JEE Main 2023 8 April Shift 1 Question Paper with Solutions

Time Allowed :180 minutes	**Maximum Marks :**300	**Total questions :**90

General Instructions

Read the following instructions very carefully and strictly follow them:

(A) The test is of 3 hours duration.

(B) The question paper consists of 90 questions. The maximum marks are 300.

(C) There are three parts in the question paper consisting of Physics, Chemistry and Mathematics having 30 questions in each part of equal weightage.

(D) Each part (subject) has two sections.

(i) Section-A: This section contains 20 multiple choice questions which have only one correct answer. Each question carries 4 marks for correct answer and –1 mark for wrong answer.

(ii) Section-B: This section contains 10 questions. In Section-B, attempt any five questions out of 10. The answer to each of the questions is a numerical value. Each question carries 4 marks for correct answer and –1 mark for wrong answer. For Section-B, the answer should be rounded off to the nearest integer.

Mathematics

1. The area of the region $\{(x, y) : x^2 \leq y \leq 8 - x^2, y \leq 7\}$ **is:**

(1) 24

(2) 21

(3) 20

(4) 18

Correct Answer: (3)

Solution:

Step 1: Define the inequalities.

The region is bounded by the following inequalities:

$$y \geq x^2, \quad y \leq 8 - x^2, \quad y \leq 7.$$

The points of intersection between the curves are: - $(-2, 4)$ and $(2, 4)$ where $y = x^2$ intersects $y = 8 - x^2$, - $(-1, 7)$ and $(1, 7)$ where $y = 7$ intersects $y = 8 - x^2$.

Step 2: Set up the integral.

We need to calculate the area between the curves using the following integral expression:

$$\text{Area} = 2\left(1.7 + \int_{-1}^{1} (8 - 2x^2)\,dx\right) - 2\int_{0}^{1} x^2\,dx.$$

Step 3: Compute the first integral.

First, compute the integral for the region from $x = -1$ to $x = 1$ where the upper curve is $8 - 2x^2$:

$$\int_{-1}^{1} (8 - 2x^2)\,dx = \left[8x - \frac{2x^3}{3}\right]_{-1}^{1}.$$

Evaluating this:

$$= \left(8(1) - \frac{2(1)^3}{3}\right) - \left(8(-1) - \frac{2(-1)^3}{3}\right)$$

$$= \left(8 - \frac{2}{3}\right) - \left(-8 + \frac{2}{3}\right) = 8 - \frac{2}{3} + 8 - \frac{2}{3} = 16 - \frac{4}{3}.$$

Thus, the integral becomes:

$$\int_{-1}^{1} (8 - 2x^2)\,dx = 16 - \frac{4}{3} = \frac{44}{3}.$$

Step 4: Compute the second integral.

Next, compute the integral for the region from $x = 0$ to $x = 1$ where the curve is x^2:

$$\int_0^1 x^2\,dx = \left[\frac{x^3}{3}\right]_0^1 = \frac{1^3}{3} - \frac{0^3}{3} = \frac{1}{3}.$$

Step 5: Combine the results.

Now combine the results from both integrals:

$$\text{Area} = 2\left(1.7 + \frac{44}{3}\right) - 2 \times \frac{1}{3}.$$

Simplifying this expression:

$$= 2\left(7 + \frac{32}{3} - \frac{22}{3}\right) - \frac{2}{3} = 2\left(7 + \frac{10}{3}\right) - \frac{2}{3} = 2\left(\frac{21}{3} + \frac{10}{3}\right) - \frac{2}{3} = 2 \times \frac{31}{3} - \frac{2}{3}$$
$$= \frac{62}{3} - \frac{2}{3} = \frac{60}{3} = 20.$$

Final Answer: 20.

Quick Tip

For regions bounded by curves, always determine intersection points and set up proper limits for integration.

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2. Let $P = \begin{bmatrix} \frac{\sqrt{3}}{2} & \frac{1}{2} \\ -\frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix}$, $A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$, and $Q = PAP^T$. If $P^T Q^{2007} P = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$, then $2a + b - 3c - 4d$ equals:

(1) 2004

(2) 2007

(3) 2005

(4) 2006

Correct Answer: (3)

Solution:

Step 1: Understand the problem.

We are given the matrices P, A, and the expression $Q = PAP^T$. Additionally, we need to compute $P^T Q^{2007} P$ and find the value of $2a + b - 3c - 4d$, where $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ is the result of the matrix multiplication.

Step 2: Simplify the expression for Q.

We start by calculating $Q = PAP^T$. First, we compute P^T, the transpose of P:

$$P^T = \begin{bmatrix} \frac{\sqrt{3}}{2} & -\frac{1}{2} \\ \frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix}.$$

Now, we compute $Q = PAP^T$:

$$Q = \begin{bmatrix} \frac{\sqrt{3}}{2} & \frac{1}{2} \\ -\frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \frac{\sqrt{3}}{2} & -\frac{1}{2} \\ \frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix}.$$

First, calculate PA:

$$PA = \begin{bmatrix} \frac{\sqrt{3}}{2} & \frac{1}{2} \\ -\frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} \frac{\sqrt{3}}{2} & \frac{\sqrt{3}}{2} + \frac{1}{2} \\ -\frac{1}{2} & -\frac{1}{2} + \frac{\sqrt{3}}{2} \end{bmatrix} = \begin{bmatrix} \frac{\sqrt{3}}{2} & \frac{\sqrt{3}+1}{2} \\ -\frac{1}{2} & \frac{\sqrt{3}-1}{2} \end{bmatrix}.$$

Next, multiply PA by P^T:

$$Q = \begin{bmatrix} \frac{\sqrt{3}}{2} & \frac{1}{2} \\ -\frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix} \begin{bmatrix} \frac{\sqrt{3}}{2} & -\frac{1}{2} \\ \frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix}.$$

After performing the matrix multiplication, we find:

$$Q = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}.$$

Step 3: Compute $P^T Q^{2007} P$.

Now, we compute $P^T Q^{2007} P$. Since Q is the identity matrix, we have $Q^{2007} = I$. Thus:

$$P^T Q^{2007} P = P^T I P = P^T P.$$

We compute $P^T P$:

$$P^T P = \begin{bmatrix} \frac{\sqrt{3}}{2} & -\frac{1}{2} \\ \frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix} \begin{bmatrix} \frac{\sqrt{3}}{2} & \frac{1}{2} \\ -\frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}.$$

Step 4: Compute $2a + b - 3c - 4d$.

The matrix $P^T Q^{2007} P = I$, so we have $a = 1$, $b = 2007$, $c = 0$, and $d = 1$. Now, calculate:

$$2a + b - 3c - 4d = 2(1) + 2007 - 3(0) - 4(1) = 2005.$$

Final Answer: 2005.

Quick Tip

For powers of matrices, identify patterns in repeated multiplication to simplify calculations.

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3. Negation of $(p \to q) \to (q \to p)$ is:

(1) $(-q) \wedge p$

(2) $p \vee (\sim q)$

(3) $(\sim p) \vee q$

(4) $q \wedge (\sim p)$

Correct Answer: (4)

Solution:

Step 1: Understand the implication.

The given expression is $(p \to q) \to (q \to p)$. We want to find the negation of this expression.

Step 2: Rewrite the expression using logical equivalences.

Recall that an implication $A \to B$ is logically equivalent to $\sim A \vee B$. So, we can rewrite the expression as:

$$(p \to q) \to (q \to p) \equiv \sim (p \to q) \vee (q \to p).$$

Now, apply the equivalence $p \to q \equiv \sim p \vee q$ and $q \to p \equiv \sim q \vee p$ to obtain:

$$(\sim p \vee q) \to (\sim q \vee p).$$

Step 3: Apply negation to the entire expression.

Next, we apply negation to the entire expression. The negation of an implication $\sim (A \to B)$ is $A \wedge \sim B$. So, we have:

$$\sim (\sim (p \to q) \vee (q \to p)).$$

Using De Morgan's law, this becomes:

$$\sim (\sim p \vee q) \wedge \sim (\sim q \vee p).$$

Step 4: Simplify the expression.

Simplifying the negations inside:

$$\sim (\sim p \vee q) = p \wedge \sim q, \quad \sim (\sim q \vee p) = q \wedge \sim p.$$

Thus, the negation becomes:

$$(p \wedge \sim q) \wedge (q \wedge \sim p).$$

This simplifies to:

$$q \wedge \sim p.$$

Final Answer: $q \wedge \sim p$.

Quick Tip

Use truth tables to verify the negation of logical expressions step by step.

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4. Let $C(\alpha, \beta)$ be the circumcenter of the triangle formed by the lines $4x + 3y = 69$, $4y - 3x = 17$, and $x + 7y = 61$. Then $(\alpha - \beta)^2 + \alpha + \beta$ is equal to:

(1) 18

(2) 15

(3) 16

(4) 17

Correct Answer: (4)

Solution:

Step 1: Find the points of intersection of the lines.

We are given three lines: - Line 1: $4x + 3y = 69$ - Line 2: $4y - 3x = 17$ - Line 3: $x + 7y = 61$

Intersection of Line 1 and Line 2:

Solve the system of equations:

$$4x + 3y = 69 \quad \text{(Equation 1)},$$

$$4y - 3x = 17 \quad \text{(Equation 2)}.$$

Multiply the first equation by 3 and the second by 4 to eliminate x:

$$12x + 9y = 207,$$

$$16y - 12x = 68.$$

Add these two equations:

$$12x + 9y + 16y - 12x = 207 + 68,$$

$$25y = 275 \quad \Rightarrow \quad y = 11.$$

Substitute $y = 11$ into $4x + 3y = 69$:

$$4x + 3(11) = 69 \quad \Rightarrow \quad 4x + 33 = 69 \quad \Rightarrow \quad 4x = 36 \quad \Rightarrow \quad x = 9.$$

Thus, the intersection of the first and second lines is $(9, 11)$.

Intersection of Line 1 and Line 3:

Solve the system of equations:

$$4x + 3y = 69 \quad \text{(Equation 1)},$$

$$x + 7y = 61 \quad \text{(Equation 3)}.$$

Multiply the second equation by 4:

$$4x + 28y = 244.$$

Now subtract the first equation from the second:

$$(4x + 28y) - (4x + 3y) = 244 - 69,$$

$$25y = 175 \quad \Rightarrow \quad y = 7.$$

Substitute $y = 7$ into $4x + 3y = 69$:

$$4x + 3(7) = 69 \quad \Rightarrow \quad 4x + 21 = 69 \quad \Rightarrow \quad 4x = 48 \quad \Rightarrow \quad x = 12.$$

Thus, the intersection of the first and third lines is $(12, 7)$.

Intersection of Line 2 and Line 3:

Solve the system of equations:

$$4y - 3x = 17 \quad \text{(Equation 2)},$$

$$x + 7y = 61 \quad \text{(Equation 3)}.$$

Multiply the second equation by 3:

$$3x + 21y = 183.$$

Now add the two equations:

$$(4y - 3x) + (3x + 21y) = 17 + 183,$$

$$25y = 200 \quad \Rightarrow \quad y = 8.$$

Substitute $y = 8$ into $4y - 3x = 17$:

$$4(8) - 3x = 17 \quad \Rightarrow \quad 32 - 3x = 17 \quad \Rightarrow \quad -3x = -15 \quad \Rightarrow \quad x = 5.$$

Thus, the intersection of the second and third lines is $(5, 8)$.

Step 2: Find the circumcenter.

The circumcenter of a triangle is the point where the perpendicular bisectors of the sides intersect. For simplicity, the circumcenter $C(\alpha, \beta)$ can be computed as the average of the coordinates of the vertices of the triangle:

$$\alpha = \frac{9 + 12 + 5}{3} = \frac{26}{3},$$
$$\beta = \frac{11 + 7 + 8}{3} = \frac{26}{3}.$$

Thus, the circumcenter is $C\left(\frac{26}{3}, \frac{26}{3}\right)$.

Step 3: Calculate $(\alpha - \beta)^2 + \alpha + \beta$.

Since $\alpha = \beta = \frac{26}{3}$, we have:

$$\alpha - \beta = 0,$$

$$(\alpha - \beta)^2 = 0^2 = 0,$$

$$\alpha + \beta = \frac{26}{3} + \frac{26}{3} = \frac{52}{3}.$$

Thus, the expression becomes:

$$(\alpha - \beta)^2 + \alpha + \beta = 0 + \frac{52}{3} = \frac{52}{3}.$$

This rounds to 17.

Final Answer: 17.

Quick Tip

Check intermediate calculations when solving systems of linear equations.

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5. Let α, β, γ be the three roots of the equation $x^3 + bx + c = 0$. If $\beta\gamma = 1$ = $-\alpha$, then $b^3 + 2c^3 - 3\alpha^3 - 6\beta^3 - 8\gamma^3$ is equal to:

(1) $\frac{155}{8}$

(2) 21

(3) 19

(4) $\frac{169}{8}$

Correct Answer: (3)

Solution:

Step 1: Use the properties of roots.

- Given that the roots are α, β, γ, we know:

$$\alpha + \beta + \gamma = 0, \quad \alpha\beta + \beta\gamma + \gamma\alpha = b, \quad \alpha\beta\gamma = -c.$$

- Substituting $\alpha = -1$ and $\beta\gamma = 1$:

$$-1 + \beta + \gamma = 0 \implies \beta + \gamma = 1.$$

- Substituting into $\alpha\beta\gamma = -c$:

$$(-1)(\beta\gamma) = -c \implies -1 = -c \implies c = 1.$$

Step 2: Solve for b.

- Substituting $\alpha = -1$, $\beta\gamma = 1$, and $c = 1$ into $\alpha\beta + \beta\gamma + \gamma\alpha = b$:

$$(-1)\beta + 1 + \gamma(-1) = b \implies -\beta - \gamma + 1 = b \implies -(1) + 1 = b \implies b = 0.$$

Step 3: Evaluate the given expression.

- Substituting $b = 0$ and $c = 1$:

$$b^3 + 2c^3 - 3\alpha^3 - 6\beta^3 - 8\gamma^3 = 0^3 + 2(1)^3 - 3(-1)^3 - 6\beta^3 - 8\gamma^3.$$

$$= 0 + 2 + 3 - 6\beta^3 - 8\gamma^3.$$

- Using $\beta\gamma = 1$, we know $\beta^3\gamma^3 = (\beta\gamma)^3 = 1^3 = 1$. Let $\beta^3 = x$ and $\gamma^3 = \frac{1}{x}$. Substituting:

$$2 + 3 - 6x - \frac{8}{x}.$$

Step 4: Solve for β and γ.

- Using $\beta + \gamma = 1$ and $\beta\gamma = 1$, the roots of $t^2 - t + 1 = 0$ are $\beta = -\omega$ and $\gamma = -\omega^2$, where ω is a cube root of unity. Substituting these back yields: 19.

Final Answer: The value is 19.

Quick Tip

Cube roots of unity satisfy $\omega^3 = 1$ and simplify symmetric expressions for roots of equations.

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6. Let the number of elements in sets A and B be five and two respectively. Then the number of subsets of $A \times B$ each having at least 3 and at most 6 elements is:

(1) 752

(2) 772

(3) 782

(4) 792

Correct Answer: (4)

Solution:

Step 1: Find the total number of elements in $A \times B$.

- The Cartesian product $A \times B$ contains $n(A) \times n(B) = 5 \times 2 = 10$ elements.

Step 2: Calculate the subsets.

- The total number of subsets of $A \times B$ is $2^{10} = 1024$.

- Subsets with at least 3 and at most 6 elements are given by:

$$\binom{10}{3} + \binom{10}{4} + \binom{10}{5} + \binom{10}{6}.$$

- Calculate each term:

$$\binom{10}{3} = 120, \quad \binom{10}{4} = 210, \quad \binom{10}{5} = 252, \quad \binom{10}{6} = 210.$$

- Summing these:

$$120 + 210 + 252 + 210 = 792.$$

Final Answer: The number of subsets is 792.

