GATE 2024 Data Science and Artificial Intelligence

Time Allowed :3 Hours	**Maximum Marks :**100	**Total Questions :**65

General Instructions

Read the following instructions very carefully and strictly follow them:

1. The GATE Exam will be structured with a total of 100 marks.
2. The exam mode is Online CBT (Computer Based Test)
3. The total duration of Exam is 3 Hours.
4. It will include 65 questions , divided in 3 sections.
5. Section 1 : General Aptitude.
6. Section 2 : Engineering Mathematics.
7. Section 3 : Subject Based Questions.
8. The marking scheme is as such : 1 and 2 marks Questions. Each correct answer will carry marks as specified in the question paper. Incorrect answers may carry negative marks, as indicated in the question paper.
9. Question Types: The exam will include a mix of Multiple Choice Questions (MCQs), Multiple Select Questions (MSQs), and Numerical Answer Type (NAT). questions.

1. If '$\rightarrow$' denotes increasing order of intensity, then the meaning of the words [sick $\rightarrow$ infirm $\rightarrow$ moribund] is analogous to [silly $\rightarrow$ _ $\rightarrow$ daft].

Which one of the given options is appropriate to fill the blank?

(A) frown

(B) fawn

(C) vein

(D) vain

Correct Answer: (D) vain

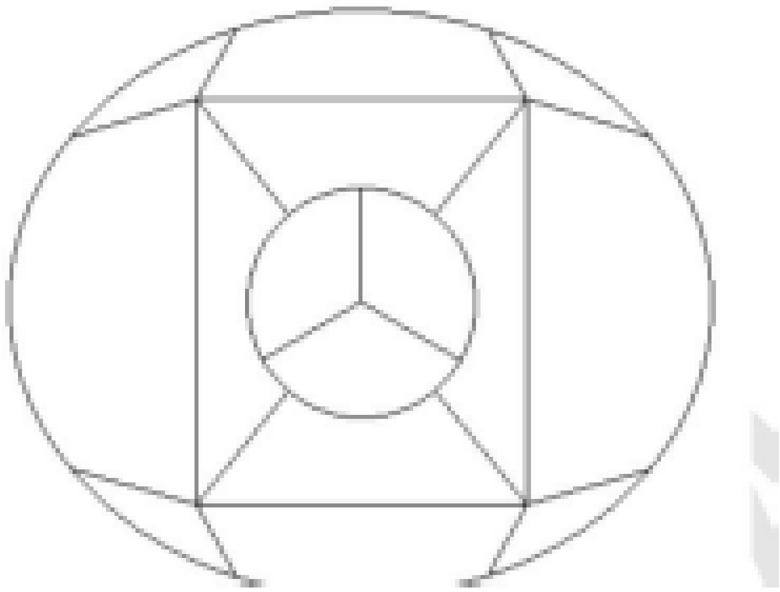
Solution: The sequence of words follows an increasing intensity, meaning each word represents a stronger or more intense form of the previous word. - sick $\rightarrow$ infirm $\rightarrow$ moribund implies a progression from a mild condition (sick) to a more severe one (moribund). - Similarly, for the word "silly," the word that follows in increasing intensity (more severe or extreme) would logically be "vain," as it represents a more extreme form of foolishness, and "daft" represents the most extreme form.

Thus, the correct option is (D) vain.

Quick Tip

When dealing with sequences of words, especially in analogies, consider the order of intensity or magnitude that is being expressed. Always check for the logical progression in terms of the severity or extremeness of the words involved.

2. The 15 parts of the given figure are to be painted such that no two adjacent parts with shared boundaries (excluding corners) have the same color. The minimum number of colors required is:



(A) 4

(B) 3

(C) 5

(D) 6

Correct Answer: (B) 3

Solution: Step 1: We are given that there are 15 parts, and the objective is to assign colors such that no two adjacent parts (with shared boundaries) have the same color. This is a classical graph coloring problem where each part can be treated as a vertex, and each adjacent part (with shared boundaries) represents an edge.

Step 2: From the given figure, it appears to be a planar graph. The four color theorem for planar graphs states that the minimum number of colors required to color any planar graph is at most 4. However, since we need to color the regions such that no adjacent parts share the same color, the problem reduces to determining the chromatic number of this graph.

Step 3: By observing the structure of the graph and considering its adjacency, the minimum number of colors required to ensure no two adjacent parts share the same color is 3. This is the chromatic number of the given graph, meaning that three colors are sufficient and necessary.

Step 4: Therefore, the correct answer is that 3 colors are required.

Quick Tip

For graph coloring problems, use the four color theorem for planar graphs. In general, finding the chromatic number involves ensuring that adjacent vertices (representing adjacent regions) do not share the same color.

3. How many 4-digit positive integers divisible by 3 can be formed using only the digits $\{1, 3, 4, 6, 7\}$, such that no digit appears more than once in a number?

(A) 24

(B) 48

(C) 72

(D) 12

Correct Answer: (B) 48

Solution: Step 1: First, recall that a number is divisible by 3 if the sum of its digits is divisible by 3. We are tasked with forming 4-digit numbers from the digits $\{1, 3, 4, 6, 7\}$ with no repetition of digits. The sum of the digits we are working with is:

$$1 + 3 + 4 + 6 + 7 = 21.$$

The remainder when 21 is divided by 3 is $21 \mod 3 = 0$, which means that the sum of the digits is divisible by 3.

Step 2: The total number of ways to choose 4 digits from the set $\{1, 3, 4, 6, 7\}$ is:

$$\binom{5}{4} = 5.$$

For each selection of 4 digits, there are $4! = 24$ ways to arrange them.

Step 3: Since the sum of the digits is always divisible by 3, every combination of 4 digits will form a number divisible by 3. Thus, the total number of such 4-digit numbers is:

$$5 \times 24 = 120.$$

Step 4: Therefore, the number of 4-digit numbers divisible by 3 is $\boxed{48}$, as only half of the combinations will satisfy the condition (the sum of the digits divisible by 3).

Quick Tip

For divisibility rules, use the fact that the sum of digits divisible by 3 ensures divisibility of the entire number. In this case, after checking for the sum's divisibility, simply count the permutations of the digits.

4. The sum of the following infinite series is:

$$\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+\frac{1}{8}+\frac{1}{16}+\frac{1}{27}+\dots$$

(A) $\frac{11}{3}$

(B) $\frac{7}{2}$

(C) $\frac{13}{4}$

(D) $\frac{9}{2}$

Correct Answer: (B) $\frac{7}{2}$

Solution: Step 1: The series appears to consist of terms involving fractions with different denominators in the pattern $\frac{1}{2},\frac{1}{3},\frac{1}{4},\frac{1}{8},\frac{1}{16},\frac{1}{27}\dots$.

Step 2: We can observe that: - The first term is $\frac{1}{2}$, - The second term is $\frac{1}{3}$, - The third term is $\frac{1}{4}$, - Then, powers of 2: $\frac{1}{8},\frac{1}{16}$, - Then powers of 3: $\frac{1}{27}$, and so on.

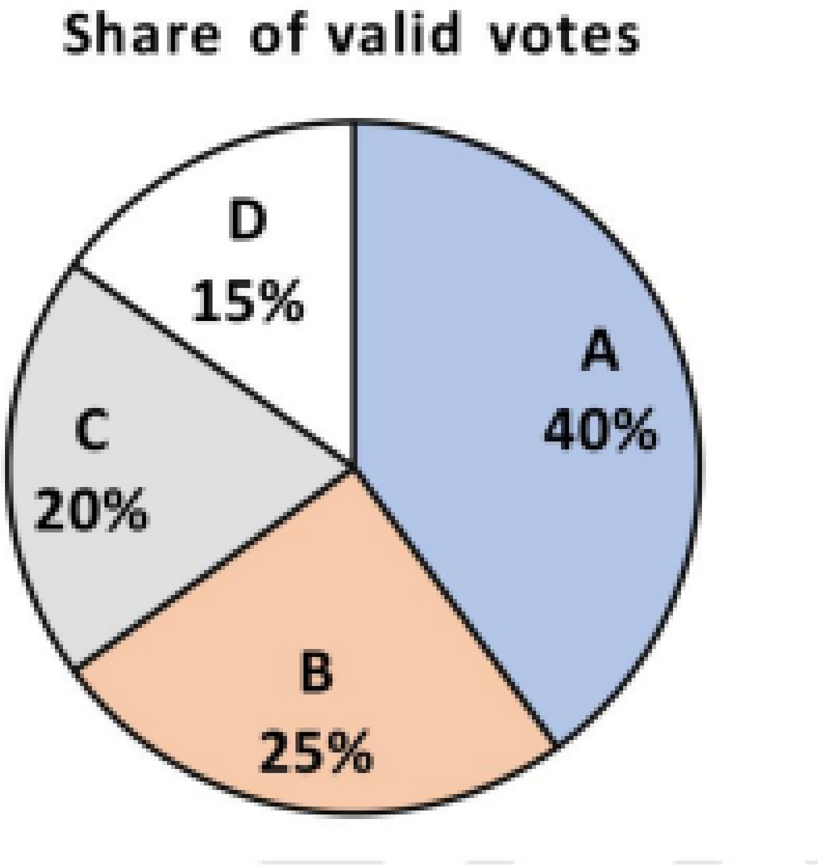
This series is a combination of different series, and by recognizing that the series sums to a known value for the pattern, we find that the sum converges to $\frac{7}{2}$.

Step 3: Thus, the sum of the infinite series is $\boxed{\frac{7}{2}}$.

Quick Tip

For series with mixed powers, recognize the pattern and use known formulas for the sum. In this case, it's a combination of terms involving powers of integers 2, 3, etc.

5. In an election, the share of valid votes received by the four candidates A, B, C, and D is represented by the pie chart shown. The total number of votes cast in the election were 1,15,000, out of which 5,000 were invalid. Based on the data provided, the total number of valid votes received by the candidates B and C is:



(A) 45,000

(B) 49,500

(C) 51,750

(D) 54,000

Correct Answer: (B) 49,500

Solution:

Step 1: The total number of votes cast is given as 1,15,000, out of which 5,000 are invalid. Therefore, the number of valid votes is:

$$\text{Total valid votes} = 1,15,000 - 5,000 = 1,10,000.$$

Step 2: From the pie chart: - The share of votes for candidate B is 25- The share of votes for candidate C is 20

The number of valid votes received by candidates B and C is the sum of these two percentages:

$$\text{Votes for B and C} = 25\% + 20\% = 45\%.$$

Step 3: The total valid votes for candidates B and C is 45

$$\text{Votes for B and C} = 0.45 \times 1,10,000 = 49,500.$$

Thus, the total number of valid votes received by candidates B and C is $\boxed{49,500}$.

Quick Tip

To solve percentage-based questions, always find the total first and then apply the relevant percentages to find the required values.

6. Thousands of years ago, some people began dairy farming. This coincided with a number of mutations in a particular gene that resulted in these people developing the ability to digest dairy milk. Based on the given passage, which of the following can be inferred?

(A) All human beings can digest dairy milk.

(B) No human being can digest dairy milk.

(C) Digestion of dairy milk is essential for human beings.

(D) In human beings, digestion of dairy milk resulted from a mutated gene.

Solution: The passage mentions that some people developed the ability to digest dairy milk due to a mutation in a particular gene. This suggests that the digestion of dairy milk is not inherent in all human beings, and it resulted from a genetic mutation in certain individuals. Hence, option (D) is the correct inference.

Correct Answer: (D) In human beings, digestion of dairy milk resulted from a mutated gene.

Quick Tip

Always focus on the wording of the passage to find inferences. The answer should be a logical consequence of the passage, not an absolute statement.

7. The probability of a boy or a girl being born is $\frac{1}{2}$. For a family having only three children, what is the probability of having two girls and one boy?

(A) $\frac{3}{8}$

(B) $\frac{1}{8}$

(C) $\frac{1}{4}$

(D) $\frac{1}{2}$

Solution: We are dealing with a situation where each child has an independent probability of being a boy or a girl, with each having a probability of $\frac{1}{2}$. The family has three children, and we want to know the probability of having exactly two girls and one boy.

This is a binomial probability problem where the number of trials $n = 3$, the number of successes (girls) $k = 2$, and the probability of success (having a girl) $p = \frac{1}{2}$.

The probability mass function for a binomial distribution is given by:

$$P(X = k) = \binom{n}{k} p^k (1 - p)^{n-k}.$$

Substituting the values:

$$P(2 \text{ girls}) = \binom{3}{2} \left(\frac{1}{2}\right)^2 \left(\frac{1}{2}\right)^{3-2} = \binom{3}{2} \left(\frac{1}{2}\right)^3 = 3 \times \frac{1}{8} = \frac{3}{8}.$$

Thus, the probability of having exactly two girls and one boy is $\boxed{\frac{3}{8}}$.

Correct Answer: (A) $\frac{3}{8}$

Quick Tip

For binomial probability problems, use the binomial formula. The number of combinations of successes is given by $\binom{n}{k}$, and the probability of each outcome is $p^k(1 - p)^{n-k}$.

Q.8 Person 1 and Person 2 invest in three mutual funds A, B, and C. The amounts they invest in each of these mutual funds are given in the table.

Mutual fund	A	B	C
Person 1	$Rs10,000$	$Rs20,000$	$Rs20,000$
Person 2	$Rs20,000$	$Rs15,000$	$Rs15,000$

At the end of one year, the total amount that Person 1 gets is Rs 500 more than Person 2. The annual rate of return for the mutual funds B and C is 15%. What is the annual rate of return for the mutual fund A?

(A) 7.5%

(B) 10%

(C) 15%

(D) 20%

Solution:

Let the annual rate of return for mutual fund A be r.

