

JEE Main 2023 Jan 31 Shift-2 Question Paper

Time Allowed :3 Hours	Maximum Marks :300	Total Questions :90
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General Instructions

Read the following instructions very carefully and strictly follow them:

1. The Duration of test is 3 Hours.
2. This paper consists of 90 Questions.
3. There are three parts in the paper consisting of Physics, Chemistry and Mathematics having 30 questions in each part of equal weightage..
4. Each part (subject) has two sections.
 - (i) Section-A: This section contains 20 multiple choice questions which have only one correct answer. Each carries 4 marks for correct answer and –1 mark for wrong answer..
 5. (ii) Section-B: This section contains 10 questions. In Section-B, attempt any five questions out of 10. The answer to each of the questions is a numerical value. Each carries 4 marks for correct answer and –1 mark for wrong answer. For Section-B, the answer should be rounded off to the nearest integer.

PHYSICS

Section-A

1. The H amount of thermal energy is developed by a resistor in 10 s when a current of 4A is passed through it. If the current is increased to 16A, the thermal energy developed by the resistor in 10 s will be:

- (1) H
 - (2) 16H
 - (3) $\frac{H}{4}$
 - (4) 4H
-

2. A body is moving with constant speed, in a circle of radius 10 m. The body completes one revolution in 4 s. At the end of the 3rd second, the displacement of the body (in m) from its starting point is:

- (1) 30
 - (2) 15π
 - (3) 5π
 - (4) $10\sqrt{2}$
-

3. A microscope is focused on an object at the bottom of a bucket. If liquid with refractive index $\frac{5}{3}$ is poured inside the bucket, then the microscope has to be raised by 30 cm to focus the object again. The height of the liquid in the bucket is:

- (1) 75 cm
 - (2) 50 cm
 - (3) 18 cm
 - (4) 12 cm
-

4. A stone of mass 1 kg is tied to the end of a massless string of length 1 m. If the breaking tension of the string is 400 N, then maximum linear velocity the stone can have without breaking the string, while rotating in horizontal plane, is:

- (1) 20 m/s
- (2) 40 m/s

- (3) 400 m/s
(4) 10 m/s
-

5. For a solid rod, the Young's modulus of elasticity is $3.2 \times 10^{11} \text{ Nm}^{-2}$ and density is $8 \times 10^3 \text{ kg m}^{-3}$. The velocity of longitudinal wave in the rod will be:

- (1) $145.75 \times 10^3 \text{ ms}^{-1}$
(2) $3.65 \times 10^3 \text{ ms}^{-1}$
(3) $18.96 \times 10^3 \text{ ms}^{-1}$
(4) $6.32 \times 10^3 \text{ ms}^{-1}$
-

6. A long conducting wire having a current I flowing through it, is bent into a circular coil of N turns. Then it is bent into a circular coil of n turns. The magnetic field is calculated at the centre of coils in both the cases. The ratio of the magnetic field in first case to that of second case is:

- (1) $n : N$
(2) $n^2 : N^2$
(3) $N^2 : n^2$
(4) $n : N$
-

7. Heat energy of 735 J is given to a diatomic gas allowing the gas to expand at constant pressure. Each gas molecule rotates around an internal axis but does not oscillate. The increase in the internal energy of the gas will be:

- (1) 525 J
(2) 441 J
(3) 572 J
(4) 735 J
-

8. Given below are two statements: Statement I: For transmitting a signal, size of antenna (l) should be comparable to wavelength of signal (at least $l = \frac{\lambda}{4}$ in dimension). Statement II: In amplitude modulation, amplitude of carrier wave remains constant (unchanged). In the light of the above statements, choose the most appropriate answer

from the options given below.

- (1) Both Statement I and Statement II are correct
 - (2) Both Statement I and Statement II are incorrect
 - (3) Statement I is incorrect but Statement II is correct
 - (4) Statement I is correct but Statement II is incorrect
-

9. The number of turns of the coil of a moving coil galvanometer is increased in order to increase current sensitivity by 50%. The percentage change in voltage sensitivity of the galvanometer will be:

- (1) 100%
 - (2) 50%
 - (3) 75%
 - (4) 0%
-

10. If the two metals A and B are exposed to radiation of wavelength 350 nm. The work functions of metals A and B are 4.8 eV and 2.2 eV. Then choose the correct option:

- (1) Metal B will not emit photo-electrons
 - (2) Both metals A and B will emit photo-electrons
 - (3) Both metals A and B will not emit photo-electrons
 - (4) Metal A will not emit photo-electrons
-

11. A body weight W , is projected vertically upwards from earth's surface to reach a height above the earth which is equal to nine times the radius of earth. The weight of the body at that height will be:

- (1) $\frac{W}{91}$
 - (2) $\frac{W}{100}$
 - (3) $\frac{W}{9}$
 - (4) $\frac{W}{3}$
-

