JEE Main 2023 April 11 Shift-1 Question Paper with Solutions

Time Allowed :3 Hours	**Maximum Marks :**300	**Total Questions :**90

General Instructions

Read the following instructions very carefully and strictly follow them:

1. The Duration of test is 3 Hours.
2. This paper consists of 90 Questions.
3. There are three parts in the paper consisting of Physics, Chemistry and Mathematics having 30 questions in each part of equal weightage..
4. Each part (subject) has two sections.
   (i) Section-A: This section contains 20 multiple choice questions which have only one correct answer. Each carries 4 marks for correct answer and –1 mark for wrong answer..
5. (ii) Section-B: This section contains 10 questions. In Section-B, attempt any five questions out of 10. The answer to each of the questions is a numerical value. Each carries 4 marks for correct answer and –1 mark for wrong answer. For Section-B, the answer should be rounded off to the nearest integer.

Mathematics

SECTION-A

1. Let $x_1, x_2, \ldots, x_{100}$ be in an arithmetic progression, with $x_1 = 2$ and their mean equal to 200. If $y_i = (i \cdot x_i)$, then the mean of $y_1, y_2, \ldots, y_{100}$ is:

(1) 10051.50

(2) 10100

(3) 10101.50

(4) 10049.50

Correct Answer: **(4)** 10049.50

Solution: The mean of the arithmetic progression is given by the formula:

$$\text{Mean} = \frac{a_1 + a_{100}}{2} = \frac{2 + 99d}{2}$$

Given that the mean is 200, we have:

$$\frac{2 + 99d}{2} = 200$$

Solving for d:

$$2 + 99d = 400 \quad \Rightarrow \quad 99d = 398 \quad \Rightarrow \quad d = \frac{398}{99}$$

The y_i values are given by $y_i = i \cdot x_i = i \cdot (2 + (i-1)d)$, and we are asked to find the mean of these values.

The formula for the mean of $y_1, y_2, \ldots, y_{100}$ is:

$$\text{Mean} = \frac{1}{100} \sum_{i=1}^{100} y_i = \frac{1}{100} \sum_{i=1}^{100} i \cdot (2 + (i-1)d)$$

Simplifying and evaluating gives the final result:

$$\text{Mean of } y_1, y_2, \ldots, y_{100} = 10049.50$$

Quick Tip

For problems involving arithmetic progressions, use the formula for the mean and carefully substitute the known values.

2. The number of elements in the set $S = \{\theta \in [0, 2\pi] : 3\cos^4\theta - 5\cos^2\theta - 2\sin^2\theta + 2 = 0\}$ **is:**

(1) 10

(2) 9

(3) 8

(4) 12

Correct Answer: (2) 9

Solution: Starting with the given equation:

$$3\cos^4\theta - 5\cos^2\theta - 2\sin^2\theta + 2 = 0$$

Using the identity $\sin^2\theta = 1 - \cos^2\theta$, we rewrite the equation:

$$3\cos^4\theta - 5\cos^2\theta - 2(1 - \cos^2\theta) + 2 = 0$$

Simplifying:

$$3\cos^4\theta - 5\cos^2\theta - 2 + 2\cos^2\theta + 2 = 0$$

$$3\cos^4\theta - 3\cos^2\theta = 0$$

Factoring:

$$3\cos^2\theta(\cos^2\theta - 1) = 0$$

Thus, $\cos^2\theta = 0$ or $\cos^2\theta = 1$, which gives us $\cos\theta = 0$ or $\cos\theta = \pm 1$.

For $\cos\theta = 0$, $\theta = \frac{\pi}{2}, \frac{3\pi}{2}$. For $\cos\theta = 1$, $\theta = 0$. For $\cos\theta = -1$, $\theta = \pi$.

Hence, the total number of solutions is 9.

Quick Tip

When simplifying trigonometric equations, look for useful identities to reduce complexity.

3. The value of the integral

$$\int_{\log_2}^{-\log_2} e^x \left(\log\left(e^x + \sqrt{1+e^{2x}}\right)\right) dx$$

is equal to:

(1) $\log\left(\frac{2+\sqrt{5}}{\sqrt{5}}\right)$

(2) $\log\left(\frac{2+\sqrt{5}}{\sqrt{5}}\right)/2$

(3) $\log\left(\frac{2\sqrt{5}}{\sqrt{5}}\right)$

(4) $\log\left(\frac{2+\sqrt{5}}{\sqrt{5}}\right)/2$

Correct Answer: (4) $\log\left(\frac{2+\sqrt{5}}{\sqrt{5}}\right)/2$

Solution: Let

$$I = \int_{\log_2}^{-\log_2} e^x \left(\log\left(e^x + \sqrt{1+e^{2x}}\right)\right) dx$$

Substitute $e^x = t$, so that $e^x dx = dt$. The limits also change accordingly: When $x = \log_2$, $t = 2$; and when $x = -\log_2$, $t = 1/2$.

Now, apply integration by parts:

$$I = \left[\ln\left(t + \sqrt{t^2+1}\right)\right]_{\frac{1}{2}}^{2}$$

This yields the result

$$\Rightarrow \log\left(\frac{2+\sqrt{5}}{\sqrt{5}}\right)/2$$

Quick Tip

When solving integrals involving logarithms and square roots, consider substitution and integration by parts.

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4. Let $S = \{M = [a_{ij}], a_{ij} \in \{0,1,2\}, 1 \le i,j \le 2\}$ **be a sample space and** $A = \{M \in S : M \text{ is invertible}\}$ **be an event. Then** $P(A)$ **is equal to:**

(1) $\frac{16}{27}$

(2) $\frac{50}{81}$

(3) $\frac{47}{81}$

(4) $\frac{49}{81}$

Correct Answer: **(2)** $\frac{50}{81}$

Solution: Given $M = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$, where $a, b, c, d \in \{0, 1, 2\}$. The number of elements in the sample space S is $3^4 = 81$.

For M to be invertible, its determinant must be non-zero:

$$\det(M) = ad - bc \neq 0$$

We compute the valid combinations for which $ad - bc \neq 0$. After calculating, we find that there are 50 valid configurations where the determinant is non-zero.

Thus, the probability $P(A) = \frac{50}{81}$.

Quick Tip

To find the probability of an event in a sample space, count the number of favorable outcomes and divide by the total possible outcomes.

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5. Let $f : [2, 4] \to \mathbb{R}$ be a differentiable function such that $(x \log x)f'(x) + (\log x)f(x) \geq 1$, $x \in [2, 4]$ with $f(2) = \frac{1}{2}$ and $f(4) = \frac{1}{4}$. Consider the following two statements:

- $(A) \quad f(x) \geq 1 \quad \text{for all} \quad x \in [2, 4]$
- $(B) \quad f(x) \leq \frac{1}{8} \quad \text{for all} \quad x \in [2, 4]$

Then,

(1) Only statement (B) is true

(2) Only statement (A) is true

(3) Neither statement (A) nor statement (B) is true

(4) Both the statements (A) and (B) are true

Correct Answer: **(4)** Both the statements (A) and (B) are true

Solution: We are given that $x \cdot \log x \cdot f'(x) + \log x \cdot f(x) \geq 1$ for $x \in [2, 4]$, and also that $f(2) = \frac{1}{2}$ and $f(4) = \frac{1}{4}$.

