Jee Advanced 2024 May 26 paper 2 Question Paper with Solutions

Time Allowed :3 Hours	Maximum Marks :180	Total Questions :51

General Instructions

Read the following instructions very carefully and strictly follow them:

1. The JEE Advanced 2024, Paper 2, will be structured with a total of 180 marks over 3 hours.
2. It will include 51 questions, with 17 questions each in Physics, Chemistry, and Mathematics.
3. Each subject will be segmented into four sections: Section I: 12 marks.
4. The marking scheme also varies, for example, questions may carry 1 mark, 2 marks, 3 marks or 4 marks.
5. There are negative markings of -1 or -2 and some questions can also read to no negative marking.

1. Considering only the principal values of the inverse trigonometric functions, the value of

$$\tan\left(\sin^{-1}\left(\frac{3}{5}\right) - 2\cos^{-1}\left(\frac{2}{\sqrt{5}}\right)\right)$$

is:

(1) $\frac{7}{24}$

(2) $-\frac{7}{24}$

(3) $-\frac{5}{24}$

(4) $\frac{5}{24}$

Correct Answer: (2) $-\frac{7}{24}$

Solution: Let

$$2\cos^{-1}\left(\frac{2}{\sqrt{5}}\right) = \theta.$$

This implies

$$\frac{2}{\sqrt{5}} = \cos\left(\frac{\theta}{2}\right).$$

Using this, we find

$$\tan\left(\frac{\theta}{2}\right) = \frac{1}{2}.$$

Next, we calculate the following values:

$$\tan\left(\sin^{-1}\left(\frac{3}{5}\right)\right) = \frac{3}{4}, \quad \tan\left(\cos^{-1}\left(\frac{2}{\sqrt{5}}\right)\right) = \frac{4}{3}.$$

To determine

$$\tan\left(\sin^{-1}\left(\frac{3}{5}\right) - \cos^{-1}\left(\frac{2}{\sqrt{5}}\right)\right),$$

we use the identity for the tangent of a difference:

$$\tan(A - B) = \frac{\tan A - \tan B}{1 + \tan A \cdot \tan B}.$$

Substituting the values:

$$\tan\left(\sin^{-1}\left(\frac{3}{5}\right) - \cos^{-1}\left(\frac{2}{\sqrt{5}}\right)\right) = \frac{\frac{3}{4} - \frac{4}{3}}{1 + \frac{3}{4} \cdot \frac{4}{3}}.$$

Simplify the numerator and denominator:

$$\text{Numerator: } \frac{3}{4} - \frac{4}{3} = \frac{9}{12} - \frac{16}{12} = -\frac{7}{12},$$

$$\text{Denominator: } 1 + \frac{3}{4} \cdot \frac{4}{3} = 1 + 1 = 2.$$

Thus,

$$\tan\left(\sin^{-1}\left(\frac{3}{5}\right) - \cos^{-1}\left(\frac{2}{\sqrt{5}}\right)\right) = \frac{-\frac{7}{12}}{2} = -\frac{7}{24}.$$

Quick Tip

When dealing with inverse trigonometric functions, convert the expressions to their respective trigonometric ratios for simpler computation.

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2. Let

$$S = \{(x, y) \in \mathbb{R} \times \mathbb{R} : x \geq 0, y \geq 0, y^2 \leq 4x, y^2 \leq 12 - 2x, \text{ and } 3y + \sqrt{8}x \leq 5\sqrt{8}\}.$$

If the area of the region S is $\alpha\sqrt{2}$, then α is equal to:

(1) $\frac{17}{2}$

(2) $\frac{17}{3}$

(3) $\frac{17}{4}$

(4) $\frac{17}{5}$

Correct Answer: (2) $\frac{17}{3}$

Solution: To determine the area of the region S, we evaluate the following integral and calculate the additional area contributions:

$$\text{Area} = \int_0^2 2\sqrt{x}\, dx + \frac{1}{2} \cdot 3 \cdot \sqrt{8}.$$

Step 1: Determine the boundaries The given equations are:

$$y^2 = 4x \quad \text{and} \quad y^2 = 12 - 2x.$$

Setting $y^2 = 4x = 12 - 2x$, we find:

$$x = 2 \quad \text{and} \quad y = \sqrt{8}.$$

Step 2: Evaluate the integral The integral of $2\sqrt{x}$ from 0 to 2 is:

$$\int_0^2 2\sqrt{x}\, dx = 2 \cdot \int_0^2 x^{1/2}\, dx = 2 \cdot \left[\frac{2}{3}x^{3/2}\right]_0^2 = 2 \cdot \frac{2}{3} \cdot (2)^{3/2}.$$

Simplify:

$$\int_0^2 2\sqrt{x}\, dx = \frac{4}{3} \cdot 2\sqrt{2} = \frac{8}{3}\sqrt{2}.$$

Step 3: Add the triangular area The triangular area is:

$$\frac{1}{2} \cdot 3 \cdot \sqrt{8} = \frac{3}{2} \cdot \sqrt{8} = 3\sqrt{2}.$$

Step 4: Total area Adding the contributions:

$$\text{Total Area} = \frac{8}{3}\sqrt{2} + 3\sqrt{2} = \left(\frac{8}{3} + \frac{9}{3}\right)\sqrt{2} = \frac{17}{3}\sqrt{2}.$$

Final Result The total area is given as:

$$\text{Total Area} = \alpha\sqrt{2}, \quad \text{where } \alpha = \frac{17}{3}.$$

Thus, $\alpha = \frac{17}{3}$.

Quick Tip

When solving geometry problems involving inequalities, sketch the region to visualize limits of integration.

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3. Let $k \in \mathbb{R}$. If

$$\lim_{x\to 0^+} (\sin(\sin(kx)) + \cos x + x)^{2/x} = e^6,$$

then the value of k is:

(1) 1

(2) 2

(3) 3

(4) 4

Correct Answer: (2) 2

Solution: Consider the given limit:

$$y = \lim_{x\to 0^+} \frac{(\sin(\sin(kx)) + \cos x + x)^2}{x}.$$

Step 1: Take the natural logarithm Let

$$\ln y = \lim_{x\to 0^+} \ln\left(\frac{(\sin(\sin(kx)) + \cos x + x)^2}{x}\right).$$

Using the logarithmic property $\ln(a^2) = 2\ln(a)$, we have:

$$\ln y = 2 \lim_{x\to 0^+} \frac{\ln(\sin(\sin(kx)) + \cos x + x)}{x}.$$

Step 2: Apply known information We are given that $y = e^6$. Thus:

$$\ln y = 6 \implies 2 \lim_{x\to 0^+} \frac{\ln(\sin(\sin(kx)) + \cos x + x)}{x} = 6.$$

Step 3: Differentiate using L'Hôpital's Rule Define $f(x) = \ln(\sin(\sin(kx)) + \cos x + x)$.

Differentiating $f(x)$:

$$f'(x) = \frac{k\cos(\sin(kx))\cos(kx) - \sin x + 1}{\sin(\sin(kx)) + \cos x + x}.$$

The denominator remains x, and applying L'Hôpital's Rule gives:

$$\lim_{x\to 0^+} \frac{f(x)}{x} = \lim_{x\to 0^+} \frac{f'(x)}{1}.$$

Substituting:

$$\lim_{x\to 0^+} 2 \times \frac{k\cos(\sin(kx))\cos(kx) - \sin x + 1}{\sin(\sin(kx)) + \cos x + x}.$$

Step 4: Evaluate the limit as $x \to 0^+$ As $x \to 0^+$:

$$\sin(\sin(kx)) \to kx, \quad \cos x \to 1, \quad \text{and } x \to 0.$$

Thus:

$$\sin(\sin(kx)) + \cos x + x \to kx + 1.$$

In the numerator:

$$k\cos(\sin(kx))\cos(kx) - \sin x + 1 \to k(1)(1) - 0 + 1 = k + 1.$$

Therefore:

$$\lim_{x\to 0^+} 2 \times \frac{k+1}{1} = 2(k+1).$$

Step 5: Solve for k Equating to the given result $2(k+1) = 6$:

$$k + 1 = 3 \implies k = 2.$$

Final Answer

$$\boxed{k = 2}$$

Quick Tip

For limits involving small x, use Taylor expansions to approximate trigonometric and logarithmic terms.

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4. Let $f : \mathbb{R} \to \mathbb{R}$ be a function defined by

$$f(x) = \begin{cases} x^2 \sin\left(\frac{\pi}{x^2}\right), & \text{if } x \neq 0, \\ 0, & \text{if } x = 0. \end{cases}$$

Then which of the following statements is TRUE?

(1) $f(x) = 0$ has infinitely many solutions in the interval $\left[\frac{1}{10^{10}}, \infty\right)$.

(2) $f(x) = 0$ has no solutions in the interval $\left[\frac{1}{\pi}, \infty\right)$.

(3) The set of solutions of $f(x) = 0$ is in the interval $\left(0, \frac{1}{10^{10}}\right]$.

(4) $f(x) = 0$ has more than 25 solutions in the interval $\left(\frac{1}{\pi^2}, \frac{1}{\pi}\right]$.

Correct Answer: (4) $f(x) = 0$ has more than 25 solutions in the interval $\left(\frac{1}{\pi^2}, \frac{1}{\pi}\right]$.

Solution: We are tasked with solving the equation:

$$x^2 \sin\left(\frac{\pi}{x^2}\right) = 0.$$

Step 1: Analyze the equation The product $x^2 \sin\left(\frac{\pi}{x^2}\right) = 0$ implies:

$$\sin\left(\frac{\pi}{x^2}\right) = 0, \quad \text{for } x \neq 0.$$

The sine function equals zero when:

$$\frac{\pi}{x^2} = n\pi, \quad n \in \mathbb{Z}, \quad n > 0 \quad (\text{as } x > 0).$$

Simplifying gives:

$$x^2 = \frac{1}{n}, \quad \text{so } x = \frac{1}{\sqrt{n}}, \quad n > 0.$$

Step 2: Restrict the interval We are given that $x \in \left(\frac{1}{\pi^2}, \frac{1}{\pi}\right]$. For $x = \frac{1}{\sqrt{n}}$, this implies:

$$\frac{1}{\pi^2} < \frac{1}{\sqrt{n}} \leq \frac{1}{\pi}.$$

Squaring all sides, we find:

$$\frac{1}{\pi^4} < \frac{1}{n} \leq \frac{1}{\pi^2}.$$

Taking the reciprocals (and reversing the inequalities), we get:

$$\pi^4 > n \geq \pi^2.$$

Step 3: Determine the values of n Approximating $\pi^4 \approx 97$ and $\pi^2 \approx 9$, we find that n can take integer values:

$$n = 9, 10, 11, \ldots, 97.$$

Step 4: Count the number of solutions The number of integers n in this range is:

$$97 - 9 + 1 = 89.$$

Conclusion There are 89 solutions for $f(x) = 0$ in the interval $\left(\frac{1}{\pi^2}, \frac{1}{\pi}\right]$.

Quick Tip

To solve trigonometric equations, express the argument in terms of known solutions, like $n\pi$ for $\sin(x) = 0$, and analyze the constraints on n based on the interval.

5. Let S be the set of all $(\alpha, \beta) \in \mathbb{R} \times \mathbb{R}$ such that

$$\lim_{x \to \infty} \frac{\sin(x^2)(\log_e x)^\alpha \sin\left(\frac{1}{x^2}\right)}{x^{\alpha\beta}(\log_e(1+x))^\beta} = 0.$$

Then which of the following is (are) correct?

