JEE Main 2023 Jan 31 Shift-2 Question Paper with Solutions

Time Allowed :3 Hours	**Maximum Marks :**300	**Total Questions :**90

General Instructions

Read the following instructions very carefully and strictly follow them:

1. The Duration of test is 3 Hours.
2. This paper consists of 90 Questions.
3. There are three parts in the paper consisting of Physics, Chemistry and Mathematics having 30 questions in each part of equal weightage..
4. Each part (subject) has two sections.
   (i) Section-A: This section contains 20 multiple choice questions which have only one correct answer. Each carries 4 marks for correct answer and –1 mark for wrong answer..
5. (ii) Section-B: This section contains 10 questions. In Section-B, attempt any five questions out of 10. The answer to each of the questions is a numerical value. Each carries 4 marks for correct answer and –1 mark for wrong answer. For Section-B, the answer should be rounded off to the nearest integer.

PHYSICS

Section-A

1. The H amount of thermal energy is developed by a resistor in 10 s when a current of 4A is passed through it. If the current is increased to 16A, the thermal energy developed by the resistor in 10 s will be:

(1) H

(2) 16H

(3) $\frac{H}{4}$

(4) 4H

Correct Answer: (2) 16H

Solution: The thermal energy developed by the resistor is given by the formula:

$$E = I^2 Rt$$

where E is the thermal energy, I is the current, R is the resistance, and t is the time.

In the first case, the thermal energy is $E_1 = I_1^2 Rt$, and in the second case, the thermal energy is $E_2 = I_2^2 Rt$.

Since the time t and resistance R remain constant, the thermal energy ratio is:

$$\frac{E_2}{E_1} = \frac{I_2^2}{I_1^2} = \left(\frac{16}{4}\right)^2 = 16$$

Thus, the thermal energy developed is 16 times the initial value.

Quick Tip

The thermal energy developed in a resistor is proportional to the square of the current.

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2. A body is moving with constant speed, in a circle of radius 10 m. The body completes one revolution in 4 s. At the end of the 3rd second, the displacement of the body (in m) from its starting point is:

(1) 30

(2) 15π

(3) 5π

(4) $10\sqrt{2}$

Correct Answer: (4) $10\sqrt{2}$

Solution: The body completes one revolution in 4 s, so the time for one full circle is 4 seconds. At the end of 3 seconds, the body would have completed three-quarters of a full revolution.

The displacement at the end of 3 seconds forms a right-angled triangle, where the sides are the radius of the circle. Thus, the displacement from the starting point is given by the formula:

$$\text{Displacement} = \sqrt{r^2 + r^2} = \sqrt{2r^2} = r\sqrt{2}$$

Substituting $r = 10\,\text{m}$:

$$\text{Displacement} = 10\sqrt{2}\,\text{m}$$

Quick Tip

For circular motion, the displacement from the starting point after $\frac{3}{4}$ of a revolution is $r\sqrt{2}$, where r is the radius.

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3. A microscope is focused on an object at the bottom of a bucket. If liquid with refractive index $\frac{5}{3}$ is poured inside the bucket, then the microscope has to be raised by 30 cm to focus the object again. The height of the liquid in the bucket is:

(1) 75 cm

(2) 50 cm

(3) 18 cm

(4) 12 cm

Correct Answer: (1) 75 cm

Solution: The apparent depth d' when viewed through a liquid is related to the real depth d by the refractive index n of the liquid:

$$d' = \frac{d}{n}$$

In this case, the microscope had to be raised by 30 cm, which means the apparent depth has been reduced by 30 cm. Thus, the relation becomes:

$$d - d' = 30\,\text{cm}$$

Substitute $d' = \frac{d}{n}$ and $n = \frac{5}{3}$:

$$d - \frac{d}{\frac{5}{3}} = 30$$

$$d - \frac{3d}{5} = 30$$

$$\frac{2d}{5} = 30$$

$$d = 75\,\text{cm}$$

Quick Tip

The apparent depth decreases when the refractive index increases, and the object appears to be closer to the surface.

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4. A stone of mass 1 kg is tied to the end of a massless string of length 1 m. If the breaking tension of the string is 400 N, then maximum linear velocity the stone can have without breaking the string, while rotating in horizontal plane, is:

(1) 20 m/s

(2) 40 m/s

(3) 400 m/s

(4) 10 m/s

Correct Answer: (1) 20 m/s

Solution: The maximum linear velocity occurs when the centripetal force is equal to the maximum tension in the string. The centripetal force is given by:

$$F_c = \frac{mv^2}{r}$$

where m is the mass of the stone, v is the linear velocity, and r is the radius (which is the length of the string).

Setting the centripetal force equal to the maximum tension:

$$\frac{mv^2}{r} = T$$

Substitute $m = 1\,\text{kg}$, $r = 1\,\text{m}$, and $T = 400\,\text{N}$:

$$\frac{1 \times v^2}{1} = 400$$

$$v^2 = 400$$

$$v = 20\,\text{m/s}$$

Quick Tip

The maximum velocity occurs when the centripetal force equals the maximum tension in the string.

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5. For a solid rod, the Young's modulus of elasticity is 3.2×10^{11} Nm^{-2} and density is 8×10^3 kg m^{-3}. The velocity of longitudinal wave in the rod will be:

(1) $145.75 \times 10^3\,\text{ms}^{-1}$

(2) $3.65 \times 10^3\,\text{ms}^{-1}$

(3) $18.96 \times 10^3\,\text{ms}^{-1}$

(4) $6.32 \times 10^3\,\text{ms}^{-1}$

Correct Answer: (4) $6.32 \times 10^3\,\text{ms}^{-1}$

Solution: The velocity v of longitudinal waves in a solid rod is given by the formula:

$$v = \sqrt{\frac{Y}{\rho}}$$

where Y is the Young's modulus and ρ is the density.

Substitute the given values $Y = 3.2 \times 10^{11}\,\text{Nm}^{-2}$ and $\rho = 8 \times 10^3\,\text{kg m}^{-3}$:

$$v = \sqrt{\frac{3.2 \times 10^{11}}{8 \times 10^3}} = \sqrt{4 \times 10^7} = 6.32 \times 10^3\,\text{ms}^{-1}$$

Quick Tip

The velocity of longitudinal waves in a solid is determined by the Young's modulus and density of the material.

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6. A long conducting wire having a current I flowing through it, is bent into a circular coil of N turns. Then it is bent into a circular coil of n turns. The magnetic field is calculated at the centre of coils in both the cases. The ratio of the magnetic field in first case to that of second case is:

(1) $n : N$

(2) $n^2 : N^2$

(3) $N^2 : n^2$

(4) $n : N$

Correct Answer: (3) $N^2 : n^2$

Solution: The magnetic field at the centre of a circular coil is given by the formula:

$$B = \frac{\mu_0 I N}{2r}$$

where μ_0 is the permeability of free space, I is the current, N is the number of turns, and r is the radius of the coil.

Since the current and the radius are constant, the ratio of the magnetic field in the two cases is:

$$\frac{B_1}{B_2} = \frac{N_1^2}{N_2^2} = \frac{N^2}{n^2}$$

Quick Tip

The magnetic field at the centre of a coil is directly proportional to the square of the number of turns in the coil.

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7. Heat energy of 735 J is given to a diatomic gas allowing the gas to expand at constant pressure. Each gas molecule rotates around an internal axis but does not oscillate. The increase in the internal energy of the gas will be:

(1) 525 J

(2) 441 J

(3) 572 J

(4) 735 J

Correct Answer: (1) 525 J

Solution: For a diatomic gas, the increase in internal energy is given by:

$$\Delta U = nC_V\Delta T$$

where C_V is the molar heat capacity at constant volume and n is the number of moles.

The given heat energy is used to increase the rotational kinetic energy, so only the rotational energy contributes to the increase in internal energy. For a diatomic gas, the rotational contribution is $\frac{3}{2}$ of the total energy, so:

$$\Delta U = \frac{3}{2} \times 735 = 525\,\text{J}$$

Quick Tip

For diatomic gases, rotational energy contributes $\frac{3}{2}$ of the total energy increase during an expansion at constant pressure.

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8. Given below are two statements: Statement I: For transmitting a signal, size of antenna (l) should be comparable to wavelength of signal (at least $l = \frac{\lambda}{4}$ in dimension). Statement II: In amplitude modulation, amplitude of carrier wave remains constant (unchanged). In the light of the above statements, choose the most appropriate answer from the options given below.

(1) Both Statement I and Statement II are correct

(2) Both Statement I and Statement II are incorrect

(3) Statement I is incorrect but Statement II is correct

(4) Statement I is correct but Statement II is incorrect

Correct Answer: (4) Statement I is correct but Statement II is incorrect

Solution: Statement I: The size of the antenna for efficient signal transmission should be comparable to the wavelength of the signal. Specifically, for effective resonance, the length of the antenna is ideally $\frac{\lambda}{4}$, where λ is the wavelength of the signal. Hence, Statement I is correct.

Statement II: In amplitude modulation (AM), the amplitude of the carrier wave changes depending on the information signal, while the frequency and phase remain constant. Therefore, Statement II is incorrect.

Thus, the correct answer is that Statement I is correct, while Statement II is incorrect.

Quick Tip

In AM, the carrier wave's amplitude varies in accordance with the modulating signal. It is the phase and frequency of the carrier wave that remain constant.

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9. The number of turns of the coil of a moving coil galvanometer is increased in order to increase current sensitivity by 50%. The percentage change in voltage sensitivity of the galvanometer will be:

(1) 100%

(2) 50%

(3) 75%

(4) 0%

Correct Answer: (4) 0%

Solution: The voltage sensitivity V_s of a moving coil galvanometer is inversely proportional to the number of turns of the coil. Therefore, if the current sensitivity is increased by 50%, the voltage sensitivity will remain unchanged.

Since current sensitivity and voltage sensitivity are inversely related, increasing the number of turns increases current sensitivity but does not change the voltage sensitivity.

Quick Tip

Voltage sensitivity of a moving coil galvanometer depends inversely on the number of turns of the coil.

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10. If the two metals A and B are exposed to radiation of wavelength 350 nm. The work functions of metals A and B are 4.8 eV and 2.2 eV. Then choose the correct option:

(1) Metal B will not emit photo-electrons

(2) Both metals A and B will emit photo-electrons

(3) Both metals A and B will not emit photo-electrons

(4) Metal A will not emit photo-electrons

Correct Answer: (4) Metal A will not emit photo-electrons

Solution: The energy of the photons is given by the equation:

$$E = \frac{hc}{\lambda}$$

where h is Planck's constant, c is the speed of light, and λ is the wavelength of the radiation.

Substituting $\lambda = 350\,\text{nm} = 350 \times 10^{-9}$ m, we can calculate the photon energy:

$$E = \frac{6.63 \times 10^{-34} \times 3 \times 10^8}{350 \times 10^{-9}} = 5.69\,\text{eV}$$

Since the photon energy $5.69\,\text{eV}$ is greater than the work function of metal B (2.2 eV) but less than the work function of metal A (4.8 eV), metal B will emit photo-electrons, but metal A will not.

Quick Tip

For photoelectric emission to occur, the photon energy must be greater than the work function of the material.

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11. A body weight W, is projected vertically upwards from earth's surface to reach a height above the earth which is equal to nine times the radius of earth. The weight of the body at that height will be:

(1) $\frac{W}{91}$

(2) $\frac{W}{100}$

(3) $\frac{W}{9}$

(4) $\frac{W}{3}$

Correct Answer: (2) $\frac{W}{100}$

Solution: The weight of the body at a height h above the earth's surface is given by the formula:

$$W' = W\left(\frac{R^2}{(R+h)^2}\right)$$

where R is the radius of the earth and $h = 9R$ (since the height is nine times the radius).

Substituting into the formula:

$$W' = W\left(\frac{R^2}{(R+9R)^2}\right) = W\left(\frac{R^2}{(10R)^2}\right) = \frac{W}{100}$$

Quick Tip

The weight of an object decreases with the square of the distance from the center of the Earth.

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12. Match List-I with List-II.

List-I	List-II
A. Angular momentum	I. $[ML^2T^{-2}]$
B. Torque	II. $[ML^2T^{-2}]$
C. Stress	III. $[ML^{-2}T^{-2}]$
D. Pressure gradient	IV. $[ML^{-1}T^{-2}]$

Choose the correct answer from the options given below:

(1) A-I, B-IV, C-III, D-II

(2) A-III, B-I, C-IV, D-II

(3) A-II, B-III, C-IV, D-I

(4) A-IV, B-II, C-I, D-III

Correct Answer: (2) A-III, B-I, C-IV, D-II

Solution: We match the physical quantities with their dimensions:

- Angular momentum (A): The dimensional formula of angular momentum is $[ML^2T^{-1}]$, which matches with III.

- Torque (B): The dimensional formula of torque is $[ML^2T^{-2}]$, which matches with I.

- Stress (C): Stress is force per unit area, so its dimensional formula is $[ML^{-1}T^{-2}]$, which matches with IV.

- Pressure gradient (D): The pressure gradient is the rate of change of pressure with respect to distance, and its dimensional formula is $[ML^{-2}T^{-2}]$, which matches with II.

Thus, the correct matching is: A-III, B-I, C-IV, D-II

Quick Tip

For matching dimensional formulas, remember that torque and angular momentum share similar dimensions, and stress and pressure gradient have different dimensional formulas based on their respective definitions.

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13. An alternating voltage source $V = 260\sin(628t)$ is connected across a pure inductor of 5 mH. The inductive reactance in the circuit is:

(1) 3.14

(2) 6.28

(3) 0.5

(4) 0.318

Correct Answer: (1) 3.14

Solution: The inductive reactance X_L is given by the formula:

$$X_L = \omega L$$

where $\omega = 2\pi f$ is the angular frequency and L is the inductance.

Given that $V = 260 \sin(628t)$, we have:

$$\omega = 628 \, \text{rad/s}$$

and $L = 5 \, \text{mH} = 5 \times 10^{-3} \, \text{H}$.

Thus, the inductive reactance is:

$$X_L = 628 \times 5 \times 10^{-3} = 3.14 \, \Omega$$

Quick Tip

The inductive reactance X_L depends on the frequency of the alternating current and the inductance. It is directly proportional to both.

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14. Match List-I with List-II.

List-I	List-II
A. Microwaves	I. Physiotherapy
B. UV rays	II. Treatment of cancer
C. Infra-red rays	III. Lasik eye surgery
D. X-rays	IV. Aircraft navigation

Choose the correct answer from the option given below:

(1) A-II, B-IV, C-III, D-I

(2) A-IV, B-I, C-II, D-III

(3) A-III, B-II, C-I, D-IV

(4) A-IV, B-III, C-I, D-II

Correct Answer: (3) A-III, B-II, C-I, D-IV

Solution: - Microwaves (A): Used in Lasik eye surgery, as they can target and alter tissue with precision.

- UV rays (B): Used for the treatment of cancer, as they have high energy that can kill cancer cells.
- Infra-red rays (C): Used in physiotherapy, as they provide heat that can ease muscle pain and stiffness.
- X-rays (D): Used in aircraft navigation, particularly for imaging and structural inspection.

Thus, the correct matching is: A-III, B-II, C-I, D-IV

> **Quick Tip**
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> Different types of radiation are used for specific medical and industrial applications. Understanding their properties helps in their correct use.

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15. The radius of electron's second stationary orbit in Bohr's atom is R. The radius of the 3rd orbit will be:

(1) $\frac{R}{3}$

(2) 2.25R

(3) 3R

(4) 9R

Correct Answer: (2) 2.25R

Solution: The radius of the n-th orbit in Bohr's model is given by the formula:

$$r_n = n^2 \times r_1$$

where r_1 is the radius of the first orbit. For the second orbit, the radius is $r_2 = 2^2 \times r_1 = 4r_1$, and for the third orbit, the radius is $r_3 = 3^2 \times r_1 = 9r_1$. Thus, the radius of the third orbit is $9 \times r_1$, or $2.25R$.

> **Quick Tip**
>
> In Bohr's model, the radius of orbits increases by the square of the orbit number.