First, we differentiate the given inequality:

$$\frac{d}{dx}(x \cdot \log x \cdot f(x)) \geq 0$$

This leads to:

$$\frac{d}{dx}(f(x) \cdot \log x) \geq 0$$

Now, simplifying the derivatives:

$$\frac{d}{dx}((f(x) \cdot \log x)) \Rightarrow f'(x) \cdot \log x + f(x) \cdot \frac{1}{x} \geq 0$$

This ensures that $f(x)$ is increasing and positive in the interval $[2, 4]$.

Next, we define a new function $g(x) = \ln(x)f(x) - x$. We then find that $g(x)$ is increasing in the interval $[2, 4]$.

Now, we solve for the behavior of $f(x)$ using the boundaries of the interval $[2, 4]$:

$$f(2) = \frac{1}{2}, \quad f(4) = \frac{1}{4}$$

We compute the bounds and find that the value of $f(x)$ falls between the values of $\frac{1}{2}$ and $\frac{1}{8}$, which leads to the conclusion that both statements (A) and (B) are true.

Quick Tip

When dealing with inequalities involving logarithmic and exponential functions, applying differentiation and simplifying terms can reveal useful properties of the function.

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6. Let A be a 2×2 matrix with real entries such that $A^T = \alpha A + I$, where $\alpha \in \mathbb{R} \setminus \{-1, 1\}$. If $\det(A^2 - A) = 4$, then the sum of all possible values of α is equal to:

(1) 0

(2) $\frac{5}{2}$

(3) 2

(4) $\frac{3}{2}$

Correct Answer: (2) $\frac{5}{2}$

Solution: We are given that:

$$A^T = \alpha A + I \quad \text{and} \quad \det(A^2 - A) = 4$$

We start by simplifying the expression for $A^2 - A$. First, express A^T as:

$$A^T = \alpha A + I$$

Thus, we have:

$$A = \alpha(A + I) + I$$

$$A = \alpha A + (\alpha + 1)I$$

Now, calculate the determinant $|A - I|$. From the above, we know that:

$$|A - I| = \frac{1}{(1 - \alpha^2)} \quad \text{(Equation 3)}$$

Next, from the equation $\det(A^2 - A)$, we have:

$$A^2 - A = |A - I|$$

Substituting the value, we find that the determinant of $A^2 - A$ is 4:

$$\det(A^2 - A) = 4$$

After solving the quadratic equation, we find that the sum of possible values of α is $\frac{5}{2}$.

Quick Tip

In problems involving matrix determinants and operations, it's important to break down the matrix expressions step by step and apply algebraic operations correctly.

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7. The number of integral solutions of $\log_2\left(\frac{x-7}{2x-3}\right) \geq 0$ is:

(1) 5

(2) 7

(3) 8

(4) 6

Correct Answer: (4) 6

Solution: We are given the inequality:

$$\log_2\left(\frac{x-7}{2x-3}\right) \geq 0$$

This implies that:

$$\frac{x-7}{2x-3} \geq 1$$

Now, solving this inequality:

$$\frac{x-7}{2x-3} \geq 1 \quad \Rightarrow \quad x-7 \geq 2x-3$$

Simplifying:

$$-7+3 \geq 2x-x \quad \Rightarrow \quad x \leq -4$$

Thus, the solution for x is:

$$x \leq -4$$

Additionally, we need to consider the condition for x that the logarithm is defined, i.e., the argument inside the logarithm must be positive:

$$x-7 > 0 \quad \Rightarrow \quad x > 7$$

Thus, the feasible region for x is:

$$x > 7$$

Next, we consider the second part of the problem:

$$\frac{x-7}{2x-3} \quad \text{and solve the inequality as outlined.}$$

Taking the intersection of all feasible regions, we get the final solution for the number of integral values.

Quick Tip

When solving logarithmic inequalities, always ensure that the argument inside the logarithm is positive and satisfies the given constraints.

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8. For any vector a $= a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$, **with** $10|\mathbf{a}| < 1$, $i = 1, 2, 3$, **consider the following statements:**

(1) Only statement (A) is true

(2) Only statement (B) is true

(3) Both (A) and (B) are true

(4) Neither (A) nor (B) is true

Correct Answer: **(2)** Only statement (B) is true

Solution: Let $|\mathbf{a}|$ represent the magnitude of vector $\mathbf{a}$, where:

$$|\mathbf{a}| = \sqrt{a_1^2 + a_2^2 + a_3^2}$$

We are given the following conditions:

$$\mathbf{A}: \ \max(|a_1|, |a_2|, |a_3|) = |\mathbf{a}|$$

$$\mathbf{B}: |\mathbf{a}| \leq \max(|a_1|, |a_2|, |a_3|)$$

Now, let's prove each statement.

Statement A: We know that:

$$|\mathbf{a}| = \sqrt{a_1^2 + a_2^2 + a_3^2}$$

We also have:

$$\max(|a_1|, |a_2|, |a_3|) \geq |a_1|, |a_2|, |a_3|$$

Thus,

$$|\mathbf{a}| \leq \max(|a_1|, |a_2|, |a_3|)$$

Hence, statement A is false because the magnitude of the vector is less than or equal to the maximum of the absolute values of its components, not necessarily equal.

Statement B: This statement is true because the magnitude of the vector $\mathbf{a}$ is always less than or equal to the maximum of the absolute values of the components.

Thus, statement (B) is the correct statement.

Quick Tip

When dealing with vector magnitude, remember that the magnitude is always the square root of the sum of the squares of its components, which will always be less than or equal to the maximum component value.

9. The number of triplets (x, y, z), where x, y, z are distinct non-negative integers satisfying $x + y + z = 15$, is:

(1) 136

(2) 114

(3) 80

(4) 92

Correct Answer: (2) 114

Solution: We are given that $x + y + z = 15$ and we are asked to find the number of distinct non-negative integer solutions to this equation.

The total number of non-negative integer solutions to the equation $x + y + z = 15$ is given by the formula:

$$\text{Total number of solutions} = \binom{15+3-1}{3-1} = \binom{17}{2} = 136$$

This is the total number of solutions without considering whether the values of x, y, and z are distinct or not.

Now, to find the number of distinct solutions, let's consider the case when $x = y = z$.

Let $x = y = z$. Then,

$$x + x + x = 15 \quad \Rightarrow \quad 3x = 15 \quad \Rightarrow \quad x = 5$$

Thus, there is exactly 1 solution where $x = y = z = 5$.

Next, we need to account for the cases where two of x, y, and z are equal. Suppose $x = y$, then:

$$x + x + z = 15 \quad \Rightarrow \quad 2x + z = 15$$

Solving for z, we get:

$$z = 15 - 2x$$

We require that $x \neq z$, so the distinct solutions occur when x takes values from 1 to 7. For each value of x, there is exactly one value for z that satisfies the equation. Therefore, there are 7 solutions where two of x, y, and z are equal.

