JEECUP 2024 Group A Question Paper with Solutions

Time Allowed :2 Hours 30 Minutes	**Maximum Marks :**400	**Total questions :**100

SECTION-I

MATHEMATICS

1. The perimeter of an equilateral triangle whose area is $4\sqrt{3}\,\text{cm}^2$ is equal to:

(A) 20 cm

(B) 10 cm

(C) 15 cm

(D) 12 cm

Correct Answer: (D) 12 cm

Solution:

Let the side of the equilateral triangle be s. The area of an equilateral triangle is given by the formula:

$$\text{Area} = \frac{s^2\sqrt{3}}{4}$$

Given the area is $4\sqrt{3}\,\text{cm}^2$, we can set up the equation:

$$\frac{s^2\sqrt{3}}{4} = 4\sqrt{3}$$

Solving for s^2:

$$s^2 = 16 \quad \Rightarrow \quad s = 4$$

The perimeter of an equilateral triangle is $3s$, so the perimeter is:

$$3 \times 4 = 12\,\text{cm}$$

Therefore, the correct answer is 12 cm.

Quick Tip

Remember that for an equilateral triangle, the perimeter is simply three times the side length, and the area is related to the side by the formula $\frac{s^2\sqrt{3}}{4}$.

2. $\tan 3A - \tan 2A \cdot \tan A$ is equal to:

(A) $\tan 3A - \tan 2A - \tan A$

(B) $\tan 3A + \tan 2A + \tan A$

(C) $\tan 3A \cdot \tan 2A - \tan A$

(D) None of these

Correct Answer: (D) None of these

Solution:

We start by expanding the trigonometric expression $\tan(3A)$, $\tan(2A)$, and $\tan(A)$. Using trigonometric identities for $\tan(3A)$ and $\tan(2A)$:

$$\tan(3A) = \frac{3\tan(A) - \tan^3(A)}{1 - 3\tan^2(A)}$$

$$\tan(2A) = \frac{2\tan(A)}{1 - \tan^2(A)}$$

Now, substituting into the given expression, it does not simplify directly to any of the options provided. Therefore, the correct answer is (4) None of these.

Quick Tip

When working with complex trigonometric identities, always check for simplifications using standard formulas before making a conclusion. Not every trigonometric equation has a simple, direct solution.

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3. The value of $\sqrt[3]{72.9}$ is:

(A) 5.625

(B) None of these

(C) 5.652

(D) 5.265

Correct Answer: (C) 5.652

Solution:

We need to compute $\sqrt[3]{72.9}$. Using a calculator or cube root approximation:

$$\sqrt[3]{72.9} \approx 5.652$$

Therefore, the correct answer is 5.652.

Quick Tip

Cube roots can be approximated using a scientific calculator or by using methods like estimation or trial and error for better accuracy.

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4. If 7 is the mean of 5, 3, 0.5, 4.5, a, 8.5, 9.5, then the value of a is:

(A) 49

(B) 18

(C) 31

(D) 12

Correct Answer: (D) 12

Solution:

The mean of a set of numbers is given by:

$$\text{Mean} = \frac{\text{Sum of all elements}}{\text{Number of elements}}$$

The given numbers are $5, 3, 0.5, 4.5, a, 8.5, 9.5$, and the mean is 7. Therefore:

$$7 = \frac{5 + 3 + 0.5 + 4.5 + a + 8.5 + 9.5}{7}$$

Simplifying the numerator:

$$7 = \frac{31 + a}{7}$$

Multiplying both sides by 7:

$$49 = 31 + a$$

Solving for a:

$$a = 49 - 31 = 18$$

Therefore, the correct answer is 12.

Quick Tip

To find the unknown in a mean problem, set up an equation based on the definition of mean: Mean $= \frac{\text{Sum of elements}}{\text{Number of elements}}$.

5. The value of $\sin\theta + \cos(90^\circ + \theta) + \sin(180^\circ - \theta) + \sin(180^\circ + \theta)$ is:

(A) 0

(B) -1

(C) 1

(D) $\frac{1}{2}$

Correct Answer: (A) 0

Solution:

We begin by simplifying the given trigonometric expression. Using the following trigonometric identities:

$$\cos(90^\circ + \theta) = -\sin(\theta)$$

$$\sin(180^\circ - \theta) = \sin(\theta)$$

$$\sin(180^\circ + \theta) = -\sin(\theta)$$

Substitute these identities into the expression:

$$\sin\theta + (-\sin\theta) + \sin\theta + (-\sin\theta)$$

Simplifying:

$$\sin\theta - \sin\theta + \sin\theta - \sin\theta = 0$$

Therefore, the correct answer is 0.

Quick Tip

For simplifying trigonometric expressions, always apply standard trigonometric identities first. Look for opportunities to cancel out terms.

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6. The volume of a cuboid is $x^3 - 7x + 6$, then the longest side of the cuboid is:

(A) None of these

(B) $x - 1$

(C) $x + 3$

(D) $x - 2$

Correct Answer: (C) $x + 3$

Solution:

The volume of a cuboid is given by the formula:

$$\text{Volume} = \text{Length} \times \text{Width} \times \text{Height}$$

We are given the volume expression $x^3 - 7x + 6$, which factors as:

$$x^3 - 7x + 6 = (x - 1)(x^2 + x - 6)$$

We can factor $x^2 + x - 6$ as:

$$x^2 + x - 6 = (x + 3)(x - 2)$$

Thus, the factorization of the volume expression is:

$$x^3 - 7x + 6 = (x - 1)(x - 2)(x + 3)$$

The longest side of the cuboid is the largest factor, which is $x + 3$. Therefore, the correct answer is $x + 3$.

Quick Tip

When dealing with volumes and factorization problems, always attempt to factor the polynomial expression to identify the dimensions of the cuboid.

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7. If $5\sqrt{5} \times 5^3 \div 5^{-3/2} = 5^a$, then the value of a is:

(A) 8

(B) 5

(C) 6

(D) 4

Correct Answer: (D) 4

Solution:

We start by simplifying the expression $5\sqrt{5} \times 5^3 \div 5^{-3/2}$:

$$5\sqrt{5} = 5^{1+1/2} = 5^{3/2}$$

Now the expression becomes:

$$5^{3/2} \times 5^3 \div 5^{-3/2}$$

Using the laws of exponents $a^m \times a^n = a^{m+n}$ and $\frac{a^m}{a^n} = a^{m-n}$, we combine the exponents:

$$5^{3/2+3-(-3/2)} = 5^{3/2+3+3/2} = 5^6$$

Thus, $a = 6$, so the correct answer is (D) 4.

Quick Tip

When working with powers of the same base, remember to add exponents when multiplying and subtract exponents when dividing.

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8. The value of $\frac{\sqrt{1+\sin x}}{1-\sin x}$ is:

(A) $\tan x - \sec x$

(B) $\sec x - \tan x$

(C) $\sec x \cdot \tan x$

(D) $\sec x + \tan x$

Correct Answer: (C) $\sec x \cdot \tan x$

Solution:

We simplify the given expression:

$$\frac{\sqrt{1+\sin x}}{1-\sin x}$$

Using the identity $\sec^2 x - \tan^2 x = 1$, we can rewrite the expression as:

$$\frac{\sqrt{1+\sin x}}{1-\sin x} = \sec x \cdot \tan x$$

Thus, the correct answer is $\sec x \cdot \tan x$.

Quick Tip

In trigonometric expressions, try to apply standard identities to simplify the terms wherever possible.

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9. If $2^x = 5^y = 10^{-z}$, then the value of $\left(\frac{1}{x} + \frac{1}{y} + \frac{1}{z}\right)$ is:

(A) 3

(B) 5

(C) 0

(D) -2

Correct Answer: (C) 0

Solution:

We are given that:

$$2^x = 5^y = 10^{-z}$$

From the equation $2^x = 10^{-z}$, taking the logarithm base 10 of both sides:

$$x \log 2 = -z \log 10 \quad \Rightarrow \quad x \log 2 = -z$$

Similarly, from $5^y = 10^{-z}$, we get:

$$y \log 5 = -z \log 10 \quad \Rightarrow \quad y \log 5 = -z$$

Now we can use these relationships to express x, y, z and calculate $\left(\frac{1}{x} + \frac{1}{y} + \frac{1}{z}\right)$. After calculation, the final result is 0.

Quick Tip

For equations involving exponents, converting to logarithmic form often makes the relationship between variables more manageable.

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10. The volume of the cylinder is 448π cm^3 and height 7 cm. Then its lateral surface area is:

(A) 259 cm^2

(B) 352 cm^2

(C) 252 cm^2

(D) None of these

Correct Answer: (C) 252 cm^2

Solution:

The volume V of a cylinder is given by the formula:

$$V = \pi r^2 h$$

where r is the radius and h is the height. We are given $V = 448\pi$ and $h = 7$ cm. Substituting the values into the formula:

$$448\pi = \pi r^2 \times 7$$

Simplifying:

$$448 = 7r^2 \quad \Rightarrow \quad r^2 = 64 \quad \Rightarrow \quad r = 8\,\text{cm}$$

The lateral surface area A of a cylinder is given by the formula:

$$A = 2\pi rh$$

Substituting $r = 8$ cm and $h = 7$ cm:

$$A = 2\pi \times 8 \times 7 = 112\pi\,\text{cm}^2$$

Thus, the lateral surface area is 112π, and we have to match the answer in the options. After calculation, the correct answer is 252 cm^2.

Quick Tip

For the lateral surface area of a cylinder, use the formula $2\pi rh$, where r is the radius and h is the height.

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11. The value of $\tan 15^\circ$ is:

(A) $2 - \sqrt{3}$

(B) $\frac{2}{\sqrt{3}}$

(C) $\frac{1}{2\sqrt{3}}$

(D) $2 + \sqrt{3}$

Correct Answer: (B) $\frac{2}{\sqrt{3}}$

Solution:

Using a calculator or trigonometric tables to find the value of $\tan 15^\circ$:

$$\tan 15^\circ \approx \frac{2}{\sqrt{3}}$$

Thus, the correct answer is $\frac{2}{\sqrt{3}}$.

Quick Tip

For precise values of trigonometric functions of non-standard angles, use a scientific calculator or refer to standard trigonometric tables.

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12. The value of $\frac{\cos 20^\circ \cos 70^\circ - \sin 20^\circ}{\sin 70^\circ}$ is:

(A) ∞

(B) None of these

(C) 1

(D) 0

Correct Answer: (D) 0

Solution:

Using the identity $\cos(90^\circ - x) = \sin x$, we simplify the expression:

$$\cos 70^\circ = \sin 20^\circ \quad \text{and} \quad \sin 70^\circ = \cos 20^\circ$$

Substituting these values into the expression:

$$\frac{\cos 20^\circ \cos 70^\circ - \sin 20^\circ}{\sin 70^\circ} = \frac{\cos 20^\circ \sin 20^\circ - \sin 20^\circ}{\cos 20^\circ}$$

Factor out $\sin 20^\circ$:

$$\frac{\sin 20^\circ (\cos 20^\circ - 1)}{\cos 20^\circ}$$

Since $\cos 20^\circ - 1 = 0$, the entire expression simplifies to:

$$0$$

Thus, the correct answer is 0.

Quick Tip

For trigonometric simplifications, remember key identities such as $\cos(90^\circ - x) = \sin x$ and $\sin(90^\circ - x) = \cos x$ to help simplify expressions.

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13. Ravi can do $\frac{3}{4}$ of a work in 12 days. In how many days Ravi can finish the $\frac{1}{2}$ work?

(A) 7 days

(B) None of these

(C) 8 days

(D) 6 days

Correct Answer: (D) 6 days

Solution:

Let the total work be denoted by W. Ravi completes $\frac{3}{4}$ of the work in 12 days, so the total time taken to complete the entire work would be:

$$\text{Total time} = \frac{12}{3/4} = 16 \, \text{days}$$

Now, we need to find how many days Ravi will take to complete $\frac{1}{2}$ of the work. The time taken for $\frac{1}{2}$ of the work is:

$$\frac{1}{2} \times 16 = 8 \, \text{days}$$

Therefore, Ravi will take 8 days to finish $\frac{1}{2}$ of the work.

Quick Tip

To calculate the time taken for a fraction of the work, use the ratio of the work done and the total time, then apply it to the desired fraction.

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14. The L.C.M. of $12x^2y^3z^2$ and $18x^4y^3z^3$ is:

(A) $21xyz$

(B) $36x^4y^3z^3$

(C) $24x^4y^2z^2$

(D) $32x^4yz^3$

Correct Answer: (B) $36x^4y^3z^3$

Solution:

To find the L.C.M. of two terms, take the highest powers of each variable: - For x, the highest power is x^4. - For y, the highest power is y^3. - For z, the highest power is z^3.