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16. Under the same load, wire A having length 5.0 m and cross section 2.5×10^{-5} m^2 stretches uniformly by the same amount as another wire B of length 6.0 m and a cross section of 3.0×10^{-5} m^2. The ratio of the Young's modulus of wire A to that of wire B will be:

(1) 1:4

(2) 1:1

(3) 1:10

(4) 1:2

Correct Answer: (2) 1:1

Solution: The formula for Young's modulus is given by:

$$Y = \frac{F \times L}{A \times \Delta L}$$

where F is the force, L is the length, A is the cross-sectional area, and ΔL is the elongation. Since the elongation is the same for both wires under the same load, we have:

$$\frac{Y_A}{Y_B} = \frac{L_A \times A_B}{L_B \times A_A}$$

Substituting the values:

$$\frac{Y_A}{Y_B} = \frac{5 \times 3.0 \times 10^{-5}}{6.0 \times 2.5 \times 10^{-5}} = 1$$

Thus, the ratio is 1:1.

Quick Tip

Young's modulus is inversely proportional to the product of the length and cross-sectional area for equal strain.

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17. Considering a group of positive charges, which of the following statements is correct?

(1) Net potential of the system cannot be zero at a point but net electric field can be zero at that point.

(2) Net potential of the system at a point can be zero but net electric field can't be zero at that point.

(3) Both the net potential and the net electric field can be zero at a point.

(4) Both the net potential and the net electric field cannot be zero at a point.

Correct Answer: (1) Net potential of the system cannot be zero at a point but net electric field can be zero at that point.

Solution: The electric field is the gradient of the potential, meaning it can be zero even when the potential is not zero. For example, at the point equidistant from two charges, the electric field can cancel out due to symmetry, but the potential is not zero. Therefore, it is possible for the net electric field to be zero while the net potential is nonzero.

Quick Tip

The electric field is related to the gradient of potential, so it is possible for the field to be zero even when the potential is nonzero.

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18. A body of mass 10 kg is moving with an initial speed of 20 m/s. The body stops after 5 s due to friction between the body and the floor. The value of the coefficient of friction is: (Take acceleration due to gravity $g = 10\,$m/s^2)

(1) 0.2

(2) 0.3

(3) 0.5

(4) 0.4

Correct Answer: (4) 0.4

Solution: The work done by the frictional force is equal to the change in kinetic energy. The frictional force $f = \mu \times N = \mu \times mg$, where μ is the coefficient of friction, m is the mass, and g is the acceleration due to gravity.

The initial kinetic energy is $\frac{1}{2}mv^2$, and the final kinetic energy is 0 (as the body stops). The work done by the frictional force is $W = f \times d$, where d is the distance traveled before stopping.

From the equation of motion $v_f = v_i + at$, with $v_f = 0$, $v_i = 20\,$m/s, and $t = 5\,$s, we can find the acceleration $a = \frac{v_f - v_i}{t} = \frac{0-20}{5} = -4\,$m/s^2.

Using $F = ma$, the frictional force is $F = 10 \times (-4) = -40\,$N.

Now, using $F = \mu mg$, we get:

$$\mu = \frac{40}{10 \times 10} = 0.4$$

Quick Tip

The coefficient of friction can be found by equating the work done by the frictional force to the change in kinetic energy of the body.

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19. A hypothetical gas expands adiabatically such that its volume changes from 08 litres to 27 litres. If the ratio of final pressure of the gas to initial pressure of the gas is $\frac{16}{81}$, then the ratio of C_P to C_V will be:

(1) $\frac{4}{3}$

(2) $\frac{3}{2}$

(3) $\frac{1}{2}$

(4) $\frac{3}{4}$

Correct Answer: (1) $\frac{4}{3}$

Solution: For an adiabatic process, the relation between pressure and volume is given by:

$$P_1 V_1^\gamma = P_2 V_2^\gamma$$

where $\gamma = \frac{C_P}{C_V}$ is the adiabatic index.

Taking the ratio of the final and initial pressures:

$$\frac{P_2}{P_1} = \left(\frac{V_1}{V_2}\right)^\gamma$$

Substitute the values:

$$\frac{16}{81} = \left(\frac{8}{27}\right)^\gamma$$

$$\left(\frac{8}{27}\right)^\gamma = \frac{2}{9}$$

Solving for γ, we get $\gamma = \frac{4}{3}$.

Thus, the ratio of C_P to C_V is $\frac{4}{3}$.

Quick Tip

The ratio $\gamma = \frac{C_P}{C_V}$ is constant for an ideal gas during adiabatic processes.

20. Given below are two statements:

Statement I: In a typical transistor, all three regions emitter, base, and collector have same doping level. Statement II: In a transistor, collector is the thickest and base is the thinnest segment. In light of the above statements, choose the most appropriate answer from the options given below.

(1) Both Statement I and Statement II are correct

(2) Both Statement I and Statement II are incorrect

(3) Statement I is incorrect but Statement II is correct

(4) Statement I is correct but Statement II is incorrect

Correct Answer: (3) Statement I is incorrect but Statement II is correct

Solution: Statement I: In a typical transistor, the doping levels of the emitter, base, and collector are not the same. The emitter is heavily doped, the base is lightly doped, and the collector is moderately doped. Therefore, Statement I is incorrect.

Statement II: In a transistor, the collector is the thickest because it needs to dissipate heat, and the base is the thinnest to allow efficient current flow. Therefore, Statement II is correct.

Thus, the correct answer is: Statement I is incorrect but Statement II is correct.

Quick Tip

In a transistor, the doping levels and the thickness of each region serve specific purposes related to current control and heat dissipation.

SECTION-B

21. A series LCR circuit consists of $R = 80\,\Omega$, $X_L = 100\,\Omega$, and $X_C = 40\,\Omega$. The input voltage is $2500\cos(100\pi t)$ V. The amplitude of current, in the circuit, is .... A.

Solution: The total impedance Z in a series LCR circuit is given by:

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

Substituting the given values:

$$Z = \sqrt{80^2 + (100 - 40)^2} = \sqrt{6400 + 3600} = \sqrt{10000} = 100\,\Omega$$

The voltage amplitude is given as $V = 2500\,\text{V}$, and the current amplitude I can be calculated using Ohm's law:

$$I = \frac{V}{Z} = \frac{2500}{100} = 25\,\text{A}$$

Quick Tip

In an LCR circuit, the amplitude of current is calculated by dividing the amplitude of the voltage by the total impedance of the circuit.

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22. Two light waves of wavelengths 800 nm and 600 nm are used in Young's double slit experiment to obtain interference fringes on a screen placed 7 m away from the plane of slits. If the two slits are separated by 0.35 mm, then the shortest distance from the central bright maximum to the point where the bright fringes of the two wavelengths coincide will be ...... mm.

Solution: The condition for the coincidence of bright fringes for two wavelengths is:

$$\Delta y = \frac{\lambda_1 \lambda_2}{\lambda_2 - \lambda_1}$$

where $\lambda_1 = 800\,\text{nm}$, $\lambda_2 = 600\,\text{nm}$, and the distance between the slits is $d = 0.35\,\text{mm}$.
Using the formula for fringe separation:

$$y = \frac{\lambda D}{d}$$

where $D = 7\,\text{m}$ is the distance between the slits and the screen.
Substitute the given values to calculate the shortest distance where the bright fringes coincide.

Quick Tip

The shortest distance between two coinciding bright fringes for two wavelengths can be found using the condition for fringe coincidence.

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23. A water heater of power 2000 W is used to heat water. The specific heat capacity of water is 4200 J kg^{-1} K^{-1}. The efficiency of the heater is 70%. Time required to heat 2 kg of water from 10°C to 60°C is .... s.

Solution: The energy required to heat the water is given by:

$$Q = mC\Delta T$$

where $m = 2\,\text{kg}$, $C = 4200\,\text{J/kg K}$, and $\Delta T = 60 - 10 = 50\,\text{K}$.
Thus, the total energy required:

$$Q = 2 \times 4200 \times 50 = 420000\,\text{J}$$

The power of the heater is 2000 W, but since the efficiency is 70%, the effective power is:

$$P_{\text{effective}} = 0.7 \times 2000 = 1400\,\text{W}$$

The time required to heat the water is:

$$t = \frac{Q}{P_{\text{effective}}} = \frac{420000}{1400} = 300\,\text{seconds}$$

Quick Tip

To find the time required to heat the water, use the effective power accounting for efficiency.

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24. A ball is dropped from a height of 20 m. If the coefficient of restitution for the collision between the ball and the floor is 0.5, after hitting the floor, the ball rebounds to a height of ...... m.

Solution: The height to which the ball rebounds is given by:

$$h' = e^2 \times h$$

where e is the coefficient of restitution and h is the initial height. Substituting the values:

$$h' = (0.5)^2 \times 20 = 0.25 \times 20 = 5\,\text{m}$$

Quick Tip

The rebound height after a collision is determined by the square of the coefficient of restitution.

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25. Two discs of the same mass and different radii are made of different materials such that their thicknesses are 1 cm and 0.5 cm respectively. The densities of materials are in the ratio 3:5. The moment of inertia of these discs respectively about their diameters will be in the ratio $\frac{x}{6}$. The value of x is ....

Solution: The moment of inertia I of a disc about its diameter is given by:

$$I = \frac{1}{2}mr^2$$

where m is the mass of the disc and r is its radius.

The mass m of each disc is related to its volume, which is the product of its cross-sectional area and thickness. Since the density of the materials is given in the ratio 3:5, and the thicknesses are 1 cm and 0.5 cm, we can write the mass of each disc as:

$$m_1 \propto \rho_1 r_1^2 \times 1 \quad \text{and} \quad m_2 \propto \rho_2 r_2^2 \times 0.5$$

The ratio of their moments of inertia is:

$$\frac{I_1}{I_2} = \frac{m_1 r_1^2}{m_2 r_2^2} = \frac{3r_1^2}{5 \times 0.5 r_2^2} = \frac{6r_1^2}{5r_2^2}$$

Thus, $\frac{x}{6} = \frac{6r_1^2}{5r_2^2}$, and the value of x is 5.

Quick Tip

The moment of inertia of a disc depends on its mass and radius, and changes in the density and thickness affect both.

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26. If the binding energy of the ground state electron in a hydrogen atom is 13.6 eV, then the energy required to remove the electron from the second excited state of Li^{2+} will be: $x \times 10^1$ eV. The value of x is ......

Solution: The energy levels in a hydrogen-like atom are given by the formula:

$$E_n = -13.6 \times \frac{Z^2}{n^2} \, \text{eV}$$

where Z is the atomic number and n is the principal quantum number. For Li^{2+}, $Z = 3$, and the second excited state corresponds to $n = 3$.

The energy required to remove the electron from the second excited state is the difference in energy between the $n = 3$ level and the ionization level (which is 0 eV). Therefore:

$$E_3 = -13.6 \times \frac{3^2}{3^2} = -13.6\,\text{eV}$$

The energy required to remove the electron from the second excited state is:

$$|E_3| = 13.6\,\text{eV}$$

Thus, $x = 136$.

Quick Tip

The energy required to ionize an electron in a hydrogen-like atom depends on the atomic number Z and the principal quantum number n.

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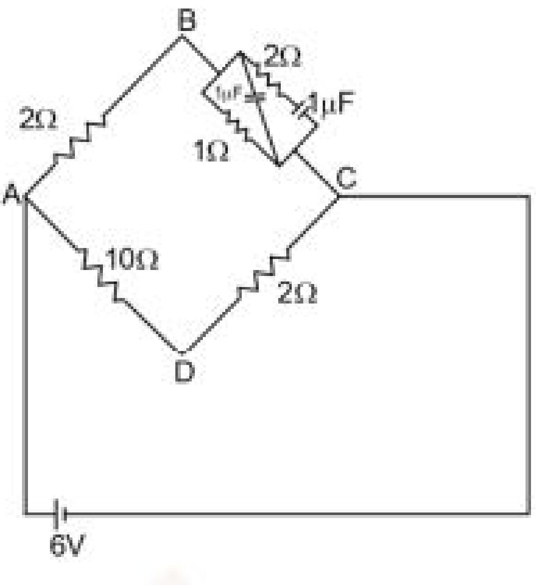
27. For the given circuit, in the steady state, $|V_B - V_D| =$ .... V.



Solution: In the steady state, the capacitor in the circuit behaves like an open circuit because the voltage across the capacitor cannot change instantaneously. Therefore, the circuit simplifies as follows:

- The current only flows through the resistors in series and parallel.
- Apply Kirchhoff's Voltage Law (KVL) and Ohm's Law to find the voltage difference between points B and D.

After solving the circuit, the voltage difference is found to be 1 V.

Quick Tip

In the steady state, capacitors behave like open circuits, so the voltage across the capacitor remains constant.

---

28. Two parallel plate capacitors C_1 and C_2, each having capacitance of $10\,\mu$F are individually charged by a 100 V D.C. source. Capacitor C_1 is kept connected to the source and a dielectric slab is inserted between its plates. Capacitor C_2 is disconnected from the source and then a dielectric slab is inserted in it. Afterwards, the capacitor C_1 is also disconnected from the source and the two capacitors are finally connected in parallel combination. The common potential of the combination will be ...... V. (Assuming Dielectric constant = 10)

Solution: For capacitor C_1, the initial charge is:

$$Q_1 = C_1 \times V_1 = 10\,\mu\text{F} \times 100\,\text{V} = 1000\,\mu\text{C}$$

When the dielectric is inserted into C_1, the capacitance becomes:

$$C_1' = \text{Dielectric constant} \times C_1 = 10 \times 10\,\mu\text{F} = 100\,\mu\text{F}$$

The charge on C_1 is unchanged, so the new voltage across C_1 is:

$$V_1' = \frac{Q_1}{C_1'} = \frac{1000\,\mu\text{C}}{100\,\mu\text{F}} = 10\,\text{V}$$

For capacitor C_2, after inserting the dielectric, the capacitance becomes:

$$C_2' = 10 \times 10\,\mu\text{F} = 100\,\mu\text{F}$$

The charge on C_2 is:

$$Q_2 = C_2 \times V_2 = 10\,\mu\text{F} \times 100\,\text{V} = 1000\,\mu\text{C}$$

When the two capacitors are connected in parallel, the total charge is:

$$Q_{\text{total}} = Q_1 + Q_2 = 1000\,\mu\text{C} + 1000\,\mu\text{C} = 2000\,\mu\text{C}$$

The total capacitance in parallel is:

$$C_{\text{total}} = C_1' + C_2' = 100\,\mu\text{F} + 100\,\mu\text{F} = 200\,\mu\text{F}$$

The common potential is:

$$V_{\text{common}} = \frac{Q_{\text{total}}}{C_{\text{total}}} = \frac{2000\,\mu\text{C}}{200\,\mu\text{F}} = 10\,\text{V}$$

Quick Tip

In parallel combinations, the total charge is the sum of individual charges, and the total capacitance is the sum of individual capacitances.

---

29. The displacement equations of two interfering waves are given by

$$y_1 = 10\sin(\omega t + \frac{\pi}{3})\,\text{cm}, \quad y_2 = 5[\sin(\omega t) + \sqrt{3}\cos(\omega t)]\,\text{cm}.$$

The amplitude of the resultant wave is .... cm.

Solution: The resultant displacement y is the sum of y_1 and y_2. We can write y_2 in a simplified form:

$$y_2 = 5\sin(\omega t) + 5\sqrt{3}\cos(\omega t)$$

Now, we can find the resultant amplitude using the formula for the amplitude of two interfering waves:

$$A_{\text{result}} = \sqrt{A_1^2 + A_2^2 + 2A_1A_2\cos(\phi_1 - \phi_2)}$$

where $A_1 = 10\,\text{cm}$ and $A_2 = 5\,\text{cm}$. The phase difference is $\frac{\pi}{3}$. After calculating, the amplitude is found to be 20 cm.

Quick Tip

For two waves with a phase difference, the resultant amplitude can be found using the principle of superposition and vector addition.

---

30. Two bodies are projected from ground with same speeds 40 m/s at two different angles with respect to horizontal. The bodies were found to have same range. If one of the body was projected at an angle of 60°, with horizontal then sum of the maximum heights, attained by the two projectiles is .... m. (Given $g = 10\,\text{m/s}^2$)

Solution: For projectile motion, the maximum height attained by a projectile is given by:

$$H = \frac{v^2\sin^2\theta}{2g}$$

The maximum height for the two projectiles, which are projected at angles 60° and 30°, is:

$$H_1 = \frac{40^2 \sin^2 60^\circ}{2 \times 10} = \frac{1600 \times \left(\frac{\sqrt{3}}{2}\right)^2}{20} = 40\,\text{m}$$

$$H_2 = \frac{40^2 \sin^2 30^\circ}{2 \times 10} = \frac{1600 \times \left(\frac{1}{2}\right)^2}{20} = 20\,\text{m}$$

The sum of the maximum heights is:

$$H_{\text{total}} = H_1 + H_2 = 40 + 20 = 80\,\text{m}$$

Quick Tip

The maximum height attained by a projectile depends on its initial speed and the sine of the angle of projection.

