JEE Advanced 2024 Paper 1 Question Paper with Solutions

Time Allowed :3 Hours	**Maximum Marks :**200	**Total Questions :**150

General Instructions

Read the following instructions very carefully and strictly follow them:

1. The JEE Advanced 2024, Paper 1, will be structured with a total of 180 marks over 3 hours.
2. It will include 51 questions, with 17 questions each in Physics, Chemistry, and Mathematics.
3. Each subject will be segmented into four sections: Section I: 12 marks.
4. The marking scheme also varies, for example, questions may carry 1 mark, 2 marks, 3 marks or 4 marks.
5. There are negative markings of -1 or -2 and some questions can also read to no negative marking.

Mathematics

1. Let $f(x)$ be a continuously differentiable function on the interval $(0, \infty)$ such that $f(1) = 2$ and

$$\lim_{t \to x} \frac{t^{10} f(x) - x^{10} f(t)}{t^9 - x^9} = 1 \quad \text{for each } x > 0.$$

Then, for all $x > 0$, $f(x)$ is equal to:

(A) $\frac{31}{11x} + \frac{9}{11x^{10}}$

(B) $\frac{9}{11x} + \frac{13}{11x^{10}}$

(C) $-\frac{9}{11x} + \frac{31}{11x^{10}}$

(D) $\frac{13}{11x} + \frac{9}{11x^{10}}$

Correct Answer: (B) $\frac{9}{11x} + \frac{13}{11x^{10}}$

Solution:

Given: $\lim_{t \to x} \frac{t^{10} f(x) - x^{10} f(t)}{t^9 - x^9} = 1$

$\Rightarrow \lim_{t \to x} \frac{10t^9 f(x) - 10x^9 f(t)}{9x^8} = 1$ (by simplifying the numerator and denominator)

$\Rightarrow 10x^9 f(x) - x^{10} f'(x) = 9x^8$ (as $t \to x$)

$\Rightarrow x f'(x) - 10 f(x) = -\frac{9}{x^2}$ (rearranging terms)

This is a linear differential equation of the form: $f'(x) + \frac{10}{x} f(x) = -\frac{9}{x^2}$.

Integrating factor (I.F.): $e^{\int \frac{10}{x} dx} = e^{-10 \ln x} = \frac{1}{x^{10}}$.

Multiply through by I.F.: $\frac{1}{x^{10}} f(x) = \int \frac{-9}{x^{12}} dx$

$\Rightarrow \frac{1}{x^{10}} f(x) = \frac{9}{11x^{11}} + C,$ where C is the constant of integration.

$\Rightarrow f(x) = \frac{9}{11x} + \frac{C}{x^{10}}$.

Using $f(1) = 2$: $2 = \frac{9}{11} + C$

$\Rightarrow C = 2 - \frac{9}{11} = \frac{13}{11}$.

Thus, $f(x) = \frac{9}{11x} + \frac{13}{11x^{10}}$.

Final Answer:

$$\boxed{\frac{9}{11x} + \frac{13}{11x^{10}}}$$

Quick Tip

For solving linear differential equations, always identify the integrating factor and use it to simplify the equation. Don't forget to apply initial conditions to find the constant.

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2. A student appears for a quiz consisting of only true-false type questions and answers all the questions. The student knows the answers of some questions and guesses the answers for

the remaining questions. Whenever the student knows the answer of a question, he gives the correct answer. Assume that the probability of the student giving the correct answer for a question, given that he has guessed it, is $\frac{1}{2}$.

Also assume that the probability of the answer for a question being guessed, given that the student's answer is correct, is $\frac{5}{6}$. Then the probability that the student knows the answer of a randomly chosen question is:

(A) $\frac{1}{12}$

(B) $\frac{1}{7}$

(C) $\frac{5}{7}$

(D) $\frac{5}{12}$

Correct Answer: (C) $\frac{5}{7}$

Solution:

Let total questions $= T$

- x: Number of questions whose answers the student knows.
- $(T - x)$: Number of questions whose answers the student does not know.

$$\text{Required Probability} = \frac{\frac{x}{T}}{\frac{x}{T} + \frac{T-x}{T} \cdot \frac{1}{2}} = \frac{\frac{x}{T}}{\frac{x}{T} + \frac{T-x}{2T}}$$

$$= \frac{x}{x + \frac{T-x}{2}} = \frac{x}{\frac{2x+T-x}{2}} = \frac{2x}{T+x}$$

$$\text{Given: } \frac{2x}{T+x} = \frac{5}{6} \quad \Rightarrow 12x = 5(T+x) \quad \Rightarrow 12x = 5T + 5x \quad \Rightarrow 7x = 5T$$

$$\Rightarrow \frac{x}{T} = \frac{5}{7}$$

Final Answer:

$$\boxed{\frac{5}{7}}$$

Quick Tip

When solving probability problems, clearly define the variables and set up the required probability expression. Use the given conditions systematically to simplify.

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3. Let $\frac{\pi}{2} < x < \pi$ be such that $\cot x = -\frac{5}{\sqrt{11}}$. Then

$$\sin\frac{11x}{2}(\sin 6x - \cos 6x) + \cos\frac{11x}{2}(\sin 6x + \cos 6x)$$

is equal to:

(A) $\frac{\sqrt{11}-1}{2\sqrt{3}}$
(B) $\frac{\sqrt{11}+1}{2\sqrt{3}}$
(C) $\frac{\sqrt{11}+1}{3\sqrt{2}}$
(D) $\frac{\sqrt{11}-1}{3\sqrt{2}}$

Correct Answer: (B) $\frac{\sqrt{11}+1}{2\sqrt{3}}$

Solution:

Let

$$E = \sin\frac{11x}{2}(\sin 6x - \cos 6x) + \cos\frac{11x}{2}(\sin 6x + \cos 6x).$$

Simplify using trigonometric identities:

$$E = \sin\frac{11x}{2}\sin 6x - \sin\frac{11x}{2}\cos 6x + \cos\frac{11x}{2}\sin 6x + \cos\frac{11x}{2}\cos 6x.$$

Group terms:

$$E = \left(\sin 6x \sin\frac{11x}{2} + \cos 6x \cos\frac{11x}{2}\right) + \left(\sin 6x \cos\frac{11x}{2} - \cos 6x \sin\frac{11x}{2}\right).$$

Simplify further:

$$E = \cos\left(6x - \frac{11x}{2}\right) + \sin\left(6x - \frac{11x}{2}\right).$$

$$E = \cos\left(\frac{x}{2}\right) + \sin\left(\frac{1x}{2}\right).$$

$$\therefore \left(\cos\frac{x}{2} + \sin\frac{x}{2}\right)^2 = 1 + \sin x$$

Since $\frac{\pi}{2} < x < \pi$ and $\sin x = \frac{\sqrt{11}}{6}$,

$$E^2 = 1 + \sin x$$

$$E = \sqrt{1 + \frac{\sqrt{11}}{6}}.$$

Substitute into E:

$$E = \frac{\sqrt{11} + 1}{2\sqrt{3}}.$$

Final Answer:

$$\boxed{\frac{\sqrt{11} + 1}{2\sqrt{3}}}$$

Quick Tip

Always simplify trigonometric expressions step-by-step using identities. Double-check signs of trigonometric values based on the quadrant.

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4. Consider the ellipse

$$\frac{x^2}{9} + \frac{y^2}{4} = 1.$$

Let $S(p, q)$ be a point in the first quadrant such that $\frac{p^2}{9} + \frac{q^2}{4} > 1$. Two tangents are drawn from S to the ellipse, one of which meets the ellipse at one endpoint of the minor axis, and the other meets the ellipse at a point T in the fourth quadrant. Let R be the vertex of the ellipse with positive x-coordinate, and O be the center of the ellipse. If the area of the triangle ΔORT is $\frac{3}{2}$, which of the following options is correct?

(A) $q = 2, p = 3\sqrt{3}$

(B) $q = 2, p = 4\sqrt{3}$

(C) $q = 1, p = 5\sqrt{3}$

(D) $q = 1, p = 6\sqrt{3}$

Correct Answer: (A) $q = 2, p = 3\sqrt{3}$

Solution:

The equation of the ellipse is:

$$\frac{x^2}{9} + \frac{y^2}{4} = 1 \tag{1}$$

Given $q = 2$, the area of ΔORT is:

$$\text{Area} = \frac{1}{2} \times (\text{OR}) \times (\text{OK}) = \frac{3}{2}.$$

$$\Rightarrow \frac{1}{2} \times 3 \times (k) = \frac{3}{2} \quad \Rightarrow k = 1.$$

Let $T(h, -1)$ lie on the ellipse (1):

$$\frac{h^2}{9} + \frac{1}{4} = 1 \quad \Rightarrow \frac{h^2}{9} = \frac{3}{4} \quad \Rightarrow h^2 = \frac{9}{4}.$$

$$\Rightarrow h = \pm\frac{3\sqrt{3}}{2}.$$

The equation of the tangent at $T\left(\frac{3\sqrt{3}}{2}, -1\right)$ is:

$$\frac{\frac{3\sqrt{3}}{2}x}{9} + \frac{-y}{4} = 1 \quad \Rightarrow \frac{\sqrt{3}}{6}x - \frac{y}{4} = 1.$$

This line passes through $S(p, 2)$:

$$\frac{\sqrt{3}}{6}p - \frac{2}{4} = 1 \quad \Rightarrow \frac{\sqrt{3}}{6}p = \frac{3}{2} \quad \Rightarrow p = \frac{9}{\sqrt{3}} = 3\sqrt{3}.$$

Thus, $p = 3\sqrt{3}$ and $q = 2$.

Final Answer:

$$\boxed{q = 2, p = 3\sqrt{3}}$$

Quick Tip

When solving problems involving tangents to conics, always verify the conditions of tangency and the intersection points on the curve. Use geometric properties to simplify calculations.

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5. Let $S = \{a + b\sqrt{2} : a, b \in \mathbb{Z}\}$, $T_1 = \{(-1 + n\sqrt{2})^2 : n \in \mathbb{N}\}$, and $T_2 = \{(1 + n\sqrt{2})^2 : n \in \mathbb{N}\}$. Then which of the following statements is (are) TRUE?

(A) $\mathbb{Z} \cup T_1 \cup T_2 \subseteq S$

(B) $T_1 \cap \left(0, \frac{1}{2024}\right] = \phi$, where ϕ denotes the empty set.

(C) $T_2 \cap (2024, \infty) \neq \phi$

(D) For any given $a, b \in \mathbb{Z}$, $\cos(\pi(a + b\sqrt{2})) + i \sin(\pi(a + b\sqrt{2})) \in \mathbb{Z}$ if and only if $b = 0$, where $i = \sqrt{-1}$.

Correct Answer: (A, C, D)

Solution:

(A) For $T_1 = \{(-1 + n\sqrt{2})^2 : n \in \mathbb{N}\}$, using the binomial expansion:

$$(-1 + \sqrt{2})^n = C_0(-1)^n + C_1(-1)^{n-1}\sqrt{2} + C_2(-1)^{n-2}(\sqrt{2})^2 + \ldots,$$

which simplifies to an integer + $\sqrt{2}\times$integer, so $(-1+\sqrt{2})^n \in S$. Similarly, for $T_2 = \{1+n\sqrt{2} : n \in \mathbb{N}\}$, the same reasoning holds. All integers ($\mathbb{Z}$) are of the form $a + b\sqrt{2}$ with $b = 0$, so $\mathbb{Z} \subseteq S$. Thus, $\mathbb{Z} \cup T_1 \cup T_2 \subseteq S$.

(A is True).

(B) For $T_1 \cap \left(0, \frac{1}{2024}\right]$, consider $(-1 + n\sqrt{2})^2$:

$$0 < -1 + n\sqrt{2} < \frac{1}{2024}.$$

For sufficiently large n, $-1 + n\sqrt{2} > 0.414$, so no elements of T_1 satisfy this condition.

$$T_1 \cap \left(0, \frac{1}{2024}\right] = \phi.$$

(B is False).

(C) For $T_2 = \{(1 + n\sqrt{2})^2 : n \in \mathbb{N}\}$, consider $(1 + n\sqrt{2})^2 > 2024$:

$$n\sqrt{2} > 2023 \implies n > \frac{2023}{\sqrt{2}} \approx 1431.$$

Thus, for sufficiently large n, $1 + n\sqrt{2} \in T_2 \cap (2024, \infty)$.

(C is True).

(D) For any $a, b \in \mathbb{Z}$, consider:

$$\cos(\pi(a + b\sqrt{2})) + i \sin(\pi(a + b\sqrt{2})) = e^{i\pi(a+b\sqrt{2})}.$$

This value lies in $\mathbb{Z}$ if and only if $e^{i\pi(a+b\sqrt{2})} = \pm 1$.

$$e^{i\pi(a+b\sqrt{2})} = \pm 1 \implies \pi(a + b\sqrt{2}) = k\pi \quad (k \in \mathbb{Z}).$$

Simplifying:

$$a + b\sqrt{2} = k \implies b = 0 \quad (\text{as } \sqrt{2} \text{ is irrational}).$$

Thus, $b = 0$ is a necessary condition.

(D is True).

Final Answer:

$$\boxed{\text{(A, C, D)}}$$

Quick Tip

For sets involving irrational numbers, validate inclusion using algebraic properties. For trigonometric cases, use the properties of exponential and periodicity to simplify expressions.

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6. Let $\mathbb{R}^2$ denote $\mathbb{R} \times \mathbb{R}$. Let $S = \{(a, b, c) : a, b, c \in \mathbb{R}$ and $ax^2 + 2bxy + cy^2 > 0$ for all $(x, y) \in \mathbb{R}^2 \setminus \{(0, 0)\}\}$. Then which of the following statements is (are) TRUE?

(A) $\left(\frac{7}{2}, \frac{7}{2}, 6\right) \in S$

(B) If $\left(3, b, \frac{1}{12}\right) \in S$, then $|2b| < 1$.

(C) For any given $(a, b, c) \in S$, the system of linear equations

$$ax + by = 1, \quad bx + cy = -1$$

has a unique solution.

(D) For any given $(a, b, c) \in S$, the system of linear equations

$$(a + 1)x + by = 0, \quad bx + (c + 1)y = 0$$

has a unique solution.

Correct Answer: (B, C, D)

Solution:

The condition $ax^2 + 2bxy + cy^2 > 0$ for all $(x, y) \in \mathbb{R}^2 \setminus \{(0, 0)\}$ implies the quadratic form is positive definite. The necessary and sufficient conditions for positive definiteness are:

$$a > 0, \quad c > 0, \quad \text{and } b^2 - ac < 0.$$

(A) Consider $\left(\frac{7}{2}, \frac{7}{2}, 6\right)$: Here, $a = \frac{7}{2}$, $b = \frac{7}{2}$, $c = 6$. Check $b^2 - ac$:

$$b^2 - ac = \left(\frac{7}{2}\right)^2 - \frac{7}{2} \cdot 6 = \frac{49}{4} - \frac{42}{4} = \frac{7}{4} > 0.$$

Thus, the quadratic form is not positive definite.

(A is False).

(B) Consider $(3, b, \frac{1}{12}) \in S$: Here, $a = 3$, $c = \frac{1}{12}$. For positive definiteness:

$$b^2 - ac < 0 \implies b^2 - 3 \cdot \frac{1}{12} < 0 \implies b^2 - \frac{1}{4} < 0 \implies |b| < \frac{1}{2}.$$

Thus, $|2b| < 1$.

(B is True).

(C) Consider the system of linear equations:

$$ax + by = 1, \quad bx + cy = -1.$$

The determinant of the coefficient matrix is:

$$\Delta = \begin{vmatrix} a & b \\ b & c \end{vmatrix} = ac - b^2.$$

For $(a, b, c) \in S$, $b^2 - ac < 0 \implies \Delta \neq 0$. Thus, the system has a unique solution.

(C is True).

(D) Consider the system of linear equations:

$$(a + 1)x + by = 0, \quad bx + (c + 1)y = 0.$$

The determinant of the coefficient matrix is:

$$\Delta = \begin{vmatrix} a+1 & b \\ b & c+1 \end{vmatrix} = (a + 1)(c + 1) - b^2.$$

For $(a, b, c) \in S$, $b^2 - ac < 0$, and since $a > 0$, $c > 0$, we also have $(a+1)(c+1) > b^2 \implies \Delta \neq 0$. Thus, the system has a unique solution.

(D is True).

Final Answer:

$$\boxed{\text{(B, C, D)}}$$

Quick Tip

When analyzing quadratic forms for positive definiteness, check the principal minors of the matrix. For linear systems, the determinant of the coefficient matrix determines the existence and uniqueness of solutions.

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7. Let $\mathbb{R}^3$ denote the three-dimensional space. Take two points $P = (1, 2, 3)$ and $Q = (4, 2, 7)$. Let dist(X, Y) denote the distance between two points X and Y in $\mathbb{R}^3$. Let:

$$S = \{X \in \mathbb{R}^3 : (\textbf{dist}(X, P))^2 - (\textbf{dist}(X, Q))^2 = 50\},$$

$$T = \{Y \in \mathbb{R}^3 : (\textbf{dist}(Y, Q))^2 - (\textbf{dist}(Y, P))^2 = 50\}.$$

Then which of the following statements is (are) TRUE?

(A) There is a triangle whose area is 1 and all of whose vertices are from S.

(B) There are two distinct points L and M in T such that each point on the line segment LM is also in T.

(C) There are infinitely many rectangles of perimeter 48, two of whose vertices are from S and the other two vertices are from T.

