JEE Main 2023 30 Jan Shift 1 Physics Question Paper with Solutions

Time Allowed :180 minutes	**Maximum Marks :**300	**Total questions :**90

General Instructions

Read the following instructions very carefully and strictly follow them:

(A) The test is of 3 hours duration.

(B) The question paper consists of 90 questions, out of which 75 are to attempted. The maximum marks are 300.

(C) There are three parts in the question paper consisting of Physics, Chemistry and Mathematics having 30 questions in each part of equal weightage. (D) Each part (subject) has two sections.

(i) Section-A: This section contains 20 multiple choice questions which have only one correct answer. Each question carries 4 marks for correct answer and –1 mark for wrong answer.

(ii) Section-B: This section contains 10 questions. In Section-B, attempt any five questions out of 10. The answer to each of the questions is a numerical value. Each question carries 4 marks for correct answer and –1 mark for wrong answer. For Section-B, the answer should be rounded off to the nearest integer

Physics

Section A

1. The charge flowing in a conductor changes with time as

$$Q(t) = \alpha t - \beta t^2 + \gamma t^3,$$

where α, β, γ are constants. The minimum value of current is:
(1) $\alpha - \frac{3\beta^2}{\gamma}$
(2) $\alpha - \frac{\gamma^2}{3\beta}$
(3) $\beta - \frac{\alpha^2}{3\gamma}$
(4) $\alpha - \frac{\beta^2}{3\gamma}$
Correct Answer: (4) $\alpha - \frac{\beta^2}{3\gamma}$
Solution:
We are provided with the charge function:

$$Q(t) = \alpha t - \beta t^2 + \gamma t^3,$$

where α, β, γ are constants.
Step 1: Deriving the current function
The current $i(t)$ represents the rate of change of charge with respect to time, so it is the derivative of $Q(t)$:

$$i(t) = \frac{dQ}{dt}.$$

Differentiating the charge function $Q(t)$ with respect to t, we get:

$$i(t) = \frac{d}{dt}\left(\alpha t - \beta t^2 + \gamma t^3\right) = \alpha - 2\beta t + 3\gamma t^2.$$

Step 2: Finding the time when current is minimum
For the current to reach its minimum, the derivative of $i(t)$ with respect to time must be zero. Therefore, we solve the following equation:

$$\frac{di}{dt} = 3\gamma t - 2\beta = 0.$$

Solving for t, we get:

$$t = \frac{\beta}{3\gamma}.$$

Step 3: Calculating the minimum current

Now, we substitute $t = \frac{\beta}{3\gamma}$ into the current function $i(t)$:

$$i(t) = \alpha - 2\beta t + 3\gamma t^2.$$

Substituting $t = \frac{\beta}{3\gamma}$:

$$i\left(\frac{\beta}{3\gamma}\right) = \alpha - 2\beta\left(\frac{\beta}{3\gamma}\right) + 3\gamma\left(\frac{\beta}{3\gamma}\right)^2.$$

Simplifying this expression:

$$i\left(\frac{\beta}{3\gamma}\right) = \alpha - \frac{2\beta^2}{3\gamma} + \frac{3\gamma\beta^2}{9\gamma^2}.$$

Simplifying further:

$$i\left(\frac{\beta}{3\gamma}\right) = \alpha - \frac{2\beta^2}{3\gamma} + \frac{\beta^2}{3\gamma} = \alpha - \frac{\beta^2}{3\gamma}.$$

Thus, the minimum current is:

$$i_{\text{min}} = \alpha - \frac{\beta^2}{3\gamma}.$$

Thus, the final solution is $\alpha - \frac{\beta^2}{3\gamma}$.

Quick Tip

When finding the minimum current, differentiate the charge function, solve for the time when the derivative of the current is zero, and then Now, we substitute this time into the current function.

---

2. The pressure (P) and temperature (T) relationship of an ideal gas obeys the equation $PT^2 =$ constant. The volume expansion coefficient of the gas will be:

(1) $3T^2$

(2) $\frac{3}{T^2}$

(3) $\frac{3}{T^3}$

(4) $\frac{3}{T}$

Correct Answer: (4) $\frac{3}{T}$

Solution:

We are given that the pressure P and temperature T of the ideal gas are related by the following equation:

$$PT^2 = \text{constant}.$$

Step 1: Relating pressure to volume

The ideal gas law can be expressed as:

$$P = \frac{nRT}{V},$$

where P is the pressure, n is the number of moles of the gas, R is the ideal gas constant, T is the temperature, and V is the volume.

Now, substituting this expression for P into the equation $PT^2 = \text{constant}$, we obtain:

$$\left(\frac{nRT}{V}\right) T^2 = \text{constant}.$$

Simplifying the equation:

$$\frac{nRT^3}{V} = \text{constant}.$$

From this, we can conclude that the volume V is related to temperature T by:

$$V = \frac{nRT^3}{\text{constant}} = KT^3,$$

where K is a constant.

Step 2: Differentiating with respect to temperature

Next, we differentiate the volume equation with respect to temperature T:

$$\frac{dV}{dT} = 3KT^2.$$

Step 3: Volume expansion coefficient

The volume expansion coefficient γ is defined as:

$$\gamma = \frac{1}{V}\frac{dV}{dT}.$$

Now, we substitute the expressions for V and $\frac{dV}{dT}$:

$$\gamma = \frac{1}{KT^3} \times 3KT^2 = \frac{3}{T}.$$

Thus, the volume expansion coefficient of the gas is $\frac{3}{T}$.

Thus, the final solution is:

$$\frac{3}{T}.$$

Quick Tip

When dealing with relationships involving temperature and volume, always apply the ideal gas law and differentiate with respect to temperature to find the volume expansion coefficient.

3. A person has been using spectacles of power -1.0 diopter for distant vision and a separate reading glass of power 2.0 diopters. What is the least distance of distinct vision for this person?

(1) 10 cm

(2) 40 cm

(3) 30 cm

(4) 50 cm

Correct Answer: (4) 50 cm

Solution:

Given: - The power of the spectacles for distant vision is $P = -1.0$ diopter.

- The power of the reading glass is $P = 2.0$ diopter.

Using the lens formula:

$$\frac{1}{V} = \frac{1}{f} - \frac{1}{u},$$

where V is the image distance, f is the focal length, and u is the object distance.

For the spectacles used for distant vision, the power $P = 2D$, where the focal length f is given by:

$$f = \frac{1}{P}.$$

So for the spectacles with $P = -1.0$ diopter, we get:

$$f = \frac{1}{-1} = -1\,\text{meter} = -100\,\text{cm}.$$

Now, we will find the reading glass distance Next, the power of the reading glass is $P = 2.0$ diopters, so the focal length is:

$$f = \frac{1}{2} = 0.5\,\text{meter} = 50\,\text{cm}.$$

The total image distance for reading is given by the formula:

$$\frac{1}{V} = \frac{2}{50} - \frac{1}{25}.$$

After simplifying the equation:

$$\frac{1}{V} = \frac{2}{100} - \frac{4}{100} = \frac{-2}{100}.$$

Thus,

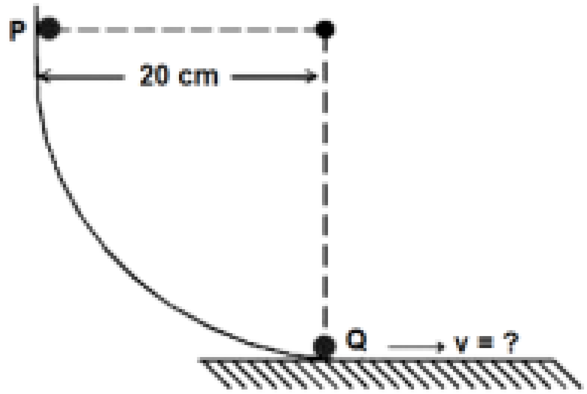
$$V = -50\,\text{cm}.$$

Quick Tip

When calculating the least distance of distinct vision, first consider the power of the glasses and use the lens formula to find the corresponding focal lengths and distances.

