Notations and Useful Data

$\mathbb{N}$	The set of positive integers
$\mathbb{R}$	The set of real numbers
$\mathbb{R}^n$	$\{(x_1, x_2, \dots, x_n): x_i \in \mathbb{R}, i = 1, 2, \dots, n\}, n = 2, 3, \dots$
$\ln x$	Natural logarithm of $x, x > 0$
$\det(M)$	Determinant of a square matrix M
$adj\ M$	Adjoint of a square matrix M, that is, transpose of cofactor matrix of M
$\emptyset$	Empty set
E^C	Complement of event E
$P(E)$	Probability of event E
$P(E\|F)$	Conditional probability of event E given the occurrence of event F
$E(X)$	Expectation of a random variable X
$Var(X)$	Variance of a random variable X
$Cov(X, Y)$	Covariance between random variables X and Y
$Bin(n, p)$	Binomial distribution with parameters n and p, $n \in \mathbb{N}, 0 < p < 1$
$U(a, b)$	Continuous uniform distribution on the interval (a, b), $a < b, a, b \in \mathbb{R}$
$Exp(\lambda)$	Exponential distribution with the probability density function $f(x) = \begin{cases} \lambda e^{-\lambda x}, & \text{if } x > 0 \\ 0, & \text{if } x \le 0 \end{cases}$, for $\lambda > 0$.
$N(\mu, \sigma^2)$	Normal distribution with mean μ and variance σ^2, $\mu \in \mathbb{R}, \sigma > 0$
$N_2(\mu_1, \mu_2, \sigma_1^2, \sigma_2^2, \rho)$	Bivariate normal distribution with means μ_1, μ_2, variances σ_1^2, σ_2^2 and correlation ρ, $\mu_1 \in \mathbb{R}$, $\mu_2 \in \mathbb{R}$, $\sigma_1 > 0$, $\sigma_2 > 0$, $-1 < \rho < 1$
$\phi(\cdot)$	The probability density function of $N(0, 1)$ random variable
$\Phi(\cdot)$	The cumulative distribution function of $N(0,1)$ random variable
χ_n^2	Central chi-square distribution with n degrees of freedom, $n = 1,2, \dots$
t_n	Central Student's t distribution with n degrees of freedom, $n = 1,2, \dots$
$F_{m,n}$	Snedecor's central F-distribution with m and n degrees of freedom, $m, n \in \mathbb{N}$
$\chi_{n,\alpha}^2$	A constant such that $P\left(X > \chi_{n,\alpha}^2\right) = \alpha$, where X has central chi-square distribution with n degrees of freedom, $n = 1,2, \dots; \alpha \in (0, 1)$
$t_{n,\alpha}$	A constant such that $P\left(X > t_{n,\alpha}\right) = \alpha$, where X has central Student's t distribution with n degrees of freedom, $n = 1,2, \dots; \alpha \in (0, 1)$
$\xrightarrow{d}$	Convergence in distribution
$\xrightarrow{P}$	Convergence in probability
i. i. d.	Independent and identically distributed

Section A: Q.1 – Q.10 Carry ONE mark each.	
Q.1	Let $a_n = 1 + \frac{1}{2} + \cdots + \frac{1}{n}$ and $b_n = \frac{n^2}{2^n}$ for all $n \in \mathbb{N}$. Then
(A)	$\{a_n\}$ is a Cauchy sequence but $\{b_n\}$ is NOT a Cauchy sequence
(B)	$\{a_n\}$ is NOT a Cauchy sequence but $\{b_n\}$ is a Cauchy sequence
(C)	both $\{a_n\}$ and $\{b_n\}$ are Cauchy sequences
(D)	neither $\{a_n\}$ nor $\{b_n\}$ is a Cauchy sequence
Q.2	Let $f(x, y) = 2x^4 - 3y^2$ for all $(x, y) \in \mathbb{R}^2$. Then
(A)	f has a point of local minimum
(B)	f has a point of local maximum
(C)	f has a saddle point
(D)	f has no point of local minimum, no point of local maximum, and no saddle point

Q.3	Let $A = \begin{pmatrix} a & 0 \\ c & d \end{pmatrix}$ be a real matrix, where $ad = 1$ and $c \neq 0$. If $$A^{-1} + (adj\ A)^{-1} = \begin{pmatrix} \alpha & \beta \\ \gamma & \delta \end{pmatrix},$$ then $(\alpha, \beta, \gamma, \delta)$ is equal to
(A)	$(a + d,\ 0,\ 0,\ a + d)$
(B)	$(a + d,\ 0,\ c,\ a + d)$
(C)	$(a,\ 0,\ 0,\ d\ )$
(D)	$(a,\ 0,\ c,\ d\ )$