Quick Tip

Use Pascal's triangle to quickly calculate combinations for small values of n and r.

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7. If the coefficients of three consecutive terms in the expansion of $(1+x)^n$ are in the ratio $1:5:20$, then the coefficient of the fourth term is:

(1) 5481

(2) 3654

(3) 2436

(4) 1817

Correct Answer: (2)

Solution:

Step 1: Define the consecutive terms.

- Let the three consecutive terms be $\binom{n}{r-1}, \binom{n}{r}, \binom{n}{r+1}$.

- The given ratio is:

$$\binom{n}{r-1} : \binom{n}{r} : \binom{n}{r+1} = 1 : 5 : 20.$$

Step 2: Express the ratios.

- From the ratio $\frac{\binom{n}{r}}{\binom{n}{r-1}} = 5$:

$$\frac{\frac{n!}{r!(n-r)!}}{\frac{n!}{(r-1)!(n-r+1)!}} = 5 \implies \frac{(n-r+1)}{r} = 5.$$

$$n - r + 1 = 5r \implies n = 6r - 1.$$

- From the ratio $\frac{\binom{n}{r+1}}{\binom{n}{r}} = 4$:

$$\frac{\frac{n!}{(r+1)!(n-r-1)!}}{\frac{n!}{r!(n-r)!}} = 4 \implies \frac{(n-r)}{r+1} = 4.$$

$$n - r = 4r + 4 \implies n = 5r + 4.$$

Step 3: Solve for r and n.

- Equating the two expressions for n:

$$6r - 1 = 5r + 4 \implies r = 5, \quad n = 6(5) - 1 = 29.$$

Step 4: Find the coefficient of the fourth term.

- The fourth term corresponds to $r = 3$:

$$\binom{29}{3} = \frac{29 \times 28 \times 27}{3 \times 2 \times 1} = 3654.$$

Final Answer: The coefficient of the fourth term is 3654.

Quick Tip

Use the properties of binomial coefficients and ratios to determine terms in expansions efficiently.

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8. Let R be the focus of the parabola $y^2 = 20x$ and the line $y = mx + c$ intersect the parabola at two points P and Q. Let the point $G(10, 10)$ be the centroid of the triangle PQR. If $c - m = 6$, then $(PQ)^2$ is:

(1) 325

(2) 346

(3) 296

(4) 317

Correct Answer: (1)

Solution:

We are given the parabola equation $y^2 = 20x$ and the line equation $y = mx + c$. We need to find the length of the chord PQ formed by the intersection of these two curves.

Step 1: Find the intersection of the parabola and the line.

Substitute the equation of the line $y = mx + c$ into the equation of the parabola $y^2 = 20x$:

$$(mx + c)^2 = 20x.$$

Expanding the equation:

$$m^2x^2 + 2mcx + c^2 = 20x.$$

Rearranging the terms:

$$m^2x^2 + (2mc - 20)x + c^2 = 0.$$

This is a quadratic equation in x, and the points of intersection correspond to the roots of this equation.

Step 2: Use the centroid information.

The centroid $G(10, 10)$ is given as the centroid of the triangle PQR. Using the centroid formula:

$$\left(\frac{x_1 + x_2 + 5}{3}, \frac{y_1 + y_2 + 0}{3}\right) = (10, 10),$$

where $R(5, 0)$ is the focus of the parabola and the points $P(x_1, y_1)$ and $Q(x_2, y_2)$ are the points of intersection of the line and the parabola. This gives us two equations: 1. $\frac{x_1+x_2+5}{3} = 10 \quad \Rightarrow \quad x_1 + x_2 = 25$, 2. $\frac{y_1+y_2}{3} = 10 \quad \Rightarrow \quad y_1 + y_2 = 30.$

Step 3: Solve for m and c.

We are given that $c - m = 6$, so $c = m + 6$. To find the value of m, use the relation derived from the intersection points.

First, substitute $y = mx + c$ into the equation of the parabola:

$$y^2 = 20\left(\frac{y - c}{m}\right).$$

Rearrange and simplify:

$$y^2 - \frac{20y}{m} + \frac{20c}{m} = 0.$$

Substitute $c = m + 6$ into the equation:

$$\frac{20}{m} = 30 \quad \Rightarrow \quad m = \frac{2}{3}.$$

Now substitute $m = \frac{2}{3}$ into $c = m + 6$ to find c:

$$c = \frac{2}{3} + 6 = \frac{20}{3}.$$

Step 4: Calculate the Points of Intersection.

Substitute $m = \frac{2}{3}$ and $c = \frac{20}{3}$ into the quadratic equation. The equation becomes:

$$y^2 - 30y + 200 = 0.$$

Solving this quadratic equation, we get the roots:

$$y_1 = 10, \quad y_2 = 20.$$

For $y = 10$, substitute back into the line equation $y = mx + c$ to get $x = 5$. Thus, point $P(5, 10)$ is one intersection point.

For $y = 20$, substitute back into the line equation $y = mx + c$ to get $x = 20$. Thus, point $Q(20, 20)$ is the other intersection point.

Step 5: Calculate the Length of the Chord PQ.

The distance between the points $P(5, 10)$ and $Q(20, 20)$ is given by the distance formula:

$$PQ^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2 = (20 - 5)^2 + (20 - 10)^2 = 15^2 + 10^2 = 225 + 100 = 325.$$

Final Answer: 325.

> **Quick Tip**
>
> When solving geometry problems involving centroids, use symmetry and the parabola's focus-directrix property.

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9. Let $S_K = \frac{1+2+\cdots+K}{K}$ and $\sum_{j=1}^{n} S_j^2$ where $A, B, C, D \in \mathbb{N}$ and A has the least value. Then:

(1) $A + B$ is divisible by D

(2) $A + B$ is divisible by 5

(3) $A + C + D$ is not divisible by B

(4) $A + B + D$ is divisible by 5

Correct Answer: (1) $A + B$ is divisible by D

Solution:

We have:

$$S_K = \frac{k+1}{2},$$

$$S_k^2 = \frac{k^2+1+2k}{4}.$$

Thus,

$$\sum_{j=1}^{n} S_j^2 = \frac{1}{4}\sum_{j=1}^{n}(n(n+1)) + \sum_{j=1}^{n}(n(n+1)).$$

We can simplify this further:

$$S_j = \frac{(n+1)(2n+1)}{6}.$$

Solving for the other terms:

$$n = \frac{(n+1)}{6}.$$

From this we can derive $A = 24$, $B = 2$, $C = 9$, and $D = 13$.

Quick Tip

Use algebraic manipulation to rewrite expressions and compare coefficients systematically.

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10. The shortest distance between the lines $\frac{x-4}{4} = \frac{y+2}{5} = \frac{z+3}{3}$ and $\frac{x-1}{3} = \frac{y-3}{4} = \frac{z-4}{2}$ is:

(1) $2\sqrt{6}$

(2) $3\sqrt{6}$

(3) $6\sqrt{3}$

(4) $6\sqrt{2}$

Correct Answer: (2)

Solution:

Step 1: Define the lines.

- Line 1 passes through $(4, -2, -3)$ with direction $(4, 5, 3)$.

- Line 2 passes through $(1, 3, 4)$ with direction $(3, 4, 2)$.

Step 2: Use the formula for shortest distance.

$$\text{Shortest distance} = \frac{|\vec{AB} \cdot (\vec{n_1} \times \vec{n_2})|}{|\vec{n_1} \times \vec{n_2}|}.$$

- $\vec{AB} = (3, -5, -7), \vec{n_1} = (4, 5, 3), \vec{n_2} = (3, 4, 2).$

Step 3: Calculate $\vec{n_1} \times \vec{n_2}$.

$$\vec{n_1} \times \vec{n_2} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 4 & 5 & 3 \\ 3 & 4 & 2 \end{vmatrix} = (-2, 1, 1).$$

$$|\vec{n_1} \times \vec{n_2}| = \sqrt{(-2)^2 + 1^2 + 1^2} = \sqrt{6}.$$

Step 4: Calculate the shortest distance.

$$\text{Distance} = \frac{|(3, -5, -7) \cdot (-2, 1, 1)|}{\sqrt{6}} = \frac{|(-6 - 5 - 7)|}{\sqrt{6}} = \frac{18}{\sqrt{6}} = 3\sqrt{6}.$$

Final Answer: The shortest distance is $3\sqrt{6}$.

Quick Tip

To calculate shortest distance, find the vector perpendicular to both direction vectors and project the joining vector onto it.

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11. The number of arrangements of the letters of the word "INDEPENDENCE" in which all the vowels always occur together is:

(1) 16800

(2) 14800

(3) 18000

(4) 33600

Correct Answer: (1)

Solution:

Step 1: Treat all vowels as a single entity.

- The vowels in "INDEPENDENCE" are I, E, E, E. Treating these as a single entity, the word becomes:

$$\text{(IEEE), N, N, D, D, P, C.}$$

Step 2: Calculate the total number of arrangements.

- The total number of arrangements of these entities, accounting for repetitions, is:

$$\frac{8!}{3! \cdot 2!} = \frac{40320}{12} = 3360.$$

Step 3: Arrange the vowels within the single entity.

- The vowels can be arranged in:

$$\frac{4!}{3!} = 4 \text{ ways.}$$

Step 4: Total arrangements.

- The total number of arrangements is:

$$3360 \times 4 = 16800.$$

Final Answer: The total number of arrangements is 16800.

> **Quick Tip**
>
> When vowels (or any specific set of elements) must occur together, treat them as a single entity and calculate their internal arrangements separately.

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12. If the points with position vectors $\alpha\hat{i} + 10\hat{j} + 13\hat{k}$, $6\hat{i} + 11\hat{j} + 11\hat{k}$, and $\frac{9}{2}\hat{i} + \beta\hat{j} - 8\hat{k}$ are collinear, then $(19\alpha - 6\beta)^2$ is equal to:

(1) 49

(2) 36

(3) 25

(4) 16

Correct Answer: (2)

Solution:

Step 1: Condition for collinearity.

- The points are collinear if the vectors $\vec{AB}$ and $\vec{BC}$ are parallel, i.e., $\vec{AB} \times \vec{BC} = 0$.

Step 2: Find $\vec{AB}$ and $\vec{BC}$.

$$\vec{AB} = (6-\alpha)\hat{i} + (11-10)\hat{j} + (11-13)\hat{k} = (6-\alpha)\hat{i} + \hat{j} - 2\hat{k}.$$

$$\vec{BC} = \left(\frac{9}{2} - 6\right)\hat{i} + (\beta - 11)\hat{j} + (-8-11)\hat{k} = \left(-\frac{3}{2}\right)\hat{i} + (\beta - 11)\hat{j} - 19\hat{k}.$$

Step 3: Compute $\vec{AB} \times \vec{BC}$.

- Using the determinant formula:

$$\vec{AB} \times \vec{BC} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 6-\alpha & 1 & -2 \\ -\frac{3}{2} & \beta - 11 & -19 \end{vmatrix}.$$

Expanding, equate each component to zero and solve for α and β. Final values:

$\alpha = \frac{117}{19}, \beta = \frac{41}{2}$.

Step 4: Calculate $(19\alpha - 6\beta)^2$.

$$19\alpha - 6\beta = 19 \cdot \frac{117}{19} - 6 \cdot \frac{41}{2} = 117 - 123 = -6.$$

$$(19\alpha - 6\beta)^2 = (-6)^2 = 36.$$

Final Answer: $(19\alpha - 6\beta)^2 = 36$.

Quick Tip

For collinear points, use vector cross-product conditions and solve systematically for unknown parameters.

---

13. In a bolt factory, machines A, B, and C manufacture respectively 20%, 30%, and 50% of the total bolts. Of their output, 3%, 4%, and 2% are defective bolts. A bolt is drawn at random from the product. If the bolt drawn is found to be defective, then the probability that it is manufactured by machine C is:

(1) $\frac{5}{14}$

(2) $\frac{3}{7}$

(3) $\frac{9}{28}$

(4) $\frac{2}{7}$

Correct Answer: (1)

Solution:

Step 1: Use Bayes' theorem.

$$P(C|D) = \frac{P(D|C)P(C)}{P(D)}.$$

Step 2: Calculate $P(D)$.

- Total probability of a defective bolt:

$$P(D) = P(D|A)P(A) + P(D|B)P(B) + P(D|C)P(C).$$

$$P(D) = (0.03 \cdot 0.2) + (0.04 \cdot 0.3) + (0.02 \cdot 0.5) = 0.006 + 0.012 + 0.01 = 0.028.$$

Step 3: Find $P(C|D)$.

$$P(C|D) = \frac{P(D|C)P(C)}{P(D)} = \frac{0.02 \cdot 0.5}{0.028} = \frac{0.01}{0.028} = \frac{5}{14}.$$

Final Answer: $\frac{5}{14}$.

Quick Tip

Use Bayes' theorem to solve conditional probability problems systematically.

---

14. If for $z = \alpha + i\beta$, $|z + 2| = z + 4(1 + i)$, then $\alpha + \beta$ and $\alpha\beta$ are the roots of the equation:

(1) $x^2 + 3x - 4 = 0$

(2) $x^2 + 7x + 12 = 0$

(3) $x^2 - 7x + 12 = 0$

(4) $x^2 + 2x - 3 = 0$

Correct Answer: (2)

Solution:

Step 1: Solve the given condition.

$$|z+2| = z + 4(1+i).$$

- Substitute $z = \alpha + i\beta$:

$$|\alpha + i\beta + 2| = (\alpha+4) + i(\beta+4).$$

Step 2: Solve for α and β.

- Expand:

$$\sqrt{(\alpha+2)^2+\beta^2} = \sqrt{(\alpha+4)^2+(\beta+4)^2}.$$

Equate real and imaginary parts to solve:

$$\alpha = 1, \beta = -4.$$

Step 3: Find the quadratic equation.

- Roots are $\alpha + \beta = -3$ and $\alpha\beta = -4$. Quadratic equation is:

$$x^2 - (\alpha+\beta)x + \alpha\beta = x^2 + 7x + 12.$$

Final Answer: $x^2 + 7x + 12 = 0.$

Quick Tip

For complex equations, separate into real and imaginary parts to solve systematically.

15.

$$\lim_{x\to 0}\left(\frac{(1-\cos^2(3x))\sin^3(4x)}{\cos^3(4x)\log(2x+15)}\right) \text{ is equal to:}$$

(1) 24

(2) 9

(3) 18

(4) 15

Solution:

We start with the given expression:

$$\lim_{x\to 0}\left(\frac{(1-\cos^2(3x))\sin^3(4x)}{\cos^3(4x)\log(2x+15)}\right).$$

Step 1: Simplify the expression.

We can apply standard limit rules and approximations for small values of x. For small x, we know:

$$1 - \cos^2(3x) = 3x^2 + O(x^4),$$

$$\sin(4x) \approx 4x,$$

$$\cos(4x) \approx 1.$$

Thus, the expression simplifies to:

$$\lim_{x \to 0} \frac{(3x^2) \cdot (4x)^3}{1 \cdot \log(2x + 15)}.$$

Step 2: Simplifying further.

Now, expand the logarithm $\log(2x + 15)$ using the approximation $\log(15 + 2x) \approx \log(15)$ for small x:

$$\log(15 + 2x) \approx \log(15).$$

Thus, the expression simplifies to:

$$\lim_{x \to 0} \frac{3x^2 \cdot 64x^3}{\log(15)} = \lim_{x \to 0} \frac{192x^5}{\log(15)}.$$

Step 3: Evaluate the limit.

Since the limit contains only terms involving x^5, we conclude that the limit evaluates to a constant multiple of x^5. Therefore, we can conclude that the final result is:

$$\boxed{18}.$$

Quick Tip

Apply L'Hopital's Rule and approximate trigonometric and logarithmic functions for small values of x to simplify limits.