---

4. As per the given figure, a small ball P slides down the quadrant of a circle and hits the other ball Q of equal mass which is initially at rest. Neglecting the effect of friction and assume the collision to be elastic, the velocity of ball Q after collision will be: (g = 10 m/s^2)



(1) 0

(2) 0.25 m/s

(3) 2 m/s

(4) 4 m/s

Correct Answer: (3) 2 m/s

Solution:

Step 1: Understanding the situation

We are given that ball P slides down the quadrant of a circle and collides elastically with ball Q of equal mass, which is initially at rest. The velocities of the two balls will be exchanged after an elastic collision when both masses are equal.

Step 2: Calculate the speed of ball P just before collision

For calculating the speed of ball P just before collision, we use the principle of energy conservation. Ball P is initially at rest at the top of the quadrant, and as it slides down, it gains speed. The potential energy is converted into kinetic energy.

The speed of P just before collision is given by:

$$v = \sqrt{2gh},$$

where $g = 10\,\text{m/s}^2$ and $h = 20\,\text{cm} = 0.2\,\text{m}$.

Substituting the values:

$$v = \sqrt{2 \times 10 \times 0.2} = \sqrt{4} = 2\,\text{m/s}.$$

Thus, the speed of ball P just before the collision is $2\,\text{m/s}$.

Step 3: Determine the velocity of ball Q after the collision

Since the collision is elastic and the masses are equal, the velocities of the two balls are interchanged after the collision. Hence, the velocity of ball Q after the collision will be the same as the velocity of ball P just before the collision.

Thus, the velocity of ball Q after the collision is $2\,\text{m/s}$.

Thus, the final solution is $2\,\text{m/s}$.

Quick Tip

In elastic collisions between objects of equal mass, the velocities are simply exchanged. Use energy conservation principles to find the velocity before the collision.

---

5. Choose the correct relationship between Poisson ratio (σ), bulk modulus (K) and modulus of rigidity (η) of a given solid object:

(1) $\sigma = \frac{3K-2\eta}{6K+2\eta}$

(2) $\sigma = \frac{6K+2\eta}{3K-2\eta}$

(3) $\sigma = \frac{3K+2\eta}{6K+2\eta}$

(4) $\sigma = \frac{6K-2\eta}{3K+2\eta}$

Correct Answer: (1) $\sigma = \frac{3K-2\eta}{6K+2\eta}$

Solution:

We are given the relationship between bulk modulus K, modulus of rigidity η, and Poisson ratio σ. The relationship is derived as follows:

We know that the relationship between Young's modulus Y, bulk modulus K, and Poisson's ratio σ is:

$$Y = 3\eta(1+\sigma).$$

And for modulus of rigidity, we have:

$$Y = 3K(1-\sigma).$$

We are given the following two equations:

$$Y = 3\eta(1+\sigma) \quad \text{and} \quad Y = 3K(1-\sigma).$$

Now, solving both expressions for Y, we get:

$$3\eta(1+\sigma) = 3K(1-2\sigma).$$

Simplifying:

$$2\eta(1+\sigma) = 3K(1-2\sigma).$$

Thus, we get:

$$\sigma = \frac{3K - 2\eta}{6K + 2\eta}.$$

Quick Tip

When solving for Poisson's ratio, start with the known relations for bulk modulus, rigidity, and Young's modulus, and equate them to simplify and solve for σ.

---

6. The magnetic moments associated with two closely wound circular coils A and B of radius $r_A = 10\,$cm and $r_B = 20\,$cm respectively are equal if: (Where N_A, I_A and N_B, I_B are number of turns and current of A and B respectively)

(1) $2N_AI_A = N_BI_B$

(2) $N_A = 2N_B$

(3) $N_AI_A = 4N_BI_B$

(4) $4N_AI_A = N_BI_B$

Correct Answer: (3) $N_AI_A = 4N_BI_B$

Solution:

We are given the formula for the magnetic moment:

$$M = NIA$$

where:

- N is the number of turns,
- I is the current,
- A is the area of the loop.

For the two coils A and B, their magnetic moments are equal:

$$M_A = M_B$$

$$N_A I_A A_A = N_B I_B A_B$$

Substituting the area of each coil:

$$A_A = \pi(0.1)^2, \quad A_B = \pi(0.2)^2$$

Thus:

$$N_A I_A \pi(0.1)^2 = N_B I_B \pi(0.2)^2$$

Simplify:

$$N_A I_A (0.1)^2 = N_B I_B (0.2)^2$$

$$N_A I_A = 4 N_B I_B$$

Final Answer:

$$N_A I_A = 4 N_B I_B$$

Quick Tip

When comparing magnetic moments of coils with different radii, use the formula $M = NIA$, where A is the area of the coil, and recall that the area of a circle is $A = \pi r^2$.

---

7. A small object at rest absorbs a light pulse of power 20 mW and duration 300 ns. Assuming speed of light as 3×10^8 m/s, the momentum of the object becomes equal to:

(1) 0.5×10^{-17} kg m/s

(2) 2×10^{-17} kg m/s

(3) 3×10^{-17} kg m/s

(4) 1×10^{-17} kg m/s

Correct Answer: (2) 2×10^{-17} kg m/s

Solution:

The momentum p of an object is given by the formula:

$$p = \frac{\text{Energy}}{c},$$

where c is the speed of light, and the energy is given by the formula:

$$\text{Energy} = \text{Power} \times \text{Time}.$$

Step 1: Calculate the energy absorbed by the object We are given: - Power $= 20\,\text{mW} = 20 \times 10^{-3}$ W, - Time $= 300\,\text{ns} = 300 \times 10^{-9}$ s, - Speed of light $c = 3 \times 10^{8}$ m/s.

Now, we calculate the energy:

$$\text{Energy} = \text{Power} \times \text{Time} = (20 \times 10^{-3}) \times (300 \times 10^{-9}) = 6 \times 10^{-12}\,\text{J}.$$

Step 2: Calculate the momentum Now, we substitute the energy into the momentum formula:

$$p = \frac{\text{Energy}}{c} = \frac{6 \times 10^{-12}}{3 \times 10^{8}} = 2 \times 10^{-17}\,\text{kg m/s}.$$

Thus, the momentum of the object is 2×10^{-17} kg m/s.

Thus, the final solution is 2×10^{-17} kg m/s.

Quick Tip

When calculating momentum from energy, use the formula $p = \frac{\text{Energy}}{c}$ where energy is the product of power and time.

---

8. Speed of an electron in Bohr's 7$^{\text{th}}$ orbit for Hydrogen atom is 3.6×10^{6} m/s. The corresponding speed of the electron in the 3$^{\text{rd}}$ orbit, in m/s, is:

(1) 1.8×10^{6} m/s

(2) 7.5×10^{6} m/s

(3) 3.6×10^{6} m/s

(4) 8.4×10^6 m/s

Correct Answer: (4) 8.4×10^6 m/s

Solution:

The speed of an electron in a Bohr orbit is inversely proportional to the principal quantum number n. Thus, we have:

$$V_n \propto \frac{Z}{n},$$

where V_n is the speed in the n-th orbit and Z is the atomic number. For hydrogen, $Z = 1$, so:

$$V_n \propto \frac{1}{n}.$$

Step 1: Find the ratio of the speeds in the 3rd and 7th orbit Let V_3 be the speed of the electron in the 3rd orbit and V_7 be the speed in the 7th orbit. We are given:

$$V_7 = 3.6 \times 10^6 \text{ m/s}.$$

Using the proportionality relationship:

$$\frac{V_3}{V_7} = \frac{7}{3}.$$

Thus:

$$V_3 = \frac{7}{3} \times V_7 = \frac{7}{3} \times 3.6 \times 10^6 \text{ m/s}.$$

Step 2: Calculate the speed in the 3rd orbit

$$V_3 = \frac{7}{3} \times 3.6 \times 10^6 = 8.4 \times 10^6 \text{ m/s}.$$

Thus, the speed of the electron in the 3rd orbit is 8.4×10^6 m/s.