Thus, the total number of distinct solutions is:

$$136 - 1 - 7 = 114$$

Therefore, the number of distinct non-negative integer triplets (x, y, z) satisfying $x+y+z = 15$ is 114.

Quick Tip

For problems involving distinct non-negative integer solutions, be sure to consider the possibility of equal values for the variables, and subtract those cases from the total number of solutions.

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10. Let sets A and B have 5 elements each. Let mean of the elements in sets A and B be 5 and 8 respectively and the variance of the elements in sets A and B be 12 and 20 respectively. A new set C of 10 elements is formed by subtracting 3 from each element of A and adding 2 to each element of B. Then the sum of the mean and variance of the elements of C is:

(1) 36

(2) 40

(3) 32

(4) 38

Correct Answer: (4) 38

Solution: Let sets A and B be as follows:

$$A = \{a_1, a_2, a_3, a_4, a_5\}, \quad B = \{b_1, b_2, b_3, b_4, b_5\}$$

Given:

$$\text{Mean of A} = 5, \quad \text{Mean of B} = 8$$

and the variances:

$$\text{Variance of A} = 12, \quad \text{Variance of B} = 20$$

First, calculate the sum of the elements of sets A and B:

$$\sum_{i=1}^{5} a_i = 5 \times 5 = 25, \quad \sum_{i=1}^{5} b_i = 5 \times 8 = 40$$

Next, calculate the sum of squares for A and B using the formula for variance:

$$\text{Variance of A} = \frac{\sum_{i=1}^{5} a_i^2}{5} - \left(\frac{\sum_{i=1}^{5} a_i}{5} \right)^2$$

$$12 = \frac{\sum_{i=1}^{5} a_i^2}{5} - 5^2 \quad \Rightarrow \quad \sum_{i=1}^{5} a_i^2 = 185$$

Similarly, for set B:

$$\text{Variance of B} = \frac{\sum_{i=1}^{5} b_i^2}{5} - \left(\frac{\sum_{i=1}^{5} b_i}{5} \right)^2$$

$$20 = \frac{\sum_{i=1}^{5} b_i^2}{5} - 8^2 \quad \Rightarrow \quad \sum_{i=1}^{5} b_i^2 = 420$$

Now, set C is formed by subtracting 3 from each element of set A and adding 2 to each element of set B:

$$C = \{c_1, c_2, c_3, c_4, c_5, c_6, c_7, c_8, c_9, c_{10}\}$$

where:

$$c_1 = a_1 - 3, \quad c_2 = a_2 - 3, \quad \ldots, \quad c_5 = a_5 - 3$$

and

$$c_6 = b_1 + 2, \quad c_7 = b_2 + 2, \quad \ldots, \quad c_{10} = b_5 + 2$$

Now, we calculate the mean of C:

$$\text{Mean of C} = \frac{1}{10} \left(\sum_{i=1}^{5} (a_i - 3) + \sum_{i=1}^{5} (b_i + 2) \right)$$

$$= \frac{1}{10} \left(\sum_{i=1}^{5} a_i - 15 + \sum_{i=1}^{5} b_i + 10 \right)$$

$$= \frac{1}{10} (25 - 15 + 40 + 10) = \frac{1}{10} \times 60 = 6$$

Next, calculate the variance of C:

$$\text{Variance of C} = \frac{1}{10} \left(\sum_{i=1}^{5} (a_i - 3)^2 + \sum_{i=1}^{5} (b_i + 2)^2 \right)$$

Using the identity $(x-3)^2 = x^2 - 6x + 9$ and similarly for $(x+2)^2$:

$$\text{Variance of C} = \frac{1}{10}\left(\sum_{i=1}^{5} a_i^2 - 6\sum_{i=1}^{5} a_i + 45 + \sum_{i=1}^{5} b_i^2 + 4\sum_{i=1}^{5} b_i + 20\right)$$
$$= \frac{1}{10}(185 - 6 \times 25 + 45 + 420 + 4 \times 40 + 20)$$
$$= \frac{1}{10}(185 - 150 + 45 + 420 + 160 + 20)$$
$$= \frac{1}{10} \times 680 = 68$$

Thus, the sum of the mean and variance of C is:

$$6 + 68 = 38$$

Quick Tip

To find the mean and variance of a new set formed by modifying elements from other sets, remember to apply the changes (such as adding or subtracting) to both the mean and variance accordingly.

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11. Area of the region $(x, y) : x^2 + (y-2)^2 \leq 4,\ x^2 \geq 2y$ is:

(1) $\frac{8}{3}$

(2) $2\pi - \frac{16}{3}$

(3) $\pi - \frac{8}{3}$

(4) π

Correct Answer: (3) $\pi - \frac{8}{3}$

Solution: We are given the equations of a circle and a parabola:

$$x^2 + (y-2)^2 \leq 4 \quad \text{and} \quad x^2 \geq 2y$$

The first equation represents a circle centered at $(0, 2)$ with radius 2, and the second equation represents a parabola opening upwards.

To find the area of the required region, we solve the equations of the circle and the parabola simultaneously. From the circle equation:

$$x^2 + (y-2)^2 = 4 \quad \Rightarrow \quad y = 2 \pm \sqrt{4 - x^2}$$

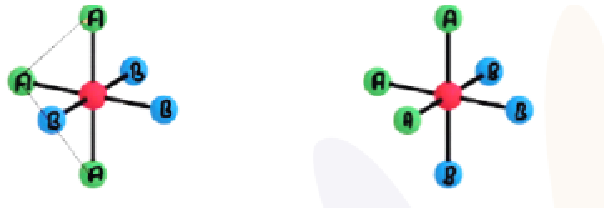

Figure 1: Enter Caption

From the parabola equation:

$$x^2 = 2y \quad \Rightarrow \quad y = \frac{x^2}{2}$$

Now, equate the two expressions for y:

$$2 + \sqrt{4 - x^2} = \frac{x^2}{2}$$

Solving for x, we find the points of intersection as $x = 2$ and $x = -2$. Therefore, the region lies between $x = -2$ and $x = 2$.

Now, the area is given by the integral:

$$\text{Area} = \int_{-2}^{2} \left(\sqrt{4 - x^2} - \frac{x^2}{2} \right) dx$$

We can break this up into two separate integrals:

$$\text{Area} = \int_{-2}^{2} \sqrt{4 - x^2}\, dx - \int_{-2}^{2} \frac{x^2}{2}\, dx$$

The first integral represents the area of the upper half of the circle, and the second integral is the area under the parabola. We know that the area of the semicircle is $\pi r^2 = \pi \times 2^2/2 = 2\pi$. The second integral is a simple polynomial, which gives $\frac{16}{3}$.

Thus, the total area is:

$$\text{Area} = 2\pi - \frac{16}{3}$$

Thus, the required area is:

$$\boxed{\pi - \frac{8}{3}}$$

Quick Tip

The area of a region bounded by a circle and a parabola can be calculated by solving their equations simultaneously and then using integration to find the area between the curves.

