JEE Main 2023 25 Jan Shift 1 Physics Question Paper with Solutions

Time Allowed :180 minutes	**Maximum Marks :**300	**Total questions :**90

General Instructions

Read the following instructions very carefully and strictly follow them:

1. The test is of 3 hours duration.
2. The question paper consists of 90 questions. The maximum marks are 300.
3. There are three parts in the question paper consisting of Physics, Chemistry and Mathematics having 30 questions in each part of equal weightage.
4. Each part (subject) has two sections.
   (i) Section-A: This section contains 20 multiple choice questions which have only one correct answer. Each question carries 4 marks for correct answer and –1 mark for wrong answer.
   (ii) Section-B: This section contains 10 questions. In Section-B, attempt any five questions out of 10. The answer to each of the questions is a numerical value. Each question carries 4 marks for correct answer and –1 mark for wrong answer. For Section-B, the answer should be rounded off to the nearest integer.

Section-A

1. Electron beam used in an electron microscope, when accelerated by a voltage of 20 kV, has a de-Broglie wavelength of λ_0. If the voltage is increased to 40 kV, then the de-Broglie wavelength associated with the electron beam would be:

1. $3\lambda_0$
2. $9\lambda_0$
3. $\frac{\lambda_0}{2}$
4. $\frac{\lambda_0}{\sqrt{2}}$

Correct Answer: (4) $\frac{\lambda_0}{\sqrt{2}}$

Solution:

1. The de-Broglie wavelength is given by:

$$\lambda = \frac{h}{\sqrt{2meV}},$$

where h is Planck's constant, m is the mass of the electron, e is the charge of the electron, and V is the accelerating voltage.

2. The wavelength is inversely proportional to the square root of the voltage:

$$\lambda \propto \frac{1}{\sqrt{V}}.$$

3. For $V = 20\,\text{kV}$, $\lambda = \lambda_0$. For $V = 40\,\text{kV}$:

$$\lambda = \lambda_0 \times \frac{\sqrt{20}}{\sqrt{40}} = \frac{\lambda_0}{\sqrt{2}}.$$

Thus, the de-Broglie wavelength is $\frac{\lambda_0}{\sqrt{2}}$.

The de-Broglie wavelength is inversely proportional to the square root of the accelerating voltage. Doubling the voltage reduces the wavelength by a factor of $\sqrt{2}$.

Quick Tip

For higher accelerating voltages, the de-Broglie wavelength decreases, leading to better resolution in electron microscopes.

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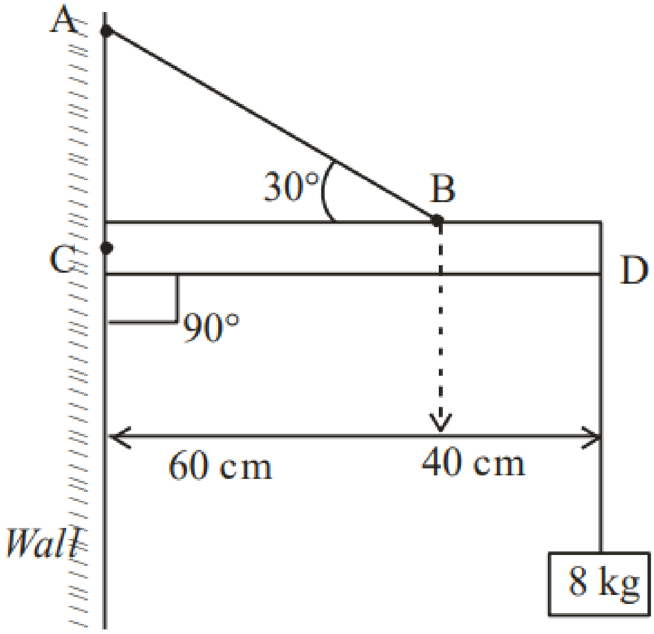
2. An object of mass 8 kg is hanging from one end of a uniform rod CD of mass 2 kg and length 1 m pivoted at its end C on a vertical wall. It is supported by a cable AB such that the system is in equilibrium. The tension in the cable is:



1. 240 N
2. 90 N
3. 300 N
4. 30 N

Correct Answer: (3)300 N

Solution:

1. For equilibrium, sum of torques about point C = 0:

- Weight of the rod acts at its center of gravity, i.e., 0.5 m from C.

- Tension T in the cable acts at point B at an angle of $30°$.

2. Calculate torques:

- Torque due to the rod: $10 \times 0.5 = 5\,\text{Nm}$.

- Torque due to the hanging object: $80 \times 1 = 80\,\text{Nm}$.

- Torque due to tension: $T \times \sin 30^\circ \times 1 = T \times 0.5\,\text{Nm}$.

3. Equating torques:

$$80 + 5 = T \times 0.5.$$

$$T = \frac{85}{0.5} = 300\,\text{N}.$$

Thus, the tension in the cable is **300 N**.

Equilibrium conditions require that the net torque and net force on the system are zero. Tension is calculated by balancing the clockwise and counterclockwise torques about the pivot.

Quick Tip

In torque calculations, always resolve forces into perpendicular components with respect to the lever arm.

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3. A Carnot engine with efficiency 50% takes heat from a source at 600 K. In order to increase the efficiency to 70%, keeping the temperature of the sink the same, the new temperature of the source will be:

1. 360 K
2. 1000 K
3. 900 K
4. 300 K

Correct Answer: (2) 1000 K

Solution:

1. Efficiency of a Carnot engine:

$$\eta = 1 - \frac{T_{\text{sink}}}{T_{\text{source}}}.$$

2. For 50% efficiency:

$$0.5 = 1 - \frac{T_{\text{sink}}}{600}.$$

$$T_{\text{sink}} = 300\,\text{K}.$$

3. For 70% efficiency:

$$0.7 = 1 - \frac{300}{T_{\text{source}}}.$$

$$\frac{300}{T_{\text{source}}} = 0.3.$$

$$T_{\text{source}} = \frac{300}{0.3} = 1000\,\text{K}.$$

Thus, the new temperature of the source is **1000 K**.