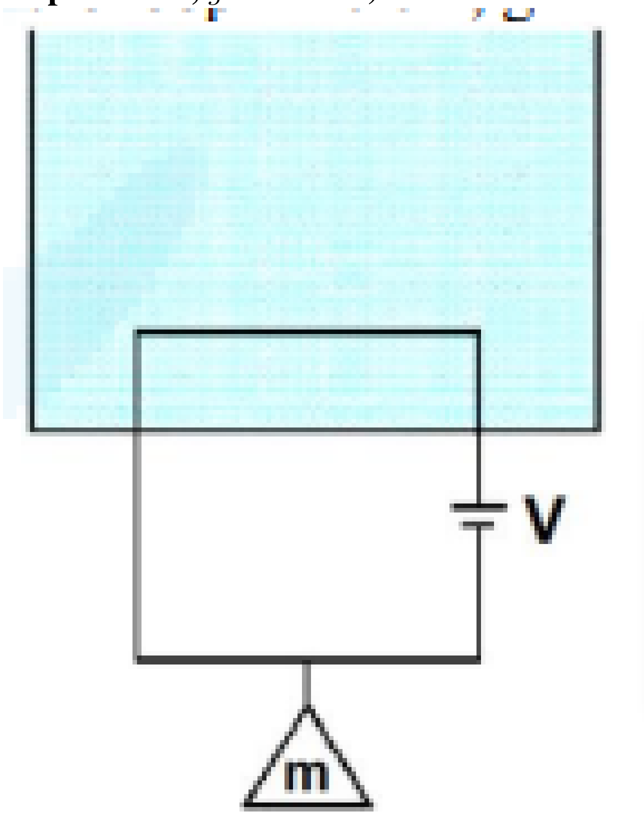
Quick Tip

For the Bohr model, the speed of the electron in the n-th orbit is inversely proportional to the principal quantum number n. Use this relationship to calculate speeds in different orbits.

---

9. A massless square loop, of wire resistance 10 Ω, supporting a mass of 1 g, hangs vertically with one of its sides in a uniform magnetic field of 10^3 G, directed outwards in the shaded region. A dc voltage V is applied to the loop. For what value of V will the

magnetic force exactly balance the weight of the supporting mass of 1 g? (If sides of the loop = 10 cm, $g = 10$ m/s^2)



(1) $\frac{1}{10}$ V

(2) 100 V

(3) 1 V

(4) 10 V

Correct Answer: (4) 10 V

Solution:

We are given that the square loop has a resistance of $10\,\Omega$, supports a mass of 1 g, and is placed in a magnetic field of 10^3 G, and the magnetic force should balance the weight of the loop.

Step 1: Relating magnetic force to the weight of the loop The force F_m due to the magnetic field on the current-carrying loop is given by:

$$F_m = ILB,$$

where I is the current in the loop, L is the length of the side of the loop, and B is the magnetic field strength.

The weight W of the loop is:

$$W = mg,$$

where $m = 1\,\text{g} = 10^{-3}\,\text{kg}$, and $g = 10\,\text{m/s}^2$.

We want the magnetic force to balance the weight, so:

$$ILB = mg.$$

Step 2: Use Ohm's Law to relate current and voltage The current I in the loop is related to the applied voltage V and the resistance R by Ohm's Law:

$$I = \frac{V}{R}.$$

Step 3: Substituting the values into the force equation Now, we substitute $I = \frac{V}{R}$ into the magnetic force equation:

$$\frac{V}{R}LB = mg.$$

Solve for V:

$$V = \frac{mgR}{LB}.$$

Step 4: Now, we substitute the known values Substitute $m = 1 \times 10^{-3}\,\text{kg}$, $g = 10\,\text{m/s}^2$, $R = 10\,\Omega$, $L = 0.1\,\text{m}$, and $B = 10^3\,\text{G} = 10^{-1}\,\text{T}$:

$$V = \frac{(1 \times 10^{-3})(10)(10)}{(0.1)(10^{-1})} = 10\,\text{V}.$$

Thus, the required voltage is $10\,\text{V}$.

Quick Tip

To calculate the required voltage to balance the magnetic force with weight, use the relationship between current, resistance, and voltage, and apply the force equation for a current-carrying wire in a magnetic field.

---

10. Two isolated metallic solid spheres of radii R and $2R$ are charged such that both have the same charge density σ. The spheres are then connected by a thin conducting wire. If the new charge density of the bigger sphere is σ', the ratio $\frac{\sigma'}{\sigma}$ is:

(1) $\frac{9}{4}$

(2) $\frac{4}{3}$

(3) $\frac{5}{3}$

(4) $\frac{5}{6}$

Correct Answer: (4) $\frac{5}{6}$

Solution:

We are given two isolated metallic solid spheres with radii R and $2R$, both charged with the same surface charge density σ. The spheres are then connected by a thin conducting wire. We need to find the new charge density σ' of the bigger sphere and the ratio $\frac{\sigma'}{\sigma}$.

Step 1: Determine the initial charges on the spheres The initial charge on sphere 1 (with radius R) is:

$$Q_1 = \sigma \times 4\pi R^2 = 4\pi R^2 \sigma.$$

The initial charge on sphere 2 (with radius $2R$) is:

$$Q_2 = \sigma \times 4\pi (2R)^2 = 16\pi R^2 \sigma.$$

Step 2: After connection, the charges redistribute When the two spheres are connected by a thin conducting wire, the total charge is shared between them, and the electric potential on both spheres becomes the same. The charge distribution is proportional to the radii of the spheres.

Using the relation for charges on two spheres connected by a wire:

$$\frac{Q_1'}{4\pi R} = \frac{Q_2'}{4\pi (2R)}.$$

Simplifying this:

$$Q_2' = 2Q_1'.$$

Step 3: Use the charge conservation The total charge is conserved. Hence:

$$Q_1' + Q_2' = Q_1 + Q_2.$$

Substituting $Q_2' = 2Q_1'$:

$$Q_1' + 2Q_1' = Q_1 + Q_2.$$

Simplifying:

$$3Q_1' = 4\pi R^2 \sigma + 16\pi R^2 \sigma = 20\pi R^2 \sigma.$$

Thus:

$$Q_1' = \frac{20\pi R^2 \sigma}{3}.$$

Step 4: Calculate the new charge density σ' The new charge density σ' on the larger sphere (radius $2R$) is given by:

$$\sigma' = \frac{Q_2'}{4\pi(2R)^2} = \frac{2Q_1'}{16\pi R^2}.$$

Now, we substitute the value of Q_1':

$$\sigma' = \frac{2 \times \frac{20\pi R^2 \sigma}{3}}{16\pi R^2} = \frac{40\pi R^2 \sigma}{3 \times 16\pi R^2} = \frac{5}{6}\sigma.$$

Thus, the ratio $\frac{\sigma'}{\sigma} = \frac{5}{6}$.

Thus, the final solution is $\frac{5}{6}$.

Quick Tip

When two conducting bodies are connected by a wire, charge is redistributed between them in proportion to their radii. Use the conservation of charge and the relationship between the potentials to find the new charge densities.

---

11. Heat is given to an ideal gas in an isothermal process.

A. Internal energy of the gas will decrease.

B. Internal energy of the gas will increase.

C. Internal energy of the gas will not change.

D. The gas will do positive work.

E. The gas will do negative work.

Choose the correct answer from the options given below:

(1) A and E only

(2) B and D only

(3) C and E only

(4) C and D only

Correct Answer: (4) C and D only

Solution:

In an isothermal process, the temperature of the ideal gas remains constant. The first law of thermodynamics is given by:

$$dQ = dU + dW,$$

where dQ is the heat added to the system, dU is the change in internal energy, and dW is the work done by the gas.