12. Match List-I with List-II.

Choose the correct answer from the options given below:

List-I	List-II
A. Angular momentum	I. $[ML^2T^{-2}]$
B. Torque	II. $[ML^2T^{-2}]$
C. Stress	III. $[ML^{-2}T^{-2}]$
D. Pressure gradient	IV. $[ML^{-1}T^{-2}]$

- (1) A-I, B-IV, C-III, D-II
(2) A-III, B-I, C-IV, D-II
(3) A-II, B-III, C-IV, D-I
(4) A-IV, B-II, C-I, D-III

13. An alternating voltage source $V = 260 \sin(628t)$ is connected across a pure inductor of 5 mH. The inductive reactance in the circuit is:

- (1) 3.14
(2) 6.28
(3) 0.5
(4) 0.318

14. Match List-I with List-II.

List-I	List-II
A. Microwaves	I. Physiotherapy
B. UV rays	II. Treatment of cancer
C. Infra-red rays	III. Lasik eye surgery
D. X-rays	IV. Aircraft navigation

Choose the correct answer from the option given below:

- (1) A-II, B-IV, C-III, D-I
(2) A-IV, B-I, C-II, D-III
(3) A-III, B-II, C-I, D-IV
(4) A-IV, B-III, C-I, D-II

15. The radius of electron's second stationary orbit in Bohr's atom is R . The radius of the 3rd orbit will be:

- (1) $\frac{R}{3}$
 - (2) $2.25R$
 - (3) $3R$
 - (4) $9R$
-

16. Under the same load, wire A having length 5.0 m and cross section $2.5 \times 10^{-5} \text{ m}^2$ stretches uniformly by the same amount as another wire B of length 6.0 m and a cross section of $3.0 \times 10^{-5} \text{ m}^2$. The ratio of the Young's modulus of wire A to that of wire B will be:

- (1) 1:4
 - (2) 1:1
 - (3) 1:10
 - (4) 1:2
-

17. Considering a group of positive charges, which of the following statements is correct?

- (1) Net potential of the system cannot be zero at a point but net electric field can be zero at that point.
 - (2) Net potential of the system at a point can be zero but net electric field can't be zero at that point.
 - (3) Both the net potential and the net electric field can be zero at a point.
 - (4) Both the net potential and the net electric field cannot be zero at a point.
-

18. A body of mass 10 kg is moving with an initial speed of 20 m/s. The body stops after 5 s due to friction between the body and the floor. The value of the coefficient of friction is: (Take acceleration due to gravity $g = 10 \text{ m/s}^2$)

- (1) 0.2
- (2) 0.3
- (3) 0.5

(4) 0.4

19. A hypothetical gas expands adiabatically such that its volume changes from 08 litres to 27 litres. If the ratio of final pressure of the gas to initial pressure of the gas is $\frac{16}{81}$, then the ratio of C_P to C_V will be:

- (1) $\frac{4}{3}$
 - (2) $\frac{3}{2}$
 - (3) $\frac{1}{2}$
 - (4) $\frac{3}{4}$
-

20. Given below are two statements:

Statement I: In a typical transistor, all three regions emitter, base, and collector have same doping level. **Statement II:** In a transistor, collector is the thickest and base is the thinnest segment. In light of the above statements, choose the most appropriate answer from the options given below.

- (1) Both Statement I and Statement II are correct
 - (2) Both Statement I and Statement II are incorrect
 - (3) Statement I is incorrect but Statement II is correct
 - (4) Statement I is correct but Statement II is incorrect
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SECTION-B

21. A series LCR circuit consists of $R = 80 \Omega$, $X_L = 100 \Omega$, and $X_C = 40 \Omega$. The input voltage is $2500 \cos(100\pi t)$ V. The amplitude of current, in the circuit, is A.

22. Two light waves of wavelengths 800 nm and 600 nm are used in Young's double slit experiment to obtain interference fringes on a screen placed 7 m away from the plane of slits. If the two slits are separated by 0.35 mm, then the shortest distance from the central bright maximum to the point where the bright fringes of the two wavelengths coincide will be mm.

23. A water heater of power 2000 W is used to heat water. The specific heat capacity of

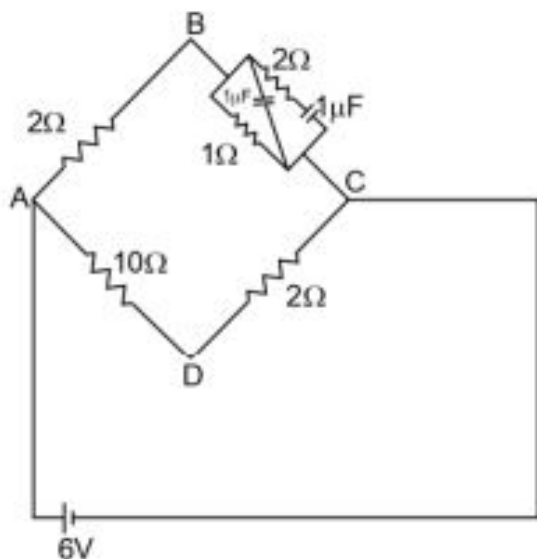
water is $4200 \text{ J kg}^{-1} \text{ K}^{-1}$. The efficiency of the heater is 70%. Time required to heat 2 kg of water from 10°C to 60°C is s.