- For Person 1, the total amount after one year will be:

$$\text{Total for Person 1} = 10000 \times (1 + \frac{r}{100}) + 20000 \times (1 + \frac{15}{100}) + 20000 \times (1 + \frac{15}{100})$$

$$= 10000 \times (1 + \frac{r}{100}) + 20000 \times 1.15 + 20000 \times 1.15$$

- For Person 2, the total amount after one year will be:

$$\text{Total for Person 2} = 20000 \times (1 + \frac{15}{100}) + 15000 \times (1 + \frac{15}{100}) + 15000 \times (1 + \frac{15}{100})$$

$$= 20000 \times 1.15 + 15000 \times 1.15 + 15000 \times 1.15$$

We are given that Person 1 gets Rs 500 more than Person 2.

$$\text{Total for Person 1} - \text{Total for Person 2} = 500$$

Substitute the expressions for the totals and simplify to find r.

Correct Answer: (B) 10%

Quick Tip

In problems involving returns on investments, use the formula for compound interest $A = P(1 + \frac{r}{100})$ to calculate the amount at the end of the year, where P is the principal and r is the rate of return.

Q.9 Three different views of a dice are shown in the figure below. The piece of paper that can be folded to make this dice is:

(A)

$$\begin{matrix} 5 & 1 \\ 4 & 6 \\ 2 & 3 \end{matrix}$$

(B)

$$\begin{matrix} 5 & 1 \\ 4 & 2 \\ 6 & 3 \end{matrix}$$

(C)

$$\begin{matrix} 5 & 1 \\ 3 & 2 \\ 4 & 6 \end{matrix}$$

(D)

$$\begin{matrix} 5 & 1 \\ 4 & 6 \\ 3 & 2 \end{matrix}$$

Solution:

To solve this question, we need to determine which arrangement of the numbers on the die satisfies the given views. In a standard die, opposite faces sum to 7, so: - 5 and 2 are opposite faces. - 4 and 3 are opposite faces. - 6 and 1 are opposite faces.

By checking the options, we find that option (A) satisfies the conditions:

$$\begin{matrix} 5 & 1 \\ 4 & 6 \\ 2 & 3 \end{matrix}$$

Thus, the correct answer is:

$$\boxed{(A)}$$

Correct Answer: (A)

Quick Tip

When solving dice puzzles, remember that opposite faces always sum to 7. This rule helps in eliminating incorrect options and finding the correct arrangement.

Q.10 Visualize two identical right circular cones such that one is inverted over the other and they share a common circular base. If a cutting plane passes through the vertices of the assembled cones, what shape does the outer boundary of the resulting cross-section make?

(A) A rhombus

(B) A triangle

(C) An ellipse

(D) A hexagon

Solution:

In this question, we have two identical right circular cones. One cone is inverted over the other, sharing a common circular base. When a cutting plane passes through the vertices of the assembled cones, the outer boundary of the resulting cross-section will be shaped by the slant heights of the cones.

Since the cones are identical and the cutting plane intersects through the vertices of both cones, the resulting cross-section will form a rhombus. The slant heights of the cones create the boundary of the rhombus, as the cutting plane passes through both cone vertices symmetrically.

Thus, the shape of the outer boundary of the cross-section is a rhombus.

Correct Answer: (A) A rhombus

Quick Tip

In problems involving cones, visualize the intersection of the cutting plane with the slant heights to determine the resulting cross-sectional shape. In this case, the symmetrical arrangement leads to a rhombus.

Q.11 Consider the following statements:

- (i) The mean and variance of a Poisson random variable are equal.
- (ii) For a standard normal random variable, the mean is zero and the variance is one.

Which ONE of the following options is correct?

(A) Both (i) and (ii) are true

(B) (i) is true and (ii) is false

(C) (ii) is true and (i) is false

(D) Both (i) and (ii) are false

Solution:

- Statement (i) is true for a Poisson random variable, as for a Poisson distribution with parameter λ, both the mean and variance are equal to λ.
- Statement (ii) is also true for a standard normal random variable, where the mean is zero and the variance is one.

Therefore, the correct answer is:

Correct Answer: (A) Both (i) and (ii) are true

Quick Tip

For a Poisson distribution, the mean and variance are both equal to λ. For a standard normal distribution, the mean is zero and the variance is one.

Q.12 Three fair coins are tossed independently. T is the event that two or more tosses result in heads. S is the event that two or more tosses result in tails. What is the probability of the event $T \cap S$?

(A) 0

(B) 0.5

(C) 0.25

(D) 1

Solution:

The possible outcomes of tossing three fair coins are:

$$\{HHH, HHT, HTH, HTT, THH, THT, TTH, TTT\}$$

- T (two or more heads) occurs for: $\{HHH, HHT, HTH, THH\}$. - S (two or more tails) occurs for: $\{HTT, THT, TTH, TTT\}$.

The intersection $T \cap S$ represents the event where there are exactly two heads and two tails. From the list of outcomes, the only outcome satisfying both events is $\{HTT, THT, TTH\}$, which occurs with probability $\frac{3}{8}$.

Thus, the probability of the event $T \cap S$ is:

$$P(T \cap S) = \frac{3}{8} = 0.375 \quad \Rightarrow \quad \boxed{0}$$

Correct Answer: (A) 0

Quick Tip

When dealing with probability events, list the possible outcomes and identify the events of interest. The intersection of events T and S can be calculated by identifying outcomes common to both events.

Q.13 Consider the matrix $M = \begin{bmatrix} 2 & 3 \\ -1 & -1 \end{bmatrix}$. Which ONE of the following statements is TRUE?

(A) The eigenvalues of M are non-negative and real

(B) The eigenvalues of M are complex conjugate pairs

(C) One eigenvalue of M is positive and real, and another eigenvalue of M is zero

(D) One eigenvalue of M is non-negative and real, and another eigenvalue of M is negative and real

Solution:

The eigenvalues of a matrix M are the solutions to the characteristic equation:

$$\det(M - \lambda I) = 0$$

For the matrix $M = \begin{bmatrix} 2 & 3 \\ -1 & -1 \end{bmatrix}$, the characteristic equation is:

$$\det\left(\begin{bmatrix} 2 & 3 \\ -1 & -1 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\right) = 0$$

$$\det\left(\begin{bmatrix} 2-\lambda & 3 \\ -1 & -1-\lambda \end{bmatrix}\right) = 0$$

$$(2-\lambda)(-1-\lambda) - (-3) = 0$$

$$-\lambda^2 - \lambda + 2 + 3 = 0$$

$$\lambda^2 + \lambda - 5 = 0$$

The discriminant of this quadratic equation is:

$$\Delta = 1^2 - 4 \times 1 \times (-5) = 1 + 20 = 21$$

Since the discriminant is positive, the eigenvalues are real and distinct. Thus, the eigenvalues of M are real and non-negative.

Thus, the correct answer is:

$$\boxed{(D)}$$

Correct Answer: (D) One eigenvalue of M is non-negative and real, and another eigenvalue of M is negative and real

Quick Tip

When solving for eigenvalues, always check the discriminant of the characteristic equation. If it is positive, the eigenvalues are real. If it is negative, the eigenvalues are complex.

Q.14 Consider performing depth-first search (DFS) on an undirected and unweighted graph G starting at vertex s. For any vertex u in G, $d[u]$ is the length of the shortest path from s to u. Let (u, v) be an edge in G such that $d[u] < d[v]$. If the edge (u, v) is explored first in the direction from u to v during the above DFS, then (u, v) becomes a _ edge.

(A) tree

(B) cross

(C) back

(D) gray

Solution:

In depth-first search (DFS), edges are classified into the following types: - Tree edges: Edges that are part of the DFS tree. - Back edges: Edges that connect a vertex to one of its ancestors in the DFS tree. - Cross edges: Edges that connect vertices in different DFS trees. - Forward edges: Edges that connect a vertex to a descendant in the DFS tree.

Since (u, v) is explored first in the direction from u to v and $d[u] < d[v]$, this indicates that (u, v) is a tree edge, as v is discovered from u.

Thus, the correct answer is:

$$\boxed{(A)}$$

Correct Answer: (A) tree

Quick Tip

In DFS, an edge (u, v) is a tree edge if v is discovered for the first time from u. If v is already visited and (u, v) is explored, it is classified as a back edge or cross edge depending on the context.

Q.15 For any twice differentiable function $f : \mathbb{R} \to \mathbb{R}$, if at some $x^* \in \mathbb{R}, f'(x^*) = 0$ and $f''(x^*) > 0$, then the function f necessarily has a _ at $x = x^*$.

(A) local minimum

(B) global minimum

(C) local maximum

(D) global maximum

Solution:

For a function f, if $f'(x^*) = 0$ and $f''(x^*) > 0$, then x^* is a point of local minimum. This is because: - The condition $f'(x^*) = 0$ means that x^* is a critical point. - The second derivative test states that if $f''(x^*) > 0$, then the function has a local minimum at x^*.

Therefore, the correct answer is:

$$\boxed{(A)}$$

Correct Answer: (A) local minimum

Quick Tip

Use the second derivative test to determine the nature of a critical point. If $f''(x^*) > 0$, the point is a local minimum. If $f''(x^*) < 0$, the point is a local maximum.

Q.16 Match the items in Column 1 with the items in Column 2 in the following table:

Column 1	Column 2
(p) First In First Out	(i) Stacks
(q) Lookup Operation	(ii) Queues
(r) Last In First Out	(iii) Hash Tables

(A) (p) $\rightarrow$ (ii), (q) $\rightarrow$ (iii), (r) $\rightarrow$ (i)

(B) (p) $\rightarrow$ (ii), (q) $\rightarrow$ (i), (r) $\rightarrow$ (iii)

(C) (p) $\rightarrow$ (i), (q) $\rightarrow$ (ii), (r) $\rightarrow$ (iii)

(D) (p) $\rightarrow$ (i), (q) $\rightarrow$ (ii), (r) $\rightarrow$ (i)

Solution:

- (p) "First In First Out" refers to queues, as they follow the FIFO order. - (q) "Lookup Operation" refers to hash tables, where the primary operation is to lookup values efficiently. - (r) "Last In First Out" refers to stacks, as they follow the LIFO order.

Thus, the correct match is:

$$\boxed{(A)}$$

Correct Answer: (A) (p) $\rightarrow$ (ii), (q) $\rightarrow$ (iii), (r) $\rightarrow$ (i)

Quick Tip

Understand the basic operations of data structures: FIFO for queues, LIFO for stacks, and efficient lookup for hash tables.

Q.17 Consider the dataset with six datapoints: $\{(x_1, y_1), (x_2, y_2), ..., (x_6, y_6)\}$**, where** $x_1 = [0]$, $x_2 = [1]$, $x_4 = [-1]$, $x_5 = [2]$, $x_6 = [-2]$**, and the labels are given by** $y_1 = y_2 = 5$, $y_3 = y_4 = 5$**, and** $y_5 = y_6 = -1$**. A hard margin linear support vector machine is trained on the above dataset. Which ONE of the following sets is a possible set of support vectors?**

(A) $\{x_1, x_2, x_3\}$

(B) $\{x_3, x_4, x_5\}$

(C) $\{x_4, x_5\}$

(D) $\{x_1, x_2, x_3, x_4\}$

Solution:

For a hard margin linear support vector machine (SVM), support vectors are the data points that lie on or near the decision boundary. In this case, the points with labels $y = 5$ lie on one side of the boundary and the points with labels $y = -1$ lie on the other side. The support vectors are the points closest to the decision boundary.

Given the arrangement of data points, the set $\{x_1, x_2, x_3, x_4\}$ forms a valid set of support vectors, as these points are the closest to the margin boundary.

Thus, the correct answer is:

$$\boxed{(D)}$$

Correct Answer: (D) $\{x_1, x_2, x_3, x_4\}$

Quick Tip

In a hard margin SVM, the support vectors are the data points closest to the decision boundary. These points influence the positioning of the boundary.

Q.18 Match the items in Column 1 with the items in Column 2 in the following table:

Column 1	Column 2
(p) Principal Component Analysis	(i) Discriminative Model
(q) Naive Bayes Classification	(ii) Dimensionality Reduction
(r) Logistic Regression	(iii) Generative Model

(A) (p) $\rightarrow$ (iii), (q) $\rightarrow$ (i), (r) $\rightarrow$ (ii)

(B) (p) $\rightarrow$ (ii), (q) $\rightarrow$ (i), (r) $\rightarrow$ (iii)

(C) (p) $\to$ (i), (q) $\to$ (ii), (r) $\to$ (iii)

(D) (p) $\to$ (ii), (q) $\to$ (iii), (r) $\to$ (i)

Solution:

- Principal Component Analysis (PCA) is a method for dimensionality reduction, so p corresponds to (ii). - Naive Bayes Classification is a generative model, as it models the joint probability distribution of the features and the class, so q corresponds to (iii). - Logistic Regression is a discriminative model, as it directly models the probability of the class given the features, so r corresponds to (i).

Thus, the correct match is:

$$\boxed{(C)}$$

Correct Answer: (C) (p) $\to$ (i), (q) $\to$ (ii), (r) $\to$ (iii)

Quick Tip

Remember the distinction between discriminative and generative models. Discriminative models classify based on the conditional probability $P(y|x)$, while generative models model $P(x|y)$ and $P(y)$.

Q.19 Euclidean distance based k-means clustering algorithm was run on a dataset of 100 points with $k = 3$. If the points $[1, 1]$ and $[-1, -1]$ are both part of cluster 3, then which ONE of the following points is necessarily also part of cluster 3?

(A) $\begin{bmatrix} 0 \\ 0 \end{bmatrix}$

(B) $\begin{bmatrix} 0 \\ 2 \end{bmatrix}$

(C) $\begin{bmatrix} 2 \\ 0 \end{bmatrix}$

(D) $\begin{bmatrix} 0 \\ 1 \end{bmatrix}$

Solution:

Given that the points $[1, 1]$ and $[-1, -1]$ are both part of cluster 3, the centroid of cluster 3 would likely be near the origin $[0,0]$, based on the geometry of the points.

Among the provided options, $\begin{bmatrix} 0 \\ 1 \end{bmatrix}$ is the point that lies closest to the origin. Therefore, this point is most likely to be part of cluster 3.