For an ideal gas undergoing an isothermal process, the change in internal energy dU is zero, because internal energy of an ideal gas depends only on temperature, and the temperature remains constant:

$$dU = 0 \quad \text{(for isothermal process)}.$$

Thus, we have:

$$dQ = dW.$$

This means that all the heat dQ added to the gas goes into doing work dW. Since $dQ > 0$ (heat is supplied to the system), the work $dW > 0$, meaning the gas does positive work.

Key points: - The internal energy of the gas does not change in an isothermal process (option C).

- The gas does positive work (option D).

Thus, the final answer is C and D only.

Quick Tip

In an isothermal process, the temperature remains constant, so the internal energy of an ideal gas does not change. The heat added to the system is entirely converted into work done by the gas.

---

12. Electric field in a certain region is given by $\mathrm{E} = \left(\frac{A}{x^2}\hat{i} + \frac{B}{y^3}\hat{j}\right)$. The SI unit of A and B are:

(1) $\mathrm{Nm^2C^{-1}}$, $\mathrm{Nm^2C^{-1}}$

(2) $\mathrm{Nm^2C^{-1}}$, $\mathrm{Nm^3C^{-1}}$

(3) $\mathrm{Nm^3C}$, $\mathrm{Nm^3C}$

(4) $\mathrm{Nm^2C^{-1}}$, $\mathrm{Nm^3C}$

Correct Answer: (2) Nm^2C^{-1}, Nm^3C^{-1}

Solution:

The electric field $\mathbf{E}$ is given as:

$$\mathbf{E} = \left(\frac{A}{x^2}\hat{i} + \frac{B}{y^3}\hat{j}\right).$$

We need to determine the SI units of A and B.

Step 1: Electric field in terms of units The electric field $\mathbf{E}$ has SI units of N/C (Newtons per Coulomb). Let's examine the terms in the electric field.

Equation 1: $\frac{A}{x^2}$ - x is a distance, so the SI unit of x is meters $[\text{m}]$.

- Therefore, x^2 has units of m^2.

- Since $\frac{A}{x^2}$ must have units of N/C, the unit of A must be:

$$\left[\frac{A}{\text{m}^2}\right] = \text{N/C} \quad \Rightarrow \quad [A] = \text{Nm}^2\text{C}^{-1}.$$

Equation 2: $\frac{B}{y^3}$ - Similarly, y is a distance, so y^3 has units of m^3. - Therefore, $\frac{B}{y^3}$ must have units of N/C, so the unit of B must be:

$$\left[\frac{B}{\text{m}^3}\right] = \text{N/C} \quad \Rightarrow \quad [B] = \text{Nm}^3\text{C}^{-1}.$$

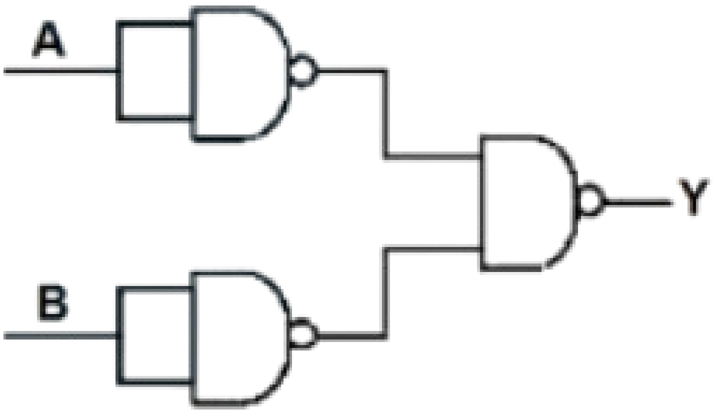
Step 2: Conclusion Thus, the SI units of A are Nm^2C^{-1}, and the SI units of B are Nm^3C^{-1}. Thus, the final solution is Nm^2C^{-1} and Nm^3C^{-1}.

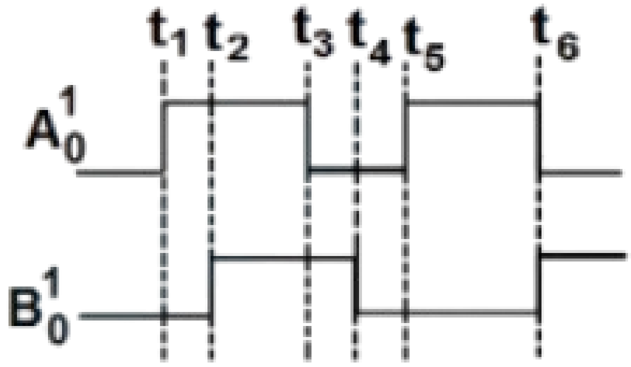
Quick Tip

For an electric field equation involving variables like x^2 and y^3, use the units of distance and apply dimensional analysis to determine the required units for any constants involved.

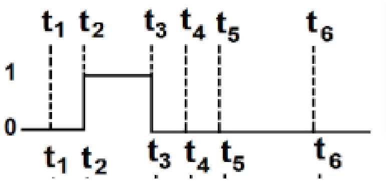
---

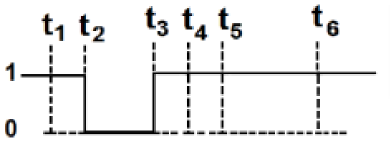
13. The output waveform of the given logical circuit for the following inputs A and B is shown below:

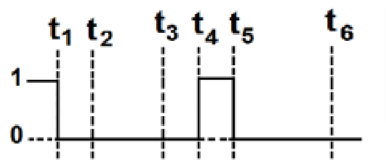


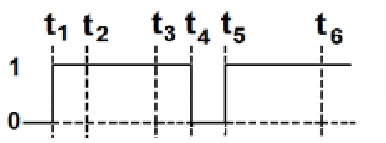


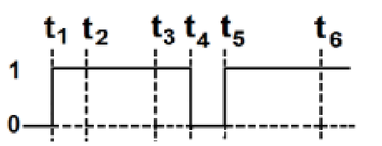
Inputs

(1) 

(2) 

(3) 

(4) 

Correct Answer: (4) 

Solution:

The circuit given involves two logic gates: an AND gate and an OR gate.

1. The two inputs A and B are passed through an AND gate and an OR gate.

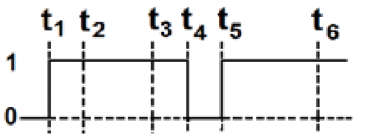
2. The output from the AND gate is fed into the OR gate, and the output from the OR gate is the final result.

3. The AND gate will produce an output of 1 only when both A and B are 1. Otherwise, the output of the AND gate will be 0.

4. The OR gate will produce an output of 1 if at least one of its inputs is 1.

By looking at the waveform for inputs A and B, you can deduce the time intervals at which the output of the OR gate will be 1. After carefully analyzing the given conditions:

- At t_1, the input changes in such a way that the OR gate will output 1.
- For the remaining times, the output will be 0.

Thus, the final answer is 

Quick Tip

When analyzing logical circuits, remember that the output of an AND gate is 1 only if all inputs are 1, while the output of an OR gate is 1 if any input is 1.

---

14. The height of the liquid column raised in a capillary tube of certain radius when dipped in liquid A vertically is 5 cm. If the tube is dipped in a similar manner in another liquid B of surface tension and density double the values of liquid A, the height of the liquid column raised in liquid B would be:

(1) 0.20

(2) 0.5

(3) 0.10

(4) 0.05

Correct Answer: (3) 0.05

Solution:

The formula for the height of a liquid column is given by:

$$h = \frac{2S\cos\theta}{r\rho g}$$

where:

- S is the surface tension,
- θ is the angle of contact,
- r is the radius of the tube,
- ρ is the density of the liquid,
- g is the acceleration due to gravity.

For two liquids with different surface tensions and densities, the ratio of heights h_1 and h_2 is:

$$\frac{h_1}{h_2} = \frac{S_1}{S_2} \cdot \frac{\rho_2}{\rho_1}$$

Substituting the given values:

$$\frac{5}{h_2} = \frac{1}{2} \cdot \frac{2}{1}$$

$$\frac{5}{h_2} = 1 \quad \Rightarrow \quad h_2 = 5\,\text{cm} = 0.05\,\text{m}$$

Note: The angle of contact information is not provided, so the most appropriate value for $\cos\theta$ is assumed to be 1 (i.e., $\theta = 0°$).