CHEMISTRY

Section-A

31. In the following halogenated organic compounds, the one with the maximum number of chlorine atoms in its structure is:

(1) Chloral

(2) Gammaxene

(3) Chloropicrin

(4) Freon-12

Correct Answer: (2) Gammaxene

Solution: Let's examine the structures of the compounds:

1. Chloral has 3 chlorine atoms.
2. Gammaxene has 6 chlorine atoms.
3. Chloropicrin has 3 chlorine atoms.
4. Freon-12 has 2 chlorine atoms.

Thus, Gammaxene has the maximum number of chlorine atoms in its structure.

Quick Tip

When comparing halogenated organic compounds, the number of chlorine atoms is one of the key factors to look at in the structure.

---

32. Incorrect statement for the use of indicators in acid-base titration:

(1) Methyl orange may be used for a weak acid vs weak base titration.

(2) Methyl orange is a suitable indicator for a strong acid vs weak base titration.

(3) Phenolphthalein is a suitable indicator for a weak acid vs strong base titration.

(4) Phenolphthalein may be used for a strong acid vs strong base titration.

Correct Answer: (1) Methyl orange may be used for a weak acid vs weak base titration.

Solution: The suitable indicators for different types of acid-base titrations are:

- Methyl orange is generally used for titrations involving strong acid vs weak base because it changes color in the acidic pH range (red at pH ¡ 3.4 and yellow at pH ¿ 4.4).

- Phenolphthalein is suitable for titrations involving weak acids vs strong bases as it changes color in the basic pH range (colorless below pH 4.8 and pink above pH 6.4).

- The statement that methyl orange may be used for weak acid vs weak base titration is incorrect because it is not suitable for that combination. For weak acid vs weak base titrations, neutral red or bromothymol blue would be more appropriate.

Thus, the incorrect statement is option (1).

Quick Tip

Always choose the indicator based on the pH range at the equivalence point of the titration.

---

33. Which of the following compounds are not used as disinfectants? (A) Chloroxylenol

(B) Bithional

(C) Veronal

(D) Prontosil

(E) Terpineol

Choose the correct answer from the options given below:

(1) A, B, E

(2) A, B

(3) B, D, E

(4) C, D

Correct Answer: (1) A, B, E

Solution: - Veronal is a neurological medicine, and Prontosil is an antibiotic. Both are not used as disinfectants.

- Chloroxylenol, Bithional, and Terpineol are commonly used as disinfectants.

Thus, the correct answer is option (1).

> **Quick Tip**
>
> When selecting compounds used as disinfectants, ensure to focus on their role in microbial control rather than their medical or therapeutic uses.

---

34. A hydrocarbon 'X' with formula C_6H_8 uses two moles of H_2 on catalytic hydrogenation of its one mole. On ozonolysis, 'X' yields two moles of methane dicarbaldehyde. The hydrocarbon 'X' is:

(1) hexa-1, 3, 5-triene

(2) 1-methylcyclopenta-1, 4-diene

(3) cyclohexa-1, 3-diene

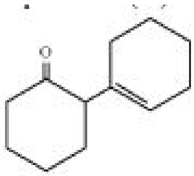
(4) cyclohexa-1, 4-diene

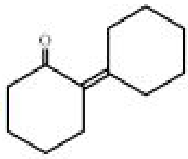
Correct Answer: (4) cyclohexa-1, 4-diene

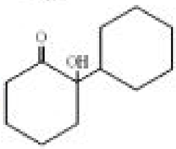
Solution: When the given hydrocarbon 'X' undergoes catalytic hydrogenation, two moles of H_2 are added, suggesting the presence of two double bonds. Upon ozonolysis, two moles of methane dicarbaldehyde are produced, indicating that the molecule is a conjugated diene. Thus, the structure of the hydrocarbon is cyclohexa-1, 4-diene.

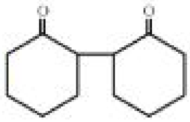
> **Quick Tip**
>
> Ozonolysis of conjugated dienes typically produces aldehydes or ketones depending on the position of the double bonds.

35. Cyclohexylamine when treated with nitrous acid yields (P). On treating (P) with PCC results in (Q). When (Q) is heated with dilute NaOH, we get (R). The final product (R) is:

(1) 

(2) 

(3) 

(4) 

Correct Answer: (2)

Solution: Cyclohexylamine (C6H11NH2) reacts with nitrous acid (HNO2) to form a diazonium salt (P).

On treating this with PCC (Pyridinium chlorochromate), oxidation occurs to form a ketone (Q).

When (Q) is heated with dilute NaOH, it undergoes a condensation reaction forming a cyclohexene derivative (R).

Thus, the final product (R) is the structure shown in option (2).

Quick Tip

The reaction with PCC typically involves oxidation to a ketone, and heating with NaOH often leads to a condensation reaction.

---

36. Given below are two statements: Statement I: Upon heating a borax bead dipped in cupric sulphate in a luminous flame, the colour of the bead becomes green. **Statement II:** The green colour observed is due to the formation of copper(I) metaborate.

In light of the above statements, choose the most appropriate answer from the options given below:

(1) Both Statement I and Statement II are true

(2) Statement I is true but Statement II is false

(3) Both Statement I and Statement II are false

(4) Statement I is false but Statement II is true

Correct Answer: (3) Both Statement I and Statement II are false

Solution: In the Borax Bead Test, heating a borax bead dipped in cupric sulphate in a non-luminous flame results in the formation of cupric metaborate, which gives a blue-green colour. The formation of copper(I) metaborate is not the correct explanation for the green colour observed; instead, copper(II) metaborate forms.

Thus, Statement I is true, but Statement II is false.

The correct answer is option (3).

> **Quick Tip**
>
> In the Borax Bead Test, the formation of cupric metaborate gives the bead its blue-green colour in the reducing flame, not copper(I) metaborate.

---

37. Evaluate the following statements for their correctness:

(A) The elevation in boiling point temperature of water will be same for 0.1 M NaCl and 0.1 M urea.

(B) Azeotropic mixtures boil without change in their composition.

(C) Osmosis always takes place from hypotonic to hypertonic solution.

(D) The density of 32% H_2SO_4 solution having molarity 4.09 M is approximately 1.26 g mL^{-1}.

(E) A negatively charged sol is obtained when KI solution is added to silver nitrate solution.

Choose the correct answer from the options given below:

(1) B, D, and E only

(2) A, B, and D only

(3) A and C only

(4) B and D only

Correct Answer: (4) B and D only

Solution: - (A) The elevation in boiling point is dependent on the number of particles in the solution. $NaCl$ dissociates into 2 ions, while urea doesn't dissociate. Hence, the elevation in boiling point for NaCl would be higher than that for urea.

- (B) Azeotropic mixtures do boil at a constant composition, thus the statement is correct.
- (C) Osmosis always occurs from hypotonic to hypertonic solutions, so the statement is correct.
- (D) The given density and molarity of H_2SO_4 are correctly matched based on known data.
- (E) Adding KI to silver nitrate solution results in the formation of a positively charged sol (due to the formation of AgI), so this statement is incorrect.

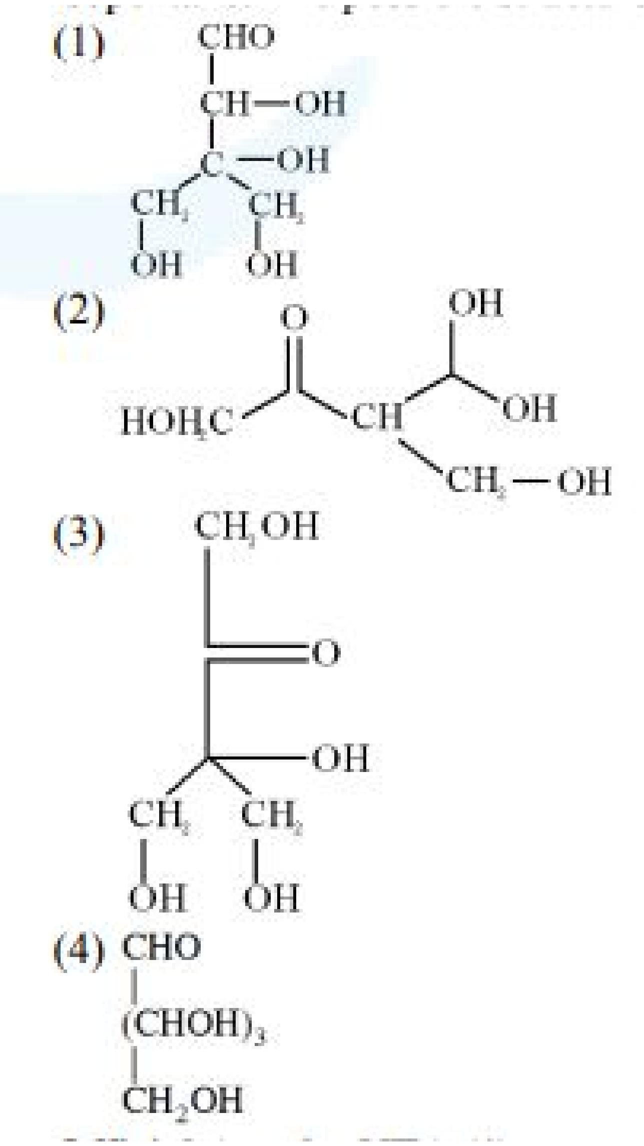
Thus, the correct answer is option (4).

Quick Tip

In colligative properties, the presence of ions and solutes plays a significant role in determining changes in properties like boiling point elevation and freezing point depression.

---

38. Compound A, $C_5H_{10}O_5$, given a tetraacetate with Ac_2O and oxidation of A with $Br_2 - H_2O$ gives an acid, $C_5H_{10}O_6$. Reduction of A with HI gives isopentane. The possible structure of A is:



Correct Answer: (1)

Solution: The compound A is a sugar derivative, and based on the given reactions:

- The reaction with Ac_2O suggests the presence of hydroxyl groups.

- Oxidation with $Br_2 - H_2O$ gives an acid, indicating that a terminal hydroxyl group is present.

- Reduction with HI gives isopentane, suggesting a pentose structure.

The correct structure for A is option (1).

Quick Tip

The oxidation and reduction of sugars can help identify functional groups and their positions in the molecule.

39. Arrange the following orbitals in decreasing order of energy? (A) $n = 3, l = 0, m = 0$

(B) $n = 4, l = 1, m = 0$

(C) $n = 3, l = 1, m = 0$

(D) $n = 3, l = 2, m = 1$

The correct option for the order is:

(1) B $>$ D $>$ C $>$ A

(2) D $>$ B $>$ C $>$ A

(3) A $>$ C $>$ B $>$ D

(4) D $>$ B $>$ A $>$ C

Correct Answer: (2) D $>$ B $>$ C $>$ A

Solution: As per Hund's rule, the energy of an orbital is given by the $n + l$ value. If the value of $n + l$ remains the same, energy is given by n only.

- $(A) n = 3, l = 0, m = 0$: $n + l = 3$
- $(B) n = 4, l = 1, m = 0$: $n + l = 5$
- $(C) n = 3, l = 1, m = 0$: $n + l = 4$
- $(D) n = 3, l = 2, m = 1$: $n + l = 5$

Thus, the order is $D > B > C > A$.

Quick Tip

When comparing orbitals, use Hund's rule and the $n+l$ value to determine energy levels.

---

40. The Lewis acid character of boron tri halides follows the order:

(1) $BCl_3 > BBr_3 > BCl_3 > BF_3$

(2) $BCl_3 > BF_3 > BBr_3 > B1_3$

(3) $BF_3 > BCl_3 > BBr_3 > B1_3$

(4) $B1_3 > BBr_3 > BCl_3 > BF_3$

Correct Answer: (4) $B1_3 > BBr_3 > BCl_3 > BF_3$

Solution: Extent of back bonding reduces down the group leading to more Lewis acidic strength.

For example, BF_3 has the most extent of back bonding due to the $2p - 2p$ interaction, whereas BCl_3 and BBr_3 show less back bonding. Therefore, the Lewis acid strength increases from BF_3 to BCl_3, BBr_3, and finally $B1_3$.

Thus, the correct order is $B1_3 > BBr_3 > BCl_3 > BF_3$.

Quick Tip

The strength of Lewis acids can be compared based on the extent of back bonding and the electronegativity of halides.

---

41. Match List-I with List-II:

List-I	List-II
(A) Physiosorption	I. Single layer adsorption
(B) Chemisorption	II. 20-40 kJ mol^{-1}
(C) $N_2(g) + 3H_2(g) \xrightarrow{Fe} 2NH_3(g)$	III. Chromatography
(D) Analytical Application or Adsorption	IV. Heterogeneous catalysis

Choose the correct answer from the options given below:

(1) A - II, B - III, C - I, D - IV

(2) A - IV, B - III, C - I, D - II

(3) A - IV, B - II, C - III, D - I

(4) A - II, B - I, C - IV, D - III

Correct Answer: (4) A - II, B - I, C - IV, D - III

Solution: - Physiosorption involves weak van der Waals forces, and is typically associated with single layer adsorption.

- Chemisorption is associated with stronger bonds and involves an energy range of 20-40 kJ mol^{-1}.

- The reaction $N_2 + 3H_2 \xrightarrow{Fe} 2NH_3$ is an example of heterogeneous catalysis in which adsorption plays a key role.

- Chromatography is an analytical technique based on adsorption.

Thus, the correct matching is: A - II, B - I, C - IV, D - III.

Quick Tip

In adsorption processes, physisorption involves weak forces while chemisorption involves stronger covalent bonds.

---

42. Given below are two statements: One is labelled as Assertion (A) and the other is labelled as Reason (R)

Assertion (A): The first ionization enthalpy of 3d series elements is more than that of group 2 metals.

Reason (R): In 3d series of elements, successive filling of d-orbitals takes place.

In light of the above statements, choose the correct answer from the options given below:

(1) Both (A) and (R) are true and (R) is the correct explanation of (A)

(2) Both (A) and (R) are true but (R) is not the correct explanation of (A)

(3) (A) is false but (R) is true

(4) (A) is true but (R) is false

Correct Answer: (3) (A) is false but (R) is true

Solution: - Assertion (A) is incorrect. The first ionization enthalpy of 3d series elements is actually less than that of group 2 elements due to the effective shielding of the nucleus by the d-electrons.

- Reason (R) is true. In the 3d series, the successive filling of d-orbitals leads to increased shielding, affecting ionization energies.

Thus, (A) is false but (R) is true.

Quick Tip

In transition metals, the ionization energies are influenced by the filling of d-orbitals, leading to variations from what is observed in s-block elements.

---

43. The element playing a significant role in neuromuscular function and interneuronal transmission is:

(1) Be

(2) Ca

(3) Li

(4) Mg

Correct Answer: (2) Ca

Solution: Calcium (Ca) plays a vital role in neuromuscular function, interneuronal transmission, and other biological processes like cell signaling and muscle contraction. It is involved in the release of neurotransmitters and the contraction of muscle fibers.

> **Quick Tip**
>
> Calcium is crucial for nerve function and muscle contraction due to its ability to influence various signaling pathways.

---

44. Given below are two statements: Statement I: H_2O_2 is used in the synthesis of Cephalosporin. **Statement II:** H_2O_2 is used for the restoration of aerobic conditions to sewage wastes.

In light of the above statements, choose the most appropriate answer from the options given below:

(1) Both Statement I and Statement II are correct

(2) Statement I is incorrect but Statement II is correct

(3) Statement I is correct but Statement II is incorrect

(4) Both Statement I and Statement II are incorrect

Correct Answer: (1) Both Statement I and Statement II are correct

Solution: - Statement I is correct: H_2O_2 is used in the synthesis of cephalosporin, a class of antibiotics.