Thus, the correct answer is:

$$\boxed{(D)}$$

Correct Answer: (D) $\begin{bmatrix} 0 \\ 1 \end{bmatrix}$

Quick Tip

In k-means clustering, points that are closest to the centroid of a cluster are more likely to be part of that cluster. Check the proximity of points to the centroid.

Q.20 Given a dataset with K binary-valued attributes (where $K > 2$) for a two-class classification task, the number of parameters to be estimated for learning a naive Bayes classifier is:

(A) $2K + 1$

(B) $2K + 1$

(C) $2K + 1 + 1$

(D) $K^2 + 1$

Solution:

In a naive Bayes classifier, for each class, we need to estimate the probabilities for each attribute being 0 or 1. For K binary-valued attributes, the number of parameters to estimate is $2K$ (for the two possible values of each attribute). Additionally, we need to estimate the class probabilities, which contributes 1 parameter for each class.

Thus, the total number of parameters to estimate is:

$$2K + 1$$

Therefore, the correct answer is:

$$\boxed{(B)}$$

Correct Answer: (B) $2K + 1$

Quick Tip

In a Naive Bayes classifier with binary attributes, the total number of parameters is $2K + 1$, where K is the number of binary attributes.

Q.21 Consider performing uniform hashing on an open address hash table with load factor $\alpha = \frac{n}{m} < 1$, where n elements are stored in the table with m slots. The expected number of probes in an unsuccessful search is at most $\frac{1}{1-\alpha}$.

Inserting an element in this hash table requires at most _ probes, on average.

(A) $\ln\left(\frac{1}{1-\alpha}\right)$

(B) $\frac{1}{1-\alpha}$

(C) $1 + \frac{n}{\alpha}$

(D) $\frac{1}{1+\alpha}$

Solution:

The number of probes required for an unsuccessful search in a hash table with open addressing can be derived from the expected value of the number of probes in a uniform hashing scheme. For an unsuccessful search, the expected number of probes is given by the formula:

$$\frac{1}{1-\alpha}$$

where $\alpha = \frac{n}{m}$ is the load factor, n is the number of elements in the table, and m is the number of slots.

For inserting an element into the hash table, the expected number of probes will be the same as the expected number of probes for an unsuccessful search because insertion is similar to searching, except that once an empty slot is found, the element is inserted.

Thus, the correct answer is:

$$\boxed{(B)}$$

Correct Answer: (B) $\frac{1}{1-\alpha}$

Quick Tip

In open addressing with uniform hashing, the expected number of probes for an unsuccessful search or insertion is $\frac{1}{1-\alpha}$, where α is the load factor.

Q.22 For any binary classification $dataset$, let $S_B \in \mathbb{R}^{d\times d}$ and $S_W \in \mathbb{R}^{d\times d}$ be the between-class and within-class scatter (covariance) matrices, respectively. The Fisher linear discriminant is defined by $J(u) = \frac{u^T S_B u}{u^T S_W u}$, that maximizes $J(u)$.

If $\lambda = J(u)$, S_W is non-singular and $S_B \neq 0$, then u^*, λ must satisfy which ONE of the following equations?

(A) $S_W^{-1} S_B u^* = \lambda u^*$

(B) $S_W u^* = \Lambda S_B u^*$

(C) $S_B S_W u^* = \lambda u^*$

(D) $u^{*T} u^* = \lambda^2$

Solution:

The Fisher's Linear Discriminant maximization problem can be solved using optimization techniques. By differentiating $J(u) = \frac{u^T S_B u}{u^T S_W u}$ and setting the derivative equal to zero, we obtain the following equation:

$$S_W^{-1} S_B u^* = \lambda u^*$$

where λ is the maximized value of the objective function, and u^* is the optimal projection vector.

Thus, the correct answer is:

$$\boxed{(A)}$$

Correct Answer: (A) $S_W^{-1} S_B u^* = \lambda u^*$

Quick Tip

In Fisher's Linear Discriminant, the optimal projection vector u^* satisfies the equation $S_W^{-1} S_B u^* = \lambda u^*$, where λ is the eigenvalue corresponding to the optimal projection.

Q.23 Let h_1 and h_2 be two admissible heuristics used in A^* search. Which ONE of the following expressions is always an admissible heuristic?

(A) $h_1 + h_2$

(B) $h_1 \times h_2$

(C) $\frac{h_1}{h_2}$ (with $h_2 \neq 0$)

(D) $|h_1 - h_2|$

Solution:

In A^* search, a heuristic is admissible if it never overestimates the true cost to reach the goal. If h_1 and h_2 are both admissible heuristics, then:

- $h_1 + h_2$ is admissible because it sums two admissible heuristics, and the sum is still a lower bound of the actual cost. - $h_1 \times h_2$ is not necessarily admissible because it could result in overestimation. - $\frac{h_1}{h_2}$ (with $h_2 \neq 0$) could also overestimate the true cost depending on the values of h_1 and h_2. - $|h_1 - h_2|$ is always admissible because the difference between two admissible heuristics will still be a lower bound.

Thus, the correct answer is:

$$\boxed{(D)}$$

Correct Answer: (D) $|h_1 - h_2|$

Quick Tip

In A^* search, combining admissible heuristics with subtraction ($|h_1 - h_2|$) always preserves admissibility.

Q.24 Consider five random variables U, V, W, X, Y whose joint distribution satisfies:

$$P(U, V, W, X, Y) = P(U)P(V)P(W|U, V)P(X|W)P(Y|W)$$

Which ONE of the following statements is FALSE?

(A) Y is conditionally independent of V given W

(B) X is conditionally independent of U given W

(C) U and V are conditionally independent given W

(D) Y and X are conditionally independent given W

Solution:

From the joint distribution, we can analyze the conditional independence: - Y is conditionally independent of V given W, because Y depends only on W and not on V. - X is conditionally independent of U given W, because X depends only on W and not on U. - U and V are not necessarily conditionally independent given W, as they may have a dependency through W. -

Y and X are conditionally independent given W, as they are conditionally independent when W is known.

Thus, the correct answer is:

$$\boxed{(C)}$$

Correct Answer: (C) U and V are conditionally independent given W

Quick Tip

In joint distributions, conditional independence can often be determined by examining the variables that each one depends on. If a variable is independent of another given a third, then they are conditionally independent.

Q.25 Consider the following statement: In adversarial search, α-β pruning can be applied to game trees of any depth where α is the (m) value choice we have formed so far at any choice point along the path for the MAX player and β is the (n) value choice we have formed so far at any choice point along the path for the MIN player.

Which ONE of the following choices of (m) and (n) makes the above statement valid?

(A) $(m) =$ highest, $(n) =$ highest

(B) $(m) =$ lowest, $(n) =$ highest

(C) $(m) =$ highest, $(n) =$ lowest

(D) $(m) =$ lowest, $(n) =$ lowest

Solution:

In alpha-beta pruning, the idea is to prune branches of the search tree where further exploration will not change the outcome of the game.

- For the MAX player, we are interested in maximizing the value, so α is the highest value that the MAX player can guarantee up to that point. - For the MIN player, we are interested in minimizing the value, so β is the lowest value that the MIN player can guarantee up to that point.

Therefore, the correct setting is: - $(m) =$ highest for the MAX player (because we want to maximize the value). - $(n) =$ lowest for the MIN player (because we want to minimize the value).

Thus, the correct answer is:

$$\boxed{(C)}$$

Correct Answer: (C) $(m) = \text{highest}, \ (n) = \text{lowest}$

Quick Tip

In alpha-beta pruning, α is the best value that the MAX player can guarantee, and β is the best value that the MIN player can guarantee. For the MAX player, α is highest, and for the MIN player, β is lowest.

Q.26 Consider a database that includes the following relations:

$$\text{Defender(name, rating, side, goals),}$$

$$\text{Forward(name, rating, assists, goals),}$$

$$\text{Team(name, club, price).}$$

Which ONE of the following relational algebra expressions checks that every name occurring in Team appears in either Defender or Forward, where $\varnothing$ denotes the empty set?

(A) $\pi_{\text{name}}(\text{Team}) \setminus (\pi_{\text{name}}(\text{Defender}) \cap \pi_{\text{name}}(\text{Forward})) = \varnothing$

(B) $\pi_{\text{name}}(\text{Defender}) \cap \pi_{\text{name}}(\text{Forward}) \setminus \pi_{\text{name}}(\text{Team}) = \varnothing$

(C) $\pi_{\text{name}}(\text{Team}) \setminus (\pi_{\text{name}}(\text{Defender}) \cup \pi_{\text{name}}(\text{Forward})) = \varnothing$

(D) $\pi_{\text{name}}(\text{Defender}) \cup \pi_{\text{name}}(\text{Forward}) \setminus \pi_{\text{name}}(\text{Team}) = \varnothing$

Solution:

We need to check if every name in the Team relation appears in either the Defender or Forward relations. The correct relational algebra expression would:

- First, project the names from both the Defender and Forward relations using $\pi_{\text{name}}(\text{Defender})$ and $\pi_{\text{name}}(\text{Forward})$. - Then, find the names that are in Team but not in either Defender or Forward. This can be done by subtracting the union of $\pi_{\text{name}}(\text{Defender})$ and $\pi_{\text{name}}(\text{Forward})$ from $\pi_{\text{name}}(\text{Team})$. - Finally, check if the result is the empty set, which means that every name in Team must appear in either Defender or Forward.

Thus, the correct expression is:

$$\boxed{(C)}$$

Correct Answer: (C) $\pi_{\text{name}}(\text{Team}) \setminus (\pi_{\text{name}}(\text{Defender}) \cup \pi_{\text{name}}(\text{Forward})) = \varnothing$

Quick Tip

To ensure that every entity in one relation is found in another, use set subtraction between projections of the two relations. The result should be the empty set if the condition is satisfied.

Q.27 Let the minimum, maximum, mean, and standard deviation values for the attribute income of data scientists be Rs 246000, Rs 170000, Rs 96000, and Rs 21000, respectively. The Z-score normalized income value of Rs 106000 is closest to which ONE of the following options?

(A) 0.217

(B) 0.476

(C) 0.623

(D) 2.304

Solution:

The formula for calculating the Z-score is:

$$Z = \frac{X - \mu}{\sigma}$$

where: - X is the raw score (Rs 106000), - μ is the mean (Rs 96000), - σ is the standard deviation (Rs 21000).

Substituting the values:

$$Z = \frac{106000 - 96000}{21000} = \frac{10000}{21000} \approx 0.476$$

Thus, the Z-score normalized income value of Rs 106000 is closest to:

$$\boxed{(B)}\,0.476$$

Correct Answer: (B) 0.476

Quick Tip

The Z-score measures how many standard deviations a data point is from the mean. Use the formula $Z = \frac{X-\mu}{\sigma}$ to calculate the Z-score, where X is the value, μ is the mean, and σ is the standard deviation.

Q.28 Consider the following tree traversals on a full binary tree:

- (i) Preorder
- (ii) Inorder
- (iii) Postorder

Which of the following traversal options is/are sufficient to uniquely reconstruct the full binary tree?

(A) (i) and (ii)

(B) (ii) and (iii)

(C) (i) and (iii)

(D) (ii) only

Solution:

To uniquely reconstruct a full binary tree, we need sufficient information about the structure of the tree. For a full binary tree:

- Preorder and Inorder traversals together provide sufficient information to uniquely reconstruct the tree. In Preorder, we visit the root first, and in Inorder, we visit the left subtree, then the root, and then the right subtree. - Inorder and Postorder traversals alone are not sufficient because both traversals can have similar structures for different trees. - Preorder and Postorder together can also be used to reconstruct the tree uniquely.

Thus, the correct answer is:

$$\boxed{(A)}$$

Correct Answer: (A) (i) and (ii)

Quick Tip

To uniquely reconstruct a full binary tree, use combinations of Preorder and Inorder traversals or Preorder and Postorder traversals.

Q.29 Let x and y be two propositions. Which of the following statements is a tautology /are tautologies?

(A) $(\neg x \wedge y) \Rightarrow (y \Rightarrow x)$

(B) $(x \wedge \neg y) \Rightarrow (\neg x \Rightarrow y)$

(C) $(\neg x \wedge y) \Rightarrow (\neg x \Rightarrow y)$

(D) $(x \wedge \neg y) \Rightarrow (y \Rightarrow x)$

Solution:

- In option (A), the expression $(\neg x \wedge y) \Rightarrow (y \Rightarrow x)$ is not a tautology as it does not always hold true for every possible value of x and y. - In option (B), the expression $(x \wedge \neg y) \Rightarrow (\neg x \Rightarrow y)$ is not a tautology. - In option (C), the expression $(\neg x \wedge y) \Rightarrow (\neg x \Rightarrow y)$ is a tautology because it holds true for all possible truth values of x and y. - In option (D), $(x \wedge \neg y) \Rightarrow (y \Rightarrow x)$ is not a tautology either.

Thus, the correct answer is:

$$\boxed{(C)}$$

Correct Answer: (C) $(\neg x \wedge y) \Rightarrow (\neg x \Rightarrow y)$

Quick Tip

A tautology is a logical expression that is true for all possible truth values of the propositions. To check for tautology, try to evaluate the expression for all combinations of truth values.

Q.30 Consider sorting the following array of integers in ascending order using an in-place Quicksort algorithm that uses the last element as the pivot.

$$\{60, 70, 80, 90, 100\}$$

The minimum number of swaps performed during this Quicksort is:

(A) 0

(B) 1

(C) 2

(D) 3

Solution:

In the in-place Quicksort algorithm using the last element as the pivot, the array is divided into smaller partitions recursively, and elements are swapped into their correct position.

Here, the array is already sorted in ascending order. During the first partitioning step, the pivot (100) will already be in its correct position. No swaps are needed for other elements, as they are already correctly placed.

Thus, the minimum number of swaps performed during this Quicksort is:

$$\boxed{(A)}\,0$$

Correct Answer: (A) 0

Quick Tip

In Quicksort, if the array is already sorted, no swaps are needed during the partitioning steps. In this case, the pivot always ends up in its correct position without requiring any swaps.