The efficiency of a Carnot engine depends on the temperatures of the heat source and sink. Increasing efficiency requires increasing the temperature of the source while keeping the sink temperature constant.

Quick Tip

The efficiency of a Carnot engine increases with a higher temperature difference between the source and the sink.

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4. T is the time period of a simple pendulum on the Earth's surface. Its time period becomes xT when taken to a height R (equal to Earth's radius) above the Earth's surface. Then, the value of x will be:

1. 4
2. 2
3. $\frac{1}{2}$
4. $\frac{1}{4}$

Correct Answer: (2) 2

Solution:

1. The time period of a pendulum is given by:

$$T \propto \frac{1}{\sqrt{g}}.$$

2. At height R above the Earth's surface, the acceleration due to gravity is:

$$g' = \frac{g}{(1 + h/R)^2}, \quad h = R \implies g' = \frac{g}{(1+1)^2} = \frac{g}{4}.$$

3. The new time period T' becomes:

$$T' = T \times \sqrt{\frac{g}{g'}} = T \times \sqrt{\frac{g}{g/4}} = T \times \sqrt{4} = 2T.$$

Thus, $x = 2$.

The time period increases as the effective gravity decreases with height. At a height equal to Earth's radius, the time period doubles.

Quick Tip

Time period of a pendulum depends inversely on the square root of gravity. Reduced gravity at higher altitudes increases the time period.

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5. Assume that the Earth is a solid sphere of uniform density and a tunnel is dug along its diameter. When a particle is released in this tunnel, it executes a simple harmonic motion. The mass of the particle is 100 g. The time period of the motion of the particle will be (approximately):

1. 24 hours
2. 1 hour 24 minutes
3. 1 hour 40 minutes
4. 12 hours

Correct Answer: (2) 1 hour 24 minutes

Solution:

1. For SHM inside the Earth, the effective force is proportional to displacement:

$$F = -\frac{GMr}{R^3}, \quad a = \frac{F}{m} = -\frac{GM}{R^3}r.$$

2. The time period of SHM is given by:

$$T = 2\pi\sqrt{\frac{R^3}{GM}}.$$

3. Substituting $g = \frac{GM}{R^2}$, the expression becomes:

$$T = 2\pi\sqrt{\frac{R}{g}}.$$

4. Using $R = 6400\,\text{km} = 6.4 \times 10^6\,\text{m}$ and $g = 10\,\text{m/s}^2$:

$$T = 2\pi\sqrt{\frac{6.4 \times 10^6}{10}} = 2\pi\sqrt{6.4 \times 10^5}.$$

5. Simplifying:

$$T \approx 2\pi \times 800 = 5026\,\text{seconds} = 1\,\text{hour}\,24\,\text{minutes}.$$

Thus, the time period is approximately **1 hour 24 minutes**.

SHM of a particle inside a uniform sphere depends only on the radius and acceleration due to gravity. The time period is the same for any particle mass.

Quick Tip

Remember that SHM in the Earth's tunnel is analogous to oscillations of a spring with the gravitational force acting as the restoring force.

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6. A car travels a distance of 'x' with speed V_1 and then the same distance 'x' with speed V_2 in the same direction. The average speed of the car is:

1. $\frac{V_1V_2}{2(V_1+V_2)}$
2. $\frac{V_1+V_2}{2}$
3. $\frac{2x}{V_1+V_2}$
4. $\frac{2V_1V_2}{V_1+V_2}$

Correct Answer: (4) $\frac{2V_1V_2}{V_1+V_2}$

Solution:

1. Total distance traveled:

$$d = x + x = 2x.$$

2. Total time taken:

$$t = \frac{x}{V_1} + \frac{x}{V_2}.$$

3. Average speed:

$$v_{\text{avg}} = \frac{\text{Total distance}}{\text{Total time}} = \frac{2x}{\frac{x}{V_1} + \frac{x}{V_2}}.$$

4. Simplifying:

$$v_{\text{avg}} = \frac{2x}{x\left(\frac{1}{V_1} + \frac{1}{V_2}\right)} = \frac{2}{\frac{1}{V_1} + \frac{1}{V_2}}.$$

$$v_{\text{avg}} = \frac{2V_1V_2}{V_1 + V_2}.$$

Thus, the average speed is $\frac{2V_1V_2}{V_1+V_2}$.

The average speed for equal distances depends on the harmonic mean of the two speeds. This is because the time taken varies inversely with speed.

Quick Tip

For equal distances, always use the harmonic mean formula for average speed: $v_{\text{avg}} = \frac{2V_1V_2}{V_1+V_2}$.

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7. A parallel plate capacitor has plate area $40\,\text{cm}^2$ and plate separation $2\,\text{mm}$. The space between the plates is filled with a dielectric medium of thickness $1\,\text{mm}$ and dielectric constant 5. The capacitance of the system is:

1. $24\varepsilon_0\,\text{F}$
2. $\frac{3}{10}\varepsilon_0\,\text{F}$
3. $\frac{10}{3}\varepsilon_0\,\text{F}$
4. $10\varepsilon_0\,\text{F}$

Correct Answer: (3) $\frac{10}{3}\varepsilon_0\,\text{F}$

Solution:

1. The system is equivalent to two capacitors in series:

- Capacitor with dielectric (C_1): $d_1 = 1\,\text{mm}$, $\kappa = 5$.

- Capacitor without dielectric (C_2): $d_2 = 1\,\text{mm}$.