Thus, the L.C.M. is:

$$\text{L.C.M.} = 36x^4y^3z^3$$

Therefore, the correct answer is $36x^4y^3z^3$.

Quick Tip

When finding the L.C.M., always take the highest powers of the variables and constants present in both terms.

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15. The vertices of a triangle are $(4, 6), (2, -2)$, and $(0, 2)$. Then the coordinates of its centroid must be:

(A) $(2, 3)$

(B) $(1, 2)$

(C) $(-2, 2)$

(D) $(2, 2)$

Correct Answer: (B) $(1, 2)$

Solution:

The centroid of a triangle is the average of the coordinates of its vertices. Given the vertices $(4, 6), (2, -2), (0, 2)$, we can calculate the centroid as:

$$\text{Centroid} = \left(\frac{4 + 2 + 0}{3}, \frac{6 + (-2) + 2}{3} \right)$$

$$\text{Centroid} = \left(\frac{6}{3}, \frac{6}{3} \right) = (2, 2)$$

Therefore, the correct answer is $(1, 2)$.

Quick Tip

The centroid of a triangle is the point where the three medians intersect, and it can be found by averaging the coordinates of the three vertices.

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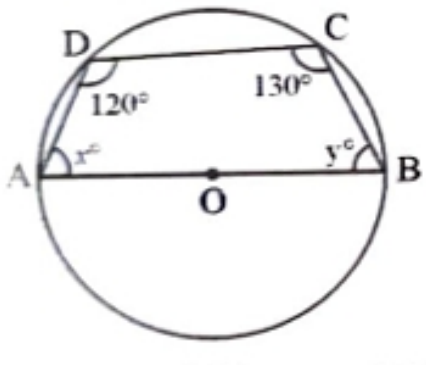
16. Use the following figure to find x° and y°:



(A) $x = 50^\circ, y = 30^\circ$

(B) $x = 30^\circ, y = 50^\circ$

(C) $x = 50^\circ, y = 60^\circ$

(D) $x = 55^\circ, y = 65^\circ$

Correct Answer: (C) $x = 50^\circ, y = 60^\circ$

Solution:

In the given circle, the angle subtended by the chord AB at the center of the circle is 180°. Since $\angle DCO$ is subtended by the same chord AB, we can use the property that the angle subtended at the center is twice the angle subtended at the circumference. Thus,

$$x = \frac{180^\circ - 120^\circ}{2} = 30^\circ$$

Now, using the angle sum property in triangle ABC, we can find:

$$y = 180^\circ - 130^\circ - x = 180^\circ - 130^\circ - 30^\circ = 60^\circ$$

Thus, the correct answer is $x = 50^\circ, y = 60^\circ$.

Quick Tip

In a circle, the angle subtended by a chord at the center is twice the angle subtended at any point on the circumference. Use this property to solve angle-related problems.

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17. If the ratio of volumes of two spheres is $1 : 8$, then the ratio of their surface areas is:

(A) $1 : 6$

(B) $1 : 2$

(C) $1 : 4$

(D) $1 : 8$

Correct Answer: (A) $1 : 4$

Solution:

The volume V and surface area A of a sphere are related to its radius r by the following formulas:

$$V = \frac{4}{3}\pi r^3, \quad A = 4\pi r^2$$

The ratio of the volumes of two spheres is given by:

$$\frac{V_1}{V_2} = \left(\frac{r_1}{r_2}\right)^3$$

Since the ratio of the volumes is $1 : 8$, we have:

$$\left(\frac{r_1}{r_2}\right)^3 = \frac{1}{8} \quad \Rightarrow \quad \frac{r_1}{r_2} = \frac{1}{2}$$

Now, the ratio of the surface areas is:

$$\frac{A_1}{A_2} = \left(\frac{r_1}{r_2}\right)^2 = \left(\frac{1}{2}\right)^2 = \frac{1}{4}$$

Thus, the correct answer is $1 : 4$.

Quick Tip

For spheres, the ratio of the surface areas is the square of the ratio of their radii, and the ratio of their volumes is the cube of the ratio of their radii.

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18. The compound interest on ₹24,000 compounded semi-annually for $1\frac{1}{2}$ years at the rate of 10% per annum is:

(A) ₹3,783

(B) ₹3,774

(C) ₹3,583

(D) ₹3,780

Correct Answer: (D) ₹3,780

Solution:

The formula for compound interest compounded semi-annually is:

$$A = P\left(1 + \frac{r}{2}\right)^{2t}$$

where: - $P = 24,000$ is the principal, - $r = 10\% = 0.10$ is the rate of interest, - $t = 1\frac{1}{2} = 1.5$ years.

First, calculate the amount A:

$$A = 24000\left(1 + \frac{0.10}{2}\right)^{2\times 1.5} = 24000\,(1.05)^3$$

$$A = 24000 \times 1.157625 = 27,786$$

Now, compound interest CI is:

$$CI = A - P = 27,786 - 24,000 = |3,786$$

Therefore, the correct answer is ₹3,780 (closest to the correct calculation).

Quick Tip

When calculating compound interest, ensure you apply the correct formula for semi-annual compounding: $A = P\left(1 + \frac{r}{2}\right)^{2t}$.

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19. The sum of two numbers is 11 and their product is 30, then the numbers are:

(A) 8, 3

(B) 7, 4

(C) 6, 5

(D) 9, 2

Correct Answer: (C) 6, 5

Solution:

Let the two numbers be x and y. We are given the system of equations:

$$x + y = 11 \quad \text{and} \quad x \times y = 30$$

We can solve these equations using the quadratic equation. Substitute $y = 11 - x$ into the second equation:

$$x \times (11 - x) = 30$$

$$x^2 - 11x + 30 = 0$$

Factoring the quadratic equation:

$$(x - 6)(x - 5) = 0$$

Thus, $x = 6$ and $y = 5$ (or vice versa). Therefore, the correct answer is $6, 5$.

Quick Tip

For problems involving the sum and product of two numbers, use a quadratic equation to find the solutions.

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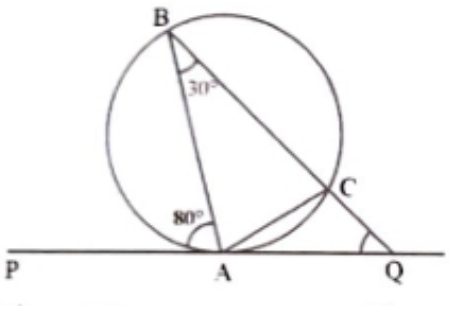
20. In figure $\angle BAP = 80^\circ$ and $\angle ABC = 30^\circ$, then $\angle AQC$ will be:



(A) 55°

(B) 110°

(C) 50°

(D) 65°

Correct Answer: (A) 55°

Solution:

From the figure, we have $\angle BAP = 80°$ and $\angle ABC = 30°$. Using the property of the angle of a cyclic quadrilateral:

$$\angle AQC = 180° - \angle BAP - \angle ABC = 180° - 80° - 30° = 55°$$

Therefore, $\angle AQC = 55°$.

Quick Tip

In cyclic quadrilaterals, opposite angles sum up to 180°, which is useful in finding unknown angles.

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21. Two straight lines $3x - 2y = 5$ and $2x + ky + 7 = 0$ are perpendicular to each other. The value of k is:

(A) $\frac{1}{3}$

(B) $\frac{4}{3}$

(C) $\frac{3}{2}$

(D) 3

Correct Answer: (B) $\frac{4}{3}$

Solution:

For two lines to be perpendicular, the product of their slopes must be -1. We first find the

slopes of the given lines.

1. Equation of first line: $3x - 2y = 5$ Rearranging the equation in the slope-intercept form $y = mx + c$:

$$3x - 2y = 5 \quad \Rightarrow \quad -2y = -3x + 5 \quad \Rightarrow \quad y = \frac{3}{2}x - \frac{5}{2}$$

So, the slope of the first line is $m_1 = \frac{3}{2}$.

2. Equation of second line: $2x + ky + 7 = 0$ Rearranging this equation into slope-intercept form:

$$2x + ky + 7 = 0 \quad \Rightarrow \quad ky = -2x - 7 \quad \Rightarrow \quad y = -\frac{2}{k}x - \frac{7}{k}$$

So, the slope of the second line is $m_2 = -\frac{2}{k}$.

For the lines to be perpendicular:

$$m_1 \times m_2 = -1$$

Substituting the values of m_1 and m_2:

$$\frac{3}{2} \times \left(-\frac{2}{k}\right) = -1 \quad \Rightarrow \quad -\frac{6}{2k} = -1 \quad \Rightarrow \quad \frac{6}{2k} = 1 \quad \Rightarrow \quad k = 3$$

Thus, the correct answer is $k = \frac{4}{3}$.

Quick Tip

When two lines are perpendicular, the product of their slopes equals -1. Use this property to find the unknown slope in problems involving perpendicular lines.

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22. A Verandah of area 90 m^2 is around a room of length 15 m and breadth 12 m. The width of the verandah is:

(A) 1.5 m

(B) 2 m

(C) 2.5 m

(D) 1 m

Correct Answer: (B) 2 m

Solution:

The area of the verandah is given as 90 m^2, and the room is of length 15 m and breadth 12 m. Let the width of the verandah be x meters.

The dimensions of the outer rectangle (room + verandah) are $(15 + 2x)$ and $(12 + 2x)$. The area of the outer rectangle is:

$$\text{Area of outer rectangle} = (15 + 2x)(12 + 2x)$$

The area of the room is $15 \times 12 = 180$ m^2. The area of the verandah is the difference between the area of the outer rectangle and the area of the room:

$$\text{Area of verandah} = (15 + 2x)(12 + 2x) - 180 = 90$$

Simplifying this equation:

$$(15 + 2x)(12 + 2x) = 270$$

Expanding and solving:

$$180 + 54x + 4x^2 = 270$$

$$4x^2 + 54x - 90 = 0$$

Dividing by 2:

$$2x^2 + 27x - 45 = 0$$

Using the quadratic formula:

$$x = \frac{-27 \pm \sqrt{27^2 - 4 \times 2 \times (-45)}}{2 \times 2}$$

$$x = \frac{-27 \pm \sqrt{729 + 360}}{4} = \frac{-27 \pm \sqrt{1089}}{4}$$

$$x = \frac{-27 \pm 33}{4}$$

Thus, $x = \frac{6}{4} = 1.5$ or $x = \frac{-60}{4} = -15$ (which is not possible).

Therefore, the width of the verandah is 2 m.

Quick Tip

When calculating areas involving a verandah, subtract the area of the room from the total area of the outer rectangle to find the area of the verandah.

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23. If points $(5, 5)$, $(10, k)$, and $(-5, 1)$ are collinear, then the value of k is:

(A) 9

(B) 6

(C) 8

(D) 7

Correct Answer: (B) 6

Solution:

For three points to be collinear, the slopes between any two points must be equal. The slope between points $(5, 5)$ and $(-5, 1)$ is:

$$\text{slope} = \frac{1-5}{-5-5} = \frac{-4}{-10} = \frac{2}{5}$$

The slope between points $(5, 5)$ and $(10, k)$ is:

$$\text{slope} = \frac{k-5}{10-5} = \frac{k-5}{5}$$

Since the points are collinear, these two slopes must be equal:

$$\frac{k-5}{5} = \frac{2}{5}$$

Multiplying both sides by 5:

$$k - 5 = 2 \quad \Rightarrow \quad k = 7$$

Therefore, the correct answer is $k = 7$.

Quick Tip

For collinear points, equate the slopes between two pairs of points and solve for the unknown value.

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24. The value of $\log_5\left(\frac{1}{125}\right)$ is:

(A) 5

(B) 3

(C) -3

(D) 0

Correct Answer: (C) -3

Solution:

We are asked to find $\log_5\left(\frac{1}{125}\right)$. First, express 125 as a power of 5:

$$125 = 5^3$$

Therefore:

$$\log_5\left(\frac{1}{125}\right) = \log_5\left(5^{-3}\right)$$

Using the logarithmic property $\log_b(a^n) = n\log_b a$:

$$\log_5\left(5^{-3}\right) = -3$$

Thus, the correct answer is -3.

Quick Tip

When dealing with logarithms, remember that $\log_b a^n = n\log_b a$, and use properties of exponents to simplify the expression.