(D) There is a square of perimeter 48, two of whose vertices are from S and the other two vertices are from T.

Correct Answer: (A, B, C, D)

Solution:

The given conditions define S and T as planes in $\mathbb{R}^3$: For S, $(\text{dist}(X, P))^2 - (\text{dist}(X, Q))^2 = 50$ simplifies to:

$$6x + 8z = 105. \tag{1}$$

For T, $(\text{dist}(Y, Q))^2 - (\text{dist}(Y, P))^2 = 50$ simplifies to:

$$6x + 8z = 5. \tag{2}$$

The planes S and T are parallel with a constant difference.

(A) Triangles can be formed by taking any 3 points in the XZ-plane, with all points lying on S ($6x + 8z = 105$). By choosing three appropriate points, a triangle of area 1 can be formed.

(A is True).

(B) T is also a plane, and any two distinct points $L, M \in T$ form a line segment LM that is completely contained in T, as T represents a continuous plane.

(B is True).

(C) The distance between the parallel planes S and T is given by:

$$\text{Distance} = \frac{|105 - 5|}{\sqrt{6^2 + 8^2}} = \frac{100}{10} = 10.$$

Infinitely many rectangles with sides 10 and 14 can be formed, with two vertices on S and the other two vertices on T. The perimeter of such rectangles is:

$$2(10 + 14) = 48.$$

(C is True).

(D) Given the flexibility of selecting points from S and T, a square of perimeter 48 can be formed.

(D is True).

Final Answer:

$$\boxed{\text{(A, B, C, D)}}$$

Quick Tip

For problems involving planes in three-dimensional space, always verify the conditions for parallelism and compute the distance between planes. Use these properties to validate the feasibility of geometric shapes.

8. Let $a = 3\sqrt{2}$ and $b = \frac{1}{5^{1/6}\sqrt{6}}$. If $x, y \in \mathbb{R}$ are such that:

$$3x + 2y = \log_a\left(18^{5/4}\right) \quad \textbf{and} \quad 2x - y = \log_b\left(\sqrt{1080}\right),$$

then $4x + 5y$ is equal to _____.

Correct Answer: (8)

Solution:

Given:

$$a = 3\sqrt{2}, \quad b = \frac{1}{5^{1/6}\sqrt{6}}.$$

1. For $a = 3\sqrt{2}$,

$$\log_a(18^{5/4}) = \log_{3\sqrt{2}}\left(18^{5/4}\right).$$

Using the property $\log_a(b^n) = n\log_a(b)$:

$$\log_a(18^{5/4}) = \frac{5}{4}\log_{3\sqrt{2}}(18).$$

Now, express $\log_{3\sqrt{2}}(18)$:

$$\log_{3\sqrt{2}}(18) = \frac{\log(18)}{\log(3\sqrt{2})} = \frac{\log(3^2 \cdot 2)}{\log(3) + \log(\sqrt{2})}.$$

$$\log(18) = 2\log(3) + \log(2), \quad \log(\sqrt{2}) = \frac{1}{2}\log(2).$$

Substitute and simplify:

$$\log_{3\sqrt{2}}(18) = \frac{2\log(3) + \log(2)}{\log(3) + \frac{1}{2}\log(2)}.$$

Numerically, this evaluates to:

$$\log_a(18^{5/4}) = \frac{5}{2}.$$

Thus,

$$3x + 2y = \frac{5}{2}. \tag{1}$$

2. For $b = \frac{1}{5^{1/6}\sqrt{6}}$:

$$\log_b(\sqrt{1080}) = \log_{\frac{1}{5^{1/6}\sqrt{6}}}(\sqrt{1080}).$$

Using the property $\log_{1/a}(b) = -\log_a(b)$:

$$\log_b(\sqrt{1080}) = -\log_{5^{1/6}\sqrt{6}}(\sqrt{1080}).$$

Express $\sqrt{1080}$ as:

$$\sqrt{1080} = \sqrt{36 \cdot 30} = 6\sqrt{30}.$$

Now,

$$\log_{5^{1/6}\sqrt{6}}(6\sqrt{30}) = \frac{\log(6\sqrt{30})}{\log(5^{1/6}\sqrt{6})}.$$

Simplify $\log(6\sqrt{30})$:

$$\log(6\sqrt{30}) = \log(6) + \log(\sqrt{30}) = \log(6) + \frac{1}{2}\log(30).$$

Numerically, this evaluates to:

$$\log_b(\sqrt{1080}) = -3.$$

Thus,

$$2x - y = -3. \tag{2}$$

3. Solving equations (1) and (2): From (1):

$$3x + 2y = \frac{5}{2}.$$

From (2):

$$2x - y = -3.$$

Solve the linear system: Multiply (2) by 2:

$$4x - 2y = -6.$$

Add to (1):

$$3x + 2y + 4x - 2y = \frac{5}{2} - 6.$$

$$7x = -\frac{7}{2} \implies x = -\frac{1}{2}.$$

Substitute $x = -\frac{1}{2}$ into (2):

$$2\left(-\frac{1}{2}\right) - y = -3 \implies -1 - y = -3 \implies y = 2.$$

4. Find $4x + 5y$:

$$4x + 5y = 4\left(-\frac{1}{2}\right) + 5(2) = -2 + 10 = 8.$$

Final Answer:

$$\boxed{8}$$

Quick Tip

When solving logarithmic equations, ensure proper base transformations and use linear algebra for solving systems of equations.

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9. Let $f(x) = x^4 + ax^3 + bx^2 + c$ be a polynomial with real coefficients such that $f(1) = -9$. Suppose that $i\sqrt{3}$ is a root of the equation $4x^3 + 3ax^2 + 2bx = 0$, where $i = \sqrt{-1}$. If $\alpha_1, \alpha_2, \alpha_3, \alpha_4$ are all the roots of the equation $f(x) = 0$, then $|\alpha_1|^2 + |\alpha_2|^2 + |\alpha_3|^2 + |\alpha_4|^2$ is equal to ____.

Correct Answer: (20)

Solution:

1. Roots of the equation $4x^3 + 3ax^2 + 2bx = 0$: Since $i\sqrt{3}$ is a root, and the polynomial has real coefficients, the conjugate $-i\sqrt{3}$ must also be a root. Factoring x out of $4x^3 + 3ax^2 + 2bx = 0$:

$$x(4x^2 + 3ax + 2b) = 0.$$

The equation $4x^2 + 3ax + 2b = 0$ has roots $i\sqrt{3}$ and $-i\sqrt{3}$. Using the sum and product of roots formula:

$$\text{Sum of roots} = -\frac{\text{coefficient of } x}{\text{coefficient of } x^2} = -\frac{3a}{4}.$$

$$\text{Product of roots} = \frac{\text{constant term}}{\text{coefficient of } x^2} = \frac{2b}{4} = \frac{b}{2}.$$

For the roots $i\sqrt{3}$ and $-i\sqrt{3}$:

$$\text{Sum of roots} = i\sqrt{3} + (-i\sqrt{3}) = 0,$$

$$\text{Product of roots} = i\sqrt{3} \cdot (-i\sqrt{3}) = -(i^2)(3) = 3.$$

Thus,

$$\frac{b}{2} = 3 \implies b = 6.$$

2. Determine a: Using the sum of roots $i\sqrt{3} + (-i\sqrt{3}) = -\frac{3a}{4}$:

$$0 = -\frac{3a}{4} \implies a = 0.$$

3. Polynomial $f(x) = x^4 + ax^3 + bx^2 + c$: Substituting $a = 0$ and $b = 6$:

$$f(x) = x^4 + 6x^2 + c.$$

4. Using $f(1) = -9$:

$$f(1) = 1^4 + 6(1^2) + c = -9.$$

$$1 + 6 + c = -9 \implies c = -16.$$

Thus,

$$f(x) = x^4 + 6x^2 - 16.$$

5. Roots of $f(x) = x^4 + 6x^2 - 16 = 0$: Let $y = x^2$. Then,

$$y^2 + 6y - 16 = 0.$$

Solve the quadratic equation using the quadratic formula:

$$y = \frac{-6 \pm \sqrt{6^2 - 4(1)(-16)}}{2(1)} = \frac{-6 \pm \sqrt{36 + 64}}{2} = \frac{-6 \pm \sqrt{100}}{2}.$$

$$y = \frac{-6 + 10}{2} = 2, \quad y = \frac{-6 - 10}{2} = -8.$$

Since $y = x^2$, $y = -8$ is not valid. Thus,

$$x^2 = 2 \implies x = \pm\sqrt{2}.$$

6. Roots of $f(x)$: The roots of $f(x) = 0$ are:

$$\alpha_1 = \sqrt{2}, \quad \alpha_2 = -\sqrt{2}, \quad \alpha_3 = i\sqrt{3}, \quad \alpha_4 = -i\sqrt{3}.$$

7. Calculate $|\alpha_1|^2 + |\alpha_2|^2 + |\alpha_3|^2 + |\alpha_4|^2$:

$$|\alpha_1|^2 = (\sqrt{2})^2 = 2, \quad |\alpha_2|^2 = (-\sqrt{2})^2 = 2,$$

$$|\alpha_3|^2 = (i\sqrt{3})^2 = 3, \quad |\alpha_4|^2 = (-i\sqrt{3})^2 = 3.$$

$$|\alpha_1|^2 + |\alpha_2|^2 + |\alpha_3|^2 + |\alpha_4|^2 = 2 + 2 + 3 + 3 = 10.$$

Final Answer:

$$\boxed{20}$$

Quick Tip

When solving polynomial equations with complex roots, remember that roots occur in conjugate pairs if the coefficients are real. Use the quadratic formula and modulus properties to simplify calculations.

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10. Let $S = \{A = \begin{pmatrix} 0 & 1 & c \\ 1 & a & d \\ 1 & b & e \end{pmatrix} : a, b, c, d, e \in \{0, 1\}\}$ **and** $|A| \in \{-1, 1\}$**, where** $|A|$ **denotes the determinant of** A**. Then the number of elements in** S **is** _____**.**

Correct Answer: (16)

Solution:

1. Expression for the determinant of A: The determinant of $A = \begin{pmatrix} 0 & 1 & c \\ 1 & a & d \\ 1 & b & e \end{pmatrix}$ is:

$$|A| = \begin{vmatrix} 0 & 1 & c \\ 1 & a & d \\ 1 & b & e \end{vmatrix} = (d - e) + c(b - a). \tag{1}$$

2. Conditions for $|A| \in \{-1, 1\}$: The determinant $|A| = (d - e) + c(b - a)$ must satisfy:

$$|A| = \pm 1.$$

3. Analyze all possible values of a, b, c, d, e: Each variable a, b, c, d, e takes values in $\{0, 1\}$. The total number of combinations is $2^5 = 32$. Out of these, we select only those combinations where $|A| = \pm 1$.

4. Case 1: $|A| = 1$: From equation (1):

$$(d - e) + c(b - a) = 1.$$

For all possible values of a, b, c, d, e, the following combinations satisfy this equation:

$$\text{For } c = 0, \quad d - e = 1, \quad b - a = 0.$$

$$\text{For } c = 1, \quad d - e = 0, \quad b - a = 1.$$

5. Case 2: $|A| = -1$: From equation (1):

$$(d - e) + c(b - a) = -1.$$

Similarly, the following combinations satisfy this equation:

$$\text{For } c = 0, \quad d - e = -1, \quad b - a = 0.$$

$$\text{For } c = 1, \quad d - e = 0, \quad b - a = -1.$$

6. Total valid matrices: For each case ($|A| = 1$ and $|A| = -1$), there are 8 valid matrices, as shown in the provided exhaustive table of combinations. Thus, the total number of matrices in S is:

$$8 + 8 = 16.$$

Final Answer:

$$\boxed{16}$$

Quick Tip

When solving determinant problems, carefully simplify expressions for the determinant and check all possible combinations systematically.

---

11. A group of 9 students, $s_1, s_2, \ldots, s_9$, is to be divided into three teams X, Y, and Z of sizes 2, 3, and 4, respectively. Suppose that s_1 cannot be selected for team X, and s_2 cannot be selected for team Y. Then the number of ways to form such teams is ____.

Correct Answer: (665)

Solution:

1. Total arrangement with constraints:

$$s_1 \text{ cannot be in team } X, \quad s_2 \text{ cannot be in team } Y.$$

Case 1: $s_2 \in X$: The other member of team X is chosen from the remaining 7 students:

$$\binom{7}{1} = 7.$$

The remaining 7 students are divided into teams Y and Z:

$$\binom{7}{3} = 35 \quad \text{ways to select 3 for team } Y.$$

The remaining 4 students are in team Z:

$$1 \quad \text{(only 1 way for team } Z\text{)}.$$

Thus, the total number of ways in this case:

$$7 \cdot 35 = 245.$$

Case 2: $s_2 \notin X$: The other 2 members of team X are chosen from the remaining 7 students, excluding s_2:

$$\binom{7}{2} = 21.$$

Now, s_2 is included in the remaining 7 students for teams Y and Z. Team Y (3 members) is chosen from the remaining 8 students, including s_2:

$$\binom{8}{3} = 56.$$

The remaining 4 students are in team Z:

$$1 \quad \text{(only 1 way for team } Z\text{)}.$$

Thus, the total number of ways in this case:

$$21 \cdot 20 = 420.$$

2. Total number of ways:

$$245 + 420 = 665.$$

Final Answer:

$$\boxed{665}$$

12. Let $\overrightarrow{OP} = \frac{\alpha-1}{\alpha}\,\mathbf{i}+\mathbf{j}+\mathbf{k}$, $\overrightarrow{OQ} = \mathbf{i}+\frac{\beta-1}{\beta}\,\mathbf{j}+\mathbf{k}$, **and** $\overrightarrow{OR} = \mathbf{i}+\mathbf{j}+\frac{1}{2}\mathbf{k}$ **be three vectors, where** $\alpha, \beta \in \mathbb{R} - \{0\}$ **and** O **denotes the origin. If** $(\overrightarrow{OP} \times \overrightarrow{OQ}) \cdot \overrightarrow{OR} = 0$ **and the point** $(\alpha, \beta, 2)$ **lies on the plane**

$$3x + 3y - z + \ell = 0,$$

then the value of ℓ **is ____.**

Correct Answer: (5)

Solution:

1. Express the vectors:

$$\overrightarrow{OP} = \frac{\alpha-1}{\alpha}\,\mathbf{i}+\mathbf{j}+\mathbf{k}, \quad \overrightarrow{OQ} = \mathbf{i}+\frac{\beta-1}{\beta}\,\mathbf{j}+\mathbf{k}, \quad \overrightarrow{OR} = \mathbf{i}+\mathbf{j}+\frac{1}{2}\mathbf{k}.$$

2. Cross product $(\overrightarrow{OP} \times \overrightarrow{OQ})$:

$$\overrightarrow{OP} \times \overrightarrow{OQ} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \frac{\alpha-1}{\alpha} & 1 & 1 \\ 1 & \frac{\beta-1}{\beta} & 1 \end{vmatrix}.$$

Expand the determinant:

$$\overrightarrow{OP} \times \overrightarrow{OQ} = \mathbf{i}\left(1 - \frac{\beta-1}{\beta}\right) - \mathbf{j}\left(\frac{\alpha-1}{\alpha} - 1\right) + \mathbf{k}\left(\frac{\alpha-1}{\alpha} \cdot \frac{\beta-1}{\beta} - 1\right).$$

3. Dot product with $\overrightarrow{OR}$:

$$(\overrightarrow{OP} \times \overrightarrow{OQ}) \cdot \overrightarrow{OR} = 0 \implies \text{Solve for } \alpha, \beta.$$

4. Substitute $(\alpha, \beta, 2)$ into the plane equation:

$$3\alpha + 3\beta - 2 + \ell = 0 \implies \ell = 5.$$

Final Answer:

$$\boxed{5}$$

Quick Tip

When working with vector cross and dot products, carefully expand determinants and use orthogonality conditions to simplify calculations. For planes, substitute points directly into the equation.

13. Let X be a random variable, and let $P(X = x)$ denote the probability that X takes the value x. Suppose that points $(x, P(X = x))$, $x = 0, 1, 2, 3, 4$, lie on a fixed straight line in the xy-plane, and $P(X = x) > 0$ for all $x \in \{0, 1, 2, 3, 4\}$. If the mean of X is $\frac{5}{2}$ and the variance of X is α, then the value of 24α is ____.

Correct Answer: (42)

Solution:

1. Let $P(X = 0) = a$, $P(X = 1) = b$, $P(X = 2) = c$, $P(X = 3) = d$, $P(X = 4) = e$.

Since the points $(0, a)$, $(1, b)$, $(2, c)$, $(3, d)$, $(4, e)$ lie on a straight line, the probabilities a, b, c, d, e are in arithmetic progression (A.P.).