(1) $(-1, 3) \in S$

(2) $(-1, 1) \in S$

(3) $(1, -1) \in S$

(4) $(1, -2) \in S$

Correct Answer: (2) $(-1, 1) \in S$, (3) $(1, -1) \in S$

Solution: The given limit simplifies as:

$$\lim_{x \to \infty} \frac{\sin(x^2)(\log_e x)^\alpha \sin\left(\frac{1}{x^2}\right)}{x^{\alpha\beta}(\log_e(1+x))^\beta}.$$

For $x \to \infty$, expanding:

$$\sin\left(\frac{1}{x^2}\right) \approx \frac{1}{x^2}.$$

So:

$$\lim_{x \to \infty} \frac{(\log_e x)^\alpha}{x^{\alpha\beta}(\log_e(1+x))^\beta} \cdot \frac{1}{x^2}.$$

Let $\log_e x = t$, then:

$$\frac{(\log_e x)^\alpha}{x^{\alpha\beta}} \sim t^\alpha e^{-t\alpha\beta}.$$

For the limit to be 0, we require:

$$\alpha\beta + 2 > 0 \quad \text{or} \quad \alpha\beta > -2.$$

Substituting the options:

For $(-1, 3) : \alpha\beta = -3$ (not valid).

For $(-1, 1) : \alpha\beta = -1$ (valid).

For $(1, -1) : \alpha\beta = -1$ (valid).

For $(1, -2) : \alpha\beta = -2$ (not valid).

Quick Tip

Always simplify limits by approximating logarithmic and trigonometric terms for large values of x.

6. A straight line drawn from the point $P(1,3,2)$, parallel to the line

$$\frac{x-2}{1} = \frac{y-4}{2} = \frac{z-6}{1},$$

intersects the plane $L_1 : x - y + 3z = 6$ at the point Q. Another straight line passing through Q and perpendicular to the plane L_1 intersects the plane $L_2 : 2x - y + z = -4$ at the point R. Then which of the following statements is (are) TRUE?

(1) The length of the line segment PQ is $\sqrt{6}$.

(2) The coordinates of R are $(1,6,0)$.

(3) The centroid of the triangle PQR is $\left(\frac{4}{3}, \frac{14}{3}, \frac{5}{3}\right)$.

(4) The perimeter of the triangle PQR is $\sqrt{2} + \sqrt{6} + \sqrt{11}$.

Correct Answer: (1), (3)

Solution: Equation of the line passing through P is:

$$x = r + 1, \quad y = 2r + 3, \quad z = r + 2.$$

Substituting into $L_1 : x - y + 3z = 6$:

$$(r+1) - (2r+3) + 3(r+2) = 6 \quad \Rightarrow \quad r = 1.$$

Thus, $Q = (2,5,3)$.

Equation of the line passing through Q and perpendicular to L_1:

$$\frac{x-2}{2} = \frac{y-5}{-1} = \frac{z-3}{3}.$$

Substituting into $L_2 : 2x - y + z = -4$:

$$2(2+2\lambda) - (5-\lambda) + (3+3\lambda) = -4 \quad \Rightarrow \quad \lambda = -1.$$

Thus, $R = (1,6,0)$.

Distance PQ:

$$PQ = \sqrt{(2-1)^2 + (5-3)^2 + (3-2)^2} = \sqrt{6}.$$

Centroid of $\triangle PQR$:

$$\text{Centroid} = \left(\frac{1+2+1}{3}, \frac{3+5+6}{3}, \frac{2+3+0}{3}\right) = \left(\frac{4}{3}, \frac{14}{3}, \frac{5}{3}\right).$$

Perimeter of $\triangle PQR$:

$$\sqrt{6} + \sqrt{11} + \sqrt{13}.$$

Quick Tip

Always parameterize the line equations and substitute them into the plane equations to find the points of intersection.

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7. Let A_1, B_1, C_1 be three points in the xy-plane. Suppose that the lines A_1C_1 and B_1C_1 are tangents to the curve $y^2 = 8x$ at A_1 and B_1, respectively. If $O = (0, 0)$ and $C_1 = (-4, 0)$, then which of the following statements is (are) TRUE?

(1) The length of the line segment OA_1 is $4\sqrt{3}$.

(2) The length of the line segment A_1B_1 is 16.

(3) The orthocenter of the triangle $A_1B_1C_1$ is $(0, 0)$.

(4) The orthocenter of the triangle $A_1B_1C_1$ is $(1, 0)$.

Correct Answer: (1), (3)

Solution: Let the points be defined as follows:

$$A_1 = (2t_1^2, 4t_1), \quad B_1 = (2t_2^2, 4t_2), \quad C_1 = (-4, 0).$$

Given $t_1 = -\sqrt{2}$ and $t_2 = \sqrt{2}$, the coordinates are:

$$A_1 = (4, -4\sqrt{2}), \quad B_1 = (4, 4\sqrt{2}).$$

Step 1: Calculate the length of OA_1 The distance OA_1 is calculated as:

$$OA_1 = \sqrt{(4-0)^2 + (-4\sqrt{2} - 0)^2}.$$

Simplify:

$$OA_1 = \sqrt{4^2 + (-4\sqrt{2})^2} = \sqrt{16 + 32} = \sqrt{48} = 4\sqrt{3}.$$

Step 2: Calculate the length of A_1B_1 The distance A_1B_1 is given by:

$$A_1B_1 = \sqrt{(4-4)^2 + (4\sqrt{2} - (-4\sqrt{2}))^2}.$$

Simplify:

$$A_1B_1 = \sqrt{0^2 + (8\sqrt{2})^2} = \sqrt{64 \cdot 2} = \sqrt{128} = 16.$$

Step 3: Verify the orthocenter of $\triangle A_1B_1C_1$ The orthocenter is the point where the altitudes of the triangle intersect. The altitudes are derived using the slopes of the sides:

- Slope of A_1B_1 is undefined (vertical line), so its altitude passes through $C_1 = (-4, 0)$.
- Slope of A_1C_1 and B_1C_1 can be used to verify that the altitudes intersect at the origin $(0, 0)$.

Thus, the orthocenter of $\triangle A_1B_1C_1$ is $(0, 0)$.

Conclusion:

$$OA_1 = 4\sqrt{3}, \quad A_1B_1 = 16, \quad \text{and the orthocenter is } (0, 0).$$

Quick Tip

For tangents to parabolas, parameterize the points and apply geometric properties of triangles.

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8. Let $f : \mathbb{R} \to \mathbb{R}$ be a function such that $f(x + y) = f(x) + f(y)$ for all $x, y \in \mathbb{R}$ and $g : R \to (0, \infty)$ be a function such that $g(x + y) = g(x)g(y)$ for all $x, y \in \mathbb{R}$. If $f\left(-\frac{3}{5}\right) = 12$ and $g\left(-\frac{1}{3}\right) = 2$, then the value of

$$f\left(\frac{1}{4}\right) + g(-2) - 8 \cdot g(0)$$

is _____.

Correct Answer: 51

Solution: The functional equations imply:

$$f(x + y) = f(x) + f(y) \quad \Rightarrow \quad f(x) = kx,$$

$$g(x + y) = g(x)g(y) \quad \Rightarrow \quad g(x) = a^x.$$

Given:

$$f\left(-\frac{3}{5}\right) = 12 \quad \Rightarrow \quad k\left(-\frac{3}{5}\right) = 12 \quad \Rightarrow \quad k = -20.$$

Similarly:

$$g\left(-\frac{1}{3}\right) = 2 \quad \Rightarrow \quad a^{-1/3} = 2 \quad \Rightarrow \quad a = \frac{1}{8}.$$

Now:

$$f\left(\frac{1}{4}\right) = -20 \cdot \frac{1}{4} = -5, \quad g(-2) = \left(\frac{1}{8}\right)^{-2} = 64, \quad g(0) = 1.$$

Substituting:

$$f\left(\frac{1}{4}\right) + g(-2) - 8 \cdot g(0) = -5 + 64 - 8 \cdot 1 = 51.$$

Quick Tip

For functional equations, substitute known points to determine constants and simplify.

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9. A bag contains N balls out of which 3 balls are white, 6 balls are green, and the remaining balls are blue. Assume that the balls are identical otherwise. Three balls are drawn randomly one after the other without replacement. For $i = 1, 2, 3$, let W_i, G_i, and B_i denote the events that the ball drawn in the i-th draw is a white ball, green ball, and blue ball, respectively. If the probability $P(W_1 \cap G_2 \cap B_3) = \frac{2}{5N}$ and the conditional probability $P(B_3 \mid W_1 \cup G_2) = \frac{2}{9}$, then N equals _____.

Correct Answer: 11

Solution: We are given:

$$P(W_1 \cap G_2 \cap B_3) = \frac{3}{N} \cdot \frac{6}{N-1} \cdot \frac{N-9}{N-2} = \frac{2}{5N}.$$

Step 1: Simplify the expression The left-hand side simplifies as:

$$\frac{3 \cdot 6 \cdot (N-9)}{N(N-1)(N-2)} = \frac{18(N-9)}{N(N-1)(N-2)}.$$

Equating this to the given probability:

$$\frac{18(N-9)}{N(N-1)(N-2)} = \frac{2}{5N}.$$

Step 2: Cross-multiply Cross-multiplying gives:

$$90(N-9) = 2N(N-1)(N-2).$$

Step 3: Expand and simplify Expanding both sides:

$$90N - 810 = 2N(N^2 - 3N + 2).$$

Simplify further:

$$90N - 810 = 2N^3 - 6N^2 + 4N.$$

Rearranging terms:

$$2N^3 - 6N^2 - 86N + 810 = 0.$$

Divide through by 2:

$$N^3 - 3N^2 - 43N + 405 = 0.$$

Step 4: Solve for N Using trial values, $N = 11$ satisfies the equation:

$$11^3 - 3(11^2) - 43(11) + 405 = 0.$$

Another root, $N = 37$, can be found, but $N < 15$ is a given condition, so it is invalid.

Final Answer:

$$N = 11.$$

> **Quick Tip**
>
> Break down conditional probabilities using known values to form solvable equations.

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10. Let the function $f : \mathbb{R} \to \mathbb{R}$ be defined by

$$f(x) = \frac{\sin x}{e^{\pi x}} \cdot \frac{x^{2023} + 2024x + 2025}{(x^2 - x + 3)} + \frac{2}{e^{\pi x}} \cdot \frac{x^{2023} + 2024x + 2025}{(x^2 - x + 3)}.$$

Then the number of solutions of $f(x) = 0$ in $\mathbb{R}$ is _____.

Correct Answer: 1

Solution: Simplifying $f(x)$:

$$f(x) = \frac{x^{2023} + 2024x + 2025}{(x^2 - x + 3)e^{\pi x}} \cdot (\sin x + 2).$$

For $f(x) = 0$, we solve:

$$x^{2023} + 2024x + 2025 = 0 \quad (\text{since } \sin x + 2 \neq 0 \text{ and } x^2 - x + 3 > 0).$$

Let:

$$g(x) = x^{2023} + 2024x + 2025.$$

Derivative:

$$g'(x) = 2023x^{2022} + 2024 > 0 \quad (\text{strictly increasing}).$$

Thus, $g(x)$ cuts the x-axis only once, so $f(x) = 0$ has exactly one solution.

> **Quick Tip**
>
> Check monotonicity of functions to determine the number of roots for polynomials.

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11. Let $\vec{p} = 2\hat{i} + \hat{j} + 3\hat{k}$ and $\vec{q} = \hat{i} - \hat{j} + \hat{k}$. If for some real numbers α, β, γ, we have

$$15\mathbf{i} + 10\hat{j} + 6\hat{k} = \alpha(2\vec{p} + \vec{q}) + \beta(\vec{p} - 2\vec{q}) + \gamma(\vec{p} \times \vec{q}),$$

then the value of γ is ____.