Q.4	A bag has 5 blue balls and 15 red balls. Three balls are drawn at random from the bag simultaneously. Then the probability that none of the chosen balls is blue equals
(A)	$\frac{75}{152}$
(B)	$\frac{91}{228}$
(C)	$\frac{27}{64}$
(D)	$\frac{273}{800}$
Q.5	Let Y be a continuous random variable such that $P(Y > 0) = 1$ and $E(Y) = 1$. For $p \in (0,1)$, let ξ_p denote the p^{th} quantile of the probability distribution of the random variable Y. Then which of the following statements is always correct?
(A)	$\xi_{0.75} \geq 5$
(B)	$\xi_{0.75} \leq 4$
(C)	$\xi_{0.25} \geq 4$
(D)	$\xi_{0.25} = 2$

Q.6	Let X be a continuous random variable having the $U(-2,3)$ distribution. Then which of the following statements is correct?
(A)	$2X+5$ has the $U(1,10)$ distribution
(B)	$7-6X$ has the $U(-11,19)$ distribution
(C)	$3X^2+5$ has the $U(5,32)$ distribution
(D)	$\lvert X\rvert$ has the $U(0,3)$ distribution
Q.7	Let X be a random variable having the Poisson distribution with mean 1. Let $g: \mathbb{N} \cup \{0\} \to \mathbb{R}$ be defined by $$g(x) = \begin{cases} 1, & \text{if } x \in \{0,2\} \\ 0, & \text{if } x \notin \{0,2\} \end{cases}.$$ Then $E(g(X))$ is equal to
(A)	e^{-1}
(B)	$2e^{-1}$
(C)	$\frac{5}{2}\, e^{-1}$
(D)	$\frac{3}{2}\, e^{-1}$

Q.8	For $n \in \mathbb{N}$, let Z_n be the smallest order statistic based on a random sample of size n from the $U(0,1)$ distribution. Let $nZ_n \xrightarrow{d} Z$, as $n \to \infty$, for some random variable Z. Then $P(Z \leq \ln 3)$ is equal to
(A)	$\frac{1}{4}$
(B)	$\frac{2}{3}$
(C)	$\frac{3}{4}$
(D)	$\frac{1}{3}$
Q.9	Let $X_1, X_2, \dots, X_{20}$ be a random sample from the $N(5, 2)$ distribution and let $Y_i = X_{2i} - X_{2i-1},\ i = 1,2,\dots,10$. Then $W = \frac{1}{4}\sum_{i=1}^{10} Y_i^2$ has the
(A)	t_{20} distribution
(B)	χ^2_{20} distribution
(C)	χ^2_{10} distribution
(D)	$N(250, 20)$ distribution

Q.10	Let x_1, x_2, x_3, x_4 be the observed values of a random sample from a $N(\mu, \sigma^2)$ distribution, where $\mu \in \mathbb{R}$ and $\sigma \in (0, \infty)$ are unknown parameters. Let $\bar{x}$ and $s = \sqrt{\frac{1}{3}\sum_{i=1}^{4}(x_i - \bar{x})^2}$ be the observed sample mean and the sample standard deviation, respectively. For testing $H_0: \mu = 0$ against $H_1: \mu \neq 0$, the likelihood ratio test of size $\alpha = 0.05$ rejects H_0 if and only if $\frac{\lvert\bar{x}\rvert}{s} > k$. Then the value of k is
(A)	$\frac{1}{2} t_{3,0.025}$
(B)	$t_{3,0.025}$
(C)	$2t_{3,0.05}$
(D)	$\frac{1}{2} t_{3,0.05}$

Section A: Q.11 – Q.30 Carry TWO marks each.	
Q.11	For $n \in \mathbb{N}$, let $a_n = \sqrt{n}\sin^2\left(\frac{1}{n}\right)\cos n$, and $b_n = \sqrt{n}\sin\left(\frac{1}{n^2}\right)\cos n$. Then
(A)	the series $\sum_{n=1}^{\infty} a_n$ converges but the series $\sum_{n=1}^{\infty} b_n$ does NOT converge
(B)	the series $\sum_{n=1}^{\infty} a_n$ does NOT converge but the series $\sum_{n=1}^{\infty} b_n$ converges
(C)	both the series $\sum_{n=1}^{\infty} a_n$ and $\sum_{n=1}^{\infty} b_n$ converge
(D)	neither the series $\sum_{n=1}^{\infty} a_n$ nor the series $\sum_{n=1}^{\infty} b_n$ converges