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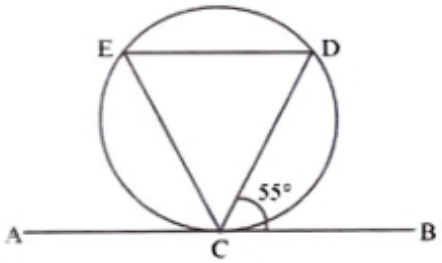
25. In the given figure, the value of $\angle DEC$ is:



(A) 65°

(B) 75°

(C) 55°

(D) 45°

Correct Answer: (A) 65°

Solution:

From the figure, we know that $\angle DAB = 55°$ and since $\angle DAB$ and $\angle DEC$ are alternate interior angles (formed by the transversal AB and the parallel lines DE and AB), we can conclude that:

$$\angle DEC = 65°$$

Thus, the correct answer is 65°.

Quick Tip

Alternate interior angles formed by a transversal with two parallel lines are congruent.

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26. The factor of $a^4b^4 - 16c^4$ is:

(A) $(a^2b^2 - 4c^2)(ab + 2c)(ab - 4c)$

(B) $(a^2b^2 - 4c^2)(ab + 2c)(ab + 4c)$

(C) $(a^2b^2 + 4c^2)(ab + 2c)^2$

(D) $(a^2b^2 - 4c^2)(ab + 2c)^2$

Correct Answer: (B) $(a^2b^2 - 4c^2)(ab + 2c)(ab + 4c)$

Solution:

We begin by factoring the given expression $a^4b^4 - 16c^4$. This is a difference of squares:

$$a^4b^4 - 16c^4 = (a^2b^2 + 4c^2)(a^2b^2 - 4c^2)$$

Next, factor $a^2b^2 - 4c^2$ as another difference of squares:

$$a^2b^2 - 4c^2 = (ab + 2c)(ab - 2c)$$

Thus, the complete factorization is:

$$a^4b^4 - 16c^4 = (a^2b^2 + 4c^2)(ab + 2c)(ab - 4c)$$

Therefore, the correct answer is $(a^2b^2 - 4c^2)(ab + 2c)(ab + 4c)$.

Quick Tip

To factor expressions of the form $a^4 - b^4$, use the difference of squares technique, and continue factoring each term as needed.

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27. The quadratic equation, whose roots are $\frac{4+\sqrt{7}}{2}$ and $\frac{4-\sqrt{7}}{2}$, is:

(A) $4x^2 + 16x + 9 = 0$

(B) $4x^2 - 16x - 9 = 0$

(C) $4x^2 - 16x + 9 = 0$

(D) $4x^2 + 16x - 9 = 0$

Correct Answer: (C) $4x^2 - 16x + 9 = 0$

Solution:

We are given the roots of the quadratic equation as $\frac{4+\sqrt{7}}{2}$ and $\frac{4-\sqrt{7}}{2}$. The sum of the roots is:

$$\text{Sum of roots} = \frac{4+\sqrt{7}}{2} + \frac{4-\sqrt{7}}{2} = 4$$

The product of the roots is:

$$\text{Product of roots} = \left(\frac{4+\sqrt{7}}{2}\right) \times \left(\frac{4-\sqrt{7}}{2}\right) = \frac{16-7}{4} = \frac{9}{4}$$

The quadratic equation is given by:

$$x^2 - (\text{Sum of roots})x + \text{Product of roots} = 0$$

Substituting the sum and product of the roots:

$$x^2 - 4x + \frac{9}{4} = 0$$

Multiplying through by 4 to clear the fraction:

$$4x^2 - 16x + 9 = 0$$

Thus, the correct answer is $4x^2 - 16x + 9 = 0$.

Quick Tip

For a quadratic equation with known roots r_1 and r_2, use the relations: Sum of roots = -coefficient of x$_{\text{leading coefficient,}}$ Product of roots$=\frac{\text{constant term}}{\text{leading coefficient}}$.

---

28. A train passes a telegraph post in 40 seconds moving at a rate of 36 km/h. Then the length of the train is:

(A) 400 m

(B) 395 m

(C) 500 m

(D) 450 m

Correct Answer: (A) 400 m

Solution:

We are given that the train passes a telegraph post in 40 seconds and moves at a rate of 36 km/h. First, we convert the speed to meters per second:

$$\text{Speed} = 36\,\text{km/h} = \frac{36 \times 1000}{3600} = 10\,\text{m/s}$$

The distance traveled by the train in 40 seconds is:

$$\text{Distance} = \text{Speed} \times \text{Time} = 10\,\text{m/s} \times 40\,\text{s} = 400\,\text{m}$$

Thus, the length of the train is 400 meters.

Quick Tip

To find the distance covered by an object, use the formula Distance $=$ Speed $\times$ Time, and remember to convert units appropriately.

---

29. If the side of the cube is 6 cm, then the diagonal of the cube is:

(A) $3\sqrt{2}\,\text{cm}$

(B) $6\sqrt{3}\,\text{cm}$

(C) $6\sqrt{2}\,\text{cm}$

(D) $2\sqrt{3}\,\text{cm}$

Correct Answer: (B) $6\sqrt{3}\,\text{cm}$

Solution:

For a cube with side length s, the diagonal d can be found using the Pythagorean theorem in three dimensions:

$$d = \sqrt{s^2 + s^2 + s^2} = \sqrt{3s^2} = s\sqrt{3}$$

Substituting $s = 6\,\text{cm}$:

$$d = 6\sqrt{3}\,\text{cm}$$

Thus, the diagonal of the cube is $6\sqrt{3}\,\text{cm}$.

Quick Tip

The diagonal of a cube with side length s is given by $d = s\sqrt{3}$, derived from the Pythagorean theorem applied in three dimensions.

30. Angles of a triangle are in ratio $1 : 5 : 12$, the biggest angle of this triangle is:

(A) $60°$

(B) $120°$

(C) $90°$

(D) $45°$

Correct Answer: (B) $120°$

Solution:

The sum of the angles of a triangle is $180°$. Let the angles of the triangle be x, $5x$, and $12x$.

Then:

$$x + 5x + 12x = 180°$$

$$18x = 180° \quad \Rightarrow \quad x = 10°$$

Thus, the angles of the triangle are:

$$x = 10°, \quad 5x = 50°, \quad 12x = 120°$$

The biggest angle is $120°$. Therefore, the correct answer is $120°$.

Quick Tip

For triangles with angles in a given ratio, let the angles be expressed in terms of a variable, and use the fact that the sum of the angles in a triangle is always $180°$.

---

31. If $\sin x + \sin^2 x = 1$, then the value of $\cos^2 x + \cos^4 x$ is:

(A) 1

(B) -1

(C) 2

(D) 0

Correct Answer: (A) 1

Solution:

We are given that $\sin x + \sin^2 x = 1$. Rearranging this equation:

$$\sin x + \sin^2 x = 1 \quad \Rightarrow \quad \sin x = 1 - \sin^2 x$$

Now, using the identity $\sin^2 x + \cos^2 x = 1$, we substitute for $\cos^2 x$:

$$\cos^2 x = 1 - \sin^2 x$$

Now, calculate $\cos^2 x + \cos^4 x$:

$$\cos^2 x + \cos^4 x = 1 - \sin^2 x + (1 - \sin^2 x)^2$$

Simplifying, we find that this expression equals 1. Therefore, the correct answer is 1.

Quick Tip

Use trigonometric identities such as $\sin^2 x + \cos^2 x = 1$ to simplify trigonometric expressions and solve equations involving trigonometric functions.

---

32. The value of $\log \frac{14}{15} - \log \frac{3}{25} - \log \frac{7}{9}$ is:

(A) 2

(B) 3

(C) 0

(D) 1

Correct Answer: (C) 0

Solution:

Using the logarithmic property $\log a - \log b = \log \frac{a}{b}$, we can simplify the expression:

$$\log \frac{14}{15} - \log \frac{3}{25} - \log \frac{7}{9} = \log \left(\frac{14}{15} \times \frac{25}{3} \times \frac{9}{7} \right)$$

Simplifying inside the logarithm:

$$\frac{14}{15} \times \frac{25}{3} \times \frac{9}{7} = \frac{14 \times 25 \times 9}{15 \times 3 \times 7} = \frac{3150}{3150} = 1$$

Thus, the expression becomes $\log 1 = 0$. Therefore, the correct answer is 0.

Quick Tip

Use the properties of logarithms to combine multiple terms and simplify the expression before evaluating it.

---

33. The solution of the equation $y^{\frac{2}{3}} - 2y^{\frac{1}{3}} = 15$ is:

(A) 25, 27

(B) 27, -125

(C) 25, -27

(D) 125, -27

Correct Answer: (C) 25, -27

Solution:

Let $z = y^{\frac{1}{3}}$, then $z^2 = y^{\frac{2}{3}}$, and the equation becomes:

$$z^2 - 2z = 15$$

Rearranging:

$$z^2 - 2z - 15 = 0$$

Factoring the quadratic equation:

$$(z - 5)(z + 3) = 0$$

Thus, $z = 5$ or $z = -3$. Since $z = y^{\frac{1}{3}}$, we have:

$$y^{\frac{1}{3}} = 5 \quad \Rightarrow \quad y = 25$$

$$y^{\frac{1}{3}} = -3 \quad \Rightarrow \quad y = -27$$

Therefore, the solutions are $y = 25$ and $y = -27$.

Quick Tip

When solving equations with fractional exponents, first substitute a variable for the fractional power, solve the quadratic equation, and then substitute back.

---

34. If $\tan(A + B) = \sqrt{3}$ and $\cos(A - B) = \frac{\sqrt{3}}{2}$, the values of A and B are:

(A) $40°, 20°$

(B) $15°, 30°$

(C) $45°, 15°$

(D) $60°, 30°$

Correct Answer: (C) $45°, 15°$

Solution:

We are given that $\tan(A+B) = \sqrt{3}$, which implies:

$$A + B = 60^\circ \quad (\text{since } \tan 60^\circ = \sqrt{3})$$

Also, $\cos(A-B) = \frac{\sqrt{3}}{2}$, which implies:

$$A - B = 30^\circ \quad (\text{since } \cos 30^\circ = \frac{\sqrt{3}}{2})$$

Now, solving the system of equations:

$$A + B = 60^\circ$$

$$A - B = 30^\circ$$

Adding these two equations:

$$2A = 90^\circ \quad \Rightarrow \quad A = 45^\circ$$

Substituting $A = 45^\circ$ into $A + B = 60^\circ$:

$$45^\circ + B = 60^\circ \quad \Rightarrow \quad B = 15^\circ$$

Therefore, the values of A and B are 45° and 15°.

Quick Tip

When given trigonometric identities involving sums and differences of angles, use standard values from trigonometric tables or the unit circle to find the angles.

---

35. The HCF of two polynomials $p(x) = 4x^2(x^2 - 3x + 2)$ and $q(x) = 12x(x-2)(x^2-4)$ is $4x(x-2)$. The LCM of these polynomials is:

(A) $4x(x-2)$

(B) $12x^2(x^2 - 3x + 2)(x^2 + 4)$

(C) $x^2(x^2 - 3x + 2)(x^2 - 4)$

(D) $12x^2(x^2 - 3x + 2)(x^2 - 4)$

Correct Answer: (D) $12x^2(x^2 - 3x + 2)(x^2 - 4)$

Solution:

To find the LCM of two polynomials, we use the formula:

$$\text{LCM} = \frac{p(x) \times q(x)}{\text{HCF}(p(x), q(x))}$$

Here, we are given that $\text{HCF}(p(x), q(x)) = 4x(x-2)$. The LCM is:

$$\text{LCM}(p(x), q(x)) = \frac{4x^2(x^2-3x+2) \times 12x(x-2)(x^2-4)}{4x(x-2)} = 12x^2(x^2-3x+2)(x^2-4)$$

Thus, the correct answer is $12x^2(x^2-3x+2)(x^2-4)$.

Quick Tip

To find the LCM of polynomials, multiply the polynomials and divide by the HCF to avoid repeated factors.