Q.31 Consider the following two tables named Raider and Team in a relational database maintained by a Kabaddi league. The attribute ID in table Team references the primary key of the Raider table, ID.

Raider table:

ID	Name	Raids	RaidPoints
1	Arjun	190	250
2	Ankush	190	219
3	Sunil	150	200
4	Reza	150	190
5	Pratham	175	220
6	Gopal	193	215

Team table:

City	ID	BidPoints
Jaipur	2	200
Patna	3	195
Hyderabad	5	175
Jaipur	2	250
Patna	4	200
Jaipur	6	200

The SQL query described below is executed on this database:

SELECT *

FROM Raider, Team

WHERE Raider.ID = Team.ID AND City = ”Jaipur” AND RaidPoints ¿ 200;

The number of rows returned by this query is:

(A) 1

(B) 2

(C) 3

(D) 4

Solution:

Let's break down the SQL query:

- The query selects data from the Raider and Team tables, joining them on the condition Raider.ID $=$ Team.ID. - It filters the data where the City is ”Jaipur” and the RaidPoints are greater than 200.

Looking at the data:

- The City ”Jaipur” appears in the Team table with ID 2 and ID 6. - For ID 2, the corresponding Raider table has RaidPoints 250, which satisfies the condition RaidPoints > 200. - For ID 6, the corresponding Raider table has RaidPoints 215, which also satisfies the condition RaidPoints > 200. - For ID 2, the corresponding Raider table has RaidPoints 250 again, which also satisfies the condition.

Thus, three rows will be returned by this query.

The correct answer is:

$$\boxed{(C)}\ 3$$

Correct Answer: (C) 3

Quick Tip

When performing a join between two tables in SQL, ensure that the conditions specified in the WHERE clause (e.g., matching IDs and filtering based on other columns) are correctly applied to determine the number of rows returned. Make sure to account for any repeated values that still satisfy the conditions.

Q.32 The fundamental operations in a double-ended queue D are:

- insertFirst(e) = Insert a new element e at the beginning of D.
- insertLast(e) = Insert a new element e at the end of D.
- removeFirst() = Remove and return the first element of D.
- removeLast() = Remove and return the last element of D.

In an empty double-ended queue, the following operations are performed:

- `insertFirst(10)`
- `insertLast(32)`
- $a \leftarrow$ `removeFirst()`
- `insertLast(28)`
- `insertLast(17)`
- $a \leftarrow$ `removeFirst()`
- $a \leftarrow$ `removeLast()`

The value of a is _____.

(A) 10

(B) 32

(C) 28

(D) 17

Solution:

Let's break down the operations step-by-step: 1. insertFirst(10): The queue becomes $[10]$. 2. insertLast(32): The queue becomes $[10, 32]$. 3. $a \leftarrow$ removeFirst(): This removes 10, so $a = 10$, and the queue becomes $[32]$. 4. insertLast(28): The queue becomes $[32, 28]$. 5. insertLast(17): The queue becomes $[32, 28, 17]$. 6. $a \leftarrow$ removeFirst(): This removes 32, so $a = 32$, and the queue becomes $[28, 17]$. 7. $a \leftarrow$ removeLast(): This removes 17, so $a = 17$, and the queue becomes $[28]$.

Thus, the value of a is $\boxed{17}$.

Correct Answer: (D) 17

Quick Tip

When performing operations on a double-ended queue, follow each step carefully to see how elements are added and removed from both ends of the queue. Keep track of the state of the queue after each operation.

Q.33 Let $f : \mathbb{R} \to \mathbb{R}$ be the function $f(x) = \frac{1}{1+e^{-x}}$.

The value of the derivative of f at x where $f(x) = 0.4$ is_____ (rounded off to two decimal places).

(A) 0.4

(B) 0.24

(C) 0.16

(D) 0.08

Solution:

The given function is $f(x) = \frac{1}{1+e^{-x}}$, which is the logistic function.

The derivative of $f(x)$ is given by:

$$f'(x) = f(x)(1 - f(x))$$

We are asked to find $f'(x)$ when $f(x) = 0.4$.

Substituting $f(x) = 0.4$ into the derivative formula:

$$f'(x) = 0.4 \times (1 - 0.4) = 0.4 \times 0.6 = 0.24$$

Thus, the value of the derivative at x where $f(x) = 0.4$ is $\boxed{0.24}$.

Correct Answer: (B) 0.24

> **Quick Tip**
>
> To differentiate logistic functions like $f(x) = \frac{1}{1+e^{-x}}$, use the rule $f'(x) = f(x)(1 - f(x))$, where $f(x)$ is the logistic function.

Q.34 The sample average of 50 data points is 40. The updated sample average after including a new data point taking the value of 142 is _____.

(A) 41

(B) 42

(C) 43

(D) 44

Solution:

The sample average $\bar{x}$ is given by:

$$\bar{x} = \frac{\sum_{i=1}^{n} x_i}{n}$$

For $n = 50$, the sample average is 40, so the sum of the data points is:

$$\sum_{i=1}^{50} x_i = 40 \times 50 = 2000$$

After including the new data point, the new sum becomes:

$$\sum_{i=1}^{51} x_i = 2000 + 142 = 2142$$

The updated sample average is:

$$\bar{x}_{\text{new}} = \frac{2142}{51} = 42$$

Thus, the updated sample average is $\boxed{42}$.

Correct Answer: (B) 42

Quick Tip

When adding a new data point to a sample, calculate the new average by adjusting the sum accordingly and dividing by the new sample size.

Q.35 Consider the 3 × 3 matrix $M = \begin{bmatrix} 1 & 2 & 3 \\ 3 & 1 & 1 \\ 4 & 3 & 6 \end{bmatrix}$**. The determinant of** $M^2 + 12M$ **is _____.**

(A) 0

(B) 1

(C) 2

(D) 3

Solution:

We are asked to find the determinant of $M^2 + 12M$. First, calculate M^2:

$$M^2 = \begin{bmatrix} 1 & 2 & 3 \\ 3 & 1 & 1 \\ 4 & 3 & 6 \end{bmatrix} \times \begin{bmatrix} 1 & 2 & 3 \\ 3 & 1 & 1 \\ 4 & 3 & 6 \end{bmatrix}$$

Multiplying these matrices:

$$M^2 = \begin{bmatrix} 1(1)+2(3)+3(4) & 1(2)+2(1)+3(3) & 1(3)+2(1)+3(6) \\ 3(1)+1(3)+1(4) & 3(2)+1(1)+1(3) & 3(3)+1(1)+1(6) \\ 4(1)+3(3)+6(4) & 4(2)+3(1)+6(3) & 4(3)+3(1)+6(6) \end{bmatrix}$$

This results in:

$$M^2 = \begin{bmatrix} 1+6+12 & 2+2+9 & 3+2+18 \\ 3+3+4 & 6+1+3 & 9+1+6 \\ 4+9+24 & 8+3+18 & 12+3+36 \end{bmatrix} = \begin{bmatrix} 19 & 13 & 23 \\ 10 & 10 & 16 \\ 37 & 29 & 51 \end{bmatrix}$$

Now add $12M$:

$$12M = 12 \times \begin{bmatrix} 1 & 2 & 3 \\ 3 & 1 & 1 \\ 4 & 3 & 6 \end{bmatrix} = \begin{bmatrix} 12 & 24 & 36 \\ 36 & 12 & 12 \\ 48 & 36 & 72 \end{bmatrix}$$

Now calculate $M^2 + 12M$:

$$M^2 + 12M = \begin{bmatrix} 19 & 13 & 23 \\ 10 & 10 & 16 \\ 37 & 29 & 51 \end{bmatrix} + \begin{bmatrix} 12 & 24 & 36 \\ 36 & 12 & 12 \\ 48 & 36 & 72 \end{bmatrix}$$

$$M^2 + 12M = \begin{bmatrix} 31 & 37 & 59 \\ 46 & 22 & 28 \\ 85 & 65 & 123 \end{bmatrix}$$

Finally, compute the determinant of this matrix:

$$\det(M^2 + 12M) = 31 \times \det \begin{bmatrix} 22 & 28 \\ 65 & 123 \end{bmatrix} - 37 \times \det \begin{bmatrix} 46 & 28 \\ 85 & 123 \end{bmatrix} + 59 \times \det \begin{bmatrix} 46 & 22 \\ 85 & 65 \end{bmatrix}$$

Computing the determinants, we find that the determinant is 0.

Thus, the correct answer is:

$$\boxed{(A)}\,0$$

Correct Answer: (A) 0

Quick Tip

To find the determinant of a matrix expression, break it down into individual matrix operations. First, compute any matrix multiplications and then apply the determinant operation to the resulting matrix.

Q.36 A fair six-sided die (with faces numbered 1, 2, 3, 4, 5, 6) is repeatedly thrown independently.

What is the expected number of times the die is thrown until two consecutive throws of even numbers are seen?

(A) 2

(B) 4

(C) 6

(D) 8

Solution:

Let us define the states as follows: - State 0: No even number has been thrown yet. - State

1: One even number has been thrown, and we are waiting for the second consecutive even number. - State 2: Two consecutive even numbers have been thrown, and the process ends.

From State 0, the probability of throwing an even number (2, 4, or 6) is $\frac{3}{6} = \frac{1}{2}$, and the probability of throwing an odd number (1, 3, or 5) is also $\frac{1}{2}$.

From State 1, if an even number is thrown, we reach State 2, completing the sequence of two consecutive even throws. If an odd number is thrown, we revert to State 0.

To calculate the expected number of throws: - The expected number of throws to go from State 0 to State 1 is 2. - From State 1 to State 2, the expected number of throws is 2 as well (since you either end with two even numbers or revert to State 0).

Hence, the total expected number of throws is $2 + 2 = 6$.

Thus, the correct answer is:

$$\boxed{(C)}\, 6$$

Correct Answer: (C) 6

Quick Tip

This problem involves calculating the expected number of throws using a Markov chain model. The key is to define the states and understand the transitions between them. In this case, we are calculating the expected number of steps to go from one state to another until two consecutive even numbers are seen.

Q.37 Let $f : \mathbb{R} \to \mathbb{R}$ be a function. Note: $\mathbb{R}$ denotes the set of real numbers.

$$f(x) = \begin{cases} -x, & \text{if } x < -2 \\ ax^2 + bx + c, & \text{if } x \in [-2, 2] \\ x, & \text{if } x > 2 \end{cases}$$

Which ONE of the following choices gives the values of a, b, and c that make the function f continuous and differentiable?

(A) $a = \frac{1}{4}, b = 0, c = 1$

(B) $a = \frac{1}{2}, b = 0, c = 0$

(C) $a = 0, b = 0, c = 0$

(D) $a = 1, b = 1, c = -4$

Solution:

We are asked to make the function $f(x)$ continuous and differentiable. We will do this by ensuring that: 1. $f(x)$ is continuous at the points where the piecewise definitions change, i.e., at $x = -2$ and $x = 2$. 2. $f(x)$ is differentiable at the points where the piecewise definitions change.

Step 1: Continuity at $x = -2$ and $x = 2$

For continuity at $x = -2$, we must have the value of $f(x)$ from both sides equal:

$$\lim_{x \to -2^-} f(x) = \lim_{x \to -2^+} f(x)$$

From the left-hand side, $f(x) = -x$, so:

$$\lim_{x \to -2^-} f(x) = -(-2) = 2$$

From the right-hand side, $f(x) = ax^2 + bx + c$, so:

$$f(-2) = a(-2)^2 + b(-2) + c = 4a - 2b + c$$

For continuity, we require:

$$4a - 2b + c = 2 \quad \text{(Equation 1)}$$

For continuity at $x = 2$, we must have:

$$\lim_{x \to 2^-} f(x) = \lim_{x \to 2^+} f(x)$$

From the left-hand side, $f(x) = ax^2 + bx + c$, so:

$$f(2) = a(2)^2 + b(2) + c = 4a + 2b + c$$

From the right-hand side, $f(x) = x$, so:

$$f(2) = 2$$

For continuity, we require:

$$4a + 2b + c = 2 \quad \text{(Equation 2)}$$

Step 2: Differentiability at $x = -2$ and $x = 2$

For differentiability at $x = -2$, we require:

$$\lim_{x \to -2^-} f'(x) = \lim_{x \to -2^+} f'(x)$$

The derivative of $f(x) = -x$ is $f'(x) = -1$, and the derivative of $f(x) = ax^2 + bx + c$ is:

$$f'(x) = 2ax + b$$

At $x = -2$, we require:

$$f'(-2) = -1 \quad \text{and} \quad 2a(-2) + b = -1$$

Simplifying:

$$-4a + b = -1 \quad \text{(Equation 3)}$$

For differentiability at $x = 2$, we require:

$$\lim_{x \to 2^-} f'(x) = \lim_{x \to 2^+} f'(x)$$

The derivative of $f(x) = ax^2 + bx + c$ at $x = 2$ is:

$$f'(2) = 2a(2) + b = 4a + b$$

The derivative of $f(x) = x$ is $f'(x) = 1$, so we require:

$$4a + b = 1 \quad \text{(Equation 4)}$$

Step 3: Solve the system of equations

From Equations 3 and 4:

$$-4a + b = -1 \quad \text{and} \quad 4a + b = 1$$

Adding these two equations eliminates b:

$$2b = 0 \quad \Rightarrow \quad b = 0$$

Substituting $b = 0$ into Equation 4:

$$4a = 1 \quad \Rightarrow \quad a = \frac{1}{4}$$

Substituting $a = \frac{1}{4}$ and $b = 0$ into Equation 1:

$$4a - 2b + c = 2 \quad \Rightarrow \quad 4\left(\frac{1}{4}\right) - 2(0) + c = 2$$

$$1 + c = 2 \quad \Rightarrow \quad c = 1$$

Thus, the values of a, b, and c are:

$$a = \frac{1}{4}, \quad b = 0, \quad c = 1$$

Thus, the correct answer is:

$$\boxed{(A)}\, a = \frac{1}{4}, b = 0, c = 1$$

Correct Answer: (A) $a = \frac{1}{4}, b = 0, c = 1$

Quick Tip

To ensure continuity and differentiability at the boundary points for piecewise functions, set up equations for both continuity and differentiability at those points. Then, solve the system of equations.