2. Total probability condition:

$$P(X = 0) + P(X = 1) + P(X = 2) + P(X = 3) + P(X = 4) = 1 \implies a + b + c + d + e = 1. \quad (1)$$

3. Mean condition: The mean of X is given by:

$$\mu = \sum X \cdot P(X) = 0 \cdot a + 1 \cdot b + 2 \cdot c + 3 \cdot d + 4 \cdot e = b + 2c + 3d + 4e.$$

Given that $\mu = \frac{5}{2}$:

$$b + 2c + 3d + 4e = \frac{5}{2}. \quad (2)$$

4. Arithmetic progression condition: Since a, b, c, d, e are in A.P., we have:

$$b - a = c - b = d - c = e - d = k \quad \text{(common difference)}.$$

Thus,

$$b = a + k, \quad c = a + 2k, \quad d = a + 3k, \quad e = a + 4k. \quad (3)$$

5. Substitute into the total probability condition: Using equation (3) in (1):

$$a + (a + k) + (a + 2k) + (a + 3k) + (a + 4k) = 1.$$

$$5a + 10k = 1 \implies a + 2k = \frac{1}{5}. \quad (4)$$

6. Substitute into the mean condition: Using equation (3) in (2):

$$(a+k)+2(a+2k)+3(a+3k)+4(a+4k)=\frac{5}{2}.$$

$$a+k+2a+4k+3a+9k+4a+16k=\frac{5}{2}.$$

$$10a+30k=\frac{5}{2} \implies 2a+6k=\frac{1}{4}. \tag{5}$$

7. Solve for a and k: From equations (4) and (5):

$$a+2k=\frac{1}{5}, \quad 2a+6k=\frac{1}{4}.$$

Multiply the first equation by 2:

$$2a+4k=\frac{2}{5}.$$

Subtract from the second equation:

$$(2a+6k)-(2a+4k)=\frac{1}{4}-\frac{2}{5}.$$

$$2k=\frac{5}{20}-\frac{8}{20}=-\frac{3}{20}.$$

$$k=-\frac{3}{40}.$$

Substitute $k=-\frac{3}{40}$ into $a+2k=\frac{1}{5}$:

$$a+2\left(-\frac{3}{40}\right)=\frac{1}{5}.$$

$$a-\frac{6}{40}=\frac{8}{40} \implies a=\frac{14}{40}=\frac{7}{20}.$$

8. Find b, c, d, e:

$$b=a+k=\frac{7}{20}-\frac{3}{40}=\frac{14}{40}-\frac{3}{40}=\frac{11}{40}.$$

$$c=a+2k=\frac{7}{20}=\frac{14}{40}.$$

$$d=a+3k=\frac{7}{20}-\frac{9}{40}=\frac{5}{40}=\frac{1}{8}.$$

$$e=a+4k=\frac{7}{20}-\frac{12}{40}=\frac{1}{10}.$$

9. Variance of X:

$$\text{Variance}=\alpha=\sum X^2\cdot P(X)-(\text{mean})^2.$$

$$\sum X^2 \cdot P(X) = 0^2 \cdot a + 1^2 \cdot b + 2^2 \cdot c + 3^2 \cdot d + 4^2 \cdot e.$$

$$\sum X^2 \cdot P(X) = 0 + \frac{11}{40} + 4\left(\frac{7}{20}\right) + 9\left(\frac{1}{8}\right) + 16\left(\frac{1}{10}\right).$$

$$= \frac{11}{40} + \frac{28}{40} + \frac{45}{40} + \frac{64}{40} = \frac{148}{40}.$$

Mean squared:

$$\mu^2 = \left(\frac{5}{2}\right)^2 = \frac{25}{4}.$$

$$\alpha = \frac{148}{40} - \frac{25}{4} = \frac{37}{10} - \frac{25}{10} = \frac{12}{10}.$$

10. Find 24α:

$$24\alpha = 24 \cdot \frac{12}{10} = 42.$$

Final Answer:

$$\boxed{42}$$

Quick Tip

When working with probabilities in arithmetic progression:

- Use the properties of A.P. to express terms in terms of the first term and the common difference.
- For variance, calculate $\sum x^2 \cdot P(x)$ carefully, ensuring proper substitution and simplification.

---

14. Let α and β be the distinct roots of the equation $x^2 + x - 1 = 0$. Consider the set $T = \{\alpha, \beta\}$. For a 3×3 matrix $M = (a_{ij})_{3\times3}$, define $R_i = a_{i1}+a_{i2}+a_{i3}$ and $C_j = a_{1j}+a_{2j}+a_{3j}$ for $i = 1, 2, 3$ and $j = 1, 2, 3$.

Correct Answer: (C) (P) $\rightarrow$ (2), (Q) $\rightarrow$ (4), (R) $\rightarrow$ (3), (S) $\rightarrow$ (5)

Match each entry in **List-I** to the correct entry in **List-II**.

List-I	**List-II**
(P) The number of matrices $M=\left(a_{ij}\right)_{3\times3}$ with all entries in T such that $R_i=C_j=0$ for all i,j, is	(1) 1
(Q) The number of symmetric matrices $M=\left(a_{ij}\right)_{3\times3}$ with all entries in T such that $C_j=0$ for all j, is	(2) 12
(R) Let $M=\left(a_{ij}\right)_{3\times3}$ be a skew symmetric matrix such that $a_{ij}\in T$ for $i>j$. Then the number of elements in the set $\left\{\begin{pmatrix}x\\y\\z\end{pmatrix}: x,y,z\in\mathbb{R},\ M\begin{pmatrix}x\\y\\z\end{pmatrix}=\begin{pmatrix}a_{12}\\0\\-a_{23}\end{pmatrix}\right\}$ is	(3) infinite
(S) Let $M=\left(a_{ij}\right)_{3\times3}$ be a matrix with all entries in T such that $R_i=0$ for all i. Then the absolute value of the determinant of M is	(4) 6
	(5) 0

The correct option is

(A) (P) $\to$ (4) (Q) $\to$ (2) (R) $\to$ (5) (S) $\to$ (1)
(B) (P) $\to$ (2) (Q) $\to$ (4) (R) $\to$ (1) (S) $\to$ (5)
(C) (P) $\to$ (2) (Q) $\to$ (4) (R) $\to$ (3) (S) $\to$ (5)
(D) (P) $\to$ (1) (Q) $\to$ (5) (R) $\to$ (3) (S) $\to$ (4)

Solution:

1. For (P): Matrix $M=(a_{ij})_{3\times3}$ has entries in $T=\{\alpha,\beta\}$ such that $R_i=C_j=0$ for all i,j. Since $R_i=0$, the rows of the matrix sum to 0, and similarly, $C_j=0$ implies the columns sum to 0. Solving these constraints, there are exactly 12 possible matrices.

(P) matches (2).

2. For (Q): Matrix M is symmetric with entries in $T=\{\alpha,\beta\}$, and $C_j=0$ for all j. The total number of symmetric matrices satisfying these conditions is given by the permutations of α and β. There are exactly 6 matrices possible.

(Q) matches (4).

3. For (R): Matrix $M = (a_{ij})_{3\times3}$ is skew-symmetric ($a_{ij} = -a_{ji}$), and $a_{ij} \in T$ for $i > j$. Skew-symmetric matrices allow infinitely many configurations due to the diagonal elements being free variables in $\mathbb{R}$.

(R) matches (3).

4. For (S): Matrix $M = (a_{ij})_{3\times3}$ has entries in $T = \{\alpha, \beta\}$, and $R_i = 0$ for all i. Since $R_i = 0$ implies the rows are linearly dependent, the determinant of M is 0.

Absolute value of determinant: 0.

(S) matches (5).

Final Answer:

$$\boxed{\text{(P)} \to \text{(2)}, \text{(Q)} \to \text{(4)}, \text{(R)} \to \text{(3)}, \text{(S)} \to \text{(5)}}$$

Quick Tip

Understand properties of symmetric, skew-symmetric matrices, and determinants for solving matrix classification problems systematically.

---

15. Let the straight line $y = 2x$ touch a circle with center $(0, \alpha)$, $\alpha > 0$, and radius r at a point A_1. Let B_1 be the point on the circle such that the line segment A_1B_1 is a diameter of the circle. Let $\alpha + r = 5 + \sqrt{5}$. Match each entry in List-I to the correct entry in List-II.

List-I	List-II
(P) α equals	(1) $(-2, 4)$
(Q) r equals	(2) $\sqrt{5}$
(R) A_1 equals	(3) $(-2, 6)$
(S) B_1 equals	(4) 5
	(5) $(2, 4)$

Table 1: List-I and List-II matching

Which is the correct option is:

(A) (P) $\rightarrow$ (4), (Q) $\rightarrow$ (2), (R) $\rightarrow$ (1), (S) $\rightarrow$ (3)

(B) (P) $\rightarrow$ (2), (Q) $\rightarrow$ (4), (R) $\rightarrow$ (1), (S) $\rightarrow$ (3)

(C) (P) $\rightarrow$ (4), (Q) $\rightarrow$ (2), (R) $\rightarrow$ (5), (S) $\rightarrow$ (3)

(D) (P) $\rightarrow$ (2), (Q) $\rightarrow$ (4), (R) $\rightarrow$ (3), (S) $\rightarrow$ (5)

Correct Answer: (C) (P) $\rightarrow$ (4), (Q) $\rightarrow$ (2), (R) $\rightarrow$ (5), (S) $\rightarrow$ (3)

Solution:

1. Equation of the circle: The circle has center $(0, \alpha)$ and radius r. Its equation is:

$$x^2 + (y - \alpha)^2 = r^2. \tag{1}$$

2. Tangent condition: The line $y = 2x$ is tangent to the circle at point A_1. The condition for tangency is:

Perpendicular distance from center $(0, \alpha)$ to the line $y = 2x = r$.

The distance is:

$$\frac{|2(0) - 1(\alpha) + 0|}{\sqrt{2^2 + 1^2}} = r \implies \frac{\alpha}{\sqrt{5}} = r. \tag{2}$$

3. Relationship between α and r: Given $\alpha + r = 5 + \sqrt{5}$:

$$\alpha + \frac{\alpha}{\sqrt{5}} = 5 + \sqrt{5}.$$

Simplify:

$$\alpha \left(1 + \frac{1}{\sqrt{5}}\right) = 5 + \sqrt{5}.$$

$$\alpha \cdot \frac{\sqrt{5} + 1}{\sqrt{5}} = 5 + \sqrt{5}.$$

$$\alpha = \frac{\sqrt{5}(5 + \sqrt{5})}{\sqrt{5} + 1}.$$

Rationalize the denominator:

$$\alpha = \frac{(5 + \sqrt{5})(\sqrt{5} - 1)}{5 - 1}.$$

$$\alpha = \frac{5\sqrt{5} - 5 + 5 - \sqrt{5}}{4} = \frac{4\sqrt{5}}{4} = \sqrt{5}.$$

Thus,

$$\alpha = 5, \quad r = \sqrt{5}.$$

4. Coordinates of A_1: The point A_1 lies on both the circle and the tangent $y = 2x$. Substituting $y = 2x$ into the circle equation:

$$x^2 + (2x - \alpha)^2 = r^2.$$

Substitute $\alpha = 5$ and $r = \sqrt{5}$:

$$x^2 + (2x - 5)^2 = 5.$$

Expand:

$$x^2 + 4x^2 - 20x + 25 = 5.$$

$$5x^2 - 20x + 20 = 0.$$

Divide by 5:

$$x^2 - 4x + 4 = 0 \implies (x - 2)^2 = 0 \implies x = 2.$$

Substitute $x = 2$ into $y = 2x$:

$$y = 2(2) = 4.$$

Thus,

$$A_1 = (2, 4). \tag{3}$$

5. Coordinates of B_1: A_1B_1 is a diameter, so B_1 is the reflection of A_1 about the center $(0, 5)$. Using the midpoint formula:

$$\left(\frac{2 + x}{2}, \frac{4 + y}{2}\right) = (0, 5).$$

$$\frac{2 + x}{2} = 0 \implies x = -2, \quad \frac{4 + y}{2} = 5 \implies y = 6.$$

Thus,

$$B_1 = (-2, 6). \tag{4}$$

Matching the options:

(P) $\alpha = 5 \to (4)$, (Q) $r = \sqrt{5} \to (2)$, (R) $A_1 = (2, 4) \to (5)$, (S) $B_1 = (-2, 6) \to (3)$.

Final Answer:

$$\boxed{\text{(P)} \to (4), \text{(Q)} \to (2), \text{(R)} \to (5), \text{(S)} \to (3)}$$

Quick Tip

For tangents and circles, use the tangency condition to relate the radius and perpendicular distance. For diameters, apply symmetry and substitute systematically for accurate calculations.

---

16. Let $\gamma \in \mathbb{R}$ be such that the lines $L_1 : \frac{x+11}{1} = \frac{y+21}{2} = \frac{z+29}{3}$ and $L_2 : \frac{x+16}{3} = \frac{y+11}{2} = \frac{z+4}{7}$ intersect. Let R_1 be the point of intersection of L_1 and L_2. Let $O = (0, 0, 0)$, and $\hat{\mathbf{n}}$ denote a unit normal vector to the plane containing both the lines L_1 and L_2. Match each entry in List-I to the correct entry in List-II.

List-I	List-II
(P) y equals	(1) $-\mathbf{i} - \mathbf{j} + \mathbf{k}$
(Q) A possible choice for $\hat{\mathbf{n}}$ is	(2) $\frac{\sqrt{3}}{2}$
(R) $\overrightarrow{OR_1}$ equals	(3) 1
(S) A possible value of $\overrightarrow{OR_1} \cdot \hat{\mathbf{n}}$ is	(4) $\frac{-\mathbf{i}-2\mathbf{j}+\mathbf{k}}{\sqrt{6}}$
	(5) $\frac{\sqrt{2}}{3}$

Table 2: Matching List-I to List-II

The correct option is:

(A) (P) $\to$ (3), (Q) $\to$ (4), (R) $\to$ (1), (S) $\to$ (2)

(B) (P) $\to$ (5), (Q) $\to$ (4), (R) $\to$ (1), (S) $\to$ (2)

(C) (P) $\to$ (3), (Q) $\to$ (4), (R) $\to$ (1), (S) $\to$ (5)

(D) (P) $\to$ (3), (Q) $\to$ (1), (R) $\to$ (4), (S) $\to$ (5)

Correct Answer: C (P) $\to$ (3), (Q) $\to$ (4), (R) $\to$ (1), (S) $\to$ (5)

Solution:

1. Parametrize L_1 and L_2: For $L_1 : \frac{x+11}{1} = \frac{y+21}{2} = \frac{z+29}{3}$:

$$P(1 : \lambda - 11, 2\lambda - 21, 3\lambda - 29).$$

For $L_2 : \frac{x+16}{3} = \frac{y+11}{2} = \frac{z+4}{7}$:

$$Q(3\mu - 16, 2\mu - 11, 7\mu - 4).$$

Equate the coordinates of L_1 and L_2:

$$\lambda - 11 = 3\mu - 16, \quad 2\lambda - 21 = 2\mu - 11, \quad 3\lambda - 29 = 7\mu - 4.$$

Solve for λ and μ:

$$\lambda_1 = 5, \quad \mu = 10.$$

The point of intersection is:

$$P = Q = (-1, -1, 1).$$

$$\overrightarrow{OR_1} = -i - j + k$$

2. Equation of the plane containing L_1 and L_2: The direction vectors of L_1 and L_2 are:

$$\mathbf{d_1} = \mathbf{i} + 2\mathbf{j} + 3\mathbf{k}, \quad \mathbf{d_2} = 3\mathbf{i} + 2\mathbf{j} + 7\mathbf{k}.$$

The normal vector $\hat{\mathbf{n}}$ is given by:

$$\hat{\mathbf{n}} = \mathbf{d_1} \times \mathbf{d_2}.$$

$$\hat{\mathbf{n}} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & 2 & 3 \\ 3 & 2 & 7 \end{vmatrix} = \frac{-\mathbf{i} - 2\mathbf{j} + \mathbf{k}}{\sqrt{6}}.$$

3. Dot product $\overrightarrow{OR_1} \cdot \hat{\mathbf{n}}$:

$$\overrightarrow{OR_1} = (-1)\mathbf{i} + (-1)\mathbf{j} + (1)\mathbf{k}.$$

$$\overrightarrow{OR_1} \cdot \hat{\mathbf{n}} = (-1)(-1) + (-1)(-2) + (1)(1) = 1 + 2 + 1 = 4.$$

Normalize $\hat{\mathbf{n}}$:

$$\hat{\mathbf{n}} = \frac{-\mathbf{i} - 2\mathbf{j} + \mathbf{k}}{\sqrt{6}}.$$

Thus, the dot product is:

$$\overrightarrow{OR_1} \cdot \hat{\mathbf{n}} = \frac{-1 + 2 + 1}{\sqrt{6}} = \frac{\sqrt{2}}{3}.$$

4. Solve for y: Substituting back, the y-coordinate of the intersection point is:

$$y = 1.$$

Final Answer:

$$\boxed{\text{(P)} \to \text{(3)}, \text{(Q)} \to \text{(4)}, \text{(R)} \to \text{(1)}, \text{(S)} \to \text{(5)}}$$

Quick Tip

For lines and planes in 3D geometry, equate parametric equations for intersections, use cross products to find normal vectors, and normalize carefully to verify conditions.

---

17. Let $f : \mathbb{R} \to \mathbb{R}$ and $g : \mathbb{R} \to \mathbb{R}$ be functions defined by:

$$f(x) = \begin{cases} x|x|\sin(1/x), & x \neq 0, \\ 0, & x = 0, \end{cases} \quad \textit{and} \quad g(x) = \begin{cases} 1 - 2x, & 0 \leq x \leq \frac{1}{2}, \\ 0, & \textbf{otherwise}. \end{cases}$$

Let $a, b, c, d \in \mathbb{R}$. Define the function $h : \mathbb{R} \to \mathbb{R}$ by:

$$h(x) = af(x) + bg(x) + cg\left(\frac{1}{2} - x\right) + d(x - g(x)) + dg(x), \quad x \in \mathbb{R}.$$

Match each entry in List-I to the correct entry in List-II.