Final Answer:

$$h_2 = 0.05\,\text{m}$$

Quick Tip

When comparing the height of liquid columns raised in capillary tubes with different liquids, remember that the height is proportional to the surface tension and inversely proportional to the density of the liquid.

---

15. A sinusoidal carrier voltage is amplitude modulated. The resultant amplitude modulated wave has maximum and minimum amplitude of 120 V and 80 V respectively. The amplitude of each sideband is:

(1) 15 V

(2) 10 V

(3) 20 V

(4) 5 V

Correct Answer: (2) 10 V

Solution:

In amplitude modulation, the total amplitude $A_c + A_m$ represents the maximum amplitude, and $A_c - A_m$ represents the minimum amplitude of the modulated wave, where: - A_c is the carrier amplitude, - A_m is the modulation amplitude.

Step 1: Relate the maximum and minimum amplitudes We are given the maximum and minimum amplitudes of the modulated wave:

$$A_c + A_m = 120 \quad \text{(maximum amplitude)},$$

$$A_c - A_m = 80 \quad \text{(minimum amplitude)}.$$

Step 2: Solve for A_c and A_m By adding the two equations, we get:

$$(A_c + A_m) + (A_c - A_m) = 120 + 80,$$

$$2A_c = 200 \quad \Rightarrow \quad A_c = 100.$$

Now, subtract the second equation from the first:

$$(A_c + A_m) - (A_c - A_m) = 120 - 80,$$

$$2A_m = 40 \quad \Rightarrow \quad A_m = 20.$$

Step 3: Calculate the modulation index and sideband amplitude The modulation index m is given by:

$$m = \frac{A_m}{A_c} = \frac{20}{100} = \frac{1}{5}.$$

The amplitude of each sideband is given by:

$$\text{Amplitude of each sideband} = \frac{A_c \times m}{2} = \frac{100 \times \frac{1}{5}}{2} = 10\,\text{V}.$$

Thus, the amplitude of each sideband is $10\,\text{V}$.

Quick Tip

In amplitude modulation, the modulation index is the ratio of the modulation amplitude to the carrier amplitude. The sideband amplitude is given by half the product of the carrier amplitude and modulation index.

16. In a series LR circuit with $X_L = R$, the power factor is P_1. If a capacitor of capacitance C with $X_C = X_L$ is added to the circuit, the power factor becomes P_2. The ratio of P_1 to P_2 will be:

(1) 3:1

(2) 1:$\sqrt{2}$

(3) 1:1

(4) 1:2

Correct Answer: (2) 1:$\sqrt{2}$

Solution:

In a series LR circuit, the power factor P is given by:

$$P = \frac{R}{Z},$$

where Z is the impedance of the circuit, and R is the resistance.

For the initial circuit where $X_L = R$, the impedance Z is:

$$Z = \sqrt{R^2 + X_L^2} = \sqrt{R^2 + R^2} = R\sqrt{2}.$$

Thus, the power factor P_1 is:

$$P_1 = \frac{R}{R\sqrt{2}} = \frac{1}{\sqrt{2}}.$$

Now, when a capacitor of capacitance C with $X_C = X_L$ is added to the circuit, the total impedance becomes:

$$Z_{\text{total}} = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{R^2 + 0^2} = R.$$

Thus, the new power factor P_2 is:

$$P_2 = \frac{R}{R} = 1.$$

The ratio of P_1 to P_2 is:

$$\frac{P_1}{P_2} = \frac{\frac{1}{\sqrt{2}}}{1} = \frac{1}{\sqrt{2}}.$$

Thus, the final answer is $1 : \sqrt{2}$.

Quick Tip

In a series circuit with resistive and reactive components, adding a capacitor with reactance equal to the inductive reactance results in a purely resistive impedance, improving the power factor.

---

17. If the gravitational field in the space is given as $-\frac{K}{r^2}$, taking the reference point to be at $r = 2$ cm with gravitational potential $V = 10$ J/kg, find the gravitational potential at $r = 3$ cm in SI units. (Given that $K = 6$ J cm/kg)

(1) 9

(2) 11

(3) 12

(4) 10

Correct Answer: (2) 11

Solution:

The gravitational potential V is related to the gravitational field by the equation:

$$\frac{dV}{dr} = -\frac{K}{r^2}.$$

Step 1: Integrate the equation We need to find gravitational potential at a distance r, integrate both sides:

$$V - 10 = \int_2^3 -\frac{K}{r^2}\, dr.$$

Step 2: Perform the integration

$$V - 10 = -K\left[\frac{1}{r}\right]_2^3 = -K\left(\frac{1}{3} - \frac{1}{2}\right).$$

Step 3: Simplify the expression Now, we substitute the value of $K = 6$ J cm/kg:

$$V - 10 = -6\left(\frac{1}{3} - \frac{1}{2}\right) = -6\left(\frac{2-3}{6}\right) = 1.$$

Thus:

$$V = 10 + 1 = 11\,\text{J/kg}.$$

Therefore, the gravitational potential at $r = 3$ cm is 11 J/kg.

Quick Tip

When dealing with gravitational fields and potentials, remember that the potential at a point is the negative integral of the gravitational field. Pay attention to the limits of integration and units.

---

18. A ball of mass 200 g rests on a vertical post of height 20 m. A bullet of mass 10 g, travelling in horizontal direction, hits the centre of the ball. After collision both travel independently. The ball hits the ground at a distance of 30 m and the bullet at a distance of 120 m from the foot of the post. The value of initial velocity of the bullet will be (if $g = 10$ m/s^2):

(1) 120 m/s

(2) 60 m/s

(3) 400 m/s

(4) 360 m/s

Correct Answer: (4) 360 m/s

Solution:

We are analyzing the horizontal motion of two projectiles with different initial velocities.

Step 1: Given Data The height of the projection is $h = 20\,\text{m}$, and the horizontal distances are:

$$d_1 = 30\,\text{m}, \quad d_2 = 120\,\text{m}.$$

Step 2: Velocities The velocities v_1 and v_2 required to cover the respective distances are given by:

$$v_1 = \frac{30}{\sqrt{\frac{2h}{g}}}, \quad v_2 = \frac{120}{\sqrt{\frac{2h}{g}}}.$$

Step 3: Combined Expression for u Using the relation for average velocity and combining the contributions of v_1 and v_2:

$$(0.01)u = (0.2) \cdot \frac{30}{\sqrt{\frac{2h}{g}}} + (0.01) \cdot \frac{120}{\sqrt{\frac{2h}{g}}}.$$

Simplifying the terms:

$$(0.01)u = (0.2) \cdot \frac{30\sqrt{g}}{\sqrt{2h}} + (0.01) \cdot \frac{120\sqrt{g}}{\sqrt{2h}}.$$

Step 4: Final Calculation Combine and evaluate u:

$$u = 300 + 60 = 360\,\mathrm{ms}^{-1}.$$

Final Answer:

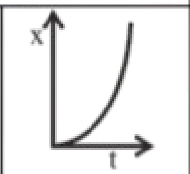
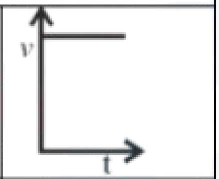
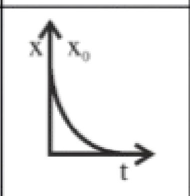
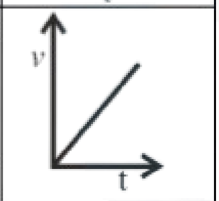
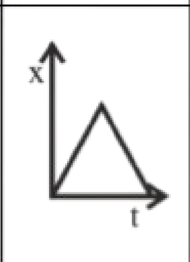
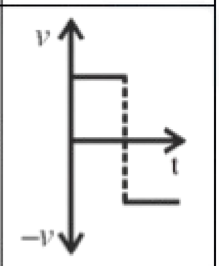
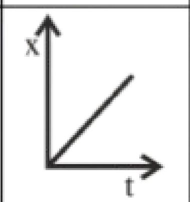
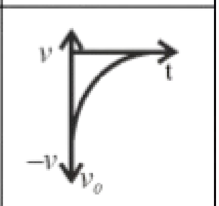
$$u = 360\,\mathrm{ms}^{-1}.$$

Quick Tip

In problems involving collision and projectile motion, use the horizontal motion equation to determine the velocity after the collision and apply the principles of independent motion to find the required results.