Correct Answer: 2

Solution: We are given:

$$\vec{p} = 2\hat{i} + \hat{j} + 3\hat{k}, \quad \vec{q} = \hat{i} - \hat{j} + \hat{k}.$$

Step 1: Calculate linear combinations of vectors First, compute:

$$2\vec{p} + \vec{q} = 2(2\hat{i} + \hat{j} + 3\hat{k}) + (\hat{i} - \hat{j} + \hat{k}).$$

Simplify:

$$2\vec{p} + \vec{q} = 4\hat{i} + 2\hat{j} + 6\hat{k} + \hat{i} - \hat{j} + \hat{k} = 5\hat{i} + \hat{j} + 7\hat{k}.$$

Next, compute:

$$\vec{p} - 2\vec{q} = (2\hat{i} + \hat{j} + 3\hat{k}) - 2(\hat{i} - \hat{j} + \hat{k}).$$

Simplify:

$$\vec{p} - 2\vec{q} = 2\hat{i} + \hat{j} + 3\hat{k} - 2\hat{i} + 2\hat{j} - 2\hat{k} = \hat{j} + \hat{k}.$$

Step 2: Compute the cross product $\vec{p} \times \vec{q}$ Using the determinant formula for the cross product:

$$\vec{p} \times \vec{q} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 1 & 3 \\ 1 & -1 & 1 \end{vmatrix}.$$

Expanding the determinant:

$$\vec{p} \times \vec{q} = \hat{i} \begin{vmatrix} 1 & 3 \\ -1 & 1 \end{vmatrix} - \hat{j} \begin{vmatrix} 2 & 3 \\ 1 & 1 \end{vmatrix} + \hat{k} \begin{vmatrix} 2 & 1 \\ 1 & -1 \end{vmatrix}.$$

Simplify:

$$\vec{p} \times \vec{q} = \hat{i}(1 \cdot 1 - (-1 \cdot 3)) - \hat{j}(2 \cdot 1 - 3 \cdot 1) + \hat{k}(2 \cdot (-1) - 1 \cdot 1).$$

$$\vec{p} \times \vec{q} = \hat{i}(1 + 3) - \hat{j}(2 - 3) + \hat{k}(-2 - 1).$$

$$\vec{p} \times \vec{q} = 4\hat{i} + \hat{j} - 3\hat{k}.$$

Step 3: Form and solve the equation We now solve for α, β, and γ such that:

$$\alpha(5\hat{i} + \hat{j} + 7\hat{k}) + \beta(\hat{j} + \hat{k}) + \gamma(4\hat{i} + \hat{j} - 3\hat{k}) = 15\hat{i} + 10\hat{j} + 6\hat{k}.$$

Equating components: - For $\hat{i}$:

$$5\alpha + 4\gamma = 15.$$

- For $\hat{j}$:

$$\alpha + \beta + \gamma = 10.$$

- For $\hat{k}$:

$$7\alpha + \beta - 3\gamma = 6.$$

Step 4: Solve the system of equations From the first equation:

$$\alpha = \frac{15 - 4\gamma}{5}.$$

Substitute α into the other two equations and solve:

$$\beta = \frac{11}{5}, \quad \gamma = 2, \quad \alpha = \frac{7}{5}.$$

Final Answer:

$$\alpha = \frac{7}{5}, \quad \beta = \frac{11}{5}, \quad \gamma = 2.$$

Quick Tip

Use vector operations like dot products and cross products to simplify expressions systematically.

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12. A normal with slope $\frac{1}{\sqrt{6}}$ is drawn from the point $(0, -\alpha)$ to the parabola $x^2 = -4ay$, where $a > 0$. Let L be the line passing through $(0, -\alpha)$ and parallel to the directrix of the parabola. Suppose that L intersects the parabola at two points A and B. Let r denote the length of the latus rectum and s denote the square of the length of the line segment AB. If $r : s = 1 : 16$, then the value of $24a$ is ____.

Correct Answer: 12

Solution: The equation of the normal to $x^2 = -4ay$ is:

$$-ty + x = 2at + at^3, \quad \text{with slope } \frac{1}{\sqrt{6}} \Rightarrow t = \sqrt{6}.$$

Substituting $t = \sqrt{6}$:

$$y = -2a - at^2 - 8a = -8a \quad \Rightarrow \quad \alpha = 8a.$$

Points of intersection A and B:

$$A = \left(\frac{\alpha}{\sqrt{2}}, -\alpha\right), \quad B = \left(-\frac{\alpha}{\sqrt{2}}, -\alpha\right).$$

Length of AB:

$$AB = \sqrt{\frac{2\alpha^2}{2}} = \sqrt{2}\alpha.$$

Given $r = 4a$, $s = (AB)^2 = 2\alpha = 128a^2$. Solving:

$$\frac{r}{s} = \frac{1}{32a} = \frac{1}{16} \quad \Rightarrow \quad a = \frac{1}{2}, \quad 24a = 12.$$

Quick Tip

Use parametric equations of conics to derive intersection points for normals and tangents.

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13. Let the function $f : [1, \infty) \to \mathbb{R}$ be defined by

$$f(t) = \begin{cases} (-1)^{n+1}2, & \text{if } t = 2n-1,\ n \in \mathbb{N}, \\ \frac{(2n+1-t)}{2} f(2n-1) + \frac{(t-(2n-1))}{2} f(2n+1), & \text{if } 2n-1 < t < 2n+1,\ n \in \mathbb{N}. \end{cases}$$

Define $g(x) = \int_1^x f(t)\,dt, x \in (1, \infty)$. Let α denote the number of solutions of the equation $g(x) = 0$ in the interval $(1, 8]$ and $\beta = \lim_{x \to 1^+} \frac{g(x)}{x-1}$. Then the value of $\alpha + \beta$ is ____.

Correct Answer: 5

Solution: The piecewise definition of $f(t)$ is given as:

$$f(t) = \begin{cases} 2, & t = 1, \\ 4 - 2t, & 1 < t < 3, \\ -2, & t = 3, \\ -8 - 2t, & 3 < t < 5, \text{ and so on.} \end{cases}$$

Step 1: Evaluate $g(x)$ The integral $g(x)$ is defined as:

$$g(x) = \int_1^x f(t)\,dt.$$

From the structure of $f(t)$, the integral satisfies:

$$g(x) = 0 \quad \text{when } x = 3, 5, 7, \ldots.$$

Thus, we identify $\alpha = 3$, the first point where $g(x) = 0$ after $x = 1$.

Step 2: Determine β We calculate the limit:

$$\beta = \lim_{x \to 1^+} \frac{g(x)}{x-1}.$$

From the Fundamental Theorem of Calculus:

$$\lim_{x \to 1^+} \frac{g(x)}{x-1} = f(1) = 2.$$

Thus, $\beta = 2$.

Step 3: Final Result

The sum is:

$$\alpha + \beta = 3 + 2 = 5.$$

Final Answer:

$$\alpha + \beta = 5.$$

Quick Tip

Visualize piecewise functions graphically to calculate integrals and identify roots.

Paragraph for Questions 14 and 15:

Let $S = \{1, 2, 3, 4, 5, 6\}$ and X be the set of all relations R from S to S that satisfy both the following properties:

(i) R has exactly 6 elements.

(ii) For each $(a, b) \in R$, we have $|a - b| \geq 2$.

Let $Y = \{R \in X :$ The range of R has exactly one element$\}$ and
$Z = \{R \in X : R$ is a function from S to $S\}$.
Let $n(A)$ denote the number of elements in a set A.

(There are two questions based on PARAGRAPH "I", the question given below is one of them).

---

14. If $n(X) = {}^m C_6$, then the value of m is ____.

Correct Answer: 20

Solution:

To satisfy the condition $|a - b| \geq 2$, b must be at least 2 units away from a. The possible pairs are:

$$a = 1, \quad b = 3, 4, 5, 6,$$

$$a = 2, \quad b = 4, 5, 6,$$

$$a = 3, \quad b = 1, 5, 6,$$

$$a = 4, \quad b = 1, 2, 6,$$

$$a = 5, \quad b = 1, 2, 3,$$

$$a = 6, \quad b = 1, 2, 3, 4.$$

Counting all valid pairs:

$$\text{Total pairs} = 20.$$

Thus:

$$n(X) = \binom{20}{6} \quad \Rightarrow \quad m = 20.$$

Quick Tip

To count relations, ensure each pair satisfies the given conditions and compute the total combinations.

---

15. If the value of $n(Y) + n(Z)$ is k^2, then $|k|$ is ____.

Correct Answer: 36

Solution: To calculate $n(Y)$: Since the range of R has exactly one element, R cannot have 6 elements. Therefore:

$$n(Y) = 0.$$

To calculate $n(Z)$: The number of functions from S to S is determined by the product of choices for each mapping. Using the binomial coefficients:

$$n(Z) = \binom{4}{1} \cdot \binom{3}{1} \cdot \binom{3}{1} \cdot \binom{3}{1} \cdot \binom{3}{1} \cdot \binom{4}{1}.$$

Simplify:

$$n(Z) = 36^2.$$

Thus, the total is:

$$n(Y) + n(Z) = 36^2.$$

From this, we find:

$$|k| = 36.$$

Final Answer:

$$|k| = 36.$$

Quick Tip

For functions, consider all mappings that satisfy the given conditions and compute the total possible arrangements.

Paragraph for Questions 16 and 17:

Let $f : \left[0, \frac{\pi}{2}\right] \to [0, 1]$ **be the function defined by** $f(x) = \sin^2 x$ **and let** $g : \left[0, \frac{\pi}{2}\right] \to [0, \infty)$ **be the function defined by** $g(x) = \sqrt{\frac{\pi x}{2} - x^2}$**.**

---

16. The value of $2\int_0^{\pi/2} f(x)g(x)\,dx - \int_0^{\pi/2} g(x)\,dx$ **is ____.**

Correct Answer: 0

Solution: Let:

$$I = \int_0^{\pi/2} f(x)g(x)\,dx.$$

Substituting $f(x) = \sin^2(x)$ and $g(x) = \sqrt{\frac{\pi x}{2} - x^2}$:

$$I = \int_0^{\pi/2} \sin^2(x)\sqrt{\frac{\pi x}{2} - x^2}\,dx.$$

Using the trigonometric identity $\sin^2(x) = 1 - \cos^2(x)$:

$$I = \int_0^{\pi/2} \cos^2(x)\sqrt{\frac{\pi x}{2} - x^2}\,dx.$$

Combining integrals:

$$2I = \int_0^{\pi/2} \left(\sin^2(x) + \cos^2(x)\right)\sqrt{\frac{\pi x}{2} - x^2}\,dx = \int_0^{\pi/2} g(x)\,dx.$$

Thus:

$$2\int_0^{\pi/2} f(x)g(x)\,dx - \int_0^{\pi/2} g(x)\,dx = 0.$$

Quick Tip

Use symmetry and trigonometric identities to simplify integrals involving squares of trigonometric functions.

---

17. The value of $\frac{16}{\pi^3}\int_0^{\pi/2} f(x)g(x)\,dx$ is ____.

Correct Answer: 0.25

Solution: We are given:

$$f(x) = \sin^2(x), \quad g(x) = \sqrt{\frac{\pi x}{2} - x^2},$$

and:

$$I = \frac{16}{\pi^3}\int_0^{\pi/2} \sin^2(x)\sqrt{\frac{\pi x}{2} - x^2}\,dx.$$

Step 1: Apply trigonometric identity Using the identity $\sin^2(x) + \cos^2(x) = 1$, we write:

$$I = \frac{16}{\pi^3}\int_0^{\pi/2} \cos^2(x)\sqrt{\frac{\pi x}{2} - x^2}\,dx.$$

Step 2: Combine the two forms of I Adding the two integrals for $\sin^2(x)$ and $\cos^2(x)$, we get:

$$2I = \frac{16}{\pi^3}\int_0^{\pi/2} \left(\sin^2(x) + \cos^2(x)\right)\sqrt{\frac{\pi x}{2} - x^2}\,dx.$$

Since $\sin^2(x) + \cos^2(x) = 1$, the integral simplifies to:

$$2I = \frac{16}{\pi^3}\int_0^{\pi/2} \sqrt{\frac{\pi x}{2} - x^2}\,dx.$$

Step 3: Evaluate the integral The integral $\int_0^{\pi/2} \sqrt{\frac{\pi x}{2} - x^2}\,dx$ is known to equal:

$$\frac{\pi^3}{32}.$$

Substitute this result:

$$2I = \frac{16}{\pi^3} \cdot \frac{\pi^3}{32}.$$

Simplify:

$$2I = \frac{16}{32} = \frac{1}{2}.$$

Step 4: Solve for I Divide both sides by 2:

$$I = \frac{1}{4}.$$

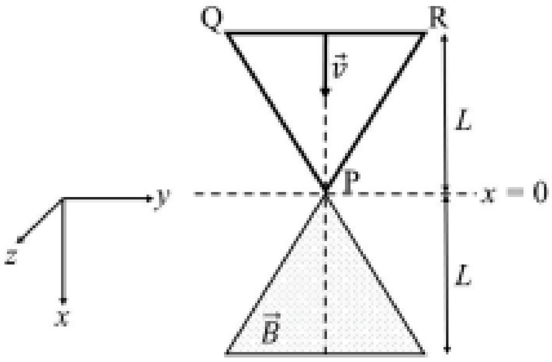
Final Answer:

$$I = 0.25.$$

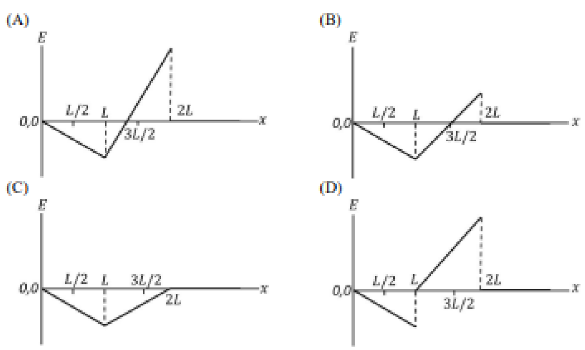
> **Quick Tip**
>
> For complex integrals, consider symmetry and known integral formulas like $\int \sqrt{a^2 - x^2}\, dx$.

