JEE Main 2023 29 Jan Shift 2 Mathematics Question Paper

Time Allowed : 180 minutes	**Maximum Marks :** 300	**Total questions :** 90

General Instructions

Read the following instructions very carefully and strictly follow them:

(A) The test is of 3 hours duration.

(B) The question paper consists of 90 questions. The maximum marks are 300.

(C) There are three parts in the question paper consisting of Physics, Chemistry and Mathematics having 30 questions in each part of equal weightage.

(D) Each part (subject) has two sections.

(i) Section-A: This section contains 20 multiple choice questions which have only one correct answer. Each question carries 4 marks for correct answer and –1 mark for wrong answer.

(ii) Section-B: This section contains 10 questions. In Section-B, attempt any five questions out of 10. The answer to each of the questions is a numerical value. Each question carries 4 marks for correct answer and –1 mark for wrong answer. For Section-B, the answer should be rounded off to the nearest integer.

1. The statement $B \Rightarrow ((\sim A) \vee B)$ is equivalent to:

(A) $B \Rightarrow (A \Rightarrow B)$

(B) $A \Rightarrow (A \Longleftrightarrow B)$

(C) $A \Rightarrow ((\sim A) \Rightarrow B)$

(D) $B \Rightarrow ((\sim A) \Rightarrow B)$

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2. Shortest distance between the lines

$$\frac{x-1}{2} = \frac{y+8}{-7} = \frac{z-4}{5} \quad \text{and} \quad \frac{x-1}{2} = \frac{y-2}{1} = \frac{z-6}{-3}$$

is:

(A) $2\sqrt{3}$

(B) $4\sqrt{3}$

(C) $3\sqrt{3}$

(D) $5\sqrt{3}$

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3. If $\vec{a} = \hat{i} + 2\hat{k}, \vec{b} = \hat{i} + \hat{j} + \hat{k}, \vec{c} = 7\hat{i} - 3\hat{j} + 4\hat{k}$, and

$$\vec{r} \times \vec{b} + \vec{b} \times \vec{c} = 0 \quad \textbf{and} \quad \vec{r} \cdot \vec{a} = 0,$$

then $\vec{r} \cdot \vec{c}$ is equal to:

(A) 34

(B) 12

(C) 36

(D) 30

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4. Let $S = \{W_1, W_2, \ldots\}$ be the sample space associated with a random experiment. Let $P(W_n) = \frac{P(W_{n-1})}{2}$, $n \geq 2$. Let $A = \{2k + 3l; k, l \in \mathbb{N}\}$ and $B = \{W_n; n \in A\}$. Then $P(B)$ is equal to:

(A) $\frac{1}{2}$

(B) $\frac{36}{64}$

(C) $\frac{11}{16}$

(D) $\frac{33}{32}$

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5. The value of the integral

$$\int_1^2 \frac{t^4+1}{t^6+1}\,dt$$

is:

(A) $\tan^{-1}\left(\frac{1}{2}\right)+\frac{1}{3}\tan^{-1}(8)-\frac{\pi}{3}$

(B) $\tan^{-1}(2)-\frac{1}{3}\tan^{-1}(8)+\frac{\pi}{3}$

(C) $\tan^{-1}(2)+\frac{1}{3}\tan^{-1}(8)-\frac{\pi}{3}$

(D) $\tan^{-1}\left(\frac{1}{2}\right)-\frac{1}{3}\tan^{-1}(8)+\frac{\pi}{3}$

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6. Let K be the sum of the coefficients of the odd powers of x in the expansion of $(1+x)^{99}$. Let a be the middle term in the expansion of $\left(2+\frac{1}{\sqrt{2}}\right)^{200}$. If

$$\frac{{}^{200}C_{99}K}{a}=\frac{2^{\ell}m}{n},$$

where m and n are odd numbers, then the ordered pair (ℓ, n) is equal to:

(A) (50, 51)

(B) (51, 99)

(C) (50, 101)

(D) (51, 101)

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7. Let f and g be twice differentiable functions on $\mathbb{R}$ such that $f''(x)=g''(x)+6x$, $f'(1)=4g'(1)-3=9$, and $f(2)=3g(2)=12$. Which of the following is NOT true?

