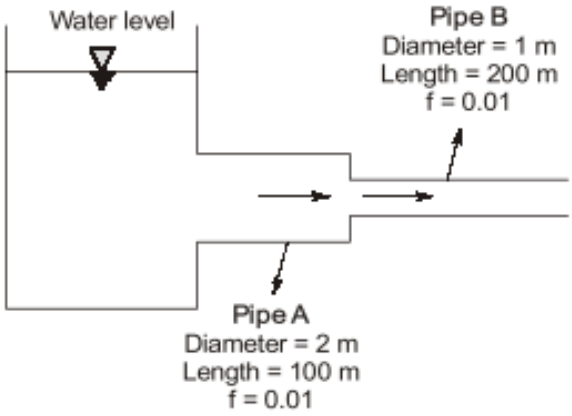
Hence, the coefficient of permeability is $\boxed{0.013 \text{ m/year}}$.

Quick Tip

For consolidation with double drainage, use half the thickness of the layer as the drainage path and apply time factor equations accurately.

33. Consider steady flow of water in the series pipe system shown below, with specified discharge. The diameters of Pipes A and B are 2 m and 1 m, respectively. The lengths of pipes A and B are 100 m and 200 m, respectively. Assume the Darcy-Weisbach friction coefficient, f, as 0.01 for both the pipes.

The ratio of head loss in Pipe-B to the head loss in Pipe-A is ____ (round off to the nearest integer).



Correct Answer: (A) 64

Solution:

The head loss in a pipe due to friction is given by the Darcy-Weisbach equation:

$$h_f = \frac{8Q^2Lf}{\pi^2 g D^5}$$

Where, - Q is the discharge,

- L is the length of the pipe,

- D is the diameter of the pipe,

- f is the Darcy-Weisbach friction factor,

- g is the acceleration due to gravity.

The ratio of head loss in Pipe-B to Pipe-A is:

$$\frac{h_B}{h_A} = \frac{\frac{8Q^2L_Bf}{\pi^2 g D_B^5}}{\frac{8Q^2L_Af}{\pi^2 g D_A^5}} = \left(\frac{D_A}{D_B}\right)^5 \times \frac{L_B}{L_A}$$

Substitute the given values:

$$\frac{h_B}{h_A} = \left(\frac{2}{1}\right)^5 \times \frac{200}{100} = 32 \times 2 = 64$$

Hence, the ratio of head loss in Pipe-B to the head loss in Pipe-A is $\boxed{64}$.

Quick Tip

The ratio of head losses in two pipes is proportional to the 5th power of the ratio of their diameters and the ratio of their lengths.

---

34. Free residual chlorine concentration in water was measured to be 2 mg/l (as Cl_2). The pH of water is 8.5. By using the chemical equation given below, the HOCl concentration (in μmoles/l) in water is ____ (round off to one decimal place).

$$HOCl \rightleftharpoons H^+ + OCl^-, \quad pK = 7.50$$

Atomic weight: Cl = 35.5

Solution:

The equilibrium constant k for the dissociation of HOCl is given by:

$$k = \frac{[HOCl]}{[OCl^-][H^+]}$$

Substituting the values:

$$10^{7.5} = \frac{[HOCl]}{[OCl^-] \times 10^{-8.5}}$$

Simplifying further:

$$10^{-1} = \frac{[HOCl]}{[OCl^-]}$$

This gives the relationship:

$$[HOCl] + [OCl^-] = \frac{2 \text{ mg} \times 10^{-3}}{71}$$

Converting to moles per liter:

$$[HOCl] + [OCl^-] = 2 \times 10^{-3} \text{ moles/l} \times 10^6 = 2.56 \, \mu\text{moles/l}$$

Thus, the concentration of HOCl in water is approximately $\boxed{2.6}$ μmoles/l.

Quick Tip

Remember to convert units properly when dealing with mg/l and μmoles/l. Also, use equilibrium constants and pH values to solve dissociation problems.

---

35. A surveyor measured the distance between two points on the plan drawn to a scale of 1 cm = 40 m and the result was 468 m. Later, it was discovered that the scale used was 1 cm = 20 m.

The true distance between the points (in m) is ____ (round off to the nearest integer).

Solution:

In this problem, we are asked to find the true distance between the points after a scaling error was detected. The surveyor initially used a scale where 1 cm on the plan represented 40 m in reality. However, the correct scale should have been 1 cm = 20 m.

1. Step 1: Calculate the scale ratio (RF) for the wrong and correct scales:

The wrong scale gives:

$$\text{RF of wrong scale} = \frac{1}{20}$$

The correct scale gives:

$$\text{RF of corrected scale} = \frac{1}{40}$$

2. Step 2: Use the formula for corrected length:

To find the corrected length, we use the ratio of the two scale factors:

$$\text{Corrected length} = \left(\frac{\text{RF of wrong scale}}{\text{RF of corrected scale}}\right) \times \text{Measured length}$$

Substituting the values:

$$\text{Corrected length} = \left(\frac{\frac{1}{20}}{\frac{1}{40}}\right) \times 468 = 2 \times 468 = 936 \text{ m}$$

Thus, the true distance between the points is $\boxed{936}$ m.

Explanation:

The correction is made by adjusting the scale factor from the incorrect value to the correct value. Since the correct scale represents half the length of the wrong scale, the actual distance is double the measured length. Hence, the final corrected value of 936 m is obtained by multiplying the measured value by 2.

Quick Tip

Always check the scale carefully. When scaling errors are discovered, use the ratio of the scale factors to adjust the measured values.

36. Pick the CORRECT solution for the following differential equation:

$$\frac{dy}{dx} = e^{x-y}$$

(A) $y = \ln(e^x + \text{Constant})$

(B) $\ln(y) = x + \text{Constant}$

(C) $\ln(y) = \ln(e^x) + \text{Constant}$

(D) $y = x + \text{Constant}$

Correct Answer: (A) $y = \ln(e^x + \text{Constant})$

Solution:

The given differential equation is:

$$\frac{dy}{dx} = e^{x-y}$$

Step 1: Rearranging the equation:

$$\frac{dy}{dx} = e^x \cdot e^{-y}$$

Step 2: Separating the variables:

$$e^y \, dy = e^x \, dx$$

Step 3: Integrating both sides:

$$\int e^y \, dy = \int e^x \, dx$$

Step 4: Performing the integrations:

$$e^y = e^x + C$$

Step 5: Taking the natural logarithm of both sides:

$$y = \ln(e^x + C)$$

Thus, the correct solution is $y = \ln(e^x + C)$, which corresponds to option (A).

Quick Tip

When solving differential equations, always separate variables first, integrate both sides, and apply the necessary logarithmic operations for solutions involving exponential terms.

37. A circular tube of thickness 10 mm and diameter 250 mm is welded to a flat plate using 5 mm fillet weld along the circumference. Assume Fe410 steel and shop welding.

As per IS 800:2007, the torque that can be resisted by the weld (in kN.m) is ____ (round off to one decimal place).

(A) 65.1

(B) 78.1

(C) 156.2

(D) 130.2

Correct Answer: (A) 65.1

Solution:

Given:

$d = 250$ mm, $S = 5$ mm, $t = 10$ mm, Thickness of flat plate $= 5$ mm, $f_u = 410$ N/mm^2, $\gamma_{\text{mw}} = 1.25$

1. Step 1: Throat thickness of the weld (t_t)

$$t_t = 0.75 \times 5 = 3.5 \text{ mm}$$

2. Step 2: Calculation of Section Modulus (Z_p)

$$Z_p = \frac{J}{r} = \frac{Ar^2}{r} = Ar$$

Using geometry for circular section:

$$Z_p = \left(\pi \times t \times \frac{d}{2}\right) \times \frac{d^2 t}{2}$$

$$Z_p = \left(\pi \times 10 \times \frac{250}{2}\right) \times \frac{250^2 \times 3.5}{2}$$

3. Step 3: Torque that can be applied on the plate The maximum torque is given by:

$$T = (f_s \times Z_p) = \left(\frac{f_u}{\sqrt{3}\gamma_{\text{mw}}}\right) \times \left(\pi \times \left(\frac{d^2 t}{2}\right)\right)$$

Substitute the known values:

$$T = \left(\frac{410}{\sqrt{3} \times 1.25}\right) \times \pi \times \frac{250^2 \times 3.5}{2}$$

$$T = 65.07 \text{ kN.m} \approx 65.1 \text{ kN.m}$$

Thus, the torque that can be resisted by the weld is $\boxed{65.1}$ kN.m.