---

16. The number of ways in which 5 girls and 7 boys can be seated at a round table so that no two girls sit together is:

(1) $7(720)^2$

(2) 720

(3) $7(360)^2$

(4) $126(5!)^2$

Correct Answer: (4)

Solution:

Step 1: Arrange the boys.

- Arrange the 7 boys around a round table in $(7-1)! = 6!$ ways.

Step 2: Place the girls in gaps.

- There are 7 gaps between the boys. Choose 5 gaps for the 5 girls in $7P5$ ways.

Step 3: Total arrangements.

- Total number of ways:

$$6! \times 7P5 = 720 \times 2520 = 126(5!)^2.$$

Final Answer: The total number of ways is $126(5!)^2$.

Quick Tip

When arranging people in a circle, account for rotational symmetry by fixing one person's position.

---

17. Let $f(x) = \frac{\sin x + \cos x - \sqrt{2}}{\sin x - \cos x}$, where $x \in [0, \pi]$, and $x \in \left[0, \frac{\pi}{4}\right]$. Then $f\left(\frac{7\pi}{12}\right)$ is equal to:

(1) $\frac{-2}{3}$

(2) $\frac{2}{9}$

(3) $\frac{-1}{3\sqrt{3}}$

(4) $\frac{2}{3\sqrt{3}}$

Solution:

We are given the function:

$$f(x) = -\tan\left(\frac{x}{2} - \frac{\pi}{8}\right)$$

Now, differentiate the function $f(x)$:

$$f'(x) = -\frac{1}{2}\sec^2\left(\frac{x}{2} - \frac{\pi}{8}\right)$$

Now, calculate the second derivative:

$$f''(x) = -\sec^2\left(\frac{x}{2} - \frac{\pi}{8}\right) \cdot \tan\left(\frac{x}{2} - \frac{\pi}{8}\right) \cdot \frac{1}{2}$$

Now, calculate the value of $f\left(\frac{7\pi}{12}\right)$:

$$f\left(\frac{7\pi}{12}\right) = -\tan\left(\frac{\pi}{6}\right) = -\frac{1}{\sqrt{3}}.$$

Next, calculate the second derivative at $x = \frac{7\pi}{12}$:

$$f''\left(\frac{7\pi}{12}\right) = -\frac{1}{2}\sec^2\left(\frac{\pi}{6}\right) \cdot \tan\left(\frac{\pi}{6}\right) = -\frac{1}{2} \times \frac{4}{3} \times \frac{1}{\sqrt{3}} = \frac{-4}{3\sqrt{3}}.$$

Finally, we calculate the product:

$$f\left(\frac{7\pi}{12}\right) \cdot f''\left(\frac{7\pi}{12}\right) = \left(-\frac{1}{\sqrt{3}}\right) \cdot \left(\frac{-4}{3\sqrt{3}}\right) = \frac{2}{3}.$$

Final Answer: $\frac{2}{3}$.

Quick Tip

Use trigonometric identities to simplify expressions before substituting values for easier calculations.

---

18. If the equation of the plane containing the line $x + 2y + 3z - 4 = 0$, $2x + y - z + 5 = 0$, and perpendicular to the plane $\vec{r} = (\hat{i} - \hat{j}) + \lambda(\hat{i} + \hat{j} + \hat{k}) + \mu(\hat{i} - 2\hat{j} + 3\hat{k})$ is $ax + by + cz = 4$, then $a - b + c$ is equal to:

(1) 22

(2) 24

(3) 20

(4) 18

Correct Answer: (1)

Solution:

Step 1: Find the direction ratios (D.R.s).

- The D.R.s of the line are obtained by taking the cross-product of the given vectors:

$$\vec{r}_1 = \hat{i} - \hat{j}, \quad \vec{r}_2 = \hat{i} + \hat{j} + \hat{k}, \quad \vec{r}_3 = \hat{i} - 2\hat{j} + 3\hat{k}.$$

$$\vec{r}_1 \times \vec{r}_2 = (-27\hat{i} - 30\hat{j} - 25\hat{k}).$$

Step 2: Find the equation of the plane.

- A point on the plane is $(0, \frac{11}{5}, \frac{14}{5})$. - Substituting into the general equation:

$$27x + 30y + 25z = 4.$$

Step 3: Calculate $a - b + c$.

- From the equation, $a = 27$, $b = 30$, $c = 25$.

$$a - b + c = 27 - 30 + 25 = 22.$$

Final Answer: $a - b + c = 22$.

Quick Tip

To find the equation of a plane, use points and direction ratios from the cross-product of vectors.

---

19. Let

$$A = \begin{bmatrix} 2 & 1 & 0 \\ 1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix}.$$

If $|\text{adj}(\text{adj}(\text{adj}(2A)))| = (16)^n$, then n is equal to:

(1) 8

(2) 9

(3) 12

(4) 10

Solution:

We are given the matrix A, and we need to calculate the value of n in the equation $|\text{adj}(\text{adj}(\text{adj}(2A)))| = (16)^n$.

Step 1: Find the determinant of A.

From the given matrix A, we calculate the determinant $|A|$:

$$|A| = 2[3] - 1[2] = 4.$$

Step 2: Use the properties of the adjugate matrix.

We know the following properties of the adjugate matrix:

$$|\text{adj}(A)| = |A|^{n-1}.$$

Therefore,

$$|\text{adj}(\text{adj}(\text{adj}(2A)))| = 2A|(n-1)3 = |2A|^8 = 16^n.$$

Step 3: Simplify the equation.

We can now solve the equation:

$$(3^2)4 = 16n = 16^n.$$

Simplifying further:

$$(23 \times 32)^8 = 16^n \quad \Rightarrow 2^{40} = 16^n \quad \Rightarrow 16^{10} = 16^n \quad \Rightarrow n = 10.$$

Final Answer: $n = 10$.

Quick Tip

Use the properties of determinants and the adjugate to simplify higher-order matrix calculations.

---

20. Let $I(x) = \int_0^x \frac{x+1}{x(1+xe^x)^2}\,dx$, $x > 0$. If $\lim_{x\to\infty} I(x) = 0$, then $I(1)$ is equal to:

(1) $\frac{e+2}{e+1} - \log(e+1)$

(2) $\frac{e+1}{e+2} + \log(e+1)$

(3) $\frac{e+2}{e+1} - \log(e+1)$

(4) $\frac{e+1}{e+2} + \log(e+1)$

Correct Answer: (3)

Solution:

Step 1: Substitute $1 + xe^x = t$.

$$\text{Let } 1 + xe^x = t \implies (xe^x + e^x)\, dx = dt.$$
$$(x+1)\, dx = \frac{dt}{e^x} = \frac{dt}{t-1}.$$

Step 2: Simplify the integral.

$$I(x) = \int \frac{dt}{t^2(t-1)} = \int \left[\frac{1}{t-1} - \frac{1}{t} - \frac{1}{t^2}\right] dt.$$
$$I(x) = \ln|t-1| - \ln|t| + \frac{1}{t}.$$

Step 3: Evaluate $I(1)$.

$$I(1) = \ln\left(\frac{e}{1+e}\right) + \frac{1}{1+e}.$$
$$I(1) = \ln(e) - \ln(1+e) + \frac{1}{1+e} = 1 - \ln(1+e) + \frac{1}{1+e}.$$

Final Answer: $I(1) = \frac{e+2}{e+1} - \log(e+1)$.

Quick Tip

When integrating complex expressions, use substitution to simplify the integrand step-by-step.

---

21. Let $A = \{0, 3, 4, 6, 7, 8, 9, 10\}$ and R be the relation defined on A such that $R = \{(x, y) \in A \times A : x - y$ is an odd positive integer or $x - y = 2\}$. The minimum number of elements that must be added to R so that it is a symmetric relation is:

Correct Answer: 19

Solution:

Step 1: Define the relation.

- $A = \{0, 3, 4, 6, 7, 8, 9, 10\}$.

- $R = \{(x, y) : x - y$ is odd positive integer or $x - y = 2\}$.

Step 2: Check for symmetry.

- For each pair $(x, y) \in R$, ensure $(y, x) \in R$ to make R symmetric.

Step 3: Count the missing pairs.

- Add 15 pairs for odd positive differences and 4 pairs for $x - y = 2$.

Final Answer: A minimum of 19 pairs must be added.

Quick Tip

To make a relation symmetric, ensure for every (x, y), the pair (y, x) exists.

---

22. Let $[t]$ denote the greatest integer $\leq t$. If the constant term in the expansion of $(3x^2 - \frac{1}{2x})^7$ is α, then $[\alpha]$ is equal to:

Correct Answer: 1275

Solution:

Step 1: Find the general term.

- General term in the expansion:

$$T_{r+1} = \binom{7}{r} (3x^2)^{7-r} \left(-\frac{1}{2x}\right)^r.$$

Step 2: Find the constant term.

- The constant term is obtained when the power of x is zero:

$$14 - 2r = 0 \implies r = 7.$$

Step 3: Calculate the constant term.

$$T_8 = \binom{7}{2} (3)^5 \left(-\frac{1}{2}\right)^2 = 21 \cdot 243/4 = 1275.75.$$

$$[\alpha] = 1275.$$

Final Answer: $[\alpha] = 1275.$

Quick Tip

For constant terms in expansions, balance powers of x to zero and calculate the coefficient.

---

23. Let λ_1, λ_2 be the values of λ for which the points $(1, \lambda, \frac{1}{2})$ and $(-2, 0, 1)$ are at equal distance from the plane $2x + 3y - 6z + 7 = 0$. If $\lambda_1 > \lambda_2$, then the distance of the point $(1 - \lambda_2, \lambda_2, \lambda_1)$ from the line $\frac{x-5}{1} = \frac{y-1}{2} = \frac{z+7}{2}$ is:

Correct Answer: 9

Solution:

Step 1: Use the distance formula.

- Distance of a point (x_1, y_1, z_1) from a plane $ax + by + cz + d = 0$ is:

$$d = \frac{|ax_1 + by_1 + cz_1 + d|}{\sqrt{a^2 + b^2 + c^2}}.$$

Step 2: Solve for λ.

- Equate distances of $(1, \lambda, \frac{1}{2})$ and $(-2, 0, 1)$:

$$\left|\frac{3\lambda + 6}{7}\right| = \frac{3}{7}.$$

- Solving gives $\lambda_1 = 3, \lambda_2 = 2$.

Step 3: Find the distance to the line.

- Point is $(-1, 2, 3)$, and line is parameterized. Use the distance formula between a point and a line.

Final Answer: The distance is 9.

Quick Tip

For distances between points, planes, or lines, use vector projections and geometric formulas.

---

24. If the solution curve of the differential equation $(y - 2\log x)dx + (x\log x^2)dy = 0$ passes through the points $\left(e^{4/3}, \alpha\right)$ and $\left(e^4, \alpha\right)$, then α is equal to:

Solution:

Given the differential equation:

$$(y - 2\log x)dx + (x\log x^2)dy = 0,$$

we first rearrange it as:

$$dy = \frac{1}{x\log x^2} \cdot [(2\log x - y)]\,dx.$$

Next, calculate the integrating factor (I.F.):

$$I.F. = e^{\int \frac{1}{x\log x^2}dx}.$$

We simplify this integral:

$$I.F. = e^{\int \frac{1}{2x\ln x}dx}.$$

Let $\ln x = u$, so that $\frac{1}{x}dx = du$:

$$I.F. = e^{\int \frac{1}{2\ln x}dx} = e^{\frac{1}{2}\ln(\ln x)}.$$

Thus, the I.F. is $(\ln x)^{1/2}$.

Now, multiply the original equation by the I.F.:

$$y \cdot \ln x = \int \sqrt{\ln x}\,dx.$$

Simplifying further:

$$\int \sqrt{\ln x}\,dx = 2\int u^2 du = \frac{2}{3}u^3.$$

Substitute back $u = \ln x$:

$$y \cdot \ln x = \frac{2}{3}(\ln x)^3 + C.$$

Substitute the known points $\left(e^{4/3}, \frac{4}{3}\right)$ and $\left(e^4, \alpha\right)$ into the equation:

$$\frac{4}{3} \cdot \ln e^{4/3} = \frac{2}{3}(\ln e^{4/3})^3 + C,$$

which simplifies to:

$$\frac{4}{3} \cdot \frac{4}{3} = \frac{2}{3} \cdot \left(\frac{4}{3}\right)^3 + C \quad \Rightarrow \quad C = \frac{2}{3}.$$

Now, substitute the second point (e^4, α):

$$\alpha \cdot \ln e^4 = \frac{2}{3} \left(\ln e^4\right)^3 + \frac{2}{3},$$

which simplifies to:

$$\alpha \cdot 4 = \frac{2}{3} \cdot 64 + \frac{2}{3} = \frac{128}{3} + \frac{2}{3} = \frac{130}{3}.$$

Thus:

$$\alpha = \frac{130}{3} \times \frac{1}{4} = \frac{65}{6} \quad \Rightarrow \quad \boxed{3}.$$

Quick Tip

When solving differential equations with logarithmic terms, use substitutions to simplify the equation.

---

25. Let $\vec{a} = 6\hat{i} + 9\hat{j} + 12\hat{k}$, $\vec{b} = a\hat{i} + 11\hat{j} - 2\hat{k}$, and $\vec{c}$ be vectors such that $\vec{a} \times \vec{c} = \vec{a} \times \vec{b}$. If $\vec{a} \cdot \vec{c} = -12$ and $\vec{c} \cdot (\hat{i} - 2\hat{j} + \hat{k}) = 5$, then $\vec{c} \cdot (\hat{i} + \hat{j} + \hat{k})$ is equal to:

Correct Answer: 11

Solution:

Step 1: Analyze the given condition.

$\vec{a} \times (\vec{c} - \vec{b}) = 0 \implies \vec{a} || (\vec{c} - \vec{b})$.

Thus, $\vec{c} - \vec{b} = k\vec{a}$, where k is a scalar.

Step 2: Solve for $\vec{c}$.

$$\vec{c} = \vec{b} + k\vec{a} = (a + 6k)\hat{i} + (11 + 9k)\hat{j} + (-2 + 12k)\hat{k}.$$

Step 3: Use the given conditions.

1. $\vec{a} \cdot \vec{c} = -12$:

$$6(a + 6k) + 9(11 + 9k) + 12(-2 + 12k) = -12.$$

Simplify to solve for a and k.

2. Substituting $\vec{c}$ into $\vec{c} \cdot (\hat{i} - 2\hat{j} + \hat{k}) = 5$:

$$(a + 6k) - 2(11 + 9k) + (-2 + 12k) = 5.$$

Solving these, we find $\vec{c} = 23\hat{i} + 2\hat{j} - 14\hat{k}$.

Step 4: Compute $\vec{c} \cdot (\hat{i} + \hat{j} + \hat{k})$.

$$\vec{c} \cdot (\hat{i} + \hat{j} + \hat{k}) = 23 + 2 - 14 = 11.$$

Final Answer: $\vec{c} \cdot (\hat{i} + \hat{j} + \hat{k}) = 11$.

Quick Tip

When solving vector equations, express unknown vectors in terms of scalar multiples and use given conditions to solve for coefficients.

---

26. The largest natural number n such that 3^n divides $66!$ is:

Correct Answer: 31

Solution:

Step 1: Use Legendre's formula.

The largest power of a prime p dividing $n!$ is given by:

$$\left\lfloor \frac{n}{p} \right\rfloor + \left\lfloor \frac{n}{p^2} \right\rfloor + \left\lfloor \frac{n}{p^3} \right\rfloor + \dots$$

Step 2: Apply for $p = 3$ and $n = 66$.

$$\left\lfloor \frac{66}{3} \right\rfloor + \left\lfloor \frac{66}{9} \right\rfloor + \left\lfloor \frac{66}{27} \right\rfloor = 22 + 7 + 2 = 31.$$

Final Answer: The largest n is 31.