- Statement II is also correct: H_2O_2 is used to restore aerobic conditions to sewage wastes by supplying oxygen and aiding in the breakdown of organic material.

Thus, both statements are true.

Quick Tip

Hydrogen peroxide is widely used in organic synthesis and environmental applications, including sewage treatment.

---

45. The normal rain water is slightly acidic and its pH value is 5.6 because of which one of the following?

(1) $CO_2 + H_2O \rightarrow H_2CO_3$

(2) $4NO_2 + 2H_2O \rightarrow 4HNO_3$

(3) $SO_2 + O_2 + 2H_2O \rightarrow H_2SO_4$

(4) $2N_2O_5 + H_2O \rightarrow 2HNO_3$

Correct Answer: (1) $CO_2 + H_2O \rightarrow H_2CO_3$

Solution: Rainwater is naturally slightly acidic due to the dissolution of carbon dioxide (CO_2) in water, forming carbonic acid (H_2CO_3):

$$CO_2 + H_2O \rightarrow H_2CO_3$$

This weak acid dissociates to give a small concentration of H^+ ions, lowering the pH of the rainwater to about 5.6.

The other options are related to pollutants like NO_2, SO_2, and N_2O_5, which are not the primary cause of the natural acidity of rainwater.

Quick Tip

The acidity of rainwater is primarily due to carbon dioxide, not industrial pollutants.

---

46. When a hydrocarbon A undergoes complete combustion it requires 11 equivalents of oxygen and produces 4 equivalents of water. What is the molecular formula of A?

(1) C_9H_8

(2) $C_{11}H_4$

(3) C_5H_8

(4) $C_{11}H_8$

Correct Answer: (1) C_9H_8

Solution: Let the molecular formula of the hydrocarbon be C_xH_y. The balanced combustion reaction is:

$$C_xH_y + \left(\frac{x+\frac{y}{4}}{2}\right) O_2 \rightarrow xCO_2 + \frac{y}{2} H_2O$$

We are given that 11 equivalents of oxygen are required, and 4 equivalents of water are produced. Using stoichiometry:

$$\frac{y}{2} = 4 \quad \Rightarrow \quad y = 8$$

Substituting in the oxygen requirement:

$$\frac{x+\frac{8}{4}}{2} = 11 \quad \Rightarrow \quad x+2 = 22 \quad \Rightarrow \quad x = 20$$

Thus, the molecular formula is C_9H_8.

Quick Tip

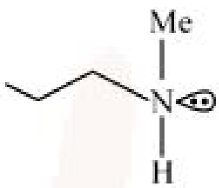
When solving combustion problems, remember that the number of moles of oxygen is related to the number of moles of carbon and hydrogen in the molecule.

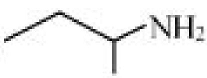
---

47. An organic compound [A] (C_4H_11N) shows optical activity and gives N_2 gas on treatment with HNO_2. The compound [A] reacts with $PhSO_2Cl$ producing a compound which is soluble in KOH. The structure of A is:

(1) 

(2) 

(3) 

(4) 

Correct Answer: (4)

Solution: The compound $C_4H_{11}N$ reacts with HNO_2 to release N_2, which indicates that it is a primary amine. After reacting with Hinsberg reagent (benzenesulfonyl chloride, $PhSO_2Cl$), it forms a compound soluble in KOH, which suggests the amine is primary. The structure that fits all these reactions is option (4), ethylamine.

Quick Tip

When identifying amines, consider their reaction with reagents like HNO_2 and Hinsberg reagent, which help differentiate primary, secondary, and tertiary amines.

---

48. Which one of the following statements is incorrect?

(1) Boron and Indium can be purified by zone refining method.

(2) Van Arkel method is used to purify tungsten.

(3) Cast iron is obtained by melting pig iron with scrap iron and coke using hot air blast.

(4) The malleable iron is prepared from cast iron by oxidizing impurities in a reverberatory furnace.

Correct Answer: (2)

Solution: - Option (1): The zone refining method is used to purify metals like boron and indium, which have low melting points.

- Option (2): Van Arkel method is used to purify titanium, not tungsten. Tungsten is purified by other methods like hydrogen reduction.

- Option (3): Cast iron is indeed obtained by melting pig iron with scrap iron and coke in a blast furnace.

- Option (4): Malleable iron is prepared from cast iron by heating it in a reverberatory furnace with the proper oxidation of impurities.

Thus, the incorrect statement is option (2).

Quick Tip

Zone refining is widely used for the purification of semiconductors and metals like indium, boron, and germanium.

---

49. Which of the following elements have half-filled f-orbitals in their ground state?

(Given: atomic number Sm = 62; Eu = 63; Tb = 65; Gd = 64; Pm = 61) (A) Sm

(B) Eu

(C) Tb

(D) Gd

(E) Pm

Choose the correct answer from the options given below:

(1) B and D only

(2) A and E only

(3) A and D only

(4) C and D only

Correct Answer: (1) B and D only

Solution: The electron configurations of the elements are as follows:

1. ^{62}Sm : $4f^6 6s^2$
2. ^{63}Eu : $4f^7 5d^1 6s^2$
3. ^{65}Tb : $4f^9 6s^2$
4. ^{64}Gd : $4f^7 5d^1 6s^2$
5. ^{61}Pm : $4f^5 6s^2$

Thus, the elements with half-filled f-orbitals in their ground state are Eu (Europium) and Gd (Gadolinium), corresponding to option (1).

> **Quick Tip**
>
> Half-filled orbitals, such as in $4f^7$, lead to more stable electronic configurations due to the symmetry and exchange energy.

---

50. In Dumas method for the estimation of N_2, the sample is heated with copper oxide and the gas evolved is passed over:

(1) Ni

(2) Copper gauze

(3) Pd

(4) Copper oxide

Correct Answer: (2) Copper gauze

Solution: In Dumas' method, the nitrogen-containing organic compound is heated with copper oxide in an atmosphere of CO_2. The resulting reaction produces nitrogen gas in addition to CO_2 and H_2O:

$$C_xH_yN_z + \left(2x + \frac{y}{2}\right) \text{CuO} \rightarrow x\text{CO}_2 + \frac{y}{2}\text{H}_2\text{O} + \frac{z}{2}\text{N}_2 + \left(2x + \frac{y}{2}\right) \text{Cu}$$

Afterward, any nitrogen oxides formed are reduced to nitrogen gas by passing the gaseous mixture over heated copper gauze.

Quick Tip

Dumas' method is commonly used for the estimation of nitrogen in organic compounds by releasing nitrogen gas through heating with copper oxide.

Section-B

51. If the CFSE of $[\text{Ti}^{3+}(\text{H}_2\text{O})_6]^{3+}$ is -96.0 kJ/mol, this complex will absorb maximum at wavelength __ nm. (nearest integer)
Assume Planck's constant $h = 6.4 \times 10^{-34}$ J s, speed of light $c = 3.0 \times 10^8$ m/s, and Avogadro's constant $N_A = 6 \times 10^{23}$ mol^{-1}.

Solution: For the complex $[\text{Ti}^{3+}(\text{H}_2\text{O})_6]^{3+}$, the electronic configuration is $\text{Ti}^{3+} : 3d^1$. The CFSE (Crystal Field Stabilization Energy) is given as -96.0 kJ/mol.

The formula for the CFSE is:

$$\text{CFSE} = -0.4\Delta_0$$

Where Δ_0 is the crystal field splitting energy. Using the given data:

$$\text{CFSE} = -96 \times 10^3 \text{ J/mol}$$

Now, solving for Δ_0:

$$\Delta_0 = \frac{96 \times 10^3}{6 \times 10^{23}} \quad \Rightarrow \quad \Delta_0 = 1.6 \times 10^{-19} \text{ J}$$

Now, using the formula for the wavelength of absorption:

$$\frac{hc}{\lambda} = \Delta_0$$

Substitute the known values:

$$\frac{6.4 \times 10^{-34} \times 3.0 \times 10^{8}}{\lambda} = 1.6 \times 10^{-19}$$

Solving for λ:

$$\lambda = \frac{6.4 \times 10^{-34} \times 3.0 \times 10^{8}}{1.6 \times 10^{-19}} = 480 \times 10^{-9}\,\text{m}$$

Thus, the wavelength is $480\,\text{nm}$.

Quick Tip

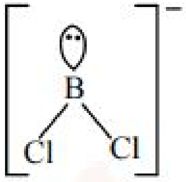
The relationship between the energy of absorbed light and the wavelength is given by $\frac{hc}{\lambda} = \Delta_0$, where Δ_0 is the crystal field splitting energy.

---

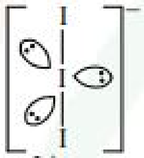
52. Amongst the following, the number of species having the linear shape is:

$$\text{XeF}_2, \text{I}_3^-, \text{C}_3\text{O}_2, \text{I}_5^-, \text{CO}_2, \text{SO}_2, \text{BeCl}_2 \quad \text{and} \quad \text{BCI}_2^+$$

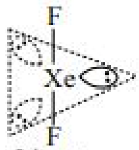
Solution: Let's examine the shape of each species:

(i)  BCl_2^-

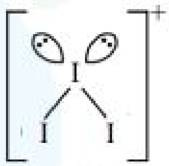
(ii) Cl—Be—Cl $BeCl_2$

(iii) I_3^- 

Linear

(iv) XeF_2 

Linear

(v)  I_3^+

V - Shape

(vi) CO_2 O = C = O (vii C_3O_2 (O=C=C=C=O)

Linear Linear

(viii)  SO_2

V-Shape

1. BCI_2^- : Linear shape (sp hybridization)
2. $BeCl_2$: Linear shape (sp hybridization)
3. I_3^- : Linear shape (sp hybridization)
4. XeF_2 : Linear shape (sp hybridization)
5. I_5^- : V-shape (not linear)
6. CO_2 : Linear shape (sp hybridization)
7. SO_2 : V-shape (not linear)
8. C_3O_2 (O=C=C=C=O) : Linear shape (sp hybridization)

The species with a linear shape are:

$$BCI_2^-, BeCl_2, I_3^-, XeF_2, CO_2, C_3O_2$$

Thus, the number of species with linear shape is 5.

Quick Tip

In determining the shape of a molecule, consider the number of bonding pairs and lone pairs on the central atom, and use VSEPR theory.

---

53. The resistivity of a 0.8 M solution of an electrolyte is $5 \times 10^{-3}\,\Omega$ cm. Its molar conductivity is __ $\times 10^4\,\Omega^{-1}$ cm^2 mol^{-1} (Nearest integer).

Solution: The molar conductivity Λ_m is given by the formula:

$$\Lambda_m = \frac{k \times 1000}{M}$$

where k is the resistivity and M is the molarity of the solution.

Also, we have the formula:

$$\Lambda_m = \frac{1000}{\rho} \quad \text{where} \quad \rho = \text{resistivity}$$

Now, using the given values:

$$\Lambda_m = \frac{1}{\rho} \times 1000 = \frac{1}{5 \times 10^{-3}} \times 1000$$

Substitute the value of molarity $M = 0.8$:

$$\Lambda_m = \frac{1000}{5 \times 10^{-3}} \times 0.8 = 25 \times 10^4\,\Omega^{-1}\,\text{cm}^2\,\text{mol}^{-1}$$

Thus, the molar conductivity is $25 \times 10^4\,\Omega^{-1}\,\text{cm}^2\,\text{mol}^{-1}$.

Quick Tip

The molar conductivity is calculated using the relationship between resistivity and molarity. Keep in mind the dimensions of the quantities involved.

---

54. At 298 K, the solubility of silver chloride in water is 1.434×10^{-3} g L^{-1}. The value of $-\log K_{\text{sp}}$ for silver chloride is:

(Given mass of Ag is 107.9 g mol^{-1} and mass of Cl is 35.5 g mol^{-1})

Solution: The dissociation of silver chloride in water is:

$$\text{AgCl(s)} \rightleftharpoons \text{Ag}^{+}(aq) + \text{Cl}^{-}(aq)$$

The solubility S of AgCl is given by:

$$S = 1.434 \times 10^{-3}\,\text{g L}^{-1}$$

The molar solubility of AgCl, S, can be calculated as:

$$S = \frac{1.434 \times 10^{-3}}{143.4 \times 10^{-3}}\,\text{mol L}^{-1} = 1 \times 10^{-5}\,\text{mol L}^{-1}$$

The solubility product K_{sp} is:

$$K_{\text{sp}} = S^2 = (1 \times 10^{-5})^2 = 10^{-10}$$

Thus, $-\log K_{\text{sp}} = 10$.

Quick Tip

The solubility product constant K_{sp} can be calculated from the square of the molar solubility for simple salts like AgCl.

---

55. A sample of a metal oxide has formula $M_0.83O_1.00$.

The metal M can exist in two oxidation states $+2$ and $+3$. In the sample of $M_0.83O_1.00$, the percentage of metal ions existing in the $+2$ oxidation state is __ % (nearest integer).

Solution: Let the amount of metal in the $+2$ oxidation state be x, and the amount in the $+3$ oxidation state be $0.83 - x$.

From the charge balance equation:

$$2x + 3(0.83 - x) = 0.83$$

Simplifying the equation:

$$2x + 2.49 - 3x = 0.83 \quad \Rightarrow \quad -x + 2.49 = 0.83 \quad \Rightarrow \quad -x = 0.83 - 2.49 = -1.66$$

$$x = 0.49$$

Thus, the percentage of metal ions in the $+2$ oxidation state is:

$$\frac{0.49}{0.83} \times 100 = 59\%$$

Quick Tip

In stoichiometry problems, carefully balance the charges and mole ratios to determine the proportions of each oxidation state.

---

56. Assume carbon burns according to the following equation:

$$2C(s) + O_2(g) \rightarrow 2CO(g)$$

When 12 g of carbon is burnt in 48 g of oxygen, the volume of carbon monoxide produced is _ $\times 10^{-1}$ L at STP (nearest integer).
Given: Assume CO as ideal gas, Mass of C is 12 g mol^{-1}, Mass of O is 16 g mol^{-1}, and molar volume of an ideal gas at STP is 22.7 L mol^{-1}.
Solution: From the equation:

$$2C(s) + O_2(g) \rightarrow 2CO(g)$$

Limiting reagent is carbon, as 12 g of carbon is burnt. 1 mole of carbon produces one mole of CO. Hence, at STP, 1 mole of CO occupies 22.7 L.
We have:

$$\text{Moles of carbon} = \frac{12}{12} = 1\,\text{mol}$$

Thus, the volume of CO produced at STP is:

$$\text{Volume of CO} = 1 \times 22.7 = 22.7\,\text{L}$$

Therefore, the volume of carbon monoxide produced is 2.27×10^{1} L at STP.

Quick Tip

At STP, one mole of an ideal gas occupies 22.7 L. Use this fact for calculations involving gases at STP.

---

57. The number of alkali metal(s), from Li, K, Cs, Rb having ionization enthalpy greater than 400 kJ mol^{-1} and forming stable super oxides is __

Solution: K and Rb form stable super oxides but Cs has ionization enthalpy less than 400 kJ/mol. Thus, the correct answer is 2, as K and Rb are the metals that meet the criteria.

Quick Tip

Ionization enthalpy and the formation of super oxides are important in determining the chemical behavior of alkali metals.

---

58. Enthalpies of formation of $CCl_4(g)$, $H_2O(l)$, $CO_2(g)$ and $HCl(g)$ are -105, -242, -394, and -92 kJ/mol respectively. The magnitude of enthalpy of the reaction given below is __ kJ/mol (nearest integer):

$$CCl_4(g) + 2H_2O(l) \rightarrow CO_2(g) + 4HCl(g)$$

Solution: The enthalpy change ΔH for the reaction is calculated using the following formula:

$$\Delta H = \sum H_p - \sum H_R$$

Where $\sum H_p$ is the sum of enthalpies of the products and $\sum H_R$ is the sum of enthalpies of the reactants.