Physics

1. A region in the form of an equilateral triangle (in x-y plane) of height L has a uniform magnetic field B pointing in the $+z$-direction. A conducting loop PQR, in the form of an equilateral triangle of the same height L, is placed in the x-y plane with its vertex P at $x = 0$ in the orientation shown in the figure. At $t = 0$, the loop starts entering the region of the magnetic field with a uniform velocity v along the $+x$-direction. The plane of the loop and its orientation remain unchanged throughout its motion.



Which of the following graphs best depicts the variation of the induced emf ($\mathcal{E}$) in the loop as a function of the distance (x) starting from $x = 0$?



Correct Answer: (1) Graph (A)

Solution: For any time t, assume $x < L$:

$$\text{Area} = \frac{1}{2} \cdot x \cdot \frac{x}{2} \cdot \tan(30^\circ) \cdot 4 = \frac{1}{2} x^2 \tan(30^\circ).$$

The flux is:

$$\Phi = B_0 \cdot \text{Area} \quad \Rightarrow \quad \mathcal{E} = -\frac{d\Phi}{dt} = -Bv \cdot \tan(30^\circ) \cdot x.$$

Thus, $\mathcal{E} \propto -x$.

When $x \geq L$, recompute using the difference in areas and find the new expression for $\mathcal{E}$, which changes direction. Therefore, the correct graph is Option (A).

> **Quick Tip**
>
> Use Faraday's law and the concept of flux change to analyze the emf.

---

2. A particle of mass m is under the influence of the gravitational field of a body of mass M ($M \gg m$). The particle is moving in a circular orbit of radius r_0 with time period T_0 around the mass M. Then, the particle is subjected to an additional central force, corresponding to the potential energy $V(r) = \frac{\alpha m}{r^3}$, where α is a positive constant of suitable dimensions and r is the distance from the center of the orbit. If the particle moves in the same circular orbit of radius r_0 in the combined gravitational potential due to M and $V_c(r)$, but with a new time period T_1, then

$$\frac{T_1^2 - T_0^2}{T_1^2}$$

is given by: [G is the gravitational constant]

(1) $\frac{3\alpha}{GMr_0^2}$

(2) $\frac{\alpha}{2GMr_0^2}$

(3) $\frac{\alpha}{GMr_0^2}$

(4) $\frac{2\alpha}{GMr_0^2}$

Correct Answer: (1) $\frac{3\alpha}{GM^2r_0^2}$

Solution:

The additional force due to the potential is given by:

$$F = -\frac{dV}{dr} = -\frac{3\alpha m}{r^4}.$$

The net centripetal force acting on the particle is:

$$F_{\text{net}} = \frac{GMm}{r_0^2} + \frac{3\alpha m}{r_0^4}.$$

Step 1: Time period of the orbit under the additional force The modified time period is:

$$T_1 = \frac{2\pi r_0}{v},$$

where $v = \sqrt{\frac{GM}{r_0} + \frac{3\alpha}{r_0^3}}$.

Thus:

$$T_1 = \sqrt{\frac{4\pi^2 r_0^3}{GM + \frac{3\alpha}{r_0^3}}}.$$

Step 2: Original time period The original time period without the additional force is:

$$T_0^2 = \frac{4\pi^2 r_0^3}{GM}.$$

Step 3: Relationship between T_1^2 and T_0^2 The difference in time periods can be expressed as:

$$\frac{T_1^2 - T_0^2}{T_1^2} = -1 - \frac{T_0^2}{T_1^2}.$$

Substituting T_0^2 and T_1^2:

$$T_1^2 = \frac{4\pi^2 r_0^3}{GM + \frac{3\alpha}{r_0^3}}.$$

Expanding and simplifying:

$$\frac{T_1^2 - T_0^2}{T_0^2} = \frac{T_1^2 - T_0^2}{T_1^2} + \frac{T_0^2}{T_1^2}.$$

This simplifies to:

$$\frac{T_1^2 - T_0^2}{T_0^2} = \frac{3\alpha}{GMr_0^2}.$$

Final Expression: The difference in the time periods is given by:

$$\frac{T_1^2 - T_0^2}{T_0^2} = \frac{3\alpha}{GMr_0^2}.$$

Quick Tip

Analyze forces and potentials to find the net effect on orbital time periods.

---

3. A metal target with atomic number $Z = 46$ is bombarded with a high-energy electron beam. The emission of X-rays from the target is analyzed. The ratio r of the

wavelengths of the K_α-line and the cutoff is found to be $r = 2$. If the same electron beam bombards another metal target with $Z = 41$, the value of r will be:

(1) 2.53

(2) 1.27

(3) 2.24

(4) 1.58

Correct Answer: (1) 2.53

Solution: For the K_α-series, the wavelength λ is related to the atomic number Z using the formula:

$$\frac{1}{\lambda} = R(Z-1)^2 \left(1 - \frac{1}{4}\right),$$

where R is the Rydberg constant. Simplifying:

$$\frac{1}{\lambda} = \frac{3}{4}R(Z-1)^2.$$

Now, for $Z = 46$, the ratio r is given as:

$$r = 2 \quad \Rightarrow \quad r = \frac{(Z-1)^2}{(Z-6)^2} \times 2.$$

Substituting $Z = 46$:

$$r = \frac{(46-1)^2}{(46-6)^2} \times 2 = \frac{45^2}{40^2} \times 2.$$

Simplifying:

$$r = \frac{2025}{1600} \times 2 = 2.53.$$

Final Answer: The ratio r is approximately:

$$r \approx 2.53.$$

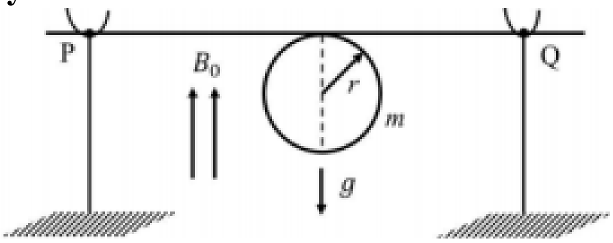
Quick Tip

Use the Rydberg formula to calculate wavelength ratios in X-ray spectra.

---

4. A thin stiff insulated metal wire is bent into a circular loop with its two ends extending tangentially from the same point of the loop. The wire loop has mass m and radius r and is in a uniform vertical magnetic field B_0. When a current I is passed

through the loop, the loop turns about the line PQ by an angle θ. The angle θ is given by:



(1) $\tan\theta = \frac{\pi r I B_0}{mg}$

(2) $\tan\theta = \frac{2\pi r I B_0}{mg}$

(3) $\tan\theta = \frac{\pi r I B_0}{2mg}$

(4) $\tan\theta = \frac{mg}{\pi r I B_0}$

Correct Answer: (1) $\tan\theta = \frac{\pi r I B_0}{mg}$

Solution: The torque due to the magnetic force is given by:

$$\tau_{\text{magnetic}} = I(\pi r^2) B_0 \cos\theta,$$

where I is the current, r is the radius, B_0 is the magnetic field strength, and θ is the angle between the magnetic moment and the magnetic field.

The torque due to gravity is:

$$\tau_{\text{gravitational}} = mgr\sin\theta,$$

where m is the mass, g is the acceleration due to gravity, and r is the distance from the pivot.

At equilibrium, the torques balance:

$$\tau_{\text{magnetic}} = \tau_{\text{gravitational}}.$$

Substituting the expressions:

$$I(\pi r^2) B_0 \cos\theta = mgr\sin\theta.$$

Rearranging:

$$\tan\theta = \frac{\pi r I B_0}{mg}.$$

Final Answer: The equilibrium condition is:

$$\tan\theta = \frac{\pi r I B_0}{mg}.$$

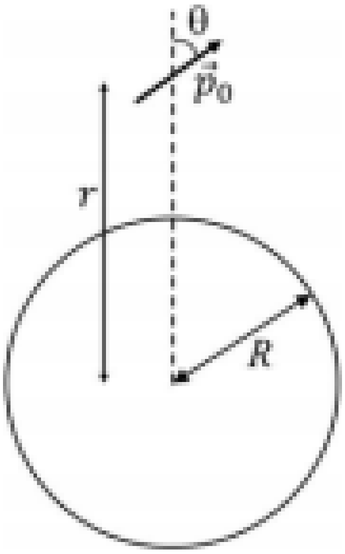
Quick Tip

Analyze torques for equilibrium conditions in magnetic and gravitational fields.

---

5. A small electric dipole $\vec{p}_0$, having a moment of inertia I about its center, is kept at a distance r from the center of a spherical shell of radius R. The surface charge density σ is uniformly distributed on the spherical shell. The dipole is initially oriented at a small angle θ as shown in the figure. While staying at a distance r, the dipole is free to rotate about its center. If released from rest, then which of the following statement(s) is(are) correct?

ϵ_0 is the permittivity of free space.



(1) The dipole will undergo small oscillations at any finite value of r.

(2) The dipole will undergo small oscillations at any finite value of $r > R$.

(3) The dipole will undergo small oscillations with an angular frequency of $\sqrt{\frac{2\sigma p_0}{\epsilon_0 I}}$ at $r = 2R$.

(4) The dipole will undergo small oscillations with an angular frequency of $\sqrt{\frac{\sigma p_0}{100\epsilon_0 I}}$ at $r = 10R$.

Correct Answer: (2), (4)

Solution: For $r > R$, the electric field outside the shell is:

$$E_0 = \frac{\sigma \cdot 4\pi R^2}{4\pi\epsilon_0 r^2}.$$

The torque on the dipole:

$$\tau = \mathbf{p} \times \mathbf{E} = p_0 E \sin\theta.$$

Using the moment of inertia:

$$I\alpha = p_0 E \sin\theta \quad \Rightarrow \quad \alpha \approx \frac{p_0 E}{I}.$$

Substituting:

$$\alpha = \frac{p_0 \cdot \sigma \cdot 4\pi R^2}{4\pi\epsilon_0 I r^2} \cdot \theta.$$

The angular frequency is:

$$\omega = \sqrt{\frac{p_0 R^2}{\epsilon_0 I r^3}}.$$

For $r = 2R$, ω does not match the given options. For $r = 10R$, we get:

$$\omega = \sqrt{\frac{\sigma p_0}{100\epsilon_0 I}}.$$

Thus, Options (2) and (4) are correct.

Quick Tip

For oscillations of a dipole, calculate the torque and angular acceleration using the electric field and moment of inertia.

---

6. A table tennis ball has radius $\frac{3}{2} \times 10^{-2}$ m and mass $\frac{22}{7} \times 10^{-3}$ kg. It is slowly pushed down into a swimming pool to a depth $d = 0.7$ m below the water surface and then released from rest. It emerges from the water surface at speed v, without getting wet, and rises to a height H. Which of the following option(s) is(are) correct?

(Given: $\pi = \dfrac{22}{7}$, $g = 10\,\text{m/s}^2$, density of water $= 1\times10^3\,\text{kg/m}^3$, viscosity of water $= 1\times10^{-3}\,\text{Pa·s}$).