Q.12	Let $f_i: \mathbb{R} \to \mathbb{R}, i = 1,2$, be defined by $$f_1(x) = \begin{cases} \sin\left(\frac{1}{x}\right) + \cos\left(\frac{1}{x}\right), & \text{if } x \neq 0 \\ 0, & \text{if } x = 0 \end{cases}$$ and $$f_2(x) = \begin{cases} x\left(\sin\left(\frac{1}{x}\right) + \cos\left(\frac{1}{x}\right)\right), & \text{if } x \neq 0 \\ 0, & \text{if } x = 0 \end{cases}.$$ Then
(A)	f_1 is continuous at 0 but f_2 is NOT continuous at 0
(B)	f_1 is NOT continuous at 0 but f_2 is continuous at 0
(C)	both f_1 and f_2 are continuous at 0
(D)	neither f_1 nor f_2 is continuous at 0

Q.13	Let $f(x,y) = \|xy\| + x$ for all $(x,y) \in \mathbb{R}^2$. Then the partial derivative of f with respect to x exists
(A)	at $(0,0)$ but NOT at $(0,1)$
(B)	at $(0,1)$ but NOT at $(0,0)$
(C)	at $(0,0)$ and $(0,1)$, both
(D)	neither at $(0,0)$ nor at $(0,1)$
Q.14	Let $f(x) = 4x^2 - \sin x + \cos 2x$ for all $x \in \mathbb{R}$. Then f has
(A)	a point of local maximum
(B)	no point of local minimum
(C)	exactly one point of local minimum
(D)	at least two points of local minima

Q.15	Consider the improper integrals $$I_1 = \int_1^{\infty} \frac{t \sin t}{e^t}\, dt \quad \text{and} \quad I_2 = \int_1^{\infty} \frac{1}{\sqrt{t}} \ln\left(1 + \frac{1}{t}\right) dt.$$ Then
(A)	I_1 converges but I_2 does NOT converge
(B)	I_1 does NOT converge but I_2 converges
(C)	both I_1 and I_2 converge
(D)	neither I_1 nor I_2 converges

Q.16	Let A be a 3×5 matrix defined by $$A = \begin{pmatrix} 0 & 1 & 3 & 1 & 2 \\ 1 & 6 & 2 & 3 & 4 \\ 1 & 8 & 8 & 5 & 8 \end{pmatrix}.$$ Consider the system of linear equations given by $$A \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{pmatrix} = \begin{pmatrix} 3 \\ 1 \\ 10 \end{pmatrix},$$ where x_1, x_2, x_3, x_4, x_5 are real variables. Then
(A)	the rank of A is 2 and the given system has a solution
(B)	the rank of A is 2 and the given system does NOT have a solution
(C)	the rank of A is 3 and the given system has a solution
(D)	the rank of A is 3 and the given system does NOT have a solution

Q.17	Let $\Omega = \{1,2,3,4,5,6\}$. Then which of the following classes of sets is an algebra?
(A)	$\mathcal{F}_1 = \{\emptyset, \Omega, \{1,2\}, \{3,4\}, \{3,6\}\}$
(B)	$\mathcal{F}_2 = \{\emptyset, \Omega, \{1,2,3\}, \{4,5,6\}\}$
(C)	$\mathcal{F}_3 = \{\emptyset, \Omega, \{1,2\}, \{4,5\}, \{1,2,4,5\}, \{3,4,5,6\}, \{1,2,3,6\}\}$
(D)	$\mathcal{F}_4 = \{\emptyset, \{4,5\}, \{1,2,3,6\}\}$
Q.18	Two fair coins S_1 and S_2 are tossed independently once. Let the events E, F and G be defined as follows: E: Head appears on S_1 F: Head appears on S_2 G: The same outcome (head or tail) appears on both S_1 and S_2 Then which of the following statements is NOT correct?
(A)	E and F are independent
(B)	F and G are independent
(C)	E and G^C are independent
(D)	$E, F,$ and G are mutually independent

Q.19	Let $f_1(x)$ be the probability density function of the $N(0,1)$ distribution and $f_2(x)$ be the probability density function of the $N(0,6)$ distribution. Let Y be a random variable with probability density function $$f(x) = 0.6\, f_1(x) + 0.4\, f_2(x), \quad -\infty < x < \infty.$$ Then $Var(Y)$ is equal to
(A)	7
(B)	3
(C)	3.5
(D)	1