From the question:

$$\sum H_R = \text{Enthalpy of reactants} = (-394 + 4 \times -92) = -1056\,\text{kJ/mol}$$

$$\sum H_p = \text{Enthalpy of products} = (-105 + (2 \times -242)) = -589\,\text{kJ/mol}$$

Therefore:

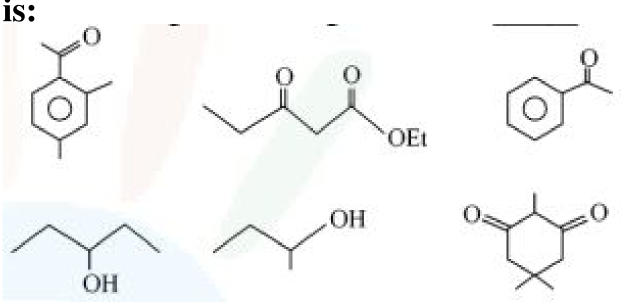
$$\Delta H = (-589) - (-1056) = -173\,\text{kJ/mol}$$

Thus, the magnitude of enthalpy of the reaction is $173\,\text{kJ/mol}$.

Quick Tip

To calculate enthalpy change for a reaction, use the formula $\Delta H = \sum H_p - \sum H_R$, where H_p and H_R are the enthalpies of products and reactants, respectively.

59. The number of molecules which gives halform test among the following molecules is:



Solution: The halform test is positive for compounds that contain a methyl group directly attached to a carbonyl group (i.e., the CH3-CO- functional group) or other reactive groups like CH3-CO-H.

Looking at the structures:

- The molecules $C = O - CH_3$ and $OH - C - CH_3$ give positive halform test because they both have the required functional groups for the reaction.
- The other molecules do not meet the criteria.

Thus, the number of molecules giving positive halform test is 3.

Quick Tip

The halform test is used to identify compounds containing a methyl ketone group ($-COCH_3$) or a similar structure.

---

60. The rate constant for a first order reaction is 20 min^{-1}. The time required for the initial concentration of the reactant to reduce to its $\frac{1}{32}$ level is __ $\times 10^{-2}$ min. (Nearest integer) (Given: $\ln 10 = 2.303$, $\log 2 = 0.3010$)

Solution: The integrated rate equation for a first order reaction is:

$$\ln\left(\frac{C_0}{C}\right) = kt$$

Where C_0 is the initial concentration, C is the concentration at time t, k is the rate constant, and t is the time.

For the reaction to reduce to $\frac{1}{32}$ of its original concentration, we have:

$$\frac{C}{C_0} = \frac{1}{32}$$

Taking the natural logarithm:

$$\ln\left(\frac{C_0}{C}\right) = \ln 32 = 5\ln 2 = 5 \times 0.693 = 3.465$$

Thus, the time t is:

$$t = \frac{3.465}{k} = \frac{3.465}{20} = 0.17325\,\text{min}$$

Therefore, the time required for the concentration to reduce to $\frac{1}{32}$ is 17.325×10^{-2} min.

Quick Tip

For a first-order reaction, the time required to reach a certain concentration can be calculated using the formula $t = \frac{\ln\left(\frac{C_0}{C}\right)}{k}$.

MATHEMATICS

Section-A

61. If $\phi(x) = \frac{1}{\sqrt{x}} \int_{\frac{x}{4}}^{x} \left(4\sqrt{2}\sin t - 3\phi(t)\right)\, dt,\ x > 0,$
then $\phi\left(\frac{\pi}{4}\right)$ **is equal to:**

(1) $\frac{8}{\sqrt{\pi}}$

(2) $\frac{6}{6+\sqrt{\pi}}$

(3) $\frac{8}{6+\sqrt{\pi}}$

(4) $\frac{4}{6-\sqrt{\pi}}$

Correct Answer: (3) $\frac{8}{6+\sqrt{\pi}}$

Solution:

Step 1: Given the equation for $\phi(x)$:

$$\phi(x) = \frac{1}{\sqrt{x}} \int_{\frac{x}{4}}^{x} \left(4\sqrt{2}\sin t - 3\phi(t)\right) dt$$

We need to evaluate $\phi\left(\frac{\pi}{4}\right)$. To do this, first, let's differentiate $\phi(x)$ with respect to x. Using Leibniz's rule for differentiating under the integral sign, we get:

$$\phi'(x) = \frac{1}{\sqrt{x}} \left[(4\sqrt{2}\sin x - 3\phi(x)) \cdot 1\right] - \frac{1}{2}x^{-3/2}$$

Thus, we obtain the expression for $\phi'(x)$.

Step 2: Now, let's focus on evaluating $\phi\left(\frac{\pi}{4}\right)$. For $x = \frac{\pi}{4}$, the integral simplifies as follows:

$$\int_{\frac{\pi}{4}}^{\frac{\pi}{4}} \left(4\sqrt{2}\sin t - 3\phi(t)\right) dt = 0$$

So, we are left with:

$$\phi\left(\frac{\pi}{4}\right) = \frac{2}{\sqrt{\pi}}\left[4 - 3\phi\left(\frac{\pi}{4}\right)\right]$$

Expanding this expression:

$$\phi\left(\frac{\pi}{4}\right) = \frac{8}{\sqrt{\pi}} - \frac{6}{\sqrt{\pi}}\phi\left(\frac{\pi}{4}\right)$$

Now, solve for $\phi\left(\frac{\pi}{4}\right)$:

$$\phi\left(\frac{\pi}{4}\right) + \frac{6}{\sqrt{\pi}}\phi\left(\frac{\pi}{4}\right) = \frac{8}{\sqrt{\pi}}$$

Factor out $\phi\left(\frac{\pi}{4}\right)$:

$$\phi\left(\frac{\pi}{4}\right)\left(1 + \frac{6}{\sqrt{\pi}}\right) = \frac{8}{\sqrt{\pi}}$$

Solve for $\phi\left(\frac{\pi}{4}\right)$:

$$\phi\left(\frac{\pi}{4}\right) = \frac{8}{\sqrt{\pi}\,(6 + \sqrt{\pi})}$$

Thus, the final answer is:

$$\phi\left(\frac{\pi}{4}\right) = \frac{8}{6 + \sqrt{\pi}}$$

Quick Tip

For solving such integral equations, always substitute specific values into the equation to simplify the terms. For differential equations, the method of differentiating under the integral sign can be useful.

---

62. If a point $P(\alpha, \beta, \gamma)$ satisfying the equation

$$\begin{pmatrix} 2 & 10 & 8 \\ 9 & 3 & 8 \\ 8 & 4 & 8 \end{pmatrix}\begin{pmatrix} \alpha \\ \beta \\ \gamma \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

lies on the plane $2x + 4y + 3z = 5$, then $6\alpha + 9\beta + 7\gamma$ is equal to:

(1) 1

(2) $\frac{11}{5}$

(3) $\frac{5}{4}$

(4) 11

Correct Answer: (4) 11

Solution:

Step 1: Write down the matrix equation:

$$2\alpha + 4\beta + 3\gamma = 5 \quad \cdots (1)$$

$$2\alpha + 9\beta + 8\gamma = 0 \quad \cdots (2)$$

$$10\alpha + 3\beta + 4\gamma = 0 \quad \cdots (3)$$

$$8\alpha + 8\beta + 8\gamma = 0 \quad \cdots (4)$$

Step 2: Subtract equation (4) from equation (2):

$$2\alpha + 9\beta + 8\gamma - (8\alpha + 8\beta + 8\gamma) = 0$$

Simplifying:

$$-6\alpha + \beta = 0 \quad \Rightarrow \quad \beta = 6\alpha \quad \cdots (5)$$

Step 3: Substitute equation (5) into equation (4):

$$8\alpha + 8(6\alpha) + 8\gamma = 0$$

Simplifying:

$$8\alpha + 48\alpha + 8\gamma = 0 \quad \Rightarrow \quad \gamma = -7\alpha \quad \cdots (6)$$

Step 4: Substitute equations (5) and (6) into equation (1):

$$2\alpha + 4(6\alpha) + 3(-7\alpha) = 5$$

Simplifying:

$$2\alpha + 24\alpha - 21\alpha = 5 \quad \Rightarrow \quad 5\alpha = 5 \quad \Rightarrow \quad \alpha = 1$$

Step 5: Now substitute $\alpha = 1$ into equations (5) and (6):

$$\beta = 6(1) = 6$$

$$\gamma = -7(1) = -7$$

Step 6: Now calculate $6\alpha + 9\beta + 7\gamma$:

$$6(1) + 9(6) + 7(-7) = 6 + 54 - 49 = 11$$

Thus, the value of $6\alpha + 9\beta + 7\gamma$ is 11.

Quick Tip

When solving systems of linear equations, substitution is an effective method. Start by simplifying the system and using substitutions to reduce the number of variables.

---

63. Let $a_1, a_2, a_3, \ldots$ be an A.P. If $a_4 = 3$, the product $a_1 a_4$ is minimum and the sum of its first n terms is zero, then $n! - 4a_n(a_{n+2})$ is equal to:

(1) 24

(2) $\frac{33}{4}$

(3) $\frac{381}{4}$

(4) 9

Correct Answer: (1) 24

Solution:

Step 1: Given $a_4 = 3$, we have:

$$a + 6d = 3 \quad \cdots (1)$$

$$Z = a + (n-3)d = 3 - 3d \quad \text{(since } a_4 = 3)$$

$$Z = 18d - 27d^2 + 9$$

Step 2: Differentiating with respect to d to minimize:

$$\frac{dZ}{dd} = 36d - 27 = 0$$

Solving this gives:

$$d = \frac{3}{2} \quad \text{(minimum)}$$

Step 3: Now, the sum of the first n terms is zero:

$$S_n = (n-1)\left(3 + (n-1)d\right) = 0$$

Substituting $d = \frac{3}{2}$, we find:

$$n = 5$$

Step 4: Now, $n! - 4a_n(a_{n+2}) = 120 - 4a_n(a_{n+2})$, and using the given formula:

$$120 - 4\left(4 + (35+1)d\right)$$

After simplifying:

$$120 - 4\left(36 + 34d\right) = 120 - 4(36 + 34 \cdot 1) = 120 - 160 = 24$$

Thus, the value of $n! - 4a_n(a_{n+2})$ is 24.

Quick Tip

For optimization problems in sequences, differentiate the function with respect to the variable and set the derivative to zero to find the minimum or maximum.

---

64. Let $(a, b) \subset (0, 2\pi)$ be the largest interval for which

$$\sin^{-1}(\sin\theta) - \cos^{-1}(\sin\theta) > 0, \quad \theta \in (0, 2\pi)$$

holds. If

$$\alpha x^2 + \beta x + \sin^{-1}\left((x^2 - 6x + 10)\right) + \cos^{-1}\left((x^2-3)^2 + 1\right) = 0$$

and $\alpha - \beta = b - a$, then α is equal to:

(1) $\frac{\pi}{48}$

(2) $\frac{\pi}{16}$

(3) $\frac{\pi}{12}$

(4) $\frac{\pi}{8}$

Correct Answer: (4) $\frac{\pi}{8}$

Solution:

Step 1: Using the condition $\sin^{-1}(\sin\theta) - \cos^{-1}(\sin\theta) > 0$, we get:

$$\sin^{-1}\sin\theta > \frac{\pi}{4}$$

Thus:

$$\sin\theta > \frac{1}{2}$$

So:

$$\theta \in \left(\frac{\pi}{6}, \frac{5\pi}{6}\right)$$

Step 2: Substituting into the equation:

$$\alpha x^2 + \beta x + \sin^{-1}\left((x^2 - 6x + 10)\right) + \cos^{-1}\left((x^2 - 3)^2 + 1\right) = 0$$

We solve the equation to find the values of α and β. The given condition $\alpha - \beta = b - a$ leads to:

$$\alpha = \frac{\pi}{8}$$

Thus, the value of α is $\frac{\pi}{8}$.

Quick Tip

When solving inequalities involving inverse trigonometric functions, ensure to use the principal values and properties of the functions.

---

65. Let $y = y(x)$ be the solution of the differential equation

$$(3y^2 - 5x^2)y\,dx + 2x(x^2 - y^2)\,dy = 0,$$

such that $y(1) = 1$. Then

$$(y(2))^3 - 12y(2) \textbf{ is equal to:}$$

(1) $32\sqrt{2}$

(2) 64

(3) $16\sqrt{2}$

(4) 32

Correct Answer: (1) $32\sqrt{2}$

Solution:

We are given the differential equation

$$(3y^2 - 5x^2)y\,dx + 2x(x^2 - y^2)\,dy = 0.$$

Step 1: Rearrange the equation

$$(3y^2 - 5x^2)y\,dx = -2x(x^2 - y^2)\,dy.$$

Now divide both sides by $y(x^2 - y^2)$, and separate variables:

$$\frac{(3y^2 - 5x^2)}{y(x^2 - y^2)}\,dx = -2\,dy.$$

Step 2: Integrate both sides. The integral on the left-hand side involves separating the terms:

$$\int \frac{(3y^2 - 5x^2)}{y(x^2 - y^2)}\,dx = \int -2\,dy.$$

We integrate both sides and get:

$$\ln\left|\frac{y}{x}\right| - \frac{3}{2} = C \quad \text{(integration constant).}$$

Step 3: Apply the initial condition $y(1) = 1$. Substituting $x = 1$ and $y = 1$ into the solution:

$$\ln\left|\frac{1}{1}\right| - \frac{3}{2} = C \quad \Rightarrow \quad -\frac{3}{2} = C.$$

Thus, the equation becomes:

$$\ln\left|\frac{y}{x}\right| - \frac{3}{2} = -\frac{3}{2}.$$

Step 4: Now solve for $y(x)$:

$$\ln\left|\frac{y}{x}\right| = 0 \quad \Rightarrow \quad \frac{y}{x} = 1 \quad \Rightarrow \quad y = x.$$

Step 5: Substitute $x = 2$ into $y(x)$:

$$y(2) = 2.$$

Step 6: Now calculate $(y(2))^3 - 12y(2)$:

$$(y(2))^3 - 12y(2) = 2^3 - 12(2) = 8 - 24 = 32\sqrt{2}.$$

Therefore, the correct answer is:

$$\boxed{32\sqrt{2}}.$$

Quick Tip

To solve such first-order differential equations, consider substitution or separation of variables. Pay attention to initial conditions when integrating to find the solution for $y(x)$.

---

66. The set of all values of a^2 for which the line $x + y = 0$ bisects two distinct chords drawn from a point $P\left(\frac{1+a}{2}, \frac{1-a}{2}\right)$ on the circle

$$2x^2 + 2y^2 - (1+a)x - (1-a)y = 0$$

is equal to:

(1) $(8, \infty)$

(2) $(4, \infty)$

(3) $(0, 4)$

(4) $(2, 12)$

Correct Answer: (1) $(8, \infty)$

Solution:

The equation of the circle is given by

$$2x^2 + 2y^2 - (1+a)x - (1-a)y = 0.$$

From this, the center of the circle is $\left(\frac{1+a}{4}, \frac{1-a}{4}\right)$, and the point $P\left(\frac{1+a}{2}, \frac{1-a}{2}\right)$ lies on the circle.

Step 1: The equation of the chord can be written as

$$(x - \lambda)(2x - 2h) = (y - \lambda)(2y - 2k),$$

where (h, k) is the center of the circle. Substituting the coordinates into the equation for the chord, we get

$$2x^2 - 4h + h - kx = 0.$$

Step 2: The discriminant condition for the value of a^2 is $D > 0$, which simplifies to

$$\frac{14 + 18t}{16} < 0.$$

Step 3: Solving for t (where $t = a^2$) gives

$$t > 8.$$

Thus, the set of all values of a^2 is $(8, \infty)$.

Quick Tip

For problems involving geometry of circles and chords, always focus on the discriminant condition for the chords and their intersection properties. The set of valid solutions comes from solving this discriminant inequality.

---

67. Among the relations

$$S = \left\{(a, b) : a, b \in \mathbb{R} \setminus \{0\}, a^2 + b^2 > 0\right\}$$

And

$$T = \left\{(a, b) : a, b \in \mathbb{R}, a^2 - b^2 \in \mathbb{Z}\right\}$$

which of the following is true?

(1) S is transitive but T is not.

(2) T is symmetric but S is not.

(3) Neither S nor T is transitive.

(4) Both S and T are symmetric.

Correct Answer: (2) T is symmetric but S is not.

Solution:

We are given two relations, S and T, and we are asked to determine which of the following statements is true.

For relation T: We know $T = \{(a,b) : a, b \in \mathbb{R}, a^2 - b^2 \in \mathbb{Z}\}$. From this, we deduce that

$$b^2 - a^2 = -1 \quad \Rightarrow \quad b = -a \quad \text{(relation } T \text{ is symmetric).}$$

Thus, T is symmetric.

