# JEE Main 2023 8 April Shift 1 Mathematics Question Paper

| **Time Allowed :**180 minutes | **Maximum Marks :**300 | **Total questions :**90 |
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## General Instructions

**Read the following instructions very carefully and strictly follow them:**

(A) The test is of 3 hours duration.

(B) The question paper consists of 90 questions. The maximum marks are 300.

(C) There are three parts in the question paper consisting of Physics, Chemistry and Mathematics having 30 questions in each part of equal weightage.

(D) Each part (subject) has two sections.

(i) Section-A: This section contains 20 multiple choice questions which have only one correct answer. Each question carries 4 marks for correct answer and –1 mark for wrong answer.

(ii) Section-B: This section contains 10 questions. In Section-B, attempt any five questions out of 10. The answer to each of the questions is a numerical value. Each question carries 4 marks for correct answer and –1 mark for wrong answer. For Section-B, the answer should be rounded off to the nearest integer.

**1. The area of the region** $\{(x, y) : x^2 \leq y \leq 8 - x^2, y \leq 7\}$ **is:**

(1) 24

(2) 21

(3) 20

(4) 18

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**2. Let** $P = \begin{bmatrix} \frac{\sqrt{3}}{2} & \frac{1}{2} \\ -\frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix}$**,** $A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$**, and** $Q = PAP^T$**. If** $P^T Q^{2007} P = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$**, then** $2a + b - 3c - 4d$ **equals:**

(1) 2004

(2) 2007

(3) 2005

(4) 2006

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**3. Negation of** $(p \to q) \to (q \to p)$ **is:**

(1) $(-q) \wedge p$

(2) $p \vee (\sim q)$

(3) $(\sim p) \vee q$

(4) $q \wedge (\sim p)$

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**4. Let** $C(\alpha, \beta)$ **be the circumcenter of the triangle formed by the lines** $4x + 3y = 69$**,** $4y - 3x = 17$**, and** $x + 7y = 61$**. Then** $(\alpha - \beta)^2 + \alpha + \beta$ **is equal to:**

(1) 18

(2) 15

(3) 16

(4) 17

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**5. Let $\alpha, \beta, \gamma$ be the three roots of the equation $x^3 + bx + c = 0$. If $\beta\gamma = 1$ = $-\alpha$, then $b^3 + 2c^3 - 3\alpha^3 - 6\beta^3 - 8\gamma^3$ is equal to:**

(1) $\frac{155}{8}$

(2) 21

(3) 19

(4) $\frac{169}{8}$

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**6. Let the number of elements in sets $A$ and $B$ be five and two respectively. Then the number of subsets of $A \times B$ each having at least 3 and at most 6 elements is:**

(1) 752

(2) 772

(3) 782

(4) 792

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**7. If the coefficients of three consecutive terms in the expansion of $(1 + x)^n$ are in the ratio $1 : 5 : 20$, then the coefficient of the fourth term is:**

(1) 5481

(2) 3654

(3) 2436

(4) 1817

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**8. Let $R$ be the focus of the parabola $y^2 = 20x$ and the line $y = mx + c$ intersect the parabola at two points $P$ and $Q$. Let the point $G(10, 10)$ be the centroid of the triangle $PQR$. If $c - m = 6$, then $(PQ)^2$ is:**

(1) 325

(2) 346

(3) 296

(4) 317

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**9. Let $S_K = \frac{1+2+\cdots+K}{K}$ and $\sum_{j=1}^{n} S_j^2$ where $A, B, C, D \in \mathbb{N}$ and $A$ has the least value. Then:**

(1) $A + B$ is divisible by $D$

(2) $A + B$ is divisible by $5$

(3) $A + C + D$ is not divisible by $B$

(4) $A + B + D$ is divisible by 5

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**10. The shortest distance between the lines $\frac{x-4}{4} = \frac{y+2}{5} = \frac{z+3}{3}$ and $\frac{x-1}{3} = \frac{y-3}{4} = \frac{z-4}{2}$ is:**

(1) $2\sqrt{6}$

(2) $3\sqrt{6}$

(3) $6\sqrt{3}$

(4) $6\sqrt{2}$

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**11. The number of arrangements of the letters of the word "INDEPENDENCE" in which all the vowels always occur together is:**

(1) 16800

(2) 14800

(3) 18000

(4) 33600

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**12. If the points with position vectors $\alpha\hat{i} + 10\hat{j} + 13\hat{k}$, $6\hat{i} + 11\hat{j} + 11\hat{k}$, and $\frac{9}{2}\hat{i} + \beta\hat{j} - 8\hat{k}$ are collinear, then $(19\alpha - 6\beta)^2$ is equal to:**

(1) 49

(2) 36

(3) 25

(4) 16

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**13. In a bolt factory, machines A, B, and C manufacture respectively 20%, 30%, and 50% of the total bolts. Of their output, 3%, 4%, and 2% are defective bolts. A bolt is drawn at random from the product. If the bolt drawn is found to be defective, then the probability that it is manufactured by machine C is:**

(1) $\frac{5}{14}$

(2) $\frac{3}{7}$

(3) $\frac{9}{28}$

(4) $\frac{2}{7}$

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**14. If for $z = \alpha + i\beta$, $|z + 2| = z + 4(1 + i)$, then $\alpha + \beta$ and $\alpha\beta$ are the roots of the equation:**