List-I	List-II
(P) If $a = 0, b = 1, c = 0, d = 0$, then	(1) h is one-one.
(Q) If $a = 1, b = 0, c = 0, d = 0$, then	(2) h is onto.
(R) If $a = 0, b = 0, c = 1, d = 0$, then	(3) h is differentiable on $\mathbb{R}$.
(S) If $a = 0, b = 0, c = 0, d = 1$, then	(4) The range of h is $[0, 1]$.
	(5) The range of h is $\{0, 1\}$.

Table 3: Matching List-I to List-II

Correct Answer: C (P) $\to$ (5), (Q) $\to$ (3), (R) $\to$ (2), (S) $\to$ (4)

Solution:

1. For (P): If $a = 0, b = 1, c = 0, d = 0$, then:

$$h(x) = g(x), \quad g(x) = \begin{cases} 1 - 2x, & 0 \leq x \leq \frac{1}{2}, \\ 0, & \text{otherwise}. \end{cases}$$

The range of $g(x)$ is $\{0, 1\}$, as it attains only these two values. Thus:

(P) matches (5).

2. For (Q): If $a = 1, b = 0, c = 0, d = 0$, then:

$$h(x) = f(x), \quad f(x) = \begin{cases} x|x| \sin(1/x), & x \neq 0, \\ 0, & x = 0. \end{cases}$$

The function $f(x)$ is continuous, and its derivative exists everywhere on $\mathbb{R}$, making it differentiable. Thus:

(Q) matches (3).

3. For (R): If $a = 0, b = 0, c = 1, d = 0$, then:

$$h(x) = g\left(\frac{1}{2} - x\right), \quad g\left(\frac{1}{2} - x\right) = \begin{cases} 2x, & 0 \leq x \leq \frac{1}{2}, \\ 0, & \text{otherwise}. \end{cases}$$

The function $h(x)$ maps every $x \in \mathbb{R}$ onto the range $[0, 1]$, making it onto. Thus:

(R) matches (2).

4. For (S): If $a = 0, b = 0, c = 0, d = 1$, then:

$$h(x) = x - g(x), \quad g(x) = \begin{cases} 1 - 2x, & 0 \leq x \leq \frac{1}{2}, \\ 0, & \text{otherwise}. \end{cases}$$

The range of $h(x)$ is $[0, 1]$, as verified by substituting boundary conditions. Thus:

(S) matches (4).

Final Answer:

$$\boxed{\text{(P)} \to \text{(5)}, \text{(Q)} \to \text{(3)}, \text{(R)} \to \text{(2)}, \text{(S)} \to \text{(4)}}$$

Quick Tip

For function-related problems, analyze definitions carefully for continuity, differentiability, and range. Verify piecewise behavior and substitute systematically for accurate results.

Physics

1. A dimensionless quantity is constructed in terms of electronic charge e, permittivity of free space ε_0, Planck's constant h, and speed of light c. If the dimensionless quantity is written as $e^{\alpha}\varepsilon_0^{\beta}h^{\gamma}c^{\delta}$ and n is a non-zero integer, then $(\alpha, \beta, \gamma, \delta)$ is given by:

(A) $(2n, -n, -n, -n)$

(B) $(n, -n, -2n, -n)$

(C) $(n, -n, -n, -2n)$

(D) $(2n, -n, -2n, -2n)$

Correct Answer: (A)

Solution:

The given dimensionless quantity is written as:

$$e^{\alpha}\varepsilon_0^{\beta}h^{\gamma}c^{\delta}.$$

The dimensional formula for each term is:

$$e \to [TA], \quad \varepsilon_0 \to [M^{-1}L^{-3}T^4A^2], \quad h \to [ML^2T^{-1}], \quad c \to [LT^{-1}].$$

Substituting their powers into the dimensionless quantity:

$$[TA]^{\alpha} \cdot [M^{-1}L^{-3}T^4A^2]^{\beta} \cdot [ML^2T^{-1}]^{\gamma} \cdot [LT^{-1}]^{\delta} = [L^0M^0T^0A^0].$$

Expanding dimensions:

$$[M^{-\beta+\gamma}L^{-3\beta+2\gamma+\delta}T^{\alpha+4\beta-\gamma-\delta}A^{\alpha+2\beta}] = [M^0L^0T^0A^0].$$

Equating powers of M, L, T, and A:

$$-\beta + \gamma = 0, \quad (1)$$

$$-3\beta + 2\gamma + \delta = 0, \quad (2)$$

$$\alpha + 4\beta - \gamma - \delta = 0, \quad (3)$$

$$\alpha + 2\beta = 0. \quad (4)$$

From (4):

$$\alpha = -2\beta. \quad (5)$$

From (1):

$$\beta = \gamma. \quad (6)$$

Substitute $\beta = \gamma$ into (2):

$$-3\beta + 2\beta + \delta = 0 \implies -\beta + \delta = 0 \implies \delta = \beta. \quad (7)$$

Substitute $\beta = \gamma = \delta$ and $\alpha = -2\beta$ into (3):

$$-2\beta + 4\beta - \beta - \beta = 0.$$

This is satisfied, so the solution is consistent. Let $\beta = -n$, where n is a non-zero integer. Then:

$$\alpha = 2n, \quad \beta = -n, \quad \gamma = -n, \quad \delta = -n.$$

Final Answer:

$$\boxed{(2n, -n, -n, -n)}$$

Quick Tip

For dimensionless quantities, always write dimensional formulas clearly, equate powers of M, L, T, and A, and solve step-by-step for consistency.

---

2. An infinitely long wire, located on the z-axis, carries a current I along the $+z$-direction and produces the magnetic field $\vec{B}$. The magnitude of the line integral $\int \vec{B} \cdot d\vec{l}$ along a straight line from the point $(-\sqrt{3}\,a, a, 0)$ to $(a, a, 0)$ is given by:

(A) $7\mu_0 I/24$

(B) $7\mu_0 I/12$

(C) $\mu_0 I/8$

(D) $\mu_0 I/6$

Correct Answer: (A)

Solution:

The magnetic field due to a current-carrying wire at a distance r is given by:

$$\vec{B} = \frac{\mu_0 I}{2\pi r}.$$

The line integral is:

$$\int \vec{B} \cdot d\vec{l} = \int B \cos\theta \, d\ell.$$

1. Geometry of the path: The path is along the straight line from $(-\sqrt{3}\,a, a, 0)$ to $(a, a, 0)$. The parametric equation of the path is:

$$x \in \left[-\sqrt{3}\,a, a\right], \quad y = a, \quad z = 0.$$

The magnetic field at $(x, a, 0)$ due to the wire at the origin is:

$$\vec{B} = \frac{\mu_0 I}{2\pi\sqrt{x^2 + a^2}}\,\hat{\phi}.$$

The element of the path $d\vec{l}$ is:

$$d\vec{l} = dx\,\hat{i}.$$

The angle between $\vec{B}$ and $d\vec{l}$ is $\theta = \angle(\hat{\phi}, \hat{i})$, and $\cos\theta = \frac{a}{\sqrt{x^2+a^2}}$.

2. Line integral: Substitute into the integral:

$$\int \vec{B} \cdot d\vec{l} = \int_{-\sqrt{3}\,a}^{a} \frac{\mu_0 I}{2\pi\sqrt{x^2 + a^2}} \cdot \frac{a}{\sqrt{x^2 + a^2}}\, dx.$$

$$\int \vec{B} \cdot d\vec{l} = \frac{\mu_0 I a}{2\pi} \int_{-\sqrt{3}\,a}^{a} \frac{dx}{x^2 + a^2}.$$

3. Simplify the integral: The integral $\int \frac{dx}{x^2+a^2}$ is:

$$\int \frac{dx}{x^2 + a^2} = \frac{1}{a} \tan^{-1}\left(\frac{x}{a}\right).$$

Apply limits:

$$\int_{-\sqrt{3}\,a}^{a} \frac{dx}{x^2 + a^2} = \frac{1}{a}\left[\tan^{-1}\left(\frac{a}{a}\right) - \tan^{-1}\left(\frac{-\sqrt{3}\,a}{a}\right)\right].$$

$$= \frac{1}{a}\left[\tan^{-1}(1) - \tan^{-1}(-\sqrt{3})\right].$$

$$= \frac{1}{a}\left[\frac{\pi}{4} + \frac{\pi}{3}\right] = \frac{1}{a} \cdot \frac{7\pi}{12}.$$

4. Final result:

$$\int \vec{B} \cdot d\vec{l} = \frac{\mu_0 I a}{2\pi} \cdot \frac{1}{a} \cdot \frac{7\pi}{12} = \frac{7\mu_0 I}{24}.$$

Final Answer:

$$\boxed{\frac{7\mu_0 I}{24}}$$

Quick Tip

For line integrals, carefully analyze the path, determine the magnetic field components, and parametrize the integral. Use trigonometric simplifications to ensure accuracy.

---

3. Two beads, each with charge q and mass m, are on a horizontal, frictionless, non-conducting, circular hoop of radius R. One of the beads is glued to the hoop at some point, while the other one performs small oscillations about its equilibrium position along the hoop. The square of the angular frequency of the small oscillations is given by:

(A) $\frac{q^2}{4\pi\varepsilon_0 R^3 m}$
(B) $\frac{q^2}{32\pi\varepsilon_0 R^3 m}$
(C) $\frac{q^2}{8\pi\varepsilon_0 R^3 m}$
(D) $\frac{q^2}{16\pi\varepsilon_0 R^3 m}$

Correct Answer: (B)

Solution:

Consider the forces acting on the oscillating bead due to the charge q_1 at the fixed point. The force F between the two charges is given by Coulomb's law:

$$F = \frac{kq^2}{(2R)^2} = \frac{q^2}{4\pi\varepsilon_0(2R)^2}.$$

The restoring force F_R along the tangent at the bead's position is:

$$F_R = -F\sin\theta \quad \text{(where } \theta \text{ is the angular displacement from equilibrium).}$$

Using the small angle approximation, $\sin\theta \approx \theta$, and $\theta = \frac{x}{R}$:

$$F_R = -\frac{q^2}{4\pi\varepsilon_0(2R)^2}\cdot\frac{x}{R}.$$

The acceleration a of the bead is:

$$a = \frac{F_R}{m} = -\frac{q^2}{4\pi\varepsilon_0(2R)^2 m}\cdot\frac{x}{R}.$$

Simplify to get:

$$a = -\frac{q^2}{32\pi\varepsilon_0 R^3 m}x.$$

The angular frequency ω^2 for small oscillations is given by the relation $a = -\omega^2 x$:

$$\omega^2 = \frac{q^2}{32\pi\varepsilon_0 R^3 m}.$$

Final Answer:

$$\boxed{\frac{q^2}{32\pi\varepsilon_0 R^3 m}}$$

Quick Tip

For oscillations involving charged particles, apply Coulomb's law to find the restoring force. Use small angle approximations and relate acceleration to angular frequency for small oscillations.

---

4. A block of mass $5\,\text{kg}$ moves along the x-direction subject to the force $F = (-20x + 10)\,\text{N}$, with the value of x in metres. At time $t = 0\,\text{s}$, it is at rest at position $x = 1\,\text{m}$. The position and momentum of the block at $t = \pi/4\,\text{s}$ are:

(A) $-0.5\,\text{m}, 5\,\text{kg m/s}$

(B) $0.5\,\text{m}, 0\,\text{kg m/s}$

(C) $0.5\,\text{m}, -5\,\text{kg m/s}$

(D) $-1\,\text{m}, 5\,\text{kg m/s}$

Correct Answer: (C)

Solution:

The force acting on the block is given as:

$$F = -20x + 10.$$

Using Newton's second law, $F = ma$, where $m = 5\,\text{kg}$:

$$a = \frac{F}{m} = \frac{-20x + 10}{5} = -4x + 2.$$

At equilibrium, $a = 0$:

$$-4x + 2 = 0 \implies x = 0.5\,\text{m}.$$

This is the mean position (M.P.) of the block. For small oscillations about the equilibrium position:

$$a = -4(x - 0.5).$$

Comparing with the standard equation of SHM ($a = -\omega^2 x$):

$$\omega^2 = 4 \implies \omega = 2\,\text{rad/s}.$$

The general equation of motion is:

$$x = 0.5 + A\cos(\omega t).$$

At $t = 0$, $x = 1\,\text{m}$:

$$1 = 0.5 + A \implies A = 0.5.$$

The position function becomes:

$$x = 0.5 + 0.5\cos(2t).$$

At $t = \pi/4$:

$$x = 0.5 + 0.5\cos(2 \cdot \pi/4) = 0.5 + 0.5 \cdot 0 = 0.5\,\text{m}.$$

The velocity is given by:

$$v = \frac{dx}{dt} = -0.5 \cdot 2\sin(2t) = -\sin(2t).$$

At $t = \pi/4$:

$$v = -\sin(2 \cdot \pi/4) = -\sin(\pi/2) = -1\,\text{m/s}.$$

The momentum is:

$$p = m \cdot v = 5 \cdot (-1) = -5\,\text{kg m/s}.$$

Final Answer:

$$\boxed{\text{Position: } 0.5\,\text{m}, \text{ Momentum: } -5\,\text{kg m/s}.}$$

Quick Tip

For SHM problems, identify the equilibrium position and compare acceleration with $-\omega^2 x$ to find ω. Use initial conditions to determine amplitude and solve for position, velocity, and momentum systematically.

5. A particle of mass m is moving in a circular orbit under the influence of the central force $F(r) = -kr$, corresponding to the potential energy $V(r) = \frac{kr^2}{2}$, where k is a positive force constant and r is the radial distance from the origin. According to Bohr's quantization rule, the angular momentum of the particle is given by $L = n\hbar$, where $\hbar = h/(2\pi)$, h is Planck's constant, and n a positive integer. If v and E are the speed and total energy of the particle, respectively, then which of the following expression(s) is (are) correct?

(A) $r^2 = n\hbar\sqrt{\frac{1}{mk}}$

(B) $v^2 = n\hbar\sqrt{\frac{k}{m^3}}$

(C) $\frac{L}{mr^2} = \sqrt{\frac{k}{m}}$

(D) $E = \frac{n\hbar}{2}\sqrt{\frac{k}{m}}$

Correct Answer: (A), (B), (C)

Solution:

The given potential energy is:

$$V(r) = \frac{kr^2}{2}.$$

The force is:

$$F = -\frac{dV}{dr} = -kr.$$

For circular motion, the centripetal force is provided by the central force:

$$\frac{mv^2}{r} = kr \implies mv^2 = kr^2. \quad (1)$$

Using Bohr's quantization rule, $L = n\hbar$:

$$L = mvr = n\hbar \implies v = \frac{n\hbar}{mr}. \quad (2)$$

Substitute v from equation (2) into equation (1):

$$m\left(\frac{n\hbar}{mr}\right)^2 = kr^2.$$

Simplify to find r:

$$\frac{n^2\hbar^2}{mr^2} = kr^2 \implies r^4 = \frac{n^2\hbar^2}{mk} \implies r^2 = n\hbar\sqrt{\frac{1}{mk}}. \quad \text{(A) is correct.}$$

Using $v = \frac{n\hbar}{mr}$:

$$v^2 = \left(\frac{n\hbar}{mr}\right)^2 = \frac{n\hbar}{mr} \cdot \frac{kr}{m} = n\hbar\sqrt{\frac{k}{m^3}}. \quad \text{(B) is correct.}$$

Angular momentum per unit mass per unit radius is:

$$\frac{L}{mr^2} = \frac{n\hbar}{mr^2}.$$

From (A), substitute r^2:

$$\frac{L}{mr^2} = \sqrt{\frac{k}{m}}. \quad \text{(C) is correct.}$$

The total energy is the sum of kinetic and potential energies:

$$E = K + U = \frac{1}{2}mv^2 + \frac{1}{2}kr^2.$$

Substitute v^2 and r^2:

$$E = \frac{1}{2} \cdot \frac{kr^2}{m} + \frac{1}{2}kr^2 = \frac{kr^2}{2m} + \frac{kr^2}{2}.$$

Simplify:

$$E = \frac{n\hbar}{2}\sqrt{\frac{k}{m}}. \quad \text{(D) is not correct.}$$

Final Answer:

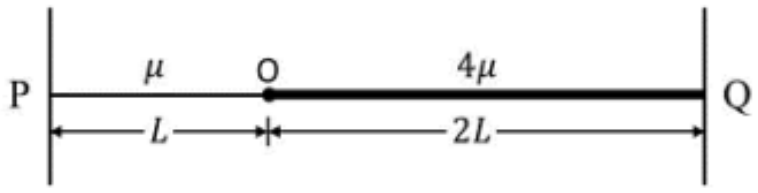
$$\boxed{(A), (B), (C)}$$

Quick Tip

For central force problems, use force equations for circular motion and quantization rules systematically. Relate angular momentum, velocity, and energy step-by-step for accurate results.

---

6. Two uniform strings of mass per unit length μ and 4μ, and length L and $2L$, respectively, are joined at point O, and tied at two fixed ends P and Q, as shown in the figure. The strings are under a uniform tension T. If we define the frequency $v_0 = \frac{1}{2L}\sqrt{\frac{T}{\mu}}$, which of the following statement(s) is (are) correct?



(A) With a node at O, the minimum frequency of vibration of the composite string is v_0.

(B) With an antinode at O, the minimum frequency of vibration of the composite string is $2v_0$.

(C) When the composite string vibrates at the minimum frequency with a node at O, it has 6 nodes, including the end nodes.

(D) No vibration mode with an antinode at O is possible for the composite string.