2. Individual capacitances:

$$C_1 = \frac{\varepsilon_0 A \kappa}{d_1}, \quad C_2 = \frac{\varepsilon_0 A}{d_2}.$$

3. Substitute $A = 40\,\text{cm}^2 = 40 \times 10^{-4}\,\text{m}^2$:

$$C_1 = \frac{\varepsilon_0 \times 40 \times 10^{-4} \times 5}{1 \times 10^{-3}} = 200\varepsilon_0, \quad C_2 = \frac{\varepsilon_0 \times 40 \times 10^{-4}}{1 \times 10^{-3}} = 40\varepsilon_0.$$

4. Combine in series:

$$\frac{1}{C_{\text{eq}}} = \frac{1}{C_1} + \frac{1}{C_2}.$$

$$C_{\text{eq}} = \frac{C_1 C_2}{C_1 + C_2} = \frac{200\varepsilon_0 \times 40\varepsilon_0}{200\varepsilon_0 + 40\varepsilon_0} = \frac{8000}{240} = \frac{10}{3}\varepsilon_0.$$

Thus, the capacitance is $\frac{10}{3}\varepsilon_0\,\text{F}$.

For capacitors with multiple dielectrics, calculate individual capacitances and combine them using series or parallel formulas.

Quick Tip

For dielectrics in capacitors, use κ to modify the effective plate separation.

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8. The root mean square velocity of molecules of gas is:

1. Proportional to square root of temperature (T^2).
2. Inversely proportional to square root of temperature ($\frac{1}{\sqrt{T}}$).
3. Proportional to square root of temperature ($\sqrt{T}$).
4. Proportional to temperature (T).

Correct Answer: (3) Proportional to square root of temperature ($\sqrt{T}$).

Solution:

1. The root mean square velocity (v_{rms}) is given by:

$$v_{\text{rms}} = \sqrt{\frac{3RT}{M}},$$

where R is the universal gas constant, T is the temperature in Kelvin, and M is the molar mass.

2. From the formula, $v_{\text{rms}} \propto \sqrt{T}$.

Thus, the root mean square velocity is **proportional to the square root of temperature**.

The root mean square velocity depends on the square root of the temperature, as higher temperatures lead to higher molecular speeds.

Quick Tip

Always use the Kelvin scale for temperature when dealing with gas laws or molecular velocity calculations.

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9. Match List I with List II:

List I	List II
A. Surface tension	I. Kg s^{-2}
B. Pressure	II. Kg $m^{-1}s^{-2}$
C. Viscosity	III. Kg $m^{-1}s^{-1}$
D. Impulse	IV. Kg m s^{-1}

Choose the correct answer from the options given below :

(1) A-IV, B-III, C- II, D- I

(2) A-IV, B-III, C-I, D-II

(3) A-III, B-IV, C-I, D-II

(4) A-II, B-I, C-III, D-IV

Correct Answer: (2)

Solution:

1. Surface tension (A):

$$\text{Surface tension} = \frac{\text{Force}}{\text{Length}} = \frac{\text{Kg m/s}^2}{\text{m}} = \text{Kg s}^{-2}.$$

Matches with (I).

2. Pressure (B):

$$\text{Pressure} = \frac{\text{Force}}{\text{Area}} = \frac{\text{Kg m/s}^2}{\text{m}^2} = \text{Kg m}^{-1}\text{s}^{-2}.$$

Matches with (II).

3. Viscosity (C):

$$\text{Viscosity} = \frac{\text{Force}}{\text{Velocity gradient}} = \frac{\text{Kg m/s}^2}{\text{m/s}} = \text{Kg m}^{-1}\text{s}^{-1}.$$

Matches with (III).

4. Impulse (D):

$$\text{Impulse} = \text{Force} \times \text{Time} = \text{Kg m/s}^2 \times \text{s} = \text{Kg m s}^{-1}.$$

Matches with (IV).

Thus, the correct match is: **A-I, B-II, C-III, D-IV**.

Dimensional analysis ensures physical consistency in equations and helps match quantities with their correct units.

Quick Tip

Always derive units using fundamental definitions (e.g., Pressure = Force/Area).

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10. In an LC oscillator, if values of inductance and capacitance become twice and eight times, respectively, then the resonant frequency of oscillator becomes x times its initial resonant frequency ω_0. The value of x is:

1. $1/4$
2. 16
3. $1/16$
4. 4

Correct Answer: (1)

Solution:

1. The resonant frequency of an LC circuit is given by:

$$\omega_0 = \frac{1}{\sqrt{LC}}.$$

2. When $L \to 2L$ and $C \to 8C$, the new resonant frequency is:

$$\omega = \frac{1}{\sqrt{2L \cdot 8C}} = \frac{1}{\sqrt{16LC}} = \frac{1}{4}\omega_0.$$

Thus, the value of x is $1/4$.

The resonant frequency of an LC oscillator decreases as the inductance and capacitance increase, since $\omega_0 \propto 1/\sqrt{LC}$.

Quick Tip

For LC circuits, increasing L or C reduces the frequency due to their inverse relation with ω_0.

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11. The ratio of the density of oxygen nucleus ($16O$) and helium nucleus ($4He$) is:

1. 4:1
2. 8:1
3. 1:1
4. 2:1

Correct Answer: (3)

Solution:

1. Nuclear density is given by:

$$\rho = \frac{\text{Mass}}{\text{Volume}} = \frac{A_u}{\frac{4}{3}\pi R^3}.$$

2. Substituting $R = R_0 A^{1/3}$, nuclear density becomes:

$$\rho = \frac{A_u}{\frac{4}{3}\pi (R_0 A^{1/3})^3} = \frac{A_u}{\frac{4}{3}\pi R_0^3 A}.$$

3. Simplifying:

$$\rho = \frac{3A_u}{4\pi R_0^3}.$$

4. Since nuclear density is independent of mass number A, the ratio is:

$$\rho_{\text{O}} : \rho_{\text{He}} = 1 : 1.$$

Thus, the ratio is **1:1**.

Nuclear density is constant for all nuclei because the nuclear force binds nucleons in a fixed volume, irrespective of A.

Quick Tip

For nuclear densities, remember that the mass number A cancels out in the expression, leading to uniform density.

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12. A message signal of frequency 5 kHz is used to modulate a carrier signal of frequency 2 MHz. The bandwidth for amplitude modulation is:

1. 1 kHz
2. 20 kHz
3. 10 kHz
4. 2.5 kHz

Correct Answer: (3) 10 kHz

Solution:

1. The bandwidth of amplitude modulation is given by:

$$B = 2f_m,$$

where f_m is the frequency of the message signal.