24. A ball is dropped from a height of 20 m. If the coefficient of restitution for the collision between the ball and the floor is 0.5, after hitting the floor, the ball rebounds to a height of m.

25. Two discs of the same mass and different radii are made of different materials such that their thicknesses are 1 cm and 0.5 cm respectively. The densities of materials are in the ratio 3:5. The moment of inertia of these discs respectively about their diameters will be in the ratio $\frac{x}{6}$. The value of x is

26. If the binding energy of the ground state electron in a hydrogen atom is 13.6 eV, then the energy required to remove the electron from the second excited state of Li^{2+} will be: $x \times 10^1 \text{ eV}$. The value of x is

27. For the given circuit, in the steady state, $|V_B - V_D| = \text{..... V}$.



28. Two parallel plate capacitors C_1 and C_2 , each having capacitance of $10 \mu\text{F}$ are individually charged by a 100 V D.C. source. Capacitor C_1 is kept connected to the source and a dielectric slab is inserted between its plates. Capacitor C_2 is disconnected from the source and then a dielectric slab is inserted in it. Afterwards, the capacitor C_1

is also disconnected from the source and the two capacitors are finally connected in parallel combination. The common potential of the combination will be V.
(Assuming Dielectric constant = 10)

29. The displacement equations of two interfering waves are given by

$$y_1 = 10 \sin\left(\omega t + \frac{\pi}{3}\right) \text{ cm}, \quad y_2 = 5[\sin(\omega t) + \sqrt{3} \cos(\omega t)] \text{ cm}.$$

The amplitude of the resultant wave is cm.

30. Two bodies are projected from ground with same speeds 40 m/s at two different angles with respect to horizontal. The bodies were found to have same range. If one of the body was projected at an angle of 60° , with horizontal then sum of the maximum heights, attained by the two projectiles is m. (Given $g = 10 \text{ m/s}^2$)

CHEMISTRY

Section-A

31. In the following halogenated organic compounds, the one with the maximum number of chlorine atoms in its structure is:

- (1) Chloral
 - (2) Gammaxene
 - (3) Chloropicrin
 - (4) Freon-12
-

32. Incorrect statement for the use of indicators in acid-base titration:

- (1) Methyl orange may be used for a weak acid vs weak base titration.
 - (2) Methyl orange is a suitable indicator for a strong acid vs weak base titration.
 - (3) Phenolphthalein is a suitable indicator for a weak acid vs strong base titration.
 - (4) Phenolphthalein may be used for a strong acid vs strong base titration.
-

**33. Which of the following compounds are not used as disinfectants? (A) Chloroxylenol
(B) Bithional**

- (C) Veronal
(D) Prontosil
(E) Terpeneol

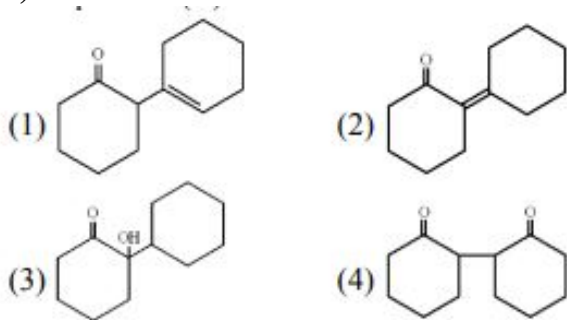
Choose the correct answer from the options given below:

- (1) A, B, E
(2) A, B
(3) B, D, E
(4) C, D

34. A hydrocarbon 'X' with formula C_6H_8 uses two moles of H_2 on catalytic hydrogenation of its one mole. On ozonolysis, 'X' yields two moles of methane dicarbaldehyde. The hydrocarbon 'X' is:

- (1) hexa-1, 3, 5-triene
(2) 1-methylcyclopenta-1, 4-diene
(3) cyclohexa-1, 3-diene
(4) cyclohexa-1, 4-diene

35. Cyclohexylamine when treated with nitrous acid yields (P). On treating (P) with PCC results in (Q). When (Q) is heated with dilute NaOH, we get (R). The final product (R) is:



36. Given below are two statements: Statement I: Upon heating a borax bead dipped in cupric sulphate in a luminous flame, the colour of the bead becomes green. **Statement II:** The green colour observed is due to the formation of copper(I) metaborate.