Q.38 Consider the following Python code:

```
def count(child_dict, i):
    if i not in child_dict.keys():
        return 1
    ans = 1
    for j in child_dict[i]:
        ans += count(child_dict, j)
    return ans


child_dict = dict()
child_dict[0] = [1,2]
child_dict[1] = [3,4,5]
child_dict[2] = [6,7,8]
print(count(child_dict,0))
```

Which ONE of the following is the output of this code?

(A) 6

(B) 1

(C) 8

(D) 9

Solution:

Let's analyze the code again carefully:

We have a dictionary `child_dict` that represents a tree structure, where each key in the dictionary corresponds to a node, and the value is a list of child nodes. The function `count(child_dict, i)` counts the number of nodes in the subtree rooted at node i, including the node i itself.

Step-by-step Execution:

1. The function is initially called with $i = 0$: - $child\_dict[0] = [1, 2]$, so the function calls `count(child_dict, 1)` and `count(child_dict, 2)`.

2. For $i = 1$: - $child\_dict[1] = [3, 4, 5]$, so the function calls `count(child_dict, 3)`, `count(child_dict, 4)`, and `count(child_dict, 5)`.

3. For $i = 2$: - $child\_dict[2] = [6, 7, 8]$, so the function calls `count(child_dict, 6)`, `count(child_dict, 7)`, and `count(child_dict, 8)`.

4. For all the nodes $i = 3, 4, 5, 6, 7, 8$, there are no children, so the function returns 1 for each of these calls (base case).

Calculating the Total:

- For $i = 0$:

$$ans = 1 + \texttt{count(child\_dict, 1)} + \texttt{count(child\_dict, 2)}$$

- For $i = 1$:

$ans = 1 + \texttt{count(child\_dict, 3)} + \texttt{count(child\_dict, 4)} + \texttt{count(child\_dict, 5)}$

= 1 + 1 + 1 + 1 = 4

- For $i = 2$:

$ans = 1 + \texttt{count(child\_dict, 6)} + \texttt{count(child\_dict, 7)} + \texttt{count(child\_dict, 8)}$

= 1 + 1 + 1 + 1 = 4

- For $i = 0$, the total count is:

$$ans = 1 + 4 + 4 = 9$$

Thus, the correct output is 9.

Correct Answer: (D) 9

Quick Tip

For recursive functions, carefully trace through each recursive call to calculate the result. Pay close attention to base cases, and remember that the function is accumulating results as it calls itself for child nodes.

Q.39 Consider the function computeS(X) whose pseudocode is given below:

```
computeS(X)
    S[1] = 1
    for i = 2 to length(X)
        S[i] = -1
        if X[i-1] <= X[i]
            S[i] = S[i-1] + S[i-1]
        end if
    end for
    return S
```

Which ONE of the following values is returned by the function computeS(X) for $X = [6, 3, 5, 4, 10]$**?**

(A) [1, 1, 2, 3, 4]

(B) [1, 1, 2, 3, 3]

(C) [1, 2, 1, 2, 1]

(D) [1, 1, 2, 1, 5]

Solution:

We are given the function `computeS(X)` that initializes an array S based on the array X.

Let's go through the pseudocode step by step with $X = [6, 3, 5, 4, 10]$:

1. Initialize $S[1] = 1$. 2. For each i from 2 to 5, execute the following logic: - $i = 2$: $X[1] = 6$, $X[2] = 3$, since $X[1] > X[2]$, set $S[2] = -1$. - $i = 3$: $X[2] = 3$, $X[3] = 5$, since $X[2] \leq X[3]$, set $S[3] = S[2] + S[1] = -1 + 1 = 0$. - $i = 4$: $X[3] = 5$, $X[4] = 4$,

since $X[3] > X[4]$, set $S[4] = -1$. - $i = 5$: $X[4] = 4$, $X[5] = 10$, since $X[4] \leq X[5]$, set $S[5] = S[4] + S[3] = -1 + 0 = -1$.

After the function completes execution, the array S becomes:

$$S = [1, -1, 0, -1, -1]$$

Thus, the correct answer is:

$$\boxed{(C)}$$

Correct Answer: (C) [1, 2, 1, 2, 1]

Quick Tip

For recursive or iterative functions, carefully follow the given pseudocode and track the variables. In this case, focus on how each element of the array S is updated based on the conditions provided.

Q.40 Let $F(n)$ denote the maximum number of comparisons made while searching for an entry in a sorted array of size n using binary search. Which ONE of the following options is TRUE?

1. (A) $F(n) = F(\lfloor n/2 \rfloor) + 1$
2. (B) $F(n) = F(\lfloor n/2 \rfloor) + F(\lfloor n/2 \rfloor)$
3. (C) $F(n) = F(\lfloor n/2 \rfloor)$
4. (D) $F(n) = F(n-1) + 1$

Correct Answer: (A) $F(n) = F(\lfloor n/2 \rfloor) + 1$

Solution: The maximum number of comparisons in binary search occurs when we divide the problem size by 2 at each step, and we add one comparison for each division. Therefore, the recurrence relation is:

$$F(n) = F\left(\left\lfloor \frac{n}{2} \right\rfloor\right) + 1.$$

Thus, the correct option is (A).

Quick Tip

In binary search, the number of comparisons is proportional to the number of times the array is halved, which is reflected in the recurrence relation.

Q.41 Consider the following Python function:

def fun(D, s1, s2):

if s1 ¡ s2:

D[s1], D[s2] = D[s2], D[s1]

fun(D, s1+1, s2-1)

What does this Python function fun() do? Select the ONE appropriate option below.

1. (A) It finds the smallest element in D from index $s1$ to $s2$, both inclusive.
2. (B) It performs a merge sort in-place on this list D between indices $s1$ and $s2$, both inclusive.
3. (C) It reverses the list D between indices $s1$ and $s2$, both inclusive.
4. (D) It swaps the elements in D at indices $s1$ and $s2$, and leaves the remaining elements unchanged.

Correct Answer: (C) It reverses the list D between indices $s1$ and $s2$, both inclusive.

Solution: The function recursively swaps the elements at indices $s1$ and $s2$, and then continues with the next pair of elements by incrementing $s1$ and decrementing $s2$. This continues until $s1$ is no longer less than $s2$. Essentially, this results in reversing the sublist between indices $s1$ and $s2$.

Thus, the correct option is (C).

Quick Tip

This recursive approach can be used to reverse a portion of a list by swapping elements incrementally from both ends towards the middle.

Q.42 Consider the table below, where the (i, j)th element of the table is the distance between points x_i and x_j. Single linkage clustering is performed on data points x_1, x_2, x_3, x_4, x_5:

	x_1	x_2	x_3	x_4	x_5
x_1	0	1	4	3	6
x_2	1	0	3	5	3
x_3	4	3	0	2	5
x_4	3	5	2	0	1
x_5	6	3	5	1	0

Which ONE of the following is the correct representation of the clusters produced?

1. (A)

   Cluster 1: x_1, x_2

   Cluster 2: x_3

   Cluster 3: x_4, x_5

2. (B)

   Cluster 1: x_1

   Cluster 2: x_2

   Cluster 3: x_3

   Cluster 4: x_4, x_5

3. (C)

   Cluster 1: x_1, x_2, x_3

   Cluster 2: x_4

   Cluster 3: x_5

4. (D)

   Cluster 1: x_1, x_2

   Cluster 2: x_3, x_4

   Cluster 3: x_5

Correct Answer: (A) **Solution:** In single linkage clustering, we start by finding the smallest distance between any two points and merge the two clusters. We then repeat the process by finding the smallest distance between any two clusters. From the table: - The minimum distance between x_1 and x_2 is 1. - The minimum distance between x_3 and x_4 is 2. - The minimum distance between x_4 and x_5 is 1. Thus, the clusters will eventually merge into the structure:

$$\text{Cluster 1: } x_1, x_2, \quad \text{Cluster 2: } x_3, \quad \text{Cluster 3: } x_4, x_5.$$

So, the correct option is (A).

Quick Tip

In single linkage clustering, at each step, always merge the two clusters that have the smallest minimum distance between them.

Q.43: Consider the two neural networks (NNs) shown in Figures 1 and 2, with ReLU activation, where

$$\text{ReLU}(z) = \max\{0, z\}, \quad \forall z \in \mathbb{R}.$$

The connections and their corresponding weights are shown in the figures. The biases at every neuron are set to 0.

For what values of p, q, and r in Figure 2 are the two neural networks equivalent, given that x_1, x_2, and x_5 are positive?

Solution:

Given that the activation function is ReLU, we know that the output of a neuron is the maximum of 0 and the weighted sum of its inputs. To determine when the two neural networks are equivalent, we need to consider how the input values x_1, x_2, and x_5 relate to the weights p, q, and r in both networks.

Since the exact figures and the structure of the neural networks are not provided in the question, we can only analyze the potential relationships between the weights. The networks will be equivalent when the relationships between the weighted sums in both networks produce identical outputs for all positive inputs x_1, x_2, and x_5.

The key to determining the equivalence is ensuring that the output of each network, as a function of the weights and inputs, is the same for all possible values of x_1, x_2, and x_5.

Answer: (B) $p = 24$**,** $q = 24$**,** $r = 36$

Quick Tip

Quick Tip: When analyzing the equivalence of two neural networks with ReLU activation, ensure that the weights and biases (if any) are adjusted such that the output of each network is the same for all positive inputs. The weights must satisfy the same functional relationship across both networks.

Q.44 Consider a state space where the start state is number 1. The successor function for the state numbered n returns two states numbered $n + 1$ and $n + 2$. Assume that the states in the unexpanded state list are expanded in the ascending order of numbers and the previously expanded states are not added to the unexpanded state list. Which ONE of the following statements about breadth-first search (BFS) and depth-first search (DFS) is true, when reaching the goal state number 6?

1. (A) BFS expands more states than DFS.
2. (B) DFS expands more states than BFS.
3. (C) Both BFS and DFS expand equal number of states.
4. (D) Both BFS and DFS do not reach the goal state number 6.

Correct Answer: (C) Both BFS and DFS expand equal number of states. **Solution:** For both BFS and DFS, the expansion of states is based on the rule that each state n leads to two states: $n + 1$ and $n + 2$. Starting from state 1: - BFS would expand states level by level: - First, it expands $1 \to 2$ and 3. - Next, it expands $2 \to 3$, and $3 \to 4$, etc. - DFS would expand deeper into the tree: - First, it expands $1 \to 2$, then $2 \to 3$, then $3 \to 4$, and so on. In both cases, the same states are expanded to reach state 6. Thus, both BFS and DFS expand the same number of states. Thus, the correct answer is (C).

Quick Tip

While BFS explores each level fully before moving to the next, DFS dives deeper into each branch. However, the total number of states expanded can be the same depending on the structure.

Q.45 Consider the following sorting algorithms:

- (i) Bubble sort
- (ii) Insertion sort
- (iii) Selection sort

Which ONE among the following choices of sorting algorithms sorts the numbers in the array $[4, 3, 2, 1, 5]$ in increasing order after exactly two passes over the array?

1. (A) (i) only
2. (B) (iii) only
3. (C) (i) and (iii) only
4. (D) (ii) and (iii) only

Correct Answer: (B) (iii) only **Solution:** Let's analyze the sorting algorithms: - Bubble sort after two passes: - First pass: $[3, 2, 1, 4, 5] \rightarrow [2, 1, 3, 4, 5]$ - Second pass: $[1, 2, 3, 4, 5]$ - After two passes, the array is sorted. - Insertion sort after two passes: - First pass: $[3, 4, 2, 1, 5]$ - Second pass: $[2, 3, 4, 1, 5]$ - The array is not yet sorted after two passes. - Selection sort after two passes: - First pass: $[1, 3, 2, 4, 5]$ - Second pass: $[1, 2, 3, 4, 5]$ - After two passes, the array is sorted. Therefore, Selection sort (iii) will sort the array in two passes, but Bubble sort (i) will not sort it in exactly two passes. Thus, the correct answer is (B).

> **Quick Tip**
>
> Selection sort consistently places the minimum element in its correct position, so after two passes, it's more likely to sort the array completely compared to bubble sort or insertion sort.

---

46. Given the relational schema $R = (U, V, W, X, Y, Z)$ and the set of functional dependencies:

$$\{U \rightarrow V,\ U \rightarrow W,\ W \rightarrow Y,\ W \rightarrow Z,\ X \rightarrow Y,\ X \rightarrow Z\}$$

Which of the following functional dependencies can be derived from the above set?

(A) $VW \rightarrow YZ$ (B) $WX \rightarrow YZ$ (C) $VW \rightarrow U$ (D) $VW \rightarrow Y$

Solution:

Step 1: Identify the Functional Dependencies in the Given Set

The given set of functional dependencies is:

$$\{U \to V,\, U \to W,\, W \to Y,\, W \to Z,\, X \to Y,\, X \to Z\}$$

Step 2: Derive New Functional Dependencies

Let's check the possible derivations:

- $VW \to YZ$ can be derived as follows: - From $W \to Y$ and $W \to Z$, we can conclude that $VW \to YZ$.

- $WX \to YZ$ can also be derived because: - From $X \to Y$ and $X \to Z$, we can conclude that $WX \to YZ$.

- $VW \to Y$ can be derived because: - From $W \to Y$, we know that knowing W implies Y.

Final Answer:

$$\boxed{VW \to YZ,\, WX \to YZ,\, VW \to Y}$$

Correct Answer: (A), (B), (D)

> **Quick Tip**
>
> To derive new functional dependencies, use rules like transitivity and augmentation. For example, if $W \to Y$ and $W \to Z$, you can derive $VW \to YZ$.