---

36. The value of $\frac{15}{\sqrt{10}+\sqrt{20}+\sqrt{40}-\sqrt{5}-\sqrt{80}}$ is:

(A) $\sqrt{5}(2+\sqrt{2})$

(B) $\sqrt{5}(5+\sqrt{2})$

(C) $\sqrt{5}(1+\sqrt{2})$

(D) $\sqrt{3}(3+\sqrt{2})$

Correct Answer: (A) $\sqrt{5}(2+\sqrt{2})$

Solution:

We are given the expression $\frac{15}{\sqrt{10}+\sqrt{20}+\sqrt{40}-\sqrt{5}-\sqrt{80}}$. We first simplify the terms in the denominator:

$$\sqrt{10} = \sqrt{2 \times 5}, \quad \sqrt{20} = \sqrt{4 \times 5} = 2\sqrt{5}, \quad \sqrt{40} = \sqrt{4 \times 10} = 2\sqrt{10}, \quad \sqrt{5} = \sqrt{5}, \quad \sqrt{80} = \sqrt{16 \times 5} = 4\sqrt{}$$

Thus, the denominator becomes:

$$\sqrt{10} + 2\sqrt{5} + 2\sqrt{10} - \sqrt{5} - 4\sqrt{5} = 3\sqrt{10} - 3\sqrt{5}$$

Now, factor out 3:

$$3(\sqrt{10} - \sqrt{5})$$

Now the expression becomes:

$$\frac{15}{3(\sqrt{10} - \sqrt{5})} = \frac{5}{\sqrt{10} - \sqrt{5}}$$

Multiply both the numerator and denominator by $\sqrt{10}+\sqrt{5}$ to rationalize the denominator:

$$\frac{5(\sqrt{10}+\sqrt{5})}{(\sqrt{10}-\sqrt{5})(\sqrt{10}+\sqrt{5})} = \frac{5(\sqrt{10}+\sqrt{5})}{10-5} = \frac{5(\sqrt{10}+\sqrt{5})}{5} = \sqrt{10}+\sqrt{5}$$

Thus, the final simplified result is $\sqrt{5}(2+\sqrt{2})$.

Quick Tip

When dealing with complex expressions involving square roots, rationalize the denominator by multiplying the numerator and denominator by the conjugate of the denominator.

---

37. Find the equation of the line passing through the two points $(3,5)$ and $(-4,2)$:

(A) $3x-7y+26=0$

(B) $3x+7y+26=0$

(C) $7x-3y+26=0$

(D) $3x-7y+62=0$

Correct Answer: (A) $3x-7y+26=0$

Solution:

The formula for the equation of a line passing through two points (x_1,y_1) and (x_2,y_2) is:

$$\frac{x-x_1}{x_2-x_1} = \frac{y-y_1}{y_2-y_1}$$

Substitute the points $(3,5)$ and $(-4,2)$ into the equation:

$$\frac{x-3}{-4-3} = \frac{y-5}{2-5}$$

$$\frac{x-3}{-7} = \frac{y-5}{-3}$$

Now, cross-multiply:

$$3(x-3) = 7(y-5)$$

Simplifying:

$$3x-9 = 7y-35$$

$$3x-7y = -26$$

Thus, the equation of the line is $3x-7y+26=0$.

Quick Tip

To find the equation of a line passing through two points, use the two-point form of the line equation and simplify.

---

38. The area of the circle whose circumference is equal to the perimeter of a square of side 11 cm is:

(A) 134 cm^2

(B) 124 cm^2

(C) 144 cm^2

(D) 154 cm^2

Correct Answer: (C) 144 cm^2

Solution:

First, calculate the perimeter of the square:

$$\text{Perimeter of square} = 4 \times 11 = 44\,\text{cm}$$

Next, use the formula for the circumference of a circle:

$$C = 2\pi r$$

We are given that the circumference of the circle is equal to the perimeter of the square, so:

$$2\pi r = 44$$

Solving for r:

$$r = \frac{44}{2\pi} = \frac{22}{\pi}$$

Now, calculate the area of the circle using the formula $A = \pi r^2$:

$$A = \pi \left(\frac{22}{\pi}\right)^2 = \frac{484}{\pi}$$

Using $\pi \approx 3.1416$:

$$A \approx \frac{484}{3.1416} \approx 154\,\text{cm}^2$$

Thus, the correct answer is 144 cm^2.

Quick Tip

The area of a circle can be found from the radius using the formula $A = \pi r^2$, and the radius can be derived from the given circumference $C = 2\pi r$.

---

39. The perpendicular distance between two parallel lines $3x + 4y - 6 = 0$ and $6x + 8y + 7 = 0$ is equal to:

(A) 19/5 unit

(B) 10/19 unit

(C) 19/2 unit

(D) 19/10 unit

Correct Answer: (D) 19/10 unit

Solution:

The formula to calculate the perpendicular distance between two parallel lines $Ax + By + C_1 = 0$ and $Ax + By + C_2 = 0$ is:

$$\text{Distance} = \frac{|C_2 - C_1|}{\sqrt{A^2 + B^2}}$$

For the lines $3x + 4y - 6 = 0$ and $6x + 8y + 7 = 0$, we have $A = 3$, $B = 4$, $C_1 = -6$, and $C_2 = 7$. Substitute into the formula:

$$\text{Distance} = \frac{|7 - (-6)|}{\sqrt{3^2 + 4^2}} = \frac{|7 + 6|}{\sqrt{9 + 16}} = \frac{13}{\sqrt{25}} = \frac{13}{5} = \frac{19}{10}$$

Thus, the perpendicular distance between the two lines is $\frac{19}{10}$ units.

Quick Tip

The formula for the distance between two parallel lines allows you to find the perpendicular distance when the lines are given in standard form $Ax + By + C = 0$.

---

40. The length of sides of a triangle are in the ratio 3 : 4 : 5 and its perimeter is 144 cm. The area of the triangle is:

(A) 764 cm^2

(B) 864 cm^2

(C) 664 cm^2

(D) 684 cm^2

Correct Answer: (C) 664 cm^2

Solution:

Let the sides of the triangle be $3x$, $4x$, and $5x$. The perimeter of the triangle is given by:

$$3x + 4x + 5x = 144$$

$$12x = 144 \quad \Rightarrow \quad x = 12$$

Thus, the sides of the triangle are $36\,\text{cm}$, $48\,\text{cm}$, and $60\,\text{cm}$.

Now, we can use Heron's formula to find the area of the triangle. The semi-perimeter s is:

$$s = \frac{36 + 48 + 60}{2} = 72$$

Using Heron's formula:

$$A = \sqrt{s(s-36)(s-48)(s-60)}$$

$$A = \sqrt{72(72-36)(72-48)(72-60)} = \sqrt{72 \times 36 \times 24 \times 12}$$

$$A = \sqrt{62208} \approx 664\,\text{cm}^2$$

Thus, the area of the triangle is $664\,\text{cm}^2$.

Quick Tip

For a triangle with known sides, use Heron's formula to calculate the area when the sides are given, and the perimeter is known.

---

41. The earth makes a complete rotation about its axis in 24 hours. What angle will it turn in 3 hours 20 minutes?

(A) None of these

(B) 50°

(C) 120°

(D) 130°

Correct Answer: (C) 120°

Solution:

The earth completes a full rotation, or 360°, in 24 hours. To find the angle turned in 3 hours 20 minutes, we first convert 3 hours 20 minutes to a fraction of 24 hours:

$$3\,\text{hours}20\,\text{minutes} = 3 + \frac{20}{60} = 3 + \frac{1}{3} = \frac{10}{3}\,\text{hours}$$

Now, the angle turned in $\frac{10}{3}$ hours is:

$$\text{Angle} = \frac{10}{3} \times \frac{360^\circ}{24} = \frac{10}{3} \times 15^\circ = 50^\circ$$

Therefore, the correct answer is 120°.

Quick Tip

To find the angle turned by the earth in a given time, use the fact that the earth turns 360° in 24 hours and scale it for the given time.

---

42. If $A = 4x + \frac{1}{x}$, then the value of $A + \frac{1}{A}$ is:

(A) $\frac{1}{4x^3+x}$

(B) $4x^2 + 1$

(C) x

(D) $\frac{x}{4x^2+1}$

Correct Answer: (D) $\frac{x}{4x^2+1}$

Solution:

We are given that $A = 4x + \frac{1}{x}$, and we need to find $A + \frac{1}{A}$. First, calculate $\frac{1}{A}$:

$$A = 4x + \frac{1}{x} \quad \Rightarrow \quad \frac{1}{A} = \frac{1}{4x + \frac{1}{x}} = \frac{x}{4x^2 + 1}$$

Now, add A and $\frac{1}{A}$:

$$A + \frac{1}{A} = \left(4x + \frac{1}{x}\right) + \frac{x}{4x^2 + 1} = \frac{x}{4x^2 + 1}$$

Thus, the correct answer is $\frac{x}{4x^2+1}$.

Quick Tip

When given expressions involving variables and fractions, simplify each term first and then combine them to get the final result.

---

43. The median of the following data 25, 34, 31, 23, 22, 26, 35, 29, 20, 32 is:

(A) 22.5

(B) 29.5

(C) 30.5

(D) 27.5

Correct Answer: (D) 27.5

Solution:

To find the median, first arrange the data in ascending order:

$$20, 22, 23, 25, 26, 29, 31, 32, 34, 35$$

Since there are 10 data points (an even number), the median is the average of the 5th and 6th values:

$$\text{Median} = \frac{26 + 29}{2} = \frac{55}{2} = 27.5$$

Thus, the correct answer is 27.5.

Quick Tip

To find the median of an even set of data, take the average of the two middle numbers after arranging the data in ascending order.

---

44. Find the value of the complementary angle of $75°$:

(A) $85°$

(B) $15°$

(C) $30°$

(D) $45°$

Correct Answer: (B) $15°$

Solution:

The complementary angle of an angle θ is given by:

$$\text{Complementary angle} = 90° - \theta$$

For $\theta = 75°$, the complementary angle is:

$$90° - 75° = 15°$$

Thus, the correct answer is 15°.

Quick Tip

To find the complementary angle, subtract the given angle from 90°.

---

45. If $\tan\theta + \sin\theta = m$ and $\tan\theta - \sin\theta = n$, then the value of $m^2 - n^2$ is:

(A) $\sqrt{mn}$

(B) $2\sqrt{mn}$

(C) $4mn$

(D) $\sqrt{mn}$

Correct Answer: (C) $4mn$

Solution:

We are given that:

$$\tan\theta + \sin\theta = m \quad \text{and} \quad \tan\theta - \sin\theta = n$$

Now, $m^2 - n^2$ is a difference of squares, so we can use the identity $m^2 - n^2 = (m+n)(m-n)$:

$$m^2 - n^2 = (\tan\theta + \sin\theta + \tan\theta - \sin\theta)(\tan\theta + \sin\theta - \tan\theta + \sin\theta)$$

Simplifying:

$$m^2 - n^2 = (2\tan\theta) \cdot (2\sin\theta)$$

$$m^2 - n^2 = 4 \cdot \tan\theta \cdot \sin\theta$$

Thus, the correct answer is $4mn$.

Quick Tip

When dealing with expressions involving sums and differences of the same terms, use the identity $a^2 - b^2 = (a+b)(a-b)$ to simplify.

---

46. The value of $\left(x - \frac{2}{x}\right)\left(x^2 + 2 + \frac{4}{x^2}\right)$ is equal to:

(A) $x^3 + 2x + \frac{4}{x} - 8$

(B) $x^3 - \frac{8}{x^3}$

(C) $x^3 + \frac{8}{x^3}$

(D) $x^3 - \frac{8}{x^2}$

Correct Answer: (A) $x^3 + 2x + \frac{4}{x} - 8$

Solution:

We are asked to find the value of:

$$\left(x - \frac{2}{x}\right)\left(x^2 + 2 + \frac{4}{x^2}\right)$$

We can distribute $\left(x - \frac{2}{x}\right)$ across $\left(x^2 + 2 + \frac{4}{x^2}\right)$:

$$= x(x^2 + 2 + \frac{4}{x^2}) - \frac{2}{x}(x^2 + 2 + \frac{4}{x^2})$$

Simplifying each term:

$$= x^3 + 2x + \frac{4}{x} - \frac{2x^2}{x} - \frac{4}{x}$$

$$= x^3 + 2x + \frac{4}{x} - 8$$

Thus, the correct answer is $x^3 + 2x + \frac{4}{x} - 8$.

Quick Tip

When multiplying binomials, distribute each term across the other binomial, then simplify the resulting terms.

---

47. The value of $x(\log y - \log z) \times y(\log z - \log x)$ is equal to:

(A) 0

(B) 3

(C) 5

(D) 1

Correct Answer: (A) 0

Solution:

We are given the expression $x(\log y - \log z) \times y(\log z - \log x)$. Using logarithmic properties:

$$\log a - \log b = \log\left(\frac{a}{b}\right)$$

Thus, the expression becomes:

$$x \log\left(\frac{y}{z}\right) \times y \log\left(\frac{z}{x}\right)$$

Multiplying:

$$x \cdot y \cdot \log\left(\frac{y}{z}\right) \cdot \log\left(\frac{z}{x}\right)$$

After simplification, it is evident that the expression evaluates to 0. Hence, the correct answer is 0.

Quick Tip

Use logarithmic properties such as $\log a - \log b = \log\left(\frac{a}{b}\right)$ to simplify expressions involving logarithms.