In light of the above statements, choose the most appropriate answer from the options given below:

- (1) Both Statement I and Statement II are true
 - (2) Statement I is true but Statement II is false
 - (3) Both Statement I and Statement II are false
 - (4) Statement I is false but Statement II is true
-

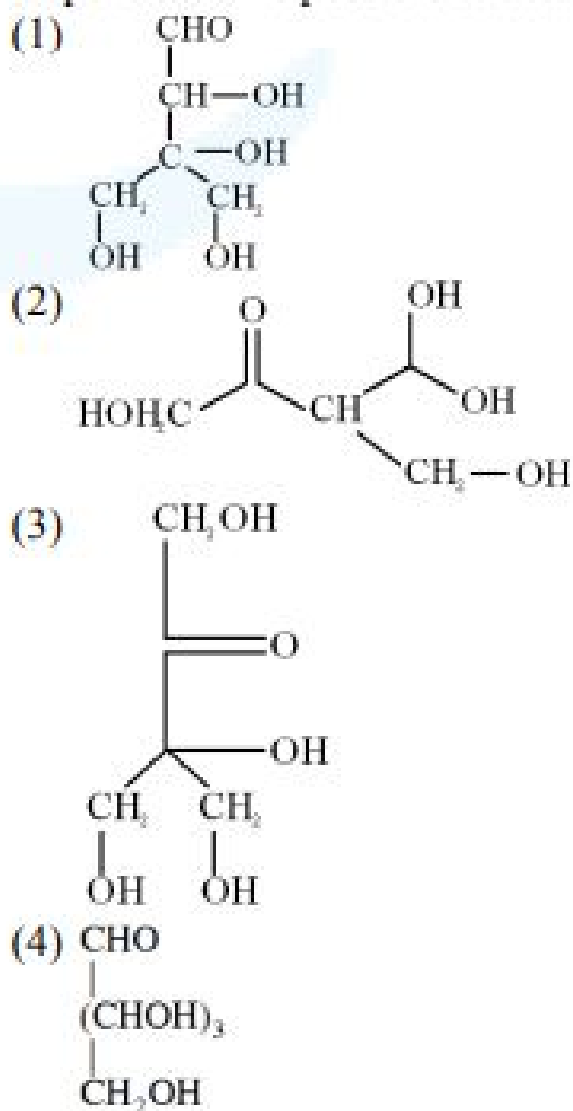
37. Evaluate the following statements for their correctness:

- (A) The elevation in boiling point temperature of water will be same for 0.1 M NaCl and 0.1 M urea.
- (B) Azeotropic mixtures boil without change in their composition.
- (C) Osmosis always takes place from hypotonic to hypertonic solution.
- (D) The density of 32% H_2SO_4 solution having molarity 4.09 M is approximately 1.26 g mL^{-1} .
- (E) A negatively charged sol is obtained when KI solution is added to silver nitrate solution.

Choose the correct answer from the options given below:

- (1) B, D, and E only
 - (2) A, B, and D only
 - (3) A and C only
 - (4) B and D only
-

38. Compound A, $C_5H_{10}O_5$, given a tetraacetate with Ac_2O and oxidation of A with $Br_2 - H_2O$ gives an acid, $C_5H_{10}O_6$. Reduction of A with HI gives isopentane. The possible structure of A is:



39. Arrange the following orbitals in decreasing order of energy? (A) $n = 3, l = 0, m = 0$

(B) $n = 4, l = 1, m = 0$

(C) $n = 3, l = 1, m = 0$

(D) $n = 3, l = 2, m = 1$

The correct option for the order is:

(1) $B > D > C > A$

(2) $D > B > C > A$

(3) $A > C > B > D$

(4) $D > B > A > C$

40. The Lewis acid character of boron tri halides follows the order:

- (1) $\text{BCl}_3 > \text{BBr}_3 > \text{BCl}_3 > \text{BF}_3$
 (2) $\text{BCl}_3 > \text{BF}_3 > \text{BBr}_3 > \text{B}_2\text{H}_6$
 (3) $\text{BF}_3 > \text{BCl}_3 > \text{BBr}_3 > \text{B}_2\text{H}_6$
 (4) $\text{B}_2\text{H}_6 > \text{BBr}_3 > \text{BCl}_3 > \text{BF}_3$

41. Match List-I with List-II:

List-I	List-II
(A) Physiosorption	I. Single layer adsorption
(B) Chemisorption	II. $20-40 \text{ kJ mol}^{-1}$
(C) $\text{N}_2(\text{g}) + 3\text{H}_2(\text{g}) \xrightarrow{\text{Fe}} 2\text{NH}_3(\text{g})$	III. Chromatography
(D) Analytical Application or Adsorption	IV. Heterogeneous catalysis

Choose the correct answer from the options given below:

- (1) A - II, B - III, C - I, D - IV
 (2) A - IV, B - III, C - I, D - II
 (3) A - IV, B - II, C - III, D - I
 (4) A - II, B - I, C - IV, D - III

42. Given below are two statements: One is labelled as Assertion (A) and the other is labelled as Reason (R)

Assertion (A): The first ionization enthalpy of 3d series elements is more than that of group 2 metals.