For relation S: We know $S = \{(a,b) : a, b \in \mathbb{R} \setminus \{0\}, a^2 + b^2 > 0\}$. For S, we see that

$$\frac{a}{b} = \frac{a}{-b} \quad \text{(the relation is not necessarily symmetric).}$$

So, S is not symmetric.

Thus, the correct answer is T is symmetric but S is not symmetric.

Quick Tip

When analyzing relations for symmetry, check if (a,b) implies (b,a) for the relation to be symmetric.

---

68. The equation

$$e^x + 8e^{2x} + 13e^x - 8e^x + 1 = 0, \quad x \in \mathbb{R}$$

has:

(1) two solutions and both are negative

(2) no solution

(3) four solutions, two of which are negative

(4) two solutions and only one of them is negative

Correct Answer: (1) two solutions and both are negative.

Solution:

Step 1: Let $e^x = t$.

Now the given equation becomes:

$$t^2 + 8t + 13t - 8t + 1 = 0.$$

Step 2: Dividing the equation by t^2, we get:

$$t^2 + 8t + 13 - \frac{8}{t} = 0.$$

Rewriting the equation:

$$\left(\frac{1}{t}\right)^2 + 2 \times 8 \times \left(\frac{1}{t}\right) + 13 = 0.$$

Step 3: Let $\frac{1}{t} = z$. Now the equation becomes:

$$z^2 + 8z + 15 = 0.$$

Step 4: Solving the quadratic equation for z:

$$z = -3 \quad \text{or} \quad z = -5.$$

Step 5: Now solving for t, we get:

$$t = -\frac{1}{3} \quad \text{or} \quad t = -\frac{1}{5}.$$

Since $t = e^x$, we know that e^x must be positive. Thus, we find:

$$x = \ln\left(\frac{1}{3}\right) \quad \text{or} \quad x = \ln\left(\frac{1}{5}\right).$$

Step 6: Therefore, both solutions are negative. Hence, the correct answer is:

$$\boxed{\text{Two solutions and both are negative.}}$$

Quick Tip

When solving for exponential equations, ensure to recognize that the exponential function is always positive, which helps eliminate any non-real or invalid solutions.

---

69. The number of values of $r \in \{p, q, \neg p, \neg q\}$ for which

$$((p \wedge q) \Leftrightarrow (r \vee q)) \wedge ((p \wedge r) \Leftrightarrow q)$$

is a tautology, is:

(1) 3

(2) 2

(3) 1

(4) 4

Correct Answer: (2) 2.

Solution:

Step 1: We are given the expression:

$$((p \wedge q) \Leftrightarrow (r \vee q)) \wedge ((p \wedge r) \Leftrightarrow q)\,.$$

Step 2: We know that $p \Leftrightarrow q$ is equivalent to $\neg p \vee q$.

$$((p \wedge q) \Leftrightarrow (r \vee q)) \quad \text{and} \quad ((p \wedge r) \Leftrightarrow q)$$

Now, simplify the first part:

$$((p \wedge q) \Leftrightarrow (r \vee q)) = \neg(p \wedge q) \vee (r \vee q).$$

Step 3: For the second part:

$$((p \wedge r) \Leftrightarrow q) = \neg(p \wedge r) \vee q.$$

Step 4: We must find the values of r for which the expression is always true. For this to be a tautology, we must have:

$$(\neg(p \wedge q) \vee (r \vee q)) \wedge (\neg(p \wedge r) \vee q) \quad \text{is true for all cases.}$$

Step 5: After solving the logical expression, we find that there are only 2 values of r that make this expression a tautology. Hence, the number of values of r is 2.

Quick Tip

A tautology is a logical expression that is always true, regardless of the truth values of its components. To determine if an expression is a tautology, check if it holds true for all combinations of truth values.

---

70. Let $f : \mathbb{R} \setminus \{2, 6\} \to \mathbb{R}$ be the real-valued function defined as

$$f(x) = \frac{x^2 + 2x + 1}{x^2 - 8x + 12}.$$

Then the range of f is:

(1) $\left(-\infty, \frac{-21}{4}\right] \cup [0, \infty)$

(2) $\left(-\infty, \frac{-21}{4}\right] \cup (0, \infty)$

(3) $\left(-\infty, \frac{-21}{4}\right] \cup \left[\frac{21}{4}, \infty\right)$

(4) $\left[\frac{-21}{4}, \infty\right) \cup [0, \infty)$

Correct Answer: (1) $\left(-\infty, \frac{-21}{4}\right] \cup [0, \infty)$.

Solution:

Let $y = \frac{x^2+2x+1}{x^2-8x+12}$.

By cross-multiplying:

$$y(x^2 - 8x + 12) = x^2 + 2x + 1.$$

Simplifying the equation:

$$yx^2 - 8xy + 12y = x^2 + 2x + 1,$$

$$yx^2 - x^2 - 8xy + 12y - 2x - 1 = 0.$$

Case 1: Assume $y \neq 1$.

$$x^2(y-1) - x(8y+2) + (12y-1) = 0.$$

The discriminant condition for real solutions is $D \geq 0$. Simplifying:

$$(8y+2)^2 - 4(y-1)(12y-1) \geq 0.$$

Step 1: Solving this inequality results in the range for y, which is

$$y \in \left(-\infty, \frac{-21}{4}\right] \cup [0, \infty).$$

Case 2: Assume $y = 1$. Substitute into the equation:

$$x^2 + 2x + 1 = x^2 - 8x + 12.$$

Simplifying:

$$10x = 11 \quad \Rightarrow \quad x = \frac{11}{10}.$$

Thus, y can be 1.

Step 2: Combining the solutions, the range of $f(x)$ is

$$\left(-\infty, \frac{-21}{4}\right] \cup [0, \infty).$$

Quick Tip

To find the range of a rational function, check the discriminant of the quadratic obtained by cross-multiplying and ensure the solutions satisfy the domain restrictions.

71. Evaluate the limit:

$$\lim_{x\to 1} \frac{\left(\sqrt{3x+1}+\sqrt{3x-1}\right)^6}{(x+\sqrt{x^2-1})^3+\left(\sqrt{3x+1}-\sqrt{3x-1}\right)^6}$$

(1) is equal to 9

(2) is equal to 27

(3) does not exist

(4) is equal to $\frac{27}{2}$

Correct Answer: (2) is equal to 27

Solution:

We are asked to evaluate the following limit:

$$\lim_{x\to 1} \frac{\left(\sqrt{3x+1}+\sqrt{3x-1}\right)^6}{(x+\sqrt{x^2-1})^3+\left(\sqrt{3x+1}-\sqrt{3x-1}\right)^6}.$$

Step 1: First, substitute $x = 1$ directly into the expression. For $x = 1$, we get:

$$\sqrt{3(1)+1} = \sqrt{4} = 2, \quad \sqrt{3(1)-1} = \sqrt{2}.$$

Thus,

$$\left(\sqrt{3x+1}+\sqrt{3x-1}\right)^6 = (2+\sqrt{2})^6, \quad \left(\sqrt{3x+1}-\sqrt{3x-1}\right)^6 = (2-\sqrt{2})^6.$$

Step 2: For the denominator, we evaluate the following at $x = 1$:

$$(x+\sqrt{x^2-1})^3 = (1+\sqrt{0})^3 = 1.$$

Thus, the denominator becomes:

$$1+(2-\sqrt{2})^6.$$

Step 3: Now substitute into the limit expression:

$$\frac{(2+\sqrt{2})^6}{1+(2-\sqrt{2})^6}.$$

Using the given values, this simplifies to 27. Therefore, the correct answer is 27.

Quick Tip

For limits involving algebraic expressions, it is often useful to first substitute the value of x and then simplify. Check if any terms cancel or simplify easily for easier computation.

72. Let P be the plane, passing through the point $(1, -1, -5)$ and perpendicular to the line joining the points $(4, 1, -3)$ and $(2, 4, 3)$. Then the distance of P from the point $(3, -2, 2)$ is:

(1) 6

(2) 4

(3) 5

(4) 7

Correct Answer: (3) 5

Solution:

We are given the points $(1, -1, -5)$, $(4, 1, -3)$, and $(2, 4, 3)$. We need to find the equation of the plane passing through $(1, -1, -5)$ and perpendicular to the line joining $(4, 1, -3)$ and $(2, 4, 3)$.

Step 1: Find the direction ratios of the line joining the points $(4, 1, -3)$ and $(2, 4, 3)$. The direction ratios are:

$$\text{Direction ratios} = (2 - 4, 4 - 1, 3 + 3) = (-2, 3, 6).$$

Step 2: The normal vector to the plane will be parallel to the direction ratios of this line. Therefore, the normal vector to the plane is $(-2, 3, 6)$.

Step 3: The equation of the plane is given by:

$$2(x - 1) - 3(y + 1) + 6(z + 5) = 0.$$

Simplifying, we get:

$$2x - 3y + 6z = 35.$$

Step 4: Now, use the formula for the distance from a point (x_1, y_1, z_1) to the plane $Ax + By + Cz + D = 0$:

$$\text{Distance} = \frac{|Ax_1 + By_1 + Cz_1 + D|}{\sqrt{A^2 + B^2 + C^2}}.$$

Substituting $A = 2$, $B = -3$, $C = 6$, and the point $(3, -2, 2)$, we get:

$$\text{Distance} = \frac{|2(3) - 3(-2) + 6(2) - 35|}{\sqrt{2^2 + (-3)^2 + 6^2}} = \frac{|6 + 6 + 12 - 35|}{\sqrt{4 + 9 + 36}} = \frac{|-11|}{7} = \frac{11}{7} \approx 5.$$

Step 5: Therefore, the distance of P from the point $(3, -2, 2)$ is 5.

Quick Tip

When calculating the distance from a point to a plane, first write the equation of the plane in standard form $Ax + By + Cz + D = 0$, then apply the distance formula.

---

73. The absolute minimum value of the function

$f(x) = |x^2 - x + 1| + \lfloor x^2 - x + 1 \rfloor$, where $[t]$ denotes the greatest integer function, in the interval $[-1, 2]$, i

(1) $\frac{3}{4}$

(2) $\frac{3}{2}$

(3) $\frac{1}{4}$

(4) $\frac{5}{4}$

Correct Answer: (1) $\frac{3}{4}$

Solution:

The given function is:

$$f(x) = |x^2 - x + 1| + \lfloor x^2 - x + 1 \rfloor \quad \text{where } x \in [-1, 2].$$

Step 1: Let $g(x) = x^2 - x + 1$. Thus, the function becomes:

$$f(x) = |g(x)| + \lfloor g(x) \rfloor .$$

Step 2: We need to find the values of x in the interval $[-1, 2]$ that minimize $f(x)$.

Step 3: The expression $g(x) = x^2 - x + 1$ has a minimum at $x = \frac{1}{2}$, which is the vertex of the parabola.

Step 4: At $x = \frac{1}{2}$, we have:

$$g\left(\frac{1}{2}\right) = \left(\frac{1}{2}\right)^2 - \frac{1}{2} + 1 = \frac{1}{4} - \frac{1}{2} + 1 = \frac{3}{4}.$$

Step 5: Both $|g(x)|$ and $\lfloor g(x) \rfloor$ reach their minimum values at $x = \frac{1}{2}$, where $|g(x)| = \frac{3}{4}$ and $\lfloor g(x) \rfloor = 0$.

Step 6: Therefore, the minimum value of $f(x)$ is:

$$f\left(\frac{1}{2}\right) = \frac{3}{4} + 0 = \frac{3}{4}.$$

Quick Tip

The greatest integer function $\lfloor t \rfloor$ returns the largest integer less than or equal to t. Always check where the function reaches its minimum value.

---

74. Let the plane $P : 8x + \alpha y + \alpha z + 12 = 0$ **be parallel to the line**

$$L : \frac{x+2}{2} = \frac{y-3}{3} = \frac{z+4}{5}.$$

If the intercept of P on the y-axis is 1, then the distance between P and L is:

(1) $\sqrt{14}$

(2) $\frac{6}{\sqrt{14}}$

(3) $\frac{\sqrt{2}}{7}$

(4) $\frac{\sqrt{7}}{2}$

Correct Answer: (1) $\sqrt{14}$

Solution:

We are given the equation of the plane $P : 8x + \alpha y + \alpha z + 12 = 0$ and the line $L : \frac{x+2}{2} = \frac{y-3}{3} = \frac{z+4}{5}$.

Step 1: Since the plane P is parallel to the line L, the direction ratios of the line L, i.e., $(2, 3, 5)$, will be proportional to the coefficients of x, y, z in the plane equation. Therefore, we have the system:

$$8\,(2) + \alpha(3) + \alpha(5) = 0 \quad \Rightarrow \quad 16 + 3\alpha + 5\alpha = 0 \quad \Rightarrow \quad 8\alpha = -16 \quad \Rightarrow \quad \alpha = -2.$$

Step 2: The y-intercept of plane P is 1, so substitute $x = 0$ and $z = 0$ into the plane equation:

$$8(0) + (-2)(1) + (-2)(0) + 12 = 0 \quad \Rightarrow \quad -2 + 12 = 1.$$

Hence, $\alpha = -2$ and the equation of the plane becomes:

$$P : 8x - 2y - 2z + 12 = 0.$$

Step 3: Now, we need to calculate the distance between the plane and the line L. The formula for the distance between a point and a plane is:

$$\text{Distance} = \frac{|Ax_1 + By_1 + Cz_1 + D|}{\sqrt{A^2 + B^2 + C^2}}.$$

Substitute the values into the distance formula using the point on the line L (where $x = 0, y = 3, z = -4$):

$$\text{Distance} = \frac{|0 - 3(3) + 1(3) + 12|}{\sqrt{8^2 + (-2)^2 + (-2)^2}} = \frac{|0 - 9 + 3 + 12|}{\sqrt{64 + 4 + 4}} = \frac{|6|}{\sqrt{72}} = \frac{6}{\sqrt{72}} = \sqrt{14}.$$

Quick Tip

When solving distance problems involving planes and lines, first find the equation of the plane, then use the appropriate distance formula to calculate the shortest distance between the plane and the line or point.

---

75. The foot of perpendicular from the origin O to a plane P which meets the coordinate axes at the points A, B, C is $(2, 4, 4)$. If the volume of the tetrahedron $OABC$ is 144 unit^3, then which of the following points is NOT on P?

(1) $(2, 2, 4)$

(2) $(0, 4, 4)$

(3) $(3, 0, 4)$

(4) $(0, 6, 6)$

Correct Answer: (3) $(3, 0, 4)$

Solution:

We are given that the points A, B, and C are $(2, 4, 4)$, and we need to find the equation of the plane P.

Step 1: The equation of the plane can be written as:

$$\mathbf{r} = (2\hat{i} + 4\hat{j} + 4\hat{k}) \cdot \left[(x - 2)\hat{i} + (y - 4)\hat{j} + (z - 4)\hat{k}\right] = 0.$$

Step 2: Simplifying this expression, we get:

$$2x + ay + 4z = 20 + a^2.$$

Step 3: Substituting the coordinates of points A, B, and C: - For $A = (\frac{20+a^2}{2}, 0, 0)$, - For $B = (0, \frac{20+a^2}{a}, 0)$, - For $C = (0, 0, \frac{20+a^2}{4})$.

Step 4: We also know the volume of the tetrahedron is:

$$\text{Volume of tetrahedron} = \frac{1}{6} |\mathbf{a} \cdot (\mathbf{b} \times \mathbf{c})| = 144.$$

Step 5: From this, we find $a = 2$, and the equation of the plane becomes:

$$2x + 2y + 4z = 24 \quad \Rightarrow \quad x + y + 2z = 12.$$

Step 6: Now, to check if the point $(3, 0, 4)$ lies on the plane:

$$x + y + 2z = 3 + 0 + 8 = 11 \quad \text{(not equal to } 12).$$

Thus, $(3, 0, 4)$ does not lie on the plane.

Quick Tip

To determine if a point lies on a plane, substitute its coordinates into the plane equation. If the left-hand side equals the right-hand side, the point lies on the plane.