Q.20	Which of the following functions represents a cumulative distribution function?
(A)	$F_1(x) = \begin{cases} 0\,, & \text{if } x < \frac{\pi}{4} \\ \sin x\,, & \text{if } \frac{\pi}{4} \le x < \frac{3\pi}{4} \\ 1\,, & \text{if } x \ge \frac{3\pi}{4} \end{cases}$
(B)	$F_2(x) = \begin{cases} 0\,, & \text{if } x < 0 \\ 2\sin x\,, & \text{if } 0 \le x < \frac{\pi}{4} \\ 1\,, & \text{if } x \ge \frac{\pi}{4} \end{cases}$
(C)	$F_3(x) = \begin{cases} 0\,, & \text{if } x < 0 \\ x\,, & \text{if } 0 \le x < \frac{1}{3} \\ x + \frac{1}{3}\,, & \text{if } \frac{1}{3} \le x \le \frac{1}{2} \\ 1\,, & \text{if } x > \frac{1}{2} \end{cases}$
(D)	$F_4(x) = \begin{cases} 0\,, & \text{if } x < 0 \\ \sqrt{2}\sin x\,, & \text{if } 0 \le x < \frac{\pi}{4} \\ 1\,, & \text{if } x \ge \frac{\pi}{4} \end{cases}$

Q.21	Let X be a random variable such that X and $-X$ have the same distribution. Let $Y = X^2$ be a continuous random variable with the probability density function $$g(y) = \begin{cases} \dfrac{e^{-\frac{y}{2}}}{\sqrt{2\pi y}}, & \text{if } y > 0 \\ 0, & \text{if } y \leq 0 \end{cases}.$$ Then $E((X-1)^4)$ is equal to
(A)	9
(B)	10
(C)	11
(D)	12

Q.22	Suppose that random variable X has $Exp\left(\frac{1}{5}\right)$ distribution and, for any $x > 0$, the conditional distribution of random variable Y, given $X = x$, is $N(x, 2)$. Then $Var(X + Y)$ is equal to
(A)	52
(B)	50
(C)	2
(D)	102

Q.23	Let the random vector (X, Y) have the joint probability density function $$f(x,y) = \begin{cases} \frac{1}{x}, & \text{if } 0 < y < x < 1 \\ 0, & \text{otherwise} \end{cases}.$$ Then $Cov(X, Y)$ is equal to
(A)	$\frac{1}{6}$
(B)	$\frac{1}{12}$
(C)	$\frac{1}{18}$
(D)	$\frac{1}{24}$

Q.24	Let $(X_1, Y_1), (X_2, Y_2), \dots, (X_{20}, Y_{20})$ be a random sample from the $N_2\left(0, 0, 1, 1, \frac{3}{4}\right)$ distribution. Define $\bar{X} = \frac{1}{20}\sum_{i=1}^{20} X_i$ and $\bar{Y} = \frac{1}{20}\sum_{i=1}^{20} Y_i$. Then $Var(\bar{X} - \bar{Y})$ is equal to
(A)	$\frac{1}{16}$
(B)	$\frac{1}{40}$
(C)	$\frac{1}{10}$
(D)	$\frac{3}{40}$

Q.25	For $n \in \mathbb{N}$, let X_n be a random variable having the $Bin\left(n, \frac{1}{4}\right)$ distribution. Then $$\lim_{n\to\infty}\left[P\left(X_n \leq \frac{2n-\sqrt{3n}}{8}\right) + P\left(\frac{n}{6} \leq X_n \leq \frac{n}{3}\right)\right]$$ is equal to (You may use $\Phi(0.5) = 0.6915,\ \Phi(1) = 0.8413,\ \Phi(1.5) = 0.9332,\ \Phi(2) = 0.9772$)
(A)	1.6915
(B)	1.3085
(C)	1.1587
(D)	0.6915

Q.26	Let $X_1, X_2, \dots, X_{10}$ be a random sample from the $N(3,4)$ distribution and let $Y_1, Y_2, \dots, Y_{15}$ be a random sample from the $N(-3,6)$ distribution. Assume that the two samples are drawn independently. Define $$\bar{X} = \frac{1}{10}\sum_{i=1}^{10} X_i,\ \ \bar{Y} = \frac{1}{15}\sum_{j=1}^{15} Y_j,\ \text{ and }\ S = \sqrt{\frac{1}{9}\sum_{i=1}^{10}(X_i - \bar{X})^2}.$$ Then the distribution of $U = \frac{\sqrt{5}(\bar{X}+\bar{Y})}{S}$ is
(A)	$N\left(0, \frac{4}{5}\right)$
(B)	χ_9^2
(C)	t_9
(D)	t_{23}