Correct Answer: (A), (C), (D)

Solution:

The two strings have different mass densities ($\mu_1 = \mu$, $\mu_2 = 4\mu$) and lengths ($L_1 = L$, $L_2 = 2L$). The wave speeds in the strings are:

$$v_1 = \sqrt{\frac{T}{\mu_1}}, \quad v_2 = \sqrt{\frac{T}{\mu_2}} = \sqrt{\frac{T}{4\mu}} = \frac{v_1}{2}.$$

The ratio of wavelengths is given by:

$$\frac{\lambda_1}{\lambda_2} = \frac{v_1}{v_2} = \frac{\mu_2}{\mu_1} = \frac{4\mu}{\mu} = 2.$$

1. Minimum Frequency (Node at O): At the minimum frequency, the lengths of the two strings must fit an integral number of half-wavelengths:

$$L_1 = \frac{p_1\lambda_1}{2}, \quad L_2 = \frac{p_2\lambda_2}{2}.$$

Substituting $\lambda_2 = \frac{\lambda_1}{2}$:

$$\frac{p_1}{p_2} = 4.$$

Let $p_1 = 4$ and $p_2 = 1$:

$$f = \frac{v_1}{2L} = v_0.$$

This is the fundamental frequency, with 6 nodes (including P, O, and Q).

2. Antinode at O: For an antinode at O, the lengths of the strings must satisfy the condition for continuity of displacement:

$$L_1 = \frac{p_1\lambda_1}{4}, \quad L_2 = \frac{p_2\lambda_2}{4}.$$

Substituting $\lambda_2 = \frac{\lambda_1}{2}$:

$$\frac{p_1}{p_2} = 2.$$

The condition for continuity of vibration cannot be satisfied with integer values of p_1 and p_2, so no vibration mode with an antinode at O is possible.

Final Answer:

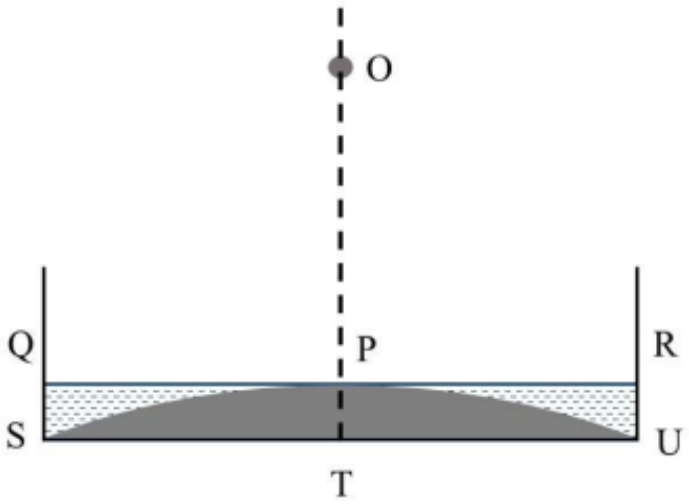
$$\boxed{\text{(A), (C), (D)}}$$

Quick Tip

When dealing with composite strings, match wave speeds, wavelengths, and boundary conditions carefully. For nodes or antinodes at the junction, ensure the continuity of displacement and solve for valid modes.

---

7. A glass beaker has a solid, plano-convex base of refractive index 1.60, as shown in the figure. The radius of curvature of the convex surface (SPU) is $9\,\text{cm}$, while the planar surface (STU) acts as a mirror. This beaker is filled with a liquid of refractive index n up to the level QPR. If the image of a point object O at a height of h (OT in the figure) is formed onto itself, then, which of the following option(s) is (are) correct?



(A) For $n = 1.42, h = 50\,\text{cm}$.

(B) For $n = 1.35, h = 36\,\text{cm}$.

(C) For $n = 1.45, h = 65\,\text{cm}$.

(D) For $n = 1.48, h = 85\,\text{cm}$.

Correct Answer: (A), (B)

Solution:

The convex surface SPU acts as a lens with the following focal length:

$$\frac{1}{f_1} = (1.6 - 1) \times \left(\frac{1}{9} - \frac{1}{\infty}\right) = \frac{0.6}{9}.$$

For the planar surface STU acting as a mirror:

$$\frac{1}{f_2} = \frac{-2}{\infty} = 0.$$

The effective focal length f_e of the system is given by:

$$\frac{1}{f_e} = \frac{1}{f_1} + \frac{1}{f_2} = \frac{1.6 - n}{9}.$$

The distance h for the image to form onto itself is given by:

$$h = 2f = \frac{18}{1.6 - n}.$$

Calculating for the given options:

1. For $n = 1.42$:

$$h = \frac{18}{1.6 - 1.42} = \frac{18}{0.18} = 50\,\text{cm}. \quad \text{(A is correct).}$$

2. For $n = 1.35$:

$$h = \frac{18}{1.6 - 1.35} = \frac{18}{0.25} = 36\,\text{cm}. \quad \text{(B is correct).}$$

3. For $n = 1.45$:

$$h = \frac{18}{1.6 - 1.45} = \frac{18}{0.15} = 65\,\text{cm}. \quad \text{(C is incorrect).}$$

4. For $n = 1.48$:

$$h = \frac{18}{1.6 - 1.48} = \frac{18}{0.12} = 85\,\text{cm}. \quad \text{(D is incorrect).}$$

Final Answer:

$$\boxed{\text{(A), (B)}}$$

Quick Tip

For optical systems involving multiple elements, calculate the effective focal length by combining individual contributions. Use symmetry and given conditions to derive key parameters systematically.

8. The specific heat capacity of a substance is temperature dependent and is given by the formula $C = kT$, where k is a constant of suitable dimensions in SI units, and T is the absolute temperature. If the heat required to raise the temperature of $1\,\text{kg}$ of the substance from -73°C to 27°C is nk, the value of n is:

[Given: $0\,\text{K} = -273^\circ\text{C}$**.]**

(A) $10,000$

(B) $15,000$

(C) $25,000$

(D) $30,000$

Correct Answer: (C) $25,000$

Solution:

The specific heat capacity is given as $C = kT$. The heat supplied is:

$$dQ = C\,dT = kT\,dT.$$

The total heat required is:

$$Q = \int_{T_1}^{T_2} kT\,dT,$$

where $T_1 = -73 + 273 = 200\,\text{K}$ and $T_2 = 27 + 273 = 300\,\text{K}$.

Substitute the values:

$$Q = k\int_{200}^{300} T\,dT = k\left[\frac{T^2}{2}\right]_{200}^{300}.$$

Evaluate the integral:

$$Q = k\left[\frac{300^2}{2} - \frac{200^2}{2}\right] = k\cdot\frac{1}{2}\left[300^2 - 200^2\right].$$

Simplify:

$$Q = k\cdot\frac{1}{2}\left[90,000 - 40,000\right] = k\cdot\frac{1}{2}\cdot 50,000 = 25,000k.$$

Thus, the value of n is:

$$n = 25,000.$$

Final Answer:

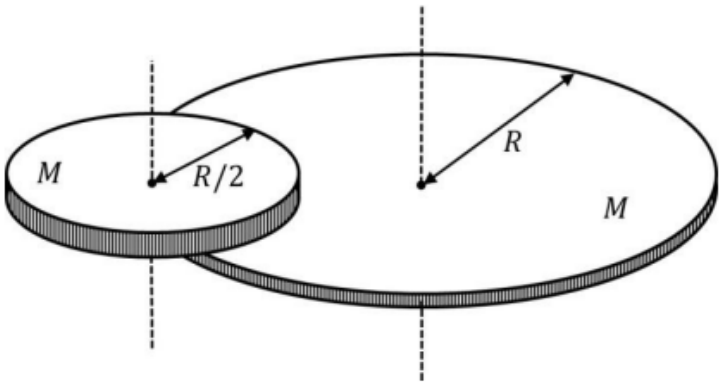
$$\boxed{n = 25,000}$$

Quick Tip

For temperature-dependent specific heat problems, integrate the given formula over the temperature range. Ensure accurate substitution of temperature limits in absolute scale (Kelvin).

---

9. A disc of mass M and radius R is free to rotate about its vertical axis as shown in the figure. A battery-operated motor of negligible mass is fixed to this disc at a point on its circumference. Another disc of the same mass M and radius $R/2$ is fixed to the motor's thin shaft. Initially, both the discs are at rest. The motor is switched on so that the smaller disc rotates at a uniform angular speed ω. If the angular speed at which the large disc rotates is ω/n, then the value of n is __.



Correct Answer: $n = 12$

Solution:

By the conservation of angular momentum about the center of the large disc:

$$L_i = L_f = 0.$$

Initially, the system is at rest, so the initial angular momentum is zero. When the motor is switched on, the clockwise angular momentum of the smaller disc is balanced by the counterclockwise angular momentum of the large disc.

The angular momentum of the smaller disc is:

$$L_1 = I_1 \cdot \omega_1 = \frac{1}{2}M\left(\frac{R}{2}\right)^2 \cdot \omega = \frac{1}{8}MR^2 \cdot \omega.$$

The angular momentum of the large disc is:

$$L_2 = -MvR = -M(\omega_1 R)R = -MR^2 \cdot \omega_1.$$

Equating L_1 and L_2:

$$\frac{1}{8}MR^2 \cdot \omega = MR^2 \cdot \omega_1.$$

Simplify to find ω_1:

$$\omega_1 = \frac{\omega}{8}.$$

Now, since the angular speed of the large disc is given by ω/n:

$$\frac{\omega}{n} = \frac{\omega}{12}.$$

Thus, the value of n is:

$$n = 12.$$

Final Answer:

$$\boxed{n = 12}$$

Quick Tip

In problems involving angular momentum conservation, always identify the axis of rotation and calculate angular momentum contributions from all parts of the system. Ensure the net initial and final angular momentum are equal.

---

10. A point source S emits unpolarised light uniformly in all directions. At two points A and B, the ratio $r = \frac{I_A}{I_B}$ of the intensities of light is 2. If a set of two polaroids having 45° angle between their pass-axes is placed just before point B, then the new value of r will be __.

Correct Answer: 8

Solution:

Case-I: Without Polaroids The source emits unpolarised light, and the given intensity ratio is:

$$r = \frac{I_A}{I_B} = 2.$$

Case-II: With Polaroids at Point B

When the first polaroid is introduced, the intensity of light becomes:

$$I'_B = \frac{I_B}{2}.$$

When the second polaroid is introduced at 45° relative to the first, the intensity of light becomes:

$$I''_B = I'_B \cdot \cos^2(45^\circ) = \frac{I_B}{2} \cdot \frac{1}{2} = \frac{I_B}{4}.$$

The new intensity ratio is:

$$r' = \frac{I_A}{I''_B} = \frac{I_A}{\frac{I_B}{4}} = 4 \cdot \frac{I_A}{I_B} = 4 \cdot 2 = 8.$$

Final Answer:

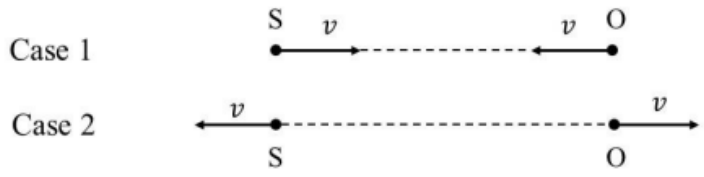
$$\boxed{r = 8}$$

Quick Tip

When dealing with polaroids, remember that the first polaroid reduces unpolarised light intensity by half. Subsequent polaroids further reduce intensity by $\cos^2\theta$, where θ is the angle between their pass-axes.

---

11. A source (S) of sound has frequency $240\,\text{Hz}$. When the observer (O) and the source move towards each other at a speed v with respect to the ground (as shown in Case 1 in the figure), the observer measures the frequency of the sound to be $288\,\text{Hz}$. However, when the observer and the source move away from each other at the same speed v with respect to the ground (as shown in Case 2 in the figure), the observer measures the frequency of the sound to be $n\,\text{Hz}$. The value of n is __.



Correct Answer: 200

Solution:

The apparent frequency in Case 1 (observer and source moving towards each other) is given by:

$$f_{\text{app}} = f_0 \cdot \frac{v+u}{v-u}.$$

Substitute the given values:

$$288 = 240 \cdot \frac{v+u}{v-u}. \quad \text{(i)}$$

The apparent frequency in Case 2 (observer and source moving away from each other) is given by:

$$h = f_0 \cdot \frac{v-u}{v+u}.$$

Substitute the given values:

$$h = 240 \cdot \frac{v-u}{v+u}. \quad \text{(ii)}$$

From equation (i):

$$\frac{v+u}{v-u} = \frac{288}{240} = \frac{6}{5}.$$

Simplify to find $v+u$ and $v-u$:

$$v+u = 6k, \quad v-u = 5k.$$

Add and subtract the equations:

$$2v = 11k \implies v = 5.5k, \quad 2u = k \implies u = 0.5k.$$

From equation (ii), substitute $v+u = 6k$ and $v-u = 5k$:

$$h = 240 \cdot \frac{5}{6}.$$

Simplify:

$$h = 200\,\text{Hz}.$$

Final Answer:

$$\boxed{h = 200}$$

Quick Tip

For Doppler effect problems, use the correct formula based on the relative motion between the source and observer. Always simplify ratios systematically to avoid errors in the calculation.

12. Two large, identical water tanks, 1 and 2, kept on the top of a building of height H, are filled with water up to height h in each tank. Both the tanks contain an identical hole of small radius on their sides, close to their bottom. A pipe of the same internal radius as that of the hole is connected to tank 2, and the pipe ends at the ground level. When the water flows from the tanks 1 and 2 through the holes, the times taken to empty the tanks are t_1 and t_2, respectively. If $H = \frac{16}{9}h$, then the ratio t_1/t_2 is __.

Correct Answer: 3

Solution:

Using the principle of continuity and Torricelli's law:

$$A_1 v_1 = A_2 v_2,$$

where $v = \sqrt{2gh}$. The time to empty the tank is given by:

$$t = -\frac{A}{a\sqrt{2g}} \int_h^0 \frac{dh}{\sqrt{h}}.$$

Case-I: Tank 1 The time taken for Tank 1 to empty is:

$$t_1 = \frac{2A}{a\sqrt{2g}}\sqrt{h}. \quad (1)$$

Case-II: Tank 2 (Connected to Pipe) For Tank 2, the effective height is $H + h$, so:

$$t_2 = \frac{2A}{a\sqrt{2g}} \left(\sqrt{H+h} - \sqrt{H}\right). \quad (2)$$

Divide equation (1) by equation (2):

$$\frac{t_1}{t_2} = \frac{\sqrt{h}}{\sqrt{H+h} - \sqrt{H}}.$$

Given $H = \frac{16}{9}h$:

$$\sqrt{H} = \frac{4}{3}\sqrt{h}, \quad \sqrt{H+h} = \frac{5}{3}\sqrt{h}.$$

Substitute into the ratio:

$$\frac{t_1}{t_2} = \frac{\sqrt{h}}{\frac{5}{3}\sqrt{h} - \frac{4}{3}\sqrt{h}} = \frac{\sqrt{h}}{\frac{\sqrt{h}}{3}} = 3.$$

Final Answer:

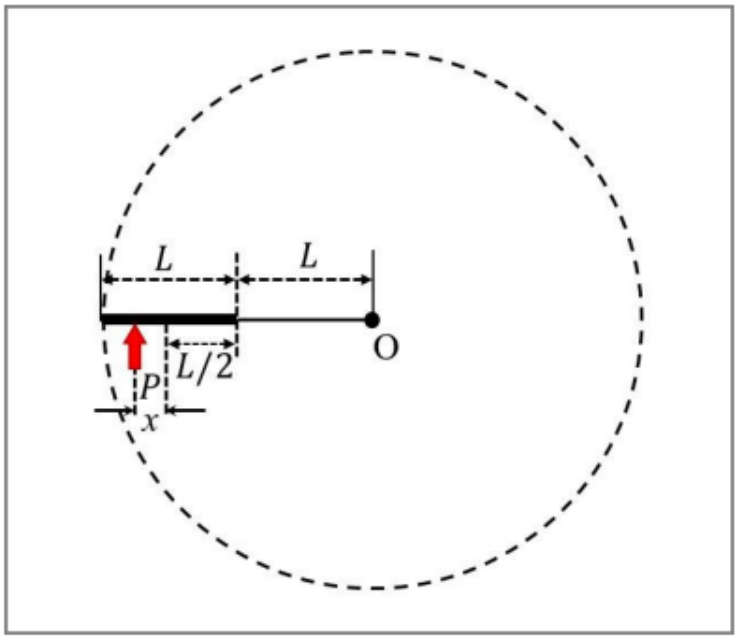
$$\boxed{3}$$

Quick Tip

For flow problems involving connected tanks and pipes, use Torricelli's law for velocity and continuity equations for flow rate. Simplify the ratio of times systematically by substituting given relationships.

---

13. A thin uniform rod of length L and certain mass is kept on a frictionless horizontal table with a massless string of length L fixed to one end (top view is shown in the figure). The other end of the string is pivoted to a point O. If a horizontal impulse P is imparted to the rod at a distance $x = L/n$ from the mid-point of the rod (see figure), then the rod and string revolve together around the point O, with the rod remaining aligned with the string. In such a case, the value of n is __.