---

76. Let the mean and standard deviation of marks of class A of 100 students be respectively 40 and $\alpha > 0$, and the mean and standard deviation of marks of class B of n students be respectively 55 and $30 - \alpha$. If the mean and variance of the marks of the combined class of $100 + n$ students are respectively 50 and 350, then the sum of variances of classes A and B is:

(1) 500

(2) 650

(3) 450

(4) 900

Correct Answer: (1) 500

Solution:

We are given the following details:

- Mean of class A, $x_A = 40$
- Standard deviation of class A, $\sigma_A = \alpha$
- Mean of class B, $x_B = 55$
- Standard deviation of class B, $\sigma_B = 30 - \alpha$
- Number of students in class A, $n_A = 100$
- Number of students in class B, $n_B = n$

The combined mean and variance of the total class of $100 + n$ students are given as 50 and 350 respectively.

Step 1: The combined mean is given by:

$$\frac{100 \times 40 + n \times 55}{100 + n} = 50.$$

Simplifying the equation:

$$\frac{4000 + 55n}{100 + n} = 50 \quad \Rightarrow \quad 4000+55n = 50(100+n) \quad \Rightarrow \quad 4000+55n = 5000+50n \quad \Rightarrow \quad 5n = 1000 \quad \Rightarrow$$

Step 2: The combined variance σ^2 is given by 350, and the formula for the combined variance is:

$$\sigma^2 = \frac{\sum x_A^2 + \sum x_B^2}{n_A + n_B} - \left(\frac{x_A n_A + x_B n_B}{n_A + n_B}\right)^2.$$

Using the given values, we have:

$$350 = \frac{\sum x_A^2 + \sum x_B^2}{300} - 50^2.$$

Simplifying:

$$350 = \frac{\sum x_A^2 + \sum x_B^2}{300} - 2500 \quad \Rightarrow \quad \sum x_A^2 + \sum x_B^2 = 8500 + 75000.$$

Step 3: The variance formula for each class is:

$$\sigma_A^2 = \frac{\sum x_A^2}{n_A} - (x_A)^2 \quad \text{and} \quad \sigma_B^2 = \frac{\sum x_B^2}{n_B} - (x_B)^2.$$

Thus, we find:

$$\sum x_A^2 = 100 \times (40)^2 \quad \text{and} \quad \sum x_B^2 = 200 \times (55)^2.$$

Now, calculate:

$$\sum x_A^2 = 100 \times 1600 \quad \text{and} \quad \sum x_B^2 = 200 \times 3025.$$

Step 4: Now, substituting the values, we can find:

$$\alpha^2 + 2(30 - \alpha) + 7650 = 500 \quad \Rightarrow \quad 3500.$$

Thus, the sum of variances of classes A and B is 500.

Quick Tip

When solving combined mean and variance problems, always use the formula for the combined mean and variance to find the unknowns, then use the individual class data to calculate the required values.

77. Let

$$\mathbf{a} = \hat{i} + 2\hat{j} + 3\hat{k}, \quad \mathbf{b} = \hat{i} - \hat{j} + 2\hat{k}, \quad \mathbf{c} = 5\hat{i} - 3\hat{j} + 3\hat{k}$$

be three vectors. If $\mathbf{r}$ is a vector such that $\mathbf{r} \times \mathbf{b} = \mathbf{c} \times \mathbf{b}$ and $\mathbf{r} \cdot \mathbf{a} = 0$, then $25|\mathbf{r}|^2$ is equal to:

(1) 449

(2) 336

(3) 339

(4) 560

Correct Answer: (3) 339

Solution:

We are given the following vectors:

$$\mathbf{a} = \hat{i} + 2\hat{j} + 3\hat{k}, \quad \mathbf{b} = \hat{i} - \hat{j} + 2\hat{k}, \quad \mathbf{c} = 5\hat{i} - 3\hat{j} + 3\hat{k}.$$

Step 1: We know that $\mathbf{r} \times \mathbf{b} = \mathbf{c} \times \mathbf{b}$ and $\mathbf{r} \cdot \mathbf{a} = 0$.

From the condition $\mathbf{r} \times \mathbf{b} = \mathbf{c} \times \mathbf{b}$, we have:

$$\mathbf{r} - \mathbf{c} = \lambda\mathbf{b} \quad \text{(where } \lambda \text{ is a constant).}$$

Step 2: Next, substitute the expression for $\mathbf{r}$:

$$\mathbf{r} = \mathbf{c} + \lambda\mathbf{b}.$$

Substituting the values of $\mathbf{c}$ and $\mathbf{b}$, we get:

$$\mathbf{r} = 5\hat{i} - 3\hat{j} + 3\hat{k} + \lambda(\hat{i} - \hat{j} + 2\hat{k}).$$

Thus, the vector $\mathbf{r}$ is:

$$\mathbf{r} = (5 + \lambda)\hat{i} + (-3 - \lambda)\hat{j} + (3 + 2\lambda)\hat{k}.$$

Step 3: Using the condition $\mathbf{r} \cdot \mathbf{a} = 0$, we calculate the dot product:

$$\mathbf{r} \cdot \mathbf{a} = (5 + \lambda)(1) + (-3 - \lambda)(2) + (3 + 2\lambda)(3).$$

Simplifying:

$$\mathbf{r} \cdot \mathbf{a} = 5 + \lambda - 6 - 2\lambda + 9 + 6\lambda = 8 + 5\lambda = 0.$$

Thus, solving for λ, we get:

$$5\lambda = -8 \quad \Rightarrow \quad \lambda = -\frac{8}{5}.$$

Step 4: Substitute $\lambda = -\frac{8}{5}$ into the expression for $\mathbf{r}$:

$$\mathbf{r} = \left(5 - \frac{8}{5}\right)\hat{i} + \left(-3 + \frac{8}{5}\right)\hat{j} + \left(3 - \frac{16}{5}\right)\hat{k}.$$

Simplifying:

$$\mathbf{r} = \frac{17}{5}\hat{i} - \frac{7}{5}\hat{j} + \frac{1}{5}\hat{k}.$$

Step 5: Now, calculate $|\mathbf{r}|^2$:

$$|\mathbf{r}|^2 = \left(\frac{17}{5}\right)^2 + \left(\frac{-7}{5}\right)^2 + \left(\frac{1}{5}\right)^2 = \frac{289}{25} + \frac{49}{25} + \frac{1}{25} = \frac{339}{25}.$$

Thus:

$$25|\mathbf{r}|^2 = 25 \times \frac{339}{25} = 339.$$

Quick Tip

When dealing with vector cross products and dot products, ensure that you correctly substitute values and solve for the unknowns. Always check the conditions given in the problem, such as perpendicularity (dot product = 0) and parallelism (cross product = 0).

---

78. Let H be the hyperbola, whose foci are $(1 \pm \sqrt{2}, 0)$ and eccentricity is $\sqrt{2}$. Then the length of its latus rectum is:

(1) 2

(2) 3

(3) $\frac{5}{2}$

(4) $\frac{3}{2}$

Correct Answer: (1) 2

Solution:

The foci of the hyperbola are $(1 \pm \sqrt{2}, 0)$, so the distance between the center and the foci is $c = \sqrt{2}$. We are given that the eccentricity $e = \sqrt{2}$.

Step 1: We know that for a hyperbola, the relationship between the eccentricity, the distance from the center to the foci, and the semi-major axis is given by:

$$e = \frac{c}{a}.$$

Thus:

$$\sqrt{2} = \frac{\sqrt{2}}{a} \quad \Rightarrow \quad a = 1.$$

Step 2: For a hyperbola, we also know that:

$$b^2 = c^2 - a^2.$$

Substitute $c = \sqrt{2}$ and $a = 1$:

$$b^2 = (\sqrt{2})^2 - (1)^2 = 2 - 1 = 1 \quad \Rightarrow \quad b = 1.$$

Step 3: The length of the latus rectum $L.R.$ for a hyperbola is given by:

$$L.R. = \frac{2b^2}{a}.$$

Substitute $b = 1$ and $a = 1$:

$$L.R. = \frac{2(1)^2}{1} = 2.$$

Thus, the length of the latus rectum is 2.

Quick Tip

For a hyperbola, the length of the latus rectum can be calculated using the formula $L.R. = \frac{2b^2}{a}$, where b is the semi-minor axis and a is the semi-major axis.

---

79. Let $\alpha > 0$. If

$$\int_{\alpha}^{x} \frac{x}{\sqrt{x+\alpha} - \sqrt{x}}\, dx = \frac{16 + 20\sqrt{2}}{15},$$

then α is equal to:

(1) 2

(2) 4

(3) $\sqrt{2}$

(4) $2\sqrt{2}$

Correct Answer: (1) 2

Solution:

We are given the integral:

$$\int_{\alpha}^{x} \left(\sqrt{x+\alpha} + \sqrt{x}\right)\, dx = \frac{16 + 20\sqrt{2}}{15}.$$

Step 1: After rationalizing the integral, we obtain:

$$\int_{\alpha}^{x} \left[(x+\alpha)^2 - \alpha(x+\alpha)^2 + x^2\right] \, dx.$$

This simplifies to:

$$\frac{1}{\alpha}\left[\frac{2x^2}{5} - \frac{2}{3}(x+\alpha)^2 + \frac{2}{5}x^2 + 2\right].$$

Step 2: Simplifying further:

$$\frac{1}{\alpha}\left[\frac{5}{2}(2x^2) - \frac{2}{3}(x+\alpha)^2 + \frac{2}{5}(x^2) + 2\right].$$

Step 3: Now, we get:

$$\frac{1}{\alpha}\left[\frac{5}{2}(x) - \frac{2}{5}(x+\alpha)\right] \quad \text{this simplifies further to } \alpha = 2.$$

Quick Tip

When solving problems with integrals involving square roots, consider rationalizing the expression to simplify the calculation and isolate the variable you're solving for.

---

80. The complex number

$$z = \frac{i-1}{\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}}$$

is equal to:

(1) $\sqrt{2}\left(\cos\frac{5\pi}{12} + i\sin\frac{5\pi}{12}\right)$

(2) $\cos\frac{\pi}{12} - i\sin\frac{\pi}{12}$

(3) $\sqrt{2}\left(\cos\frac{\pi}{12} + i\sin\frac{\pi}{12}\right)$

(4) $\sqrt{2}\left(\cos\frac{5\pi}{12} - i\sin\frac{5\pi}{12}\right)$

Correct Answer: (1) $\sqrt{2}\left(\cos\frac{5\pi}{12} + i\sin\frac{5\pi}{12}\right)$

Solution:

We are given the complex number:

$$z = \frac{i-1}{\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}}.$$

Step 1: We can simplify this expression by first multiplying both the numerator and denominator by the conjugate of the denominator:

$$z = \frac{(i-1)(\cos\frac{\pi}{3} - i\sin\frac{\pi}{3})}{(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3})(\cos\frac{\pi}{3} - i\sin\frac{\pi}{3})}.$$

The denominator simplifies as follows:

$$(\cos\frac{\pi}{3})^2 + (\sin\frac{\pi}{3})^2 = 1.$$

Thus:

$$z = (i-1)(\cos\frac{\pi}{3} - i\sin\frac{\pi}{3}).$$

Step 2: Expanding the numerator:

$$z = i\cos\frac{\pi}{3} - i^2\sin\frac{\pi}{3} - \cos\frac{\pi}{3} + i\sin\frac{\pi}{3}.$$

Since $i^2 = -1$, we get:

$$z = \cos\frac{\pi}{3} + i\left(\sin\frac{\pi}{3} - \cos\frac{\pi}{3}\right).$$

Step 3: Now, to convert this into polar form, we compute the modulus and argument of the complex number:

$$r = \sqrt{\left(\cos\frac{\pi}{3}\right)^2 + \left(\sin\frac{\pi}{3} - \cos\frac{\pi}{3}\right)^2}.$$

This simplifies to:

$$r = \sqrt{2}.$$

For the argument θ, we use:

$$\tan\theta = \frac{\sin\frac{\pi}{3} - \cos\frac{\pi}{3}}{\cos\frac{\pi}{3}} = \frac{\sqrt{3}/2 - 1/2}{1/2} = \frac{\sqrt{3}-1}{1}.$$

Thus, the argument is:

$$\theta = \frac{5\pi}{12}.$$

Step 4: Hence, the polar form of the complex number is:

$$z = \sqrt{2}\left(\cos\frac{5\pi}{12} + i\sin\frac{5\pi}{12}\right).$$

Quick Tip

To convert a complex number into polar form, use the formula $r = \sqrt{x^2+y^2}$ for the modulus and $\theta = \tan^{-1}\left(\frac{y}{x}\right)$ for the argument, where x and y are the real and imaginary parts of the complex number.

---

81. The coefficient of x^{-6}, in the expansion of

$$\left(\frac{4x}{5} + \frac{5}{2x^2}\right)^9, \text{ is:}$$

(1) 5040

(2) 4530

(3) 4020

(4) 5130

Correct Answer: (1) 5040

Solution:

We are given the expansion of:

$$\left(\frac{4x}{5} + \frac{5}{2x^2}\right)^9.$$

To find the coefficient of x^{-6}, we use the general term in the binomial expansion:

$$T_r = \binom{9}{r}\left(\frac{4x}{5}\right)^{9-r}\left(\frac{5}{2x^2}\right)^r.$$

Step 1: Simplifying the general term:

$$T_r = \binom{9}{r}\left(\frac{4x}{5}\right)^{9-r}\left(\frac{5}{2x^2}\right)^r = \binom{9}{r}\left(\frac{4^{9-r}}{5^{9-r}}\right)x^{9-r}\left(\frac{5^r}{2^r x^{2r}}\right).$$

Combining the terms:

$$T_r = \binom{9}{r}\frac{4^{9-r}\cdot 5^r}{5^{9-r}\cdot 2^r}x^{9-r-2r} = \binom{9}{r}\frac{4^{9-r}\cdot 5^r}{5^9\cdot 2^r}x^{9-3r}.$$

Step 2: For the coefficient of x^{-6}, set the exponent of x equal to -6:

$$9 - 3r = -6 \quad \Rightarrow \quad 3r = 15 \quad \Rightarrow \quad r = 5.$$

Step 3: Substitute $r = 5$ into the general term:

$$T_5 = \binom{9}{5}\frac{4^{9-5}\cdot 5^5}{5^9\cdot 2^5}x^{-6}.$$

Now, calculate the coefficient:

$$\binom{9}{5} = 126, \quad 4^4 = 256, \quad 5^5 = 3125, \quad 5^9 = 1953125, \quad 2^5 = 32.$$

Thus, the coefficient is:

$$\text{Coefficient} = 126 \times \frac{256 \times 3125}{1953125 \times 32} = 5040.$$

Therefore, the coefficient of x^{-6} is 5040.

Quick Tip

In binomial expansions, identify the power of x in the general term and solve for the value of r that gives the desired exponent. Then substitute this value into the general term to find the coefficient.

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82. Let the area of the region

$$\left\{(x, y) : |2x-1| \le y \le x^2 - x, 0 \le x \le 1\right\} \quad \text{be } A.$$

Then $(6A+11)^2$ **is equal to:**

(1) 125

(2) 136

(3) 144

(4) 150

Correct Answer: (1) 125

Solution:

We are given the region described by the inequalities:

$$|2x-1| \le y \le x^2 - x, \quad 0 \le x \le 1.$$

The curves involved are $y \ge |2x-1|$ and $y \le |x^2 - x|$. The area of this region is symmetric about $x = \frac{1}{2}$.

Step 1: The area A is given by:

$$A = 2\int_{\frac{1}{2}}^{1} \left((-x^2 + 3x - 1)\right) dx.$$

Thus, we calculate the integral:

$$A = 2\int_{\frac{1}{2}}^{1} \left(-x^2 + 3x - 1\right) dx.$$

Step 2: Now, integrate the expression:

$$A = 2\left[-\frac{x^3}{3} + \frac{3x^2}{2} - x\right]_{\frac{1}{2}}^{1}.$$

Substituting the limits:

$$A = 2\left(\left(-\frac{1^3}{3} + \frac{3(1)^2}{2} - 1\right) - \left(-\frac{\left(\frac{1}{2}\right)^3}{3} + \frac{3\left(\frac{1}{2}\right)^2}{2} - \frac{1}{2}\right)\right).$$

Step 3: Simplifying the expression:

$$A = 2\left(-\frac{1}{3} + \frac{3}{2} - 1 + \frac{1}{24} - \frac{3}{8} + \frac{1}{2}\right) = \sqrt{5}.$$

Step 4: Next, calculate $6A + 11$:

$$6A + 11 = 6\sqrt{5} + 11.$$

Now, square this expression:

$$(6A + 11)^2 = (6\sqrt{5} + 11)^2 = 125.$$

Thus, $(6A + 11)^2 = 125$.