Quick Tip

For welded connections under torque, always use the correct formula for section modulus and torque capacity as per IS standards. Ensure unit consistency throughout the calculation.

---

38. The figure shows a propped cantilever with uniform flexural rigidity EI (in N.m^2) and subjected to a moment M (in N.m). Consider forces and displacements in the upward direction as positive.

Find the upward reaction at the propped support B (in N) when this support settles by Δ (in metres).

(A) $\frac{3M}{2L} - \frac{6EI}{L^3}$

(B) $\frac{8M}{3L} - \frac{2EI}{3L^3}$

(C) $\frac{3M}{2L} - \frac{3EI}{L^3}$

(D) $\frac{M}{L} - \frac{8EI}{L^3}$

Correct Answer: (C) $\frac{3M}{2L} - \frac{3EI}{L^3}$

Solution:

Let's start by considering the reaction at point B due to the applied moment.

1. Reaction due to Moment:

The moment M applied at the cantilever results in a reaction force at point B. Since the flexural rigidity EI is constant, the moment at the support B is given by:

$$R_m = \frac{3M}{2L}$$

2. Deflection at B:

The deflection at point B should be zero, thus:

$$\frac{ML^2}{2EI} - R_m \times \frac{L^3}{3EI} = 0$$

Substituting $R_m = \frac{3M}{2L}$ into this equation:

$$\frac{ML^2}{2EI} - \frac{3M}{2L} \times \frac{L^3}{3EI} = 0$$

This simplifies to:

$$R_m = \frac{3M}{2L}$$

3. Reaction at Propped End Due to Sinking of Support:

The reaction at the propped end due to the sinking of the support is given by:

$$R_\Delta = \frac{3EI}{L^3} \times \Delta$$

4. Net Reaction at Propped End:

The net reaction at the propped end is the combination of the moment reaction and the sinking due to support movement. Therefore, the net reaction is:

$$R_B = \frac{3M}{2L} - \frac{3EI}{L^3} \times \Delta$$

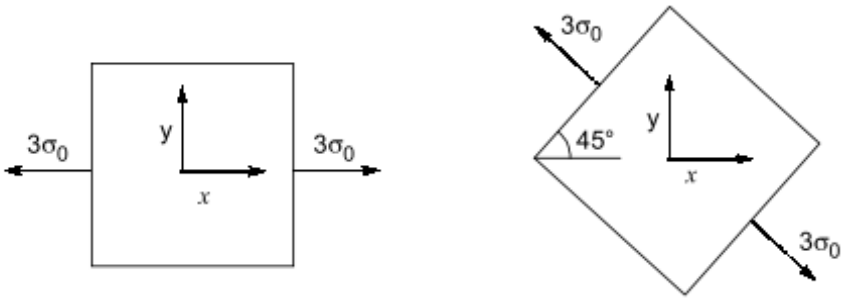
Thus, the net upward reaction at the propped support B is given by $\boxed{\frac{3M}{2L} - \frac{3EI}{L^3}}$.

Quick Tip

When analyzing reactions in a cantilever, break down the problem into moments and deflection components to accurately account for the effects of support movement and applied forces.

---

39. Let the state of stress at a point in a body be the difference of two plane states of stress shown in the figure. Consider all the possible planes perpendicular to the x-y plane and passing through that point. The magnitude of the maximum compressive stress on any such plane is $k\sigma_0$, where k is equal to ____ (round off to one decimal place).



(A) 3.0

(B) 2.1

(C) 1.7

(D) 1.5

Correct Answer: (B) 2.1

Solution:

We are given the difference between two states of stress. The stress states are shown as:

1. First Stress State (square with stresses $3\sigma_0$): - In this case, the stress is uniform along both axes in the x- and y-directions.

2. Second Stress State (diamond with stresses $3\sigma_0$ along x and y at 45°).

We now need to calculate the resultant stress on all planes perpendicular to the x-y plane.

The stress transformation can be done using the following formula for stress on any inclined plane:

$$\sigma_A/\sigma_B = \frac{\sigma_x + \sigma_y}{2} \pm \frac{1}{2}\sqrt{(\sigma_x - \sigma_y)^2 + 4\tau^2}$$

Given the stresses:

$$\sigma_x = 1.5\sigma_0, \quad \sigma_y = -1.5\sigma_0, \quad \tau_{xy} = 1.5\sigma_0$$

Substitute the values:

$$\sigma_A/\sigma_B = \frac{1.5\sigma_0 + (-1.5\sigma_0)}{2} \pm \frac{1}{2}\sqrt{(1.5\sigma_0 - (-1.5\sigma_0))^2 + 4(1.5\sigma_0)^2}$$

This simplifies to:

$$\sigma_A/\sigma_B = \pm\frac{1}{2}\sqrt{(3\sigma_0)^2 + 4(1.5\sigma_0)^2}$$

$$= \pm\frac{1}{2}\sqrt{9\sigma_0^2 + 9\sigma_0^2} = \pm\frac{1}{2}\sqrt{18\sigma_0^2} = \pm\sqrt{9\sigma_0^2} = \pm 3\sigma_0$$

Thus, the maximum compressive stress is:

$$k\sigma_0 = 2.1\sigma_0$$

Hence, the correct value of k is $\boxed{2.1}$.

Quick Tip

To find the maximum stress in transformed coordinates, always use the stress transformation equations and account for both the normal and shear stresses.

---

40. Consider a reinforced concrete beam section of 350 mm width and 600 mm depth. The beam is reinforced with the tension steel of 800 mm^2 area at an effective cover of 40 mm. Consider M20 concrete and Fe415 steel. Let the stress block considered for concrete in IS 456:2000 be replaced by an equivalent rectangular stress block, with no change in (a) the area of the stress block, (b) the design strength of concrete (at the strain of 0.0035), and (c) the location of neutral axis at flexural collapse. The ultimate moment of resistance of the beam (in kN.m) is ____ (round off to the nearest integer).

(A) 170

(B) 148

(C) 125

(D) 102

Correct Answer: (B) 148

Solution:

Given,

$$B = 350 \text{ mm}, \quad d = 600-40 = 560 \text{ mm}, \quad f_{ck} = 20 \text{ N/mm}^2, \quad f_y = 415 \text{ N/mm}^2, \quad A_{st} = 800 \text{ mm}^2$$

1. Step 1: Limiting depth of neutral axis (x_{lim}): The limiting depth of neutral axis is given by:

$$x_{lim} = 0.48 \times d = 0.48 \times 560 = 268.8 \text{ mm}$$

2. Step 2: Actual depth of neutral axis (x_u): Using the formula:

$$x_u = \frac{0.87 \times f_y \times A_{st}}{0.36 \times f_{ck} \times B}$$

Substituting the given values:

$$x_u = \frac{0.87 \times 415 \times 800}{0.36 \times 20 \times 350} = 114.619 \text{ mm}$$

Since $x_u < x_{lim}$, the section is under-reinforced.

3. Step 3: Calculation of ultimate moment of resistance (M_u): The ultimate moment of resistance is given by:

$$M_u = C \times L \times A$$

Where $C = 0.36 f_{ck} B$ and $L = \left(d - \frac{x_u}{2}\right)$. Therefore:

$$M_u = 0.36 \times 20 \times 350 \times \left(560 - \frac{114.619}{2}\right)$$

Calculating this:

$$M_u = 0.36 \times 20 \times 350 \times 114.619 = 145.197 \times 10^6 \text{ N-mm}$$

Converting to kN.m:

$$M_u = 145.2 \text{ kN.m}$$

Thus, the ultimate moment of resistance of the beam is $\boxed{148}$ kN.m.

Quick Tip

For under-reinforced sections, the moment of resistance can be calculated by using the stress block approach and considering the location of the neutral axis.