(1) $x^2 + 3x - 4 = 0$

(2) $x^2 + 7x + 12 = 0$

(3) $x^2 - 7x + 12 = 0$

(4) $x^2 + 2x - 3 = 0$

**15.**

$$\lim_{x \to 0} \left( \frac{(1 - \cos^2(3x)) \sin^3(4x)}{\cos^3(4x) \log(2x + 15)} \right) \text{ is equal to:}$$

(1) 24

(2) 9

(3) 18

(4) 15

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**16. The number of ways in which 5 girls and 7 boys can be seated at a round table so that no two girls sit together is:**

(1) $7(720)^2$

(2) 720

(3) $7(360)^2$

(4) $126(5!)^2$

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**17. Let $f(x) = \frac{\sin x + \cos x - \sqrt{2}}{\sin x - \cos x}$, where $x \in [0, \pi]$, and $x \in \left[0, \frac{\pi}{4}\right]$. Then $f\left(\frac{7\pi}{12}\right)$ is equal to:**

(1) $\frac{-2}{3}$

(2) $\frac{2}{9}$

(3) $\frac{-1}{3\sqrt{3}}$

(4) $\frac{2}{3\sqrt{3}}$

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**18. If the equation of the plane containing the line $x + 2y + 3z - 4 = 0$, $2x + y - z + 5 = 0$, and perpendicular to the plane $\vec{r} = (\hat{i} - \hat{j}) + \lambda(\hat{i} + \hat{j} + \hat{k}) + \mu(\hat{i} - 2\hat{j} + 3\hat{k})$ is $ax + by + cz = 4$, then $a - b + c$ is equal to:**

(1) 22

(2) 24

(3) 20

(4) 18

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**19. Let**

$$A = \begin{bmatrix} 2 & 1 & 0 \\ 1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix}.$$

**If** $|\text{adj}(\text{adj}(\text{adj}(2A)))| = (16)^n$, then $n$ is equal to:

(1) 8

(2) 9

(3) 12

(4) 10

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**20. Let $I(x) = \int_0^x \frac{x+1}{x(1+xe^x)^2}\,dx$, $x > 0$. If $\lim_{x\to\infty} I(x) = 0$, then $I(1)$ is equal to:**

(1) $\frac{e+2}{e+1} - \log(e+1)$

(2) $\frac{e+1}{e+2} + \log(e+1)$

(3) $\frac{e+2}{e+1} - \log(e+1)$

(4) $\frac{e+1}{e+2} + \log(e+1)$

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**21. Let $A = \{0, 3, 4, 6, 7, 8, 9, 10\}$ and $R$ be the relation defined on $A$ such that $R = \{(x, y) \in A \times A : x - y$ is an odd positive integer or $x - y = 2\}$. The minimum number of elements that must be added to $R$ so that it is a symmetric relation is:**

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**22. Let $[t]$ denote the greatest integer $\leq t$. If the constant term in the expansion of $(3x^2 - \frac{1}{2x})^7$ is $\alpha$, then $[\alpha]$ is equal to:**

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**23. Let $\lambda_1, \lambda_2$ be the values of $\lambda$ for which the points $(1, \lambda, \frac{1}{2})$ and $(-2, 0, 1)$ are at equal distance from the plane $2x + 3y - 6z + 7 = 0$. If $\lambda_1 > \lambda_2$, then the distance of the point $(1 - \lambda_2, \lambda_2, \lambda_1)$ from the line $\frac{x-5}{1} = \frac{y-1}{2} = \frac{z+7}{2}$ is:**

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**24.** If the solution curve of the differential equation $(y - 2\log x)dx + (x \log x^2)dy = 0$ passes through the points $\left(e^{4/3}, \alpha\right)$ and $\left(e^4, \alpha\right)$, then $\alpha$ is equal to:

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**25. Let $\vec{a} = 6\hat{i} + 9\hat{j} + 12\hat{k}$, $\vec{b} = a\hat{i} + 11\hat{j} - 2\hat{k}$, and $\vec{c}$ be vectors such that $\vec{a} \times \vec{c} = \vec{a} \times \vec{b}$. If $\vec{a} \cdot \vec{c} = -12$ and $\vec{c} \cdot (\hat{i} - 2\hat{j} + \hat{k}) = 5$, then $\vec{c} \cdot (\hat{i} + \hat{j} + \hat{k})$ is equal to:**

**26. The largest natural number $n$ such that $3^n$ divides $66!$ is:**

**27. If $a_n = \frac{n^3}{n^4+147}$, $n = 1, 2, 3, \ldots$, and $a_n$ is the greatest term in the sequence, then $a_n$ is equal to:**

**28. Let the mean and variance of 8 numbers $x, y, 10, 12, 6, 12, 4, 8$ be $9$ and $9.25$ respectively. If $x > y$, then $3x - 2y$ is equal to:**

**29. Consider a circle $C_1 : x^2 + y^2 - 4x - 2y = \alpha - 5$. Let its mirror image in the line $y = 2x + 1$ be another circle $C_2 : 5x^2 + 5y^2 - 10x - 10y + 36 = 0$. Let $r$ be the radius of $C_2$. Then $\alpha + r$ is equal to:**

**30. Let $[t]$ denote the greatest integer $\leq t$. The $\int_{\frac{\pi}{6}}^{\frac{5\pi}{6}} (8[\csc x] - 5[\cot x])\, dx$ is equal to:**