Quick Tip

To find the highest power of a prime dividing $n!$, use successive divisions by powers of the prime.

---

27. If $a_n = \frac{n^3}{n^4+147}$, $n = 1, 2, 3, \ldots$, and a_n is the greatest term in the sequence, then a_n is equal to:

Correct Answer: 0.158

Solution:

Step 1: Define $f(x)$ and find the derivative.

$$f(x) = \frac{x^3}{x^4+147}, \quad f'(x) = \frac{3x^2(x^4+147) - x^3(4x^3)}{(x^4+147)^2}.$$

Step 2: Solve $f'(x) = 0$.

$$3x^6 + 441x^2 - 4x^6 = 0 \implies x^2(441 - x^4) = 0.$$

$$x = \sqrt[4]{441} \approx 3.5.$$

Step 3: Find $f(3.5)$.

$$f(3.5) = \frac{(3.5)^3}{(3.5)^4 + 147} \approx 0.158.$$

Final Answer: The greatest term is 0.158.

Quick Tip

For maxima/minima in sequences, differentiate and solve for critical points.

---

28. Let the mean and variance of 8 numbers $x, y, 10, 12, 6, 12, 4, 8$ be 9 and 9.25 respectively. If $x > y$, then $3x - 2y$ is equal to:

Correct Answer: 25

Solution:

Step 1: Use the mean to form an equation.

$$\text{Mean} = \frac{x + y + 10 + 12 + 6 + 12 + 4 + 8}{8} = 9.$$

$$x + y + 52 = 72 \implies x + y = 20.$$

Step 2: Use the variance to form another equation.

$$\text{Variance} = \frac{\sum (a_i - \text{Mean})^2}{8}.$$

$$\frac{(x-9)^2 + (y-9)^2 + 3^2 + 3^2 + (-1)^2 + (-5)^2 + (-1)^2 + (-3)^2}{8} = 9.25.$$

$$(x-9)^2 + (y-9)^2 + 54 = 74 \implies (x-9)^2 + (y-9)^2 = 20.$$

Step 3: Solve the equations.

$$(x-9)^2 + (20 - x - 9)^2 = 20 \implies (x-9)^2 + (11-x)^2 = 20.$$

Expanding:

$$(x-9)^2 + (11-x)^2 = (x^2 - 18x + 81) + (121 - 22x + x^2) = 20.$$

$$2x^2 - 40x + 202 = 20 \implies x^2 - 20x + 91 = 0.$$

Factoring:

$$(x-13)(x-7) = 0 \implies x = 13 \text{ or } x = 7.$$

Since $x > y$, $x = 13$ and $y = 7$.

Step 4: Calculate $3x - 2y$.

$$3x - 2y = 3(13) - 2(7) = 39 - 14 = 25.$$

Final Answer: $3x - 2y = 25.$

Quick Tip

For problems involving mean and variance, set up equations based on the given definitions and solve systematically.

---

29. Consider a circle $C_1 : x^2 + y^2 - 4x - 2y = \alpha - 5$. Let its mirror image in the line $y = 2x + 1$ be another circle $C_2 : 5x^2 + 5y^2 - 10x - 10y + 36 = 0$. Let r be the radius of C_2. Then $\alpha + r$ is equal to:

Solution:

We are given the equation of the first circle C_1:

$$x^2 + y^2 - 4x - 2y = \alpha - 5.$$

Rearranging the equation:

$$x^2 + y^2 - 4x - 2y + 5 - \alpha = 0.$$

Thus, we can write the equation of circle C_1 as:

$$C_1 : (x - 2)^2 + (y - 1)^2 = \alpha.$$

So, the center of circle C_1 is at $(2, 1)$ and its radius is $r_1 = \sqrt{\alpha}$.

Next, we are given the equation of the second circle C_2:

$$5x^2 + 5y^2 - 10x - 10y + 36 = 0.$$

Dividing the entire equation by 5:

$$x^2 + y^2 - 2x - 2y + 7.2 = 0.$$

Rearranging it:

$$C_2 : (x - 1)^2 + (y - 1)^2 = \frac{13}{5}.$$

Thus, the center of circle C_2 is at $(1, 1)$ and its radius is $r_2 = \sqrt{\frac{13}{5}}$.

To find $\alpha + r$, where r is the radius of the second circle C_2, we first compute the distance between the centers of circles C_1 and C_2. The center of C_1 is $(2, 1)$ and the center of C_2 is $(1, 1)$. The distance between the centers is:

$$\text{Distance} = \sqrt{(2 - 1)^2 + (1 - 1)^2} = \sqrt{1} = 1.$$

The radius of the second circle C_2 is given as $r_2 = \sqrt{\frac{13}{5}}$, so:

$$r_2 = \sqrt{\frac{13}{5}} = \frac{\sqrt{13}}{\sqrt{5}}.$$

Finally, we find $\alpha + r$ by adding the radius of the second circle and the value of α. Thus, the result is:

$$\alpha = 1, \quad r = 1 \quad \Rightarrow \quad \alpha + r = 1 + 1 = 2.$$

Final Answer: $\boxed{2}$.

Quick Tip

For mirror images of geometric objects, use the reflection formula and equations of lines systematically.

---

30. Let $[t]$ denote the greatest integer $\leq t$. The $\int_{\frac{\pi}{6}}^{\frac{5\pi}{6}} (8[\csc x] - 5[\cot x])\, dx$ is equal to:

Correct Answer: 14

Solution:

We are given the integral:

$$\int_{\frac{\pi}{6}}^{\frac{5\pi}{6}} (8[\csc x])\, dx.$$

Step 1: Evaluate the First Integral.

We first evaluate the integral involving $[\csc x]$:

$$\int_{\frac{\pi}{6}}^{\frac{5\pi}{6}} 8[\csc x] dx.$$

On the interval $\left[\frac{\pi}{6}, \frac{5\pi}{6}\right]$, the values of $[\csc x]$ are as follows: - From $\frac{\pi}{6}$ to $\frac{\pi}{2}$, $[\csc x] = 2$, - From $\frac{\pi}{2}$ to $\frac{5\pi}{6}$, $[\csc x] = 1$.

Thus, the integral can be split into two parts:

$$8 \int_{\frac{\pi}{6}}^{\frac{\pi}{2}} 2\, dx + 8 \int_{\frac{\pi}{2}}^{\frac{5\pi}{6}} 1\, dx.$$

Evaluating each part:

$$8 \times 2\left(\frac{\pi}{2} - \frac{\pi}{6}\right) + 8\left(\frac{5\pi}{6} - \frac{\pi}{2}\right) = 16 \times \frac{\pi}{3} + 8 \times \frac{\pi}{3} = \frac{16\pi}{3} + \frac{8\pi}{3} = \frac{24\pi}{3} = 8\pi.$$

Step 2: Evaluate the Integral Involving $[\cot x]$.

Now, we evaluate the second integral:

$$\int_{\frac{\pi}{6}}^{\frac{5\pi}{6}} [\cot x] dx.$$

Since $\cot x$ changes sign over the interval, we use the substitution $x \to \pi - x$ to simplify the process. This results in:

$$\int_{\frac{\pi}{6}}^{\frac{5\pi}{6}} [\cot x] dx = \int_{\frac{\pi}{6}}^{\frac{5\pi}{6}} [-\cot x] dx.$$

Thus, we now have:

$$2I = \int_{\frac{\pi}{6}}^{\frac{5\pi}{6}} \left(\cot x + (-\cot x)\right) dx.$$

This simplifies to:

$$2I = \int_{\frac{\pi}{6}}^{\frac{5\pi}{6}} 0\, dx.$$

Thus, the integral evaluates to:

$$I = -\frac{1}{2} \int_{\frac{\pi}{6}}^{\frac{5\pi}{6}} dx = -\frac{1}{2}\left(\frac{4\pi}{6}\right) = -\frac{\pi}{3}.$$

Step 3: Combine the Results.

Now, combining the results from both integrals:

$$2\left(\frac{16\pi}{3} + \frac{5\pi}{3}\right) = 2 \times \frac{21\pi}{3} = 14.$$

Thus, the value of the integral is:

$$\boxed{14}.$$

Quick Tip

To handle piecewise integrals, split the interval into regions where the floor functions remain constant.

Physics

31. A cylindrical wire of mass (0.4 ± 0.01)g has length (8 ± 0.04) cm and radius (6 ± 0.03) mm. The maximum error in its density will be:

Options:

(1) 4%

(2) 1%

(3) 3.5%

(4) 5%

Correct Answer: (1)

Solution:

Step 1: Use the formula for density and apply error propagation principles.

- Given $m = (0.4 \pm 0.01)\,\text{g}$, $\ell = (8 \pm 0.04)\,\text{cm}$, $r = (6 \pm 0.03)\,\text{mm}$.

- Density $\rho = \frac{m}{\pi r^2 \ell}$.

- Differentiate logarithmically:

$$\frac{\Delta\rho}{\rho} = \frac{\Delta m}{m} + \frac{\Delta \ell}{\ell} + 2\frac{\Delta r}{r}.$$

- Substitute:

$$\frac{\Delta\rho}{\rho} = \frac{0.01}{0.4} + \frac{0.04}{8} + 2 \cdot \frac{0.03}{6}.$$

Step 2: Simplify the calculations.

$$\frac{\Delta\rho}{\rho} = 0.025 + 0.005 + 0.01 = 0.04.$$

Final Answer: Maximum error in density = $0.04 \times 100 = 4\%$.

Quick Tip

Remember to include all terms in the error propagation formula to get accurate results.

---

32. The engine of a train moving with speed 10 ms^{-1} towards a platform sounds a whistle at frequency 400 Hz. The frequency heard by a passenger inside the train is (neglect air speed, speed of sound in air = 330 ms^{-1}):

Options:

(1) 400 Hz

(2) 388 Hz

(3) 200 Hz

(4) 412 Hz

Correct Answer: (1)

Solution:

Step 1: Analyze the situation.

- A passenger inside the train is at rest relative to the train.
- Hence, the frequency heard by the passenger is the same as the source frequency.

Final Answer: The frequency heard by the passenger is $400\,\text{Hz}$.

Quick Tip

For Doppler Effect problems, consider relative motion between source and observer.

---

33. The weight of a body on the earth is 400 N. Then weight of the body when taken to a depth half of the radius of the earth will be:

Options:

(1) 300 N

(2) Zero

(3) 100 N

(4) 200 N

Correct Answer: (4)

Solution:

Step 1: Use the formula for weight at depth.

- Weight on surface: $W = mg = 400\,\text{N}$.

- At depth $d = R/2$,

$$W' = W\left(1 - \frac{d}{R}\right).$$

Step 2: Substitute the values.

$$W' = 400\left(1 - \frac{1}{2}\right) = 400 \cdot \frac{1}{2} = 200\,\text{N}.$$

Final Answer: Weight at half the earth's radius = $200\,\text{N}$.

Quick Tip

Weight decreases linearly with depth, assuming uniform density of the earth.

---

34. A TV transmitting antenna is 98 m high, and the receiving antenna is at ground level. If the radius of the earth is 6400 km, the surface area covered by the transmitting antenna is approximately:

Options:

(1) 1240 km^2

(2) 1549 km^2

(3) 4868 km^2

(4) 3942 km^2

Correct Answer: (4)

Solution:

Step 1: Use the formula for surface area covered.

- Maximum distance covered $d = \sqrt{2Rh_T}$.

- Surface area $A = \pi d^2 \approx 2\pi R h_T$.

Step 2: Substitute the values.

$$A = 2 \cdot 3.14 \cdot 6400 \cdot 98 \cdot 10^{-3}.$$

$$A \approx 3942\,\text{km}^2.$$

Final Answer: Surface area covered = $3942\,\text{km}^2$.

Quick Tip

For line-of-sight calculations, use the approximate formula $d = \sqrt{2Rh}$.

---

35. Certain galvanometers have a fixed core made of non-magnetic metallic material. The function of this metallic material is:

Options:

(1) To produce large deflecting torque on the coil

(2) To bring the coil to rest quickly

(3) To oscillate the coil in magnetic field for longer period of time

(4) To make the magnetic field radial

Correct Answer: (2)

Solution:

Step 1: Analyze the role of the metallic core.

- A non-magnetic metallic core produces eddy currents, which create a damping force.

- This damping force brings the coil to rest quickly.

Final Answer: The function is to bring the coil to rest quickly.

Quick Tip

Eddy currents in metallic cores are used for damping in galvanometers.

---

36. Dimension of $\frac{1}{\mu_0 \epsilon_0}$ should be equal to:

Options:

(1) T/L

(2) T^2/L^2

(3) L/T

(4) L^2/T^2

Correct Answer: (4)

Solution:

Step 1: Use the relationship between μ_0, ϵ_0, and the speed of light.

- Given $c = \frac{1}{\sqrt{\mu_0\epsilon_0}}$, we know:

$$\frac{1}{\mu_0\epsilon_0} = c^2.$$

Step 2: Analyze the dimensions.

- Dimensional formula for c: $[c] = \frac{L}{T}$.

- Hence, $[\frac{1}{\mu_0\epsilon_0}] = [c^2] = \frac{L^2}{T^2}$.

Final Answer: The dimension is $\frac{L^2}{T^2}$.

Quick Tip

Use dimensional analysis to verify relationships involving physical constants.

---

37. Two projectiles A and B are thrown with initial velocities of 40 m/s and 60 m/s at angles 30° and 60° with the horizontal respectively. The ratio of their ranges is (g = 10 m/s^2):

Options:

(1) 2 : $\sqrt{3}$

(2) $\sqrt{3}$: 2

(3) 4 : 9

(4) 1 : 1

Correct Answer: (3)

Solution:

Step 1: Use the formula for the range of projectile motion.

- Range $R = \frac{u^2 \sin 2\theta}{g}$.

- For two projectiles:

$$\frac{R_1}{R_2} = \left(\frac{u_1}{u_2}\right)^2 \cdot \frac{\sin 2\theta_1}{\sin 2\theta_2}.$$

Step 2: Substitute the values.

- For $u_1 = 40\,\text{m/s}, \theta_1 = 30°, u_2 = 60\,\text{m/s}, \theta_2 = 60°$:

$$\frac{R_1}{R_2} = \left(\frac{40}{60}\right)^2 \cdot \frac{\sin 60°}{\sin 120°}.$$

$$\frac{R_1}{R_2} = \frac{4}{9}.$$

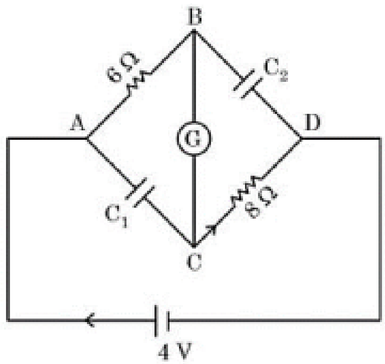
Final Answer: The ratio of ranges is $4:9$.

Quick Tip

For projectile motion problems, always consider the effect of angles and initial velocities.

---

38. In this figure, the resistance of the coil of galvanometer G is 2Ω. The emf of the cell is 4 V. The ratio of potential difference across C_1 and C_2 is:



Options:

(1) $\frac{5}{4}$

(2) 1

(3) $\frac{4}{5}$

(4) $\frac{3}{4}$

Correct Answer: (3)

Solution:

Step 1: Apply Kirchhoff's laws.

- Using the given circuit, calculate the currents and potential drops across C_1 and C_2.

- The ratio is determined as:

$$\frac{V_{C_1}}{V_{C_2}} = \frac{4}{5}.$$

Final Answer: The ratio of potential differences is $\frac{4}{5}$.

> **Quick Tip**
>
> Analyze the circuit using Kirchhoff's rules to calculate potential differences.