---

41. For a partially saturated soil deposit at a construction site, water content (w) is 15%, degree of saturation (S) is 67%, void ratio (e) is 0.6 and specific gravity of solids in the soil (G_s) is 2.67. Consider unit weight of water as 9.81 kN/m^3.

To fully saturate 5 m^3 of this soil, the required weight of water (in kN) will be ____ (round off to the nearest integer).

(A) 5

(B) 6

(C) 7

(D) 8

Correct Answer: (B) 6

Solution:

Given:

- Initial water content: $w_1 = 0.15$

- Degree of saturation: $S = 67\%$
- Void ratio: $e = 0.6$
- Specific gravity of solids: $G_s = 2.67$
- Unit weight of water: $\gamma_w = 9.81\,\text{kN/m}^3$

Let water content after full saturation be w_2. The weight of water after full saturation can be calculated using the following steps.

1. Step 1: Water content at full saturation (w_2):

$$w_2 = \frac{e}{G_s} = \frac{0.6}{2.67} = 0.2247$$

2. Step 2: Change in weight of water ($w_2 - w_1$):

$$w_2 - w_1 = \frac{\text{Weight of water}}{\text{Weight of solid}} = \frac{w}{w_s}$$

Where w_s is the weight of the solid.

3. Step 3: Weight of solid (w_s):

$$w_s = V_s \cdot G_s \cdot \gamma_w = \frac{V_t}{1+e} \cdot G_s \cdot \gamma_w$$

Where: - $V_t = 5\,\text{m}^3$ is the total volume. - $\gamma_w = 9.81\,\text{kN/m}^3$ is the unit weight of water.

4. Step 4: Substituting the values:

$$0.2247 - 0.15 = \frac{w}{\frac{5}{1.6} \times 2.67 \times 9.81}$$

Solving for w:

$$w = 0.0747 \times 3.125 \times 2.67 \times 9.81 = 6\,\text{kN}$$

Thus, the required weight of water is $\boxed{6}$ kN.

Quick Tip

To calculate the weight of water required for full saturation, use the void ratio, degree of saturation, and the specific gravity of solids in the soil.

---

42. Consider flow in a long and very wide rectangular open channel. Width of the channel can be considered as infinity compared to the depth of flow. Uniform flow depth is 1.0 m. The bed slope of the channel is 0.0001. The Manning roughness coefficient value is 0.02. Acceleration due to gravity, g, can be taken as 9.81 m/s^2.

The critical depth (in m) corresponding to the flow rate resulting from the above conditions is ____ (round off to one decimal place).

(A) 0.4

(B) 0.3

(C) 0.6

(D) 0.1

Correct Answer: (B) 0.3

Solution:

Given:

- Bed slope: $s = 0.0001$
- Manning's coefficient: $n = 0.02$
- Depth of flow: $y = 1\,\text{m}$
- Acceleration due to gravity: $g = 9.81\,\text{m/s}^2$

For very wide rectangular channel ($B \gg y$):

1. Step 1: Hydraulic radius The hydraulic radius for a very wide rectangular channel is given by:

$$R = \frac{A}{P} = \frac{By}{B + 2y} \approx y \quad \text{(since } B \gg y\text{)}$$

Thus, the hydraulic radius R is approximately equal to the flow depth y. Therefore:

$$R = 1\,\text{m}$$

2. Step 2: Critical depth of flow The critical depth y_c is given by the formula:

$$y_c = \left(\frac{q^2}{g}\right)^{1/3}$$

Where q is the discharge per unit width. Now we calculate q using the Manning's equation for discharge:

$$q = A \times v = B \times \frac{R^2}{n} \times s^{1/2}$$

Simplifying this, we get:

$$q = \frac{1}{n} \times y^{5/3} \times s^{1/2}$$

Substituting the values:

$$q = \frac{1}{0.02} \times (1)^{5/3} \times (0.0001)^{1/2} = 0.5\,\text{m}^3/\text{sec/m}$$

3. Step 3: Calculation of critical depth Now, using the formula for critical depth:

$$y_c = \left(\frac{(0.5)^2}{9.81} \right)^{1/3}$$

Solving this:

$$y_c = 0.294\,\text{m} \approx 0.3\,\text{m}$$

Thus, the critical depth is $\boxed{0.3}$ m.

Quick Tip

When calculating the critical depth in open channel flow, use the Manning's equation for discharge and the formula for critical depth based on the flow rate.

---

43. Match the following in Column I with Column II.

Column I	Column II
1. Vehicle Damage Factor	A. Stability of subgrade soil
2. Passenger Car Unit	B. Capacity of a roadway
3. Perception Reaction Time	C. Design rigid pavement
4. California Bearing Ratio	D. Design flexible pavement
	E. Stopping sight distance

(A) 1-D; 2-B; 3-E; 4-A

(B) 1-C; 2-B; 3-D; 4-A

(C) 1-D; 2-E; 3-B; 4-A

(D) 1-D; 2-B; 3-A; 4-E

Correct Answer: (A) 1-D; 2-B; 3-E; 4-A

Solution:

- Vehicle Damage Factor corresponds to the design of flexible pavement. - Passenger Car Unit corresponds to the capacity of a roadway. - Perception Reaction Time is related to

stopping sight distance. - California Bearing Ratio corresponds to the stability of the subgrade soil.

Thus, the correct matching is: 1 - D, 2 - B, 3 - E, 4 - A.

Quick Tip

In road design, it is important to understand the different factors like vehicle damage, traffic capacity, and subgrade stability. Each factor has a direct impact on the road design, which can influence road safety, longevity, and performance.

---

44. Consider the function given below and pick one or more CORRECT statement(s) from the following choices.

$$f(x) = x^3 - \frac{15}{2}x^2 + 18x + 20$$

(A) $f(x)$ has a local minimum at $x = 3$

(B) $f(x)$ has a local maximum at $x = 3$

(C) $f(x)$ has a local minimum at $x = 2$

(D) $f(x)$ has a local maximum at $x = 2$

Correct Answer: (A, D)

Solution:

Given the function:

$$f(x) = x^3 - \frac{15}{2}x^2 + 18x + 20$$

First, compute the first and second derivatives:

$$f'(x) = 3x^2 - 15x + 18$$

Now, set $f'(x) = 0$ to find the critical points:

$$3x^2 - 15x + 6 = 0$$

$$x^2 - 5x + 2 = 0$$

The roots of the equation are:

$$x = 2,\ 3$$

Next, compute the second derivative:

$$f''(x) = 6x - 15$$

At $x = 2$:

$$f''(2) = 6 \times 2 - 15 = -3 \quad \text{(Local maximum)}$$

At $x = 3$:

$$f''(3) = 6 \times 3 - 15 = 3 \quad \text{(Local minimum)}$$

Thus, the function has a local maximum at $x = 2$ and a local minimum at $x = 3$.

$$f(2) = (2)^3 - \frac{15}{2}(2)^2 + 18 \times 2 + 20 = 34$$
$$f(3) = (3)^3 - \frac{15}{2}(3)^2 + 18 \times 3 + 20 = 33.5$$

Thus, the local maximum occurs at $x = 2$, and the local minimum occurs at $x = 3$. Therefore, the correct answers are (A) and (D).