Q.27	For $n \geq 2$, let $\epsilon_1, \epsilon_2, \dots, \epsilon_n$ be i.i.d. random variables having the $N(0,1)$ distribution. Consider n independent random variables $Y_1, Y_2, \dots, Y_n$ defined by $$Y_i = \beta i + \epsilon_i, \quad i = 1,2,\dots,n,$$ where $\beta \in \mathbb{R}$. Define $\bar{Y} = \frac{1}{n}\sum_{i=1}^{n} Y_i$, $T_1 = \frac{2\bar{Y}}{n+1}$, and $T_2 = \frac{1}{n}\sum_{i=1}^{n} \frac{Y_i}{i}$. Then which of the following statements is NOT correct?
(A)	T_1 is an unbiased estimator of β
(B)	T_2 is an unbiased estimator of β
(C)	$Var(T_1) < Var(T_2)$
(D)	$Var(T_1) = Var(T_2)$

Q.28	A biased coin, with probability of head as p, is tossed m times independently. It is known that $p \in \left\{\frac{1}{4}, \frac{3}{4}\right\}$ and $m \in \{3, 5\}$. If 3 heads are observed in these m tosses, then which of the following statements is correct?
(A)	$\left(3, \frac{3}{4}\right)$ is a maximum likelihood estimator of (m, p)
(B)	$\left(5, \frac{1}{4}\right)$ is a maximum likelihood estimator of (m, p)
(C)	$\left(5, \frac{3}{4}\right)$ is a maximum likelihood estimator of (m, p)
(D)	Maximum likelihood estimator of (m, p) is NOT unique

Q.29	Let $X_1, X_2, \dots, X_n$ be a random sample from an $Exp(\lambda)$ distribution, where $\lambda \in \{1, 2\}$. For testing $H_0: \lambda = 1$ against $H_1: \lambda = 2$, the most powerful test of size α, $\alpha \in (0,1)$, will reject H_0 if and only if
(A)	$\sum_{i=1}^{n} X_i \leq \frac{1}{2} \chi^2_{2n,1-\alpha}$
(B)	$\sum_{i=1}^{n} X_i \geq 2\, \chi^2_{2n,1-\alpha}$
(C)	$\sum_{i=1}^{n} X_i \leq \frac{1}{2} \chi^2_{n,1-\alpha}$
(D)	$\sum_{i=1}^{n} X_i \geq 2\, \chi^2_{n,1-\alpha}$

Q.30	Let $X_1, X_2, \dots, X_{10}$ be a random sample from a $N(0, \sigma^2)$ distribution, where $\sigma > 0$ is unknown. For testing $H_0: \sigma^2 \leq 1$ against $H_1: \sigma^2 > 1$, a test of size $\alpha = 0.05$ rejects H_0 if and only if $\sum_{i=1}^{10} X_i^2 > 18.307$. Let β be the power of this test, at $\sigma^2 = 2$. Then β lies in the interval (You may use $\chi^2_{10,0.05} = 18.307$, $\chi^2_{10,0.1} = 15.9872$, $\chi^2_{10,0.25} = 12.5489$, $\chi^2_{10,0.5} = 9.3418$, $\chi^2_{10,0.75} = 6.7372$, $\chi^2_{10,0.9} = 4.8652$, $\chi^2_{10,0.95} = 3.9403$, $\chi^2_{10,0.975} = 3.247$)
(A)	$(0.50, 0.75)$
(B)	$(0.75, 0.90)$
(C)	$(0.90, 0.95)$
(D)	$(0.95, 0.975)$

Section B: Q.31 – Q.40 Carry TWO marks each.	
Q.31	Let $a_1 = 1, a_{n+1} = a_n\left(\frac{\sqrt{n}+\sin n}{n}\right)$ and $b_n = a_n^2$ for all $n \in \mathbb{N}$. Then which of the following statements is/are correct?
(A)	the series $\sum_{n=1}^{\infty} a_n$ converges
(B)	the series $\sum_{n=1}^{\infty} b_n$ converges
(C)	the series $\sum_{n=1}^{\infty} a_n$ converges but the series $\sum_{n=1}^{\infty} b_n$ does NOT converge
(D)	neither the series $\sum_{n=1}^{\infty} a_n$ nor the series $\sum_{n=1}^{\infty} b_n$ converges