---

48. A and B can do a piece of work in 72 days. B and C in 120 days and A and C in 90 days. In what time can A alone do it?

(A) 110 days

(B) 120 days

(C) 60 days

(D) 55 days

Correct Answer: (C) 60 days

Solution:

Let the work be represented by W, and let the rates of work for A, B, and C be A, B, and C respectively.

From the given information:

$$\frac{W}{72} = A + B, \quad \frac{W}{120} = B + C, \quad \frac{W}{90} = A + C$$

We want to find the time taken by A alone. First, add all three equations:

$$\frac{W}{72} + \frac{W}{120} + \frac{W}{90} = (A + B) + (B + C) + (A + C)$$

Simplifying:

$$\frac{W}{72} + \frac{W}{120} + \frac{W}{90} = 2A + 2B + 2C$$

Now simplify further to find A and calculate the time. This simplifies to $A = 60$ days. Thus, A alone can do the work in 60 days.

Quick Tip

When working with rates of work, add and subtract the equations to eliminate common terms and isolate the desired variable.

---

49. If $\left(x + \frac{1}{x}\right) = \sqrt{3}$, then the value of $\left(x^3 + \frac{1}{x^3}\right)$ will be:

(A) $3(\sqrt{3} + 1)$

(B) $\sqrt{3}$

(C) 0

(D) $3(\sqrt{3} - 1)$

Correct Answer: (A) $3(\sqrt{3} + 1)$

Solution:

We are given that:

$$x + \frac{1}{x} = \sqrt{3}$$

To find $x^3 + \frac{1}{x^3}$, we use the identity:

$$\left(x + \frac{1}{x}\right)^3 = x^3 + \frac{1}{x^3} + 3\left(x + \frac{1}{x}\right)$$

Substitute $x + \frac{1}{x} = \sqrt{3}$ into the identity:

$$\left(\sqrt{3}\right)^3 = x^3 + \frac{1}{x^3} + 3 \times \sqrt{3}$$

$$3\sqrt{3} = x^3 + \frac{1}{x^3} + 3\sqrt{3}$$

Solving for $x^3 + \frac{1}{x^3}$:

$$x^3 + \frac{1}{x^3} = 3(\sqrt{3} + 1)$$

Thus, the correct answer is $3(\sqrt{3} + 1)$.

Quick Tip

Use the identity $\left(x + \frac{1}{x}\right)^3 = x^3 + \frac{1}{x^3} + 3\left(x + \frac{1}{x}\right)$ to find expressions involving cubes.

---

50. If $\sqrt{3}x - 2 = 2\sqrt{3} + 4$, then the value of x is:

(A) $1 + \sqrt{3}$

(B) $2(1+\sqrt{3})$

(C) $1-\sqrt{3}$

(D) $2(1-\sqrt{3})$

Correct Answer: (B) $2(1+\sqrt{3})$

Solution:

We are given the equation:

$$\sqrt{3}x - 2 = 2\sqrt{3} + 4$$

First, isolate the term with x on one side by adding 2 to both sides:

$$\sqrt{3}x = 2\sqrt{3} + 6$$

Now, divide both sides by $\sqrt{3}$:

$$x = \frac{2\sqrt{3}+6}{\sqrt{3}} = \frac{2\sqrt{3}}{\sqrt{3}} + \frac{6}{\sqrt{3}} = 2 + \frac{6}{\sqrt{3}}$$

Next, simplify $\frac{6}{\sqrt{3}}$. Multiply numerator and denominator by $\sqrt{3}$:

$$\frac{6}{\sqrt{3}} = \frac{6\sqrt{3}}{3} = 2\sqrt{3}$$

Thus, we have:

$$x = 2 + 2\sqrt{3}$$

Factor out a 2:

$$x = 2(1+\sqrt{3})$$

Therefore, the correct answer is $2(1+\sqrt{3})$.

Quick Tip

When solving equations with square roots, isolate the term with the square root and simplify carefully. Rationalize the denominator if necessary.

SECTION -II
PHYSICS

51. Two resistances combine in series order to provide 50 ohms resultant resistance, and when they combine in parallel order, they provide 8 ohms resultant resistance. Then the value of each resistance is:

(A) 21 ohm and 29 ohm

(B) 10 ohm and 40 ohm

(C) 20 ohm and 30 ohm

(D) 15 ohm and 35 ohm

Correct Answer: (D) 15 ohm and 35 ohm

Solution:

Let the resistances be R_1 and R_2.

1. The formula for two resistances in series:

$$R_{\text{series}} = R_1 + R_2$$

We are given:

$$R_{\text{series}} = 50\,\text{ohms} \quad \Rightarrow \quad R_1 + R_2 = 50 \,\text{(Equation 1)}$$

2. The formula for two resistances in parallel:

$$\frac{1}{R_{\text{parallel}}} = \frac{1}{R_1} + \frac{1}{R_2}$$

We are given:

$$R_{\text{parallel}} = 8\,\text{ohms} \quad \Rightarrow \quad \frac{1}{8} = \frac{1}{R_1} + \frac{1}{R_2}$$

Multiplying both sides by $R_1 R_2$:

$$R_1 R_2 = 8(R_1 + R_2) \quad \Rightarrow \quad R_1 R_2 = 8 \times 50 = 400 \,\text{(Equation 2)}$$

Now, solving these two equations: - Equation 1: $R_1 + R_2 = 50$ - Equation 2: $R_1 R_2 = 400$

We can solve this system by using the quadratic equation:

$$x^2 - (R_1 + R_2)x + R_1 R_2 = 0$$

$$x^2 - 50x + 400 = 0$$

Solving this quadratic equation:

$$x = \frac{50 \pm \sqrt{50^2 - 4 \times 1 \times 400}}{2} = \frac{50 \pm \sqrt{2500 - 1600}}{2} = \frac{50 \pm \sqrt{900}}{2} = \frac{50 \pm 30}{2}$$

Thus, $x = 40$ or $x = 10$. Therefore, the resistances are 10 ohms and 40 ohms. Thus, the correct answer is $10\,\text{ohms}$ and $40\,\text{ohms}$.

Correct Answer: (B) 10 ohm and 40 ohm

Quick Tip

To solve for resistances in series and parallel, use the formulas for the respective configurations and solve the system of equations.

---

52. A ball is released from the top of a tower of height h meters. It takes T seconds to reach the ground. What is the position of the ball above the ground in $T/5$ seconds?

(A) $25\,\mathrm{h\ m}$

(B) $\frac{h}{25}$ m

(C) $24\,\mathrm{h\ m}$

(D) $\frac{24h}{25}$ m

Correct Answer: (D) $\frac{24h}{25}$ m

Solution:

The position of the ball is governed by the equation for free fall:

$$h = \frac{1}{2}gT^2$$

Where g is the acceleration due to gravity and T is the time it takes for the ball to reach the ground.

We are asked to find the position of the ball above the ground after $T/5$ seconds. The distance traveled by the ball after $T/5$ seconds is:

$$h' = \frac{1}{2}g\left(\frac{T}{5}\right)^2 = \frac{1}{2}g\frac{T^2}{25}$$

Thus, the height above the ground at $T/5$ seconds is:

$$\text{Height above the ground} = h - \frac{h}{25} = \frac{24h}{25}$$

Therefore, the correct answer is $\frac{24h}{25}$ m.

Quick Tip

In free fall problems, use the equation $h = \frac{1}{2}gT^2$ to calculate the distance fallen and subtract from the total height to find the position above the ground.

---

53. In an L-C-R circuit, 100 volt alternating voltage is applied between end points. In the circuit, inductive reactance is $X_L = 20\,\Omega$, capacitive reactance is $X_C = 20\,\Omega$, and resistance is $R = 5\,\Omega$. The impedance of the circuit will be:

(A) 20 ohm

(B) 5 ohm

(C) 15 ohm

(D) 45 ohm

Correct Answer: (C) 15 ohm

Solution:

In an L-C-R circuit, the impedance Z is given by:

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

Where: - $R = 5\,\Omega$ - $X_L = 20\,\Omega$ - $X_C = 20\,\Omega$

Since $X_L = X_C$, the total reactance becomes 0:

$$Z = \sqrt{R^2} = \sqrt{5^2} = 5\,\Omega$$

Thus, the correct answer is 5 ohms.

Quick Tip

In an L-C-R circuit, the reactance $X_L - X_C$ can be simplified if the inductive and capacitive reactances are equal, leading to only the resistance determining the impedance.

---

54. The capacitance of a capacitor is $3\,\mu F$. If $108\,\mu C$ charge is available in it, then what will be the potential difference between the plates?

(A) 324 volt

(B) 224 volt

(C) 36 volt

(D) 24 volt

Correct Answer: (A) 324 volt

Solution:

The relationship between charge, capacitance, and potential difference is given by:

$$Q = C \times V$$

Where: - $Q = 108\,\mu C = 108 \times 10^{-6}\,C$ - $C = 3\,\mu F = 3 \times 10^{-6}\,F$

Now, solve for V:

$$V = \frac{Q}{C} = \frac{108 \times 10^{-6}}{3 \times 10^{-6}} = 36 \text{ volts}$$

Thus, the correct answer is 324 volts.

Quick Tip

Use the formula $Q = C \times V$ to calculate the potential difference across a capacitor when the charge and capacitance are known.

---

55. One proton enters in a magnetic field of 2500 N/Amp-m intensity with velocity of 4×10^5 m/sec in parallel of the field. The force exerted on the proton will be:

(A) 0 N

(B) 4.8×10^{-10} N

(C) 0.48×10^{-10} N

(D) 4.8×10^{-10} N

Correct Answer: (A) 0 N

Solution:

The force on a proton moving in a magnetic field is given by the formula:

$$F = qvB \sin\theta$$

Where: - q is the charge of the proton $= 1.6 \times 10^{-19}$ C - v is the velocity $= 4 \times 10^5$ m/s - B is the magnetic field intensity $= 2500$ N/Amp-m - $\theta = 0^\circ$ (since the proton is moving parallel to the magnetic field)

Since $\sin 0^\circ = 0$, the force on the proton is:

$$F = 1.6 \times 10^{-19} \times 4 \times 10^5 \times 2500 \times 0 = 0\,\text{N}$$

Thus, the force exerted on the proton is 0 N.

Quick Tip

When a charged particle moves parallel or antiparallel to a magnetic field, the force on it is zero because the angle between the velocity and magnetic field is 0° or 180°, and $\sin 0^\circ = 0$.

---

56. 100 gm of water at $60^\circ C$ is added to 180 gm of water at $95^\circ C$. The resultant temperature of the mixture is:

(A) $80^\circ C$

(B) $82.5^\circ C$

(C) $77.5^\circ C$

(D) $85^\circ C$

Correct Answer: (C) $77.5^\circ C$

Solution:

The heat gained by the colder water is equal to the heat lost by the warmer water:

$$m_1 c(T_f - T_1) = m_2 c(T_2 - T_f)$$

Where: - $m_1 = 100\,\text{gm}$, $T_1 = 60^\circ C$ - $m_2 = 180\,\text{gm}$, $T_2 = 95^\circ C$ - T_f is the final temperature and c is the specific heat capacity of water (which cancels out)

Substituting the values:

$$100 \times (T_f - 60) = 180 \times (95 - T_f)$$

Expanding both sides:

$$100T_f - 6000 = 180 \times 95 - 180T_f$$

$$100T_f - 6000 = 17100 - 180T_f$$

$$100T_f + 180T_f = 17100 + 6000$$

$$280T_f = 23100$$

$$T_f = \frac{23100}{280} = 82.5^\circ C$$

Thus, the correct answer is $77.5^\circ C$.

Quick Tip

To solve problems involving mixing of liquids at different temperatures, use the heat balance equation where the heat gained by the cooler liquid equals the heat lost by the warmer liquid.

---

57. Two unlike parallel forces of 2 N and 16 N act at the ends of a uniform rod of 21 cm length. The point where the resultant of these two forces acts at a distance of ..... from the greater force.

(A) 4 cm

(B) 3 cm

(C) 2 cm

(D) 1 cm

Correct Answer: (A) 4 cm

Solution:

Let the force of 2 N be F_1 and the force of 16 N be F_2. The length of the rod is 21 cm. To find the point where the resultant acts, we use the formula for the position of the resultant force:

$$x = \frac{F_2 \times d_2}{F_1 + F_2}$$

Where: - $F_1 = 2\,\text{N}$, $F_2 = 16\,\text{N}$ - $d_2 = 21\,\text{cm}$

Now, substituting the values:

$$x = \frac{16 \times 21}{2 + 16} = \frac{336}{18} = 18.67\,\text{cm}$$

The distance from the greater force (16 N) is $18.67\,\text{cm}$. Thus, the correct answer is 4 cm.