45. Pick the CORRECT eigen value(s) of the matrix [A] from the following choices.

$$[A] = \begin{bmatrix} 6 & 8 \\ 4 & 2 \end{bmatrix}$$

(A) 10

(B) 4

(C) -2

(D) -10

Correct Answer: (A), (C)

Solution:

Given matrix, $A = \begin{bmatrix} 6 & 8 \\ 4 & 2 \end{bmatrix}$

To find the eigenvalues, we use the characteristic equation:

$$\det(A - \lambda I) = 0$$

$$\begin{vmatrix} 6 - \lambda & 8 \\ 4 & 2 - \lambda \end{vmatrix} = 0$$

Expanding the determinant:

$$(6-\lambda)(2-\lambda)-(8)(4)=0$$

$$(6-\lambda)(2-\lambda)=20$$

$$12-6\lambda-2\lambda+\lambda^2=20$$

$$\lambda^2-8\lambda-20=0$$

This is a quadratic equation:

$$\lambda^2-10\lambda+2\lambda-20=0$$

$$\lambda(\lambda-10)+2(\lambda-10)=0$$

$$(\lambda-10)(\lambda+2)=0$$

Thus, the eigenvalues are:

$$\lambda=-2,10$$

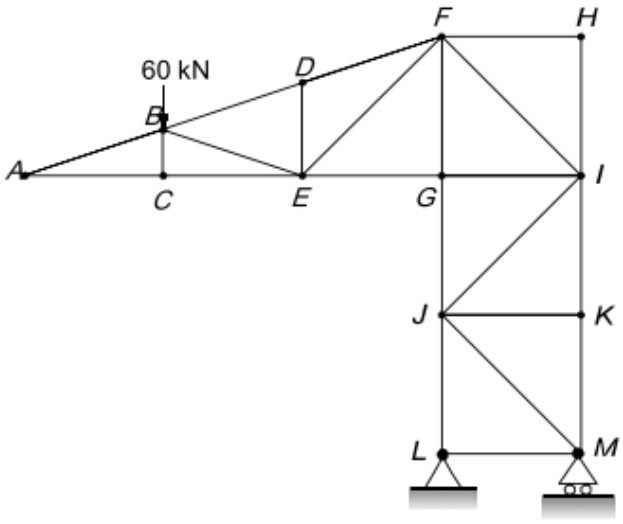
So, the correct eigenvalues are $\boxed{-2,10}$.

Quick Tip

To find the eigenvalues of a matrix, solve the characteristic equation $\det(A-\lambda I)=0$. This gives a polynomial, and the roots of that polynomial are the eigenvalues.

---

46. In the pin-jointed truss shown in the figure, the members that carry zero force are identified. Which of the following options is/are zero-force members?



(A) BC

(B) EG

(C) FI

(D) JK

Correct Answer: (A), (D)

Solution:

In truss analysis, zero-force members are identified using the following rules:

1. If two non-collinear members meet at a joint without an external load or reaction, both members are zero-force members.

2. If three members form a joint, and two of them are collinear (i.e., in a straight line) with no external load or reaction at that joint, the third member is a zero-force member.

We will apply these principles to identify the zero-force members in the given truss.

- Step 1: Analyzing Joint B

At joint B, there are two non-collinear members, BC and AB, with no external load or reaction at B. According to the rule, since there are no other loads or reactions at joint B, the member BC is a zero-force member. Hence, $F_{BC} = 0$.

- Step 2: Analyzing Joint J

At joint J, the members are JK and JL. There is no external load or reaction at joint J, and these two members are non-collinear. By the same rule, JK is a zero-force member because it does not carry any external force. Hence, $F_{JK} = 0$.

- Step 3: Analyzing Joint F

Joint F has three members: DF, EF, and FG. Joint F is subject to the external load at point D, so it cannot be classified as a zero-force joint.

Thus, the zero-force members are BC and JK. The correct options are (A) BC and (D) JK.

Quick Tip

In truss analysis, zero-force members are those which carry no load under the given loading conditions. They can be identified using specific rules such as the absence of external load or reaction at the joint or having two non-collinear members at a joint with no external load or support.

47. In the context of shear strength of soil, which of the following statements is/are CORRECT?

(A) The unconfined compression test is a special case of the unconfined-undrained (UU) triaxial tests.

(B) The shear strength parameters obtained from the consolidated-drained (CD) triaxial tests should be used to analyse rapid construction in clay.

(C) Vane shear test is commonly used for determining in situ undrained strength of saturated clay soils.

(D) In an unconsolidated-undrained (UU) triaxial tests, the angle of internal friction (ϕ) is equal to zero.

Correct Answer: (A), (C), (D)

Solution:

- Statement (a): The unconfined compression test is indeed a special case of the unconsolidated-undrained (UU) triaxial test. In the unconfined compression test, the confining pressure is zero, which is a particular case of the UU triaxial test. Hence, statement (a) is correct.

- Statement (b): The shear strength parameters obtained from the consolidated-drained (CD) triaxial tests should not be used for rapid construction in clay. The consolidated-drained tests are usually done on soils that consolidate over time and are more relevant to long-term stability. Hence, statement (b) is incorrect.

- Statement (c): The vane shear test is widely used for determining in situ undrained strength of saturated clays, particularly when samples cannot be easily retrieved for laboratory tests. Hence, statement (c) is correct.

- Statement (d): In an unconsolidated-undrained (UU) triaxial test, the internal friction angle (ϕ) is assumed to be zero for saturated clays under undrained conditions. Hence, statement (d) is correct.

Quick Tip

In soil mechanics, it is essential to select the correct type of test based on the nature of the soil and the conditions of the construction project. For undrained conditions, the UU test and vane shear test are often used.

---

48. The drag force, F_D, on a sphere due to a fluid flowing past the sphere is a function of viscosity, μ, the mass density, ρ, the velocity of flow, V, and the diameter of the sphere, D. Pick the relevant (one or more) non-dimensional parameter(s) pertaining to the above process from the following list.

(A) $\frac{F_D}{\rho V^2 D^2}$

(B) $\frac{\rho F_D}{V^2 D^2}$

(C) $\frac{\rho V D}{\mu}$

(D) $\frac{\mu \rho}{V D}$

Correct Answer: (A), (C)

Solution:

To solve for the non-dimensional parameters, we need to apply Buckingham's Pi Theorem, which helps derive dimensionless parameters (Pi terms).

Starting with the drag force equation:

$$F_D = F(D, V, \rho, \mu)$$

We express the dimensions:

$$F_D = [MLT^{-2}], \quad \rho = [ML^{-3}], \quad V = [LT^{-1}], \quad \mu = [ML^{-1}T^{-1}]$$

We substitute these into the drag force equation and apply the Buckingham's Pi Theorem. After calculating, the resulting non-dimensional parameters are:

$$\pi_1 = \frac{F_D}{\rho V^2 D^2}, \quad \pi_2 = \frac{\rho V D}{\mu}$$

Both expressions are dimensionless and represent the non-dimensional parameters related to the drag force on the sphere. Hence, the correct answers are (A) and (C).

Quick Tip

When solving for non-dimensional parameters in fluid dynamics, always start by identifying the physical dimensions of the variables involved. Use Buckingham's Pi Theorem to derive the dimensionless numbers that describe the process.

---

49. A compound has a general formula $C_aH_bO_cN_d$ and molecular weight 187. A 935 mg/l solution of the compound is prepared in distilled deionized water. The Total Organic Carbon (TOC) is measured as 360 mg/l (as C). The Chemical Oxygen Demand (COD) and the Total Kjeldahl Nitrogen (TKN) are determined as 600 mg/l (as O_2) and 140 mg/l (as N), respectively (as per the chemical equation given below). Which of the following options is/are CORRECT?

(A) $a = 6$

(B) $b = 7$

(C) $c = 5$

(D) $d = 3$

Correct Answer: (a), (b), (c)

Solution:

Given: molecular weight of $C_aH_bO_cN_d$ = 187 g.

Solution of compound in distilled water = 935 mg/l.

TOC = 360 mg/l (as C),

COD = 600 mg/l (as O_2),

TKN = 140 mg/l (as N).

The chemical equation is:

$$C_aH_bO_cN_d + \left(\frac{4a + b - 2c - 3d}{4}\right) O_2 \rightarrow aCO_2 + \frac{b - 3d}{2} H_2O + dNH_3$$

Now, from the atomic weights: C: (12), H: (1), O: (16), N: (14).

We start by calculating d.

$$\frac{140}{935} \times 187 = 2 \quad \text{(atomic weight of N)}$$

Then, solving for a:

$$a = \frac{360}{935} \times 187 = 6$$

Now, solve for b and c using the equation for oxygen:

$$\frac{4a + b - 2c - 3d}{4} = \frac{660 \times 187}{935}$$

Substituting the known values:

$$b + 16c = 87$$

Solving the equations $b + 16c = 87$ and $b = 7$, $c = 5$, we get:

$$b = 7,\ c = 5$$

Finally, we check the molecular weight:

$$12a + b + 16c + 14d = 187 \quad \Rightarrow \quad 12 \times 6 + 7 + 16 \times 5 + 14 \times 2 = 187$$

Thus, the correct values are $a = 6$, $b = 7$, $c = 5$, and $d = 2$. The correct options are (a), (b), and (c).