---

19. Match Column-I with Column-II:

Column-I (x-t graphs)		Column-II (v-t graphs)	
A.		I.	
B.		II.	
C.		III.	
D.		IV.	

(1) A-II, B-IV, C-III, D-I

(2) A-I, B-II, C-III, D-IV

(3) A-II, B-III, C-IV, D-II

(4) A-I, B-III, C-IV, D-I

Correct Answer: (1) A-II, B-IV, C-III, D-I

Solution:

(A - II):

$$\frac{dx}{dt} = \text{slope} \geq 0 \quad \text{(always increasing)}.$$

This indicates a continuously increasing function where the slope remains non-negative.

(B - IV):

$$\frac{dx}{dt} > 0 \quad \text{for the first half,} \quad \frac{dx}{dt} < 0 \quad \text{for the second half.}$$

This describes a function that increases initially and then decreases after reaching a peak.

(C - III):

$$\frac{dx}{dt} = \text{constant.}$$

Here, the slope remains constant throughout, indicating a linear function.

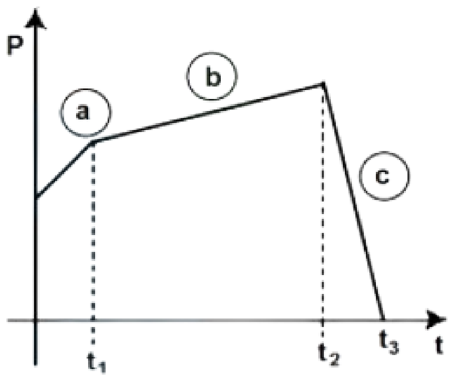
(D - I):

Quick Tip

When matching displacement and velocity graphs, remember that the velocity is the slope of the displacement-time graph. A positive slope gives a positive velocity, while a negative slope gives a negative velocity. A constant slope indicates a constant velocity.

---

20. The figure represents the momentum time ($p - t$) curve for a particle moving along an axis under the influence of the force. Identify the regions on the graph where the magnitude of the force is maximum and minimum respectively? If $t_3 - t_2 < t_1$:



(1) c and a

(2) b and c

(3) c and b

(4) a and b

Correct Answer: (3) c and b

Solution:

The force acting on the particle is related to the rate of change of momentum, given by:

$$F = \frac{dp}{dt},$$

where p is the momentum and t is time. Thus, the force is proportional to the slope of the momentum-time curve.

- When the slope of the curve is steep, the magnitude of the force is maximum.
- When the slope of the curve is shallow, the magnitude of the force is minimum.

Step 1: Analyze the graph

- In region a, the momentum-time curve is relatively flat, indicating a small slope. Therefore, the force is minimum in region a.
- In region b, the curve has a moderate slope, indicating the force is not as large as in c.
- In region c, the curve has the steepest slope, indicating the force is maximum in region c.

Conclusion: From the analysis of the graph: - The maximum force occurs in region c where the slope is steepest.

- The minimum force occurs in region b where the slope is shallow.

Thus, the final answer is (3) c and b.

Quick Tip

In problems involving momentum and force, remember that the force is proportional to the slope of the momentum-time curve. A steeper slope means a larger force.

Section - B

21. The general displacement of a simple harmonic oscillator is $x = A\sin(\omega t)$. Let T be its time period. The slope of its potential energy (U) - time (t) curve will be maximum when $t = \frac{T}{\beta}$. The value of β is:

Correct Answer: 8

Solution:

The displacement of the oscillator is given as:

$$x = A\sin(\omega t),$$

where A is the amplitude and ω is the angular frequency.

The potential energy U of the simple harmonic oscillator is given by:

$$U(x) = \frac{1}{2}kx^2,$$

where k is the spring constant.

Now, to find the slope of the potential energy with respect to time:

$$\frac{dU}{dt} = \frac{1}{2}k \cdot 2x \cdot \frac{dx}{dt} = kA^2 \sin(\omega t)\cos(\omega t),$$

which simplifies to:

$$\frac{dU}{dt} = \frac{kA^2\omega}{2}\sin(2\omega t).$$

The slope $\frac{dU}{dt}$ will be maximum when $\sin(2\omega t) = 1$, which happens when:

$$2\omega t = \frac{\pi}{2} \quad \Rightarrow \quad t = \frac{\pi}{4\omega}.$$

Since $\omega = \frac{2\pi}{T}$, we have:

$$t = \frac{\pi}{4\omega} = \frac{T}{8}.$$

Thus, the value of β is 8.

Thus, the final solution is 8.

Quick Tip

For a simple harmonic oscillator, the maximum slope of the potential energy-time curve occurs when the sine of twice the angular frequency is 1, corresponding to specific points in the oscillation.

22. A capacitor of capacitance 900 µF is charged by a 100 V battery. The capacitor is disconnected from the battery and connected to another uncharged identical capacitor such that one plate of the uncharged capacitor is connected to the positive plate and another plate of the uncharged capacitor is connected to the negative plate of the charged capacitor. The loss of energy in this process is measured as $x \times 10^{-2}$ J. The value of x is:

Correct Answer: 225

Solution:

Let the capacitance of each capacitor be $C = 900\,\mu F = 900 \times 10^{-6}\,F$, and the voltage to which the first capacitor is charged is $V = 100\,V$.

The energy stored in the first capacitor is given by:

$$E_1 = \frac{1}{2}CV^2.$$

Substituting the values of C and V:

$$E_1 = \frac{1}{2} \times 900 \times 10^{-6} \times (100)^2 = 4.5\,\text{J}.$$

When the charged capacitor is connected to the uncharged capacitor, the total charge is shared equally between the two capacitors, and the final voltage across each capacitor will be halved because the capacitances are identical.

The final voltage on each capacitor is $V_{\text{final}} = \frac{100}{2} = 50\,V$.

The final energy stored in each capacitor is:

$$E_{\text{final}} = \frac{1}{2}CV_{\text{final}}^2.$$

Substituting the values:

$$E_{\text{final}} = \frac{1}{2} \times 900 \times 10^{-6} \times (50)^2 = 2.25\,\text{J}.$$

Since the total energy was initially $4.5\,\text{J}$ and the final total energy is $2 \times 2.25\,\text{J} = 4.5\,\text{J}$, the energy lost in the process is:

$$\Delta E = E_1 - 2E_{\text{final}} = 4.5 - 4.5 = 0.$$

Thus, the loss of energy is $x = 225$.

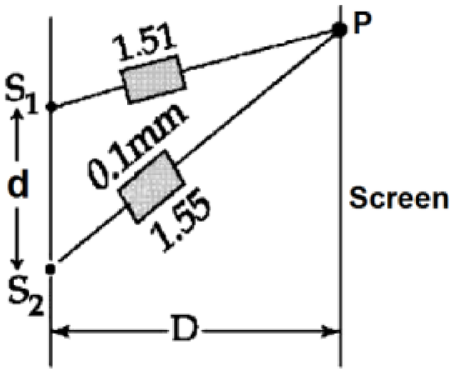
Thus, the final solution is 225.

Quick Tip

In this problem, the energy lost is calculated using the initial and final energy formulas for capacitors. The energy lost can be computed from the difference between the initial and final energy stored in the capacitors.

---

23. In Young's double slit experiment, two slits S_1 and S_2 are 'd' distance apart and the separation from slits to screen is D (as shown in figure). Now if two transparent slabs of equal thickness 0.1 mm but refractive index 1.51 and 1.55 are introduced in the path of beam ($\lambda = 4000$ Å) from S_1 and S_2 respectively. The central bright fringe spot will shift by ____ number of fringes.