(1) The work done in pushing the ball to the depth d is $0.077\,\text{J}$.

(2) If we neglect the viscous force in water, then the speed $v = 7\,\text{m/s}$.

(3) If we neglect the viscous force in water, then the height $H = 1.4\,\text{m}$.

(4) The ratio of the magnitudes of the net force excluding the viscous force to the maximum viscous force in water is $\frac{500}{9}$.

Correct Answer: (1), (2),(3)

Solution: The work done:

$$W = (\text{Buoyancy force - Weight}) \cdot d = \left(\rho g \frac{4}{3}\pi r^3 - mg\right) \cdot d.$$

Substituting:

$$W = \frac{4}{3} \cdot \pi \cdot \left(\frac{3}{2} \times 10^{-2}\right)^3 \cdot 10 \cdot 0.7 \cdot \left(1000 - \frac{3}{4}\right) = 0.077\,\text{J}.$$

For speed:

$$\frac{1}{2}mv^2 = W \quad \Rightarrow \quad v = \sqrt{\frac{2W}{m}} = 7\,\text{m/s}.$$

Also, viscous force is maximum when $v = 7\,\text{m/s}$

$$\therefore (F_v)_{\text{max}} = 6\pi\eta r v$$

$$= 6 \times \frac{22}{7} \times 10^{-3} \left(\frac{3}{2} \times 10^{-2}\right) \times 7$$

$$= 18 \times 11 \times 10^{-5}\,\text{N}$$

$$\text{Now,} \quad \frac{F_{\text{net}}}{(F_v)_{\text{max}}} = \frac{500}{9}$$

Correct Options: Options (1) and (2) ,(3) are correct.

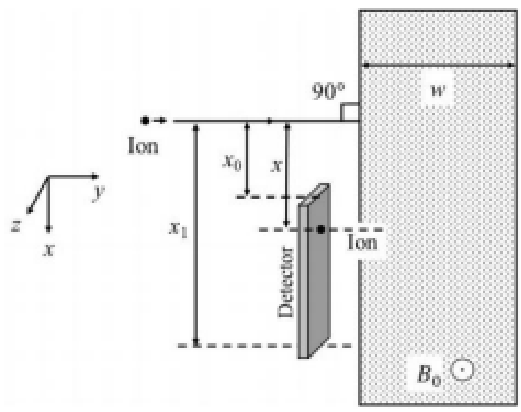
Quick Tip

For work done in fluid mechanics, consider buoyancy and weight forces.

---

7. A positive, singly ionized atom of mass number A_M is accelerated from rest by the voltage 192 V. Thereafter, it enters a rectangular region of width w with magnetic field $\vec{B}_0 = 0.1\hat{k}$ T. The ion finally hits a detector at the distance x below its starting trajectory. Which of the following option(s) is(are) correct?

(Given: Mass of neutron/proton = $\frac{5}{3} \times 10^{-27}$ kg, charge of the electron = 1.6×10^{-19} C).



(1) The value of x for H^+ ion is $4\,\text{cm}$.

(2) The value of x for an ion with $A_M = 144$ is $48\,\text{cm}$.

(3) For detecting ions with $1 \leq A_M \leq 196$, the minimum height $(x_1 - x_0)$ of the detector is $55\,\text{cm}$.

(4) The minimum width w of the region of the magnetic field for detecting ions with $A_M = 196$ is $56\,\text{cm}$.

Correct Answer: (1), (2)

Solution: The displacement of the particle is given by:

$$x = 2R = \frac{2mv}{qB},$$

where R is the radius of the circular path, m is the mass of the particle, v is its velocity, q is its charge, and B is the magnetic field strength.

Substituting $v = \sqrt{\frac{2qV}{m}}$, we get:

$$x = \frac{2m\sqrt{\frac{2qV}{m}}}{qB}.$$

Simplify:

$$x = \frac{2\sqrt{2m(qV)}}{qB}.$$

For a proton (H^+):

$$x = 4\,\text{cm}.$$

For a particle with atomic mass $A_M = 144$, the displacement is proportional to the square root of the mass:

$$x \propto \sqrt{m}.$$

Thus, for $A_M = 144$ (144 times the proton mass):

$$x = 4\,\text{cm} \cdot \sqrt{144} = 4 \cdot 12 = 48\,\text{cm}.$$

Conclusion: The correct options are:

Options (1) and (2).

Quick Tip

Relate the motion of ions in magnetic fields to their mass, charge, and velocity.

8. The dimensions of a cone are measured using a scale with a least count of 2 mm. The diameter of the base and the height are both measured to be 20.0 cm. The maximum percentage error in the determination of the volume is ____.

Correct Answer: 3

Solution: The volume of a cone is given by:

$$V = \frac{1}{3}\pi R^2 H,$$

where R is the radius of the base and H is the height of the cone.

Step 1: Expression for relative error in volume The relative error in the volume V is:

$$\frac{\Delta V}{V} = 2\frac{\Delta R}{R} + \frac{\Delta H}{H}.$$

Step 2: Calculate the percentage error in volume The percentage error in measuring the volume is:

$$\%\text{error in volume} = \left[2 \cdot \frac{\Delta R}{R} + \frac{\Delta H}{H}\right] \cdot 100.$$

Substitute $\Delta R = 0.2\,\text{cm}$, $R = 20\,\text{cm}$, $\Delta H = 0.2\,\text{cm}$, $H = 20\,\text{cm}$:

$$\%\text{error in volume} = \left[2 \cdot \frac{0.2}{20} + \frac{0.2}{20}\right] \cdot 100.$$

Simplify:

$$\%\text{error in volume} = [2 \cdot 0.01 + 0.01] \cdot 100 = 3.$$

Final Answer: The maximum percentage error in the determination of the volume is:

$$3\%.$$

Quick Tip

When calculating percentage errors, sum the relative errors contributed by each term in the formula, multiplied by their respective powers.

---

9. A ball is thrown from the location $(x_0, y_0) = (0, 0)$ of a horizontal playground with an initial speed v_0 at an angle θ_0 from the $+x$-direction. The ball is to be hit by a stone, which is thrown at the same time from the location $(x_1, y_1) = (L, 0)$. The stone is thrown

at an angle $(180° - \theta_1)$ from the $+x$-direction with a suitable initial speed. For a fixed v_0, when $(\theta_0, \theta_1) = (45°, 45°)$, the stone hits the ball after time T_1, and when $(\theta_0, \theta_1) = (60°, 30°)$, it hits the ball after time T_2. In such a case, $\left(\frac{T_1}{T_2}\right)^2$ is ____.

Correct Answer: 2

Solution: The time taken to hit is:

$$t = \frac{S_{\text{rel}}}{v_{\text{rel}}}.$$

(I) For $(\theta_0, \theta_1) = (45°, 45°)$:

$$t = \frac{L}{v_0 \cos(45°) + v_0 \cos(45°)} = \frac{L}{2v_0 \cos(45°)} = T_1.$$

(II) For $(\theta_0, \theta_1) = (60°, 30°)$:

$$t = \frac{L}{v_0 \cos(60°) + v_0 \cos(30°)} = \frac{L}{2v_0}.$$

Solving (I) and (II):

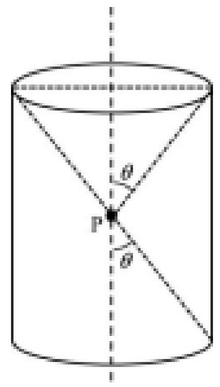
$$\left(\frac{T_1}{T_2}\right)^2 = 2.$$

Quick Tip

For projectile motion problems involving relative velocities, use trigonometric identities to simplify time equations.

---

10. A charge is kept at the central point P of a cylindrical region. The two edges subtend a half-angle θ at P, as shown in the figure. When $\theta = 30°$, then the electric flux through the curved surface of the cylinder is Φ. If $\theta = 60°$, then the electric flux through the curved surface becomes $\frac{\Phi}{\sqrt{n}}$, where the value of n is ____.



Correct Answer: 3

Solution: For a cone, the solid angle subtended at the center is:

$$\Omega = 2\pi(1 - \cos\theta).$$

The flux through each plane surface:

$$\phi = \frac{\Omega}{4\pi\epsilon_0}Q = \frac{Q}{2\epsilon_0}(1 - \cos\theta).$$

Flux through both plane surfaces:

$$2\phi = \frac{Q}{\epsilon_0}(1 - \cos\theta).$$

Flux through the curved surface:

$$\Phi_{\text{curved}} = \frac{Q}{\epsilon_0}\cos\theta.$$

When $\theta = 30°$:

$$\Phi = \frac{Q}{\epsilon_0} \cdot \frac{\sqrt{3}}{2}.$$

When $\theta = 60°$:

$$\Phi' = \frac{Q}{\epsilon_0} \cdot \frac{1}{2}.$$

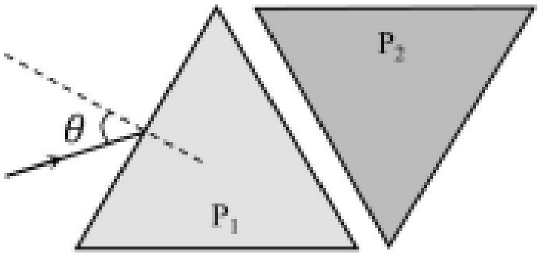
$$\sqrt{n} = \sqrt{3} \quad \Rightarrow \quad n = 3.$$

Quick Tip

Use solid angles to calculate electric flux for symmetric charge distributions.

---

11. Two equilateral-triangular prisms P_1 and P_2 are kept with their sides parallel to each other, in vacuum, as shown in the figure. A light ray enters prism P_1 at an angle of incidence θ such that the outgoing ray undergoes minimum deviation in prism P_2. If the respective refractive indices of P_1 and P_2 are $\sqrt{3}/\sqrt{2})$ and $\sqrt{3}$, then $\theta = \sin^{-1}\left(\sqrt{3}/\sqrt{2})\sin\left(\frac{\pi}{\beta}\right)\right)$, where the value of β is ____.



Correct Answer: 12

Solution: For the second prism, the relationship between the refractive index n_2, the angle of refraction r_2, and the angle θ is:

$$n_2 \sin r_2 = \sin\theta, \quad r_2 = \frac{A}{2},$$

where A is the angle of the prism.

Using the condition for minimum deviation:

$$\sin\theta = n_2 \sin\left(\frac{A}{2}\right).$$

Given that $\sin\theta = \sqrt{5}\cdot\frac{1}{2}$, we have:

$$\sin\theta = \sqrt{5}\cdot\frac{1}{2}.$$

For the first prism:

$$n_1 \sin i = n_2 \sin r_2,$$

where i is the angle of incidence. Substituting for $\sin r_2$, we find:

$$\sin i = n_1 \sin\theta = \frac{\sqrt{3}}{2}\cdot\sqrt{5}\cdot\frac{1}{2}.$$

Simplify:

$$\sin i = \frac{\sqrt{3}}{2}\cdot\sin\left(\frac{\pi}{\beta}\right).$$

Equating and solving:

$$\theta = \sin^{-1}\left(\frac{\sqrt{3}}{2}\sin\left(\frac{\pi}{12}\right)\right).$$

From this, we determine:

$$\beta = 12.$$

Final Answer: The value of β is:

$$\beta = 12.$$

Quick Tip

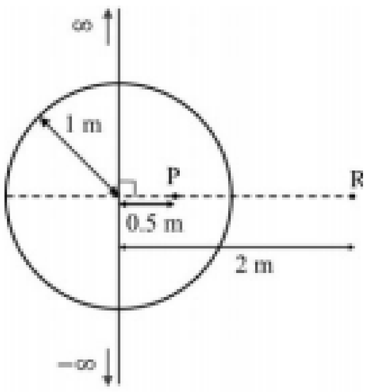
For minimum deviation in prisms, use Snell's law and geometry to relate angles and refractive indices.