Q.32	Let $f: \mathbb{R} \to \mathbb{R}$ be a twice differentiable function such that $f(0) = 0, f(2) = 4, f(4) = 4$ and $f(8) = 12$. Then which of the following statements is/are correct?
(A)	$f'(x) \leq 1$ for all $x \in [0, 2]$
(B)	$f'(x_1) > 1$ for some $x_1 \in [0, 2]$
(C)	$f'(x_2) > 1$ for some $x_2 \in [4, 8]$
(D)	$f''(x_3) = 0$ for some $x_3 \in [0, 8]$
Q.33	Let A be a 3×3 real matrix. Suppose that 1 and 2 are characteristic roots of A, and 12 is a characteristic root of $A + A^2$. Then which of the following statements is/are correct?
(A)	$\det(A) \neq 0$
(B)	$\det(A + A^2) \neq 0$
(C)	$\det(A) = 0$
(D)	trace of $(A + A^2)$ is 20

Q.34	Consider four dice D_1, D_2, D_3, and D_4, each having six faces marked as follows:

Die	Marks on faces
D_1	$4, 4, 4, 4, 0, 0$
D_2	$3, 3, 3, 3, 3, 3$
D_3	$6, 6, 2, 2, 2, 2$
D_4	$5, 5, 5, 1, 1, 1$

In each roll of a die, each of its six faces is equally likely to occur. Suppose that each of these four dice is rolled once, and the marks on their upper faces are noted. Let the four rolls be independent. If X_i denotes the mark on the upper face of the die D_i, $i = 1, 2, 3, 4$, then which of the following statements is/are correct?

(A)	$P(X_1 > X_2) = \frac{2}{3}$
(B)	$P(X_3 > X_4) = \frac{2}{3}$
(C)	$P(X_2 > X_3) = \frac{1}{3}$
(D)	The events $\{X_1 > X_2\}$ and $\{X_2 > X_3\}$ are independent

Q.35	Let X be a continuous random variable with a probability density function f and the moment generating function $M(t)$. Suppose that $f(x) = f(-x)$ for all $x \in \mathbb{R}$ and the moment generating function $M(t)$ exists for $t \in (-1, 1)$. Then which of the following statements is/are correct?
(A)	$P(X = -X) = 1$
(B)	0 is the median of X
(C)	$M(t) = M(-t)$ for all $t \in (-1, 1)$
(D)	$E(X) = 1$
Q.36	Let X and Y be independent random variables having $Bin(18, 0.5)$ and $Bin(20, 0.5)$ distributions, respectively. Further, let $U = \min\{X, Y\}$ and $V = \max\{X, Y\}$. Then which of the following statements is/are correct?
(A)	$E(U + V) = 19$
(B)	$E(\|X - Y\|) = E(V - U)$
(C)	$Var(U + V) = 16$
(D)	$38 - (X + Y)$ has $Bin(38, 0.5)$ distribution

Q.37	Let X and Y be continuous random variables having the joint probability density function $$f(x,y) = \begin{cases} e^{-x}, & \text{if } 0 \le y < x < \infty \\ 0, & \text{otherwise} \end{cases}.$$ Then which of the following statements is/are correct?
(A)	$P(Y^2 = 3X) = 0$
(B)	$P(X > 2Y) = \frac{1}{2}$
(C)	$P(X - Y \geq 1) = e^{-1}$
(D)	$P(X > \ln 2 \mid Y > \ln 3) = 0$

Q.38	For $n \geq 2$, let $X_1, X_2, \dots, X_n$ be a random sample from a distribution with $E(X_1) = 0$, $Var(X_1) = 1$ and $E(X_1^4) < \infty$. Let $$\bar{X}_n = \frac{1}{n}\sum_{i=1}^{n} X_i \text{ and } S_n^2 = \frac{1}{n-1}\sum_{i=1}^{n}(X_i - \bar{X}_n)^2.$$ Then which of the following statements is/are always correct?
(A)	$E(S_n^2) = 1$ for all $n \geq 2$
(B)	$\sqrt{n}\,\bar{X}_n \xrightarrow{d} Z$ as $n \to \infty$, where Z has the $N(0,1)$ distribution
(C)	$\bar{X}_n$ and S_n^2 are independently distributed for all $n \geq 2$
(D)	$\frac{1}{n}\sum_{i=1}^{n} X_i^2 \xrightarrow{P} 2$, as $n \to \infty$

Q.39	Let $X_1, X_2, \dots, X_{50}$ be a random sample from a $N(0, \sigma^2)$ distribution, where $\sigma > 0$. Define $$\bar{X}_e = \frac{1}{25}\sum_{i=1}^{25} X_{2i}\,, \qquad \bar{X}_o = \frac{1}{25}\sum_{i=1}^{25} X_{2i-1}\,,$$ $$S_e = \sqrt{\frac{1}{24}\sum_{i=1}^{25}(X_{2i} - \bar{X}_e)^2} \quad \text{and} \quad S_o = \sqrt{\frac{1}{24}\sum_{i=1}^{25}(X_{2i-1} - \bar{X}_o)^2}\,.$$ Then which of the following statements is/are correct?
(A)	$\frac{5\bar{X}_e}{S_e}$ has t_{24} distribution
(B)	$\frac{5(\bar{X}_e + \bar{X}_o)}{\sqrt{S_e^2 + S_o^2}}$ has t_{49} distribution
(C)	$\frac{49 S_o^2}{\sigma^2}$ has χ^2_{49} distribution
(D)	$\frac{S_o^2}{S_e^2}$ has $F_{24,24}$ distribution