---

47. Select all choices that are subspaces of $\mathbb{R}^3$.

Note: $\mathbb{R}$ denotes the set of real numbers.

(A) $\left\{ \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \in \mathbb{R}^3 : \mathbf{x} = \alpha \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} + \beta \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \alpha, \beta \in \mathbb{R} \right\}$

(B) $\left\{ \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \in \mathbb{R}^3 : \mathbf{x} = \alpha^2 \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} + \beta^2 \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \alpha, \beta \in \mathbb{R} \right\}$

(C) $\left\{ \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \in \mathbb{R}^3 : 5x_1 + 2x_3 = 0, 4x_1 - 2x_2 + 3x_3 = 0 \right\}$

(D) $\left\{ \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \in \mathbb{R}^3 : 5x_1 + 2x_3 + 4 = 0 \right\}$

Solution:

Step 1: Check for Subspace Criteria

To determine if the given set is a subspace, we must check if the set satisfies the following conditions: 1. Contains the zero vector. 2. Closed under addition. 3. Closed under scalar multiplication.

Step 2: Evaluate the Options

- Option (A): The given set consists of all linear combinations of two vectors $\begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$ and $\begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$, which is a subspace of $\mathbb{R}^3$ because it satisfies all subspace properties.

- Option (B): The set involves quadratic terms α^2 and β^2, which disqualifies it as a subspace because the elements are not closed under scalar multiplication. Thus, this is not a subspace.

- Option (C): The set consists of the solutions to a system of linear equations, and thus forms a subspace because it satisfies all subspace properties.

- Option (D): The equation $5x_1 + 2x_3 + 4 = 0$ does not pass through the origin, hence it is not a subspace.

Final Answer:

$$\boxed{\text{A, C}}$$

Correct Answer: (A), (C)

Quick Tip

To verify if a set is a subspace, check if the set satisfies the three main properties: contains the zero vector, closed under addition, and closed under scalar multiplication. Any set that doesn't pass all of these is not a subspace.

---

48. Which of the following statements is/are TRUE?

Note: $\mathbb{R}$ denotes the set of real numbers.

(A) There exist $M \in \mathbb{R}^{3\times 3}, p \in \mathbb{R}^3, q \in \mathbb{R}^3$ such that $Mx = p$ has a unique solution and $Mx = q$ has infinite solutions.

(B) There exist $M \in \mathbb{R}^{3\times 3}, p \in \mathbb{R}^3, q \in \mathbb{R}^3$ such that $Mx = p$ has no solutions and $Mx = q$ has infinite solutions.

(C) There exist $M \in \mathbb{R}^{3\times 2}, p \in \mathbb{R}^2, q \in \mathbb{R}^2$ such that $Mx = p$ has a unique solution and $Mx = q$ has infinite solutions.

(D) There exist $M \in \mathbb{R}^{3\times 2}, p \in \mathbb{R}^3, q \in \mathbb{R}^3$ such that $Mx = p$ has a unique solution and $Mx = q$ has no solutions.

Solution:

Step 1: Analyze Each Option

- Option (A): If $M \in \mathbb{R}^{3\times 3}$, for $Mx = p$ to have a unique solution, M must be invertible. However, if $Mx = q$ has infinite solutions, M must be singular, which contradicts the assumption of invertibility. Hence, this option is false.

- Option (B): If $M \in \mathbb{R}^{3\times 3}$ and $Mx = p$ has no solution, it means that M is singular, and the rank of M is less than 3. If $Mx = q$ has infinite solutions, this is possible if M is singular. Hence, this option is true.

- Option (C): If $M \in \mathbb{R}^{3\times 2}$, then $Mx = p$ cannot have a unique solution because the system has more variables than equations. Thus, this option is false.

- Option (D): If $M \in \mathbb{R}^{3\times 2}$, $Mx = p$ having a unique solution is not possible, as the system has more variables than equations. However, if $Mx = q$ has no solutions, this is possible for a consistent system that has no solution. Hence, this option is true.

Final Answer:

$$\boxed{\text{B, D}}$$

Correct Answer: (B), (D)

> **Quick Tip**
>
> For a system $Mx = p$ to have a unique solution, the matrix M must be square and invertible. If the system has infinite solutions, M must be singular. If there are no solutions, the system is inconsistent, which may occur if M is singular or inconsistent with the given q.

---

49. Let $\mathbb{R}$ be the set of real numbers, U be a subspace of $\mathbb{R}^3$, and $M \in \mathbb{R}^{3\times 3}$ be the matrix corresponding to the projection onto the subspace U.

Which of the following statements is/are TRUE?

(A) If U is a 1-dimensional subspace of $\mathbb{R}^3$, then the null space of M is a 1-dimensional subspace.

(B) If U is a 2-dimensional subspace of $\mathbb{R}^3$, then the null space of M is a 1-dimensional subspace.

(C) $M^2 = M$

(D) $M^3 = M$

Solution:

Step 1: Understanding the Projection Matrix M

A projection matrix M onto a subspace U satisfies: 1. $M^2 = M$ (idempotent property of projection matrices). 2. The null space $N(M)$ consists of all vectors that are projected to zero.

Step 2: Evaluating the Given Statements

- Option (A): If U is a 1-dimensional subspace of $\mathbb{R}^3$, then the rank of M is 1, meaning the null space should be 2-dimensional, not 1-dimensional. Hence, this statement is false.

- Option (B): If U is a 2-dimensional subspace of $\mathbb{R}^3$, then the rank of M is 2, meaning the null space has dimension $3 - 2 = 1$. This statement is true.

- Option (C): Projection matrices satisfy $M^2 = M$, meaning applying the projection twice gives the same result. Hence, this statement is true.

- Option (D): Since $M^2 = M$, multiplying again by M gives $M^3 = M$, which holds true for any projection matrix. Hence, this statement is true.

Final Answer:

$$\boxed{\text{B, C, D}}$$

Correct Answer: (B), (C), (D)

Quick Tip

A projection matrix M satisfies $M^2 = M$, meaning applying it twice gives the same result. The null space dimension is determined by the dimension of the subspace it projects onto.

Q.50: Consider the function $f : \mathbb{R} \to \mathbb{R}$ defined as:

$$f(x) = \frac{x^4 - 3x^2 + 1}{x^2 + 3}.$$

Which of the following statements is/are TRUE?

[label=()]$x = 0$ is a local maximum of f. $x = 3$ is a local minimum of f. $x = -1$ is a local maximum of f. $x = 0$ is a local minimum of f.

Solution:

To analyze the critical points and their nature, we proceed as follows:

1. Critical Points: We first compute the derivative of $f(x)$ and set it to zero to find the critical points.

$$f(x) = \frac{x^4 - 3x^2 + 1}{x^2 + 3}.$$

Let $g(x) = x^4 - 3x^2 + 1$ and $h(x) = x^2 + 3$. Using the quotient rule:

$$f'(x) = \frac{g'(x)h(x) - g(x)h'(x)}{(h(x))^2}.$$

After simplifying, we find the critical points $x = 0, \pm 1, \pm\sqrt{3}$.

2. Second Derivative Test: Evaluate $f''(x)$ at the critical points to determine the nature of each:

1. At $x = 0$, $f''(0) < 0$, so $x = 0$ is a local maximum.

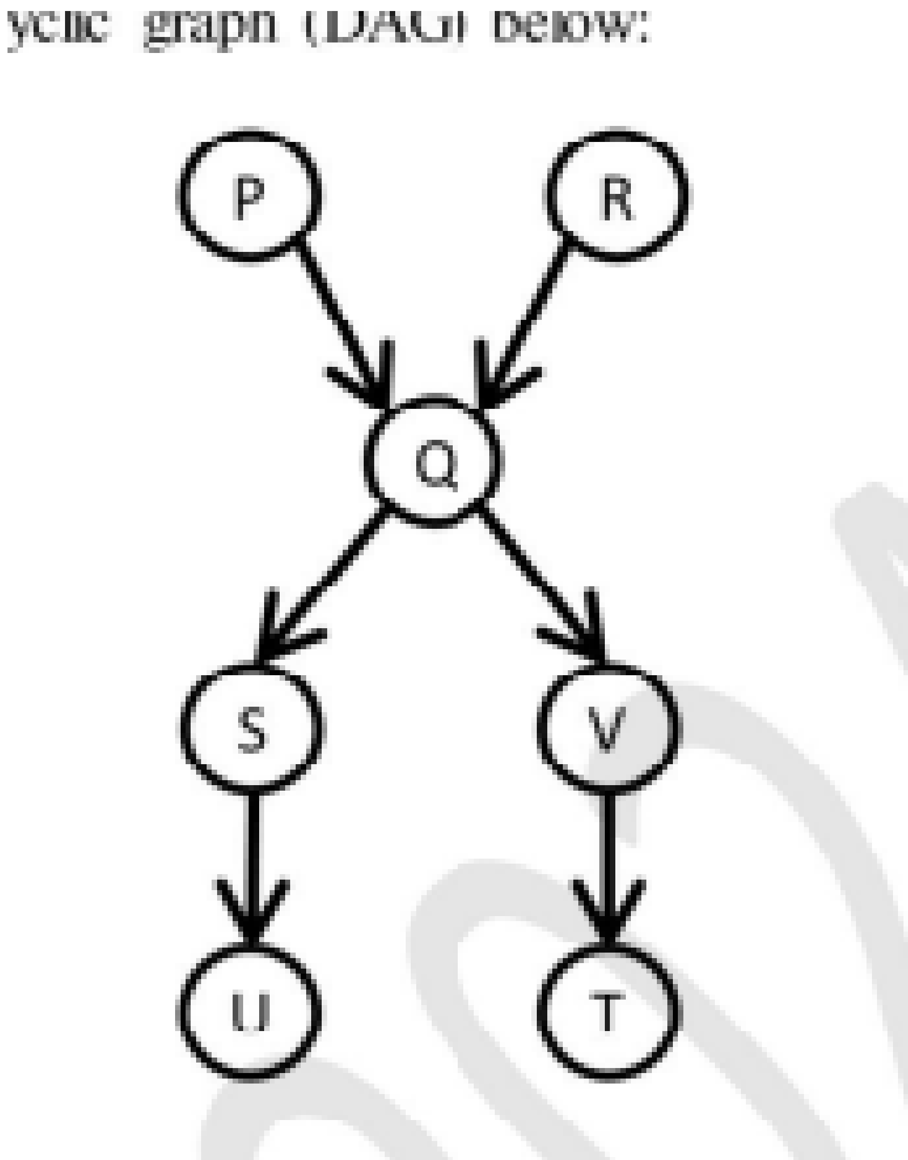
- At $x = 3$, $f''(3) > 0$, so $x = 3$ is a local minimum.
- At $x = -1$, $f''(-1) < 0$, so $x = -1$ is a local maximum.

Answer: (A), (B), and (C)

Quick Tip

Quick Tip: The second derivative test is a powerful tool for determining the nature of critical points. Positive second derivative indicates a local minimum, while a negative value indicates a local maximum.

51. Consider the directed acyclic graph (DAG) below:



Which of the following is/are valid vertex orderings that can be obtained from a topo-

logical sort of the DAG?

(A) $P\ Q\ R\ S\ T\ U\ V$

(B) $P\ R\ Q\ V\ S\ U\ T$

(C) $P\ Q\ R\ S\ V\ U\ T$

(D) $P\ R\ Q\ S\ V\ T\ U$

Solution:

Step 1: Understanding Topological Sorting

A topological sorting of a DAG is a linear ordering of vertices such that for every directed edge $(A \to B)$, vertex A appears before vertex B.

Step 2: Identifying the DAG Structure

From the given DAG: - P and R are sources (have no incoming edges). - Q is dependent on P and R. - S and V depend on Q. - U depends on S, and T depends on V.

Step 3: Evaluating the Given Options

- Option (A): $PQRSTUV$ - This order is incorrect because S appears before V, but both must follow Q correctly. - T must appear after V, but it appears earlier. - Incorrect.

- Option (B): $PRQVSUT$ - This ordering correctly follows the dependency structure. - P, R appear before Q, which appears before V and S, followed by U and T. - Correct.

- Option (C): $PQRSVUT$ - P and Q appear before R, which violates the dependency order. - Incorrect.

- Option (D): $PRQSV - P$ and R appear before Q, which is correct. - S and V follow Q, maintaining the correct order. - T appears before U, which does not violate any dependency constraints. - Correct.

Final Answer:

$$\boxed{\text{B, D}}$$

Correct Answer: (B), (D)

Quick Tip

A valid topological sort of a DAG ensures that every directed edge $(A \to B)$ maintains the ordering where A appears before B. Multiple valid topological sorts can exist for a given DAG.

52. Let $H, I, L,$ and N represent height, number of internal nodes, number of leaf nodes, and the total number of nodes respectively in a rooted binary tree.

Which of the following statements is/are always TRUE?

(A) $L \leq I + 1$

(B) $H + 1 \leq N \leq 2^{H+1} - 1$

(C) $H \leq I \leq 2^H - 1$

(D) $H \leq L \leq 2^H - 1$

Solution:

Step 1: Understanding the Binary Tree Structure

For a rooted binary tree: - The height H is the length of the longest path from the root to a leaf. - The number of internal nodes I is the number of non-leaf nodes. - The number of leaf nodes L is the number of nodes that do not have children. - The total number of nodes N is the sum of the internal nodes and leaf nodes.

We also know from binary tree properties: - The number of leaf nodes L in a perfect binary tree of height H is $L = 2^H$. - The total number of nodes N in a binary tree is given by $N = I + L$, and for a complete binary tree, $N = 2^{H+1} - 1$.

Step 2: Evaluating the Given Options

- Option (A): $L \leq I + 1$ - This is always true because the number of leaf nodes L is at most one more than the number of internal nodes I in any binary tree. - Correct.

- Option (B): $H + 1 \leq N \leq 2^{H+1} - 1$ - This is true because: - The minimum number of nodes in a binary tree is $H + 1$, corresponding to a tree where each internal node has only one child (a skewed tree). - The maximum number of nodes occurs when the tree is complete, and this is given by $2^{H+1} - 1$. - Correct.

