JEE Main 2023 April 10 Shift 2 Mathematics Question Paper

Time Allowed :3 Hours	**Maximum Marks :**300	**Total Questions :**90

General Instructions

Read the following instructions very carefully and strictly follow them:

1. The test is of 3 hours duration.
2. The question paper consists of 90 questions, out of which 75 are to attempted. The maximum marks are 300.
3. There are three parts in the question paper consisting of Physics, Chemistry and Mathematics having 30 questions in each part of equal weightage.
4. Each part (subject) has two sections.
   (i) Section-A: This section contains 20 multiple choice questions which have only one correct answer. Each question carries 4 marks for correct answer and –1 mark for wrong answer.
   (ii) Section-B: This section contains 10 questions. In Section-B, attempt any five questions out of 10. The answer to each of the questions is a numerical value. Each question carries 4 marks for correct answer and –1 mark for wrong answer. For Section-B, the answer should be rounded off to the nearest integer

Mathematics

Section-A

1. If the coefficients of x and x^2 in $(1+x)^p(1-x)^q$ are 4 and -5 respectively, then $2p+3q$ is equal to:

(1) 60

(2) 63

(3) 66

(4) 69

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2. Let $A = \{2, 3, 4\}$ and $B = \{8, 9, 12\}$. Then the number of elements in the relation $R = \{((a_1, b_1), (a_2, b_2)) \in (A \times B, A \times B) : a_1$ divides b_2 and a_2 divides $b_1\}$ is:

(1) 18

(2) 24

(3) 12

(4) 36

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3. Let time image of the point $P(1, 2, 6)$ in the plane passing through the points A(1, 2, 0), B(1, 4, 1), and C(0, 5, 1) be $Q(\alpha, \beta, \gamma)$. Then $\alpha^2 + \beta^2 + \gamma^2$ is equal to:

(1) 70

(2) 76

(3) 62

(4) 65

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4. The statement $\sim [p \vee (\sim (p \wedge q))]$ is equivalent to:

(1) $\sim (p \wedge q) \wedge q$

(2) $\sim (p \vee q)$

(3) $\sim (p \wedge q)$

(4) $(p \wedge q) \wedge (\sim p)$

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5. Let $S = \left\{x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right) : 9^{1-\tan^2 x} + 9^{\tan^2 x} = 10\right\}$

$$b = \sum_{x \in S} \tan^2\left(\frac{x}{3}\right), \text{ then } (\beta - 14)^2 \text{ is equal to:}$$

(1) 16

(2) 32

(3) 8

(4) 64

6. If the points P and Q are respectively the circumcenter and the orthocenter of a $\triangle ABC$**, the** $\overrightarrow{PA} + \overrightarrow{PB} + \overrightarrow{PC}$ **is equal to:**

(1) $2\overrightarrow{PQ}$

(2) $\overrightarrow{PQ}$

(3) $2\overrightarrow{PQ}$

(4) $\overrightarrow{PQ}$

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7. Let A be the point (1, 2) and B be any point on the curve $x^2 + y^2 = 16$**. If the centre of the locus of the point P, which divides the line segment AB in the ratio 3:2 is the point C** (α, β)**, then the length of the line segment AC is:**

(1) $\frac{6\sqrt{5}}{5}$

(2) $\frac{2\sqrt{5}}{5}$

(3) $\frac{3\sqrt{5}}{5}$

(4) $\frac{4\sqrt{5}}{5}$

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8. Let m **be the mean and** σ **be the standard deviation of the distribution:**

x_i	0	1	2	3	4	5
f_i	$k+2$	$2k$	k^2-1	k^2-1	k^2+1	$k-3$

where $\sum f_i = 62$. If $[x]$ denotes the greatest integer $\leq x$, then $[\mu^2 + \sigma^2]$ is equal to:

(1) 8

(2) 7

(3) 6

(4) 9

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9. If $S_n = 4 + 11 + 21 + 34 + 50 + \ldots$ to n terms, then $\frac{1}{60}(S_{29} - S_9)$ is equal to:

(1) 220

(2) 227

(3) 226

(4) 223

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10. Eight persons are to be transported from city A to city B in three cars of different makes. If each car can accommodate at most three persons, then the number of ways in which they can be transported is:

(1) 1120

(2) 560

(3) 3360

(4) 1680

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11. If $A = \begin{bmatrix} 5! & 6! & 7! \\ 6! & 7! & 8! \\ 7! & 8! & 9! \end{bmatrix}$, then $|\text{adj}(\text{adj}(2A))|$ is equal to:

(1) 2^{16}

(2) 2^8

(3) 2^{12}

(4) 2^{20}

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12. Let the number $(22)^{2022} + (2022)^{22}$ leave the remainder α when divided by 3 and β when divided by 7. Then $(\alpha^2 + \beta^2)$ is equal to:

(1) 13

(2) 20

(3) 10

(4) 5

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13. Let $g(x) = f(x) + f(1-x)$ and $f^{(n)}(x) > 0$, $x \in (0,1)$. If g is decreasing in the interval $(0, \alpha)$ and increasing in the interval $(\alpha, 1)$, then $\tan^{-1}(2\alpha) + \tan^{-1}\left(\frac{\alpha+1}{\alpha}\right)$ is equal to:

(1) $\frac{5\pi}{4}$

(2) π

(3) $\frac{3\pi}{4}$

(4) $\frac{3\pi}{2}$

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14. For $\alpha, \beta, \gamma, \delta \in \mathbb{N}$, if

$$\int \left(\frac{x}{e}\right)^{2x} + \left(\frac{e}{x}\right)^{2x} \log x \, dx = \frac{1}{\alpha}\left(\frac{x}{e}\right)^{\beta x} - \frac{1}{\gamma}\left(\frac{e}{x}\right)^{\delta x} + C$$

where $e = \sum_{n=0}^{\infty} \frac{1}{n!}$ and C is the constant of integration, then $\alpha + 2\beta + 3\gamma - 4\delta$ is equal to:

(1) 4

(2) -4

(3) 8

(4) 1

15. Let f be a continuous function satisfying

$$\int_0^{t^2} \left(f(x) + x^2\right) \, dx = \frac{4}{3}t^3, \, \forall t > 0.$$

Then $f\left(\frac{\pi^2}{4}\right)$ is equal to:

(1) $-\pi^2\left(1 + \frac{\pi^2}{16}\right)$

(2) $\pi\left(1 - \frac{\pi^3}{16}\right)$

(3) $-\pi\left(1 + \frac{\pi^3}{16}\right)$

(4) $\pi^2\left(1 - \frac{\pi^3}{16}\right)$

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16. Let a die be rolled n times. Let the probability of getting odd numbers seven times be equal to the probability of getting odd numbers nine times. If the probability of getting even numbers twice is $\frac{k}{2^{15}}$, then k is equal to:

(1) 60

(2) 30

(3) 90

(4) 15

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17. Let a circle of radius 4 be concentric to the ellipse $15x^2 + 19y^2 = 285$. Then the common tangents are inclined to the minor axis of the ellipse at the angle:

(1) $\frac{\pi}{6}$

(2) $\frac{\pi}{12}$

(3) $\frac{\pi}{3}$

(4) $\frac{\pi}{4}$

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18. Let $\vec{a} = 2\hat{i} + 7\hat{j} - \hat{k}$, $\vec{b} = 3\hat{i} + 5\hat{k}$, $\vec{c} = \hat{i} - \hat{j} + 2\hat{k}$. Let $\vec{d}$ be a vector which is perpendicular to both $\vec{a}$ and $\vec{b}$, and $\vec{c} \cdot \vec{d} = 12$. The value of $\left(\hat{i} + \hat{j} - \hat{k}\right) \cdot \left(\vec{c} \times \vec{d}\right)$ is:

(1) 24

(2) 42

(3) 48

(4) 44

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19. Let $S = \left\{z = x + iy : \frac{2z-3i}{4z+2i} \text{ is a real number}\right\}$

Then which of the following is NOT correct?

(1) $y \in \left(-\infty, -\frac{1}{2}\right) \cup \left(-\frac{1}{2}, \infty\right)$

(2) $(x, y) = (0, -\frac{1}{2})$

(3) $x = 0$

(4) $y + x^2 + y^2 \neq -\frac{1}{4}$

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20. Let the line

$$\frac{x}{1} = \frac{6-y}{2} = \frac{z+8}{5}$$

intersect the lines

$$\frac{x-5}{4} = \frac{y-7}{3} = \frac{z+2}{1} \quad \text{and} \quad \frac{x+3}{6} = \frac{3-y}{3} = \frac{z-6}{1}$$

at the points A and B respectively. Then the distance of the mid-point of the line segment AB from the plane $2x - 2y + z = 14$ is:

(1) 3

(2) $\frac{10}{3}$

(3) 4

(4) $\frac{11}{3}$

SECTION-B

21. The sum of all the four-digit numbers that can be formed using all the digits 2, 1, 2, 3 is equal to ______.

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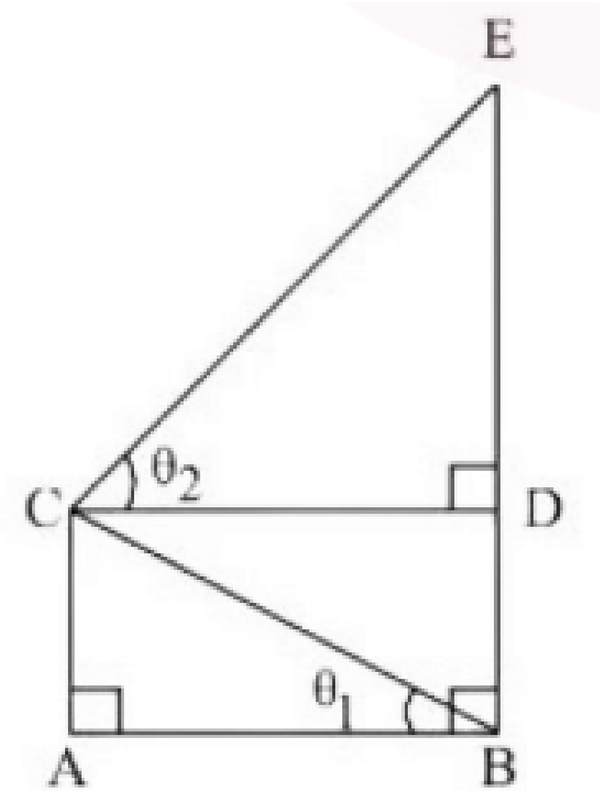
22. In the figure, $\theta_1 + \theta_2 = \frac{\pi}{2}$ and $\sqrt{3}\,BE = 4\,AB$. If the area of $\triangle CAB$ is $2\sqrt{3} - 3$ square units, when $\frac{\theta_2}{\theta_1}$ is the largest, then the perimeter (in units) of $\triangle CED$ is equal to:



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23. Let the tangent at any point P on a curve passing through the points (1, 1) and $\left(\frac{1}{10}, 100\right)$, intersect positive x-axis and y-axis at the points A and B respectively. If $PA : PB = 1 : k$ and $y = y(x)$ is the solution of the differential equation $e^{\frac{dy}{dx}} = kx + \frac{k}{2}$, $y(0) = k$, then $4y(1) - 5\log 3$ is equal to:

(1) 1

(2) 2

(3) 3

(4) 4

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24. Suppose a_1, a_2, a_3, a_4 be in an arithmetico-geometric progression. If the common ratio of the corresponding geometric progression in 2 and the sum of all 5 terms of the arithmetico-geometric progression is $\frac{49}{2}$, then a_4 is equal to ______.

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25. If the area of the region $\{(x, y) : |x^2 - 2| \leq x\}$ is A, then $6A + 16\sqrt{2}$ is equal to ____.

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26. Let the foot of perpendicular from the point A(4, 3, 1) on the plane $P : x - y + 2z + 3 = 0$ be N. If B(5, α, β) is a point on plane P such that the area of triangle ABN is $3\sqrt{2}$, then $\alpha^2 + \beta^2 + \alpha\beta$ is equal to:

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27. Let S be the set of values of λ, for which the system of equations

$$6\lambda x - 3y + 3z = 4\lambda^2, \quad 2x + 6\lambda y + 4z = 1, \quad 3x + 2y + 3\lambda z = \lambda$$

has no solution. Then $12 \sum_{\lambda \in S} |\lambda|$ is equal to:

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28. If the domain of the function $f(x) = \sec^{-1}\left(\frac{2x}{5x+3}\right)$ is $[\alpha, \beta] \cup (\gamma, \delta)$, then $|3\alpha + 10(\beta + \gamma) + 21\delta|$ is equal to:

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29. Let the quadratic curve passing through the point $(-1, 0)$ and touching the line $y = x$ at $(1, 1)$ be $y = f(x)$. Then the x-intercept of the normal to the curve at the point $(\alpha, \alpha + 1)$ in the first quadrant is:

30. Let the equations of two adjacent sides of a parallelogram ABCD be $2x - 3y = -23$ and $5x + 4y = 23$. If the equation of its one diagonal AC is $3x + 7y = 23$ and the distance of A from the other diagonal is d, then $50d^2$ is equal to:

(1) 529

(2) 625

(3) 490

(4) 512