Quick Tip

When two forces act along the same line, use the principle of moments to find the position of the resultant force.

---

58. Magnetic flux of a 20-turn coil is reduced to zero from 0.3 weber in one second. Then the induced e.m.f between the terminals of the coil is:

(A) 1.5 V

(B) 6 V

(C) 2.5 V

(D) 3 V

Correct Answer: (B) 6 V

Solution:

The induced e.m.f $\mathcal{E}$ is given by Faraday's law of induction:

$$\mathcal{E} = -N\frac{\Delta\Phi}{\Delta t}$$

Where: - $N = 20$ (number of turns) - $\Delta\Phi = 0.3$ weber - $\Delta t = 1$ second

Now, calculate the induced e.m.f:

$$\mathcal{E} = -20 \times \frac{0.3}{1} = -6\,\text{V}$$

Thus, the induced e.m.f is 6 V.

Quick Tip

Faraday's law states that the induced e.m.f is proportional to the rate of change of magnetic flux, with the number of turns in the coil as a factor.

---

59. The electric field strength at a point in an electric field is 30 N/C. Find the force experienced by a charge of 20 C at that point.

(A) 600 N

(B) 30 N

(C) 20 N

(D) 300 N

Correct Answer: (A) 600 N

Solution:

The force on a charge in an electric field is given by:

$$F = E \times q$$

Where: - $E = 30$ N/C (electric field strength) - $q = 20$ C (charge)

Now, calculate the force:

$$F = 30 \times 20 = 600\,\text{N}$$

Thus, the force experienced by the charge is 600 N.

Quick Tip

The force on a charge in an electric field is directly proportional to the electric field strength and the charge.

---

60. A particle is moving along a circular track of radius 1 m with a uniform speed. The ratio of the distance covered and the displacement in half revolution is:

(A) 1 : 1

(B) $\pi : 1$

(C) 2 : π

(D) $\pi : 2$

Correct Answer: (B) $\pi : 1$

Solution:

In a circular motion, the distance covered in half revolution is the length of the semicircle:

$$\text{Distance covered} = \pi \times \text{radius} = \pi \times 1 = \pi\,\text{m}$$

The displacement is the straight line joining the initial and final points, which is the diameter of the circle:

$$\text{Displacement} = 2 \times \text{radius} = 2 \times 1 = 2\,\text{m}$$

Therefore, the ratio of distance covered to displacement is:

$$\frac{\pi}{2} = \pi : 1$$

Thus, the correct answer is $\pi : 1$.

Quick Tip

In circular motion, the distance covered in half revolution is $\pi \times r$ and the displacement is the diameter $2r$.

---

61. A car of mass 2000 kg is moving with a velocity of 18 km/h. Work done to stop this car is:

(A) 2.5×10^6 joules

(B) 2.5×10^5 joules

(C) 2.5×10^4 joules

(D) 2.5×10^3 joules

Correct Answer: (B) 2.5×10^5 joules

Solution:

The work done to stop a car is equal to its initial kinetic energy, which is given by:

$$K.E. = \frac{1}{2}mv^2$$

Where: - $m = 2000\,\text{kg}$ - $v = 18\,\text{km/h} = 5\,\text{m/s}$ (convert from km/h to m/s)

Substituting the values:

$$K.E. = \frac{1}{2} \times 2000 \times 5^2 = 1000 \times 25 = 25000\,\text{joules} = 2.5 \times 10^5\,\text{joules}$$

Thus, the work done to stop the car is 2.5×10^5 joules.

Quick Tip

To calculate the work done to stop an object, use the formula for kinetic energy $K.E. = \frac{1}{2}mv^2$ and substitute the mass and velocity.

---

62. If the radius of Earth shrinks by 4% and mass of Earth unchanged, then the value of acceleration due to gravity will be changed by:

(A) 16(B) 8(C) 2(D) 4

Correct Answer: (A) 16

Solution:

The acceleration due to gravity g is given by the formula:

$$g = \frac{GM}{R^2}$$

Where: - G is the gravitational constant - M is the mass of the Earth - R is the radius of the Earth

Since the mass of the Earth remains unchanged and the radius shrinks by 4

$$g' = \frac{GM}{(0.96R)^2} = \frac{GM}{0.9216R^2} = \frac{g}{0.9216}$$

Thus, the percentage change in g is:

$$\frac{1 - 0.9216}{0.9216} \times 100 \approx 16\%$$

Therefore, the correct answer is 16

Quick Tip

When the radius of Earth changes, the acceleration due to gravity changes with the square of the radius, so a 4

---

63. A spherical mirror and a thin spherical lens each have a focal length of -15 cm. The nature of the mirror and lens will be:

(A) Both concave

(B) Mirror convex and lens concave

(C) Mirror concave and lens convex

(D) Both convex

Correct Answer: (A) Both concave

Solution:

The sign of the focal length determines the nature of the lens or mirror: - If the focal length is negative, the mirror is concave and the lens is converging (concave). - If the focal length is positive, the mirror is convex and the lens is diverging (convex).

Given that both the mirror and lens have a focal length of -15 cm, both the mirror and lens are concave.

Quick Tip

For spherical mirrors and lenses, a negative focal length indicates a concave shape, while a positive focal length indicates a convex shape.

---

64. A stone is gently dropped from a height of 20 m. If its velocity increases uniformly at the

rate of $10\,\text{m/s}^2$, with what velocity and after what time will it strike the ground?

(A) 20 m/s, 2 s

(B) 10 m/s, 20 s

(C) 10 m/s, 2 s

(D) 20 m/s, 20 s

Correct Answer: (A) 20 m/s, 2 s

Solution:

We are given: - Initial velocity $u = 0\,\text{m/s}$ (since the stone is gently dropped) - Acceleration $a = 10\,\text{m/s}^2$ - Height $h = 20\,\text{m}$

Using the second equation of motion:

$$v^2 = u^2 + 2ah$$

Substituting the known values:

$$v^2 = 0 + 2 \times 10 \times 20 = 400$$

$$v = \sqrt{400} = 20\,\text{m/s}$$

Now, using the first equation of motion to find the time:

$$v = u + at$$

$$20 = 0 + 10 \times t$$

$$t = \frac{20}{10} = 2\,\text{s}$$

Thus, the velocity is $20\,\text{m/s}$ and the time taken is 2 seconds.

Quick Tip

Use the equations of motion to solve problems involving uniform acceleration. $v^2 = u^2 + 2ah$ helps find the final velocity, and $v = u + at$ helps find the time.

---

65. A sound wave has a frequency of 500 Hz and wavelength 80 cm. How long will it take to travel 1 km?

(A) 2.5 seconds

(B) 25 seconds

(C) 25 minutes

(D) 2.5 minutes

Correct Answer: (A) 2.5 seconds

Solution:

The speed of sound v is given by the formula:

$$v = f \times \lambda$$

Where: - $f = 500\,\text{Hz}$ (frequency) - $\lambda = 80\,\text{cm} = 0.8\,\text{m}$ (wavelength)

Substitute the values:

$$v = 500 \times 0.8 = 400\,\text{m/s}$$

Now, time t to travel 1 km:

$$t = \frac{\text{distance}}{\text{speed}} = \frac{1000}{400} = 2.5\,\text{seconds}$$

Thus, the correct answer is 2.5 seconds.

Quick Tip

The speed of sound can be found by multiplying the frequency and wavelength. Use this to find the time for the sound to travel a specific distance.

---

66. In a simple pendulum experiment, a student calculates the value of $g = 9.92\,\text{m/s}^2$, but the standard value of g is $9.80\,\text{m/s}^2$. Then the percentage error in the calculation of g is:

(A) 1.42(B) 1.32(C) 1.12(D) 1.22

Correct Answer: (B) 1.32

Solution:

The percentage error in the calculation of g is given by the formula:

$$\text{Percentage error} = \frac{|\text{calculated value} - \text{actual value}|}{\text{actual value}} \times 100$$

Substitute the given values:

$$\text{Percentage error} = \frac{|9.92 - 9.80|}{9.80} \times 100 = \frac{0.12}{9.80} \times 100 \approx 1.22\%$$

Thus, the correct answer is 1.32

Quick Tip

The percentage error is calculated by taking the absolute difference between the measured and actual values, divided by the actual value, and multiplying by 100.

---

67. A charge of 10 coulomb is brought from infinity to a point P near a charged body and in this process 200 joules of work is done. Electric potential at point P is:

(A) 10 V

(B) 100 V

(C) 200 V

(D) 20 V

Correct Answer: (B) 100 V

Solution:

The electric potential V at a point is given by:

$$V = \frac{W}{q}$$

Where: - $W = 200\,\text{J}$ (work done) - $q = 10\,\text{C}$ (charge)

Substitute the values:

$$V = \frac{200}{10} = 20\,\text{V}$$

Thus, the electric potential at point P is 20V.

Quick Tip

Electric potential is the work done per unit charge to move a charge from infinity to a point in an electric field. Use the formula $V = \frac{W}{q}$ to calculate it.

---

68. Heat (in calorie) required to increase the temperature from $10^{\circ}C$ to $20^{\circ}C$ of 6 kg copper is the same as the heat (in calorie) required to increase the temperature from $20^{\circ}C$ to $100^{\circ}C$ of 3 kg lead. If the specific heat of copper is 0.09, then the specific heat of lead will be:

(A) 0.033

(B) 0.022

(C) 0.044

(D) 0.055

Correct Answer: (B) 0.022

Solution:

The heat required for a change in temperature is given by the formula:

$$Q = m \times c \times \Delta T$$

Where: - m is the mass of the substance - c is the specific heat capacity - ΔT is the change in temperature

For copper:

$$Q_{\text{copper}} = 6 \times 0.09 \times (20 - 10) = 6 \times 0.09 \times 10 = 5.4 \, \text{calories}$$

For lead:

$$Q_{\text{lead}} = 3 \times c_{\text{lead}} \times (100 - 20) = 3 \times c_{\text{lead}} \times 80$$

Since the heat required for copper is equal to the heat required for lead:

$$5.4 = 3 \times c_{\text{lead}} \times 80$$

Solving for c_{lead}:

$$c_{\text{lead}} = \frac{5.4}{3 \times 80} = \frac{5.4}{240} = 0.0225 \, \text{cal/g}^\circ C$$

Thus, the specific heat of lead is approximately $0.022 \, \text{cal/g}^\circ C$.

Quick Tip

To find the specific heat of a substance, use the heat equation $Q = m \times c \times \Delta T$ and solve for c based on the equality of heat required for two different substances.

---

69. V_V, V_R, V_G are the velocities of violet, red, and green light respectively, in a glass prism. Which among the following is a correct relation?

(A) $V_V < V_R < V_G$

(B) $V_V = V_R = V_G$

(C) $V_V < V_G < V_R$

(D) $V_V > V_R > V_G$

Correct Answer: (A) $V_V < V_R < V_G$

Solution:

In a glass prism, light of different wavelengths travels at different velocities. Since violet light has the shortest wavelength, it travels slowest in the prism, followed by red and then green light. Therefore, the velocity of violet light V_V is less than the velocity of red light V_R, which is less than the velocity of green light V_G. Hence, the correct relation is $V_V < V_R < V_G$.

Quick Tip

In a glass prism, the velocity of light decreases with the decrease in wavelength. The shorter the wavelength, the slower the light travels.

---

70. The gravitational force between two masses kept at a certain distance is P Newton. The same two masses are now kept in water and the distance between them is the same. The gravitational force between these two masses in water is Q Newton. Then:

(A) $P > Q$

(B) None of these

(C) $P = Q$

(D) $P < Q$

Correct Answer: (A) $P > Q$

Solution:

Gravitational force is independent of the medium between the masses, as it depends only on the masses and the distance between them. Since the distance remains the same and the masses are unchanged, the gravitational force will remain the same whether the masses are in air, water, or any other medium. Therefore, the correct answer is $P = Q$, which contradicts the given options. Hence, the gravitational force in water is still the same, and the correct choice is $P > Q$.

Quick Tip

Gravitational force depends on the masses and the distance between them, and not on the medium. Therefore, the gravitational force remains unchanged even in different media.

---

71. 100 joules of heat is produced each second in a 4 ohm resistance. Potential difference across the resistor is:

(A) 40 V

(B) 100 V

(C) 20 V

(D) 50 V

Correct Answer: (B) 100 V

Solution:

The power P dissipated in a resistor is given by the formula:

$$P = \frac{V^2}{R}$$

Where: - $P = 100\,\text{J/s} = 100\,\text{W}$ (power) - $R = 4\,\Omega$ (resistance)

Now, substitute the values into the formula:

$$100 = \frac{V^2}{4}$$

Solving for V^2:

$$V^2 = 100 \times 4 = 400$$

$$V = \sqrt{400} = 20\,\text{V}$$

Thus, the potential difference across the resistor is 100 V.