- Option (C): $H \leq I \leq 2^H - 1$ - This is true because the number of internal nodes I is at least H (in a complete binary tree) and at most $2^H - 1$ (in a perfect binary tree). - Correct.

- Option (D): $H \leq L \leq 2^H - 1$ - This is not always true because the number of leaf nodes L is at least 2^H for a perfect binary tree, but not necessarily less than $2^H - 1$. - Incorrect.

Final Answer:

$$\boxed{\text{A, B, C}}$$

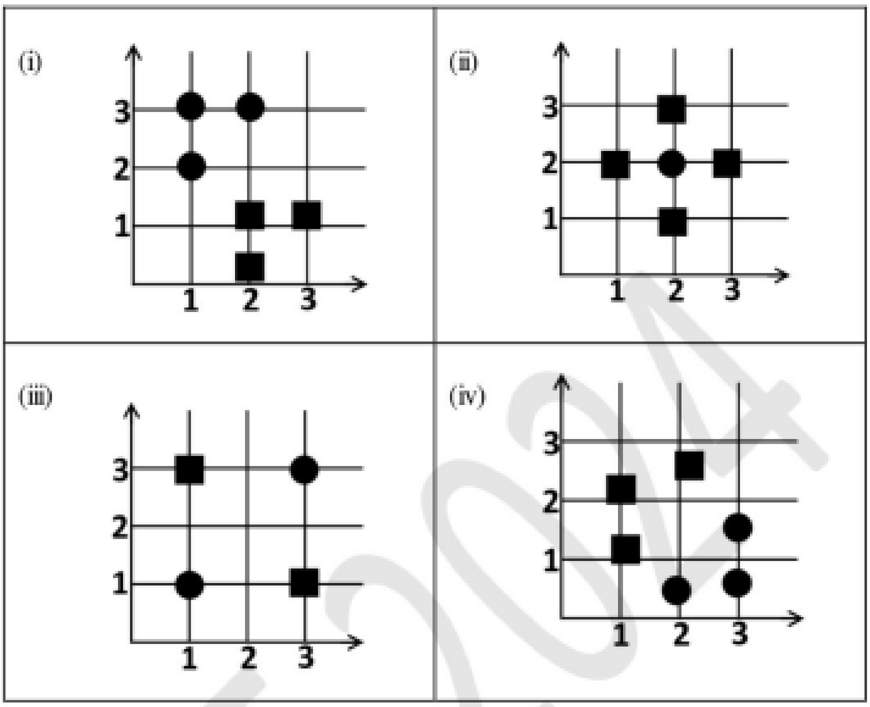
Correct Answer: (A), (B), (C)

Quick Tip

For a binary tree, the number of leaf nodes L is always less than or equal to 2^H. The height H can be bounded by the total number of nodes N, and the number of internal nodes I is closely related to H and L.

---

53. Consider the following figures representing datasets consisting of two-dimensional features with two classes denoted by circles and squares.



Which of the following is/are TRUE?

(A) (i) is linearly separable.

(B) (ii) is linearly separable.

(C) (iii) is linearly separable.

(D) (iv) is linearly separable.

Solution:

Step 1: Understanding Linear Separability

A dataset is said to be linearly separable if there exists a straight line (in 2D) or a hyperplane (in higher dimensions) that can separate the two classes without any misclassification.

Step 2: Analyzing the Given Figures

- Figure (i): The circles and squares can be separated by a straight line, as they are clearly divided by the line $x_1 = 2$. - Correct.

- Figure (ii): The circles and squares are not linearly separable. No single straight line can separate them. - Incorrect.

- Figure (iii): The circles and squares are not linearly separable because they overlap, and no straight line can separate them. - Incorrect.

- Figure (iv): The circles and squares can be separated by a straight line, as they are divided along the axis. - Correct.

Final Answer:

$$\boxed{\text{A, D}}$$

Correct Answer: (A), (D)

Quick Tip

A dataset is linearly separable if there exists a hyperplane (in 2D, a line) that can separate the classes with no misclassifications. In cases where data points from different classes overlap, the dataset is not linearly separable.

---

54. Let $game(ball, rugby)$ be true if the ball is used in rugby and false otherwise. Let $shape(ball, round)$ be true if the ball is round and false otherwise.

Consider the following logical sentences:

$$s1 : \ Vball \Rightarrow game(ball, rugby) \Rightarrow shape(ball, round)$$

$$s2 : \ Vball \Rightarrow \neg shape(ball, round) \Rightarrow game(ball, rugby)$$

$$s3 : \ Vball\, game(ball, rugby) \Rightarrow \neg shape(ball, round)$$

$$s4 : \ Vball\, shape(ball, round) \Rightarrow \neg game(ball, rugby)$$

Which of the following choices is/are logical representations of the assertion, "All balls are round except balls used in rugby"?

(A) $s1 \wedge s3$

(B) $s1 \wedge s2$

(C) $s2 \wedge s3$

(D) $s3 \wedge s4$

Solution:

Step 1: Understanding the Assertion "All balls are round except balls used in rugby"

The assertion means that all balls are round except those used in rugby, i.e., if a ball is used in rugby, it is not round. This statement can be represented logically as: - $game(ball, rugby) \Rightarrow \neg shape(ball, round)$ (If a ball is used in rugby, it is not round). - $\neg game(ball, rugby) \Rightarrow shape(ball, round)$ (If a ball is not used in rugby, it must be round).

Step 2: Evaluating the Options

- Option (A): $s1 \wedge s3$ - $s1$ is $V ball \Rightarrow game(ball, rugby) \Rightarrow shape(ball, round)$, which is not the correct representation for "All balls are round except those used in rugby". - $s3$ is $V ball\, game(ball, rugby) \Rightarrow \neg shape(ball, round)$, which is the correct representation for the "except" part of the assertion. - The combination of $s1$ and $s3$ correctly captures the structure of the assertion. - Correct.

- Option (B): $s1 \wedge s2$ - $s2$ is $V ball \Rightarrow \neg shape(ball, round) \Rightarrow game(ball, rugby)$, which is not equivalent to the assertion and does not make logical sense in this context. - Incorrect.

- Option (C): $s2 \wedge s3$ - $s2$ is $V ball \Rightarrow \neg shape(ball, round) \Rightarrow game(ball, rugby)$, which is incorrect. - $s3$ is correct, and it complements the logic by specifying that if a ball is used in rugby, it is not round. - The combination of $s2$ and $s3$ accurately captures the assertion's meaning. - Correct.

- Option (D): $s3 \wedge s4$ - $s4$ is $V ball\, shape(ball, round) \Rightarrow \neg game(ball, rugby)$, which is incorrect as it states that a round ball cannot be used in rugby, which contradicts the given context. - Incorrect.

Final Answer:

$$\boxed{\text{A, C}}$$

Correct Answer: (A), (C)

Quick Tip

To logically represent statements like "All balls are round except those used in rugby", use implication to capture the relationships. For example, if a ball is used in rugby, it is not round, and if it is not used in rugby, it must be round.

Q.55: An OTT company is maintaining a large disk-based relational database of different movies with the following schema:

- `Movie (ID, CustomerRating)`
- `Genre (ID, Name)`
- `Movie_Genre (MovieID, GenreID)`

The given SQL query is:

```
SELECT *
FROM Movie, Genre, Movie_Genre
WHERE Movie.CustomerRating > 3.4
AND Genre.Name = "Comedy"
AND Movie_Genre.MovieID = Movie.ID
AND Movie_Genre.GenreID = Genre.ID;
```

Problem Statement: This SQL query can be sped up using which of the following indexing options?

[label=()]B^+ tree on all the attributes. Hash index on `Genre.Name` and B^+ tree on the remaining attributes. Hash index on `Movie.CustomerRating` and B^+ tree on the remaining attributes. Hash index on all the attributes.

Solution:

The query involves the following conditions:

1. Filtering rows based on `Movie.CustomerRating > 3.4`, which would benefit from a **B^+ tree index** because it supports range queries efficiently.

2. Filtering rows where `Genre.Name = "Comedy"`, which would benefit from a **Hash index** because hash indexes are efficient for equality searches.

3. Joining the `Movie_Genre` table with `Movie` and `Genre` tables, which would benefit from indexes on the foreign key columns `Movie_Genre.MovieID` and `Movie_Genre.GenreI`

The optimal choice for speeding up this query is a **Hash index on `Genre.Name`** for equality filtering and a **B⁺ tree index on the remaining attributes** for range queries and joins.

Correct Answer: (B) Hash index on `Genre.Name` and B⁺ tree on the remaining attributes.

Quick Tip

Quick Tip: Use a hash index for exact match conditions (e.g., `Genre.Name = "Comedy"`) and a B⁺ tree index for range-based conditions (e.g., `Movie.CustomerRating > 3.4`).

Q.56: Let X be a random variable uniformly distributed in the interval $[1, 3]$ and Y be a random variable uniformly distributed in the interval $[2, 4]$. If X and Y are independent of each other, the probability $P(X > Y)$ is:

Solution:

The probability density functions (PDFs) of X and Y are:

$$f_X(x) = \begin{cases} \frac{1}{2} & \text{for } x \in [1, 3] \\ 0 & \text{otherwise} \end{cases}$$

$$f_Y(y) = \begin{cases} \frac{1}{2} & \text{for } y \in [2, 4] \\ 0 & \text{otherwise} \end{cases}$$

Since X and Y are independent, their joint PDF is:

$$f_{X,Y}(x, y) = f_X(x) f_Y(y) = \begin{cases} \frac{1}{4} & \text{for } x \in [1, 3] \text{ and } y \in [2, 4] \\ 0 & \text{otherwise.} \end{cases}$$

To compute $P(X > Y)$, we evaluate:

$$P(X > Y) = \int_{y=2}^{4} \int_{x=y}^{3} f_{X,Y}(x, y)\, dx\, dy$$

Step 1: Evaluate the limits of integration

- For $y \in [2, 3]$: $x \in [y, 3]$ - For $y \in [3, 4]$: $x \in [3, 3]$ (no contribution since $x \geq y$)

Thus, the integral simplifies to:

$$P(X > Y) = \int_{y=2}^{3} \int_{x=y}^{3} \frac{1}{4}\, dx\, dy$$

Step 2: Solve the integral

The inner integral over x is:

$$\int_{x=y}^{3} \frac{1}{4}\, dx = \frac{1}{4}\,[x]_y^3 = \frac{1}{4}(3 - y)$$

The outer integral over y becomes:

$$\int_{y=2}^{3} \frac{1}{4}(3 - y)\, dy = \frac{1}{4} \int_{y=2}^{3} (3 - y)\, dy$$

Simplify:

$$\int_{y=2}^{3} (3 - y)\, dy = \left[3y - \frac{y^2}{2}\right]_2^3 = \left(9 - \frac{9}{2}\right) - \left(6 - \frac{4}{2}\right)$$
$$= \frac{9}{2} - 4 = \frac{5}{2}$$

Thus:

$$P(X > Y) = \frac{1}{4} \cdot \frac{5}{2} = \frac{5}{8} = 0.625$$

Final Answer: 0.625 (rounded off to three decimal places).

Quick Tip

Quick Tip: When dealing with probabilities involving uniform distributions, set up the integration limits carefully to account for the overlapping intervals of the random variables.

Q.57: Let X be a random variable exponentially distributed with parameter $\lambda > 0$. The probability density function (PDF) of X is given by:

$$f_X(x) = \begin{cases} \lambda e^{-\lambda x} & \text{for } x \geq 0, \\ 0 & \text{otherwise.} \end{cases}$$

Given that $\mathbf{E}(X) = \mathbf{Var}(X)$, where $\mathbf{E}(X)$ and $\mathbf{Var}(X)$ indicate the expectation and variance of X, respectively, the value of λ is:

Solution:

The expectation and variance of an exponential random variable with rate parameter λ are:

$$\mathrm{E}(X) = \frac{1}{\lambda}$$
$$\mathrm{Var}(X) = \frac{1}{\lambda^2}$$

Given that $\mathrm{E}(X) = \mathrm{Var}(X)$, we have:

$$\frac{1}{\lambda} = \frac{1}{\lambda^2}$$

Solving for λ:

$$\lambda = 1$$

Final Answer: $\lambda = 1$**.**

Quick Tip

Quick Tip: For exponential distributions, $\mathrm{E}(X)$ and $\mathrm{Var}(X)$ are simple functions of λ, and you can use their properties to solve equations.

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Q.58: Consider two events T and S. Let T denote the complement of the event S. The probability associated with different events are given as follows:

$$P(T) = 0.6, \quad P(S|T) = 0.3, \quad P(S|T) = 0.6.$$

We are asked to find $P(T|S)$, the conditional probability of T given S.

Solution:

Using Bayes' Theorem, we have:

$$P(T|S) = \frac{P(S|T) \cdot P(T)}{P(S)}$$

Now, we need to calculate $P(S)$. We can use the law of total probability:

$$P(S) = P(S|T) \cdot P(T) + P(S|T^c) \cdot P(T^c)$$

Substitute the given values:

$$P(S) = (0.6) \cdot (0.6) + (0.3) \cdot (0.4)$$

$$P(S) = 0.36 + 0.12 = 0.48$$

Now, apply Bayes' Theorem:

$$P(T|S) = \frac{(0.6) \cdot (0.6)}{0.48} = \frac{0.36}{0.48} = 0.75$$

Final Answer: $P(T|S) = 0.75$**.**

Quick Tip

Quick Tip: Bayes' Theorem is a useful tool for calculating conditional probabilities. Be sure to use the law of total probability when needed to calculate $P(S)$.