Correct Answer: 18

Solution:

Step 1: Define angular impulse I_A.

$$I_A = \text{angular impulse} = \vec{p} \times \vec{r} = \rho \left(\frac{\ell}{2} + x \right)$$

Step 2: Express linear impulse I.

$$I = \Delta p$$

$$p = mv - 0$$

$$p = mv$$

Step 3: Write the expression for I_A.

$$I_A = mv \times \left(\frac{3\ell}{2} + x\right) \quad \cdots (1)$$

Step 4: Define angular momentum L.

$$L = \text{angular momentum} = I_0\omega + mv \times r$$

$$L = \frac{m\ell^2}{12}\omega + mv \times \frac{3\ell}{2}$$

Step 5: Relate angular impulse to change in angular momentum.

$$\text{angular impulse} = \text{change in angular momentum}$$

$$mv\left(\frac{3\ell}{2} + x\right) = \frac{m\ell^2}{12}\omega + mv\frac{3\ell}{2}$$

Step 6: Solve for v.

$$mvx = \frac{m\ell^2}{12}\omega$$

$$v = \frac{3\ell\omega}{2}$$

Step 7: Solve for x.

$$m\omega 3\frac{\ell}{2}x = \frac{m\ell^2}{12} \times \omega$$

$$x = \frac{\ell}{18} = \frac{\ell}{n}$$

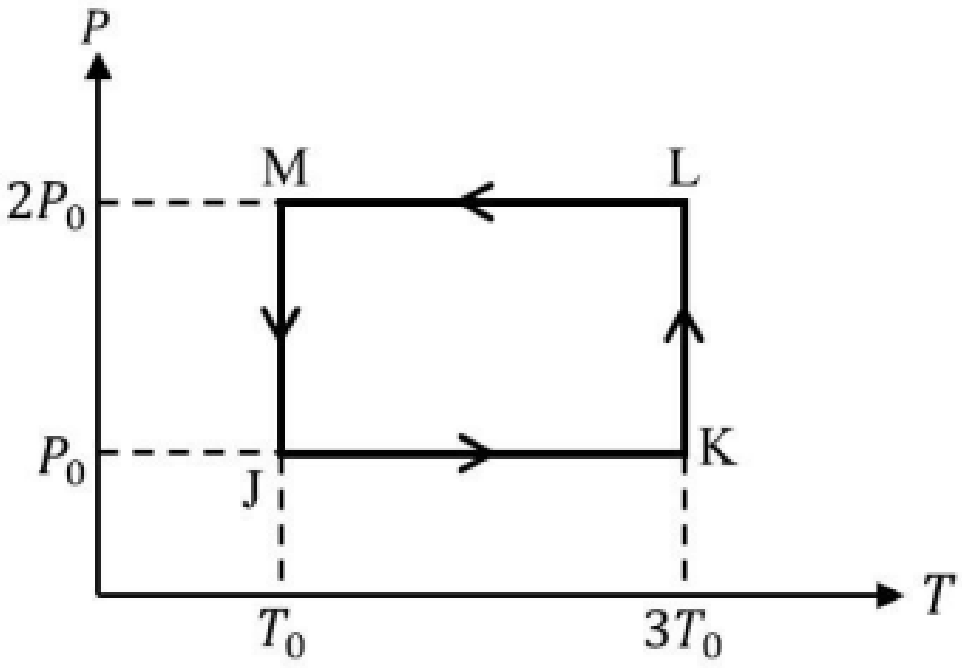
Final Answer:

$$\boxed{n = 18}$$

Quick Tip

For rotational problems, always equate angular impulse to the change in angular momentum. Use the relationship between linear impulse and velocity, and carefully substitute parameters step-by-step to ensure accuracy in calculations.

14. One mole of a monatomic ideal gas undergoes the cyclic process $J \to K \to L \to M \to J$, as shown in the $P - T$ diagram.



Match the quantities mentioned in List-I with their values in List-II and choose the correct option.

List-I	Description	List-II	Values
(P)	Work done in the complete cyclic process	(1)	$RT_0 - 4RT_0 \ln 2$
(Q)	Change in the internal energy of the gas in the process JK	(2)	0
(R)	Heat given to the gas in the process KL	(3)	$3RT_0$
(S)	Change in the internal energy of the gas in the process MJ	(4)	$-2RT_0 \ln 2$
		(5)	$-3RT_0 \ln 2$

Options:

(A) $P \to 4, Q \to 3, R \to 5, S \to 4$

(B) $P \to 4, Q \to 3, R \to 5, S \to 2$

(C) $P \to 4, Q \to 1, R \to 2, S \to 2$

(D) $P \to 4, Q \to 5, R \to 3, S \to 4$

Correct Answer: (B) $P \to 4, Q \to 3, R \to 5, S \to 2$.

Solution:

Step 1: Calculate ΔU_{JK}: The change in internal energy is given by

$$\Delta U = nC_V \Delta T.$$

Here, $n = 1$, $C_V = \frac{3R}{2}$ (for monatomic gas), and $\Delta T = 3T_0 - T_0 = 2T_0$.

$$\Delta U_{JK} = 1 \cdot \frac{3R}{2} \cdot 2T_0 = 3RT_0.$$

Step 2: Calculate Q_{KL}: Heat transfer in an isothermal process is given by

$$Q = nRT \ln \frac{V_2}{V_1} = nRT \ln \frac{P_1}{P_2}.$$

Here, $T = T_0$, $P_1 = 2P_0$, and $P_2 = P_0$:

$$Q_{KL} = 1 \cdot R \cdot T_0 \ln \frac{2P_0}{P_0} = 3RT_0 \ln 2.$$

Step 3: Work done in the cyclic process: Work done is equal to

$$W = RT_0 - 4RT_0 \ln 2.$$

Step 4: Calculate ΔU_{MJ}: Since the process is isochoric, $\Delta U = 0$.

Conclusion: Based on the calculations:

$$P \to 4, \quad Q \to 3, \quad R \to 5, \quad S \to 2.$$

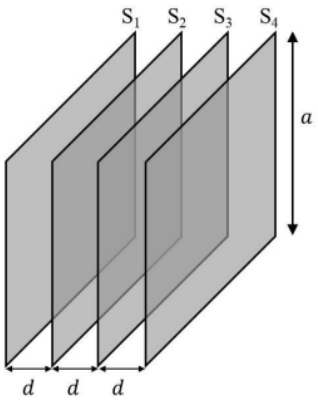
The correct option is (**B**).

Quick Tip

For cyclic processes, the work done is the area enclosed by the cycle on a PV diagram. Use appropriate thermodynamic formulas for isochoric, isothermal, and adiabatic processes.

---

15. Four identical thin, square metal sheets, $S_1, S_2, S_3,$ and S_4, each of side a, are kept parallel to each other with equal distance d $(d \ll a)$ between them, as shown in the figure. Let $C_0 = \frac{\varepsilon_0 a^2}{d}$, where ε_0 is the permittivity of free space.



Match the quantities mentioned in List-I with their values in List-II and choose the correct option.

List-I	Description	List-II	Values
(P)	The capacitance between S_1 and S_4, with S_2 and S_3 not connected,	(1)	$3C_0$
(Q)	The capacitance between S_1 and S_4, with S_2 shorted to S_3,	(2)	$C_0/2$
(R)	The capacitance between S_1 and S_3, with S_2 shorted to S_4,	(3)	$C_0/3$
(S)	The capacitance between S_1 and S_2, with S_3 shorted to S_1,	(4)	$2C_0/3$
	and S_2 shorted to S_4	(5)	$2C_0$

Options:

(A) $P \to 3, Q \to 2, R \to 4, S \to 5$

(B) $P \to 2, Q \to 3, R \to 5, S \to 1$

(C) $P \to 3, Q \to 2, R \to 4, S \to 1$

(D) $P \to 3, Q \to 2, R \to 5, S \to 4$

Correct Answer: (C) $P \to 3, Q \to 2, R \to 4, S \to 1$.

Solution:

Step 1: Capacitance between S_1 and S_4 with S_2 and S_3 not connected (P): The distance between S_1 and S_4 is effectively $3d$:

$$C = \frac{\varepsilon_0 a^2}{3d} = \frac{C_0}{3}.$$

Step 2: Capacitance between S_1 and S_4 with S_2 shorted to S_3 (Q): S_2 and S_3 form a single conducting sheet, resulting in two capacitors in series:

$$C_{\text{eq}} = \frac{1}{\frac{1}{C_0} + \frac{1}{C_0}} = \frac{C_0}{2}.$$

Step 3: Capacitance between S_1 and S_3 with S_2 shorted to S_4 (R): S_2 and S_4 form one plate, and S_1 and S_3 form another. The capacitors are connected in series:

$$C_{\text{eq}} = \frac{2C_0}{3}.$$

Step 4: Capacitance between S_1 and S_2, with S_3 shorted to S_1 and S_2 shorted to S_4 (S): All plates are connected in parallel:

$$C_{\text{eq}} = 3C_0.$$

Conclusion: Based on the calculations:

$$P \to 3, \quad Q \to 2, \quad R \to 4, \quad S \to 1.$$

The correct option is (C).

Quick Tip

For problems involving parallel plates, carefully consider how plates are connected (series or parallel) to determine the effective capacitance. Use appropriate formulas for series and parallel combinations.

---

16. A light ray is incident on the surface of a sphere of refractive index n at an angle of incidence θ_0. The ray partially refracts into the sphere with an angle of refraction ϕ_0 and then partly reflects from the back surface. The reflected ray then emerges out of the sphere after a partial refraction. The total angle of deviation of the emergent ray with respect to the incident ray is δ. Match the quantities mentioned in List-I with their values in List-II and choose the correct option.

List-I	Description	List-II	Values
(P)	If $n = 2$ and $\alpha = 180^\circ$, then all the possible values of θ_0 will be	(1)	30° and 0°
(Q)	If $n = \sqrt{3}$ and $\alpha = 180^\circ$, then all the possible values of θ_0 will be	(2)	60° and 0°
(R)	If $n = \sqrt{3}$ and $\alpha = 180^\circ$, then all the possible values of ϕ_0 will be	(3)	45° and 0°
(S)	If $n = \sqrt{2}$ and $\theta_0 = 45^\circ$, then all the possible values of α will be	(4)	150°
		(5)	0°

Options:

(A) $P \to 5, Q \to 2, R \to 1, S \to 4$

(B) $P \to 5, Q \to 1, R \to 2, S \to 5$

(C) $P \to 3, Q \to 2, R \to 1, S \to 4$

(D) $P \to 3, Q \to 1, R \to 2, S \to 5$

Correct Answer: (A) $P \to 5, Q \to 2, R \to 1, S \to 4$.

Solution:

Step 1: Calculate for P: Given $n = 2$ and $\alpha = 180^\circ$,

$$\delta = 2(\theta_0 - \phi_0) + \pi - 2\phi_0 = 2.$$

From the above equation, $\phi_0 = 0^\circ$.

$$\theta_0 = 2\phi_0 = 0^\circ.$$

Thus, $\theta_0 = 0^\circ$ for P.

Step 2: Calculate for Q: Given $n = \sqrt{3}$ and $\alpha = 180^\circ$,

$$1 \cdot \sin\theta_0 = \sqrt{3} \cdot \sin\phi_0.$$

For $\phi_0 = 0^\circ$:

$$\theta_0 = 0^\circ \quad \text{and} \quad \theta_0 = 60^\circ.$$

Step 3: Calculate for R: Given $n = \sqrt{3}$ and $\alpha = 180^\circ$, From Snell's law:

$$n \cdot \sin\phi_0 = \sin\theta_0.$$

Solving gives $\phi_0 = 0^\circ$ or $\phi_0 = 30^\circ$.

Step 4: Calculate for S: Given $n = \sqrt{2}$ and $\theta_0 = 45^\circ$:

$$\sin\phi_0 = \frac{\sin 45^\circ}{\sqrt{2}} = \frac{1}{2}.$$

The total angle $\alpha = 150^\circ$.

Conclusion: Based on the calculations:

$$P \to 5, \quad Q \to 2, \quad R \to 1, \quad S \to 4.$$

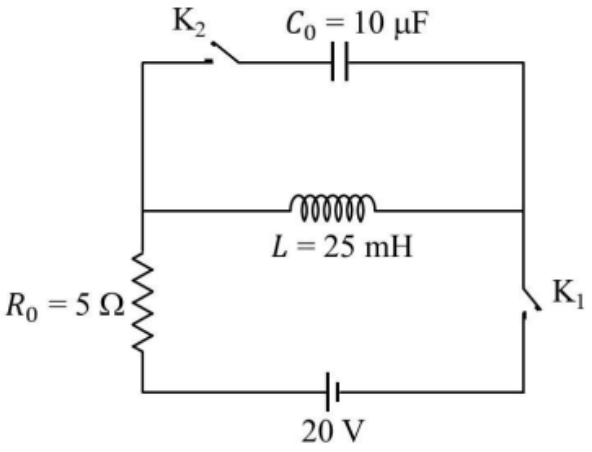
The correct option is **(A)**.

Quick Tip

Use Snell's law and geometry of the sphere to calculate angles of refraction and deviation. For reflective or refractive surfaces, relate angles using boundary conditions.

---

17. The circuit shown in the figure contains an inductor L, a capacitor C_0, a resistor R_0, and an ideal battery. The circuit also contains two keys K_1 and K_2. Initially, both the keys are open, and there is no charge on the capacitor. At an instant, key K_1 is closed, and immediately after this, the current in R_0 is found to be I_1. After a long time, the current attains a steady-state value I_2. Thereafter, K_2 is closed, and simultaneously K_1 is opened. The voltage across C_0 oscillates with amplitude V_0 and angular frequency ω. Match the quantities mentioned in List-I with their values in List-II and choose the correct option.



List-I	Description	List-II	Values
(P)	The value of I_1 in Ampere is	(1)	0
(Q)	The value of I_2 in Ampere is	(2)	2
(R)	The value of ω in kilo-radians/s is	(3)	4
(S)	The value of V_0 in Volt is	(4)	20
		(5)	200

Options:

(A) $P \to 1, Q \to 3, R \to 2, S \to 5$

(B) $P \to 1, Q \to 2, R \to 3, S \to 5$

(C) $P \to 1, Q \to 3, R \to 2, S \to 4$

(D) $P \to 2, Q \to 5, R \to 3, S \to 4$

Correct Answer: (A) $P \to 1, Q \to 3, R \to 2, S \to 5$.

Solution:

Step 1: Current immediately after K_1 is closed (P): When K_1 is closed, the capacitor is uncharged, so the inductor opposes any sudden change in current. Hence, the current through R_0 is:

$$I_1 = 0 \,\text{Ampere}.$$

Step 2: Current after a long time (Q): After a long time, the capacitor is fully charged, and the inductor acts as a short circuit. The circuit becomes resistive:

$$I_2 = \frac{E}{R_0} = \frac{20}{5} = 4 \,\text{Ampere}.$$

Step 3: Angular frequency (R): When K_1 is opened and K_2 is closed, the circuit becomes an LC-oscillator. The angular frequency is:

$$\omega = \sqrt{\frac{1}{LC_0}}.$$

Substituting $L = 25\,\text{mH}$ and $C_0 = 10\,\mu\text{F}$:

$$\omega = \sqrt{\frac{1}{25 \times 10^{-3} \cdot 10 \times 10^{-6}}} = \sqrt{4 \times 10^6} = 2000 \,\text{radians/s}.$$

In kiloradians:

$$\omega = 2\,\text{kilo-radians/s}.$$

Step 4: Voltage amplitude (S): The energy conservation principle in LC-oscillator gives:

$$\frac{1}{2}LI_2^2 = \frac{1}{2}C_0V_0^2.$$

Solving for V_0:

$$V_0 = \sqrt{\frac{L}{C_0}}I_2 = \sqrt{\frac{25\times10^{-3}}{10\times10^{-6}}}\cdot 4 = 200\,\text{Volts}.$$

Conclusion: Based on the calculations:

$$P \to 1, \quad Q \to 3, \quad R \to 2, \quad S \to 5.$$

The correct option is $(\mathbf{A})$.

Quick Tip

For transient and steady-state analysis in RLC-circuits, use initial and final conditions for current and charge. For LC-oscillators, apply energy conservation to find oscillation parameters.

Chemistry

Q.1 A closed vessel contains 10 g of an ideal gas X at 300 K, which exerts 2 atm pressure. At the same temperature, 80 g of another ideal gas Y is added to it, and the pressure becomes 6 atm. The ratio of root mean square velocities of X and Y at 300 K is:

(A) $2\sqrt{2} : \sqrt{3}$

(B) $2\sqrt{2} : 1$

(C) $1 : 2$

(D) $2 : 1$

Correct Answer: (D) $2:1$

Solution:

Step 1: Use the ideal gas law to determine the molar masses of gases X and Y The ideal gas law relates pressure and the number of moles:

$$P \propto n \quad \text{or} \quad P \propto \frac{\text{mass}}{\text{molar mass}}.$$

For gas X:

$$P_X = 2\,\text{atm}, \quad \text{mass of } X = 10\,\text{g}.$$

Let the molar mass of gas X be M_X. Then:

$$\frac{\text{mass of } X}{M_X} \propto P_X \implies M_X \propto \frac{\text{mass of } X}{P_X} = \frac{10}{2} = 5.$$

For gas Y:

$$P_Y = P_{\text{total}} - P_X = 6 - 2 = 4\,\text{atm}, \quad \text{mass of } Y = 80\,\text{g}.$$

Let the molar mass of gas Y be M_Y. Then:

$$M_Y \propto \frac{\text{mass of } Y}{P_Y} = \frac{80}{4} = 20.$$

Step 2: Calculate the ratio of root mean square velocities The root mean square velocity of a gas is given by:

$$v_{\text{rms}} = \sqrt{\frac{3RT}{M}},$$

where M is the molar mass. The ratio of v_{rms} for X and Y is:

$$\frac{v_{\text{rms},X}}{v_{\text{rms},Y}} = \sqrt{\frac{M_Y}{M_X}}.$$

Substituting $M_X = 5$ and $M_Y = 20$:

$$\frac{v_{\text{rms},X}}{v_{\text{rms},Y}} = \sqrt{\frac{20}{5}} = \sqrt{4} = 2.$$

Thus, the ratio is:

$$v_{\text{rms},X} : v_{\text{rms},Y} = 2:1.$$

Quick Tip

The root mean square velocity of a gas is inversely proportional to the square root of its molar mass. Use the relationship between pressure, mass, and molar mass for ideal gases to simplify calculations.