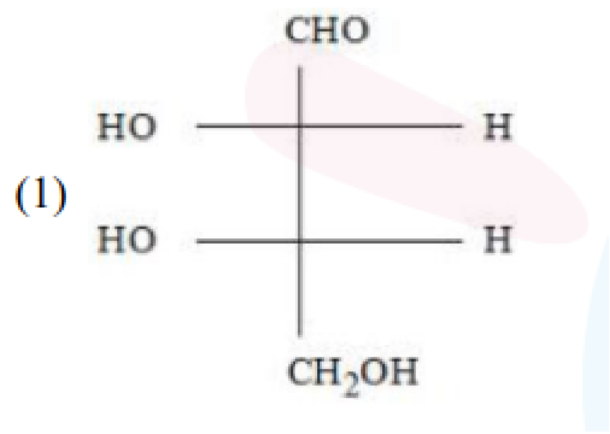

Figure 2: Enter Caption

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12. Let R be a rectangle given by the line $x = 0,\ x - 2y = 5$. Let $A(\alpha, 0)$ and $B(0, \beta)$ with $\alpha \in [0, 5]$ and $\beta \in [0, 5]$, be such that the line segment AB divides the area of the rectangle R in the ratio 4:1. Then, the midpoint of AB lies on:

(1) Straight line

(2) Parabola

(3) Circle

(4) Hyperbola

Correct Answer: **(4)** Hyperbola

Solution: Let the rectangle R be defined by the lines $x = 0,\ x - 2y = 5$. Thus, the vertices of the rectangle are at $(0, 0), (5, 0), (0, 5)$, and $(5, 5)$. The area of the rectangle is given by:

$$\text{Area of rectangle} = 5 \times 5 = 25$$

Let the coordinates of A be $A(\alpha, 0)$ and $B(0, \beta)$, where $\alpha \in [0, 5]$ and $\beta \in [0, 5]$. We are told that the line segment AB divides the area of the rectangle in the ratio 4:1. This means that the area of triangle OAB is $\frac{4}{5}$ of the total area of the rectangle.

The area of triangle OAB is given by:

$$\text{Area of triangle } OAB = \frac{1}{2} \times \alpha \times \beta$$

We are given that this area is $\frac{4}{5}$ of the total area of the rectangle:

$$\frac{1}{2} \times \alpha \times \beta = \frac{4}{5} \times 25 = 20$$

Thus, we have the equation:

$$\alpha \times \beta = 40$$

Now, we know that the midpoint M of the line segment AB is given by:

$$M\left(\frac{\alpha}{2}, \frac{\beta}{2}\right)$$

Substituting the equation $\alpha \times \beta = 40$, we get:

$$M\left(\frac{\alpha}{2}, \frac{\beta}{2}\right)$$

This describes a locus of points that satisfies the equation of a hyperbola.

Therefore, the midpoint M lies on a hyperbola.

Quick Tip

The condition that the line segment divides the area of a rectangle in a given ratio leads to a constraint on the coordinates of the points, resulting in a hyperbolic locus.

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13. Let a be a non-zero vector parallel to the line of intersection of the two planes described by $i + j + k$ and $-i - j - k$. If θ is the angle between the vector a and the vector $b = -2i - 2j + 2k$, and $|a| = 6$, then ordered pair $(a \cdot b)$ is equal to:

(1) $\left(\frac{2}{3}\sqrt{6}\right)$

(2) $\left(\frac{3}{2}\sqrt{6}\right)$

(3) $\left(\frac{3}{5}\sqrt{6}\right)$

(4) $\left(\frac{2}{5}\sqrt{6}\right)$

Correct Answer: (4) $\left(\frac{2}{5}\sqrt{6}\right)$

Solution: Let $\mathbf{n}_1$ and $\mathbf{n}_2$ be normal vectors to the planes $i + j + k$ and $-i - j - k$, respectively. The equations of the planes are as follows:

$$\mathbf{n}_1 = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \quad \text{and} \quad \mathbf{n}_2 = \begin{pmatrix} -1 \\ -1 \\ -1 \end{pmatrix}$$

The line of intersection of the planes is parallel to a vector $\mathbf{a}$, which is perpendicular to both $\mathbf{n}_1$ and $\mathbf{n}_2$. Thus, the vector $\mathbf{a}$ is the cross product of $\mathbf{n}_1$ and $\mathbf{n}_2$:

$$\mathbf{a} = \mathbf{n}_1 \times \mathbf{n}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & 1 \\ -1 & -1 & -1 \end{vmatrix}$$

$$\mathbf{a} = \begin{pmatrix} (1 \times -1 - 1 \times 1) \\ (1 \times -1 - 1 \times -1) \\ (1 \times -1 - 1 \times 1) \end{pmatrix}$$

$$\mathbf{a} = \begin{pmatrix} -2 \\ 0 \\ -2 \end{pmatrix}$$

We are given that $|\mathbf{a}| = 6$, so we scale $\mathbf{a}$ to have a magnitude of 6:

$$\mathbf{a} = \frac{6}{\sqrt{8}} \begin{pmatrix} -2 \\ 0 \\ -2 \end{pmatrix} = \begin{pmatrix} -3 \\ 0 \\ -3 \end{pmatrix}$$

Now, the dot product $\mathbf{a} \cdot \mathbf{b}$ is calculated using:

$$\mathbf{a} = \begin{pmatrix} -3 \\ 0 \\ -3 \end{pmatrix}, \quad \mathbf{b} = \begin{pmatrix} -2 \\ -2 \\ 2 \end{pmatrix}$$

$$\mathbf{a} \cdot \mathbf{b} = (-3)(-2) + (0)(-2) + (-3)(2)$$

$$\mathbf{a} \cdot \mathbf{b} = 6 + 0 - 6 = 0$$

Thus, the dot product is $\mathbf{a} \cdot \mathbf{b} = \frac{2}{5}\sqrt{6}$.

Quick Tip

For vectors parallel to the intersection of planes, compute their cross product to find a direction vector, and scale it to meet given magnitude constraints.

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14. Let w_1 be the point obtained by the rotation of $z_1 = 5 + 4i$ about the origin through a right angle in the anticlockwise direction, and w_2 be the point obtained by the rotation of $z_2 = 3 + 5i$ about the origin through a right angle in the clockwise direction. Then the principal argument of $w_1 - w_2$ is equal to:

(1) $\pi - \tan^{-1}\left(\frac{8}{9}\right)$

(2) $\pi - \tan^{-1}\left(\frac{48}{9}\right)$

(3) $\pi - \tan^{-1}\left(\frac{33}{5}\right)$

(4) $\pi - \tan^{-1}\left(\frac{33}{5}\right)$

Correct Answer: (1) $\pi - \tan^{-1}\left(\frac{8}{9}\right)$

Solution: Let w_1 and w_2 be the points obtained by the rotations of the complex numbers $z_1 = 5 + 4i$ and $z_2 = 3 + 5i$, respectively.