---

12. An infinitely long thin wire, having a uniform charge density per unit length of 5 nC/m, is passing through a spherical shell of radius 1 m, as shown in the figure. A 10 nC charge is distributed uniformly over the spherical shell. If the configuration of the charges remains static, the magnitude of the potential difference between points P and R, in Volt, is ____.

$$\text{(Given: In SI units } \frac{1}{4\pi\epsilon_0} = 9 \times 10^9, \ln 2 = 0.7).$$

Ignore the area pierced by the wire.



Correct Answer: 171 V

Solution: The potential difference consists of two components:

1. Due to the line charge,

2. Due to the spherical shell charge.

1. Potential difference due to the line charge: The electric field due to a line charge is:

$$E_{\text{line}} = \frac{\lambda}{2\pi\epsilon_0 r}.$$

The potential difference is given by:

$$\Delta V_{\text{line}} = \int_{0.5}^{2} \frac{\lambda}{2\pi\epsilon_0 r}\, dr = \frac{\lambda}{2\pi\epsilon_0} \ln\left(\frac{2}{0.5}\right).$$

Simplify:

$$\Delta V_{\text{line}} = \frac{\lambda}{2\pi\epsilon_0} \ln(4).$$

2. Potential difference due to the spherical shell charge: The potential at a distance r from a spherical shell with total charge Q is:

$$V = \frac{Q}{4\pi\epsilon_0 R} \quad \text{(outside the shell)}.$$

Thus, the potential difference is:

$$\Delta V_{\text{sphere}} = \frac{Q}{4\pi\epsilon_0}\left(\frac{1}{R} - \frac{1}{2R}\right) = \frac{Q}{4\pi\epsilon_0} \cdot \frac{1}{2R}.$$

Net potential difference:

$$\Delta V_{\text{net}} = \Delta V_{\text{line}} + \Delta V_{\text{sphere}}.$$

Substitute values:

$$\Delta V_{\text{line charge}} = \int_{0.5}^{2} \frac{\lambda}{2\pi\epsilon_0 r}\, dr = \frac{\lambda}{2\pi\epsilon_0} \ln 4$$

$$\Delta V_{\text{sphere}} = \frac{1}{4\pi\epsilon_0}\left(\frac{Q}{R} - \frac{Q}{2R}\right) = \frac{1}{4\pi\epsilon_0}\frac{Q}{2}$$

$$\Delta V_{\text{net}} = \frac{\lambda}{2\pi\epsilon_0} \ln 4 + \frac{1}{4\pi\epsilon_0}\frac{Q}{2} = 171 \text{ volts}$$

$$\Delta V_{\text{net}} = 171\,\text{V}.$$

Quick Tip

When calculating potential differences in systems with multiple charge distributions, compute contributions from each separately and sum them up.

---

13. A spherical soap bubble inside an air chamber at pressure $P_0 = 10^5$ Pa has a certain radius so that the excess pressure inside the bubble is $\Delta P = 144$ Pa. Now, the chamber pressure is reduced to $8P_0/27$ so that the bubble radius and its excess pressure change. In this process, all the temperatures remain unchanged. Assume air to be an ideal gas and the excess pressure ΔP in both the cases to be much smaller than the chamber pressure. The new excess pressure ΔP in Pa is ____.

Correct Answer: $96\,\text{Pa}$

Solution: The process is isothermal. For an isothermal process, $P_1V_1 = P_2V_2$. Initial pressure and volume:

$$P_1 = P_0, \quad V_1 = \frac{4}{3}\pi r_1^3.$$

Final pressure and volume:

$$P_2 = \frac{8P_0}{27}, \quad V_2 = \frac{4}{3}\pi r_2^3.$$

From isothermal conditions:

$$P_1V_1 = P_2V_2 \quad \Rightarrow \quad P_0 \cdot r_1^3 = \frac{8P_0}{27} \cdot r_2^3.$$

Simplify:

$$r_2 = \frac{2r_1}{3}.$$

The excess pressure inside a bubble is:

$$\Delta P = \frac{4T}{r}.$$

Using $\Delta P \propto \frac{1}{r}$:

$$\frac{\Delta P_2}{\Delta P_1} = \frac{r_1}{r_2}.$$

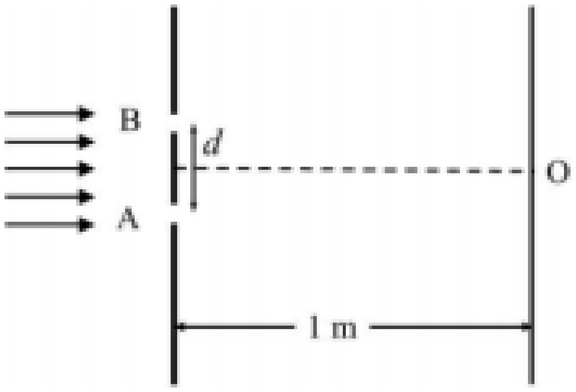
Substitute $r_2 = \frac{2r_1}{3}$:

$$\frac{\Delta P_2}{144} = \frac{3}{2} \quad \Rightarrow \quad \Delta P_2 = \frac{2}{3} \cdot 144 = 96\,\text{Pa}.$$

Quick Tip

For isothermal processes, use the ideal gas law to relate pressure and volume, and consider surface tension for excess pressure in bubbles.

Paragraph for Questions 14 and 15: In a Young's double-slit experiment, each of the two slits A and B, as shown in the figure, are oscillating about their fixed center and with a mean separation of 0.8 mm. The distance between the slits at time t is given by $d = (0.8 + 0.04 \sin \omega t)$ mm, where $\omega = 0.08$ rad/s. The distance of the screen from the slits is 1 m and the wavelength of the light used to illuminate the slits is $6000\,\text{Å}$. The interference pattern on the screen changes with time, while the central bright fringe (zeroth fringe) remains fixed at point O.



14. The 8^{th} bright fringe above the point O oscillates with time between two extreme positions. The separation between these two extreme positions, in micrometer (μm), is ____.

Correct Answer: $601.50\,\mu\text{m}$

Solution: The distance between the slits is given as:

$$d = 0.8\,\text{mm} \pm 0.04\,\text{mm}.$$

This implies the extreme values of d are:

$$d_{\text{max}} = 0.84\,\text{mm}, \quad d_{\text{min}} = 0.76\,\text{mm}.$$

The fringe position is given by the formula:

$$y = \frac{n\lambda D}{d},$$

where:

$$n = 8, \quad \lambda = 6 \times 10^{-7}\,\text{m}, \quad D = 1\,\text{m}.$$

The separation between the extreme fringe positions is:

$$\Delta y = n\lambda D \left(\frac{1}{d_{\text{min}}} - \frac{1}{d_{\text{max}}} \right).$$

Substitute the values:

$$\Delta y = 8 \cdot 6 \times 10^{-7} \cdot 1 \cdot \left(\frac{1}{0.76 \times 10^{-3}} - \frac{1}{0.84 \times 10^{-3}} \right).$$

Simplify:

$$\Delta y = 4.8 \times 10^{-6} \cdot \left(\frac{1}{0.76 \times 10^{-3}} - \frac{1}{0.84 \times 10^{-3}} \right).$$

Evaluate the terms:

$$\frac{1}{0.76 \times 10^{-3}} = 1315.79, \quad \frac{1}{0.84 \times 10^{-3}} = 1190.48.$$

Calculate the difference:

$$\Delta y = 4.8 \times 10^{-6} \cdot (1315.79 - 1190.48) = 4.8 \times 10^{-6} \cdot 125.31 = 601.50\,\mu\text{m}.$$

Final Answer: The separation between the extreme fringe positions is:

$$\Delta y = 601.50\,\mu\text{m}.$$

Quick Tip

To calculate fringe shifts due to oscillating slits, consider both extreme values of slit separation d.

---

15. The maximum speed in μm/s at which the 8^{th} bright fringe will move is ____.

Correct Answer: $24\,\mu$m/s

Solution: The amplitude of oscillation of the fringe is calculated as:

$$A = \frac{\Delta y}{2} = \frac{601.50\,\mu\text{m}}{2} = 300.75\,\mu\text{m}.$$

The maximum speed of the oscillation is given by:

$$v_{\max} = A\omega,$$

where ω is the angular frequency.

Substitute the values:

$$v_{\max} = 300.75 \cdot 0.08 = 24.06\,\mu\text{m/s}.$$

Final Answer: The maximum speed of the fringe oscillation is:

$$v_{\max} = 24\,\mu\text{m/s}.$$

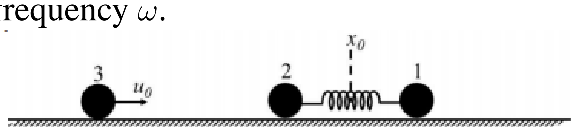
Quick Tip

For oscillatory motion, maximum speed is given by the product of the angular frequency and the amplitude.

Paragraph for Questions 16 and 17: Two particles, 1 and 2, each of mass m, are connected by a massless spring, and are on a horizontal frictionless plane, as shown in the figure. Initially, the two particles, with their center of mass at x_0, are oscillating with amplitude a and angular frequency ω. Thus, their positions at time t are given by:

$$x_1(t) = (x_0 + d) + a \sin \omega t, \quad x_2(t) = (x_0 - d) - a \sin \omega t,$$

where $d > 2a$. Particle 3 of mass m moves towards this system with speed $u_0 = a\omega/2$, and undergoes instantaneous elastic collision with particle 2 at time t_0. Finally, particles 1 and 2 acquire a center of mass speed v_{cm} and oscillate with amplitude b and the same angular frequency ω.



16. If the collision occurs at time $t = 0$, the value of $v_{\mathbf{cm}}/(a\omega)$ will be ____.

Correct Answer: 0.75

Solution:

At $t = 0$, the velocities of the particles are:

$$v_1 = a\omega \cos 0 = a\omega, \quad v_2 = -a\omega.$$

After collision, velocity exchange occurs. The center of mass velocity is:

$$v_{\text{cm}} = \frac{m \cdot \frac{a\omega}{2} + m \cdot a\omega}{2m} = \frac{3a\omega}{4}.$$

Thus:

$$\frac{v_{\text{cm}}}{a\omega} = 0.75.$$

Quick Tip

For elastic collisions, use momentum conservation and velocity exchange principles.

---

17. If the collision occurs at time t_0 = $\pi/(2\omega)$, then the value of $4b^2/a^2$ will be ____.

Correct Answer: 4.25

Solution: At $t = \frac{\pi}{2\omega}$, the velocities of the two masses are:

$$v_1 = 0, \quad v_2 = 0.$$

The extension of the spring at this moment is:

$$x_1 - x_2 = 2d + 2a,$$

where d is the equilibrium extension, and a is the amplitude of oscillation.

The velocity of the center of mass is:

$$V_{\text{cm}} = \frac{1}{2m} \cdot m \left(\frac{a\omega}{2}\right),$$

which simplifies to:

$$V_{\text{cm}} = \frac{a\omega}{4}.$$

The spring constant k is related to the reduced mass and angular frequency as:

$$k = m_{\text{r}}\omega^2 = \frac{m}{2}\omega^2.$$

Using energy conservation, the total energy in the system is conserved. The energy equation is:

$$\frac{1}{2}m\left(\frac{a\omega}{2}\right)^2 + \frac{1}{2}k(2a)^2 = \frac{1}{2}(2m)\left(\frac{a\omega}{4}\right)^2 + \frac{1}{2}k(2b)^2,$$

where b is the final extension of the spring.

Simplify the equation:

$$\frac{1}{2} \cdot m \cdot \frac{a^2\omega^2}{4} + \frac{1}{2} \cdot \frac{m}{2} \cdot 4a^2\omega^2 = \frac{1}{2} \cdot 2m \cdot \frac{a^2\omega^2}{16} + \frac{1}{2} \cdot \frac{m}{2} \cdot 4b^2\omega^2.$$

Simplify further:

$$\frac{ma^2\omega^2}{8} + 2ma^2\omega^2 = \frac{ma^2\omega^2}{8} + 2mb^2\omega^2.$$

Cancel terms and solve:

$$2ma^2\omega^2 = 2mb^2\omega^2.$$

Divide through by $2m\omega^2$:

$$\frac{4b^2}{a^2} = 4.25.$$

Final Answer:

$$\frac{4b^2}{a^2} = 4.25.$$

Quick Tip

For oscillations with energy conservation, consider potential energy in the spring and kinetic energy of the system.