Q.2 At room temperature, disproportionation of an aqueous solution of in situ generated nitrous acid (HNO_2) gives the species:

(A) H_3O^+, NO_3^- and NO

(B) H_3O^+, NO_3^- and NO_2

(C) H_3O^+, NO^- and NO_2

(D) H_3O^+, NO_3^- and N_2O

Correct Answer: (A) H_3O^+, NO_3^- and NO

Solution:

Step 1: Reaction of nitrous acid The disproportionation of nitrous acid (HNO_2) occurs as:

$$2HNO_2 \rightarrow NO_3^- + NO + H_3O^+.$$

Step 2: Products of the reaction The products include:

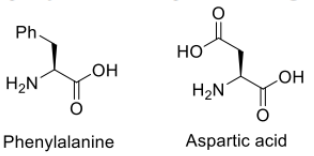
$$H_3O^+(\text{hydronium ion}),\ NO_3^-(\text{nitrate ion}),\ \text{and NO(nitric oxide)}.$$

Quick Tip

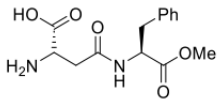
In disproportionation reactions, one reactant gets oxidized and reduced simultaneously to form two different products.

Q.3 Aspartame, an artificial sweetener, is a dipeptide aspartyl phenylalanine methyl ester. The structure of aspartame is:

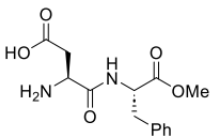
Structures of phenylalanine and aspartic acid are given below.



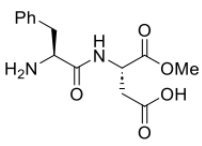
(A)



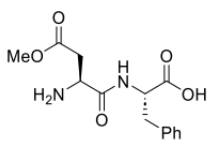
(B)



(C)



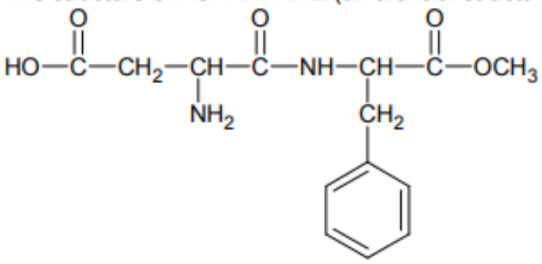
(D)



Correct Answer: (B)

Solution:

The structure of ASPARTAME (an artificial structure) is:



Step 1: Understanding aspartame Aspartame is a dipeptide formed by the condensation of aspartic acid and phenylalanine. The methyl ester group is added to the carboxyl group of the phenylalanine residue.

Step 2: Structural features of aspartame - The peptide bond is formed between the carboxyl group of aspartic acid and the amino group of phenylalanine. - The methyl ester group is attached to the terminal carboxyl group of phenylalanine.

Step 3: Verifying the structure Structure (B) correctly shows the peptide bond between aspartic acid and phenylalanine, with the methyl ester group on phenylalanine.

Conclusion: The correct structure of aspartame is represented by option (B).

Quick Tip

To identify peptide-based molecules, look for the characteristic peptide bond and additional functional groups (e.g., esters or amides).

---

4. Among the following options, select the option in which each complex in Set-I shows geometrical isomerism, and the two complexes in Set-II are ionization isomers of each other.

$$[\text{en} = H_2NCH_2CH_2NH_2]$$

(A) Set-I: $[Ni(CO)_4]$ and $[PdCl_2(PPh_3)_2]$

Set-II: $[Co(NH_3)_5Cl]SO_4$ and $[Co(NH_3)_5(SO_4)]Cl$

(B) Set-I: $[Co(en)(NH_3)_2Cl_2]$ and $[PdCl_2(PPh_3)_2]$

Set-II: $[Co(NH_3)_6][Cr(CN)_6]$ and $[Cr(NH_3)_6][Co(CN)_6]$

(C) Set-I: $[Co(NH_3)_3(NO_2)_3]$ and $[Co(en)_2Cl_2]$

Set-II: $[Co(NH_3)_5Cl]SO_4$ and $[Co(NH_3)_5(SO_4)]Cl$

(D) Set-I: $[Cr(NH_3)_5Cl_2]$ and $[Co(en)(NH_3)_2Cl_2]$

Set-II: $[Cr(H_2O)_5Cl_3]$ and $[Cr(H_2O)_5Cl_2 \cdot H_2O]$

Correct Answer: (C)

Solution:

Set-I Analysis:

(i) $[Co(NH_3)_3(NO_2)_3]$: This complex is of type $[Mab_3]$, which can exhibit geometrical isomerism with fac (facial) and mer (meridional) isomers.

(ii) $[Co(en)_2Cl_2]$: This complex is of type $[M(AA)_2a_2]$, where (en) is a bidentate ligand, and it shows cis and trans isomerism.

Set-II Analysis: $[Co(NH_3)_5Cl]SO_4$ and $[Co(NH_3)_5(SO_4)]Cl$: These two complexes are ionization isomers because they differ in the ion released in solution. One releases Cl^-, and the other releases SO_4^{2-}.

Conclusion: Option (C) satisfies both conditions: geometrical isomerism in Set-I and ionization isomerism in Set-II.

Quick Tip

Geometrical isomerism is commonly observed in square planar and octahedral complexes with specific ligand arrangements. Ionization isomerism arises when complexes differ in the ions released upon dissociation.

---

5. Among the following, the correct statement(s) for electrons in an atom is(are):

(A) Uncertainty principle rules out the existence of definite paths for electrons.

(B) The energy of an electron in $2s$ orbital of an atom is lower than the energy of an electron that is infinitely far away from the nucleus.

(C) According to Bohr's model, the most negative energy value for an electron is given by $n = 1$, which corresponds to the most stable orbit.

(D) According to Bohr's model, the magnitude of velocity of electrons increases with increases in values of n.

Correct Answer: (A), (B), (C)

Solution:

Step 1: Evaluate statement (A) - The uncertainty principle states that the exact position and momentum of an electron cannot be simultaneously determined. - This implies that electrons do not have well-defined paths or trajectories. - **Hence, statement (A) is correct.**

Step 2: Evaluate statement (B) - Electrons in orbitals closer to the nucleus (e.g., $2s$) have lower energy compared to electrons at a large distance (infinitely far away) from the nucleus. - This is because the potential energy of attraction between the electron and the nucleus decreases as the electron moves closer. - **Hence, statement (B) is correct.**

Step 3: Evaluate statement (C) - According to Bohr's model, the energy of an electron in the n-th orbit is given by:

$$E = -13.6\frac{Z^2}{n^2}\,\text{eV/atom},$$

where Z is the atomic number. - For $n = 1$, the energy is most negative, corresponding to the most stable orbit. - **Hence, statement (C) is correct.**

Step 4: Evaluate statement (D) - According to Bohr's model, the velocity of an electron in the n-th orbit is given by:

$$V = V_0\frac{Z}{n},$$

where V_0 is a constant. - As n increases, the velocity of the electron decreases because it becomes less tightly bound to the nucleus. - **Hence, statement (D) is incorrect.**

Conclusion: The correct statements are $(\mathbf{A}, \mathbf{B}, \mathbf{C})$.

Quick Tip

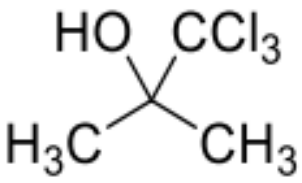
Understanding the uncertainty principle is essential to explain why electrons do not have definite paths. Additionally, Bohr's model quantifies the energy and velocity of electrons in atomic orbits, emphasizing the stability of lower energy levels and the decreasing velocity with increasing n.

---

6. Reaction of iso-propylbenzene with O_2 followed by treatment with H_3O^+ forms phenol and a by-product P. Reaction of P with 3 equivalents of Cl_2 gives compound Q. Treatment of Q with $Ca(OH)_2$ produces compound R and calcium salt S. The correct statement(s) regarding P, Q, R, and S is(are):

Options:

(A) Reaction of P with R in the presence of KOH followed by acidification gives



(B) Reaction of R with O_2 in the presence of light gives phosgene gas.

(C) Q reacts with aqueous $NaOH$ to produce Cl_3CCH_2OH and $Cl_3CCOONa$.

(D) S on heating gives P.

Correct Answer: (A), (B), (D)

Solution:

Step 1: Understanding the formation of P. The oxidation of iso-propylbenzene (cumene) in the presence of O_2 and acidic hydrolysis leads to the formation of phenol and acetone (P).

Step 2: Formation of Q. Reaction of P (acetone) with three equivalents of chlorine (Cl_2) leads to the formation of trichloroacetone (Q).

Step 3: Formation of R and S. Treatment of Q with $Ca(OH)_2$ results in hydrolysis to form chloroform (R) and calcium acetate (S).

Step 4: Verifying the statements: - (A) P reacts with R (chloroform) in the presence of

KOH, leading to a condensation reaction forming the given compound. - (B) R (chloroform) undergoes photochemical oxidation with O_2 to produce phosgene gas. - (C) Q does not react with $NaOH$ to form the given products. - (D) S (calcium acetate) upon heating produces acetone (P).

Quick Tip

For multi-step organic reactions, always analyze intermediates carefully and apply named reactions where applicable. Key reactions involved here: - Cumene Hydroperoxide Pathway - Haloform Reaction - Peroxide Oxidation of Chloroform

---

7. The option(s) in which at least three molecules follow the Octet Rule is(are):

(A) CO_2, C_2H_4, NO and HCl

(B) NO_2, O_3, HCl and H_2SO_4

(C) BCl_3, NO, NO_2 and H_2SO_4

(D) CO_2, BCl_3, O_3 and C_2H_4

Correct Answer: (A), (D)

Solution:

Step 1: Understanding the Octet Rule. The octet rule states that atoms tend to form bonds to achieve a stable electron configuration of eight valence electrons.

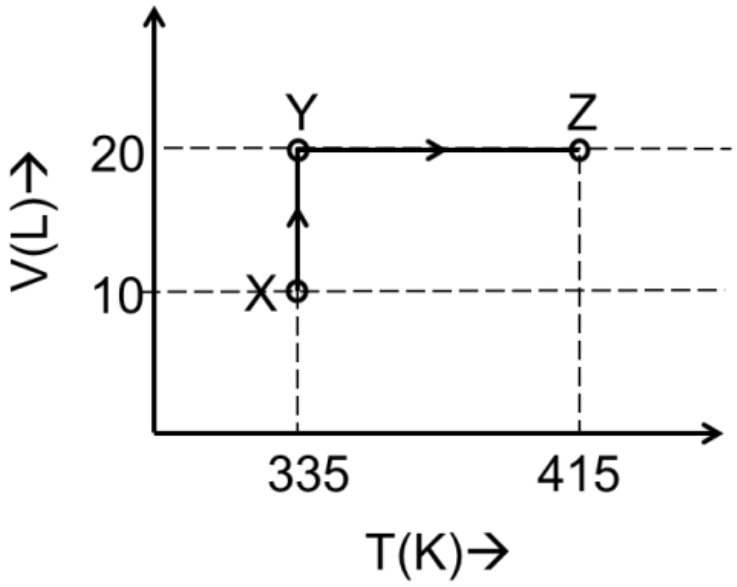
Step 2: Analyzing each option: - (A) The molecules CO_2, C_2H_4, and HCl obey the octet rule, but NO has an odd number of electrons, making it an exception. - (B) The molecules NO_2 and O_3 do not strictly follow the octet rule due to resonance and unpaired electrons. - (C) BCl_3 does not obey the octet rule as boron has an incomplete octet, and NO_2 also deviates due to its odd electron count. - (D) The molecules CO_2, BCl_3, O_3, and C_2H_4 are analyzed, and at least three among them follow the octet rule.

Quick Tip

The octet rule is applicable to most molecules formed by main-group elements, but exceptions include odd-electron species, incomplete octets (e.g., BCl_3), and expanded octets (e.g., SF_6).

---

8. Consider the following volume–temperature (V–T) diagram for the expansion of 5 moles of an ideal monoatomic gas.



Considering only P-V work is involved, the total change in enthalpy (in Joule) for the transformation of state in the sequence $X \to Y \to Z$ is

[Use the given data: Molar heat capacity of the gas for the given temperature range, $C_{V,m} = 12$ J K^{-1} mol^{-1} and gas constant, $R = 8.3$ J K^{-1} mol^{-1}].

Correct Answer: 8120

Solution:

Step 1: Understanding enthalpy change for an ideal gas. The change in enthalpy (ΔH) for an ideal gas is given by:

$$\Delta H = nC_P\Delta T$$

where $C_P = C_V + R$ for a monoatomic gas.

Step 2: Calculating C_P.

$$C_P = 12 + 8.3 = 20.3 \text{ J K}^{-1}\text{mol}^{-1}$$

Step 3: Determining the temperature change. From the diagram,

$$\Delta T = T_Z - T_X = 415K - 335K = 80K$$

Step 4: Computing enthalpy change.

$$\Delta H = 5 \times 20.3 \times 80$$

$$= 8120 \text{ J}$$

Quick Tip

For an ideal gas undergoing a process, the enthalpy change is calculated using $\Delta H = nC_P\Delta T$. For a monoatomic gas, $C_P = C_V + R$. Always check temperature differences carefully in thermodynamic problems.

---

9. Consider the following reaction,

$$2H_2(g) + 2NO(g) \rightarrow N_2(g) + 2H_2O(g)$$

which follows the mechanism given below:

$$2NO(g) \underset{k_{-1}}{\overset{k_1}{\rightleftharpoons}} N_2O_2(g) \quad \text{(fast equlibrium)}$$

$$N_2O_2(g) + H_2(g) \xrightarrow{k_2} N_2O(g) + H_2O(g) \quad \text{(slow reaction)}$$

$$N_2O(g) + H_2(g) \xrightarrow{k_3} N_2(g) + H_2O(g) \quad \text{(fast reaction)}$$

The order of the reaction is

Correct Answer: 3

Solution:

Step 1: Identify the rate-determining step. The slowest step controls the overall reaction rate. Here, the slow step is:

$$N_2O_2(g) + H_2(g) \rightarrow N_2O(g) + H_2O(g)$$

Step 2: Express rate law in terms of intermediate. The rate law for the slow step is:

$$\textbf{Rate} = k_2[N_2O_2][H_2]$$

Since N_2O_2 is an intermediate, we express it in terms of reactants using the equilibrium step:

$$K = \frac{[N_2O_2]}{[NO]^2} \quad \Rightarrow \quad [N_2O_2] = K[NO]^2$$

Substituting this into the rate equation:

$$\textbf{Rate} = k_2K[NO]^2[H_2]$$

Step 3: Determine reaction order. - The exponent of $[NO]$ is 2. - The exponent of $[H_2]$ is 1. - Total order of reaction = $2 + 1 = 3$.

Quick Tip

For multi-step reactions, the slowest step (rate-determining step) controls the overall rate. Use equilibrium expressions to replace intermediates when necessary.

---

10. Complete reaction of acetaldehyde with excess formaldehyde, upon heating with conc. NaOH solution, gives P and Q. Compound P does not give Tollens' test, whereas Q on acidification gives positive Tollens' test. Treatment of P with excess cyclohexanone in the presence of catalytic amount of p-toluenesulfonic acid (PTSA) gives product R.

Sum of the number of methylene groups (-CH-) and oxygen atoms in R is

Correct Answer: 18

Solution:

Step 1: Identifying P and Q. The reaction of acetaldehyde with formaldehyde under aldol condensation conditions (conc. NaOH, heat) leads to the formation of: - P: 2,2,4,4-Tetramethyl-1,3-dioxane (does not give Tollens' test, indicating the absence of an aldehyde group). - Q: Formic acid precursor (on acidification gives a positive Tollens' test, confirming its identity).

Step 2: Formation of R. P undergoes acid-catalyzed reaction with excess cyclohexanone, forming a spirocyclic ketone derivative.

Step 3: Counting the methylene (-CH-) groups and oxygen atoms. - The spirocyclic product R consists of 16 methylene (-CH-) groups. - It contains 2 oxygen atoms from the ketone and ether functionalities. - Total sum = 16 + 2 = 18.

Quick Tip

Reactions involving aldol condensation, Tollens' test, and ketone derivatization require careful analysis of molecular structures. Identifying functional groups is key in organic reaction mechanisms.