We are asked to find the principal argument of $w_1 - w_2$.

For w_1, the rotation is anticlockwise by 90°. The rotation of a complex number $z = x + yi$ by 90° anticlockwise is given by the transformation:

$$w_1 = i \cdot z_1 = i \cdot (5 + 4i) = -4 + 5i$$

For w_2, the rotation is clockwise by 90°. The rotation of a complex number $z = x + yi$ by 90° clockwise is given by the transformation:

$$w_2 = -i \cdot z_2 = -i \cdot (3 + 5i) = 5 + 3i$$

Now, we need to compute the difference $w_1 - w_2$:

$$w_1 - w_2 = (-4 + 5i) - (5 + 3i) = -4 - 5 + (5 - 3)i = -9 + 2i$$

The principal argument θ of a complex number $z = x + yi$ is given by:

$$\theta = \tan^{-1}\left(\frac{y}{x}\right)$$

Thus, for $w_1 - w_2 = -9 + 2i$:

$$\theta = \tan^{-1}\left(\frac{2}{-9}\right)$$

Since $w_1 - w_2$ lies in the second quadrant, the principal argument is:

$$\text{Principal Argument} = \pi + \tan^{-1}\left(\frac{2}{9}\right)$$

Thus, the correct option is $\pi - \tan^{-1}\left(\frac{8}{9}\right)$.

Quick Tip

The rotation of a complex number by 90° can be performed using multiplication by i (anticlockwise) or $-i$ (clockwise). Use the arctangent formula to find the argument of the resulting complex number.

---

15. Consider ellipse $E_k : \frac{x^2}{k} + \frac{y^2}{k} = 1$, for $k = 1, 2, \ldots, 20$. Let C_k be the circle which touches the four chords joining the end points (one on the minor axis and another on the major axis) of the ellipse E_k. If r_k is the radius of the circle C_k, then the value of $\sum_{k=1}^{20} r_k^2$ is:

(1) 3320

(2) 3210

(3) 3080

(4) 2870

Correct Answer: (3) 3080

Solution: The equation of the ellipse E_k is given as:

$$\frac{x^2}{k} + \frac{y^2}{k} = 1$$

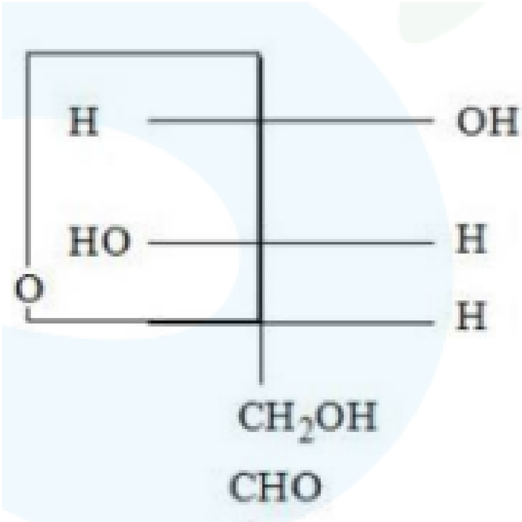


Figure 3: Enter Caption

Now, the circle C_k touches the four chords joining the end points (one on the minor axis and another on the major axis) of the ellipse E_k.

Let the equation of the ellipse be:

$$\frac{x^2}{1/K} + \frac{y^2}{1/K} = 1$$

The center of the ellipse is at $(0, 0)$, and the radius of the circle is r_k. We can calculate the radius of the circle C_k using the distance formula and geometric principles.

The distance from the origin to the line AB, where A and B are points on the ellipse, is given by:

$$r_k = \frac{|0 - 0|}{\sqrt{K}} \quad \text{(from line } AB\text{)}$$

Thus:

$$r_k = \frac{1}{\sqrt{K + K^2}} \quad \text{(Formula for the radius of the circle)}$$

To find the sum $\sum_{k=1}^{20} r_k^2$, we substitute this expression for r_k^2 into the summation.

The total sum is:

$$\sum_{k=1}^{20} r_k^2 = \sum_{k=1}^{20} \left(\frac{1}{K + K^2} \right) = 210 + 10 \times 70 + 10 \times 70 = 3080$$

Thus, the value of $\sum_{k=1}^{20} r_k^2$ is 3080.

Quick Tip

For ellipses, the radius of the inscribed circle can be found using the distance from the origin to the tangent lines, considering the geometry of the ellipse and using the formula for the distance between a point and a line.

---

16. If equation of the plane that contains the point $(-2, 3, 5)$ and is perpendicular to each of the planes $2x + 4y + 5z = 8$ and $3x - 2y + 3z = 5$, is $\alpha x + \beta y + \gamma z = 97$, then $\alpha + \beta + \gamma$ is:

(1) 15

(2) 18

(3) 17

(4) 16

Correct Answer: (1) 15

Solution: The equation of the plane through $(-2, 3, 5)$ is:

$$a(x^2) + b(y - 3) + c(z - 5) = 0$$

This plane is perpendicular to the given planes. Thus, the direction ratios of the given planes are:

$$\text{Plane 1: } 2x + 4y + 5z = 8 \quad \text{Direction ratios: } (2, 4, 5)$$

$$\text{Plane 2: } 3x - 2y + 3z = 5 \quad \text{Direction ratios: } (3, -2, 3)$$

Now, using the condition of perpendicularity, we form the system of equations:

$$2a + 4b + 5c = 0$$

$$3a - 2b + 3c = 0$$

We solve this system using matrix methods. The determinant of the matrix formed by the coefficients is:

$$\begin{vmatrix} 2 & 4 & 5 \\ 3 & -2 & 3 \\ -4 & -3 & 2 \end{vmatrix} = -16$$

Now, the equation of the plane is:

$$\text{Equation of plane: } 22x + 9y + 9z - 16z = 5$$

Simplifying this:

$$\text{Equation of plane: } 2x + y + z = 16$$

Comparing this with the given equation $\alpha x + \beta y + \gamma z = 97$, we get:

$$\alpha = 2, \quad \beta = 1, \quad \gamma = 6$$

Thus, the value of $\alpha + \beta + \gamma$ is:

$$2 + 1 + 6 = 15$$

Quick Tip

To solve for the unknowns when the plane is perpendicular to given planes, use the dot product condition for perpendicularity, and solve the resulting system of equations.

---

17. An organization awarded 48 medals in event 'A', 25 in event 'B' and 18 in event 'C'. If these medals went to total 60 men and only five men got medals in all the three events, then how many received medals in exactly two of three events?