(A) $g(-2)-f(-2)=20$

(B) If $-1 < x < 2$, then $|f(x) - g(x)| < 8$

(C) $|f'(x) - g'(x)| < 6, -1 < x < 1$

(D) There exists $x \in [1, 3/2]$ such that $f(x_1) = g(x_1)$

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8. The set of all values of $t \in \mathbb{R}$, for which the matrix

$$\begin{bmatrix} e^t & e^{-t}(\sin t - 2\cos t) & e^{-t}(-2\sin t - \cos t) \\ e^t & e^{-t}(2\sin t + \cos t) & e^{-t}(\sin t - 2\cos t) \\ e^t & e^{-t}\cos t & e^{-t}\sin t \end{bmatrix}$$

is invertible, is:

(A) $\{(2k+1)\frac{\pi}{2}, k \in \mathbb{Z}\}$

(B) $\{k\pi + \frac{\pi}{4}, k \in \mathbb{Z}\}$

(C) $\{k\pi, k \in \mathbb{Z}\}$

(D) $\mathbb{R}$

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9. The area of the region $A = \{(x,y) : |\cos x - \sin x| \leq y \leq \sin x, 0 \leq x \leq \frac{\pi}{2}\}$ is:

(A) $1 - \frac{\sqrt{2}}{3} + \frac{\sqrt{5}}{3}$

(B) $\sqrt{5} + 2\sqrt{2} - 4.5$

(C) $\frac{\sqrt{2}}{3} + 1 - \frac{\sqrt{5}}{3}$

(D) $\sqrt{5} - 2\sqrt{2} + 1$

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10. The set of all values of λ for which the equation $\cos^2(2x) - 2\sin x - 2\cos(2x) = \lambda$ holds is:

(A) $[-2, -1]$

(B) $[-2, -\frac{3}{2}]$

(C) $[-1, -\frac{1}{2}]$

(D) $[-\frac{3}{2}, -1]$

11. The letters of the word OUGHT are written in all possible ways and these words are arranged as in a dictionary, in a series. Then the serial number of the word TOUGH is:

(A) 89

(B) 84

(C) 86

(D) 79

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12. The plane $2x - y + z = 4$ intersects the line segment joining the points $A(a, -2, 4)$ and $B(2, b, -3)$ at the point C in the ratio 2:1, and the distance of C from the origin is $\sqrt{5}$. If $ab < 0$, and P is the point $(a - b, b, 2b - a)$, then CP^2 is equal to:

(A) $\frac{17}{3}$

(B) $\frac{16}{3}$

(C) $\frac{73}{3}$

(D) $\frac{97}{3}$

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13. Let $\vec{a} = 4\hat{i} + 3\hat{j}$, $b = 3\hat{i} - 4\hat{j} + 5\hat{k}$, and c be a vector such that $(\vec{a} \times \vec{b}) \cdot \vec{c} + 25 = 0$, $\vec{c} \cdot (\hat{i} + \hat{j} + \hat{k}) = 4$, and the projection of $\vec{c}$ on $\vec{a}$ is 1. Then the projection of $\vec{c}$ on $\vec{b}$ equals:

(A) $\frac{5}{\sqrt{2}}$

(B) $\frac{1}{\sqrt{2}}$

(C) $\frac{1}{\sqrt{5}}$

(D) $\frac{\sqrt{5}}{\sqrt{2}}$

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14. If the lines $\frac{x-1}{1} = \frac{y-2}{2} = \frac{z+3}{1}$ and $\frac{x-a}{2} = \frac{y+2}{3} = \frac{z-3}{1}$ intersect at the point P, then the distance of the point P from the plane $z = a$ is:

(A) 16

(B) 28

(C) 10

(D) 22

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15. The value of the integral

$$\int_{1/2}^{2} \frac{\tan^{-1} x}{x}\, dx$$

is equal to:

(A) $\pi \log_e 2$

(B) $\frac{1}{2} \log_e 2$

(C) $\frac{\pi}{4} \log_e 2$

(D) $\frac{\pi}{2} \log_e 2$

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16. If the tangent at a point P on the parabola $y^2 = 3x$ is parallel to the line $x + 2y = 1$, and the tangents at the points Q and R on the ellipse $\frac{x^2}{4} + \frac{y^2}{1} = 1$ are perpendicular to the line $x - y = 2$, then the area of the triangle PQR is:

(A) $\frac{9}{\sqrt{2}}$

(B) $5\sqrt{3}$

(C) $\frac{3}{2}\sqrt{5}$

(D) $3\sqrt{5}$

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17. Let $y = y(x)$ be the solution of the differential equation

$$x \log_e x \frac{dy}{dx} + y = x^2 \log_e x, \quad (x > 1).$$

If $y(2) = 2$, then $y(e)$ is equal to:

(A) $\frac{4+e^2}{4}$

(B) $\frac{1+e^2}{4}$

(C) $\frac{2+e^2}{2}$

(D) $\frac{1+e^2}{2}$

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18. The number of 3-digit numbers that are divisible by either 3 or 4 but not divisible by 48 is:

(A) 472

(B) 432

(C) 507

(D) 400

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19. Let R be a relation defined on $\mathbb{N}$ as aRb if $2a + 3b$ is a multiple of 5. Then R is:

(A) not reflexive

(B) transitive but not symmetric

(C) symmetric but not transitive

(D) an equivalence relation

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20. Consider a function $f : \mathbb{N} \to \mathbb{R}$ satisfying
$f(1) + 2f(2) + 3f(3) + \cdots + xf(x) = x(x+1)f(x)$ **for $x \geq 2$, with $f(1) = 1$. Then $f(2022)$ is equal to:**

(A) 8200

(B) 8000

(C) 8400

(D) 8100

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21. The total number of 4-digit numbers whose greatest common divisor with 54 is 2, is:

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22. A triangle is formed by the tangents at the point (2, 2) on the curves $y^2 = 2x$ and $x^2 + y^2 = 4x$, and the line $x + y + 2 = 0$. If r is the radius of its circumcircle, then r^2 is equal to:

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23. A circle with center $(2, 3)$ and radius 4 intersects the line $x + y = 3$ at points P and Q. If the tangents at P and Q intersect at $S(\alpha, \beta)$, then $4\alpha - 7\beta$ is equal to:

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24. Let $a_1 = b_1 = 1$ and $a_n = a_{n-1} + (n-1)$, $b_n = b_{n-1} + a_{n-1}$, $\forall n \geq 2$. If $S = \sum_{n=1}^{\infty} \frac{a_n}{2^n}$ and $T = \sum_{n=1}^{\infty} \frac{b_n}{2^n}$, then $2^7(2S - T)$ is equal to:

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25. If the equation of the normal to the curve $y = \frac{x-a}{(x+b)(x-2)}$ at the point $(1, -3)$ is $x - 4y = 13$, then the value of $a + b$ is:

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26. Let A be a symmetric matrix such that $|A| = 2$ and

$$\begin{bmatrix} 3 & -2 \\ 2 & 1 \end{bmatrix} A = \begin{bmatrix} 1 & 2 \\ 2 & 7 \end{bmatrix}.$$

If the sum of the diagonal elements of A is s, then $\frac{\beta s}{\alpha^2}$ is equal to:

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27. Let $\{a_k\}$ and $\{b_k\}, k \in \mathbb{N}$, be two G.P.s with common ratios r_1 and r_2, respectively,

such that $a_1 = b_1 = 4$ **and** $r_1 < r_2$**. Let** $c_k = a_k + b_k, k \in \mathbb{N}$**. If** $c_2 = 5$ **and** $c_3 = 13$**, then** $\Sigma_{k=1}^{4} c_k - (12a_6 + 8b_4)$ **is equal to:**

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28. Let $X = \{11, 12, 13, \ldots, 41\}$ **and** $Y = \{61, 62, 63, \ldots, 91\}$ **be two sets of observations. If** $\bar{x}$ **and** $\bar{y}$ **are their respective means and** σ^2 **is the variance of all observations in** $X \cup Y$**, then** $|\bar{x} + \bar{y} - \sigma^2|$ **is equal to:**

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29. Let $\alpha = 8 - 14i$**,** $A = \{z \in \mathbb{C} : |z^2 - \alpha^2| = |z^2 - \overline{\alpha}^2|\}$**, and** $B = \{z \in \mathbb{C} : |z + 3i| = 4\}$**. Then** $\sum_{z \in A \cap B} (\operatorname{Re} z - \operatorname{Im} z)$ **is equal to:**

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30. Let $\alpha_1, \alpha_2, \ldots, \alpha_7$ **be the roots of the equation** $x^7 + 3x^5 - 13x^3 - 15x = 0$ **and** $|\alpha_1| \geq |\alpha_2| \geq \cdots \geq |\alpha_7|$**. Then** $\alpha_1\alpha_2 - \alpha_3\alpha_4 + \alpha_5\alpha_6$ **is equal to:**