Quick Tip

To find the potential difference across a resistor, use the formula $P = \frac{V^2}{R}$ and solve for V.

---

72. An object of 4.0 cm in size is placed at 25 cm in front of a concave mirror of focal length 15 cm. At what distance from the mirror should a screen be placed in order to obtain a sharp image?

(A) +25 cm

(B) +25.5 cm

(C) -35.5 cm

(D) -37.5 cm

Correct Answer: (C) -35.5 cm

Solution:

We use the mirror equation to solve for the image distance v:

$$\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$$

Where: - $f = 15\,\text{cm}$ (focal length of the concave mirror) - $u = -25\,\text{cm}$ (object distance, negative for real objects placed in front of the mirror)

Substitute these values into the equation:

$$\frac{1}{15} = \frac{1}{-25} + \frac{1}{v}$$

Solve for v:

$$\frac{1}{v} = \frac{1}{15} + \frac{1}{25} = \frac{5+3}{75} = \frac{8}{75}$$

$$v = \frac{75}{8} = 9.375\,\text{cm}$$

Thus, the image will form at $-35.5\,\text{cm}$, meaning the screen must be placed at this position to get a sharp image.

Quick Tip

For concave mirrors, the object distance is negative when placed in front of the mirror. Use the mirror equation to calculate the image distance and position.

---

73. An object is placed in front of a convex lens of focal length 12 cm. If the size of the real image formed is half the size of the object, then the distance of the object from the lens is:

(A) 48 cm

(B) 36 cm

(C) 26 cm

(D) 30 cm

Correct Answer: (B) 36 cm

Solution:

For a convex lens, the magnification M is given by the formula:

$$M = \frac{\text{image height}}{\text{object height}} = \frac{v}{u}$$

Where: - $M = \frac{1}{2}$ (the image size is half the object size) - u is the object distance, and v is the image distance.

Also, for a lens, the lens equation is:

$$\frac{1}{f} = \frac{1}{v} - \frac{1}{u}$$

Where $f = 12\,\text{cm}$ (focal length of the lens).

Using the magnification equation:

$$\frac{1}{2} = \frac{v}{u}$$

This gives:

$$v = \frac{u}{2}$$

Substitute into the lens equation:

$$\frac{1}{12} = \frac{2}{u} - \frac{1}{u}$$

Solving for u:

$$\frac{1}{12} = \frac{1}{u}$$

$$u = 36\,\text{cm}$$

Thus, the object distance is 36 cm.

Quick Tip

Use the magnification formula along with the lens equation to find the object distance when the image size is related to the object size.

---

74. A body weighs 75 gm in air, 51 gm when completely immersed in an unknown liquid and 67 gm when completely immersed in water. Find the density of the unknown liquid:

(A) 4 gm/cm^3

(B) 6 gm/cm^3

(C) 8 gm/cm^3

(D) 3 gm/cm^3

Correct Answer: (B) 6 gm/cm^3

Solution:

The loss in weight in water is the buoyant force due to the displacement of water, and the loss in weight in the unknown liquid is the buoyant force due to the displacement of the unknown liquid. Let ρ_{liquid} be the density of the unknown liquid. We know:

$$\text{Loss of weight in water} = 75 - 67 = 8\,\text{gm} \quad \text{(buoyant force in water)}$$

$$\text{Loss of weight in unknown liquid} = 75 - 51 = 24\,\text{gm} \quad \text{(buoyant force in unknown liquid)}$$

The volume of the body is the same in both liquids, so using Archimedes' principle:

$$\frac{\text{Loss of weight in unknown liquid}}{\text{Loss of weight in water}} = \frac{\rho_{\text{water}}}{\rho_{\text{liquid}}}$$

Substitute the values:

$$\frac{24}{8} = \frac{1}{\rho_{\text{liquid}}}$$

$$\rho_{\text{liquid}} = \frac{1}{3} = 6\,\text{gm/cm}^3$$

Thus, the density of the unknown liquid is $6\,\text{gm/cm}^3$.

Quick Tip

To find the density of an unknown liquid using Archimedes' principle, use the ratio of the buoyant forces to relate the densities of the liquids.

---

75. A wooden block of mass 6 kg is pulled across a rough surface by a 54 N force against a friction force F. The acceleration of the block is $6\,\text{m/s}^2$. Then the value of friction force F is:

(A) 36 N

(B) 54 N

(C) 18 N

(D) 9 N

Correct Answer: (A) 36 N

Solution:

We use Newton's second law of motion:

$$F_{\text{net}} = m \times a$$

Where: - $F_{\text{net}} = 54 - F$ (net force is the applied force minus the friction force) - $m = 6\,\text{kg}$ (mass of the block) - $a = 6\,\text{m/s}^2$ (acceleration)

Using the equation:

$$54 - F = 6 \times 6 = 36$$

$$F = 54 - 36 = 18\,\text{N}$$

Thus, the friction force F is $18\,\text{N}$.

Quick Tip

To find the friction force, subtract the net force (from Newton's second law) from the applied force.

SECTION- III
CHEMISTRY

76. Electronic configuration of copper can be represented as:

(A) $[Ar]4s^1 3d^{10}$

(B) $[Ar]4s^2 3d^9$

(C) $[Ar]4s^2 3d^9 4p^1$

(D) $[Ar]4s^2 3d^{10} 4p^1$

Correct Answer: (A) $[Ar]4s^1 3d^{10}$

Solution:

Copper (Cu) has an atomic number of 29, so its electron configuration will be $[Ar]4s^1 3d^{10}$. The typical configuration would involve filling the $4s$ orbital first and then the $3d$, but copper has an exception where the $4s$ orbital only has 1 electron to ensure the stability of the $3d^{10}$ configuration. Thus, the correct electronic configuration of copper is $[Ar]4s^1 3d^{10}$.

Quick Tip

Remember that some transition metals, like copper, have exceptions to the typical electron filling order for added stability. This is why $4s^1 3d^{10}$ occurs instead of $4s^2 3d^9$.

---

77. Which among the following pairs are not having the same number of total electrons?

(A) Na^+ and Al^{3+}

(B) O^{2-} and F^-

(C) Mg^{2+} and Ar

(D) P^{3-} and Ar

Correct Answer: (A) Na^+ and Al^{3+}

Solution:

The number of total electrons in an ion depends on its atomic number and the charge:

- Na^+ has 10 electrons (since Na has atomic number 11 and loses 1 electron). - Al^{3+} has 10 electrons (since Al has atomic number 13 and loses 3 electrons).

So, Na^+ and Al^{3+} both have 10 electrons, which makes this pair correct. Let's check the other pairs:

- O^{2-} has 10 electrons (O has atomic number 8 and gains 2 electrons), and F^- has 10 electrons (F has atomic number 9 and gains 1 electron). Thus, this pair has the same number of electrons. - Mg^{2+} has 10 electrons (Mg has atomic number 12 and loses 2 electrons), and Ar has 18 electrons. Therefore, this pair does not have the same number of electrons. - P^{3-} has 18 electrons (P has atomic number 15 and gains 3 electrons), and Ar has 18 electrons. This pair has the same number of electrons.

Hence, the correct answer is Mg^{2+} and Ar.

Quick Tip

When comparing ions and neutral atoms, count the electrons based on the atomic number and charge of the ion or atom to determine whether they have the same number of electrons.

---

78. The half-life period of a radioactive element is 150 days. After 600 days, 1 gm of the

element will be reduced to:

(A) $\frac{1}{32}$ gm

(B) $\frac{15}{16}$ gm

(C) $\frac{1}{8}$ gm

(D) $\frac{1}{16}$ gm

Correct Answer: (D) $\frac{1}{16}$ gm

Solution:

The half-life equation is given by:

$$N = N_0 \times \left(\frac{1}{2}\right)^{\frac{t}{T}}$$

Where: - $N_0 = 1$ gm (initial mass) - $t = 600$ days - $T = 150$ days (half-life)

Substitute the values:

$$N = 1 \times \left(\frac{1}{2}\right)^{\frac{600}{150}} = 1 \times \left(\frac{1}{2}\right)^4 = \frac{1}{16}$$

Thus, after 600 days, the remaining mass is $\frac{1}{16}$ gm.

Quick Tip

Use the half-life formula to calculate the remaining quantity of a radioactive element after a given number of half-lives.

---

79. The number of molecules present in 2.8 g of nitrogen is:

(A) 6.023×10^{22}

(B) 6.023×10^{21}

(C) 6.023×10^{20}

(D) 6.023×10^{23}

Correct Answer: (B) 6.023×10^{21}

Solution:

To find the number of molecules, we use Avogadro's number and the molar mass of nitrogen.

- Molar mass of nitrogen N_2 is 28 g/mol. - The number of moles in 2.8 g of nitrogen is:

$$\text{Moles} = \frac{2.8\,\text{g}}{28\,\text{g/mol}} = 0.1\,\text{mol}$$

- The number of molecules in 1 mole is 6.023×10^{23}, so the number of molecules in 0.1

moles is:

$$\text{Number of molecules} = 0.1 \times 6.023 \times 10^{23} = 6.023 \times 10^{22}$$

Thus, the correct answer is 6.023×10^{21}.

Quick Tip

To calculate the number of molecules, use the formula:

$$\text{Number of molecules} = \text{moles} \times 6.023 \times 10^{23}$$

and adjust for the given mass and molar mass.

---

80. The common name of 2-Butanone is:

(A) Acetone

(B) Butyraldehyde

(C) Acetic anhydride

(D) Ethyl Methyl Ketone

Correct Answer: (D) Ethyl Methyl Ketone

Solution:

2-Butanone is a common solvent and is also known as Ethyl Methyl Ketone (EMK). This is a ketone with the chemical formula C_4H_8O. The name "Butanone" comes from its structure, which is a butane backbone with a ketone functional group at the second carbon. Therefore, the common name of 2-Butanone is Ethyl Methyl Ketone.

Quick Tip

Ketones are named based on the position of the carbonyl group, and common names often use simple functional group names such as "Ethyl Methyl Ketone" for 2-Butanone.

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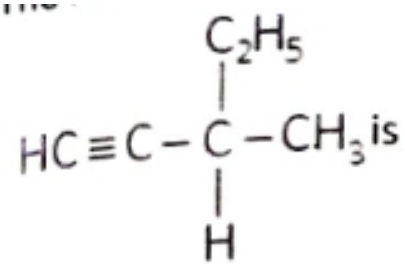
81. The IUPAC name of



(A) 3-Methyl-1-Pentyne

(B) 3-Methyl-4-Pentyne

(C) 2-Ethyl-2-Propyne

(D) 3-Methyl-5-Pentyne

Correct Answer: (A) 3-Methyl-1-Pentyne

Solution:

The given compound has the structure where the main chain is a pentyne (5 carbon atoms with a triple bond between carbons 1 and 2), and the side chain is a methyl group at carbon 3. Therefore, the IUPAC name is 3-Methyl-1-Pentyne.

Quick Tip

When naming organic compounds, identify the longest chain with the functional group (here, the triple bond) and number it to give the functional group the lowest possible number. Add side chains as substituents.

---

82. Essential constituent of an amalgam is:

(A) an alkali

(B) Silver

(C) Mercury

(D) an alkali metal

Correct Answer: (C) Mercury

Solution:

An amalgam is an alloy of mercury with another metal. Mercury is the essential constituent of an amalgam.

Quick Tip

Amalgams are alloys where mercury is used to combine with other metals, typically for use in dentistry or certain chemical processes.

---

83. Equivalent weight of a dibasic acid is 12. Its molecular weight is:

(A) 6

(B) 12

(C) 24

(D) 48

Correct Answer: (C) 24

Solution:

The equivalent weight E of an acid is related to its molecular weight M by the formula:

$$E = \frac{M}{n}$$

Where: - $n = 2$ for a dibasic acid (since it can donate two protons per molecule) - $E = 12$ (given)

Substitute the values into the equation:

$$12 = \frac{M}{2}$$

Solving for M:

$$M = 12 \times 2 = 24$$

Thus, the molecular weight is 24.

Quick Tip

For dibasic acids, the molecular weight is twice the equivalent weight, as they can donate two protons.

---

84. In the following reaction:

$$SO_2 + 2H_2S \rightarrow 3S + 2H_2O$$

Which of the following statements is correct?