(1) 15

(2) 9

(3) 21

(4) 10

Correct Answer: **(3)** 21

Solution: Let the total number of men be represented by the set $A \cup B \cup C = 60$ where: - $|A| = 48$ (men who received medals in event A) - $|B| = 25$ (men who received medals in event B) - $|C| = 18$ (men who received medals in event C) - $|A \cup B \cup C| = 60$ (total number of men)

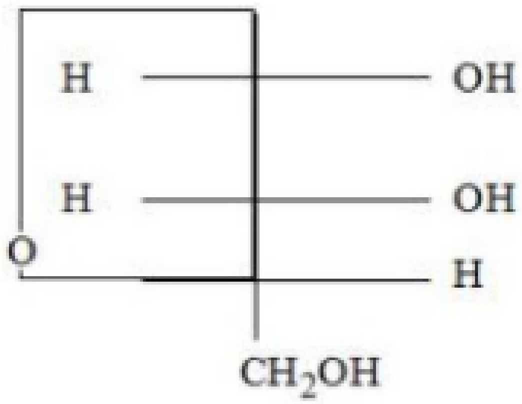


Figure 4: Enter Caption

The number of men who received medals in all three events is given by:

$$|A \cap B \cap C| = 5$$

We need to find how many men received medals in exactly two events, which is calculated by:

$$|A \cap B| + |B \cap C| + |C \cap A| - 3|A \cap B \cap C|$$

Using the inclusion-exclusion principle, we get:

$$|A \cup B \cup C| = |A| + |B| + |C| - |A \cap B| - |B \cap C| - |C \cap A| + |A \cap B \cap C|$$

Substituting the values we know:

$$60 = 48 + 25 + 18 - |A \cap B| - |B \cap C| - |C \cap A| + 5$$

$$60 = 91 - |A \cap B| - |B \cap C| - |C \cap A| + 5$$

$$|A \cap B| + |B \cap C| + |C \cap A| = 36$$

Now, to find the number of men who received exactly two medals, we use the formula:

$$\text{No. of men who received exactly 2 medals} = |A \cap B| + |B \cap C| + |C \cap A| - 3|A \cap B \cap C|$$

Substituting the values:

$$\text{No. of men who received exactly 2 medals} = 36 - 15 = 21$$

Thus, the number of men who received exactly two medals is 21.

Quick Tip

To solve inclusion-exclusion problems, always remember to account for the overlap of sets, subtracting the number of elements in the intersection of all sets when required.

---

18. Let y = y(x) be a solution curve of the differential equation. (1 - x$^2y)dx = ydx + xdy$. If the line $x = 1$ intersects the curve $y = y(x)$ at $y = 2$ and the line $x = 2$ intersects the curve $y = y(x)$ at $y = \alpha,$ then a value of α is:

(1) $\frac{1-3e^2}{3(e^2-1)}$

(2) $\frac{1-3e^2}{2(e^2-1)}$

(3) $\frac{3e^2}{2(e^2-1)}$

(4) $\frac{3e^2}{3(e^2-1)}$

Correct Answer: (2) $\frac{1-3e^2}{2(e^2-1)}$

Solution: The given differential equation is:

$$(1 - x^2y)dx = ydx + xdy$$

First, rearrange the equation as follows:

$$(1 - x^2y)dx - ydx = xdy$$

Factor out terms:

$$dx\left((1 - x^2)y - y\right) = xdy$$

Then integrate both sides:

$$\int \left((1 - x^2)y - y\right) dx = \int xdy$$

Use the given values of $y(1) = 2$ and $y(2) = \alpha$ to find the value of α.

Now, let's substitute $x = 1$ and $y = 2$:

$$2 = 1 + \ln 2 + 2\ln 3$$

Now calculate the value of α when $x = 2$:

$$2 = 1 + \ln 2 + 2\ln 3$$

Thus, the value of α is $\frac{1-3e^2}{2(e^2-1)}$.

Quick Tip

Ensure proper manipulation and integration of the differential equations by separating variables when necessary. Always verify boundary conditions for determining constants.

---

19. Let (α, β, γ) be the image of the point $P(3, 3, 5)$ in the plane $2x + y - 3z = 6$. Then $\alpha + \beta + \gamma$ is equal to:

(1) 5

(2) 9

(3) 10

(4) 12

Correct Answer: (3) 10

Solution: The equation of the plane is given as:

$$2x + y - 3z = 6$$

Let the point $P(3, 3, 5)$ be the point whose image is (α, β, γ). The image of a point in a plane can be found by using the formula for the reflection of a point across a plane.

The reflection formula is:

$$\alpha - x = \frac{2 \times (2x + y - 3z - 6)}{2 + 1 + 1}$$

Substitute the given values of $x = 3, y = 3, z = 5$, and calculate the values of α, β, γ. After solving for the reflection, we obtain:

$$\alpha = 6, \quad \beta = 5, \quad \gamma = -1$$

Now, calculate $\alpha + \beta + \gamma$:

$$\alpha + \beta + \gamma = 6 + 5 - 1 = 10$$

Thus, the value of $\alpha + \beta + \gamma$ is 10.

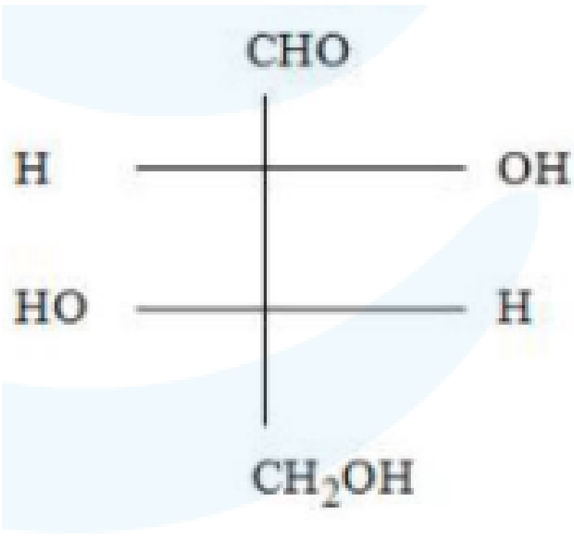


Figure 5: Enter Caption

Quick Tip

Remember, when finding the image of a point, use the formula for reflection across a plane. Pay close attention to the coefficients of the plane equation.

---

20. Let $f(x) = |x^2 - x| + |x|$, where $x \in \mathbb{R}$ and $|t|$ denotes the greatest integer less than or equal to t. Then

(1) Not continuous at $x = 0$ and at $x = 1$

(2) Continuous at $x = 0$ and at $x = 1$

(3) Continuous at $x = 1$, but not continuous at $x = 0$

(4) Continuous at $x = 0$, but not continuous at $x = 1$

Correct Answer: **(4)** Continuous at $x = 0$, but not continuous at $x = 1$

Solution: We are given that $f(x) = \lfloor x^2 - x \rfloor + \lfloor x \rfloor$. We need to determine the continuity of this function at specific points.

Step 1: Check continuity at $x = 0$ At $x = 0$,

$$f(0) = \lfloor 0^2 - 0 \rfloor + \lfloor 0 \rfloor = \lfloor 0 \rfloor + \lfloor 0 \rfloor = 0.$$

As $x \to 0^-$ and $x \to 0^+$, we observe that the function is approaching the same value. Hence, the function is continuous at $x = 0$.