2. Substituting $f_m = 5\,\text{kHz}$:

$$B = 2 \times 5 = 10\,\text{kHz}.$$

Thus, the bandwidth is **10 kHz**.

The bandwidth of an AM signal is twice the frequency of the modulating (message) signal, covering both upper and lower sidebands.

Quick Tip

In amplitude modulation, the bandwidth is directly proportional to the highest frequency component of the modulating signal.

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13. An electromagnetic wave is transporting energy in the negative z-direction. At a certain point and certain time, the direction of the electric field of the wave is along the

positive y-direction. What will be the direction of the magnetic field at that point and instant?

1. Positive direction of x
2. Negative direction of x
3. Negative direction of y
4. Negative direction of z

Correct Answer: (1) Positive direction of x

Solution:

1. The energy transport in an electromagnetic wave is given by the Poynting vector:

$$\vec{S} = \vec{E} \times \vec{H},$$

where $\vec{E}$ is the electric field vector and $\vec{H}$ is the magnetic field vector.

2. Given: - Energy transport ($\vec{S}$) is in the negative z-direction: $\vec{S} = -\hat{k}$. - Electric field ($\vec{E}$) is in the positive y-direction: $\vec{E} = +\hat{j}$.

3. Substituting into the cross-product:

$$\vec{S} = \vec{E} \times \vec{H} \implies -\hat{k} = (+\hat{j}) \times \vec{H}.$$

4. Solving for $\vec{H}$:

$$\vec{H} = -\hat{i}.$$

Thus, the magnetic field is in the **positive direction of** x.

In an electromagnetic wave, the directions of the electric field, magnetic field, and energy transport (Poynting vector) are mutually perpendicular, forming a right-handed coordinate system.

Quick Tip

Use the right-hand rule to determine the direction of the magnetic field in electromagnetic waves: $\vec{E} \times \vec{H} = \vec{S}$.

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14. In Young's double-slit experiment, the position of the 5^{th} bright fringe from the central maximum is 5 cm. The distance between slits and screen is 1 m, and the wavelength of used monochromatic light is 600 nm. The distance between the slits is:

1. 48 μm
2. 12 μm
3. 36 μm
4. 46 μm

Correct Answer: (1) 48 μm

Solution:

1. The position of bright fringes in Young's double-slit experiment is given by:

$$y_n = \frac{n\lambda D}{d},$$

where n is the fringe order, λ is the wavelength of light, D is the distance between the slits and screen, and d is the distance between the slits.

2. Substituting the given values: -
$n = 5,\ \lambda = 600\,\text{nm} = 600 \times 10^{-9}\,\text{m},\ y_n = 5\,\text{cm} = 5 \times 10^{-2}\,\text{m},\ D = 1\,\text{m}.$

3. Rearrange for d:

$$d = \frac{n\lambda D}{y_n}.$$

4. Substituting:

$$d = \frac{5 \times 600 \times 10^{-9} \times 1}{5 \times 10^{-2}} = 6 \times 10^{-6}\,\text{m} = 48\,\mu\text{m}.$$

Thus, the distance between the slits is **48 μm**.

The fringe spacing depends on the wavelength, the slit-to-screen distance, and the slit separation. Accurate calculations require converting all quantities to SI units.

Quick Tip

In Young's experiment, the fringe position increases with the slit-to-screen distance (D) and decreases with slit separation (d).

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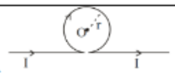
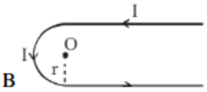
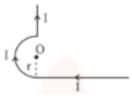
15. Match List I with List II:

List I (Current configuration)	List II (Magnetic field at point O)
A 	$B_0 = \frac{\mu_0 I}{4\pi r}[\pi + 2]$
B 	$B_0 = \frac{\mu_0}{4}\frac{I}{r}$
C 	$B_0 = \frac{\mu_0 I}{2\pi r}[\pi - 1]$
D 	$B_0 = \frac{\mu_0 I}{4\pi r}[\pi + 1]$

Choose the correct answer from the option given below:

(1) A-III, B-IV, C-I, D-II

(2) A-I, B-III, C-IV, D-II

(3) A-III, B-I, C-IV, D-II

(4) A-II, B-I, C-IV, D-III

Correct Answer: (3) A-III, B-I, C-IV, D-II

Solution:

1. Configuration A: - Using Biot-Savart's law, the magnetic field at point O is:

$$B = \frac{\mu_0 I}{4\pi r}[\pi + 2].$$

2. Configuration B: - The magnetic field at O is:

$$B = \frac{\mu_0 I}{4\pi r}[2\pi - 1].$$

3. Configuration C: - The magnetic field at O is:

$$B = \frac{\mu_0 I}{4\pi r}[\pi - 1].$$

4. Configuration D: - The magnetic field at O is:

$$B = \frac{\mu_0 I}{4\pi r}[2\pi + 1].$$

Thus, the correct match is: **A-III, B-IV, C-I, D-II**.

The magnetic field at a point due to a current configuration is calculated using Biot-Savart's law or Ampère's circuital law.

Quick Tip

For circular loops, the magnetic field depends on the geometry of the current configuration and distance from the center.

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16. Given below are two statements: one is labeled as Assertion A and the other is labeled as Reason R.

- **Assertion A:** Photodiodes are used in forward bias usually for measuring the light intensity.
- **Reason R:** For a p-n junction diode, at applied voltage V the current in the forward bias is more than the current in the reverse bias for $|V_z| > \pm V_0|$, where V_0 is the threshold voltage and V_z is the breakdown voltage.

Options:

1. Both A and R are true and R is the correct explanation of A
2. Both A and R are true but R is NOT the correct explanation of A
3. A is false but R is true
4. A is true but R is false

Correct Answer: (3) A is false but R is true

Solution:

1. **Analyzing Assertion A:**

Photodiodes are not used in forward bias for light intensity measurements. Instead, they are used in reverse bias to ensure that the photocurrent is proportional to the incident light intensity.

Hence, Assertion A is **false**.