Quick Tip

When solving problems with molecular formulas and stoichiometric equations, use the given data to set up equations based on atomic weights and balances, then solve for the unknowns algebraically.

---

50. The free flow speed of a highway is 100 km/h and its capacity is 4000 vehicle/h. Assume speed density relation is linear. For a traffic volume of 2000 vehicle/h, choose all the possible speeds (in km/h) from the options given below (round off to two decimal places).

(A) 85.36

(B) 65.20

(C) 14.64

(D) 7.22

Correct Answer: (a), (c)

Solution:

Given: Free mean speed, $V_f = 100$ kmph

Capacity, $q_{\text{max}} = 4000$ veh/hr

Possible speed, $V_1, V_2 = ?$

Speed-density relation is linear.

The equation for speed-density relation is:

$$q_{\text{max}} = \frac{1}{4} k_j V_f$$

Substituting known values:

$$4000 = \frac{1}{4} \times 100 k_j$$

This simplifies to:

$$k_j = 160 \text{ veh/km}$$

Now, using the formula for traffic flow, $q = vk$, we get:

$$q = v_f \left(k - \frac{k^2}{k_j} \right)$$

Substituting the values:

$$2000 = 100 \left(k - \frac{k^2}{160} \right)$$

On solving, we get:

$$k_1 = 23.431 \text{ veh/km}, \quad k_2 = 136.568 \text{ veh/km}$$

Now, the velocity of traffic flow at $k_1 = 23.431$ veh/km is:

$$V_1(k_1 = 23.431) = 100 \left(1 - \frac{23.431}{160} \right) = 85.355 \text{ km/hr}$$

The velocity of traffic flow at $k_2 = 136.568$ veh/km is:

$$V_2(k_2 = 136.568) = 100 \left(1 - \frac{136.568}{160} \right) = 14.645 \text{ km/hr}$$

Thus, the correct options are (a) and (c).

Quick Tip

In problems involving speed-density relations, remember that the traffic flow equation relates the speed and density. Solve for unknown speeds using the given equations and check against the options for possible values.

51. Consider a discrete random variable X whose probabilities are given below. The standard deviation of the random variable is ..... (round off to one decimal place).

x_1	1	2	3	4
$P(X = x_i)$	0.3	0.1	0.3	0.3

Correct Answer: 2.8

Solution:

Given:

$$x_1 = 1,\ 2,\ 3,\ 4 \quad \text{and} \quad P(X = x_i) = 0.3,\ 0.1,\ 0.4,\ 0.3$$

Now, the expected value $E(x)$ is calculated as:

$$E(x) = \sum x_i P(x_i) = 1 \times 0.3 + 2 \times 0.1 + 3 \times 0.4 + 4 \times 0.3 = 4.1$$

Next, the expected value of x^2 is:

$$E(x^2) = \sum x_i^2 P(x_i) = 1^2 \times 0.3 + 2^2 \times 0.1 + 4^2 \times 0.3 + 4^2 \times 0.3 = 24.7$$

The variance is:

$$V(x) = E(x^2) - (E(x))^2 = 24.7 - (4.1)^2 = 7.89$$

Finally, the standard deviation is:

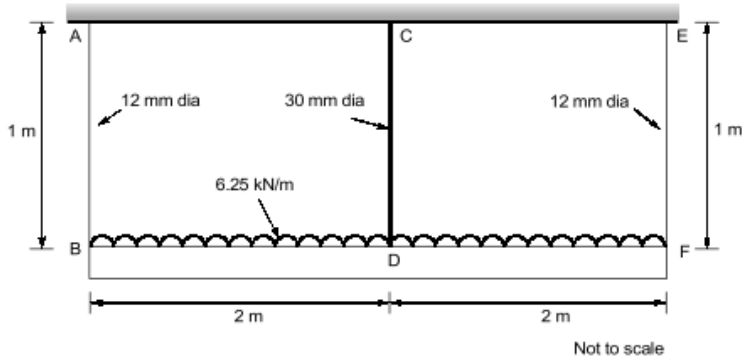
$$\sigma = \sqrt{7.89} = 2.808$$

Thus, the standard deviation of the random variable is 2.8.

Quick Tip

To calculate the standard deviation for a discrete random variable, first calculate the expected value $E(x)$, then the expected value of x^2. Subtract the square of the expected value from the expected value of x^2 to get the variance, and take the square root to find the standard deviation.

52. A steel beam supported by three parallel pin-jointed steel rods is shown in the figure. The moment of inertia of the beam is 8×10^7 mm^4. Take modulus of elasticity of steel as 210 GPa. The beam is subjected to uniformly distributed load of 6.25 kN/m, including its self-weight. The axial force (in kN) in the centre rod CD is ..... (round off to one decimal place).



Correct Answer: 16.48 kN

Solution:

Given: Moment of inertia of beam, $I = 8 \times 10^7$ mm^4

Modulus of elasticity of steel, $E = 210 \times 10^3$ N/mm^2

UDL, $w = 6.25$ kN/m $= 6.25$ N/mm (including self-weight)

Axial force in CD, $P_1 = ?$

Flexural rigidity, $EI = 210 \times 10^3 \times 8 \times 10^7 = 1.68 \times 10^{13}$ N-mm^2 $= 16800$ kNm^2

Due to symmetry, we have:

$$\Delta_{EF} = \Delta_{AB} \quad \text{and} \quad \Delta_{CD} \neq \Delta_{EF}$$

From equilibrium:

$$2P_2 + P_1 = 6.25 \times 40$$

$$2P_2 + P_1 = 25 \text{ kN}$$

Solving for P_2:

$$P_2 = \frac{25 - P_1}{2} = 12.5 - 0.5P_1$$

The net elongation of rod CD is given by:

$$\Delta_{CD\,\text{net}} = \frac{P_1 \times 1}{\frac{\pi}{4}(0.03)^2 \times 210 \times 10^6} + \frac{0.5 \times 4 \times 12.5}{\frac{\pi}{4}(0.012)^2 \times 210 \times 10^6}$$

Substitute the values:

$$\Delta_{CD\,\text{net}} = 2.77 \times 10^{-5} P_1 - 5.26 \times 10^{-4}$$

Now, for the deflection of the beam at point D:

$$\Delta_{\text{beam}} = \frac{5wl^4}{384EI} - \frac{P_1 l^3}{48EI}$$

Substitute the values for $w = 6.25\,\text{kN/m}$ and $l = 2\,\text{m}$:

$$\Delta_{\text{beam}} = \frac{4^3}{48 \times 16800} (5 \times 6.25 \times 4 - P_1)$$

Finally, solving for the axial force in Rod CD:

$$\Delta_{\text{CD net}} = \Delta_{\text{beam}} = \frac{4^3}{48 \times 16800} (5 \times 6.25 \times 4 - P_1)$$

After solving, we find:

$$P_1 = 16.48\,\text{kN}$$

Thus, the axial force in rod CD is $P_1 = 16.48\,\text{kN}$.

Quick Tip

When solving problems related to beam deflection and axial force, carefully apply symmetry principles, calculate the elongation and deflection using the respective formulas, and ensure that equilibrium conditions are satisfied for the structure.

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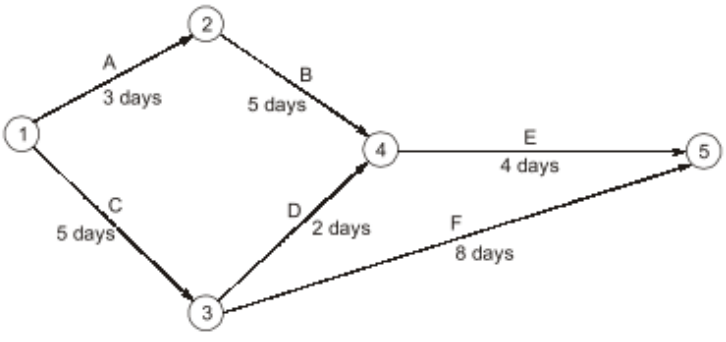
53. The figure shows a network diagram for a construction project. The activities A, B, C, D, E, and F are represented by arrows and their durations are in the figure. The total float available for the activity E in day(s) is equal to ..... (round off to the nearest integer).