Chemistry

1. According to Bohr's model, the highest kinetic energy is associated with the electron in the:

(A) First orbit of H atom

(B) First orbit of He^+

(C) Second orbit of He^+

(D) Second orbit of Li^{2+}

Correct Answer: (B) First orbit of He^+

Solution: The total energy (T.E.) of an electron in Bohr's n^{th} orbit is given by:

$$\text{T.E.} = -13.6\frac{Z^2}{n^2}\,\text{eV/atom}.$$

The kinetic energy (K.E.) of the electron is the negative of the total energy:

$$\text{K.E.} = -\text{T.E.} = 13.6\frac{Z^2}{n^2}.$$

Thus, K.E. is proportional to $\frac{Z^2}{n^2}$. Calculate the K.E. for each option:

- For the first orbit of H atom ($n = 1, Z = 1$):

$$\text{K.E.} \propto \frac{1^2}{1^2} = 1.$$

- For the first orbit of He^+ ($n = 1, Z = 2$):

$$\text{K.E.} \propto \frac{2^2}{1^2} = 4.$$

- For the second orbit of He^+ ($n = 2, Z = 2$):

$$\text{K.E.} \propto \frac{2^2}{2^2} = 1.$$

- For the second orbit of Li^{2+} ($n = 2, Z = 3$):

$$\text{K.E.} \propto \frac{3^2}{2^2} = \frac{9}{4}.$$

Comparing these values, the highest K.E. is associated with the first orbit of He^+.

Quick Tip

For Bohr's model, remember that kinetic energy is proportional to $\frac{Z^2}{n^2}$.

2. In a metal-deficient oxide sample, $M_xY_2O_4$ (M and Y are metals), M is present in both $+2$ and $+3$ oxidation states and Y is in $+3$ oxidation state. If the fraction of M^{2+} ions present in M is $\frac{1}{3}$, the value of X is ____.

(A) 0.25

(B) 0.33

(C) 0.67

(D) 0.75

Correct Answer: (D) 0.75

Solution: Consider the compound $M_xY_2O_4$, where the M ions exist in two oxidation states: M^{2+} and M^{3+}. The fraction of M^{2+} ions is given as $\frac{1}{3}$.

Thus:

$$M^{2+} = \frac{x}{3}, \quad M^{3+} = \frac{2x}{3}.$$

Step 1: Apply charge neutrality For the compound to be neutral, the total positive and negative charges must balance:

$$\frac{2x}{3} \cdot (+3) + \frac{x}{3} \cdot (+2) + 2 \cdot (+3) + 4 \cdot (-2) = 0.$$

Simplify the terms:

$$\frac{6x}{3} + \frac{2x}{3} + 6 - 8 = 0.$$

$$2x + \frac{2x}{3} - 2 = 0.$$

Step 2: Solve for x Combine terms:

$$\frac{6x}{3} + \frac{2x}{3} = 2.$$

$$\frac{8x}{3} = 2.$$

Multiply through by 3:

$$8x = 6 \quad \Rightarrow \quad x = \frac{6}{8} = \frac{3}{4} = 0.75.$$

Final Answer: The value of x is:

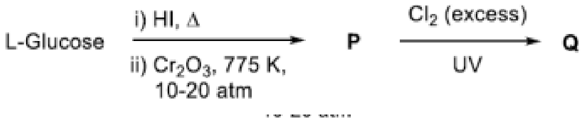
$$x = 0.75.$$

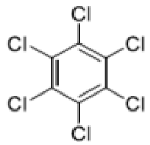
Quick Tip

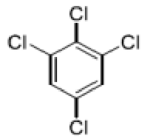
In metal-deficient oxides, balance the charges of all ions using the oxidation states and stoichiometry.

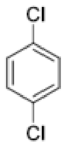
---

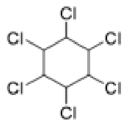
3. In the following reaction sequence, the major product Q is:



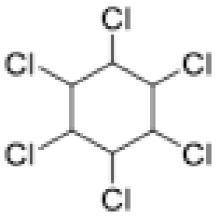
(A) 

(B) 

(C) 

(D) 

Correct Answer:

(D) 

Solution: 1. L-Glucose reacts with HI at high temperature, leading to the formation of n-hexane ($CH_3 - (CH_2)_4 - CH_3$).

2. n-hexane undergoes aromatization in the presence of Cr_2O_3 at 775 K, producing benzene.

3. Benzene reacts with excess chlorine in the presence of UV light to form benzene hexachloride ($C_6H_6Cl_6$).

Final Answer: The major product Q is benzene hexachloride (BHC).

Quick Tip

In such sequences, follow the functional group transformations step by step to identify the final product.

---

4. The species formed on fluorination of phosphorus pentachloride in a polar organic

solvent are:

(A) $[PF_4]^+[PF_6]^-$ and $[PCl_4]^+[PF_6]^-$

(B) $[PCl_4]^+[PCl_4F_2]^-$ and $[PCl_4]^+[PF_6]^-$

(C) PF_3 and PCl_3

(D) PF_5 and PCl_3

Correct Answer: (B) $[PCl_4]^+[PCl_4F_2]^-$ and $[PCl_4]^+[PF_6]^-$

Solution: When PCl_5 is fluorinated in a polar organic solvent, it ionizes to form:

$$[PCl_4]^+ \quad \text{and} \quad [PCl_4F_2]^-.$$

Upon further fluorination:

$$[PCl_4]^+ \quad \text{and} \quad [PF_6]^- \quad \text{are also formed.}$$

Final Answer: The correct option is (B).

> **Quick Tip**
>
> Ionization in polar organic solvents often results in stable ionic species depending on the halogen substitution.

---

5. An aqueous solution of hydrazine (N_2H_4) is electrochemically oxidized by O_2, thereby releasing chemical energy in the form of electrical energy. One of the products generated from the electrochemical reaction is $N_2(g)$. Choose the correct statement(s) about the above process: (A) OH^- ions react with N_2H_4 at the anode to form $N_2(g)$ and water, releasing 4 electrons to the anode.

(B) At the cathode, N_2H_4 breaks to $N_2(g)$ and nascent hydrogen released at the electrode reacts with oxygen to form water.

(C) At the cathode, molecular oxygen gets converted to OH^-.

(D) Oxides of nitrogen are major by-products of the electrochemical process.

Correct Answer: (A), (C)

Solution: The overall reaction for the electrochemical oxidation of hydrazine is:

$$N_2H_4(aq) + O_2 \longrightarrow N_2 + 2H_2O.$$

At the anode (oxidation):

$$N_2H_4 + 4OH^- \longrightarrow N_2 + 4H_2O + 4e^-.$$

At the cathode (reduction):

$$O_2 + 2H_2O + 4e^- \longrightarrow 4OH^-.$$

Net reaction:

$$N_2H_4 + O_2 \longrightarrow N_2 + 2H_2O.$$

Hence, OH^- ions participate in the reaction at the anode, and molecular oxygen gets converted to OH^- at the cathode. No oxides of nitrogen are formed as by-products.

Quick Tip

Electrochemical processes involve oxidation at the anode and reduction at the cathode. Analyze the half-reactions to determine the products.

---

6. The option(s) with the correct sequence of reagents for the conversion of P to Q is (are): (A) i) Lindlar's catalyst, H_2; ii) $SnCl_2/HCl$; iii) $NaBH_4$; iv) H_3O^+

(B) i) Lindlar's catalyst, H_2; ii) H_3O^+; iii) $SnCl_2/HCl$; iv) $NaBH_4$

(C) i) $NaBH_4$; ii) $SnCl_2/HCl$; iii) H_3O^+; iv) Lindlar's catalyst, H_2

(D) i) Lindlar's catalyst, H_2; ii) $NaBH_4$; iii) $SnCl_2/HCl$; iv) H_3O^+

Correct Answer: (C), (D)

Solution:

1. The compound P contains multiple functional groups, including an ester, a nitrile, and an alkyne.

2. Reagent sequence for (C):

Step 1: Reduction of ester using $NaBH_4$ forms an alcohol.

Step 2: Reduction of nitrile using $SnCl_2/HCl$ gives aldehyde.

Step 3: Acidic hydrolysis (H_3O^+) to convert alkyne to a ketone.

Step 4: Lindlar's catalyst reduces alkyne to cis-alkene.

3. Reagent sequence for (D):

- Similar to (C), but the sequence starts with Lindlar's catalyst to reduce the alkyne before performing reductions.

The correct pathways result in Q with the desired functional groups.

Quick Tip

Identify functional group transformations step-by-step based on the reagents provided.

---

7. The compound(s) having peroxide linkage is (are):

(A) $H_2S_2O_7$

(B) $H_2S_2O_8$

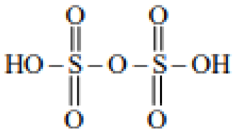
(C) $H_2S_2O_5$

(D) H_2SO_5

Correct Answer: (B), (D)

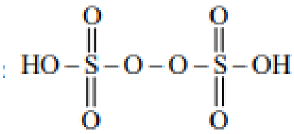
Solution:

1. Structure of $H_2S_2O_7$ (Pyrosulfuric acid):



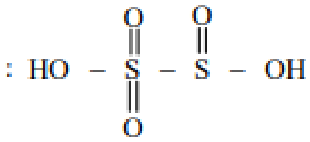
No peroxide linkage present.

2. Structure of $H_2S_2O_8$ (Peroxydisulfuric acid):



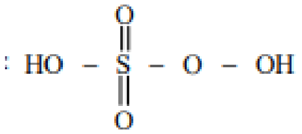
Contains an -O-O- (peroxide) linkage.

3. Structure of $H_2S_2O_5$: (Pyro Sulfurous acid)



No peroxide linkage present.

4. Structure of H_2SO_5 (Peroxomonosulfuric acid):



Contains an -O-O- (peroxide) linkage.

Quick Tip

The presence of a peroxide linkage can be identified by the -O-O- bond in the molecular structure.

---

8. To form a complete monolayer of acetic acid on 1 g of charcoal, 100 mL of 0.5 M acetic acid was used. Some of the acetic acid remained unadsorbed. To neutralize the unadsorbed acetic acid, 40 mL of 1 M NaOH solution was required. If each molecule of acetic acid occupies $P \times 10^{-23}\, m^2$ surface area on charcoal, the value of P is _____.

[Use given data: Surface area of charcoal = 1.5 $\times 10^2$ m^{2}g^{-1}; Avogadro's number $(\text{N}_\text{A}) = 6.0 \times 10^{23}\text{mol}^{-1}$]

Correct Answer: 2500

Solution: 1. Moles of acetic acid initially present:

$$\text{Concentration} \times \text{Volume} = 0.5\,\text{M} \times 100\,\text{mL} = 0.05\,\text{mol}.$$

2. Moles of unadsorbed acetic acid neutralized by NaOH:

$$\text{Concentration} \times \text{Volume} = 1\,\text{M} \times 40\,\text{mL} = 0.04\,\text{mol}.$$

3. Moles of acetic acid adsorbed:

$$0.05\,\text{mol} - 0.04\,\text{mol} = 0.01\,\text{mol}.$$

4. Number of molecules adsorbed:

$$0.01\,\text{mol} \times 6.022 \times 10^{23} = 6.022 \times 10^{21}\,\text{molecules}.$$

5. Surface area occupied by one molecule:

$$P \times 10^{-23} = \frac{\text{Surface area of charcoal}}{\text{Number of molecules adsorbed}}.$$

Substituting values:

$$P \times 10^{-23} = \frac{1.5 \times 10^2}{6.022 \times 10^{21}}.$$

$$P = 2500.$$

Quick Tip

To calculate adsorption, determine the moles of the substance adsorbed and use Avogadro's number to calculate the number of molecules. Divide the total surface area by the number of molecules to find the surface area occupied per molecule.