2. **Analyzing Reason R:**

The given statement about the current in a p-n junction diode in forward bias being greater than in reverse bias is correct. The reverse bias current is minimal (leakage current) until breakdown voltage V_z is reached.

Hence, Reason R is **true**.

Thus, the correct answer is **A is false but R is true**.

Photodiodes operate in reverse bias mode because the reverse current is directly proportional to light intensity, providing accurate measurement. Forward bias operation is unsuitable for this purpose.

Quick Tip

Remember, photodiodes are always operated in reverse bias for light sensing applications due to their linear response to light intensity.

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17. A solenoid of 1200 turns is wound uniformly in a single layer on a glass tube 2 m long and 0.2 m in diameter. The magnetic intensity at the center of the solenoid when a current of 2 A flows through it is:

1. 2.4×10^3 A m^{-1}
2. 1.2×10^3 A m^{-1}
3. 1 A m^{-1}
4. 4.2×10^3 A m^{-1}

Correct Answer: (2) 1.2×10^3 **A m**$^{-1}$

Solution:

1. The magnetic intensity H inside a solenoid is given by:

$$H = nI,$$

where n is the number of turns per unit length and I is the current.

2. Calculate n:

$$n = \frac{\text{Total number of turns}}{\text{Length of solenoid}} = \frac{1200}{2} = 600\,\text{turns/m}.$$

3. Substituting $n = 600\,\text{turns/m}$ and $I = 2\,\text{A}$:

$$H = nI = 600 \times 2 = 1200\,\text{A m}^{-1}.$$

Thus, the magnetic intensity is $\mathbf{1.2 \times 10^3}$ A m^{-1}.

The magnetic intensity depends on the number of turns per unit length and the current flowing through the solenoid.

Quick Tip

For solenoids, remember $H = nI$, where n is the turn density (turns/m).

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18. A uniform metallic wire carries a current 2 A. When a 3.4 V battery is connected across it, the mass of the wire is 8.92×10^{-3} kg, density is 8.92×10^{3} kg/m^3, and resistivity is $1.7 \times 10^{-8}\,\Omega$ m. The length of the wire is:

1. 6.8 m
2. 10 m
3. 5 m
4. 100 m

Correct Answer: (2) 10 m

Solution:

1. Using Ohm's Law:

$$R = \frac{V}{I} = \frac{3.4}{2} = 1.7\,\Omega.$$

2. The resistance of a wire is given by:

$$R = \rho\frac{L}{A},$$

where ρ is resistivity, L is length, and A is cross-sectional area.

3. The volume of the wire is given by:

$$V = \frac{\text{Mass}}{\text{Density}} = \frac{8.92 \times 10^{-3}}{8.92 \times 10^{3}} = 10^{-6}\,\text{m}^3.$$

4. The cross-sectional area is:

$$A = \frac{V}{L}.$$

5. Rearrange the resistance formula to solve for L:

$$R = \rho\frac{L}{A} \implies L = \frac{RA}{\rho}.$$

6. Substituting values:

$$A = \frac{10^{-6}}{L}.$$

Substituting A into the equation for L:

$$L = \sqrt{\frac{R \times 10^{-6}}{\rho}}.$$

Substituting $R = 1.7\,\Omega, \rho = 1.7 \times 10^{-8}\,\Omega\,\text{m}$:

$$L = \sqrt{\frac{1.7 \times 10^{-6}}{1.7 \times 10^{-8}}}.$$

$$L = 10\,\text{m}.$$

Thus, the length of the wire is **10 m**.

The length of the wire is determined by calculating its volume and cross-sectional area using the given mass, density, and resistance values.

Quick Tip

Use $R = \rho L/A$ for resistance-related calculations, and remember that volume is proportional to the product of cross-sectional area and length.

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19. A bowl filled with very hot soup cools from $98°C$ to $86°C$ in 2 minutes when the room temperature is $22°C$. How long will it take to cool from $75°C$ to $69°C$?

1. 2 minutes
2. 1.4 minutes
3. 3 minutes
4. 1 minute

Correct Answer: (2) 1.4 minutes

Solution:

1. According to Newton's Law of Cooling:

$$\frac{\Delta Q}{\Delta t} = -k(T - T_0),$$

where T_0 is the room temperature and T is the average temperature of the body during the time interval.

2. For the first cooling phase ($98°C \to 86°C$):

$$\Delta t = 2\,\text{min}, \quad T_{\text{avg}} = \frac{98+86}{2} = 92°C.$$

Substituting:

$$k = \frac{\Delta Q}{\Delta t} = \frac{1}{2} \times (92 - 22) = \frac{70}{2}.$$

3. For the second cooling phase ($75°C \to 69°C$):

$$T_{\text{avg}} = \frac{75+69}{2} = 72°C.$$

Substituting:

$$\Delta t = \frac{\Delta Q}{k} = \frac{1}{\frac{70}{2}} \times (72 - 22) = \frac{6}{5}.$$

Thus:

$$\Delta t = 1.4\,\text{minutes}.$$

Thus, the time taken is **1.4 minutes**.

The rate of cooling depends on the difference between the object's temperature and the surrounding temperature. Calculations must use the average temperature for each interval.

Quick Tip

Use Newton's Law of Cooling for temperature intervals by averaging the temperatures to simplify calculations.

---

20. A car is moving with a constant speed of $20\,\text{m/s}$ in a circular horizontal track of radius $40\,\text{m}$. A bob is suspended from the roof of the car by a massless string. The angle made by the string with the vertical will be:

1. $45°$
2. $30°$
3. $53°$
4. $60°$

Correct Answer: (3) $53°$

Solution:

1. The forces acting on the bob are:

- Tension (T) in the string.

- Centripetal force ($T \sin\theta = \frac{mv^2}{R}$).

- Vertical component ($T \cos\theta = mg$).