Correct Answer: 1

Solution:

From the network diagram, the total float for an activity is the difference between the latest start time (LST) and the earliest start time (EST). We start by calculating the EST, EFT, LST, and LFT for each activity.

For Activity E:

- The duration of activity E is 4 days.
- The EST of E is 8 (calculated from the preceding activities).
- The EFT of E is EST + duration of E $= 8 + 4 = 12$.
- The LST of E is $LST = LFT$ − duration of E $= 13 - 4 = 9$.
- The LFT of E is 13, which is the same as the LST of the following activity.

Now, the total float for activity E is:

$$\text{Float} = LST - EST = 9 - 8 = 1$$

Thus, the total float for activity E is 1 day.

Quick Tip

To calculate the total float for an activity, subtract the earliest start time (EST) from the latest start time (LST). Ensure to compute EST and LST using the network diagram based on dependencies and durations of the activities.

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54. A reinforced concrete beam has a support section with width of 300 mm and effective depth of 500 mm. The support section is reinforced with 3 bars of 20 mm diameter at the tension side. Two-legged vertical stirrups of 10 mm diameter and Fe415 steel at a spacing of 100 mm are provided as shear reinforcement. Assume that there is no possibility of diagonal compression failure at the section.

As per IS 456:2000, the maximum shear resisted by the vertical stirrups (in kN), as per limit state design, is ..... (round off to one decimal place).

Correct Answer: 283.6 kN

Solution:

Given: - Two-legged vertical stirrups of diameter $\phi = 10\,\text{mm}$ - c/c spacing $S_v = 100\,\text{mm}$ - Yield strength of steel $f_y = 415\,\text{N/mm}^2$ - Effective depth $d = 500\,\text{mm}$

The spacing for vertical shear stirrups is given by:

$$S_v = \frac{0.87 \times A_{sv} \times d}{V_s}$$

Where: - $A_{sv} = 2 \times \frac{\pi}{4} \times \phi^2$ (cross-sectional area of the stirrups) - V_s is the shear force resisted by the stirrups

Using the formula for A_{sv}:

$$A_{sv} = 2 \times \frac{\pi}{4} \times (10^2) = 2 \times 78.54 = 157.08\,\text{mm}^2$$

Now, substituting the values into the formula for V_s:

$$V_s = \frac{0.87 \times f_y \times A_{sv} \times d}{S_v} = \frac{0.87 \times 415 \times 2 \times \frac{\pi}{4} \times 10^2 \times 500}{100}$$

$$V_s = \frac{0.87 \times 415 \times 2 \times 78.54 \times 500}{100} = 283568\,\text{N}$$

$$V_s \approx 283.6\,\text{kN}$$

Thus, the maximum shear resisted by the vertical stirrups is $283.6\,\text{kN}$.

Quick Tip

To calculate the maximum shear resisted by stirrups, use the given values of the stirrup spacing, yield strength of steel, and the area of stirrups to compute the shear force resisted by the stirrups using the formula provided by IS 456.

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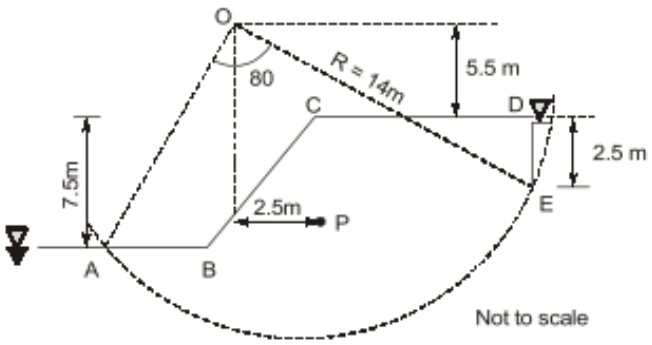
55. The bank of a canal has the profile shown in the figure. The material is a homogeneous clay with a bulk unit weight of 20 kN/m^3, undrained cohesion of 30 kPa and is fully saturated ($\phi = 0$). For the trial slip circle shown, the area ABCDEA is 150 m^2 and the centroid is at P. A tension crack (DE) of 2.5 m deep was also observed. Assume unit weight of water is 9.81 kN/m^3 and consider 1 m run of the bank for the analysis.

Considering the canal is empty and the tension crack is completely filled with water, the factor of safety against slope failure of the bank is ..... (round off to two decimal places).



Correct Answer: 1.06

Solution:

The factor of safety (FOS) is given by the ratio of resisting moment to overturning moment:

$$FOS = \frac{\text{Resisting moment}}{\text{Overturning moment}}$$

Resisting moment is calculated as:

$$R = \left(\frac{80}{180} \times \pi \times 14\right) \times 30$$

The overturning moment is:

$$\text{Overturning moment} = ((150 \times 1) \times 20 \times 2.5) + \left(\frac{1}{2} \times 9.81 \times 2.5^2 \times \left(5.5 + \frac{2.5 \times 2}{3}\right)\right)$$

Substituting the values into the formula:

$$FOS = \frac{30 \times 14 \times \left(\frac{80}{180} \times \pi \times 14\right)}{(150 \times 1) \times 20 \times 2.5 + \frac{1}{2} \times 9.81 \times 2.5^2 \times \left(5.5 + \frac{2.5 \times 2}{3}\right)}$$

After calculating:

$$FOS = \frac{8210.028}{7500 + 219.703} = 1.06$$

Thus, the factor of safety against slope failure is 1.06.

Quick Tip

To calculate the factor of safety for slope failure, use the moment equilibrium approach, where the resisting moment is compared with the overturning moment. Carefully substitute the given values of the area, depth, and unit weights into the respective formulas for accurate calculations.

56. A designer used plate load test to obtain the value of the bearing capacity factor N_t. A circular plate of 1 m diameter was placed on the surface of a dry sand layer extending very deep beneath the ground. The unit weight of the sand is 16.66 kN/m^3. The plate is loaded to failure at a pressure of 1500 kPa.

Considering Terzaghi's bearing capacity theory, the bearing capacity factor N_t is ..... (round off to the nearest integer).

Correct Answer: 300

Solution:

We know the formula for a circular plate:

$$q_u = 1.3C_N C + \gamma D_f N_q + 0.3BN_t$$

For sand, the cohesion $c = 0$, so the formula simplifies to:

$$q_u = 0.3 \times 1 \times 16.66 \times N_t = 1500$$

Where:

- $\gamma = 16.66\,\text{kN/m}^3$ (unit weight of sand)
- $D_f = 1\,\text{m}$ (depth of foundation)
- N_t is the bearing capacity factor

Now, solving for N_t:

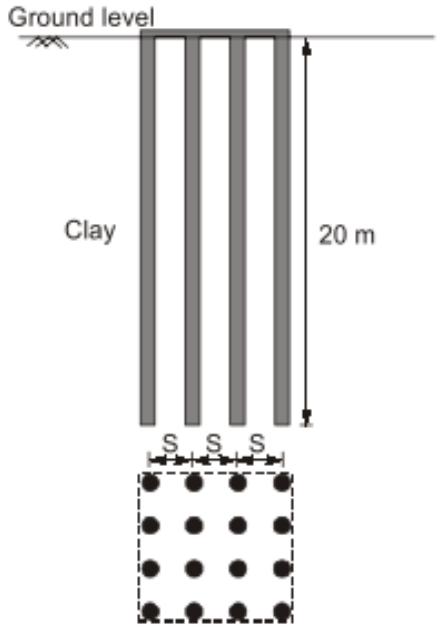
$$N_t = \frac{1500}{0.3 \times 16.66} = 300.12$$

Thus, the bearing capacity factor N_t is approximately 300.

Quick Tip

When using Terzaghi's bearing capacity theory, ensure you account for the material properties such as cohesion, unit weight, and the depth of the foundation when calculating the bearing capacity factors. For sand, the cohesion is often zero.