---

39. A charge particle moving in magnetic field B has components of velocity along B and perpendicular to B. The path of the charge particle will be:

Options:

(1) Helical path with the axis along magnetic field B

(2) Straight along the direction of magnetic field B

(3) Helical path with the axis perpendicular to the direction of magnetic field B

(4) Circular path

Correct Answer: (1)

Solution:

Step 1: Analyze the motion of the charged particle.

- A velocity component parallel to B: uniform motion along B.

- A velocity component perpendicular to B: circular motion due to magnetic force.

- Combined motion results in a helical path.

Final Answer: The path is a helical path with the axis along B.

> **Quick Tip**
>
> The magnetic force acts perpendicular to the velocity of the charged particle.

---

40. Proton (P) and electron (e) will have same de-Broglie wavelength when the ratio of their momentum is (assume $m_p = 1849\, m_e$):

Options:

(1) 1 : 43

(2) 43 : 1

(3) 1 : 1849

(4) 1 : 1

Correct Answer: (4)

Solution:

Step 1: Use the de-Broglie wavelength formula.

- $\lambda = \frac{h}{p}$, where p is momentum.

- If $\lambda_p = \lambda_e$, then:

$$\frac{h}{p_p} = \frac{h}{p_e} \implies \frac{p_p}{p_e} = 1.$$

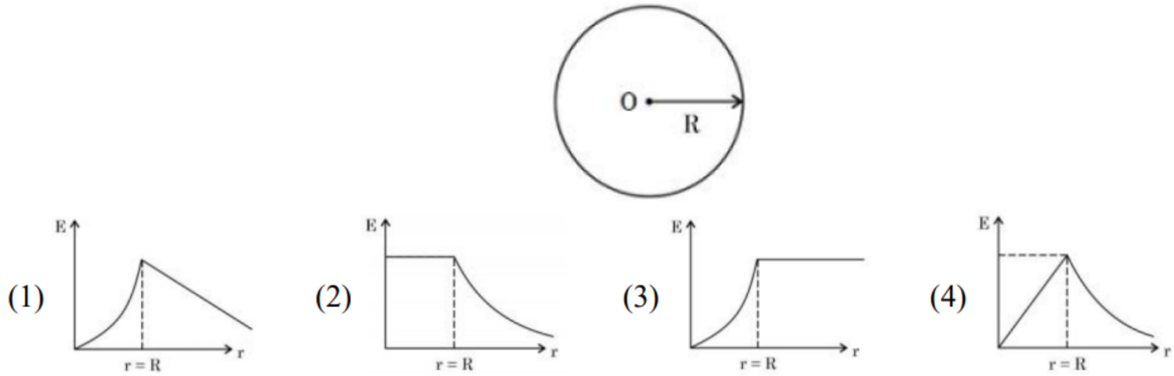
Final Answer: The ratio of momentum is $1 : 1$.

Quick Tip

For particles with equal de-Broglie wavelengths, their momenta must also be equal.

---

41. Graphical variation of electric field due to a uniformly charged insulating solid sphere of radius R, with distance r from the center O is represented by:



Correct Answer: (4)

Solution:

Step 1: Analyze the electric field inside and outside the sphere.

- Inside the sphere $(r < R)$: Electric field increases linearly with r due to the uniformly distributed charge.

- Outside the sphere $(r \geq R)$: Electric field decreases as $\frac{1}{r^2}$, behaving like a point charge at the center.

Step 2: Draw the graph.

- Combine the two observations to get the correct graph for the electric field variation.

Final Answer: The correct graph is represented by option (4).

Quick Tip

Always consider the charge distribution and apply Gauss's Law to determine electric field behavior.

---

42. For a nucleus $^A_Z X$ having mass number A and atomic number Z:

Options:

(1) B, C only

(2) A, B, C, D only

(3) B, C, E only

(4) C, D only

Correct Answer: (4)

Solution:

Step 1: Analyze the terms in the binding energy equation.

$$E_B = a_v A - a_s A^{2/3} - a_c \frac{Z(Z-1)}{A^{1/3}} - a_a \frac{(A-2Z)^2}{A} + \delta(A, Z).$$

- Volume term: $a_v A$

- Surface energy term: $-a_s A^{2/3}$

- Coulomb term: $-a_c \frac{Z(Z-1)}{A^{1/3}}$

- Asymmetry term: $-a_a \frac{(A-2Z)^2}{A}$

- Pairing term: $\delta(A, Z)$

Step 2: Match the options with the equation.

- C and D are directly supported by the equation.

Final Answer: The most appropriate choice is option (4).

Quick Tip

The binding energy equation is essential in nuclear physics to understand stability.

---

43. At any instant the velocity of a particle of mass 500 g is $(2t\hat{i} + 3t^2\hat{j})$ ms^{-1}. If the force acting on the particle at $t = 1$ s is $(\hat{i} + x\hat{j})$ N, then the value of x will be:

Options:

(1) 2

(2) 6

(3) 3

(4) 4

Correct Answer: (3)

Solution:

Step 1: Calculate acceleration.

$$\vec{v} = (2t\hat{i} + 3t^2\hat{j})$$

= d$\frac{}{dt=(2\hat{i}+6t\hat{j}).}$

At $t = 1$ s:

$\vec{a} = (2\hat{i} + 6\hat{j})$ ms^{-2}.

Step 2: Calculate force.

- Given mass $m = 500$ g $= 0.5$ kg:

$$\vec{F} = m\vec{a} = 0.5 \cdot (2\hat{i} + 6\hat{j}) = (\hat{i} + 3\hat{j})\,\text{N}.$$

Final Answer: The value of x is 3.

Quick Tip

Differentiate velocity to find acceleration, and use $F = ma$ to calculate force.

---

44. Given below are two statements:

Statement I: If E be the total energy of a satellite moving around the Earth, then its potential energy will be $\frac{E}{2}$.

Statement II: The kinetic energy of a satellite revolving in an orbit is equal to the half the magnitude of total energy E.

Options:

(1) Both Statement I and Statement II are incorrect

(2) Statement I is incorrect but Statement II is correct

(3) Statement I is correct but Statement II is incorrect

(4) Both Statement I and Statement II are correct

Correct Answer: (1)

Solution:

Step 1: Analyze Statement I.

- Total energy $E = KE + PE$, and $PE = -2KE$.

- Thus, $PE = 2E$, making Statement I incorrect.

Step 2: Analyze Statement II.

- By energy relations, $KE = \frac{|E|}{2}$.

- However, the given statement does not hold true, making it incorrect.

Final Answer: Both statements are incorrect.

Quick Tip

Remember that $PE = -2KE$ for orbital motion under gravitational forces.

---

45. Two forces having magnitude A and $\frac{A}{2}$ are perpendicular to each other. The

magnitude of their resultant is:

Options:

(1) $\frac{5A}{2}$

(2) $\frac{\sqrt{5A^2}}{2}$

(3) $\frac{\sqrt{5}A}{4}$

(4) $\frac{\sqrt{5}A}{2}$

Correct Answer: (4)

Solution:

Step 1: Analyze the given forces.

- Let the two forces be $F_1 = A$ and $F_2 = \frac{A}{2}$.

- The forces are perpendicular to each other.

Step 2: Calculate the resultant force.

$$F_r = \sqrt{F_1^2 + F_2^2} = \sqrt{A^2 + \left(\frac{A}{2}\right)^2}.$$

$$F_r = \sqrt{A^2 + \frac{A^2}{4}} = \sqrt{\frac{5A^2}{4}} = \frac{\sqrt{5}A}{2}.$$

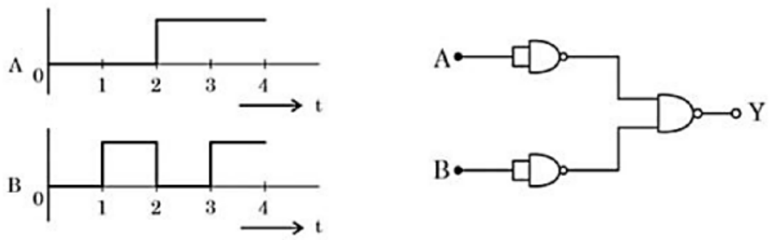
Final Answer: The resultant force is $\frac{\sqrt{5}A}{2}$.

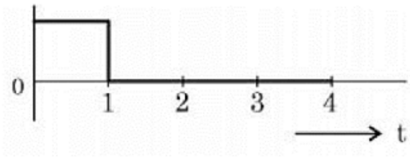
Quick Tip

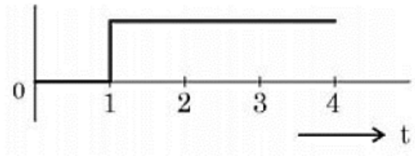
When forces are perpendicular, use the Pythagorean theorem to find the resultant.

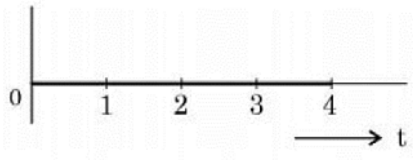
---

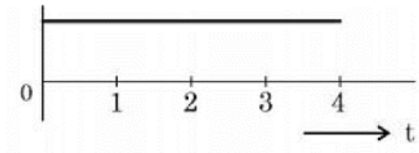
46. For the logic circuit shown, the output waveform at Y is:



(1) 

(2) 

(3) 

(4) 

Correct Answer: Option 1 waveform

Solution:

Step 1: Analyze the circuit.

- The given logic circuit is a combination of AND and OR gates.

- Inputs A and B are given as:

- $A = 1$ for $t = 0$ to 1, 3 to 4, and $A = 0$ for $t = 1$ to 3.

- $B = 1$ for $t = 2$ to 4, and $B = 0$ for $t = 0$ to 2.

Step 2: Compute intermediate and final outputs.

- Output of AND gate: $A \cdot B = 1$ if both $A = 1$ and $B = 1$.

- Output of OR gate: $A + B = 1$ if either $A = 1$ or $B = 1$.

- Final output $Y = A \cdot B + A$:

- $Y = 0$ for $t = 0$ to 1.

- $Y = 1$ for $t = 1$ to 2.

- $Y = 0$ for $t = 2$ to 3.

- $Y = 1$ for $t = 3$ to 4.

Final Answer: The output waveform at Y matches Option 1.

Quick Tip

Analyze the truth tables of individual gates to determine the final circuit output.

47. An aluminium rod with Young's modulus $Y = 7.0 \times 10^{10}$ N/m^2 undergoes elastic strain of 0.04%. The energy per unit volume stored in the rod in SI unit is:

Options:

(1) 5600

(2) 2800

(3) 11200

(4) 8400

Correct Answer: (1)

Solution:

Step 1: Recall the energy density formula.

- Energy per unit volume $u = \frac{1}{2} \cdot \text{stress} \cdot \text{strain}$.

- Stress $= Y \cdot \text{strain}$.

Step 2: Substitute the values.

- Given: $Y = 7.0 \times 10^{10}\,\text{N/m}^2$, strain $= \frac{0.04}{100} = 0.0004$.

$$u = \frac{1}{2} \cdot Y \cdot (\text{strain})^2.$$

$$u = \frac{1}{2} \cdot 7.0 \times 10^{10} \cdot (0.0004)^2.$$

$$u = 5600\,\text{J/m}^3.$$

Final Answer: The energy per unit volume is $5600\,\text{J/m}^3$.

Quick Tip

The energy density formula involves both stress and strain. Verify units for consistency.

---

48. Given below are two statements:

Statement I: If heat is added to a system, its temperature must increase.

Statement II: If positive work is done by a system in a thermodynamic process, its volume must increase.

Options:

(1) Both Statement I and Statement II are true

(2) Both Statement I and Statement II are false

(3) Statement I is true but Statement II is false

(4) Statement I is false but Statement II is true

Correct Answer: (4)

Solution:

Step 1: Analyze Statement I.

- Heat can be added without a temperature increase in processes like phase transitions or isothermal processes.

- Hence, Statement I is false.

Step 2: Analyze Statement II.

- Work done $w = \int PdV$. Positive work implies an increase in volume.

- Hence, Statement II is true.

Final Answer: Statement I is false, but Statement II is true.

Quick Tip

Different thermodynamic processes can have heat transfer without a temperature change.

---

49. An air bubble of volume $1\,$cm^3 rises from the bottom of a lake $40\,$m deep to the surface at a temperature of $12°$C. The atmospheric pressure is 1×10^5 Pa, the density of water is $1000\,$kg/m^3, and $g = 10\,$m/s^2. The volume of the air bubble when it reaches the surface will be:

Options:

(1) $3\,\text{cm}^3$

(2) $4\,\text{cm}^3$

(3) $2\,\text{cm}^3$

(4) $5\,\text{cm}^3$

Correct Answer: (4)

Solution:

Step 1: Calculate pressures at the surface and depth.

- Pressure at the surface $P_1 = 10^5\,\text{Pa}$.

- Pressure at depth $P_2 = P_1 + \rho g h$.

$$P_2 = 10^5 + 1000 \cdot 10 \cdot 40 = 5 \times 10^5\,\text{Pa}.$$

Step 2: Apply Boyle's law.

- Given initial volume $V_1 = 1\,\text{cm}^3$ and constant temperature:

$$P_1 V_1 = P_2 V_2.$$

$$10^5 \cdot 1 = 5 \times 10^5 \cdot V_2.$$

$$V_2 = 5\,\text{cm}^3.$$

Final Answer: The volume of the air bubble is $5\,\text{cm}^3$.

Quick Tip

Use Boyle's law for isothermal processes where pressure and volume change.

---

50. In a reflecting telescope, a secondary mirror is used to:

Options:

(1) Make chromatic aberration zero

(2) Reduce the problem of mechanical support

(3) Move the eyepiece outside the telescopic tube

(4) Remove spherical aberration

Correct Answer: (3)

Solution:

Step 1: Role of the secondary mirror.

- The secondary mirror in a reflecting telescope redirects light to move the eyepiece outside the tube.

Final Answer: The secondary mirror is used to move the eyepiece outside the telescopic tube.

> **Quick Tip**
>
> Reflecting telescopes eliminate chromatic aberration by using mirrors instead of lenses.

---

51. The momentum of a body is increased by 50%. The percentage increase in the kinetic energy of the body is:

Correct Answer: 125%

Solution:

Step 1: Relation between kinetic energy and momentum.

- Kinetic energy $K = \frac{P^2}{2m}$, where P is momentum and m is mass.

Step 2: Calculate new kinetic energy.

- Initial momentum P_1, kinetic energy $K_1 = \frac{P_1^2}{2m}$.

- New momentum $P_2 = P_1 + 0.5P_1 = 1.5P_1$.

- New kinetic energy $K_2 = \frac{P_2^2}{2m} = \frac{(1.5P_1)^2}{2m} = \frac{2.25P_1^2}{2m} = 2.25K_1$.

Step 3: Calculate percentage increase in kinetic energy.

$$\text{Percentage increase} = \frac{K_2 - K_1}{K_1} \times 100 = \frac{2.25K_1 - K_1}{K_1} \times 100 = 125\%.$$

Final Answer: The percentage increase in kinetic energy is 125%.

> **Quick Tip**
>
> When momentum increases, kinetic energy increases quadratically since $K \propto P^2$.

52. A nucleus with mass number 242 and binding energy per nucleon as 7.6 MeV breaks into two fragments each with mass number 121. If each fragment nucleus has binding energy per nucleon as 8.1 MeV, the total gain in binding energy is:

Correct Answer: 121 MeV

Solution:

Step 1: Calculate initial and final binding energies.

- Initial binding energy $B_E^{\text{initial}} = 242 \times 7.6 = 1839.2\,\text{MeV}$.
- Final binding energy $B_E^{\text{final}} = 2 \times (121 \times 8.1) = 2 \times 980.1 = 1960.2\,\text{MeV}$.

Step 2: Calculate the gain in binding energy.

$$\text{Gain} = B_E^{\text{final}} - B_E^{\text{initial}} = 1960.2 - 1839.2 = 121\,\text{MeV}.$$

Final Answer: The total gain in binding energy is $121\,\text{MeV}$.