Reason (R): In 3d series of elements, successive filling of d-orbitals takes place.

In light of the above statements, choose the correct answer from the options given below:

- (1) Both (A) and (R) are true and (R) is the correct explanation of (A)
 (2) Both (A) and (R) are true but (R) is not the correct explanation of (A)
 (3) (A) is false but (R) is true
 (4) (A) is true but (R) is false

43. The element playing a significant role in neuromuscular function and interneuronal transmission is:

- (1) Be

- (2) Ca
 - (3) Li
 - (4) Mg
-

44. Given below are two statements: Statement I: H_2O_2 is used in the synthesis of Cephalosporin. **Statement II:** H_2O_2 is used for the restoration of aerobic conditions to sewage wastes.

In light of the above statements, choose the most appropriate answer from the options given below:

- (1) Both Statement I and Statement II are correct
 - (2) Statement I is incorrect but Statement II is correct
 - (3) Statement I is correct but Statement II is incorrect
 - (4) Both Statement I and Statement II are incorrect
-

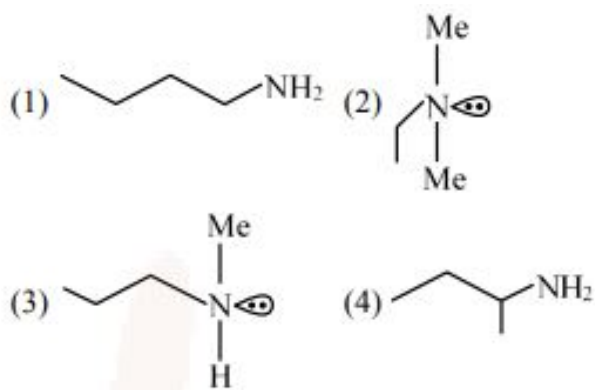
45. The normal rain water is slightly acidic and its pH value is 5.6 because of which one of the following?

- (1) $CO_2 + H_2O \rightarrow H_2CO_3$
 - (2) $4NO_2 + 2H_2O \rightarrow 4HNO_3$
 - (3) $SO_2 + O_2 + 2H_2O \rightarrow H_2SO_4$
 - (4) $2N_2O_5 + H_2O \rightarrow 2HNO_3$
-

46. When a hydrocarbon A undergoes complete combustion it requires 11 equivalents of oxygen and produces 4 equivalents of water. What is the molecular formula of A?

- (1) C_9H_8
 - (2) $C_{11}H_4$
 - (3) C_5H_8
 - (4) $C_{11}H_8$
-

47. An organic compound [A] ($C_4H_{11}N$) shows optical activity and gives N_2 gas on treatment with HNO_2 . The compound [A] reacts with $PhSO_2Cl$ producing a compound which is soluble in KOH. The structure of A is:



48. Which one of the following statements is incorrect?

- (1) Boron and Indium can be purified by zone refining method.
- (2) Van Arkel method is used to purify tungsten.
- (3) Cast iron is obtained by melting pig iron with scrap iron and coke using hot air blast.
- (4) The malleable iron is prepared from cast iron by oxidizing impurities in a reverberatory furnace.

49. Which of the following elements have half-filled f-orbitals in their ground state?

(Given: atomic number Sm = 62; Eu = 63; Tb = 65; Gd = 64; Pm = 61) (A) Sm

- (B) Eu
- (C) Tb
- (D) Gd
- (E) Pm

Choose the correct answer from the options given below:

- (1) B and D only
- (2) A and E only
- (3) A and D only
- (4) C and D only

50. In Dumas method for the estimation of N_2 , the sample is heated with copper oxide and the gas evolved is passed over:

- (1) Ni
- (2) Copper gauze

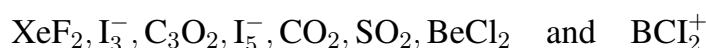
- (3) Pd
(4) Copper oxide
-

Section-B

51. If the CFSE of $[\text{Ti}^{3+}(\text{H}_2\text{O})_6]^{3+}$ is -96.0 kJ/mol, this complex will absorb maximum at wavelength __ nm. (nearest integer)

Assume Planck's constant $h = 6.4 \times 10^{-34}$ J s, speed of light $c = 3.0 \times 10^8$ m/s, and Avogadro's constant $N_A = 6 \times 10^{23} \text{ mol}^{-1}$.

52. Amongst the following, the number of species having the linear shape is:



53. The resistivity of a 0.8 M solution of an electrolyte is $5 \times 10^{-3} \Omega \text{ cm}$. Its molar conductivity is $__ \times 10^4 \Omega^{-1} \text{ cm}^2 \text{ mol}^{-1}$ (Nearest integer).