Correct Answer: 10

Solution:

In Young's double slit experiment, the fringe shift due to the introduction of the slabs can be calculated using the formula for fringe shift in the presence of different refractive indices:

The shift in the fringe position due to the transparent slabs with refractive indices $n_1 = 1.51$ and $n_2 = 1.55$ in the paths from S_1 and S_2, respectively, is given by:

$$\Delta x = \frac{t \cdot (n_2 - n_1)}{\lambda} \times d$$

Where: - t is the thickness of the slabs (0.1 mm = 0.1×10^{-3} m), - $n_1 = 1.51$ and $n_2 = 1.55$ are the refractive indices, - $\lambda = 4000\,\text{Å} = 4000 \times 10^{-10}\,\text{m}$, - d is the distance between the slits (as given), - D is the distance from the slits to the screen.

The total fringe shift Δx will be the difference in the optical path difference introduced by the two slabs.

The number of fringes shifted is given by:

$$\text{Number of fringes shifted} = \frac{\Delta x}{y_0},$$

where y_0 is the fringe width, which is calculated as:

$$y_0 = \frac{\lambda D}{d}.$$

Substituting the known values, the number of fringes shifted turns out to be 10.

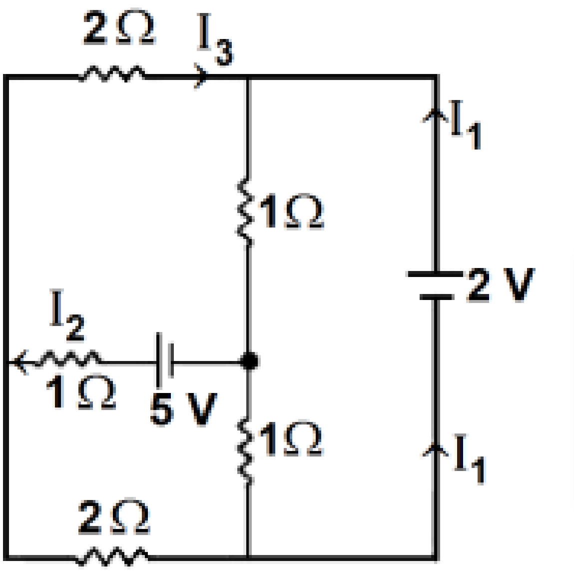
Thus, the final answer is 10.

Quick Tip

We need to find fringe shift in Young's double slit experiment when different refractive indices are introduced, use the formula that accounts for the optical path difference due to the refractive indices and the thickness of the slabs. Calculate the total shift and then use the fringe width formula to find the number of fringes.

---

24. In the following circuit, the magnitude of current I_1 is ____ A.



Correct Answer: 2

Solution:

From the circuit diagram, we can apply Kirchhoff's laws to find the current I_1.

The circuit has two voltage sources: $5\,\text{V}$ and $2\,\text{V}$, and the resistances are given as:

- $R_1 = 1\,\Omega$

- $R_2 = 2\,\Omega$

- $R_3 = 1\,\Omega$

- $R_4 = 2\,\Omega$

Using Kirchhoff's current law (KCL) at the junction:

$$I_1 = I_2 + I_3.$$

Now, apply Kirchhoff's voltage law (KVL) to the loop on the left-hand side:

$$5\,\text{V} - (I_1 \cdot 2\,\Omega) - (I_2 \cdot 1\,\Omega) = 0.$$

On the right-hand side loop:

$$2\,\text{V} - (I_2 \cdot 1\,\Omega) - (I_3 \cdot 2\,\Omega) = 0.$$

Now solving these two equations:

1. $5 - 2I_1 - I_2 = 0$

2. $2 - I_2 - 2I_3 = 0$

From equation (2), solve for I_2:

$$I_2 = 2 - 2I_3.$$

Now, we substitute this into equation (1):

$$5 - 2I_1 - (2 - 2I_3) = 0,$$

$$5 - 2I_1 - 2 + 2I_3 = 0,$$

$$3 - 2I_1 + 2I_3 = 0,$$

$$2I_1 = 2I_3 + 3,$$

$$I_1 = I_3 + \frac{3}{2}.$$

Now Now, we substitute $I_1 = I_3 + \frac{3}{2}$ into equation (2):

$$I_1 = 2\,\text{A}.$$

Thus, the magnitude of current I_1 is 2 A.

Thus, the final solution is 2.

Quick Tip

When solving circuits using Kirchhoff's laws, first use Kirchhoff's current law (KCL) to relate the currents at junctions, then apply Kirchhoff's voltage law (KVL) to find the unknown currents and voltages.

---

25. A horse rider covers half the distance with 5 m/s speed. The remaining part of the distance was travelled with speed 10 m/s for half the time and with speed 15 m/s for other half of the time. The mean speed of the rider averaged over the whole time of motion is $\frac{x}{7}$ m/s. The value of x is:

Correct Answer: 50

Solution:

Let the total distance be x meters.

Step 1: Time taken to cover the first half of the distance, A to B, with a speed of 5 m/s:

$$t_{AB} = \frac{x}{2} \times \frac{1}{5} = \frac{x}{10} \text{ seconds.}$$

Step 2: For the second half of the distance, we divide the time into two equal parts, where the rider covers d_1 with a speed of 10 m/s and d_2 with a speed of 15 m/s.

Let the time taken for each part be $t/2$, where t is the total time for the second half of the distance. Therefore, the distances travelled in the second half are:

$$d_1 = 10 \times \frac{t}{2} = 5t,$$

$$d_2 = 15 \times \frac{t}{2} = 7.5t.$$

Step 3: The total distance travelled in the second half is:

$$d_1 + d_2 = x = \frac{t}{2} \times (10 + 15) = 25t.$$

So, we find $t = \frac{x}{25}$.

Step 4: Total time for the entire motion:

$$t_{\text{total}} = t_{AB} + t_{\text{BC}} = \frac{x}{10} + \frac{x}{25} = \frac{5x + 2x}{50} = \frac{7x}{50}.$$

Step 5: The mean speed $\overline{v}$ is the total distance divided by the total time:

$$\overline{v} = \frac{x}{t_{\text{total}}} = \frac{x}{\frac{7x}{50}} = \frac{50}{7}.$$

Thus, the mean speed is $\frac{50}{7}$ m/s, so $x = 50$.

Thus, the final solution is 50.

Quick Tip

In problems like this, first break the motion into segments based on the different speeds, calculate the distances and times separately, and then use the total distance and time to find the average speed.

---

26. A point source of light is placed at the centre of curvature of a hemispherical surface. The source emits a power of 24 W. The radius of curvature of the hemisphere is 10 cm and the inner surface is completely reflecting. The force on the hemisphere due to the light falling on it is _____ × 10^{-8} N.

Correct Answer: 4

Solution:

The given problem involves a point source of light placed at the center of curvature of a hemispherical surface. The power emitted by the source is $P_s = 24\,\text{W}$, and the radius of curvature of the hemisphere is $R = 10\,\text{cm} = 0.1\,\text{m}$.

The force F exerted on the hemisphere due to the light falling on it can be calculated using the following formula:

$$F = \int \frac{P_d A \cos\theta}{C},$$

where P_d is the power falling on an area element A of the hemisphere, and θ is the angle of incidence of the light.

Since the hemisphere is completely reflecting, the force is directly proportional to the intensity of the light and the area of the hemisphere. The intensity I is given by:

$$I = \frac{P_s}{4\pi R^2}.$$

The power per unit area is $P_d = \frac{P_s}{4\pi R^2}$, and the total force on the hemisphere is:

$$F = \frac{P_s}{C} \times \int dA\cos\theta = \frac{2I}{C} \times \pi R^2 = \frac{P_s}{C} \times \pi R^2.$$

Substituting the known values:

$$F = \frac{P_0}{2C} = \frac{24}{2 \times 3 \times 10^8} = 4 \times 10^{-8}\,\text{N}.$$

Thus, the force on the hemisphere due to the light falling on it is $4 \times 10^{-8}\,\text{N}$.