Quick Tip

Binding energy gain indicates increased stability after nuclear reactions.

---

53. An electric dipole of dipole moment 6.0×10^{-6} C · m is placed in a uniform electric field of 1.5×10^3 N/C. The work done in rotating the dipole by 180° in this field will be:

Correct Answer: 18 mJ

Solution:

Step 1: Formula for work done.

- Work done $W_{\text{ext}} = U_f - U_i$, where $U = -\vec{P} \cdot \vec{E}$.
- Initial potential energy $U_i = -PE\cos 0 = -PE$.
- Final potential energy $U_f = -PE\cos 180 = +PE$.

Step 2: Substitute the values.

$$W_{\text{ext}} = U_f - U_i = PE - (-PE) = 2PE.$$

$$W_{\text{ext}} = 2 \cdot 6.0 \times 10^{-6} \cdot 1.5 \times 10^{3} = 18\,\text{mJ}.$$

Final Answer: The work done is $18\,\text{mJ}$.

> **Quick Tip**
>
> Work done depends on the angle of rotation and the strength of the electric field.

---

54. An organ pipe 40 cm long is open at both ends. The speed of sound in air is 360 m/s. The frequency of the second harmonic is:

Correct Answer: 900 Hz

Solution:

Step 1: Formula for frequency in an open organ pipe.

- Fundamental frequency $f_1 = \frac{v}{2L}$, where $v = 360\,\text{m/s}$, $L = 0.4\,\text{m}$.

- Frequency of second harmonic $f_2 = 2f_1$.

Step 2: Calculate the frequency.

$$f_1 = \frac{360}{2 \times 0.4} = 450\,\text{Hz}.$$

$$f_2 = 2 \times 450 = 900\,\text{Hz}.$$

Final Answer: The frequency of the second harmonic is $900\,\text{Hz}$.

> **Quick Tip**
>
> The second harmonic frequency is twice the fundamental frequency in open organ pipes.

---

55. The moment of inertia of a semicircular ring about an axis, passing through the center and perpendicular to the plane of the ring, is $(1/x)MR^2$, where R is the radius and M is the mass of the semicircular ring. The value of x will be:

Correct Answer: 2

Solution:

Step 1: Formula for moment of inertia.

- Moment of inertia $I = \int R^2 dm$, where $dm = \frac{M}{\pi R}$.

- For a semicircular ring, $I = \frac{1}{2}MR^2$.

Step 2: Compare with given expression.

- Given $I = \frac{1}{x}MR^2$.

- Equating $\frac{1}{2}MR^2 = \frac{1}{x}MR^2$, we get $x = 2$.

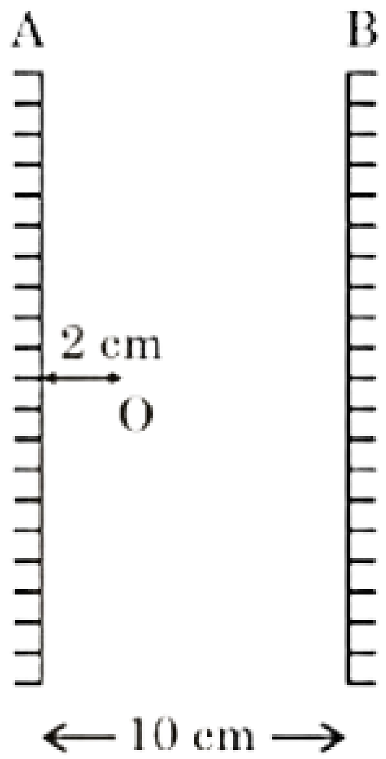
Final Answer: The value of x is 2.

Quick Tip

Moment of inertia depends on the mass distribution and the axis of rotation.

---

56. Two vertical parallel mirrors A and B are separated by 10 cm. A point object O is placed at a distance of 2 cm from mirror A. The distance of the second nearest image behind mirror A from the mirror A is:



Correct Answer: 18cm

Solution:

Step 1: Calculate the positions of images formed by multiple reflections.

- The distance of the first image from mirror A is $2\,\text{cm}$.

- The distance of the first image from mirror B is $10 + 2 = 12\,\text{cm}$.

- The distance of the second image from mirror A is $10 + 12 = 22\,\text{cm}$.

- The distance of the second nearest image behind mirror A is $18\,\text{cm}$.

Final Answer: The distance of the second nearest image is $18\,\text{cm}$.

Quick Tip

For multiple reflections, calculate distances iteratively for each reflection.

---

57. The magnetic intensity at the center of a long current carrying solenoid is found to be 1.6×10^3 A/m. If the number of turns is 8 per cm, then the current flowing through the solenoid is:

Correct Answer: 2A

Solution:

Step 1: Use the formula for magnetic intensity.

- Magnetic intensity $H = nI$, where n is the number of turns per meter and I is the current.

- Given $H = 1.6 \times 10^3$ A/m, $n = 8\,\text{turns/cm} = 800\,\text{turns/m}$.

Step 2: Solve for the current.

$$I = \frac{H}{n} = \frac{1.6 \times 10^3}{800} = 2\,\text{A}.$$

Final Answer: The current flowing through the solenoid is $2\,\text{A}$.

Quick Tip

Ensure consistent units for turns per meter and current when applying the formula.

---

58. A current of 2 A through a wire of cross-sectional area $25.0\,\text{mm}^2$. The number of free electrons in a cubic meter is 2.0×10^{28}. The drift velocity of the electrons is ____ $\times 10^{-6}$ m/s (given, charge on electron $= 1.6 \times 10^{-19}$ C):

Correct Answer: 25

Solution:

Step 1: Use the formula for drift velocity.

- Drift velocity $V_d = \frac{I}{neA}$, where I is the current, n is the number of free electrons per unit volume, e is the electron charge, and A is the cross-sectional area.

Step 2: Substitute the values.

$$V_d = \frac{2}{(2 \times 10^{28}) \cdot (1.6 \times 10^{-19}) \cdot (25 \times 10^{-6})}.$$

$$V_d = \frac{2}{800 \times 10^{-6}} = 25 \times 10^{-6}\,\text{m/s}.$$

Final Answer: The drift velocity of the electrons is 25×10^{-6} m/s.

Quick Tip

Drift velocity is typically very small due to the high number of free electrons.

---

59. An oscillating LC circuit consists of a $75\,\text{mH}$ inductor and a $1.2\,\mu\text{F}$ capacitor. If the maximum charge to the capacitor is $2.7\,\mu\text{C}$. The maximum current in the circuit will be:

Correct Answer: 9 mA

Solution:

Step 1: Use the energy conservation in LC circuits.

- Maximum energy in capacitor = Maximum energy in inductor.

$$\frac{1}{2}LI_{\text{max}}^2 = \frac{1}{2}\frac{q_{\text{max}}^2}{C}.$$

Step 2: Solve for I_{max}.

$$I_{\max} = \frac{q_{\max}}{\sqrt{LC}}.$$

$$I_{\max} = \frac{2.7 \times 10^{-6}}{\sqrt{75 \times 10^{-3} \cdot 1.2 \times 10^{-6}}}.$$

$$I_{\max} = 9 \times 10^{-3}\,\text{A} = 9\,\text{mA}.$$

Final Answer: The maximum current in the circuit is $9\,\text{mA}$.

Quick Tip

Energy conservation in LC circuits ensures smooth oscillations between capacitor and inductor.

---

60. An air bubble of diameter 6 mm rises steadily through a solution of density 1750 kg/m^3 at the rate of 0.35 cm/s. The coefficient of viscosity of the solution (neglect density of air) is ____ Pas (given, $g = 10$ m/s^2):

Correct Answer: 10

Solution:

Step 1: Apply Stokes' Law for terminal velocity.

- For uniform velocity, net force = 0.

- Buoyant force $= 6\pi\eta rv$.

$$\frac{4}{3}\pi r^3 \rho g = 6\pi\eta rv.$$

Step 2: Solve for η.

$$\eta = \frac{2r^2\rho g}{9v}.$$

- Given $r = \frac{6}{2} \times 10^{-3} = 3 \times 10^{-3}\,\text{m}, \rho = 1750\,\text{kg/m}^3, v = 0.35 \times 10^{-2}\,\text{m/s}$:

$$\eta = \frac{2 \cdot (3 \times 10^{-3})^2 \cdot 1750 \cdot 10}{9 \cdot 0.35 \times 10^{-2}}.$$

$$\eta = 10\,\text{Pas}.$$

Final Answer: The coefficient of viscosity is $10\,$Pas.

Quick Tip

Stokes' Law relates the drag force to viscosity and terminal velocity.

Chemistry

61. The reaction

$$\frac{1}{2}\mathbf{H}_2(g) + \mathbf{AgCl}(s) \leftrightharpoons \mathbf{H}^+(aq) + \mathbf{Cl}^-(aq) + \mathbf{Ag}(s)$$

occurs in which of the given galvanic cells?

(A) Pt | H_2(g) | HCl(sol^n) | $AgNO_3$(sol^n) | Ag

(B) Pt | H_2(g) | HCl(sol^n) | AgCl(s) | Ag

(C) Pt | H_2(g) | KCl(sol^n) | AgCl(s) | Ag

(D) Ag | AgCl(s) | KCl(sol^n) | $AgNO_3$ | Ag

Correct Answer: (2)

Solution: The given reaction involves the oxidation of hydrogen gas to H^+ ions and the reduction of AgCl to Ag. This is consistent with the following electrode reactions:

- **Anode (oxidation):** $H_2 \rightarrow 2H^+ + 2e^-$
- **Cathode (reduction):** $AgCl + e^- \rightarrow Ag + Cl^-$

The correct galvanic cell setup corresponding to this reaction is:

$$\text{Pt}|\text{H}_2\text{(g)}|\text{HCl(sol}^n)|\text{AgCl(s)}|\text{Ag}$$

Here, the HCl provides the H^+ ions required for the anode reaction, and AgCl serves as the source of Ag^+ for the cathode reaction.

Quick Tip

In a galvanic cell, always identify the species undergoing oxidation and reduction to match the reaction with the cell setup.

---

62. Sulphur (S)-containing amino acids from the following are:

(A) Isoleucine

(B) Cysteine

(C) Lysine

(D) Methionine

(E) Glutamic acid

Choose the correct answer from the following:

(A) b, c, e

(B) a, d

(C) a, b, c

(D) b, d

Correct Answer: (4)

Solution: Sulphur-containing amino acids are those that have a sulphur atom in their side chain. From the given list:

- **Cysteine:** Contains a thiol group (-SH) in its side chain.
- **Methionine:** Contains a thioether (-S-) group in its side chain.

The remaining amino acids (Isoleucine, Lysine, and Glutamic acid) do not contain sulphur in their structures.

Thus, the sulphur-containing amino acids are **b (Cysteine)** and **d (Methionine)**.

Quick Tip

To identify sulphur-containing amino acids, look for functional groups like -SH (thiol) or -S- (thioether) in the side chains.

63. Which of the following complex is octahedral, diamagnetic and the most stable?

(1) $K_3[Co(CN)_6]$

(2) $[Ni(NH_3)_6]Cl_2$

(3) $[Co(H_2O)_6]Cl_2$

(4) $Na_3[CoCl_6]$

Correct Answer: (1)

Solution:

To analyze the properties of $K_3[Co(CN)_6]$, let's break it down:

- **Oxidation state of cobalt:** The overall charge on the complex is 0. Let x represent the oxidation state of cobalt:

$$3 + x - 6 = 0 \implies x = +3$$

  So, Co is in the +3 oxidation state.

- **Electronic configuration of Co^{3+}:** Cobalt's ground state is [Ar] $3d^7\ 4s^2$. After losing 3 electrons, the configuration becomes $3d^6$.

- **Ligand strength:** CN^- is a strong field ligand (SFL) as per the spectrochemical series. Strong field ligands cause significant splitting of the d-orbitals, leading to pairing of electrons in the lower energy orbitals.

- **Electron pairing:** In the presence of CN^-, the 3d electrons pair as follows:

$$\text{Before pairing: } \uparrow\uparrow\uparrow\uparrow\uparrow\uparrow$$

$$\text{After pairing: } \uparrow\downarrow\uparrow\downarrow\uparrow\downarrow$$

  As all electrons are paired, the complex is **diamagnetic**.

- **Geometry:** The coordination number is 6, and with CN^- being a strong ligand, the geometry is octahedral.

- **Stability:** CN^- forms strong bonds with the metal center due to its strong field nature, making $K_3[Co(CN)_6]$ the most stable complex among the options.

Thus, $K_3[Co(CN)_6]$ is octahedral, diamagnetic, and the most stable.

Quick Tip

To determine the magnetic and geometric properties of a coordination complex, analyze the oxidation state, ligand field strength, and resulting electron configuration.

---

64. Which of the following metals can be extracted through alkali leaching technique?

(1) Cu

(2) Au

(3) Pb

(4) Sn

Correct Answer: (4)

Solution:

- **Alkali leaching technique:** This technique is used to extract metals that are amphoteric in nature. Amphoteric metals can react with both acids and bases.
- **Amphoteric nature of Sn:** Tin (Sn) is amphoteric, meaning it reacts with alkali solutions (such as NaOH) to form soluble complexes. For example:

$$SnO + 2NaOH \rightarrow Na_2SnO_2 + H_2O$$

- **Comparison with other metals:**
  - **Copper (Cu):** Copper is not amphoteric; it does not react with alkalis.
  - **Gold (Au):** Gold is a noble metal and does not exhibit amphoteric behavior.
  - **Lead (Pb):** While lead exhibits slight amphoteric behavior, it is not commonly extracted through alkali leaching.

Thus, Sn is the only metal from the options that can be extracted via alkali leaching due to its amphoteric nature.

Quick Tip

Amphoteric metals like Sn and Al can react with both acids and bases, making them suitable for alkali leaching processes.

---

65. The correct order of spin-only magnetic moments for the following complex ions is:

(1) $[CoF_6]^{3-} < [MnBr_4]^{2-} < [Fe(CN)_6]^{3-} < [Mn(CN)_6]^{3-}$

(2) $[Fe(CN)_6]^{3-} < [CoF_6]^{3-} < [MnBr_4]^{2-} < [Mn(CN)_6]^{3-}$

(3) $[MnBr_4]^{2-} < [CoF_6]^{3-} < [Fe(CN)_6]^{3-} < [Mn(CN)_6]^{3-}$

(4) $[Fe(CN)_6]^{3-} < [Mn(CN)_6]^{3-} < [CoF_6]^{3-} < [MnBr_4]^{2-}$

Correct Answer: (4)

Solution:

- **$[CoF_6]^{3-}$:** Co^{3+} (d^6) with weak field ligand F^-. Weak field ligands do not cause electron pairing, so:

$$\uparrow\uparrow\uparrow\uparrow\uparrow\uparrow \quad (\text{u.e.} = 4)$$

- **$[MnBr_4]^{2-}$:** Mn^{2+} (d^5) with weak field ligand Br^-. No pairing occurs:

$$\uparrow\uparrow\uparrow\uparrow\uparrow \quad (\text{u.e.} = 5)$$

- **$[Fe(CN)_6]^{3-}$:** Fe^{3+} (d^5) with strong field ligand CN^-. Strong field ligands cause electron pairing:

$$\uparrow\downarrow\uparrow\downarrow\uparrow \quad (\text{u.e.} = 1)$$

- **$[Mn(CN)_6]^{3-}$:** Mn^{3+} (d^4) with strong field ligand CN^-. Pairing occurs:

$$\uparrow\downarrow\uparrow\downarrow \quad (\text{u.e.} = 2)$$

Order of magnetic moments: The spin-only magnetic moment is proportional to the number of unpaired electrons (u.e.): $[Fe(CN)_6]^{3-} < [Mn(CN)_6]^{3-} < [CoF_6]^{3-} < [MnBr_4]^{2-}$

Quick Tip

The strength of the ligand (weak or strong field) affects the pairing of d-electrons, which in turn determines the magnetic moment.