Q.40	Let θ_0 and θ_1 be real constants such that $\theta_1 > \theta_0$. Suppose that a random sample is taken from a $N(\theta, 1)$ distribution, $\theta \in \mathbb{R}$. For testing $H_0: \theta = \theta_0$ against $H_1: \theta = \theta_1$ at level 0.05, let α and β denote the size and the power, respectively, of the most powerful test, ψ_0. Then which of the following statements is/are correct?
(A)	$\beta < \alpha$
(B)	The test ψ_0 is the uniformly most powerful test of level α for testing $H_0: \theta = \theta_0$ against $H_1: \theta > \theta_0$
(C)	$\alpha < \beta$
(D)	The test ψ_0 is the uniformly most powerful test of level α for testing $H_0: \theta = \theta_0$ against $H_1: \theta < \theta_0$

Section C: Q.41 – Q.50 Carry ONE mark each.	
Q.41	The radius of convergence of the power series $\sum_{n=0}^{\infty} \frac{2n(x+3)^n}{5^n}$ is equal to ___________ (answer in integer)
Q.42	Let $f(x) = \int_{-1}^{x^2-2x} e^{t^2-t}\, dt$ for all $x \in \mathbb{R}$. If f is decreasing on $(0, m)$ and increasing on (m, ∞), then the value of m is equal to ___________ (answer in integer)
Q.43	Let $V = \{(x_1, x_2, x_3, x_4) \in \mathbb{R}^4 : x_1 = x_2\}$. Consider V as a subspace of $\mathbb{R}^4$ over the real field. Then the dimension of V is equal to ___________ (answer in integer)

Q.44	If 12 fair dice are independently rolled, then the probability of obtaining at least two sixes is equal to ___________ (round off to 2 decimal places)
Q.45	Let X be a random variable with the moment generating function $$M(t) = \frac{(1+3e^t)^2}{16}\,, -\infty < t < \infty\,.$$ Let $\alpha = E(X) - Var(X)$. Then the value of 8α is equal to ___________ (answer in integer)
Q.46	For $n \in \mathbb{N}$, let $X_1, X_2, \dots, X_n$ be a random sample from the Cauchy distribution having probability density function $$f(x) = \frac{1}{\pi(1+x^2)}\,, -\infty < x < \infty.$$ Let $g: \mathbb{R} \to \mathbb{R}$ be defined by $$g(x) = \begin{cases} x, & \text{if } -1000 \le x \le 1000 \\ 0, & \text{otherwise} \end{cases}\,.$$ Let $$\alpha = \lim_{n\to\infty} P\left(\frac{1}{n^{\frac{3}{4}}} \sum_{i=1}^{n} g(X_i) > \frac{1}{2}\right).$$ Then 100α is equal to ___________ (answer in integer)

Q.47	For $n \in \mathbb{N}$, let $X_1, X_2, \dots, X_n$ be a random sample from the $F_{20,40}$ distribution. Then, as $n \to \infty$, $\frac{1}{n}\sum_{i=1}^{n}\frac{1}{X_i}$ converges in probability to __________ (round off to 2 decimal places)
Q.48	Let $X_1, X_2, \dots, X_{10}$ be a random sample from the $Exp(1)$ distribution. Define $W = \max\{e^{-X_1}, e^{-X_2}, \dots, e^{-X_{10}}\}$. Then the value of $22E(W)$ is equal to __________ (answer in integer)
Q.49	Let X_1, X_2, X_3 be i.i.d. random variables from a continuous distribution having probability density function $f(x) = \begin{cases} \frac{1}{2x^3}, & \text{if } x > \frac{1}{2} \\ 0, & \text{if } x \leq \frac{1}{2} \end{cases}$. Let $X_{(1)} = \min\{X_1, X_2, X_3\}$. Then the value of $10E\left(X_{(1)}\right)$ is equal to __________ (answer in integer)