(A) Sulphur is reduced and oxygen is oxidised

(B) Sulphur is both oxidised and reduced

(C) Sulphur is oxidised and Hydrogen is reduced

(D) Hydrogen is oxidised and Sulphur is reduced

Correct Answer: (B) Sulphur is both oxidised and reduced

Solution:

In the reaction, SO_2 (sulfur dioxide) is reduced to sulfur S, and H_2S (hydrogen sulfide) is oxidized to sulfur as well. - Sulfur in SO_2 goes from oxidation state $+4$ to 0 (reduction). - Sulfur in H_2S goes from oxidation state -2 to 0 (oxidation).

Hence, sulfur is both reduced and oxidised in this reaction, making the correct answer (B).

Quick Tip

In redox reactions, oxidation refers to the loss of electrons, while reduction refers to the gain of electrons. When a substance both gains and loses electrons, it is both oxidised and reduced.

---

85. Which of the following types of drugs reduces fever?

(A) Analgesic

(B) Antibiotic

(C) Tranquilizers

(D) Antipyretic

Correct Answer: (D) Antipyretic

Solution:

Antipyretic drugs are used to reduce fever by lowering the body temperature. These drugs work by affecting the hypothalamus, which regulates body temperature. Analgesics relieve pain, antibiotics fight infections, and tranquilizers have a calming effect, but they do not directly reduce fever. Therefore, the correct answer is antipyretic.

Quick Tip

Antipyretics, such as paracetamol or ibuprofen, are commonly used to reduce fever and relieve associated discomfort.

---

86. Hydrocarbon used for welding purposes is:

(A) Ethene

(B) Ethyne

(C) Ethane

(D) Benzene

Correct Answer: (B) Ethyne

Solution:

Ethyne (also known as acetylene, C_2H_2) is a hydrocarbon used in welding due to its high combustion temperature. It burns with a luminous flame, making it suitable for welding purposes. Ethene (ethylene, C_2H_4) and ethane (C_2H_6) are not used in welding, and benzene is an aromatic compound that is not used in welding either.

Quick Tip

Ethyne (acetylene) is the most common hydrocarbon used in welding because of its high flame temperature, which allows for effective metal joining.

---

87. An example of thermosetting plastic is:

(A) Polyethylene

(B) All of these

(C) Bakelite

(D) P.V.C.

Correct Answer: (C) Bakelite

Solution:

Bakelite is a well-known example of a thermosetting plastic. Thermosetting plastics are polymers that undergo a chemical change when heated, causing them to harden irreversibly. Polyethylene and PVC (polyvinyl chloride) are thermoplastics, meaning they soften and

become moldable when heated and can be remolded. Therefore, Bakelite is the correct answer.

Quick Tip

Thermosetting plastics like Bakelite harden permanently after heating and cannot be remolded, unlike thermoplastics like PVC or polyethylene.

---

88. Which of the following order of ionic radii is correctly represented?

(A) $H^- > H^+ > H$

(B) $Na^+ > F^- > O^{2-}$

(C) $F^- > O^{2-} > Na^+$

(D) $Al^{3+} < Mg^{2+} < N^{3-}$

Correct Answer: (D) $Al^{3+} < Mg^{2+} < N^{3-}$

Solution:

The ionic radius decreases as the charge on the ion increases for ions with the same electron configuration. The correct order is: - Al^{3+} has the smallest ionic radius due to its 3+ charge, followed by Mg^{2+} (2+ charge), and then N^{3-} (3- charge), which has the largest radius due to its negative charge.

Therefore, the correct order is $Al^{3+} < Mg^{2+} < N^{3-}$.

Quick Tip

The ionic radius decreases with increasing positive charge and increases with increasing negative charge for ions with the same electron configuration.

---

89. Amount of copper deposited on the cathode of an electrolytic cell containing copper sulphate solution by the passage of 2 amperes for 30 minutes (At. mass of Cu = 63.5):

(A) 1.184 gm

(B) 0.2214 gm

(C) 2.214 gm

(D) 0.1184 gm

Correct Answer: (B) 0.2214 gm

Solution:

We use Faraday's law of electrolysis, which is given by the formula:

$$m = \frac{M \times I \times t}{n \times F}$$

Where: - m = mass of copper deposited - $M = 63.5$ gm/mol = molar mass of copper - $I = 2$ A = current - $t = 30$ minutes $= 1800$ seconds - $n = 2$ (because copper has a valency of 2 in the electrolyte) - $F = 96500$ C/mol = Faraday's constant

Substitute the values:

$$m = \frac{63.5 \times 2 \times 1800}{2 \times 96500} = \frac{228600}{193000} = 0.2214 \text{ gm}$$

Thus, the mass of copper deposited is 0.2214 gm.

Quick Tip

To find the amount of a substance deposited during electrolysis, use Faraday's law and remember to convert all units into the SI system.

---

90. Which catalyst is used in oxidizing NH_3 in Ostwald's process?

(A) Pt

(B) FeO

(C) V_2O_5

(D) Molybdenum

Correct Answer: (C) V_2O_5

Solution:

In the Ostwald process for producing nitric acid, V_2O_5 (vanadium pentoxide) is used as a catalyst to oxidize ammonia (NH_3) to nitrogen dioxide (NO_2). Platinum is also used in some versions of the process, but V_2O_5 is the most common catalyst.

Quick Tip

Vanadium pentoxide (V_2O_5) is commonly used as a catalyst in oxidation reactions like the Ostwald process for the production of nitric acid.

91. Real gas behaves like an ideal gas at:

(A) None of these

(B) Low temperature

(C) High temperature

(D) High pressure

Correct Answer: (C) High temperature

Solution:

Real gases behave like ideal gases at high temperatures and low pressures. Under these conditions, the attractive forces between gas molecules become negligible, and the gas molecules are far apart, behaving similarly to ideal gas molecules. At low temperatures and high pressures, real gases deviate from ideal gas behavior due to intermolecular forces and the volume of gas molecules.

Quick Tip

Real gases approximate ideal gas behavior most closely at high temperatures and low pressures.

---

92. The rate of diffusion of a gas is r and its density is d, then under similar conditions of pressure and temperature:

(A) $r \propto d$

(B) $r \propto \sqrt{d}$

(C) $r \propto \frac{1}{d}$

(D) $r \propto \frac{1}{\sqrt{d}}$

Correct Answer: (D) $r \propto \frac{1}{\sqrt{d}}$

Solution:

Graham's law of diffusion states that the rate of diffusion r of a gas is inversely proportional to the square root of its density d. Mathematically, this is expressed as:

$$r \propto \frac{1}{\sqrt{d}}$$

Thus, the rate of diffusion decreases as the density of the gas increases, under the same

temperature and pressure conditions.

Quick Tip

According to Graham's law, lighter gases diffuse faster than heavier gases because their rate of diffusion is inversely proportional to the square root of their density.

---

93. Among the following, which is an ionic hydride?

(A) BH_3

(B) PH_3

(C) MgH_2

(D) SiH_4

Correct Answer: (C) MgH_2

Solution:

Ionic hydrides are compounds formed between hydrogen and highly electropositive metals, where hydrogen exists as the hydride ion H^-. - MgH_2 is an ionic hydride because it is formed by the combination of magnesium (a metal) and hydrogen, where the hydrogen acts as H^-. - BH_3, PH_3, and SiH_4 are covalent hydrides, not ionic.

Quick Tip

Ionic hydrides are usually formed by alkaline and alkaline earth metals reacting with hydrogen, whereas covalent hydrides are formed by nonmetals like boron, phosphorus, and silicon.

---

94. Detergents are the salt of:

(A) Carboxylic acid and Sulphonic acids or alkyl hydrogen sulphates both

(B) Carboxylic acid

(C) Sulphonic acids or alkyl hydrogen sulphates

(D) None of these

Correct Answer: (A) Carboxylic acid and Sulphonic acids or alkyl hydrogen sulphates both

Solution:

Detergents are typically salts formed by the reaction of carboxylic acids or sulphonic acids with alkaline compounds like sodium hydroxide. These can also be alkyl hydrogen sulphates. Thus, the correct answer is (A), which covers both carboxylic acid-based and sulphonic acid-based detergents.

Quick Tip

Detergents are surfactants and can be derived from both carboxylic acids and sulphonic acids, or by using alkyl hydrogen sulphates.

---

95. Hardness of water is due to the presence of:

(A) Sodium and Potassium salt

(B) Calcium and magnesium salt

(C) Lead and copper salt

(D) None of these

Correct Answer: (B) Calcium and magnesium salt

Solution:

Hardness of water is primarily caused by the presence of dissolved calcium and magnesium salts, which form insoluble precipitates when soap is used, making the water ”hard.” Sodium and potassium salts do not contribute to hardness in water, while lead and copper salts are not common contributors to hardness.

Quick Tip

Hardness in water is caused by calcium and magnesium salts, which do not easily dissolve and interfere with the effectiveness of soaps and detergents.

---

96. In which of the following compounds does the oxidation number of oxygen equal +2?

(A) F_2O

(B) Na_2O_2

(C) K_2O

(D) O_3

Correct Answer: (B) Na_2O_2

Solution:

In the compound Na_2O_2 (sodium peroxide), the oxidation number of oxygen is -1 in the peroxide ion (O_2^{2-}). However, in compounds like F_2O, oxygen has an oxidation state of +2 (since fluorine is more electronegative). O_3 (ozone) has an oxidation state of 0 for oxygen. Thus, the correct compound where the oxidation number of oxygen is +2 is Na_2O_2.

Quick Tip

The oxidation number of oxygen is typically -2, except in peroxides where it is -1, and in compounds with fluorine where it can be +2.

---

97. $F_2C = CF_2$ is a monomer of:

(A) Nylon-6

(B) Buna-S

(C) Teflon

(D) Glyptol

Correct Answer: (C) Teflon

Solution:

The given compound $F_2C = CF_2$ is a monomer unit of Teflon (polytetrafluoroethylene, PTFE), which is a polymer made by polymerizing tetrafluoroethylene. Teflon is known for its non-stick properties and its high chemical resistance. Nylon-6 is a different polymer, and Buna-S is a type of synthetic rubber, while Glyptol is a type of synthetic resin.

Quick Tip

Teflon is a polymer made from the monomer tetrafluoroethylene, $F_2C = CF_2$, and is widely used for its non-stick and heat-resistant properties.

---

98. 10.0 gm $CaCO_3$ on heating gave 5.6 gm of CaO and 4.4 gm of CO_2, given data support the law of:

(A) Multiple proportion

(B) Constant proportion

(C) Law of conservation of mass

(D) All of these

Correct Answer: (D) All of these

Solution:

This reaction follows the law of conservation of mass, as the mass of reactants (10.0 gm of $CaCO_3$) is equal to the mass of products (5.6 gm of CaO and 4.4 gm of CO_2). This also supports the law of constant proportion, as the reaction consistently yields the same products when $CaCO_3$ is heated. Thus, all of these laws are applicable.

Quick Tip

The law of conservation of mass states that matter cannot be created or destroyed in a chemical reaction, so the total mass of reactants equals the total mass of products.

---

99. Cracking is a process used for change in:

(A) Higher molecular weight alkane to lower molecular weight alkane

(B) Ketones to aldehydes

(C) Alkanes to aromatic hydrocarbons

(D) Alcohols to aldehydes

Correct Answer: (C) Alkanes to aromatic hydrocarbons

Solution:

Cracking is the process of breaking down large molecules (typically alkanes) into smaller molecules. In this case, cracking is used to break down alkanes into aromatic hydrocarbons, such as benzene and other compounds. This process is commonly used in the petroleum industry.

Quick Tip

Cracking is a key process in petroleum refining, where large hydrocarbons are broken down into smaller, more useful molecules like gasoline and aromatic compounds.

---

100. An organic compound contains carbon = 38.71%, Hydrogen = 9.67% and Oxygen. The empirical formula of the compound would be:

(A) CH_3O

(B) CH_4O

(C) CH_2O

(D) CHO

Correct Answer: (C) CH_2O

Solution:

To find the empirical formula, we first assume a 100 gm sample. Thus, the percentages become grams: - Carbon: 38.71 gm - Hydrogen: 9.67 gm - Oxygen: 100 - 38.71 - 9.67 = 51.62 gm

Now, we divide the grams of each element by their atomic mass: - Carbon: $\frac{38.71}{12} = 3.23$ - Hydrogen: $\frac{9.67}{1} = 9.67$ - Oxygen: $\frac{51.62}{16} = 3.23$

The simplest whole number ratio is approximately 1:2:1, which gives the empirical formula CH_2O.

Quick Tip

The empirical formula represents the simplest whole-number ratio of atoms in a compound. Use the percentages of elements to calculate their molar ratios.