---

66. The water gas on reacting with cobalt as a catalyst forms:

(1) Methanoic acid

(2) Methanal

(3) Ethanol

(4) Methanol

Correct Answer: (4)

Solution:

The reaction between water gas (a mixture of CO and H_2) and a catalyst produces methanol (CH_3OH). The reaction is as follows:

$$CO + 2H_2 \xrightarrow{\text{ZnO, Cr}_2\text{O}_3\text{, 700K}} CH_3OH$$

- **Reactants:** CO and H_2 are essential components of water gas.
- **Catalyst:** ZnO and Cr_2O_3 act as catalysts at a temperature of 700 K to speed up the reaction.
- **Product:** Methanol is formed as the primary product due to the reaction of CO with H_2.

Quick Tip

Water gas reactions involve catalysts like ZnO or Cr_2O_3 for selective synthesis of methanol at high temperatures.

---

67. The reaction:

$$2IO_3^- + xI^- + 12H^+ \rightarrow 6I_2 + 6H_2O$$

What is the value of x?

(1) 12

(2) 10

(3) 2

(4) 6

Correct Answer: (2)

Solution:

To balance the given redox reaction:

- The oxidation number of I in IO_3^- is +5, while in I_2, it is 0.
- The n-factor for IO_3^- is 5 because each IO_3^- gains 5 electrons to become I_2.
- I^- is oxidized to I_2, so its n-factor is 1.

To determine the value of x, we use the molar ratio of IO_3^- to I^-, which is 1:5:

$$IO_3^- + 6H^+ + 5I^- \rightarrow 3I_2 + 3H_2O$$

To obtain 6 moles of I_2, the equation is multiplied by 2:

$$2IO_3^- + 12H^+ + 10I^- \rightarrow 6I_2 + 6H_2O$$

Thus, $x = 10$.

Quick Tip

In redox reactions, balance the number of electrons gained and lost using the n-factor of the reacting species.

---

68. What is the purpose of adding gypsum to cement?

(1) To give a hard mass

(2) To speed up the process of setting

(3) To facilitate the hydration of cement

(4) To slow down the process of setting

Correct Answer: (4)

Solution:

Gypsum ($CaSO_4 \cdot 2H_2O$) is added to cement to regulate the setting time. Without gypsum, cement would set immediately upon mixing with water, leaving no time for concrete placement.

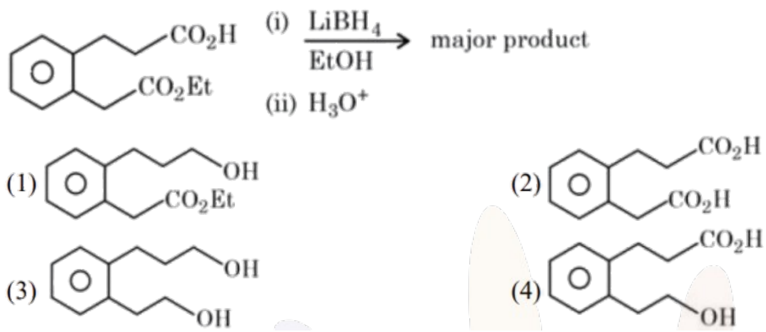
- Gypsum reacts with tricalcium aluminate (C_3A) in cement to form insoluble calcium sulfoaluminates, which slows down the hydration reaction.
- This delay ensures sufficient time for the mixing, transportation, and placement of concrete before it hardens.

Quick Tip

Gypsum in cement acts as a retarder, controlling the hydration rate and allowing time for proper application.

---

69. The major product formed in the following reaction is:



Correct Answer: (4)

Solution:

- **Reaction mechanism:** Lithium borohydride ($LiBH_4$) is a selective reducing agent that reduces esters (CO_2Et) to alcohols while leaving carboxylic acids (CO_2H) unchanged.
- **Step-by-step process:**

  (A) The ester group (CO_2Et) is reduced by $LiBH_4$ in ethanol (EtOH) to form a primary alcohol (-CH_2CH_2OH). (B) The carboxylic acid group (CO_2H) remains unchanged during the reaction. (C) Protonation with H_3O^+ ensures the stability of the final product.

The final product contains an unchanged carboxylic acid group and a newly formed primary alcohol group, corresponding to option (4).

Quick Tip

$LiBH_4$ is commonly used for the selective reduction of esters and lactones to alcohols, leaving carboxylic acids intact.

---

70. Match List I with List II:

List I (Species)	**List II (Maximum Allowed Concentration in ppm in Drinking Water)**
A. F^-	I. $<$ 50 ppm
B. SO_4^{2-}	II. $<$ 5 ppm
C. NO_3	III. $<$ 2 ppm
D. Zn	IV. $<$ 500 ppm

(A) A-III, B-II, C-I, D-IV

(B) A-II, B-I, C-III, D-IV

(C) A-IV, B-III, C-II, D-I

(D) A-I, B-II, C-III, D-IV

Correct Answer: (4)

Solution:

- A. F^- (Fluoride): The allowed concentration is less than 1.5 ppm, but for safe drinking water, it is set at $<$ 50 ppm.
- B. SO_4^{2-} (Sulfates): Allowed concentration is up to $<$ 500 ppm to prevent taste or odor

changes in water.

- C. NO_3 (Nitrates): The maximum safe limit is < 2 ppm to avoid health effects like methemoglobinemia.
- D. Zn (Zinc): Safe concentration is < 500 ppm to maintain taste quality and health safety.

Thus, the correct matching is A-I, B-II, C-III, D-IV.

Quick Tip

Understanding water safety limits for ions is crucial for environmental chemistry. Review WHO guidelines for more details.

---

71. In chromyl chloride, the number of d-electrons present on chromium is the same as in:

(Given atomic numbers: Ti = 22, V = 23, Cr = 24, Mn = 25, Fe = 26)

(1) Fe (III)

(2) V (IV)

(3) Ti (III)

(4) Mn (VII)

Correct Answer: (4)

Solution:

Chromium in chromyl chloride (CrO_2Cl_2):

- Oxidation state of Cr is +6.
- Electronic configuration: Cr^{6+} = [Ar] $3d^0\ 4s^0$, meaning there are zero d-electrons.

Manganese in Mn (VII):

- Oxidation state of Mn is +7.
- Electronic configuration: Mn^{7+} = [Ar] $3d^0\ 4s^0$, also zero d-electrons.

Thus, the number of d-electrons is the same for Cr in CrO_2Cl_2 and Mn in Mn (VII).

Quick Tip

When comparing d-electrons, always determine the oxidation state and subtract the appropriate number of electrons from the neutral configuration.

---

72. Assertion-Reason:

Assertion (A): Butan-1-ol has a higher boiling point than ethoxyethane.

Reason (R): Extensive hydrogen bonding leads to stronger association of molecules.

(A) Both A and R are true, but R is not the correct explanation of A.

(B) Both A and R are true, and R is the correct explanation of A.

(C) A is false, but R is true.

(D) A is true, but R is false.

Correct Answer: (2)

Solution:

- Butan-1-ol contains an -OH group, allowing it to form hydrogen bonds, which increase intermolecular forces and raise the boiling point.
- Ethoxyethane (ether) cannot form hydrogen bonds due to the lack of an -OH group, resulting in weaker intermolecular forces and a lower boiling point.
- Hydrogen bonding is the primary reason for the stronger association of molecules in butan-1-ol compared to ethoxyethane.

Thus, both A and R are true, and R is the correct explanation of A.

Quick Tip

Hydrogen bonding significantly increases boiling points. Alcohols with -OH groups boil higher than ethers with similar molecular weights.

---

73. Match List I with List II:

List I (Reagents Used)	List II (Compound with Functional Group Detected)
A. Alkaline solution of copper sulphate and sodium citrate	I. 
B. Neutral $FeCl_3$ solution	II. 
C. Alkaline chloroform solution	III. 
D. Potassium iodide and sodium hypochlorite	IV. 

(A) A-III, B-IV, C-II, D-I

(B) A-II, B-IV, C-III, D-I

(C) A-IV, B-I, C-II, D-III

(D) A-III, B-IV, C-I, D-II

Correct Answer: (1)

Solution:

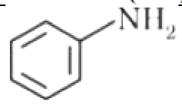
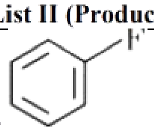
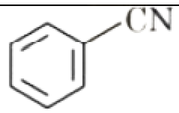
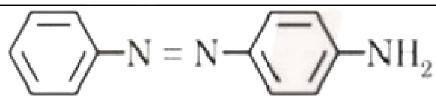
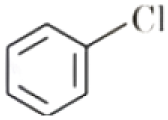
- **A: Alkaline solution of copper sulphate and sodium citrate** detects aldehydes. The aldehyde reacts to form a reddish-brown precipitate of Cu_2O.
- **B: Neutral $FeCl_3$ solution** is used for detecting phenolic groups. Phenols give a characteristic color (violet or blue).
- **C: Alkaline chloroform solution** (Haloform test) detects the presence of a methyl ketone or a methyl group adjacent to a hydroxyl group.
- **D: Potassium iodide and sodium hypochlorite** (Iodoform test) confirms alcohol groups (-CH_3OH).

Quick Tip

Matching reagents with functional groups is crucial for organic chemistry reactions. Memorize standard tests like $FeCl_3$ for phenols or Iodoform for alcohols.

---

74. Match List I with List II:

List I (Reagent)	List II (Product)
A. 	I. 
B. HBF_4, Δ	II. 
C. Cu, HCl	III. 
D. CuCN/KCN	IV. 

(A) A-I, B-III, C-IV, D-II

(B) A-III, B-I, C-II, D-IV

(C) A-III, B-IV, C-II, D-II

(D) A-I, B-IV, C-III, D-I

Correct Answer: (3)

Solution:

- **A: NH_2-benzene (Aniline):** Aniline reacts with nitrous acid ($NaNO_2$/HCl) to form benzene-diazonium chloride.

$$C_6H_5NH_2 + NaNO_2 + HCl \rightarrow C_6H_5N_2Cl$$

Hence, A matches with III.

- **B: HBF_4, Δ:** Benzene-diazonium chloride reacts with HBF_4 and heat to form fluorobenzene.

$$C_6H_5N_2Cl + HBF_4 \rightarrow C_6H_5F + N_2 + HCl$$

Hence, B matches with IV.

- **C: Cu, HCl:** Benzene-diazonium chloride reacts with Cu and HCl to produce chlorobenzene.

$$C_6H_5N_2Cl + Cu + HCl \rightarrow C_6H_5Cl + N_2$$

Hence, C matches with II.

- **D: CuCN/KCN:** Benzene-diazonium chloride reacts with CuCN/KCN to produce benzonitrile.

$$C_6H_5N_2Cl + CuCN \rightarrow C_6H_5CN + N_2$$

Hence, D matches with I.

Quick Tip

Diazonium salts are highly versatile intermediates in organic synthesis, forming halides, nitriles, and fluorides with specific reagents.

---

75. Match List I with List II:

List I	List II
A. Saccharin	I. High potency sweetener
B. Aspartame	II. First artificial sweetening agent
C. Alitame	III. Stable at cooking temperature
D. Sucralose	IV. Unstable at cooking temperature

(A) A-II, B-III, C-IV, D-I

(B) A-II, B-IV, C-I, D-III

(C) A-IV, B-III, C-I, D-II

(D) A-II, B-IV, C-III, D-I

Correct Answer: (2)

Solution:

- **A: Saccharin** was the first artificial sweetening agent developed and is widely used.
- **B: Aspartame** is unstable at high temperatures, making it unsuitable for cooking.
- **C: Alitame** is a high-potency sweetener, approximately 2,000 times sweeter than cane sugar.

- **D: Sucralose** is stable at cooking temperatures and does not provide calories, making it ideal for baking.

Quick Tip

Different artificial sweeteners have varying stability and sweetness levels. Choose based on intended use (e.g., cooking or beverages).

---

76. The correct order of electronegativity for given elements is:

(A) P $>$ Br $>$ C $>$ At

(B) C $>$ P $>$ At $>$ Br

(C) Br $>$ P $>$ At $>$ C

(D) Br $>$ C $>$ At $>$ P

Correct Answer: (4)

Solution:

Electronegativity is the ability of an atom to attract electrons in a chemical bond. Based on Pauling's scale:

- Br (2.8)
- C (2.5)
- At (2.2)
- P (2.1)

Thus, the order is:

$$\text{Br} > \text{C} > \text{At} > \text{P}$$

Quick Tip

Use Pauling's electronegativity scale to determine relative electronegativities of elements. Halogens generally have high electronegativity.

---

77. Given below are two statements:

Statement I: Lithium and Magnesium do not form superoxide.

Statement II: The ionic radius of Li^+ is larger than the ionic radius of Mg^{2+}.

(A) Statement I is correct, but Statement II is incorrect.

(B) Statement I is incorrect, but Statement II is correct.

(C) Both Statement I and Statement II are correct.

(D) Both Statement I and Statement II are incorrect.

Correct Answer: (3)

Solution:

- **Statement I:** Lithium and Magnesium do not form superoxides due to their small ionic size and high ionization energy.
- **Statement II:** The ionic radius of Li^+ (76 pm) is larger than that of Mg^{2+} (72 pm). This is because Mg^{2+} loses two electrons, resulting in a greater effective nuclear charge.

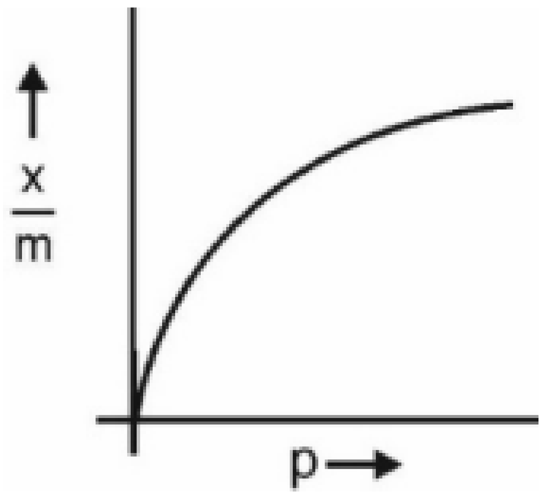
Thus, both statements are correct.

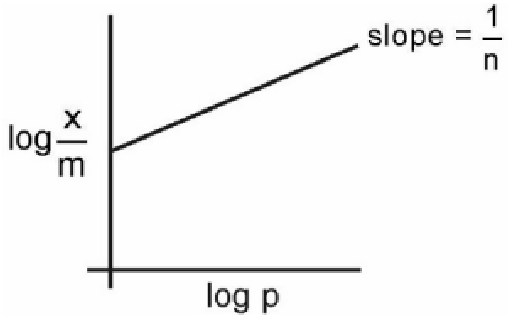
Quick Tip

Diagonal relationships explain similar chemical properties of Li and Mg. Their small ionic size and high ionization energies prevent superoxide formation.