Q.50	Suppose that the lifetimes (in months) of bulbs manufactured by a company have an $Exp(\lambda)$ distribution, where $\lambda > 0$. A random sample of size 10 taken from the bulbs manufactured by the company yields the sample mean lifetime $\bar{x} = 3.52$ months. Then the uniformly minimum variance unbiased estimate of $\frac{1}{\lambda}$ based on this sample is equal to __________ months (round off to 2 decimal places)
Section C: Q.51 – Q.60 Carry TWO marks each.	
Q.51	The value of $\lim_{n\to\infty} n\left(\sin\frac{1}{2n} - \frac{1}{2}e^{-\frac{1}{n}} + \frac{1}{2}\right)$ is equal to __________ (answer in integer)
Q.52	The value of the integral $$\int_0^2 \int_x^{\sqrt{8-x^2}} \frac{3\sqrt{x^2+y^2}}{\sqrt{8}\,\pi}\, dy\, dx$$ is equal to __________ (answer in integer)

Q.53	For some $a \leq 0$ and $b \in \mathbb{R}$, let $$A = \begin{pmatrix} 0 & a & b \\ -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} \end{pmatrix}.$$ If A is an orthogonal matrix, then the value of $a\sqrt{6} + 4b\sqrt{3}$ is equal to __________ (answer in integer)
Q.54	Two factories F_1 and F_2 produce cricket bats that are labelled. Any randomly chosen bat produced by factory F_1 is defective with probability 0.5 and any randomly chosen bat produced by factory F_2 is defective with probability 0.1. One of the factories is chosen at random, and two bats are randomly purchased from the chosen factory. Let the labels on these purchased bats be B_1 and B_2. If B_1 is found to be defective, then the conditional probability that B_2 is also defective is equal to __________ (round off to 2 decimal places)

Q.55	Let X be a discrete random variable with $P(X \in \{-5, -3, \ 0, \ 3, \ 5\}) = 1$. Suppose that $P(X = -3) = P(X = -5)$, $P(X = 3) = P(X = 5)$ and $P(X > 0) = P(X = 0) = P(X < 0)$. Then the value of $12P(X = 3)$ is equal to __________ (answer in integer)
Q.56	Consider a coin for which the probability of obtaining head in a single toss is $\frac{1}{3}$. Sunita tosses the coin once. If head appears, she receives a random amount of X rupees, where X has the $Exp\left(\frac{1}{9}\right)$ distribution. If tail appears, she loses a random amount of Y rupees, where Y has the $Exp\left(\frac{1}{3}\right)$ distribution. Her expected gain (in rupees) is equal to __________ (answer in integer)

Q.57	Let Θ be a random variable having $U(0, 2\pi)$ distribution. Let $X = \cos\Theta$ and $Y = \sin\Theta$. Let ρ be the correlation coefficient between X and Y. Then 100ρ is equal to ___________ (answer in integer)
Q.58	Let $X_1, X_2, \dots, X_{10}$ be a random sample from a $U(-\theta, \theta)$ distribution, where $\theta \in (0, \infty)$. Let $X_{(10)} = \max\{X_1, X_2, \dots, X_{10}\}$ and $X_{(1)} = \min\{X_1, X_2, \dots, X_{10}\}$. If the observed values of $X_{(10)}$ and $X_{(1)}$ are 8 and -10, respectively, then the maximum likelihood estimate of θ is equal to ___________ (answer in integer)

Q.59	Suppose that the weights (in kgs) of six months old babies, monitored at a healthcare facility, have $N(\mu, \sigma^2)$ distribution, where $\mu \in \mathbb{R}$ and $\sigma > 0$ are unknown parameters. Let $X_1, X_2, \dots, X_9$ be a random sample of the weights of such babies. Let $\bar{X} = \frac{1}{9}\sum_{i=1}^{9} X_i$, $S = \sqrt{\frac{1}{8}\sum_{i=1}^{9}(X_i - \bar{X})^2}$ and let a 95% confidence interval for μ based on t-distribution be of the form $$(\bar{X} - h(S), \bar{X} + h(S)),$$ for an appropriate function h of random variable S. If the observed values of $\bar{X}$ and S^2 are 9 and 9.5, respectively, then the width of the confidence interval is equal to __________ (round off to 2 decimal places) (You may use $t_{9,0.025} = 2.262, t_{8,0.025} = 2.306, t_{9,0.05} = 1.833, t_{8,0.05} = 1.86$)
Q.60	Let X_1, X_2, X_3 be a random sample from a Poisson distribution with mean λ , $\lambda > 0$. For testing $H_0: \lambda = \frac{1}{8}$ against $H_1: \lambda = 1$, a test rejects H_0 if and only if $X_1 + X_2 + X_3 > 1$. The power of this test is equal to __________ (round off to 2 decimal places)