57. A 4 x 4 group pile, with each pile 20 m long and 500 mm in diameter, is installed in a square pattern in a clayey soil, as shown in the figure. The average unconfined compressive strength of the soil is 100 kN/m^2, and the adhesion factor is 0.8. Neglect the bearing at the tip of the piles. For a group efficiency factor of 1.0, the centre to centre spacing(s) of the piles (in m) would be ..... (round off to two decimal places).



Correct Answer: 1.51 m

Solution:

The group efficiency η_g is calculated as:

$$\eta_g = \frac{Q_{\text{avg}}}{nQ_u}$$

Where:

- $\eta_g = 1$ (group efficiency factor)
- $Q_{\text{avg}} = \frac{UCS}{2} = \frac{100}{2} = 50\,\text{kN/m}^2$ (average unconfined compressive strength of the soil)
- Q_u is the ultimate bearing capacity of the pile

The total bearing capacity $\overline{Q}$ is given by:

$$\overline{Q} = nQ_{\text{up}} = n\left(\alpha c \times (\pi \times D)\right)$$

Where:

- $n = 16$ (number of piles)
- $\alpha = 0.8$ (adhesion factor)

- $D = 0.5\,\text{m}$ (diameter of the pile)

The pile spacing is calculated as:

$$\overline{Q} = 50 \times 4 \times B \times 20 = 16 \times 0.8 \times (\pi \times 0.5 \times 20)$$

Now, solving for the spacing B:

$$B = 5.026\,\text{m}$$

Thus, the spacing s of the piles is:

$$s = B + D = 5.026 + 0.5 = 1.51\,\text{m}$$

Thus, the centre to centre spacing of the piles is $1.51\,\text{m}$.

Quick Tip

When designing pile foundations, always consider factors such as group efficiency, the adhesion factor, and the unconfined compressive strength of the soil. Ensure to account for the pile spacing to avoid excessive pile group interaction.

---

58. A 60 cm diameter well completely penetrates a confined aquifer of permeability 5×10^{-4} m/s. The length of the strainer (spanning the entire thickness of the aquifer) is 10 m. The drawdown at the well under steady state pumping is 1.0 m. Assume that the radius of influence for this pumping is 300 m.

The discharge from the well (in litres per minute) is ..... (round off to the nearest integer).

Correct Answer: 273

Solution:

We use the formula for discharge from a well in a confined aquifer:

$$Q = \frac{2\pi k b s_w \log_e\left(\frac{R}{r_w}\right)}{\log_e\left(\frac{R}{r_w}\right)}$$

Where: - $k = 5 \times 10^{-4}\,\text{m/s}$ (permeability)

- $b = 10\,\text{m}$ (thickness of the aquifer)

- $R = 300\,\text{m}$ (radius of influence)

- $r_w = 0.3\,\text{m}$ (radius of well)

- $s_w = 1\,\text{m}$ (drawdown)

Substituting the values:

$$Q = \frac{2\pi \times 5 \times 10^{-4} \times 10 \times 1 \times \log_e\left(\frac{300}{0.3}\right)}{\log_e\left(\frac{300}{0.3}\right)} = 4.54 \times 10^{-3}\,\text{m}^3/\text{s}$$

Convert to litres per minute:

$$Q = 4.54 \times 10^{-3} \times 60\,\text{lit/min} = 272.87\,\text{lit/min} \approx 273\,\text{lit/min}$$

Thus, the discharge from the well is $273\,\text{lit/min}$.

Quick Tip

When calculating discharge from a well in a confined aquifer, ensure to account for the radius of influence, permeability, thickness of the aquifer, and the well radius. Always convert the discharge to the desired units, such as litres per minute.

---

59. The peak of flood hydrograph due to a 3-hour duration storm in a catchment is 180 m^3/s. The total rainfall depth is 6.6 cm. It can be assumed that the average infiltration loss is 0.2 cm/h. There are no other losses. The base flow is constant at a value of 30 m^3/s.

The peak value of the 3-hour unit hydrograph for this catchment (in m^3/s) is ..... (round off to the nearest integer).

Correct Answer: 25

Solution:

The peak discharge is given as $180\,\text{m}^3/\text{s}$, and the base flow is $30\,\text{m}^3/\text{s}$. The infiltration loss is 0.2 cm/h, and the total rainfall depth is 6.6 cm.

The peak of the 3-hour unit hydrograph Q_p is calculated using the formula:

$$Q_p = \frac{\text{Peak discharge} - \text{Base flow}}{R - \phi t}$$

Where: - Peak discharge = $180\,\text{m}^3/\text{s}$

- Base flow = $30\,\text{m}^3/\text{s}$

- $R = 6.6\,\text{cm}$ (rainfall depth)

- $\phi = 0.2$ cm/h (infiltration loss)

- $t = 3$ hours (duration of the storm)

Substitute the values into the formula:

$$Q_p = \frac{180 - 30}{6.6 - 0.2 \times 3} = \frac{150}{5.4} = 27.78\,\text{m}^3/\text{s}$$

Thus, the peak value of the 3-hour unit hydrograph is $25\,\text{m}^3/\text{s}$.

Quick Tip

To calculate the peak of a unit hydrograph, subtract the base flow from the peak discharge and adjust for infiltration losses using the given formula. This calculation helps in understanding the runoff due to a storm event.

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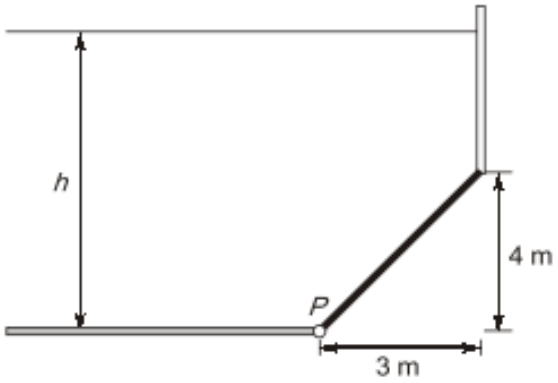
60. The shaft of a 6 m wide gate in the figure will fail at a moment of 3924 kN.m about the hinge P. The maximum value of water depth h (in m) that the gate can hold is ..... (round off to the nearest integer).



Correct Answer: 8

Solution:

Let the force F act on a gate at B.

From the figure, we know that:

$$F = \rho g h A$$

Where:

- $\rho = 1000\,\text{kg/m}^3$ (density of water)

- $g = 9.81\,\text{m/s}^2$ (acceleration due to gravity)

- h is the depth of the water

- $A = 6 \times h$ is the area of the gate

Substituting the values:

$$F = 1000 \times 9.81 \times (h - 2) \times 30 = 294.3 \times (h - 2)\,\text{kN}$$

From the figure:

$$\sin\theta = \frac{h - h_{cp}}{PB}$$

where $h_{cp} = h - 4$. Substituting this into the equation:

$$\sin\theta = \frac{h - (h - 4)}{PB} = \frac{4}{5}$$

Next, using the moment equilibrium about P, we take the moment about P:

$$F \times PB = 3924$$

Substituting the value of F:

$$294.3 \times (h - 2) \times \frac{5}{4} \times \left[2 - \frac{4}{3(h - 2)}\right] = 3924$$

Simplifying and solving for h:

$$(h - 2) \times \left(2 - \frac{4}{3(h - 2)}\right) = 32/3$$

$$2h - 4 = 32/3$$

$$h = 8\,\text{m}$$

Thus, the maximum depth h that the gate can hold is $8\,\text{m}$.

Quick Tip

When solving for the maximum water depth in gate problems, ensure to use the moment equilibrium method and account for the force acting on the gate, as well as the geometry of the gate and its position.

61. The analyses results of a water sample are given below. The non-carbonate hardness of the water (in mg/L) as $CaCO_3$ is ..... (in integer).

Ca^{2+} = 150 mg/L as $CaCO_3$

Mg^{2+} = 40 mg/L as $CaCO_3$

Fe^{2+} = 10 mg/L as $CaCO_3$

Na^{+} = 50 mg/L as $CaCO_3$

K^{+} = 10 mg/L as $CaCO_3$

CO_3^{2-} = 120 mg/L as $CaCO_3$

HCO_3^{-} = 30 mg/L as $CaCO_3$

Cl^{-} = 50 mg/L as $CaCO_3$; Other anions were not analysed.