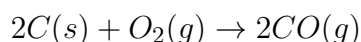
54. At 298 K, the solubility of silver chloride in water is $1.434 \times 10^{-3} \text{ g L}^{-1}$. The value of $-\log K_{\text{sp}}$ for silver chloride is:

(Given mass of Ag is 107.9 g mol^{-1} and mass of Cl is 35.5 g mol^{-1})

55. A sample of a metal oxide has formula $M_{0.83}O_{1.00}$.

The metal M can exist in two oxidation states +2 and +3. In the sample of $M_{0.83}O_{1.00}$, the percentage of metal ions existing in the +2 oxidation state is __ % (nearest integer).

56. Assume carbon burns according to the following equation:

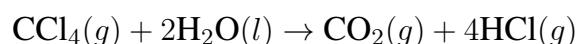


When 12 g of carbon is burnt in 48 g of oxygen, the volume of carbon monoxide produced is $_ \times 10^{-1} \text{ L}$ at STP (nearest integer).

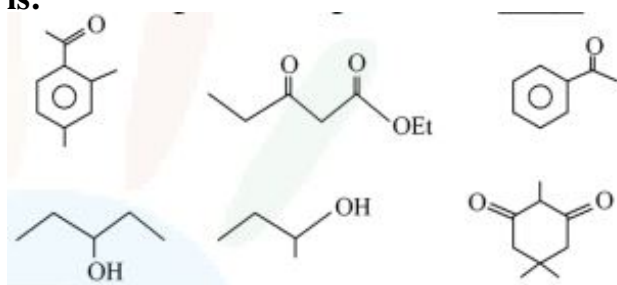
Given: Assume CO as ideal gas, Mass of C is 12 g mol^{-1} , Mass of O is 16 g mol^{-1} , and molar volume of an ideal gas at STP is 22.7 L mol^{-1} .

57. The number of alkali metal(s), from Li, K, Cs, Rb having ionization enthalpy greater than 400 kJ mol^{-1} and forming stable super oxides is __

58. Enthalpies of formation of $\text{CCl}_4(g)$, $\text{H}_2\text{O}(l)$, $\text{CO}_2(g)$ and $\text{HCl}(g)$ are -105, -242, -394, and -92 kJ/mol respectively. The magnitude of enthalpy of the reaction given below is __ kJ/mol (nearest integer):



59. The number of molecules which gives halform test among the following molecules is:



60. The rate constant for a first order reaction is 20 min^{-1} . The time required for the initial concentration of the reactant to reduce to its $\frac{1}{32}$ level is __ $\times 10^{-2}$ min. (Nearest integer) (Given: $\ln 10 = 2.303$, $\log 2 = 0.3010$)

MATHEMATICS

Section-A

61. If $\phi(x) = \frac{1}{\sqrt{x}} \int_{\frac{x}{4}}^x (4\sqrt{2} \sin t - 3\phi(t)) dt$, $x > 0$, then $\phi\left(\frac{\pi}{4}\right)$ is equal to:

- (1) $\frac{8}{\sqrt{\pi}}$
 - (2) $\frac{6}{6+\sqrt{\pi}}$
 - (3) $\frac{8}{6+\sqrt{\pi}}$
 - (4) $\frac{4}{6-\sqrt{\pi}}$
-

62. If a point $P(\alpha, \beta, \gamma)$ satisfying the equation

$$\begin{pmatrix} 2 & 10 & 8 \\ 9 & 3 & 8 \\ 8 & 4 & 8 \end{pmatrix} \begin{pmatrix} \alpha \\ \beta \\ \gamma \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

lies on the plane $2x + 4y + 3z = 5$, then $6\alpha + 9\beta + 7\gamma$ is equal to:

- (1) 1
 - (2) $\frac{11}{5}$
 - (3) $\frac{5}{4}$
 - (4) 11
-

63. Let $a_1, a_2, a_3, \dots$ be an A.P. If $a_4 = 3$, the product $a_1 a_4$ is minimum and the sum of its first n terms is zero, then $n! - 4a_n(a_{n+2})$ is equal to:

- (1) 24
 - (2) $\frac{33}{4}$
 - (3) $\frac{381}{4}$
 - (4) 9
-

64. Let $(a, b) \subset (0, 2\pi)$ be the largest interval for which

$$\sin^{-1}(\sin \theta) - \cos^{-1}(\sin \theta) > 0, \quad \theta \in (0, 2\pi)$$

holds. If

$$\alpha x^2 + \beta x + \sin^{-1}((x^2 - 6x + 10)) + \cos^{-1}((x^2 - 3)^2 + 1) = 0$$

and $\alpha - \beta = b - a$, then α is equal to:

- (1) $\frac{\pi}{48}$
 - (2) $\frac{\pi}{16}$
 - (3) $\frac{\pi}{12}$
 - (4) $\frac{\pi}{8}$
-