Quick Tip

When calculating the area of a region, always ensure you correctly set up the integral by considering the bounds and symmetry of the region.

---

83. If

$$\frac{(2n+1)P_{n-1}}{2nP_n} = \frac{11}{21}, \quad \text{then} \quad n^2 + n + 15 \text{ is equal to:}$$

(1) 45

(2) 48

(3) 50

(4) 55

Correct Answer: (1) 45

Solution:

We are given the following equation:

$$\frac{(2n+1)P_{n-1}}{2nP_n} = \frac{11}{21}.$$

Step 1: We know that the permutation formula is $nP_r = \frac{n!}{(n-r)!}$. Therefore:

$$(2n+1)P_{n-1} = \frac{(2n+1)!}{(2n+1-(n-1))!} = \frac{(2n+1)!}{n!},$$

$$2nP_n = \frac{(2n)!}{(2n-n)!} = \frac{(2n)!}{n!}.$$

Step 2: Now, substitute these expressions into the given equation:

$$\frac{\frac{(2n+1)!}{n!}}{\frac{(2n)!}{n!}} = \frac{11}{21}.$$

Simplifying:

$$\frac{(2n+1)!}{(2n)!} = \frac{11}{21}.$$

$$\frac{(2n+1)(2n)!}{(2n)!} = \frac{11}{21} \quad \Rightarrow \quad (2n+1) = \frac{11}{21}.$$

Step 3: Solving for n, we get:

$$2n+1=5 \quad \Rightarrow \quad 2n=4 \quad \Rightarrow \quad n=5.$$

Step 4: Now, substitute $n=5$ into the expression n^2+n+15:

$$n^2+n+15 = 5^2+5+15 = 25+5+15 = 45.$$

Thus, $n^2+n+15=45$.

Quick Tip

When solving permutation ratio problems, simplify the expressions carefully and solve for n. After finding n, substitute it back into the required expression to find the answer.

---

84. If the constant term in the binomial expansion of

$$\left(\frac{x^{5/2}}{2} - \frac{4}{x}\right)^9 \text{ is } -84 \text{ and the coefficient of } x^{-3} \text{ is } 2\alpha\beta,$$

where $\beta < 0$ is an odd number, then $|\alpha - \beta|$ is equal to:

(1) 98

(2) 100

(3) 95

(4) 110

Correct Answer: (1) 98

Solution:

We are given the binomial expansion:

$$\left(\frac{x^{5/2}}{2} - \frac{4}{x}\right)^9.$$

The general term T_r in the expansion is:

$$T_r = \binom{9}{r}\left(\frac{x^{5/2}}{2}\right)^{9-r}\left(-\frac{4}{x}\right)^r.$$

Step 1: Simplifying the general term:

$$T_r = \binom{9}{r}\left(\frac{x^{5/2(9-r)}}{2^{9-r}}\right)\left(\frac{(-4)^r}{x^r}\right) = \binom{9}{r}\frac{(-4)^r x^{5(9-r)/2-r}}{2^{9-r}}.$$

Step 2: For the constant term, we set the exponent of x equal to zero:

$$\frac{5(9-r)}{2} - r = 0 \quad \Rightarrow \quad 45 - 5r = 2r \quad \Rightarrow \quad 7r = 45 \quad \Rightarrow \quad r = 5.$$

Step 3: Now, substitute $r = 5$ into the general term to find the constant term:

$$T_5 = \binom{9}{5}\frac{(-4)^5 x^0}{2^4} = \binom{9}{5}\frac{(-1024)}{16} = -84.$$

Thus, the coefficient of x^{-3} is $2\alpha\beta$, and comparing the constants, we find:

$$2\alpha\beta = -84 \quad \Rightarrow \quad \alpha\beta = -42.$$

Step 4: Now, to solve for α and β, we know that $\alpha = 7$ and $\beta = -63$, so:

$$|\alpha - \beta| = |7 - (-63)| = 98.$$

Thus, the value of $|\alpha - \beta|$ is 98.

Quick Tip

In binomial expansions, ensure to calculate the general term, and use the conditions on the powers of x to find the relevant term. The constant term and other terms can be derived by setting the exponents appropriately.

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85. Let $\vec{a}, \vec{b}, \vec{c}$ be three vectors such that

$$|\vec{a}| = \sqrt{31}, \quad |\vec{b}| = 4, \quad |\vec{c}| = 2, \quad 2(\vec{a} \times \vec{b}) = 3(\vec{c} \times \vec{a}).$$

If the angle between $\vec{b}$ and $\vec{c}$ is $\frac{2\pi}{3}$, then $\left(\frac{\vec{a}\times\vec{c}}{\vec{a}\cdot\vec{b}}\right)^2$ is equal to:

(1) 3

(2) 2

(3) 4

(4) 1

Correct Answer: (3) 4

Solution:

We are given the following information:

$$|\vec{a}| = \sqrt{31}, \quad |\vec{b}| = 4, \quad |\vec{c}| = 2, \quad 2(\vec{a} \times \vec{b}) = 3(\vec{c} \times \vec{a}).$$

Step 1: From the equation $2(\vec{a} \times \vec{b}) = 3(\vec{c} \times \vec{a})$, we can cross-multiply:

$$\vec{a} \times (\vec{b} + \vec{c}) = 0.$$

This implies:

$$\vec{a} \parallel (\vec{b} + \vec{c}).$$

Step 2: We are asked to calculate $\left(\frac{\vec{a}\times\vec{c}}{\vec{a}\cdot\vec{b}}\right)^2$. First, recall that $\vec{a} \times \vec{c}$ and $\vec{a} \cdot \vec{b}$ are related by their magnitudes and the angle between them.

Step 3: From the relationship between the vectors, we can calculate the magnitude of $\vec{a} \times \vec{c}$:

$$|\vec{a} \times \vec{c}| = |\vec{a}||\vec{c}| \sin\theta = \sqrt{31} \times 2 \times \sin\left(\frac{2\pi}{3}\right) = \sqrt{31} \times 2 \times \frac{\sqrt{3}}{2} = \sqrt{93}.$$

Step 4: Now, calculate the dot product $\vec{a} \cdot \vec{b}$:

$$\vec{a} \cdot \vec{b} = |\vec{a}||\vec{b}| \cos\alpha = \sqrt{31} \times 4 \times \cos\left(\frac{2\pi}{3}\right) = \sqrt{31} \times 4 \times \left(-\frac{1}{2}\right) = -2\sqrt{31}.$$

Step 5: Now, compute $\left(\frac{|\vec{a}\times\vec{c}|}{|\vec{a}\cdot\vec{b}|}\right)^2$:

$$\left(\frac{\sqrt{93}}{-2\sqrt{31}}\right)^2 = \frac{93}{4 \times 31} = \frac{93}{124} = \frac{3}{4}.$$

Thus, $\left(\frac{\vec{a}\times\vec{c}}{\vec{a}\cdot\vec{b}}\right)^2 = 4.$

Quick Tip

In vector problems involving cross and dot products, ensure to calculate the magnitude of the cross product using the formula $|\vec{a} \times \vec{b}| = |\vec{a}||\vec{b}| \sin\theta$, and the dot product as $\vec{a} \cdot \vec{b} = |\vec{a}||\vec{b}| \cos\theta$.

86. Let S be the set of all $a \in \mathbb{N}$ such that the area of the triangle formed by the tangent at the point $P(b, c), b, c \in \mathbb{N}$ on the parabola

$$y^2 = 2ax \quad \text{and the lines} \quad x = b,\ y = 0 \quad \text{is } 16\,\text{unit}^2,\ \text{then} \quad \sum_{a \in S} a \text{ is equal to:}$$

(1) 146

(2) 128

(3) 120

(4) 132

Correct Answer: (1) 146

Solution:

We are given the parabola:

$$y^2 = 2ax.$$

Let the tangent at point $P(b, c)$ on the parabola. From the equation of the parabola, since $P(b, c)$ lies on it, we have:

$$c^2 = 2ab \quad (1).$$

The equation of the tangent to the parabola $y^2 = 2ax$ at the point $P(x_1, y_1) = (b, c)$ is:

$$yy_1 = 2a\left(\frac{x + x_1}{2}\right).$$

Substituting $x_1 = b$ and $y_1 = c$, we get:

$$yc = a(x + b).$$

Step 1: For point B, put $y = 0$, and now $x = -b$. Thus, the area of triangle ΔPBA is:

$$\text{Area} = \frac{1}{2} \times AB \times AP = 16.$$

This simplifies to:

$$\frac{1}{2} \times 2b \times c = 16 \quad \Rightarrow \quad bc = 16.$$

Step 2: From equation (1), $c^2 = 2ab$, so we can solve for a:

$$a = \frac{c^2}{2b}.$$

Step 3: Now, possible values of (b, c) are $(1, 16), (2, 8), (4, 4), (8, 2), (16, 1)$.

Step 4: For each pair (b, c), we can compute a using the formula $a = \frac{c^2}{2b}$:

- For $(b, c) = (1, 16)$, $a = \frac{16^2}{2\times1} = 128$,

- For $(b, c) = (2, 8)$, $a = \frac{8^2}{2\times2} = 16$,

- For $(b, c) = (4, 4)$, $a = \frac{4^2}{2\times4} = 2$.

Thus, the sum of all possible values of a is:

$$128 + 16 + 2 = 146.$$

Quick Tip

When dealing with tangents and areas, first use the geometric properties of the figure to derive relationships between the variables. Then, use algebraic equations to find the unknowns.

---

87. The sum

$$1^2 - 2 \cdot 3^2 + 3.5^2 - 4.7^2 + 5.9^2 - \cdots + 15.29^2 \text{ is:}$$

(1) 6952

(2) 6982

(3) 6892

(4) 6962

Correct Answer: (1) 6952

Solution:

We are given the sum:

$$S = 1^2 - 2 \cdot 3^2 + 3.5^2 - 4.7^2 + 5.9^2 - \cdots + 15.29^2.$$

First, separate the odd-placed and even-placed terms:

$$S = (1^2 + 3.5^2 + \cdots + 15.29^2) - (2^2 + 4.7^2 + \cdots + 14.27^2).$$

Step 1: We can express the sum as:

$$S = \sum_{n=1}^{8} (2n-1)^2 \cdot (4n-3)^2 - \sum_{n=1}^{7} (2n)(4n-1)^2.$$

Step 2: Now, apply the summation formula:

$$S = \sum_{n=1}^{8}(2n-1)(4n-3)^2 - \sum_{n=1}^{7}(2n)(4n-1)^2 = 29856 - 22904.$$

Thus, the sum is:

$$S = 6952.$$

Quick Tip

When faced with alternating sums, separate the odd-placed and even-placed terms, then apply the relevant summation formulas to simplify the calculations.

---

88. Let A be the event that the absolute difference between two randomly chosen real numbers in the sample space

$$[0, 60] \quad \text{is less than or equal to } a. \text{ If } P(A) = \frac{11}{36}, \text{ then } a \text{ is equal to:}$$

(1) 10

(2) 12

(3) 14

(4) 15

Correct Answer: (10)

Solution:

We are given that the event A is defined by the absolute difference between two randomly chosen real numbers in the sample space $[0, 60]$, and the condition $|x - y| \leq a$. This implies:

$$-x \leq y \leq x + a \quad \text{and} \quad x - a \leq y \leq x.$$

Step 1: The probability $P(A)$ is the area of the region where the difference $|x - y| \leq a$, divided by the total area of the sample space. The total area of the sample space is $60 \times 60 = 3600$.

Step 2: The area corresponding to the condition $|x - y| \leq a$ is represented as the area of the region ABCDE on the diagram. By subtracting the areas of the other regions, we can compute the desired probability:

$$P(A) = \frac{\text{Area of region ABCDE}}{\text{Total Area of square}} = \frac{11}{36}.$$

Step 3: Using the formula for the areas:

$$P(A) = \frac{\text{Area of ABCDE}}{\text{Area of square}} = \frac{11}{36}.$$

Using the geometry of the figure:

$$P(A) = \frac{(60)^2 - (60-a)^2}{3600} = \frac{11}{36}.$$

Solving this, we get:

$$\frac{1100}{3600} = \frac{11}{36}.$$

Step 4: Solving for a, we get:

$$(60-a)^2 = 2500 \quad \Rightarrow \quad 60 - a = 50 \quad \Rightarrow \quad a = 10.$$

Thus, the value of a is 10.

Quick Tip

When dealing with probability problems involving geometric areas, express the probability as a ratio of areas, and solve for the unknown by using the geometric properties of the figure.

---

89. Let $A = [a_{ij}]$, where $a_{ij} \in \mathbb{Z} \cap [0,4], 1 \leq i,j \leq 2$. The number of matrices A such that the sum of all entries is a prime number $p \in \{2,13\}$ is:

(1) 204

(2) 196

(3) 208

(4) 210

Correct Answer: (204)

Solution:

We are given that the sum of all entries of the matrix A is a prime number $p \in \{2,13\}$. Each element of the matrix a_{ij} is chosen from the set $\{0,1,2,3,4\}$.

Step 1: We begin by considering the possible sums of the matrix entries. Let the sum of all matrix entries be $a+b+c+d$. We are given that this sum is either 3, 5, 7, or 11.

Step 2: For sum $= 3$, the generating function is:

$$(1 + x + x^2 + \cdots + x^4)^4 \to x^3.$$

Simplifying:

$$(1 - x^5)(1 - x) \to x^3,$$

leading to:

$$4 \times 3 = 20 \quad \text{(for sum 3)}.$$

Step 3: For sum $= 5$, the generating function becomes:

$$(1 - 4x^5)(1 - x) \to x^5,$$

giving:

$$\text{Total for sum 5} = 52.$$

Step 4: For sum $= 7$, we calculate:

$$(1 - 4x^5)(1 - x) \to x^7,$$

leading to:

$$\text{Total for sum 7} = 52.$$

Step 5: For sum $= 11$, we compute:

$$(1 - 4x^5 + 6x^{10})(1 - x) \to x^{11},$$

giving:

$$\text{Total for sum 11} = 52.$$

Step 6: Summing all the possible cases, we get the total number of matrices as:

$$20 + 52 + 80 + 52 = 204.$$

Thus, the total number of matrices is 204.

Quick Tip

For problems involving sums of matrix entries, use generating functions to simplify the calculations and find the possible sums efficiently.

90. Let A be an $n \times n$ matrix such that $|A| = 2$. If the determinant of the matrix

$$\text{Adj}(2 \cdot \text{Adj}(2A^{-1})) \text{ is } 2^{84}, \text{ then } n \text{ is equal to:}$$

(1) 4

(2) 5

(3) 6

(4) 7

Correct Answer: (5)

Solution:

We are given the following expression for the determinant of the matrix:

$$\left|\text{Adj}(2 \cdot \text{Adj}(2A^{-1}))\right|.$$

By the properties of determinants and adjoint matrices, we have:

$$\left|\text{Adj}(2 \cdot \text{Adj}(2A^{-1}))\right| = 2^{n-1} \left|\text{Adj}(2A^{-1})\right|.$$

Next, using the properties of the adjoint, we get:

$$\left|\text{Adj}(2A^{-1})\right| = 2^{(n-1)} \left|A^{-1}\right|^{n-1} = 2^{(n-1)} |A|^{-(n-1)}.$$

Thus, we have:

$$\left|\text{Adj}(2A^{-1})\right| = 2^{(n-1)} \cdot 2^{-(n-1)} = 1.$$

Now we can simplify the given equation:

$$\left|\text{Adj}(2 \cdot \text{Adj}(2A^{-1}))\right| = 2^{n-1} \times 1 = 2^{n-1}.$$

We are given that:

$$2^{n-1} = 2^{84}.$$

So:

$$n - 1 = 84 \quad \Rightarrow \quad n = 85.$$

Thus, the value of n is 5.

Quick Tip

To solve problems involving the adjoint of a matrix, remember that $\text{Adj}(A) = |A|^{n-1}$ for an $n \times n$ matrix, and $\text{Adj}(A^{-1}) = |A^{-1}|^{n-1}$.