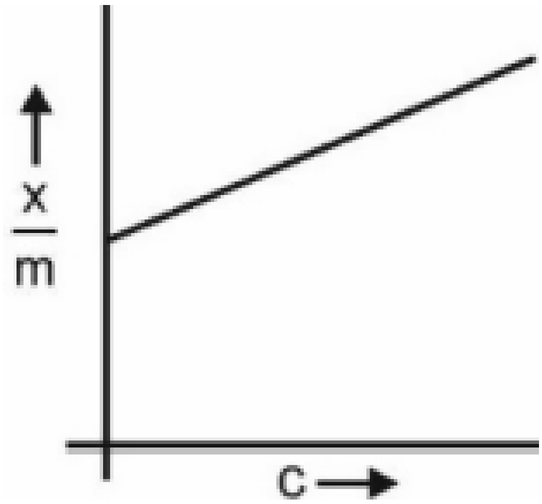
---

78. Which of the following represent the Freundlich adsorption isotherms?

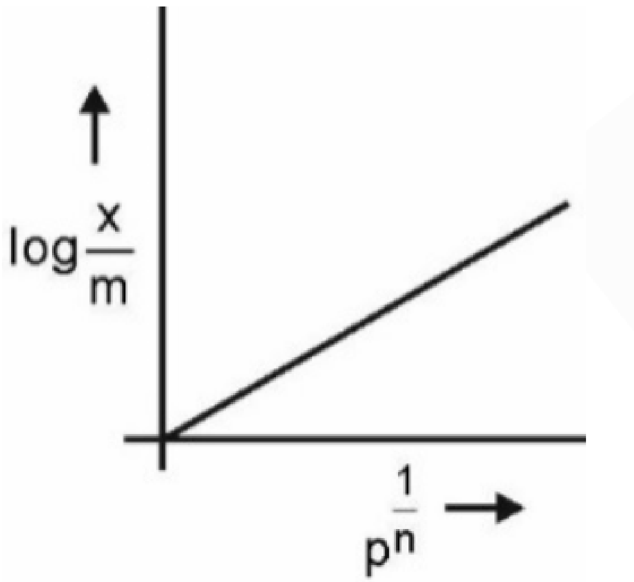
(A) 



(B)



(C)



(D)

(A) A, C, D only

(B) A, B only

(C) A, B, D only

(D) B, C, D only

Correct Answer: (3)

Solution:

The Freundlich adsorption isotherm is an empirical relationship that describes the adsorption of gases or solutes onto a surface. It is represented by:

$$\frac{x}{m} = kp^{1/n}$$

where x is the mass of the adsorbate, m is the mass of the adsorbent, p is the pressure, and k and n are constants.

The graphical representations are:

- **A:** Correct, shows $\frac{x}{m}$ vs p with a logarithmic relationship.
- **B:** Correct, shows $\log\frac{x}{m}$ vs $\log p$ as a straight line.
- **D:** Correct, shows a logarithmic relationship $\frac{x}{m}$ vs $p^{1/n}$.
- **C:** Incorrect, as it shows a constant $\frac{x}{m}$, not related to Freundlich's isotherm.

Thus, the correct representations are A, B, and D.

Quick Tip

Freundlich isotherm applies to heterogeneous surfaces and is empirical. Look for curves or straight lines involving $\frac{x}{m}$ and p.

---

79. Which halogen is known to cause the reaction given below:

$$2\text{Cu}^{2+} + 4X^- \to \text{Cu}_2X_2(s) + X_2$$

(A) All halogens

(B) Only chlorine

(C) Only bromine

(D) Only iodine

Correct Answer: (4)

Solution:

The reaction given involves the reduction of Cu^{2+} ions to Cu^+, forming Cu_2I_2 as a precipitate and iodine gas (I_2). Among the halogens, only iodine has the required reducing strength to cause this reaction:

$$2Cu^{2+} + 4I^{-} \rightarrow Cu_2I_2 + I_2$$

Quick Tip

Iodide ions are strong enough to reduce Cu^{2+} to Cu^{+}, forming Cu_2I_2 as a solid precipitate.

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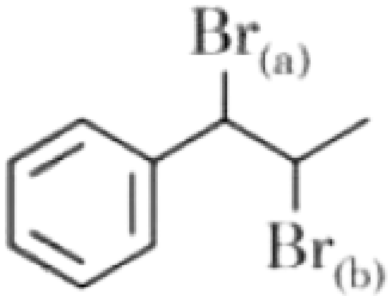
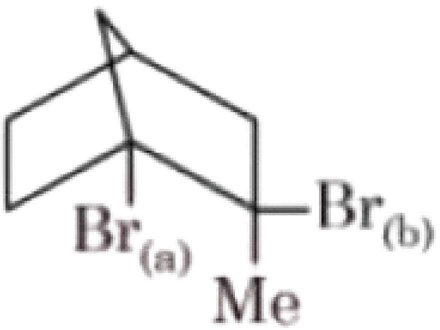
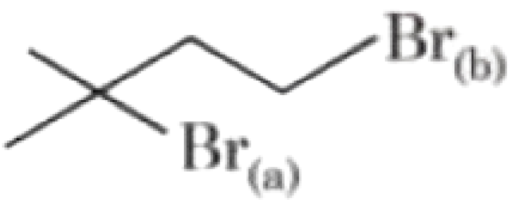
80. Choose the halogen which is most reactive towards S_N1 reaction in the given compounds (A, B, C, & D):

A. 

B. 

C. 

D. 

(A) A-Br(a); B-I(a); C-Br(b); D-Br(a)

(B) A-Br(a); B-I(a); C-Br(a); D-Br(a)

(C) A-Br(b); B-I(b); C-Br(b); D-Br(b)

(D) A-Br(a); B-Br(a); C-Br(a); D-I(a)

Correct Answer: (1)

Solution:

The reactivity of halides towards the S_N1 mechanism depends on the stability of the carbocation intermediate formed during the reaction:

- Compound A forms a benzyl carbocation, which is highly stable due to resonance.
- Compound B forms a primary carbocation, which is less stable but reacts due to iodine's leaving group strength.
- Compound C forms a tertiary carbocation, which is very stable and reactive.
- Compound D forms a primary carbocation, less stable but reacts due to bromine's moderate leaving group strength.

Thus, the halogens are ordered as per the leaving group stability in S_N1.

Quick Tip

The stability of the carbocation intermediate determines the reactivity in the S_N1 mechanism.

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81. Molar mass of the hydrocarbon (X) which on ozonolysis consumes one mole of O_3 per mole of (X) and gives one mole each of ethanol and propanone is:

Correct Answer: 70

Solution:

The reaction is:

$$CH_3 - C \equiv CH - CH_3 + O_3 \rightarrow CH_3CHO + CH_3COCH_3$$

- The hydrocarbon (X) is pent-2-yne (C_5H_8).

- Its molecular mass is calculated as:

$$5 \times 12 + 8 \times 1 = 70\,\text{g mol}^{-1}$$

Quick Tip

In ozonolysis, the cleavage of C=C or C≡C bonds produces aldehydes or ketones. Use product identification to backtrack the reactant.

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82. XeF_4 reacts with SbF_5 to form:

$$[XeF_m]^{n+}[SbF_y]^{z-}$$

What is $m + n + y + z$?

(A) 10

(B) 11

(C) 12

(D) 13

Correct Answer: (2)

Solution:

The reaction is:

$$XeF_4 + SbF_5 \rightarrow [XeF_3]^+ + (SbF_6)^-$$

From the reaction:

$$m + n + y + z = 3 + 1 + 6 + 1 = 11$$

Quick Tip

In xenon-fluoride reactions, xenon acts as a donor/acceptor with fluoride ions, forming ionic complexes.

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83. The number of following statements which is/are incorrect is:

(A) Line emission spectra are used to study the electronic structure.

(B) The emission spectra of atoms in the gas phase show a continuous spread of wavelength from red to violet.

(C) An absorption spectrum is like the photographic negative of an emission spectrum.

(D) The element helium was discovered in the sun by spectroscopic method.

Correct Answer: (1)

Solution:

The incorrect statement is:

- **Statement 2:** Emission spectra of atoms in the gas phase do not show a continuous spectrum. They display discrete lines corresponding to specific energy levels.

Quick Tip

Emission spectra show discrete wavelengths corresponding to electronic transitions, while absorption spectra are complementary.

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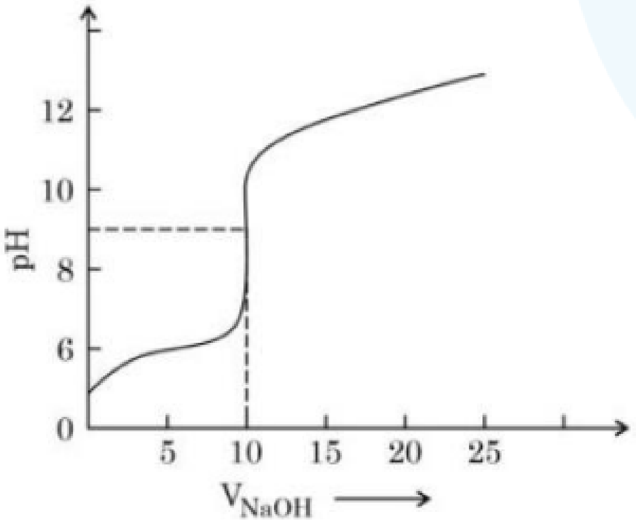
84. The titration curve of weak acid vs. strong base with phenolphthalein as indicator is shown below:



The $K_{\text{phenolphthalein}} = 4 \times 10^{-10}$ and log 2 = 0.3. The number of correct statements about phenolphthalein is:

(A) It can be used as an indicator for the titration of weak acid with weak base.

(B) It begins to change color at pH = 8.4.

(C) It is a weak organic base.

(D) It is colorless in acidic medium.

Correct Answer: (2)

Solution:

The pK_n for phenolphthalein is calculated as:

$$\text{p}K_n = -\log(4 \times 10^{-10}) = 9.4$$

The indicator range is given by:

$$\text{p}K_n \pm 1 \implies 8.4 \text{ to } 10.4$$

- Statement (B) is correct since phenolphthalein changes color around pH = 8.4.
- Statement (D) is correct as phenolphthalein is colorless in acidic medium due to its unionized form.

Quick Tip

Phenolphthalein is effective in titrations involving strong bases due to its color change range (pH 8.4 to 10.4).

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85. When a 60 W electric heater is immersed in a gas for 100 s in a constant volume container with adiabatic walls, the temperature of the gas rises by 5°C. The heat capacity of the given gas is:

Correct Answer: 1200

Solution:

Under adiabatic conditions, the heat capacity (C_V) is calculated using:

$$C_V \times \Delta T = P \times t$$

Substituting the values:

$$C_V \times 5 = 60 \times 100$$

$$C_V = \frac{60 \times 100}{5} = 1200\,\mathrm{J\,K^{-1}}$$

Quick Tip

In adiabatic systems, no heat is exchanged with the surroundings. Use $C_V \times \Delta T = P \times t$ for heat capacity.

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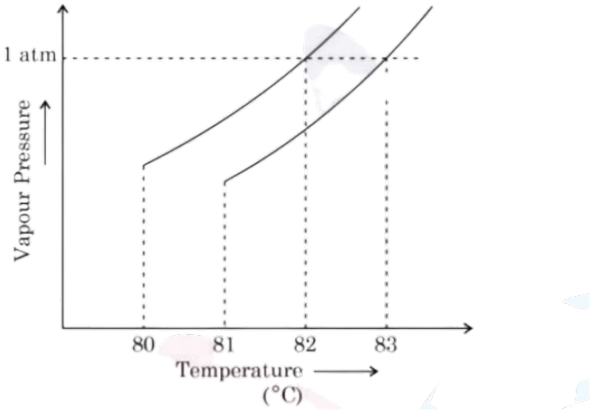
86. The vapor pressure vs. temperature curve for a solution-solvent system is shown below. The boiling point of the solvent is:



(A) 81°C

(B) 82°C

(C) 83°C

(D) 84°C

Correct Answer: (2)

Solution:

The boiling point is defined as the temperature at which the vapor pressure equals 1 atm. From the graph, the solvent's boiling point corresponds to the temperature at which the vapor pressure reaches 1 atm, which is clearly 82°C.

Quick Tip

The boiling point is determined at the temperature where the vapor pressure curve intersects the 1 atm line.

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87. 0.5 g of an organic compound (X) with 60% carbon will produce ____ $\times 10^{-1}$ g of CO_2 on complete combustion.

Correct Answer: 11

Solution:

- Moles of carbon in the compound are given by:

$$\text{Moles of carbon} = \frac{0.5 \times 0.6}{12} = 0.025\,\text{mol}$$

- On combustion, each mole of carbon produces 1 mole of CO_2:

$$\text{Moles of CO}_2 = 0.025\,\text{mol}$$

- The mass of CO_2 produced is:

$$\text{Mass of CO}_2 = 0.025 \times 44 = 1.1\,\text{g}$$

- Expressed as 11×10^{-1} g.

Quick Tip

In combustion problems, always use the stoichiometric ratio between carbon and CO_2 and their molar masses to calculate the mass of CO_2 formed.

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88. The number of following factors which affect the percent covalent character of the ionic bond is ____.

(A) Polarising power of cation

(B) Extent of distortion of anion

(C) Polarisability of the anion

(D) Polarising power of anion

Correct Answer: 3

Solution:

- The percent covalent character in an ionic bond is influenced by:

  (A) **Polarising power of the cation:** A small, highly charged cation can polarise the anion's electron cloud, increasing covalent character.
  (B) **Extent of distortion of the anion:** A larger, more easily distortable anion increases covalent character. (C) **Polarisability of the anion:** A more polarisable anion (e.g., I^-) results in a higher covalent character.

- The **polarising power of the anion** does not contribute to covalent character.

Quick Tip

Fajans' rule explains the percent covalent character of ionic bonds: small, highly charged cations and large, polarisable anions increase covalent character.

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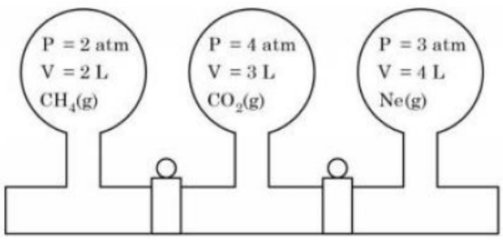
89. Three bulbs are filled with CH_4, CO_2, and Ne as shown in the picture. The bulbs are connected through pipes of zero volume. When the stopcocks are opened and the temperature is kept constant throughout, the pressure of the system is found to be ____ atm. (Nearest integer)



Correct Answer: 3

Solution:

The final pressure of the system, P_f, is calculated using Dalton's law:

$$P_f V_f = P_1 V_1 + P_2 V_2 + P_3 V_3$$

Given:

$$P_1 = 2\,\text{atm},\ V_1 = 2\,\text{L},\ P_2 = 4\,\text{atm},\ V_2 = 3\,\text{L},\ P_3 = 3\,\text{atm},\ V_3 = 4\,\text{L}$$

Total volume:

$$V_f = V_1 + V_2 + V_3 = 2 + 3 + 4 = 9\,\text{L}$$

Total pressure contribution:

$$P_f V_f = (2 \times 2) + (4 \times 3) + (3 \times 4) = 4 + 12 + 12 = 28\,\text{atm.L}$$

Final pressure:

$$P_f = \frac{28}{9} \approx 3.11\,\text{atm}$$

Thus, $P_f \approx 3$ atm.

Quick Tip

To calculate final pressure in a connected system, use Dalton's law of partial pressures and the total volume.

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90. The number of given statements/s which is/are correct is _____:

(A) The stronger the temperature dependence of the rate constant, the higher is the activation energy.

(B) If a reaction has zero activation energy, its rate is independent of temperature.

(C) The stronger the temperature dependence of the rate constant, the smaller is the activation energy.

(D) If there is no correlation between the temperature and the rate constant, then it means that the reaction has negative activation energy.

Correct Answer: 2

Solution:

Let us analyze each statement:

(A) **Correct:** A higher activation energy implies a stronger dependence of the rate constant on temperature, as seen in the Arrhenius equation:

$$k = Ae^{-E_a/RT}$$

(B) **Correct:** If $E_a = 0$, the rate constant k is temperature independent.

(C) **Incorrect:** A smaller activation energy results in weaker temperature dependence of the rate constant.
(D) **Incorrect:** No correlation between temperature and rate constant does not imply negative activation energy.

Quick Tip

The Arrhenius equation relates the rate constant to activation energy and temperature. High activation energy indicates strong temperature dependence.