65. Let $y = y(x)$ be the solution of the differential equation

$$(3y^2 - 5x^2)y \, dx + 2x(x^2 - y^2) \, dy = 0,$$

such that $y(1) = 1$. Then

$(y(2))^3 - 12y(2)$ is equal to:

- (1) $32\sqrt{2}$
 - (2) 64
 - (3) $16\sqrt{2}$
 - (4) 32
-

66. The set of all values of a^2 for which the line $x + y = 0$ bisects two distinct chords drawn from a point $P\left(\frac{1+a}{2}, \frac{1-a}{2}\right)$ on the circle

$$2x^2 + 2y^2 - (1+a)x - (1-a)y = 0$$

is equal to:

- (1) $(8, \infty)$
 - (2) $(4, \infty)$
 - (3) $(0, 4)$
 - (4) $(2, 12)$
-

67. Among the relations

$$S = \{(a, b) : a, b \in \mathbb{R} \setminus \{0\}, a^2 + b^2 > 0\}$$

And

$$T = \{(a, b) : a, b \in \mathbb{R}, a^2 - b^2 \in \mathbb{Z}\}$$

which of the following is true?

- (1) S is transitive but T is not.
 - (2) T is symmetric but S is not.
 - (3) Neither S nor T is transitive.
 - (4) Both S and T are symmetric.
-

68. The equation

$$e^x + 8e^{2x} + 13e^x - 8e^x + 1 = 0, \quad x \in \mathbb{R}$$

has:

- (1) two solutions and both are negative
 - (2) no solution
 - (3) four solutions, two of which are negative
 - (4) two solutions and only one of them is negative
-

69. The number of values of $r \in \{p, q, \neg p, \neg q\}$ for which

$$((p \wedge q) \Leftrightarrow (r \vee q)) \wedge ((p \wedge r) \Leftrightarrow q)$$

is a tautology, is:

- (1) 3
 - (2) 2
 - (3) 1
 - (4) 4
-

70. Let $f : \mathbb{R} \setminus \{2, 6\} \rightarrow \mathbb{R}$ be the real-valued function defined as

$$f(x) = \frac{x^2 + 2x + 1}{x^2 - 8x + 12}.$$

Then the range of f is:

- (1) $(-\infty, \frac{-21}{4}] \cup [0, \infty)$
 - (2) $(-\infty, \frac{-21}{4}] \cup (0, \infty)$
 - (3) $(-\infty, \frac{-21}{4}] \cup [\frac{21}{4}, \infty)$
 - (4) $[\frac{-21}{4}, \infty) \cup [0, \infty)$
-

71. Evaluate the limit:

$$\lim_{x \rightarrow 1} \frac{(\sqrt{3x+1} + \sqrt{3x-1})^6}{(x + \sqrt{x^2-1})^3 + (\sqrt{3x+1} - \sqrt{3x-1})^6}$$

- (1) is equal to 9
 - (2) is equal to 27
 - (3) does not exist
 - (4) is equal to $\frac{27}{2}$
-

72. Let P be the plane, passing through the point $(1, -1, -5)$ and perpendicular to the

line joining the points $(4, 1, -3)$ and $(2, 4, 3)$. Then the distance of P from the point $(3, -2, 2)$ is:

- (1) 6
 - (2) 4
 - (3) 5
 - (4) 7
-

73. The absolute minimum value of the function

$f(x) = |x^2 - x + 1| + [x^2 - x + 1]$, where $[t]$ denotes the greatest integer function, in the interval $[-1, 2]$, is

- (1) $\frac{3}{4}$
 - (2) $\frac{3}{2}$
 - (3) $\frac{1}{4}$
 - (4) $\frac{5}{4}$
-

74. Let the plane $P : 8x + \alpha y + \alpha z + 12 = 0$ be parallel to the line

$$L : \frac{x+2}{2} = \frac{y-3}{3} = \frac{z+4}{5}.$$

If the intercept of P on the y-axis is 1, then the distance between P and L is:

- (1) $\sqrt{14}$
 - (2) $\frac{6}{\sqrt{14}}$
 - (3) $\frac{\sqrt{2}}{7}$
 - (4) $\frac{\sqrt{7}}{2}$
-

75. The foot of perpendicular from the origin O to a plane P which meets the coordinate axes at the points A, B, C is $(2, 4, 4)$. If the volume of the tetrahedron $OABC$ is 144 unit^3 , then which of the following points is NOT on P ?