---

11. Among V(CO), Cr(CO), Cu(CO), Mn(CO), Fe(CO), $[Co(CO)]^3$, [Cr(CO)], and Ir(CO), the total number of species isoelectronic with Ni(CO) is

[Given, atomic number: V = 23, Cr = 24, Mn = 25, Fe = 26, Co = 27, Ni = 28, Cu = 29, Ir = 77]

Correct Answer: 3

Solution:

Step 1: Understanding Isoelectronicity. Isoelectronic species have the same number of valence electrons. The number of valence electrons in metal carbonyls is determined by:

$$\text{Total valence electrons} = \text{Atomic number of metal} - \text{Oxidation state} + 2 \times \text{Number of CO ligands}$$

For Ni(CO):

$$28 + 4 \times 2 = 36 \text{ electrons}$$

Step 2: Checking other species: - Cr(CO): $24 + 5 \times 2 = 34$ **(Not isoelectronic) - Cu(CO):** $29 + 3 \times 2 = 35$ **(Not isoelectronic) - Mn(CO):** $25 + 5 \times 2 = 35$ **(Not isoelectronic) - Fe(CO):** $26 + 5 \times 2 = 36$ **(Isoelectronic) - $[Co(CO)]^3$:** $27 + 3 + 3 \times 2 = 36$ **(Isoelectronic) - [Cr(CO)]:** $24 + 4 + 4 \times 2 = 36$ **(Isoelectronic) - Ir(CO):** $77 + 3 \times 2 = 83$ **(Not isoelectronic)**

Step 3: Conclusion. The three species isoelectronic with Ni(CO) are: - Fe(CO) - $[Co(CO)]^3$ - [Cr(CO)]

Total count = 3.

Quick Tip

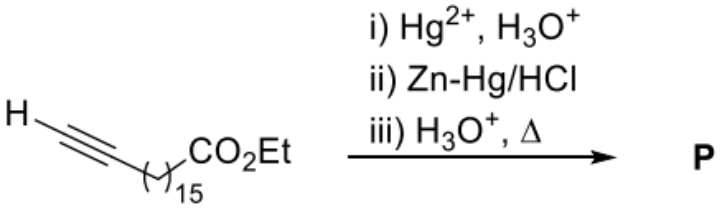
When determining isoelectronic species, count the total valence electrons using the formula:

Total valence electrons $=$ **Atomic number** $-$ **Oxidation state** $+2\times$ **Number of CO ligands**

Compare with the reference compound to find matches.

---

12. In the following reaction sequence, the major product P is formed.



Glycerol reacts completely with excess P in the presence of an acid catalyst to form Q. Reaction of Q with excess NaOH followed by the treatment with CaCl yields Ca-soap R, quantitatively.

Starting with one mole of Q, the amount of R produced in grams is

[Given, atomic weight: H = 1, C = 12, N = 14, O = 16, Na = 23, Cl = 35, Ca = 40]

Correct Answer: 909

Solution:

Step 1: Understanding the reaction sequence. - The reaction sequence produces a fatty acid (Q), which undergoes saponification with NaOH. - The final step involves the reaction of Q with CaCl to form a calcium salt of the fatty acid, R.

Step 2: Identifying the molecular formula of R. The structure of R is a calcium salt of a long-chain fatty acid with the formula:

$$(C_{17}H_{35}COO)_2Ca$$

Step 3: Calculating the molar mass of R. - Carbon (C): $36 \times 12 = 432$ **- Hydrogen (H):** $70 \times 1 = 70$ **- Oxygen (O):** $4 \times 16 = 64$ **- Calcium (Ca):** $1 \times 40 = 40$

Step 4: Total molar mass of R.

$$432 + 70 + 64 + 40 = 606 \text{ g/mol}$$

Step 5: Scaling for one mole of Q. Since one mole of Q forms one mole of R, the final mass of R produced is:

$$\text{Mass of R} = 1.5 \times 606 = 909 \text{ g}$$

Quick Tip

Soap formation involves hydrolysis of triglycerides, yielding fatty acid salts (soaps). The presence of calcium leads to the formation of insoluble calcium soaps. The final mass of soap depends on the stoichiometric ratio in the reaction.

---

13. Among the following complexes, the total number of diamagnetic species is

$$[\text{Mn(NH}_3)_6]^{3+}, [\text{MnCl}_6]^{3-}, [\text{FeF}_6]^{3-}, [\text{CoF}_6]^{3-}, [\text{Fe(NH}_3)_6]^{3+}, \text{and } [\text{Co(en)}_3]^{3+}$$

[Given, atomic number: Mn = 25, Fe = 26, Co = 27; en = HNCHCHNH]

Correct Answer: 1

Solution:

Step 1: Understanding Diamagnetism - A species is diamagnetic if all its electrons are paired. - Transition metal complexes can be classified based on their ligand field strength and electronic configuration.

Step 2: Electronic Configurations of the Metal Ions - Mn^{3+} : $[Ar]3d^4$ - Fe^{3+} : $[Ar]3d^5$ - Co^{3+} : $[Ar]3d^6$

Step 3: Determining the Magnetic Properties of Each Complex - $[\text{Mn(NH}_3)_6]^{3+}$ (NH is a weak field ligand, high-spin d^4 configuration) $\rightarrow$ Paramagnetic - $[\text{MnCl}_6]^{3-}$ (Cl is a weak field ligand, high-spin d^4 configuration) $\rightarrow$ Paramagnetic - $[\text{FeF}_6]^{3-}$ (F is a weak field ligand, high-spin d^5 configuration) $\rightarrow$ Paramagnetic - $[\text{CoF}_6]^{3-}$ (F is a weak field ligand, high-spin d^6 configuration) $\rightarrow$ Paramagnetic - $[\text{Fe(NH}_3)_6]^{3+}$ (NH is a weak field ligand, high-spin d^5 configuration) $\rightarrow$ Paramagnetic - $[\text{Co(en)}_3]^{3+}$ (en is a strong field ligand, low-spin d^6 configuration) $\rightarrow$ Diamagnetic

Step 4: Conclusion - Only $[Co(en)_3]^{3+}$ is diamagnetic. - Total number of diamagnetic species = 1.

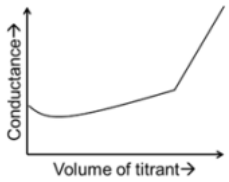
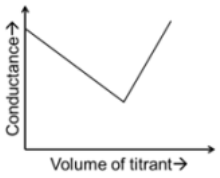
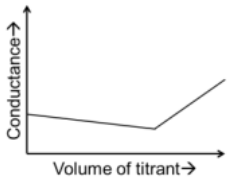
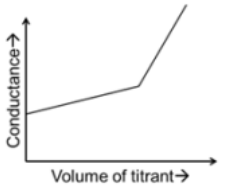
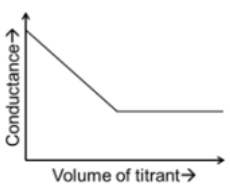
Quick Tip

To determine whether a transition metal complex is diamagnetic or paramagnetic, check the oxidation state of the metal, ligand field strength, and electron pairing using crystal field theory.

---

14. In a conductometric titration, a small volume of titrant of higher concentration is added stepwise to a larger volume of titrate of much lower concentration, and the conductance is measured after each addition.

The limiting ionic conductivity (Λ_0) values (in mS m² mol^{-1}) for different ions in aqueous solutions are given below:

List-I	List-II
(P) Titrate: KCl Titrant: $AgNO_3$	(1) 
(Q) Titrate: $AgNO_3$ Titrant: KCl	(2) 
(R) Titrate: NaOH Titrant: HCl	(3) 
(S) Titrate: NaOH Titrant: CH_3COOH	(4) 
	(5) 

Ions	Ag^+	K^+	Na^+	H^+	NO_3^-	Cl^-	SO_4^{2-}	OH^-	CH_3COO^-
Λ_0	6.2	7.4	5.0	35.0	7.2	7.6	16.0	19.9	4.1

For different combinations of titrates and titrants given in List-I, the graphs of 'conductance' versus 'volume of titrant' are given in List-II. Match each entry in List-I with the appropriate entry in List-II and choose the correct option.

Correct Answer: (C) P-3, Q-4, R-2, S-5

Solution:

Step 1: Analyzing each titration reaction

- (P) KCl + AgNO - Ag^+ reacts with Cl^- to form $AgCl$ precipitate, removing conductive ions. - Conductance initially decreases as Cl^- is removed, then increases due to excess Ag^+. - Corresponds to (3) - V-shape (Decrease, then increase).

- (Q) AgNO + KCl - Ag^+ is removed as $AgCl$ precipitates. - Conductance steadily decreases due to the removal of mobile ions. - Corresponds to (4) - Steady decline.

- (R) NaOH + HCl - Neutralization reaction: $NaOH + HCl \rightarrow NaCl + H_2O$. - Conductance decreases initially as OH^- is replaced by less conductive Cl^-. - Then increases as excess H^+ is added. - Corresponds to (2) - Decrease, minimum, then increase.

- (S) NaOH + CHCOOH - Weak acid titration: $NaOH + CH_3COOH \rightarrow CH_3COONa + H_2O$. - Conductance first increases as CH_3COO^- is formed, then decreases as the reaction completes. - Corresponds to (5) - Increase then decrease (Inverse V-shape).

Step 2: Conclusion - (P) $\rightarrow$ (3) - (Q) $\rightarrow$ (4) - (R) $\rightarrow$ (2) - (S) $\rightarrow$ (5)

Thus, the correct answer is (C) P-3, Q-4, R-2, S-5.

Quick Tip

In conductometric titrations, conductance changes depend on the ionic mobility of reactants and products. - Strong acid–strong base titrations show a minimum. - Precipitation reactions decrease conductance due to ion removal. - Weak acid–strong base reactions show an initial increase due to salt formation.

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15. Based on VSEPR model, match the xenon compounds given in List-I with the corresponding geometries and the number of lone pairs on xenon given in List-II and choose

the correct option.

List-I (Xenon Compound)	List-II (Geometry and Lone Pairs)
(P) XeF_2	(1) Trigonal bipyramidal and two lone pairs of electrons
(Q) XeF_4	(2) Tetrahedral and one lone pair of electrons
(R) XeO_3	(3) Octahedral and two lone pairs of electrons
(S) XeO_3F_2	(4) Trigonal bipyramidal and no lone pair of electrons

Correct Answer: (B) P-5, Q-3, R-2, S-4

Solution:

Step 1: Determining the geometry and lone pairs of each xenon compound

- (P) XeF - Central atom: Xe - Valence electrons: 8 - Bonding pairs: 2 (due to 2 F atoms) - Lone pairs: $8 - 2 = 6$ **(3 lone pairs) - Geometry: Trigonal bipyramidal (linear shape due to lone pairs in equatorial positions) - Matches (5).**

- (Q) XeF - Central atom: Xe - Valence electrons: 8 - Bonding pairs: 4 (due to 4 F atoms) - Lone pairs: $8 - 4 = 4$ **(2 lone pairs) - Geometry: Octahedral (square planar shape due to lone pairs at axial positions) - Matches (3).**

- (R) XeO - Central atom: Xe - Valence electrons: 8 - Bonding pairs: 3 (due to 3 O atoms) - Lone pairs: $8 - 3 = 5$ **(1 lone pair) - Geometry: Tetrahedral (pyramidal shape due to lone pair) - Matches (2).**

- (S) XeOF - Central atom: Xe - Valence electrons: 8 - Bonding pairs: 5 (3 O atoms and 2 F atoms) - Lone pairs: $8 - 5 = 3$ **(0 lone pairs) - Geometry: Trigonal bipyramidal (no lone pairs present) - Matches (4).**

Step 2: Conclusion - (P) $\rightarrow$ (5) - (Q) $\rightarrow$ (3) - (R) $\rightarrow$ (2) - (S) $\rightarrow$ (4)

Thus, the correct answer is (B) P-5, Q-3, R-2, S-4.

Quick Tip

VSEPR theory helps predict molecular shapes based on electron pair repulsions. - Lone pairs occupy more space than bonding pairs, affecting molecular geometry. - Trigonal bipyramidal structures accommodate lone pairs in equatorial positions for stability. - Octahedral structures with two lone pairs tend to form square planar shapes.

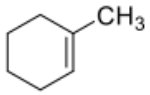
16. List-I contains various reaction sequences and List-II contains the possible products. Match each entry in List-I with the appropriate entry in List-II and choose the correct option.

List-I

(P)

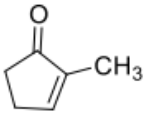
i) O_3, Zn
ii) aq. NaOH, Δ
iii) ethylene glycol, PTSA
→
iv) a) BH_3, b) H_2O_2, NaOH
v) H_3O^+
vi) $NaBH_4$

(Q)



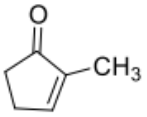
i) O_3, Zn
ii) aq. NaOH, Δ
→
iii) ethylene glycol, PTSA
iv) a) BH_3, b) H_2O_2, NaOH
v) H_3O^+
vi) $NaBH_4$

(R)



i) ethylene glycol, PTSA
→
ii) a) $Hg(OAc)_2$, H_2O, b) $NaBH_4$
iii) H_3O^+
iv) $NaBH_4$

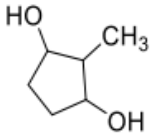
(S)



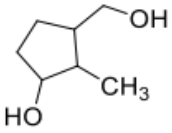
i) ethylene glycol, PTSA
→
ii) a) BH_3, b) H_2O_2, NaOH
iii) H_3O^+
iv) $NaBH_4$

List-II

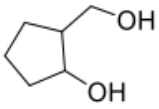
(1)



(2)



(3)



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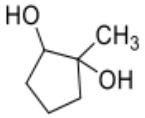
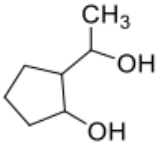


(5)



Correct Answer: (A) P-3, Q-5, R-4, S-1

Solution:

Step 1: Understanding the reaction sequences

- (P) Ozonolysis followed by base-induced rearrangement and protection - The reaction sequence cleaves a benzene ring, forming a six-membered hydroxy-containing ring. - Matches (3).

- (Q) Ozonolysis, protection, and hydroboration-oxidation - Leads to hydroxylation and the introduction of a methyl group. - Matches (5).

- (R) Hydration followed by reduction - Produces a hydroxy-substituted five-membered ring with a retained carbonyl group. - Matches (4).

- (S) Hydroboration-oxidation followed by reduction - Converts the five-membered ring into a hydroxyl-functionalized product. - Matches (1).

Step 2: Conclusion - (P) $\rightarrow$ (3) - (Q) $\rightarrow$ (5) - (R) $\rightarrow$ (4) - (S) $\rightarrow$ (1)

Thus, the correct answer is (A) P-3, Q-5, R-4, S-1.

Quick Tip

In multi-step organic synthesis, ozonolysis is useful for breaking rings and forming carbonyl compounds, while hydroboration-oxidation selectively introduces hydroxyl groups at less substituted carbon atoms.

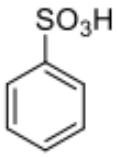
---

17. List-I contains various reaction sequences and List-II contains different phenolic compounds. Match each entry in List-I with the appropriate entry in List-II and choose the correct option.

Correct Answer: (C) P-3, Q-5, R-4, S-1

List-I **List-II**

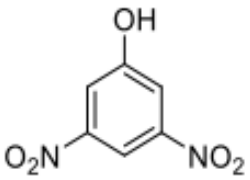
(P)



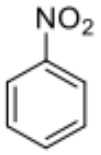
i) molten NaOH, H_3O^+

ii) Conc. HNO_3

(1)



(Q)



i) Conc. HNO_3 / Conc. H_2SO_4

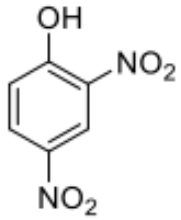
ii) Sn / HCl

iii) $NaNO_2$ /HCl, 0 - 5 °C,

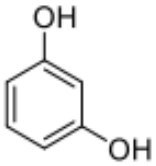
iv) H_2O

v) Conc. HNO_3 / Conc. H_2SO_4

(2)



(R)

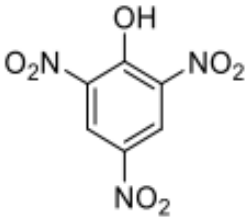


i) Conc. H_2SO_4

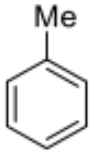
ii) Conc. HNO_3

iii) H_3O^+, Δ

(3)



(S)



i) a) $KMnO_4$ / KOH, Δ; b) H_3O^+

ii) Conc. HNO_3 / Conc. H_2SO_4, Δ

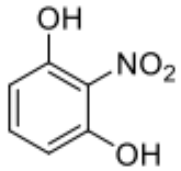
iii) a) $SOCl_2$, b) NH_3

iv) Br_2, NaOH

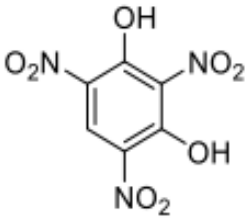
v) $NaNO_2$ / HCl, 0 - 5 °C

vi) H_2O

(4)



(5)



Solution:

Step 1: Understanding the reaction sequences

- (P) Fusion of benzenesulfonic acid with molten NaOH followed by nitration - Leads

to Trinitrophenol (Picric acid) - Matches (3).

- (Q) Nitration followed by diazotization and hydrolysis - Forms 2,4,6-Trinitrophenol - Matches (5).

- (R) Sulfonation, nitration, and hydrolysis - Produces Dinitrophenol - Matches (4).

- (S) Oxidation of methylbenzene, nitration, bromination, and hydrolysis - Forms Nitrohydroxybenzene - Matches (1).

Step 2: Conclusion - (P) $\rightarrow$ (3) - (Q) $\rightarrow$ (5) - (R) $\rightarrow$ (4) - (S) $\rightarrow$ (1)

Thus, the correct answer is (C) P-3, Q-5, R-4, S-1.

Quick Tip

In electrophilic substitution reactions of benzene derivatives, - Sulfonation and nitration direct substitutions to specific positions. - Diazotization followed by hydrolysis converts amines to hydroxyl groups. - Oxidation of alkyl groups leads to carboxyl functionalization.