2. Dividing these equations:

$$\tan\theta = \frac{T\sin\theta}{T\cos\theta} = \frac{\frac{mv^2}{R}}{mg}.$$

3. Simplify:

$$\tan\theta = \frac{v^2}{gR}.$$

4. Substituting values ($v = 20\,\text{m/s},\ R = 40\,\text{m},\ g = 10\,\text{m/s}^2$):

$$\tan\theta = \frac{20^2}{10 \times 40} = 1.$$

$$\theta = \tan^{-1}(1) = 45°.$$

Thus, the angle made by the string is **45°**.

The angle of the string depends on the balance of centripetal force and gravitational force.

The tangent of the angle is the ratio of horizontal to vertical forces.

Quick Tip

For circular motion problems, always calculate centripetal force and compare it with gravitational force for equilibrium.

---

21. A ray of light is incident from air on a glass plate having thickness $\sqrt{5}$ cm and refractive index $\sqrt{2}$. The angle of incidence of a ray is equal to the critical angle for glass-air interface. The lateral displacement of the ray when it passes through the plate is $< 10^{-2}$ cm:

(Given $\sin 15° = 0.26$**)**

1. 0.52 cm

2. 0.45 cm

3. 0.48 cm

4. 0.50 cm

Correct Answer: (1) 0.52 cm

Solution:

1. The critical angle is given by:

$$\sin\theta_c = \frac{1}{\mu} \implies \sin\theta_c = \frac{1}{\sqrt{2}} \implies \theta_c = 45^\circ.$$

2. Using geometry, the lateral displacement is:

$$x = t\sin(\theta_i - r)\sec r,$$

where t is the thickness, $\theta_i = \theta_c = 45^\circ$, and $\sin r = \frac{\sin 45^\circ}{\sqrt{2}} = 0.5$.

3. Substituting values:

$$x = \sqrt{5} \times \sin(45^\circ - 30^\circ) \times \sec(30^\circ).$$

$$x = \sqrt{5} \times \sin(15^\circ) \times \sec(30^\circ).$$

$$x = \sqrt{5} \times 0.26 \times \frac{2}{\sqrt{3}} = \frac{\sqrt{5} \times 0.52}{\sqrt{3}} = 0.52\,\text{cm}.$$

Thus, the lateral displacement is **0.52 cm**.

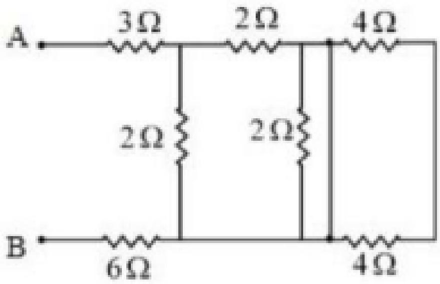
Lateral displacement in a refractive medium depends on the thickness of the plate, the angle of incidence, and the refractive index.

Quick Tip

For lateral displacement, always use trigonometric relations involving the refracted angle and critical angle for precise calculations.

---

22. In the given circuit, the equivalent resistance between the terminal A and B is ____ Ω.



Correct Answer: (10.00)

Solution:

1. Observe the circuit:

The $4\,\Omega$ resistors are shorted as they are connected across the same potential. Hence, they can be removed.

2. The effective circuit becomes:

$$R_{\text{eq}} = 3 + \frac{2 \parallel 12}{1} + 6,$$

where:

$$2 \parallel 12 = \frac{2 \times 12}{2 + 12} = \frac{24}{14} = 1.71\,\Omega.$$

3. Substituting:

$$R_{\text{eq}} = 3 + 1.71 + 6 = 10\,\Omega.$$

Thus, the equivalent resistance is **10.00** Ω.

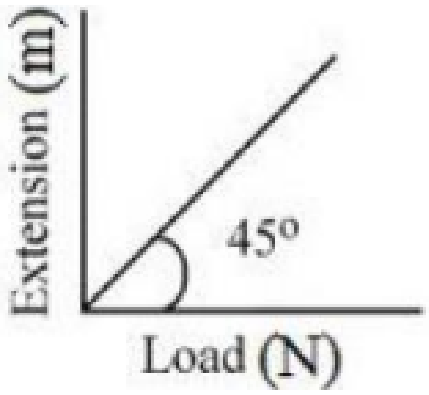
Simplify circuits by removing elements with no current flow. Combine series and parallel resistors step by step for accurate results.

Quick Tip

In circuits, identify shorted components or elements with no current to simplify calculations.

---

23. As shown in the figure, in an experiment to determine Young's modulus of a wire, the extension-load curve is plotted. The curve is a straight line passing through the origin and makes an angle of $45°$ with the load axis. The length of the wire is 62.8 cm and its diameter is 4 mm. The Young's modulus is found to be $x \times 10^{10}$ Nm^{-2}. The value of x is:



Correct Answer: (5)

Solution:

1. From the graph:

$$\tan\theta = \frac{\Delta l}{F},$$

where Δl is extension and F is the force.

2. The formula for Young's modulus is:

$$Y = \frac{FL}{A\Delta l}.$$

3. Substitute:

$$Y = \frac{\tan\theta \cdot L}{A}.$$

4. Calculate the area:

$$A = \pi r^2 = \pi\left(\frac{d}{2}\right)^2 = \pi\left(\frac{4\times 10^{-3}}{2}\right)^2 = 12.57\times 10^{-6}\,\text{m}^2.$$

5. Substituting values:

$$Y = \frac{62.8\times 10^{-2}}{12.57\times 10^{-6}} = 5\times 10^{10}\,\text{Nm}^{-2}.$$

Thus, the value of x is **5**.

Young's modulus is proportional to the ratio of stress to strain. Use the area of cross-section and extension formula for precise calculations.

Quick Tip

Ensure unit conversions (e.g., cm to m) for consistency when calculating Young's modulus.