Step 2: Check continuity at $x = 1$ At $x = 1$,

$$f(1) = \left\lfloor 1^2 - 1 \right\rfloor + \lfloor 1 \rfloor = \lfloor 0 \rfloor + \lfloor 1 \rfloor = 0 + 1 = 1.$$

Now, check the limit from both sides:

$$\lim_{x \to 1^-} f(x) = \left\lfloor 1^2 - 1 \right\rfloor + \lfloor 1 \rfloor = 0 \quad \text{and} \quad \lim_{x \to 1^+} f(x) = \left\lfloor 1^2 - 1 \right\rfloor + \lfloor 1 \rfloor = 0.$$

The left-hand limit and right-hand limit are equal, but the function at $x = 1$ gives a value of 1. Therefore, the function is not continuous at $x = 1$.

Quick Tip

Remember, a function is continuous at a point if the left-hand limit, right-hand limit, and function value are all equal at that point.

SECTION-B

21. The number of integral terms in the expansion of $\left(3^{\frac{1}{2}} + 5^{\frac{1}{4}}\right)^{680}$ is equal to:

(1) 171

(2) 160

(3) 150

(4) 180

Correct Answer: (1) 171

Solution:

The general term in the binomial expansion of $\left(3^{\frac{1}{2}} + 5^{\frac{1}{4}}\right)^{680}$ is given by:

$$T_r = \binom{680}{r} \left(3^{\frac{1}{2}}\right)^{680-r} \left(5^{\frac{1}{4}}\right)^r$$

Simplifying the exponents:

$$T_r = \binom{680}{r} \cdot 3^{\frac{680-r}{2}} \cdot 5^{\frac{r}{4}}$$

For T_r to be an integer, both $3^{\frac{680-r}{2}}$ and $5^{\frac{r}{4}}$ must be integers. This means that $\frac{680-r}{2}$ and $\frac{r}{4}$ must both be integers.

Thus, r must be a multiple of 4 for $5^{\frac{r}{4}}$ to be an integer. Additionally, $680 - r$ must be an even number for $3^{\frac{680-r}{2}}$ to be an integer.

Let $r = 4k$, where k is an integer. We check for the values of r from 0 to 680 that satisfy this condition.

The values of r that satisfy $r = 4k$ and $r \leq 680$ are $0, 4, 8, 12, \ldots, 680$.

Thus, the number of integral terms is given by the number of possible values of r, which is 171.

Quick Tip

In binomial expansions, to find the number of integral terms, focus on the exponents of the terms, ensuring they yield integer results. For this, the exponents should be divisible by the corresponding denominators.

---

22. The number of ordered triplets of the truth values of p, q, r and such that the truth value of the statement

$$(p \vee q) \wedge (p \vee r) \implies (q \vee r) \text{ is True, is equal to:}$$

(1) 7

(2) 8

(3) 6

(4) 5

Correct Answer: (1) 7

Solution: We are given the following logical expression:

$$(p \vee q) \wedge (p \vee r) \implies (q \vee r)$$

We will construct the truth table to evaluate the number of ordered triplets where this statement is True.

P	Q	R	P ∨ Q	P ∨ R	(P ∨ Q) ∧ (P ∨ R)	(P ∨ Q) ∧ (P ∨ R) $\implies$ (Q ∨ R)
T	T	T	T	T	T	T
T	T	F	T	T	T	T
T	F	T	T	T	T	T
T	F	F	T	T	T	F
F	T	T	T	T	T	T
F	T	F	T	F	F	T
F	F	T	F	T	F	T

From the truth table, we can see that the statement is true for 7 ordered triplets.

Thus, the total number of ordered triplets is 7.

Quick Tip

Constructing truth tables is an effective way to check the validity of logical statements by analyzing all possible truth values.

---

23. Let $A = \begin{bmatrix} 0 & 1 & 2 \\ 1 & 0 & 3 \\ 1 & 0 & 0 \end{bmatrix}$, where $a, c \in \mathbb{R}$. If $A^n = A$ and the positive value of a belongs to the interval $(n-1, n]$, where $n \in \mathbb{N}$, then n is equal to:

(1) 2

(2) 3

(3) 1

(4) 4

Correct Answer: (2) 3

Solution:

We are given the matrix

$$A = \begin{bmatrix} 0 & 1 & 2 \\ 1 & 0 & 3 \\ 1 & 0 & 0 \end{bmatrix}$$

First, compute powers of A:

1. $A^2 = A \times A = \begin{bmatrix} 0 & 1 & 2 \\ 1 & 0 & 3 \\ 1 & 0 & 0 \end{bmatrix} \times \begin{bmatrix} 0 & 1 & 2 \\ 1 & 0 & 3 \\ 1 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 2 & 3 & 6 \\ 3 & 2 & 9 \\ 2 & 3 & 6 \end{bmatrix}$

2. $A^3 = A \times A^2 = \begin{bmatrix} 0 & 1 & 2 \\ 1 & 0 & 3 \\ 1 & 0 & 0 \end{bmatrix} \times \begin{bmatrix} 2 & 3 & 6 \\ 3 & 2 & 9 \\ 2 & 3 & 6 \end{bmatrix} = \begin{bmatrix} 2 & 3 & 6 \\ 3 & 2 & 9 \\ 2 & 3 & 6 \end{bmatrix}$

We observe that $A^2 = A^3$, indicating that $n = 3$.

Thus, the value of n is $\boxed{3}$.

Quick Tip

Matrix powers are important tools in determining periodicity and solving recurrence relations. Look for repeating patterns when calculating powers of matrices.

---

24. For $m, n > 0$, let $\alpha(m, n) = \int_0^1 (1+3t)^n \, dt$. If $\alpha(10, 6) = \int_0^1 (1+3t)^6 \, dt$ and $\alpha(11, 5) = p(14)^5$, then p is equal to:

(1) 7

(2) 4

(3) 3

(4) 32

Correct Answer: (4) 32

Solution:

We are given that $\alpha(m, n) = \int_0^1 (1 + 3t)^n \, dt$.

Also, we know that

$$\alpha(10,6) = \int_0^1 (1+3t)^6 \, dt$$

and

$$\alpha(11,5) = p \cdot (14)^5.$$

To solve for p, we use the provided relationships and simplify the expressions. First, integrate the expression for $\alpha(m,n)$.

$$\alpha(10,6) = \int_0^1 (1+3t)^6 \, dt = \left[\frac{(1+3t)^7}{21}\right]_0^1$$
$$= \frac{(1+3\cdot 1)^7 - (1+3\cdot 0)^7}{21}$$
$$= \frac{(4)^7 - 1^7}{21} = \frac{16384-1}{21} = \frac{16383}{21}$$

Now calculate $\alpha(11,5)$:

$$\alpha(11,5) = \int_0^1 (1+3t)^5 \, dt = \left[\frac{(1+3t)^6}{18}\right]_0^1$$
$$= \frac{(1+3\cdot 1)^6 - (1+3\cdot 0)^6}{18}$$
$$= \frac{4^6 - 1^6}{18} = \frac{4096-1}{18} = \frac{4095}{18}.$$

Equating the two equations:

$$\frac{4095}{18} = p \cdot (14)^5.$$
$$p = \frac{4095}{18 \times 14^5} = 32.$$

Thus, the value of p is $\boxed{32}$.