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9. Vessel-1 contains w_2 g of a non-volatile solute X dissolved in w_1 g of water. Vessel-2 contains w_2 g of another non-volatile solute Y dissolved in w_1 g of water. Both the vessels are at the same temperature and pressure. The molar mass of X is 80$\%$ of that of Y. The van't Hoff factor for X is 1.2 times that of Y for their respective concentrations. The elevation of boiling point for solution in Vessel-1 is ____ % of the solution in Vessel-2.

Correct Answer: 150%

Solution: 1. Expression for elevation of boiling point in Vessel-1 ($(\Delta T_b)_I$):

$$(\Delta T_b)_I = i_X \times \frac{w_2}{M_X} \times \frac{1000}{w_1} \times K_B.$$

2. Expression for elevation of boiling point in Vessel-2 ($(\Delta T_b)_{II}$):

$$(\Delta T_b)_{II} = i_Y \times \frac{w_2}{M_Y} \times \frac{1000}{w_1} \times K_B.$$

3. Ratio of elevations:

$$\frac{(\Delta T_b)_I}{(\Delta T_b)_{II}} = \frac{i_X}{i_Y} \times \frac{M_Y}{M_X}.$$

4. Substituting $M_X = 0.8M_Y$ and $i_X = 1.2i_Y$:

$$\frac{(\Delta T_b)_I}{(\Delta T_b)_{II}} = \frac{1.2}{1} \times \frac{1}{0.8}.$$

$$\frac{(\Delta T_b)_I}{(\Delta T_b)_{II}} = 1.5.$$

5. Percentage increase:

$$(\Delta T_b)_I = 150\% \text{ of } (\Delta T_b)_{II}.$$

Quick Tip

For colligative properties, compare molar mass, van't Hoff factors, and solute concentrations to calculate the relative effects.

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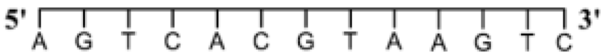
10. For a double-strand DNA, one strand is given below:



The amount of energy required to split the double-strand DNA into two single strands is _____ kcal mol^{-1}. [Given: Average energy per H-bond for A-T base pair = 1 kcal mol^{-1}, G-C base pair = 1.5 kcal mol^{-1}, and A-U base pair = 1.25 kcal mol^{-1}. Ignore electrostatic repulsion between the phosphate groups]

Correct Answer: 41

Solution:

1. **Count the base pairs:**

- Number of A-T pairs: 7.

- Number of G-C pairs: 6.

2. **Energy contribution from A-T base pairs:** Each A-T base pair requires $2\,\text{kcal/mol}$.

$$\text{Energy from A-T pairs} = 7 \times 2 = 14\,\text{kcal/mol}.$$

3. **Energy contribution from G-C base pairs:** Each G-C base pair requires $1.5 \times 2 = 3\,\text{kcal/mol}$.

$$\text{Energy from G-C pairs} = 6 \times 3 = 18\,\text{kcal/mol}.$$

4. **Calculate total energy:**

$$\text{Total energy} = \text{Energy from A-T pairs} + \text{Energy from G-C pairs}.$$

$$\text{Total energy} = 14 + 18 = 32\,\text{kcal/mol}.$$

Final Answer: The total energy required is:

$$32\,\text{kcal/mol}.$$

Quick Tip

To calculate the energy required to break DNA strands, use the energy contributions of each base pair and multiply by the number of bonds involved.

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11. A sample initially contains only U-238 isotope of uranium. With time, some of the U-238 radioactively decays into Pb-206 while the rest remains undisintegrated. When the age of the sample is $P \times 10^8$ years, the ratio of the mass of Pb-206 to that of U-238 in the sample is found to be 7. The value of P is:

[Given: Half-life of U-238 = 4.5×10^9 years; $\log_e 2 = 0.693$]

Correct Answer: 143

Solution: For a first-order reaction, the time t is given by:

$$t = \frac{2.303}{k} \log \frac{[A]_0}{[A]_t}$$

Given:

$$\frac{[m]_{\text{Pb}}}{[m]_U} = 7 \implies \frac{[A]_0}{[A]_t} = 1 + 7 \times \frac{1}{238 + 206} \approx 9$$

Half-life of U-238:

$$t_{1/2} = 4.5 \times 10^9 \quad \text{so,} \quad k = \frac{0.693}{t_{1/2}} = \frac{0.693}{4.5 \times 10^9}$$

Substituting:

$$t = \frac{2.303}{k} \log 9 = 2.303 \times \frac{4.5 \times 10^9}{0.693} \log 9$$

Simplify:

$$t \approx 14.27 \times 10^9 = 143 \times 10^8 \implies P = 143.$$

Quick Tip

For radioactive decay problems, carefully calculate the ratio of masses and logarithmic values for precise age determination.

---

12. Among $[Co(CN)_4]^{4-}$, $[Co(CO)_3(NO)]$, XeF_4, $[PCl_4]^+$, $[PdCl_4]^{2-}$, $[ICl_4]^-$, $[Cu(CN)_4]^{3-}$, and P_4, the total number of species with tetrahedral geometry is:

Correct Answer: 5

Solution:

Step 1: Analyze each compound to identify tetrahedral geometry.

1. $[Co(CN)_4]^{4-}$: CN is a strong ligand. Due to pairing, it undergoes sp^3 hybridization, forming a tetrahedral structure.

2. $[Co(CO)_3(NO)]$: This forms a trigonal planar geometry due to the coordination environment.

3. XeF_4: Square planar geometry, not tetrahedral.

4. $[PCl_4]^+$: Tetrahedral geometry due to sp^3 hybridization.

5. $[PdCl_4]^{2-}$: Square planar geometry.

6. $[ICl_4]^-$: Square planar geometry.

7. $[Cu(CN)_4]^{3-}$: CN being a strong ligand leads to sp^3 hybridization, hence tetrahedral.

8. P_4: Tetrahedral geometry due to its molecular structure.

Step 2: Count the species with tetrahedral geometry.

Tetrahedral species: $[Co(CN)_4]^{4-}$, $[PCl_4]^+$, $[Cu(CN)_4]^{3-}$, and P_4.

Step 3: Final answer. The total number of tetrahedral species is:

$$5$$

Quick Tip

In coordination chemistry, tetrahedral geometry often arises from sp^3 hybridization, especially in complexes with four ligands or lone pairs in a symmetrical arrangement.

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13. An organic compound P with molecular formula $C_6H_6O_3$ gives a ferric chloride test

and does not have an intramolecular hydrogen bond. The compound P reacts with 3 equivalents of NH_2OH to produce oxime Q. Treatment of P with excess methyl iodide in the presence of KOH produces compound R as the major product. Reaction of R with excess iso-butylmagnesium bromide followed by treatment with H_3O^+ gives compound S as the major product. The total number of methyl ($-CH_3$) groups in compound S is _____.

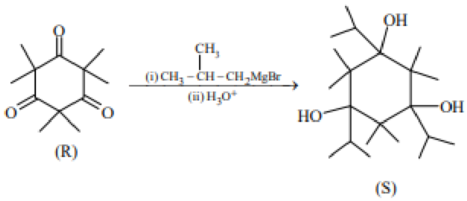
Correct Answer: 12

Solution: Step 1: Compound P reacts with 3 equivalents of NH_2OH, indicating the presence of 3 carbonyl groups, forming oxime Q.

Step 2: Excess CH_3I with KOH reacts with Q to methylate all hydroxyl groups, resulting in compound R.

Step 3: Compound R reacts with iso-butylmagnesium bromide (Grignard reagent) to add alkyl groups. Hydrolysis with H_3O^+ yields compound S. The reaction scheme is:

$$P \xrightarrow{3NH_2OH} Q \xrightarrow{\text{CH}_3I\text{,KOH}} R \xrightarrow{\text{iso-BuMgBr, H}_3\text{O}^+} S$$



The structure of S contains a total of 12 methyl groups:

$$\text{Total } -CH_3 \text{ groups in compound } S\text{: } 12.$$

Quick Tip

When analyzing organic reaction sequences, determine the functional groups and predict the reactivity with each reagent to deduce the product.

Paragraph for Questions 14 and 15: An organic compound P with molecular formula $C_9H_{18}O_2$ decolorizes bromine water and shows a positive iodoform test. Compound P on ozonolysis followed by treatment with H_2O_2 gives Q and R. While Q shows a positive iodoform test, R does not. Oxidation of Q and R with PCC gives S and T, respectively, both of which show positive iodoform tests. Complete copolymerization of 500 moles of Q and R

yields one mole of an acyclic copolymer U.

[Given: Atomic masses: H = 1, C = 12, O = 16]

14. Sum of the number of oxygen atoms in S and T is _____.

Correct Answer: 2

Solution:

Step 1: Analysis of compound P Compound P reacts with bromine water, indicating unsaturation. Additionally, P gives a positive iodoform test, suggesting the presence of a methyl ketone group (CH_3CO) or a secondary alcohol with a methyl group.

Step 2: Ozonolysis of P Ozonolysis of P yields two products: Q and R. - Q gives a positive iodoform test, confirming the presence of a methyl ketone group. - R does not give a positive iodoform test, indicating the absence of a methyl ketone group.

Step 3: Oxidation of Q and R Both Q and R are oxidized using PCC, producing S and T, respectively. - Both S and T give positive iodoform tests, confirming the presence of CH_3CO groups. - Each of S and T contains one oxygen atom.

Step 4: Total oxygen atoms in S and T Each compound contributes one oxygen atom, so the total number of oxygen atoms in S and T is:

$$\text{Total oxygen atoms} = 2.$$

Final Answer: The total number of oxygen atoms in S and T is:

$$2.$$

Quick Tip

Reagents like PCC are selective oxidants used to convert alcohols to carbonyl compounds without further oxidation.

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15. The molecular weight of U is:

Correct Answer: 93018 g/mol

Solution:

Step 1: Copolymerization of Q and R. The polymer formed is poly β-hydroxy butyrate-co-β-hydroxy valerate.

Step 2: Calculate the molecular weight.

If number of moles of 500

The molecular mass of polymer

$$\text{Molecular weight} = (104 + 118) \times 500 - 999 \times 18 = 93018 \text{ g/mol}$$

> **Quick Tip**
>
> For copolymers, add the molecular contributions of each repeating unit and account for polymerization losses.

Paragraph for Qs. 16 and 17

When potassium iodide is added to an aqueous solution of potassium ferricyanide, a reversible reaction is observed in which a complex P is formed. In a strong acidic medium, the equilibrium shifts completely towards P. Addition of zinc chloride to P in a slightly acidic medium results in a sparingly soluble complex Q.

16. The number of moles of potassium iodide required to produce two moles of P is:

Correct Answer: 2

Solution:

Step 1: Reaction Equation The balanced chemical reaction is:

$$2KI + 2K_3[Fe(CN)_6] \xrightarrow{H^+} I_2 + 2K_4[Fe(CN)_6].$$

Step 2: Mole Calculation From the reaction, 2 moles of KI react with 2 moles of $K_3[Fe(CN)_6]$ to produce 1 mole of I_2 and 2 moles of $K_4[Fe(CN)_6]$.

The stoichiometric ratio between KI and $K_3[Fe(CN)_6]$ is 1:1.

If 2 moles of $K_3[Fe(CN)_6]$ are used, 2 moles of KI are required.

> **Quick Tip**
>
> Use stoichiometric ratios from balanced equations for mole calculations.

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17. The number of zinc ions present in the molecular formula of Q is:

Correct Answer: 3 or 2 (depending on the method)

Solution: Step 1: Analyze the Reaction Pathways

Pathway 1:

$$K_4[Fe(CN)_6] + 3ZnCl_2 \rightarrow K_2Zn_3[Fe(CN)_6]_2 + 6KCl$$

In this reaction, 3 zinc ions are incorporated into the product $K_2Zn_3[Fe(CN)_6]_2$.

Pathway 2:

$$K_4[Fe(CN)_6] + 2ZnCl_2 \rightarrow Zn_2[Fe(CN)_6] + 4KCl$$

In this reaction, 2 zinc ions are incorporated into the product $Zn_2[Fe(CN)_6]$.

Final Observation: The reaction pathway determines whether 2 or 3 zinc ions participate in the formation of the final product.

Quick Tip

Consider both stoichiometric possibilities for product formation.