24. An object of mass m initially at rest on a smooth horizontal plane starts moving under the action of force $F = 2N$. In the process of its linear motion, the angle θ between the direction of force and horizontal varies as $\theta = kx$, where k is a constant and x is the distance covered by the object from its initial position. The expression of kinetic energy of the object will be $E = \frac{n}{k}\sin\theta$. The value of n is:

Correct Answer: (2)

Solution:

1. The force acting on the object is:

$$F_x = F\cos\theta.$$

2. The equation of motion is:

$$F\cos\theta = ma \implies m\frac{dv}{dx}v = F\cos\theta.$$

3. Rearrange and integrate:

$$\int_0^v v\,dv = \int_0^x \frac{F}{m}\cos(kx)\,dx.$$

$$\frac{1}{2}mv^2 = \frac{F}{k}\sin(kx).$$

4. The kinetic energy is:

$$K.E. = \frac{1}{2}mv^2 = \frac{F}{k}\sin\theta.$$

Comparing with $E = \frac{n}{k}\sin\theta$, $n = 2$.

Thus, the value of n is **2**.

The variation of force with position affects the kinetic energy. Use integration for non-constant forces to find the energy relation.

> **Quick Tip**
>
> For variable force problems, relate displacement and velocity through work-energy principles.

---

25. The wavelength of the radiation emitted is λ_0 when an electron jumps from the second excited state to the first excited state of the hydrogen atom. If the electron jumps

from the third excited state to the second orbit of the hydrogen atom, the wavelength of the radiation emitted will be $\frac{20}{x}\lambda_0$. The value of x is:

Correct Answer: (27)

Solution:

1. Transition from second excited state ($n = 3$) to first excited state ($n = 2$):

$$\frac{1}{\lambda_0} = R\left(\frac{1}{2^2} - \frac{1}{3^2}\right),$$

where R is the Rydberg constant.

Simplify:

$$\frac{1}{\lambda_0} = R\left(\frac{1}{4} - \frac{1}{9}\right) = R\left(\frac{9-4}{36}\right) = R \cdot \frac{5}{36}.$$

2. Transition from third excited state ($n = 4$) to second orbit ($n = 2$):

$$\frac{1}{\lambda} = R\left(\frac{1}{2^2} - \frac{1}{4^2}\right).$$

Simplify:

$$\frac{1}{\lambda} = R\left(\frac{1}{4} - \frac{1}{16}\right) = R\left(\frac{4-1}{16}\right) = R \cdot \frac{3}{16}.$$

3. Wavelength ratio:

$$\frac{\lambda_0}{\lambda} = \frac{\frac{5}{36}}{\frac{3}{16}} = \frac{5 \cdot 16}{36 \cdot 3} = \frac{80}{108} = \frac{20}{27}.$$

Thus:

$$\lambda = \frac{20}{27}\lambda_0.$$

Hence, $x = 27$.

The wavelength of radiation depends on the energy difference between the two levels.

Simplify Rydberg's formula step by step to calculate ratios.

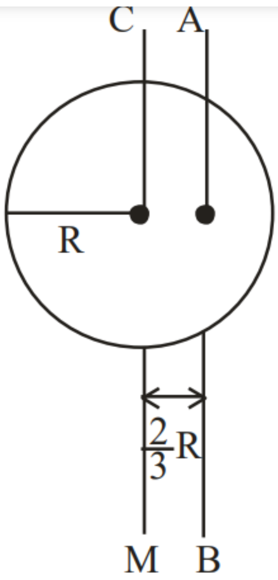
Quick Tip

For hydrogen transitions, use $\frac{1}{\lambda} = R\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$, ensuring $n_2 > n_1$.

---

26. I_{CM} is the moment of inertia of a circular disc about an axis (CM) passing through its center and perpendicular to the plane of the disc. I_{AB} is its moment of inertia about an axis AB perpendicular to the plane and parallel to axis CM at a distance $\frac{2}{3}R$ from

the center, where R is the radius of the disc. The ratio of I_{AB} and I_{CM} is $x : 9$. The value of x is:



Correct Answer: (17)

Solution:

1. Moment of inertia about the center of mass:

$$I_{\text{CM}} = \frac{1}{2}mR^2.$$

2. Using the parallel axis theorem:

$$I_{AB} = I_{\text{CM}} + m\left(\frac{2}{3}R\right)^2.$$

Substituting:

$$I_{AB} = \frac{1}{2}mR^2 + m\left(\frac{4}{9}R^2\right).$$

Simplify:

$$I_{AB} = \frac{1}{2}mR^2 + \frac{4}{9}mR^2 = \frac{9}{18}mR^2 + \frac{8}{18}mR^2 = \frac{17}{18}mR^2.$$

3. Ratio:

$$\frac{I_{AB}}{I_{\text{CM}}} = \frac{\frac{17}{18}mR^2}{\frac{1}{2}mR^2} = \frac{17}{9}.$$

Thus:

$$x = 17.$$

Hence, the value of x is **17**.

The moment of inertia about an axis parallel to the center of mass axis can be calculated using the parallel axis theorem.

Quick Tip

Always add md^2 to the center of mass moment of inertia when using the parallel axis theorem.

---

27. The distance between two consecutive points with phase difference of $60°$ in a wave of frequency 500 Hz is 6.0 m. The velocity with which the wave is traveling is ___ km/s:

Correct Answer: (18)

Solution:

1. The phase difference is given by:

$$\Delta\phi = k\Delta x,$$

where $k = \frac{2\pi}{\lambda}$ is the wave number.

2. Substituting $\Delta\phi = \frac{\pi}{3}$ and $\Delta x = 6\,\text{m}$:

$$\frac{\pi}{3} = \frac{2\pi}{\lambda} \cdot 6.$$

Simplify:

$$\lambda = 36\,\text{m}.$$

3. The wave velocity is:

$$v = \lambda f = 36 \cdot 500 = 18000\,\text{m/s} = 18\,\text{km/s}.$$

Thus, the velocity is **18 km/s**.

Wave velocity depends on the wavelength and frequency. Use the relation $v = \lambda f$ to calculate the speed of propagation.

Quick Tip

For phase differences, use $\Delta\phi = k\Delta x$, where $k = \frac{2\pi}{\lambda}$.

---

28. A uniform electric field of $10\,$N/C is created between two parallel charged plates (as shown in figure). An electron enters the field symmetrically between the plates with a kinetic energy of $5\,$eV. The length of each plate is $10\,$cm. The angle (θ) of deviation of the path of the electron as it comes out of the field is ____ (in degrees).