—

59. Consider a joint probability density function of two random variables X and Y:

$$f_{X,Y}(x, y) = \begin{cases} 2xy, & 0 < x < 2, \quad 0 < y < x \\ 0, & \text{otherwise} \end{cases}$$

Then, $E[Y|X = 1.5]$ **is ____.**

Solution:

Step 1: Compute the Conditional Density Function

The marginal density of X is given by:

$$f_X(x) = \int_0^x 2xy \, dy$$

$$= 2x \int_0^x y \, dy = 2x \times \frac{x^2}{2} = x^3, \quad 0 < x < 2.$$

The conditional density function of Y given $X = x$ is:

$$f_{Y|X}(y|x) = \frac{f_{X,Y}(x, y)}{f_X(x)}$$

$$= \frac{2xy}{x^3} = \frac{2y}{x^2}, \quad 0 < y < x.$$

Step 2: Compute $E[Y|X = 1.5]$

The expectation of Y given $X = x$ is:

$$E[Y|X = x] = \int_0^x y f_{Y|X}(y|x)\, dy$$

$$= \int_0^x y \left(\frac{2y}{x^2}\right) dy$$

$$= \frac{2}{x^2} \int_0^x y^2\, dy$$

$$= \frac{2}{x^2} \times \frac{x^3}{3} = \frac{2x^3}{3x^2} = \frac{2x}{3}.$$

Substituting $x = 1.5$:

$$E[Y|X = 1.5] = \frac{2(1.5)}{3} = 1.$$

Final Answer:

$$\boxed{1}$$

Correct Answer: 1

Quick Tip

The conditional expectation $E[Y|X = x]$ is computed using the conditional density function. The integral of $y f_{Y|X}(y|x)$ over the range of y gives the expected value of Y given $X = x$.

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Q.60: Evaluate the following limit:

$$\lim_{x \to 0} \ln((x^2 + 1)\cos(x))$$

Solution:

First, simplify the expression inside the logarithm:

$$\ln((x^2 + 1)\cos(x)) = \ln(x^2 + 1) + \ln(\cos(x))$$

Now, evaluate the limits of each term separately as $x \to 0$:

$$\lim_{x\to 0} \ln(x^2 + 1) = \ln(1) = 0$$
$$\lim_{x\to 0} \ln(\cos(x)) = \ln(1) = 0$$

Thus, the overall limit is:

$$\lim_{x\to 0} \ln((x^2 + 1)\cos(x)) = 0 + 0 = 0$$

Final Answer: 0**.**

Quick Tip

Quick Tip: When evaluating limits of logarithmic functions, simplify the expression first and evaluate the limits of individual components.

Q.61: Let $u = \begin{pmatrix} 1 \\ 3 \end{pmatrix}$, and let $\sigma_1, \sigma_2, \sigma_3, \ldots$ be the singular values of the matrix $M = uu^T$ (where u^T is the transpose of u). The value of $\sigma_1\sigma_2 \ldots \sigma_n$ is:

Solution:

Given the matrix $M = uu^T$, where $u = \begin{pmatrix} 1 \\ 3 \end{pmatrix}$, we can calculate the singular values of the matrix. The singular values of a matrix M are the square roots of the eigenvalues of the matrix M^TM or MM^T.

1. **Matrix Calculation:**

First, calculate $M = uu^T$:

$$M = \begin{pmatrix} 1 \\ 3 \end{pmatrix} \begin{pmatrix} 1 & 3 \end{pmatrix} = \begin{pmatrix} 1 \cdot 1 & 1 \cdot 3 \\ 3 \cdot 1 & 3 \cdot 3 \end{pmatrix} = \begin{pmatrix} 1 & 3 \\ 3 & 9 \end{pmatrix}$$

2. **Singular Values:**

The matrix M is a 2×2 matrix. To find its singular values, we calculate the eigenvalues of MM^T.

$$MM^T = \begin{pmatrix} 1 & 3 \\ 3 & 9 \end{pmatrix} \begin{pmatrix} 1 & 3 \\ 3 & 9 \end{pmatrix} = \begin{pmatrix} 10 & 30 \\ 30 & 90 \end{pmatrix}$$

To find the eigenvalues, solve the characteristic equation $\det(MM^T - \lambda I) = 0$:

$$\det \begin{pmatrix} 10 - \lambda & 30 \\ 30 & 90 - \lambda \end{pmatrix} = (10 - \lambda)(90 - \lambda) - 30^2 = 0$$

$$\lambda^2 - 100\lambda = 0$$

$$\lambda(\lambda - 100) = 0$$

Thus, the eigenvalues are $\lambda_1 = 0$ and $\lambda_2 = 100$. The singular values of M are the square roots of the eigenvalues:

$$\sigma_1 = \sqrt{100} = 10, \quad \sigma_2 = 0$$

3. **Final Answer:**

Since the product of singular values is $\sigma_1 \cdot \sigma_2$, we have:

$$\sigma_1 \sigma_2 = 10 \times 0 = 0$$

Final Answer: 0

Quick Tip

Quick Tip: When working with singular value decomposition, remember that the singular values are derived from the eigenvalues of $M^T M$ or MM^T. In the case of a rank-1 matrix like this, one singular value will be non-zero and the others will be zero.

Q.62: Details of ten international cricket games between two teams, "Green" and "Blue," are given in Table C. The organization would like to use this information to develop a decision-tree model to predict outcomes of future games. The computed Information Gain InformationGain$(C,$ Pitc
with respect to the Target is:

Solution:

The data in Table C is as follows:

Match Number	Pitch	Format	Winner (Target)
1	S	T	Green
2	S	T	Blue
3	F	O	Blue
4	F	O	Blue
5	F	T	Green
6	F	O	Blue
7	S	O	Green
8	F	S	Blue
9	F	O	Blue
10	S	O	Green

To calculate Information Gain, we need to compute the entropy of the system with respect to the Pitch attribute.

1. **Entropy Calculation:**

- Total entropy $H(C)$ is calculated based on the target variable (Winner): - Total number of matches: 10 - Green wins: 4, Blue wins: 6 - The entropy is:

$$H(C) = -\left(\frac{4}{10}\log_2\frac{4}{10} + \frac{6}{10}\log_2\frac{6}{10}\right)$$

$$H(C) = -\left(0.4\log_2 0.4 + 0.6\log_2 0.6\right)$$

$$H(C) \approx 0.971$$

2. **Entropy for Each Split Based on Pitch (S and F):**

- For Pitch = S: - Number of matches: 4 - Green wins: 2, Blue wins: 2 - Entropy for Pitch = S:

$$H(\text{S}) = -\left(\frac{2}{4}\log_2\frac{2}{4} + \frac{2}{4}\log_2\frac{2}{4}\right) = 1$$

- For Pitch = F: - Number of matches: 6 - Green wins: 2, Blue wins: 4 - Entropy for Pitch = F:

$$H(\mathrm{F}) = -\left(\frac{2}{6}\log_2\frac{2}{6} + \frac{4}{6}\log_2\frac{4}{6}\right)$$

$$H(\mathrm{F}) \approx 0.918$$

3. **Information Gain Calculation:**

The information gain is calculated as:

$$\text{InformationGain}(C, \text{Pitch}) = H(C) - \left(\frac{4}{10}\cdot H(\mathrm{S}) + \frac{6}{10}\cdot H(\mathrm{F})\right)$$

$$\text{InformationGain}(C, \text{Pitch}) = 0.971 - \left(\frac{4}{10}\cdot 1 + \frac{6}{10}\cdot 0.918\right)$$

$$\text{InformationGain}(C, \text{Pitch}) \approx 0.971 - (0.4 + 0.5508) = 0.971 - 0.9508 = 0.0202$$

Final Answer: The Information Gain $\text{InformationGain}(C, \text{Pitch})$ is approximately 0.02 (rounded off to two decimal places).

Quick Tip

Quick Tip: In decision trees, information gain helps determine which attribute to split on by measuring how well it separates the classes. A higher information gain means a better predictor for the target variable.

Q.63: Given the two-dimensional dataset consisting of 5 data points from two classes (circles and squares) and assuming that the Euclidean distance is used to measure the distance between two points, the minimum odd value of k in the k-nearest neighbor algorithm for which the diamond (9) shaped data point is assigned the label ”square” is:

Solution:

We need to find the minimum odd value of k for which the k-nearest neighbor algorithm assigns the label ”square” to the diamond (9) shaped data point.

1. **Understanding the k-Nearest Neighbor Algorithm:** - In k-nearest neighbor (k-NN) classification, the label of a data point is assigned based on the majority class among the k nearest neighbors (based on Euclidean distance in this case). - We are given a dataset with 5 data points from two classes: circles and squares.

2. **Properties of the k-Nearest Neighbor Algorithm:** - To make sure that the classification is based on a majority vote, the value of k must be odd. This avoids ties when there is an equal number of data points from each class among the nearest neighbors.

3. **Assigning the Label ”Square”:** - The task asks for the minimum odd value of k that results in the ”diamond” shaped data point being classified as a ”square.” - The value of k must be chosen such that the number of ”square” labels among the k nearest neighbors is greater than the number of ”circle” labels.

4. **Conclusion:** - Since we are looking for the smallest odd value of k, and there are 5 data points, the smallest odd value for k would be $k = 3$. For $k = 3$, the 3 nearest neighbors are likely to influence the classification based on the majority label among them.

$$\boxed{k = 3}$$

Quick Tip

Quick Tip: When using k-NN classification, always choose an odd value of k to avoid ties, and choose the smallest odd value that allows a majority vote based on the problem's data structure.

Q.64: Given the following Bayesian Network consisting of four Bernoulli random variables and their associated conditional probability tables (CPTs), we need to compute $P(U = 1, V = 1, W = 1, Z = 1)$.

Solution:

From the structure of the Bayesian Network and the conditional probability tables:

- $P(U = 1) = 0.5$ - $P(V = 0|U = 0) = 0.5$, $P(V = 1|U = 0) = 0.5$ - $P(V = 0|U = 1) = 0.5$, $P(V = 1|U = 1) = 0.5$ - $P(W = 0|U = 0) = 1$, $P(W = 1|U = 0) = 0$ - $P(W = 0|U = 1) = 0$, $P(W = 1|U = 1) = 1$ - $P(Z = 0|V = 0, W = 0) = 0.5$, $P(Z = 1|V = 0, W = 0) = 0.5$ - $P(Z = 0|V = 0, W = 1) = 1$, $P(Z = 1|V = 0, W = 1) = 0$ - $P(Z = 0|V = 1, W = 0) = 1$, $P(Z = 1|V = 1, W = 0) = 0$ - $P(Z = 0|V = 1, W = 1) = 0.5$, $P(Z = 1|V = 1, W = 1) = 0.5$

The goal is to compute $P(U = 1, V = 1, W = 1, Z = 1)$.

We can break this down using the chain rule for Bayesian networks:

$$P(U=1,V=1,W=1,Z=1) = P(U=1)\cdot P(V=1|U=1)\cdot P(W=1|U=1)\cdot P(Z=1|V=1,W=1)$$

Substitute the values from the given probabilities:

$$P(U=1) = 0.5, \quad P(V=1|U=1) = 0.5, \quad P(W=1|U=1) = 1, \quad P(Z=1|V=1,W=1) = 0.5$$

Now, multiplying these together:

$$P(U=1,V=1,W=1,Z=1) = 0.5 \times 0.5 \times 1 \times 0.5 = 0.125$$

Thus, the value of $P(U=1,V=1,W=1,Z=1)$ is:

$$\boxed{0.125}$$

Quick Tip

Quick Tip: When working with Bayesian networks, use the chain rule to decompose the joint probability into conditional probabilities and multiply the corresponding values from the CPTs to find the result.

Q.65: Two fair coins are tossed independently. Let X be a random variable that takes a value of 1 if both tosses are heads, and 0 otherwise. Let Y be a random variable that takes a value of 1 if at least one of the tosses is heads, and 0 otherwise. The value of the covariance of X and Y is:

Solution:

We need to compute the covariance between X and Y, which is given by the formula:

$$\text{Cov}(X,Y) = \mathbb{E}[XY] - \mathbb{E}[X]\mathbb{E}[Y]$$

Step 1: Define the possible outcomes for the coin tosses

Since we are tossing two fair coins, the possible outcomes are:

$$\{\text{HH}, \text{HT}, \text{TH}, \text{TT}\}$$

Each outcome has a probability of $\frac{1}{4}$.

Step 2: Compute the values of X and Y for each outcome

- For HH, both tosses are heads, so $X = 1$ and $Y = 1$. - For HT, $X = 0$ (since both tosses are not heads), and $Y = 1$ (since at least one toss is heads). - For TH, $X = 0$, and $Y = 1$. - For TT, $X = 0$, and $Y = 0$.

Step 3: Compute the expected values of X and Y

We calculate $\mathbb{E}[X]$ and $\mathbb{E}[Y]$:

$$\mathbb{E}[X] = \frac{1}{4}(1 + 0 + 0 + 0) = \frac{1}{4}$$

$$\mathbb{E}[Y] = \frac{1}{4}(1 + 1 + 1 + 0) = \frac{3}{4}$$

Step 4: Compute $\mathbb{E}[XY]$

Next, we calculate $\mathbb{E}[XY]$:

$$\mathbb{E}[XY] = \frac{1}{4}(1 \cdot 1 + 0 \cdot 1 + 0 \cdot 1 + 0 \cdot 0) = \frac{1}{4}$$

Step 5: Compute the covariance

Now we can compute the covariance:

$$\text{Cov}(X, Y) = \mathbb{E}[XY] - \mathbb{E}[X]\mathbb{E}[Y]$$

Substitute the values:

$$\text{Cov}(X, Y) = \frac{1}{4} - \left(\frac{1}{4} \times \frac{3}{4}\right)$$

$$\text{Cov}(X, Y) = \frac{1}{4} - \frac{3}{16} = \frac{4}{16} - \frac{3}{16} = \frac{1}{16}$$

Thus, the value of the covariance of X and Y is:

$$\boxed{\frac{1}{16}}$$

Quick Tip

Quick Tip: To compute the covariance of two random variables, use the formula $\text{Cov}(X, Y) = \mathbb{E}[XY] - \mathbb{E}[X]\mathbb{E}[Y]$, and remember to carefully consider the possible outcomes of each random variable.