- (1) $(2, 2, 4)$
 - (2) $(0, 4, 4)$
 - (3) $(3, 0, 4)$
 - (4) $(0, 6, 6)$
-

76. Let the mean and standard deviation of marks of class A of 100 students be respectively 40 and $\alpha > 0$, and the mean and standard deviation of marks of class B of n students be respectively 55 and $30 - \alpha$. If the mean and variance of the marks of the combined class of $100 + n$ students are respectively 50 and 350, then the sum of variances of classes A and B is:

- (1) 500
 - (2) 650
 - (3) 450
 - (4) 900
-

77. Let

$$\mathbf{a} = \hat{i} + 2\hat{j} + 3\hat{k}, \quad \mathbf{b} = \hat{i} - \hat{j} + 2\hat{k}, \quad \mathbf{c} = 5\hat{i} - 3\hat{j} + 3\hat{k}$$

be three vectors. If $\mathbf{r}$ is a vector such that $\mathbf{r} \times \mathbf{b} = \mathbf{c} \times \mathbf{b}$ and $\mathbf{r} \cdot \mathbf{a} = 0$, then $25|\mathbf{r}|^2$ is equal to:

- (1) 449
 - (2) 336
 - (3) 339
 - (4) 560
-

78. Let H be the hyperbola, whose foci are $(1 \pm \sqrt{2}, 0)$ and eccentricity is $\sqrt{2}$. Then the length of its latus rectum is:

- (1) 2
 - (2) 3
 - (3) $\frac{5}{2}$
 - (4) $\frac{3}{2}$
-

79. Let $\alpha > 0$. If

$$\int_{\alpha}^x \frac{x}{\sqrt{x+\alpha} - \sqrt{x}} dx = \frac{16 + 20\sqrt{2}}{15},$$

then α is equal to:

- (1) 2
- (2) 4

(3) $\sqrt{2}$

(4) $2\sqrt{2}$

80. The complex number

$$z = \frac{i - 1}{\cos \frac{\pi}{3} + i \sin \frac{\pi}{3}}$$

is equal to:

(1) $\sqrt{2} \left(\cos \frac{5\pi}{12} + i \sin \frac{5\pi}{12} \right)$

(2) $\cos \frac{\pi}{12} - i \sin \frac{\pi}{12}$

(3) $\sqrt{2} \left(\cos \frac{\pi}{12} + i \sin \frac{\pi}{12} \right)$

(4) $\sqrt{2} \left(\cos \frac{5\pi}{12} - i \sin \frac{5\pi}{12} \right)$

Section - B

81. The coefficient of x^{-6} , in the expansion of

$$\left(\frac{4x}{5} + \frac{5}{2x^2} \right)^9, \text{ is:}$$

82. Let the area of the region

$$\{(x, y) : |2x - 1| \leq y \leq x^2 - x, 0 \leq x \leq 1\} \text{ be } A.$$

Then $(6A + 11)^2$ is equal to:

83. If

$$\frac{(2n+1)P_{n-1}}{2nP_n} = \frac{11}{21}, \quad \text{then } n^2 + n + 15 \text{ is equal to:}$$

84. If the constant term in the binomial expansion of

$$\left(\frac{x^{5/2}}{2} - \frac{4}{x} \right)^9 \text{ is } -84 \text{ and the coefficient of } x^{-3} \text{ is } 2\alpha\beta,$$

where $\beta < 0$ is an odd number, then $|\alpha - \beta|$ is equal to:

85. Let $\vec{a}, \vec{b}, \vec{c}$ be three vectors such that

$$|\vec{a}| = \sqrt{31}, \quad |\vec{b}| = 4, \quad |\vec{c}| = 2, \quad 2(\vec{a} \times \vec{b}) = 3(\vec{c} \times \vec{a}).$$

If the angle between $\vec{b}$ and $\vec{c}$ is $\frac{2\pi}{3}$, then $\left(\frac{\vec{a} \times \vec{c}}{\vec{a} \cdot \vec{b}}\right)^2$ is equal to:

86. Let S be the set of all $a \in \mathbb{N}$ such that the area of the triangle formed by the tangent at the point $P(b, c)$, $b, c \in \mathbb{N}$ on the parabola

$$y^2 = 2ax \quad \text{and the lines } x = b, y = 0 \quad \text{is } 16 \text{ unit}^2, \text{ then } \sum_{a \in S} a \text{ is equal to:}$$

87. The sum

$$1^2 - 2 \cdot 3^2 + 3 \cdot 5^2 - 4 \cdot 7^2 + 5 \cdot 9^2 - \dots + 15 \cdot 29^2 \text{ is:}$$

88. Let A be the event that the absolute difference between two randomly chosen real numbers in the sample space

$$[0, 60] \quad \text{is less than or equal to } a. \text{ If } P(A) = \frac{11}{36}, \text{ then } a \text{ is equal to:}$$

89. Let $A = [a_{ij}]$, where $a_{ij} \in \mathbb{Z} \cap [0, 4]$, $1 \leq i, j \leq 2$. The number of matrices A such that the sum of all entries is a prime number $p \in \{2, 13\}$ is:

90. Let A be an $n \times n$ matrix such that $|A| = 2$. If the determinant of the matrix

$$\text{Adj}(2 \cdot \text{Adj}(2A^{-1})) \text{ is } 2^{84}, \text{ then } n \text{ is equal to:}$$