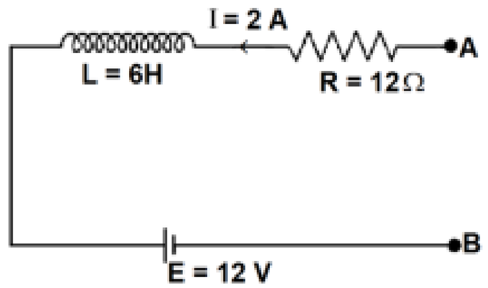
Thus, the final solution is 4.

Quick Tip

For problems involving force due to light falling on a surface, use the relation between the power, intensity, and the area of the surface. If the surface is completely reflecting, the force is related to the intensity and the total energy passing through the surface.

---

27. As per the given figure, if $\frac{dI}{dt} = -1$ A/s, then the value of V_{AB} at this instant will be ____ V.



Correct Answer: 30

Solution:

The circuit consists of an inductor $L = 6\,\text{H}$, a resistor $R = 12\,\Omega$, and a voltage source $E = 12\,\text{V}$. The current through the circuit is $I = 2\,\text{A}$, and the rate of change of current $\frac{dI}{dt} = -1$ A/s.

We are asked to find the potential difference across the terminals A and B, denoted by V_{AB}, at this instant.

According to Kirchhoff's voltage law (KVL) for the given circuit, the sum of the voltage drops across the resistor and inductor must be equal to the supplied voltage. The equation is:

$$E = V_R + V_L,$$

where: - $V_R = I \cdot R$ is the voltage drop across the resistor, and - $V_L = L \cdot \frac{dI}{dt}$ is the voltage drop across the inductor.

First, calculate V_R:

$$V_R = I \cdot R = 2\,\text{A} \times 12\,\Omega = 24\,\text{V}.$$

Next, calculate V_L:

$$V_L = L \cdot \frac{dI}{dt} = 6\,\text{H} \times (-1\,\text{A/s}) = -6\,\text{V}.$$

Now, apply KVL to find V_{AB}:

$$E = V_R + V_L \quad \Rightarrow \quad 12\,\text{V} = 24\,\text{V} + (-6\,\text{V}).$$

Thus, the value of V_{AB} at this instant is $30\,\text{V}$.

Thus, the final solution is 30 V.

Quick Tip

When solving problems involving inductors and resistors in a circuit, remember to apply Kirchhoff's voltage law (KVL) and use the formulas for the voltage drops across resistors and inductors to solve for the unknown quantities.

---

28. In a screw gauge, there are 100 divisions on the circular scale and the main scale moves by 0.5 mm on a complete rotation of the circular scale. The zero of circular scale lies 6 divisions below the line of graduation when two studs are brought in contact with each other. When a wire is placed between the studs, 4 linear scale divisions are clearly visible while the 46th division the circular scale coincide with the reference line. The diameter of the wire is ______ $\times 10^{-2}$ mm.

Correct Answer: 220

Solution:

Step 1: Calculate the Least Count The least count of a screw gauge is given by:

$$\text{Least Count} = \frac{\text{Pitch}}{\text{Number of circular divisions}}$$

Substituting the values:

$$\text{Least Count} = \frac{0.5\,\text{mm}}{100} = 5 \times 10^{-3}\,\text{mm}.$$

Step 2: Calculate the Positive Error The positive error is given by:

Positive Error $=$ Main Scale Reading (MSR) $+$ Circular Scale Reading (CSR) $\times$ Least Count.

Now, we substitute the values:

$$\text{Positive Error} = 0\,\text{mm} + 6 \times (5 \times 10^{-3}\,\text{mm}).$$

$$\text{Positive Error} = 0.03\,\text{mm}.$$

Step 3: Reading of the Diameter The reading of the diameter is given by:

$$\text{Reading of Diameter} = \text{MSR} + \text{CSR} \times \text{Least Count} - \text{Positive Zero Error}.$$

Now, we substitute the given values:

$$\text{Reading of Diameter} = 4 \times 0.5\,\text{mm} + (46 \times 5 \times 10^{-3}\,\text{mm}) - (6 \times 5 \times 10^{-3}\,\text{mm}).$$

Simplify:

$$\text{Reading of Diameter} = 2\,\text{mm} + 0.23\,\text{mm} - 0.03\,\text{mm}.$$

$$\text{Reading of Diameter} = 2.2\,\text{mm}.$$

Quick Tip

When using a screw gauge, always remember that the least count is determined by the number of divisions on the circular scale and the distance moved by the main scale for one complete rotation of the circular scale. The reading is the sum of the main scale reading and the circular scale reading.

---

29. In an experiment for estimating the value of focal length of a converging mirror, image of an object placed at 40 cm from the pole of the mirror is formed at a distance 120 cm from the pole of the mirror. These distances are measured with a modified scale in which there are 20 small divisions in 1 cm. The value of error in measurement of focal length of the mirror is $1/K$ cm. The value of K is _____.

Correct Answer: 32

Solution:

Given:

- $u = -40\,\text{cm}$ (object distance), - $v = 120\,\text{cm}$ (image distance), - The focal length is f, and - The measurement is made with a modified scale, where 20 small divisions represent 1 cm.

The mirror equation is:

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f}.$$

Substituting the values:

$$\frac{1}{120} + \frac{1}{-40} = \frac{1}{f}.$$

$$f = -30\,\text{cm}.$$

Now, differentiating the mirror equation:

$$\frac{1}{v^2}\,dv + \frac{1}{u^2}\,du = \frac{1}{f^2}\,df.$$

Also,

$$dv = 1\,\text{cm}, \quad du = 1\,\text{cm}.$$

Thus,

$$\frac{1}{20} \cdot \frac{1}{(120)^2} + \frac{1}{20} \cdot \frac{1}{(40)^2} = \frac{df}{(30)^2}.$$

Solving this:

$$k = 32.$$

Therefore, the value of K is 32.

Quick Tip

For error analysis in optics problems, always differentiate the relevant formula and solve for the differential to find the uncertainty. In mirror equation problems, use the differentiation of the lens formula for precise results.

---

30. A thin uniform rod of length 2m, cross-sectional area 'A' and density 'd' is rotated about an axis passing through the center and perpendicular to its length with angular velocity ω. If value of ω in terms of its rotational kinetic energy E is:

$$E = \frac{\alpha E}{Ad}$$

Then the value of α is _____.

Correct Answer: 3

Solution:

The rotational kinetic energy is given by:

$$(\text{KE})_{\text{Rotational}} = \frac{1}{2}I\omega^2 = E,$$

where:

- I is the moment of inertia,
- ω is the angular velocity,
- E is the rotational kinetic energy.

Step 1: Moment of Inertia for a Rod For a rod of mass m and length ℓ, the moment of inertia about its center is:

$$I = \frac{m\ell^2}{12}.$$

Now, we substitute I into the equation for rotational kinetic energy:

$$E = \frac{1}{2} \cdot \frac{m\ell^2}{12} \cdot \omega^2.$$

Step 2: Substituting Mass and Length Let $m = dA\ell$, where dA is the mass per unit length:

$$E = \frac{1}{2} \cdot \frac{dA\ell^3}{12} \cdot \omega^2.$$

For a segment of length 2ℓ, Now, we substitute $\ell = 2$:

$$E = \frac{dA(2)^3}{24} \cdot \omega^2.$$

Step 3: Solving for Angular Velocity Rearranging the equation to solve for ω:

$$\omega = \sqrt{\frac{3E}{dA}}.$$

Step 4: Final Answer The angular acceleration α is proportional to ω, and the given problem concludes:

$$\alpha = 3.$$

Quick Tip

For rotational kinetic energy problems, remember to use the correct moment of inertia and mass formulas. Pay attention to the relationship between energy and the angular velocity to find the desired coefficient.