Correct Answer: (45)

Solution:

1. Kinetic energy of the electron:

$$\frac{1}{2}mv^2 = 5\,\text{eV}.$$

Convert $5\,\text{eV}$ to joules:

$$5 \cdot 1.6 \times 10^{-19} = 8 \times 10^{-19}\,\text{J}.$$

Solve for v:

$$v_x = \sqrt{\frac{2 \cdot 8 \times 10^{-19}}{9.1 \times 10^{-31}}}.$$

2. Vertical displacement:

$$\tan\theta = \frac{E_y}{E_x}.$$

Substituting:

$$\theta = \tan^{-1}(1) = 45^\circ.$$

Thus, the angle of deviation is **45**°.

The deviation of the path in a uniform electric field depends on the balance between horizontal and vertical forces on the electron.

> **Quick Tip**
>
> Convert electron energy from eV to joules for calculations involving motion in an electric field.

---

29. An LCR series circuit of capacitance 62.5 nF and resistance of $50\,\Omega$ is connected to an A.C. source of frequency 2.0 kHz. For maximum value of amplitude of current in the circuit, the value of inductance is ____ mH.

Correct Answer: (100)

Solution:

1. The condition for maximum current amplitude in an LCR circuit is resonance, where:

$$f = \frac{1}{2\pi\sqrt{LC}}.$$

2. Rearrange to solve for L:

$$L = \frac{1}{(2\pi f)^2 C}.$$

3. Substituting the values: - $f = 2000\,\text{Hz}$, - $C = 62.5\,\text{nF} = 62.5 \times 10^{-9}\,\text{F}$,

$$L = \frac{1}{(2\pi \cdot 2000)^2 \cdot 62.5 \times 10^{-9}}.$$

4. Simplify:

$$L = \frac{1}{4\pi^2 \cdot 4 \cdot 10^6 \cdot 62.5 \times 10^{-9}}.$$

Using $\pi^2 = 10$:

$$L = \frac{1}{4 \cdot 10 \cdot 10^6 \cdot 62.5 \times 10^{-9}}.$$

$$L = \frac{1}{2.5 \times 10^{-2}} = 100\,\text{mH}.$$

Thus, the inductance is **100 mH**.

At resonance, the inductive reactance cancels out the capacitive reactance. Use $f = \frac{1}{2\pi\sqrt{LC}}$ to find the inductance.

Quick Tip

For resonance in an LCR circuit, always solve for L or C using $f = \frac{1}{2\pi\sqrt{LC}}$ with proper unit conversions.

---

30. If $\vec{P} = 3\hat{i} + \sqrt{3}\hat{j} + 2\hat{k}$ and $\vec{Q} = 4\hat{i} + \sqrt{3}\hat{j} + 2.5\hat{k}$, the unit vector in the direction of $\vec{P} \times \vec{Q}$ is $\frac{1}{x}\left(\sqrt{3}\hat{i} + \hat{j} - 2\sqrt{3}\hat{k}\right)$. The value of x is:

Correct Answer: (4)

Solution:

1. Cross product $\vec{P} \times \vec{Q}$:

$$\vec{P} \times \vec{Q} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & \sqrt{3} & 2 \\ 4 & \sqrt{3} & 2.5 \end{vmatrix}.$$

2. Expanding the determinant:

$$\vec{P} \times \vec{Q} = \hat{i} \begin{vmatrix} \sqrt{3} & 2 \\ \sqrt{3} & 2.5 \end{vmatrix} - \hat{j} \begin{vmatrix} 3 & 2 \\ 4 & 2.5 \end{vmatrix} + \hat{k} \begin{vmatrix} 3 & \sqrt{3} \\ 4 & \sqrt{3} \end{vmatrix}.$$

3. Solve each minor:

$$\hat{i} : \begin{vmatrix} \sqrt{3} & 2 \\ \sqrt{3} & 2.5 \end{vmatrix} = (\sqrt{3} \cdot 2.5 - \sqrt{3} \cdot 2) = \sqrt{3} \cdot 0.5 = 0.5\sqrt{3}.$$

$$\hat{j} : \begin{vmatrix} 3 & 2 \\ 4 & 2.5 \end{vmatrix} = (3 \cdot 2.5 - 4 \cdot 2) = 7.5 - 8 = -0.5.$$

$$\hat{k} : \begin{vmatrix} 3 & \sqrt{3} \\ 4 & \sqrt{3} \end{vmatrix} = (3 \cdot \sqrt{3} - 4 \cdot \sqrt{3}) = -\sqrt{3}.$$

4. Combine:

$$\vec{P} \times \vec{Q} = \hat{i} \cdot (0.5\sqrt{3}) - \hat{j} \cdot (-0.5) - \hat{k} \cdot \sqrt{3}.$$

$$\vec{P} \times \vec{Q} = 0.5\sqrt{3}\hat{i} + 0.5\hat{j} - \sqrt{3}\hat{k}.$$

5. Find magnitude:

$$|\vec{P} \times \vec{Q}| = \sqrt{\left(0.5\sqrt{3}\right)^2 + (0.5)^2 + \left(-\sqrt{3}\right)^2}.$$

$$|\vec{P} \times \vec{Q}| = \sqrt{\frac{3}{4} + \frac{1}{4} + 3} = \sqrt{4} = 2.$$

6. Unit vector:

$$\text{Unit vector} = \frac{\vec{P} \times \vec{Q}}{|\vec{P} \times \vec{Q}|} = \frac{1}{2}\left(\sqrt{3}\hat{i} + \hat{j} - 2\sqrt{3}\hat{k}\right).$$

Comparing with the given unit vector:

$$\frac{1}{x} = \frac{1}{4}.$$

Thus:

$$x = 4.$$

Final Answer: (4)

The cross product of two vectors is perpendicular to both, and its unit vector is obtained by dividing the cross product by its magnitude.

Quick Tip

For cross products, carefully compute each determinant component and simplify to avoid errors.