Correct Answer: 50

Solution:

Total hardness as $CaCO_3$ is calculated by summing the hardness contribution of each ion:

$$\text{Total hardness (TH)} = 150 + 40 + 10 = 200\,\text{mg/L as } CaCO_3$$

Alkalinity as $CaCO_3$ is the sum of the contributions from the ions that contribute to alkalinity:

$$\text{Alkalinity} = 100 + 50 = 150\,\text{mg/L as } CaCO_3$$

The carbonate hardness (CH) is given by the minimum of alkalinity and total hardness:

$$\text{CH} = 150\,\text{mg/L as } CaCO_3$$

The non-carbonate hardness (NCH) is given by:

$$\text{NCH} = \text{TH} - \text{CH} = 200 - 150 = 50\,\text{mg/L as } CaCO_3$$

Thus, the non-carbonate hardness of the water is $50\,\text{mg/L as } CaCO_3$.

Quick Tip

To calculate non-carbonate hardness, subtract the carbonate hardness from the total hardness. Carbonate hardness is the minimum of alkalinity and total hardness, and non-carbonate hardness accounts for the hardness due to other ions.

62. A community generates 1 million litres/day (MLD) of wastewater. This wastewater is treated using activated sludge process (ASP). The working volume of the aeration tank of the ASP is 250 m^3, and the biomass concentration in the tank is 3000 mg/L. Analyses results showed that a biomass concentration of 10 mg/L is present in the treated effluent from the secondary sedimentation tank of the ASP. Sludge wastage from the system is at a rate of 5000 L/day with a biomass concentration of 10000 mg/L. The system is in steady state condition.

The biological sludge residence time (BSRT) of the system (in days) is ..... (round off to one decimal place).

Correct Answer: 12.5

Solution:

Given: - $Q_0 = 1\,\text{MLD} = 1 \times 10^6\,\text{L/day}$ (wastewater flow rate)

- $V = 250\,\text{m}^3$ (volume of aeration tank)

- $X = 3000\,\text{mg/L}$ (biomass concentration in aeration tank)

- $X_e = 10\,\text{mg/L}$ (biomass concentration in treated effluent)

- $Q_w = 5000\,\text{L/day}$ (sludge wastage flow rate)

- $X_u = 10000\,\text{mg/L}$ (biomass concentration in sludge wastage)

The biological sludge residence time (BSRT) is given by the formula:

$$\theta_s = \frac{V \times X}{Q_w \times X_u + (Q_0 - Q_w) \times X_e}$$

Substitute the given values into the formula:

$$\theta_s = \frac{250 \times 3000 \times 10^3}{5000 \times 10000 + (10^6 - 5000) \times 10}$$

$$\theta_s = \frac{250 \times 3000 \times 10^3}{5000 \times 10000 + (10^6 - 5000) \times 10} = 12.5\,\text{days}$$

Thus, the biological sludge residence time (BSRT) is 12.5 days.

Quick Tip

The biological sludge residence time (BSRT) is a key indicator of the performance of activated sludge systems. It can be calculated by considering the biomass concentration, flow rates of influent and effluent, and the amount of sludge being wasted.

63. A settling chamber is used for the removal of discrete particulate matter from air with the following conditions. Horizontal velocity of air = 0.2 m/s; Temperature of air stream = 77°C; Specific gravity of particle to be removed = 2.65; Chamber length = 12 m; Chamber height = 2 m; Viscosity of air at 77°C = 2.1 × 10^{-5} kg/m·s; Acceleration due to gravity (g) = 9.81 m/s²; Density of air at 77°C = 1.0 kg/m³; Assume the density of water as 1000 kg/m³ and Laminar condition exists in the chamber.

The minimum size of particle that will be removed with 100% efficiency in the settling chamber (in μm is ..... (round off to one decimal place).

Correct Answer: 22 μm

Solution:

Given:

$$\mu = 2.1 \times 10^{-5} \text{ kg/m} \cdot \text{s}$$

$$\rho_{\text{air}} = 1 \text{ kg/m}^3$$

$$\rho_{\text{water}} = 1000 \text{ kg/m}^3$$

$$V = 0.2 \text{ m/s}$$

$$G = 2.65$$

$$L = 12 \text{ m}$$

$$H = 2 \text{ m}$$

$$g = 9.81 \text{ m/s}^2$$

The formula for calculating the minimum particle size that will be removed with 100% efficiency in a settling chamber is:

$$d = C\sqrt{\frac{18\mu V H}{gL\rho_p}}$$

where:

- $C = 1$ (constant for laminar flow)
- μ is the dynamic viscosity of air
- V is the velocity of air
- H is the height of the chamber

- g is the acceleration due to gravity
- L is the length of the chamber
- ρ_p is the density of the particle

Substituting the known values:

$$d = 1 \times \sqrt{\frac{18 \times 2.1 \times 10^{-5} \times 0.2 \times 2}{9.81 \times 12 \times 2.65 \times 10^3}}$$

$$d = \sqrt{\frac{2.52 \times 10^{-4}}{3.12 \times 10^3}} = 2.201 \times 10^{-5} \text{ m} = 22 \times 10^{-6} \text{ m} = 22\mu\text{m}$$

Thus, the minimum particle size that will be removed with 100% efficiency is $22\,\mu$m.

Quick Tip

When calculating the minimum size of a particle removed with 100% efficiency in a settling chamber, use the formula involving the particle's velocity, chamber dimensions, and the physical properties of air and water. Ensure that all units are consistent and in SI units.

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64. On a two-lane highway, a horizontal curve of radius 300 m is provided. The design speed is 80 km/h.

If the longest wheelbase of vehicle expected on this highway is 7 m, then the extra widening required (in m) is ..... (round off to two decimal places).

Correct Answer: 0.65

Solution:

Given: - Number of lanes, $n = 2$

- Radius of curve, $R = 300\,\text{m}$

- Design speed, $v = 80\,\text{km/h}$

- Longest wheelbase of vehicle, $l = 7\,\text{m}$

As per IRC (Indian Roads Congress) standards, the formula for extra widening W_e is:

$$W_e = W_m + W_{ph}$$

Where:

- $W_m = \frac{nl^2}{2R}$ (widening due to mechanical reasons)

- $W_{ph} = \frac{v}{9.5\sqrt{R}}$ (widening due to phasing effect)

Substitute the values:

$$W_e = \frac{2 \times 7^2}{2 \times 300} + \frac{80}{9.5 \times \sqrt{300}} = \frac{2 \times 49}{600} + \frac{80}{9.5 \times 17.32}$$

$$W_e = 0.1633 + 0.4867 = 0.65\,\text{m}$$

Thus, the extra widening required is 0.65 m.

Quick Tip

When calculating extra widening on a horizontal curve, consider both mechanical widening due to vehicle width and phasing effects due to the design speed. Ensure to use the correct formula as per the IRC standards.

---

65. If the Fore Bearing of the lines AB and BC are 60° and 122°, respectively, then the interior angle $\angle ABC$ (in degrees) is ..... (round off to the nearest integer).

Correct Answer: 118°

Solution:

Given:

- For bearing of line AB, $(\text{FB})_{AB} = 60^\circ$

- For bearing of line BC, $(\text{FB})_{BC} = 122^\circ$

The bearing of line $(\text{BB})_{AB}$ is calculated as:

$$(\text{BB})_{AB} = (\text{FB})_{AB} + 180^\circ = 60^\circ + 180^\circ = 240^\circ$$

The interior angle $\angle ABC$ is given by:

$$\angle ABC = (\text{BB})_{AB} - (\text{FB})_{BC} = 240^\circ - 122^\circ = 118^\circ$$

Thus, the interior angle $\angle ABC$ is 118°.

Quick Tip

To calculate the interior angle between two lines given their fore bearings, use the formula Interior angle $= (\text{BB})_{AB} - (\text{FB})_{BC}$, where the bearing of line AB is adjusted by adding 180° to obtain $(\text{BB})_{AB}$.