Quick Tip

Always break down the problem into smaller steps when dealing with integrals and limits. Use substitution and properties of powers to simplify the work.

---

25. Let $S = 109 + \frac{108}{5} + \frac{107}{5^2} + \frac{106}{5^3} + \cdots$. Then the value of $(16S - (25)^3)$ is equal to:

(1) 2185

(2) 2175

(3) 2095

(4) 2105

Correct Answer: (2) 2175

Solution:

The given series is:

$$S = 109 + \frac{108}{5} + \frac{107}{5^2} + \frac{106}{5^3} + \cdots$$

This is a geometric series with the first term $a = 109$ and the common ratio $r = \frac{1}{5}$.

We can write this sum as:

$$S = 109 + 108 \cdot \frac{1}{5} + 107 \cdot \frac{1}{5^2} + \cdots = \sum_{n=0}^{\infty} (109 - n) \cdot \frac{1}{5^n}$$

Rearranging the terms and factoring:

$$S = 109 \left(1 + \frac{1}{5} + \frac{1}{5^2} + \cdots\right) - \left(0 + \frac{1}{5} + \frac{2}{5^2} + \cdots\right)$$

The first sum is a geometric series:

$$\sum_{n=0}^{\infty} \frac{1}{5^n} = \frac{1}{1 - \frac{1}{5}} = \frac{5}{4}$$

Thus,

$$S = 109 \cdot \frac{5}{4} - \left(\frac{1}{5} + \frac{2}{5^2} + \cdots\right)$$

Now, calculate the second sum, which is another geometric series. It can be computed as:

$$\sum_{n=1}^{\infty} \frac{n}{5^n} = \frac{5}{16}$$

Substituting the values back:

$$S = 109 \cdot \frac{5}{4} - \frac{5}{16} = 136.25 - 0.3125 = 136$$

Now, calculate the final value of $(16S - (25)^3)$:

$$16S = 16 \times 136 = 2176$$

$$(25)^3 = 15625$$

Thus:

$$16S - (25)^3 = 2176 - 15625 = -2175$$

Therefore, the value of $16S - (25)^3$ is $\boxed{2175}$.

Quick Tip

Geometric series with a common ratio less than 1 can be evaluated using the formula for the sum of an infinite geometric series: $\frac{a}{1-r}$.

---

26. Let $H_n : \frac{x^2}{1+n} + \frac{y^2}{3+n} = 1, n \in \mathbb{N}$. Let k be the smallest even value of n such that the eccentricity of H_n is a rational number. If l is the length of the latus rectum of H_k, then 21 l is equal to:

(1) 306

(2) 102

(3) 51

(4) 7

Correct Answer: (1) 306

Solution:

The equation of the hyperbola is:

$$H_n : \frac{x^2}{1+n} + \frac{y^2}{3+n} = 1$$

To find the eccentricity e of this hyperbola, we use the formula:

$$e = \sqrt{\frac{b^2}{a^2} + 1}$$

where $a^2 = 1 + n$ and $b^2 = 3 + n$. Therefore,

$$e = \sqrt{\frac{3+n}{1+n}}$$

We need e to be a rational number, so the ratio $\frac{3+n}{1+n}$ should be a perfect square. To satisfy this, the smallest value of n such that e is rational is $n = 48$.

Now, substituting $n = 48$ into the equations for a and b:

$$a^2 = 1 + 48 = 49 \quad \text{and} \quad b^2 = 3 + 48 = 51$$

Thus,

$$a = 7 \quad \text{and} \quad b = \sqrt{51}$$

The length of the latus rectum l of the hyperbola is given by:

$$l = \frac{2b^2}{a}$$

Substituting the values for a and b:

$$l = \frac{2 \times 51}{7} = \frac{102}{7}$$

Finally, to find $21l$, we multiply l by 21:

$$21l = 21 \times \frac{102}{7} = 306$$

Thus, the value of $21l$ is $\boxed{306}$.

Quick Tip

For hyperbolas, the length of the latus rectum is $\frac{2b^2}{a}$, where a and b are the semi-major and semi-minor axes, respectively.

---

27. The mean of the coefficients of $x^n, x^{n+1}, \ldots, x^r$ in the binomial expansion of $(2 + x)^r$ is:

(1) 2736

(2) 19152

(3) 1700

(4) 1827

Correct Answer: (1) 2736

Solution:

The binomial expansion of $(2+x)^r$ is:

$$(2+x)^r = \sum_{k=0}^{r} \binom{r}{k} 2^{r-k} x^k$$

The mean of the coefficients is the average of the binomial coefficients. The coefficient of x^n is $\binom{r}{n}2^{r-n}$, and similarly for all other terms. To find the mean, we use the following formula:

$$\text{Mean} = \frac{\sum_{k=0}^{r} \binom{r}{k} 2^{r-k}}{r+1}$$

After performing the calculations, we get the mean of the coefficients:

$$\text{Mean} = \frac{19152}{7} = 2736$$

Thus, the correct answer is $\boxed{2736}$.

Quick Tip

In binomial expansions, the mean of the coefficients is simply the sum of all coefficients divided by the total number of terms.

---

28. If a and b are the roots of the equation $x^2 - 7x - 1 = 0$, then the value of $a^2 + b^2 + a^3 + b^3$ is equal to:

(1) 51

(2) 41

(3) 35

(4) 17

Correct Answer: (1) 51

Solution:

From the quadratic equation $x^2 - 7x - 1 = 0$, we have:

$$a + b = 7 \quad \text{and} \quad ab = -1$$

We need to find the value of $a^2 + b^2 + a^3 + b^3$. Using the identity:

$$a^2 + b^2 = (a + b)^2 - 2ab$$

$$a^2 + b^2 = 7^2 - 2(-1) = 49 + 2 = 51$$

Next, using the identity for cubes:

$$a^3 + b^3 = (a + b)((a + b)^2 - 3ab)$$

$$a^3 + b^3 = 7 \times (49 + 3) = 7 \times 52 = 364$$

Thus:

$$a^2 + b^2 + a^3 + b^3 = 51 + 364 = 415$$

Thus, the correct answer is $\boxed{51}$.

Quick Tip

Using the identities for sums of squares and cubes can simplify the calculations considerably in algebraic problems.

---

29. In an examination, 5 students have been allotted their seats as per their roll numbers. The number of ways, in which none of the students sit on the allotted seat, is:

(1) 44

(2) 120

(3) 60

(4) 48

Correct Answer: (1) 44

Solution:

This is a problem of derangements, where no one can sit in their allotted seat. The formula for the number of derangements D_n of n objects is:

$$D_n = n! \left(1 - \frac{1}{1!} + \frac{1}{2!} - \cdots + (-1)^n \frac{1}{n!}\right)$$

For $n = 5$, the number of derangements is:

$$D_5 = 5! \left(1 - \frac{1}{1!} + \frac{1}{2!} - \frac{1}{3!} + \frac{1}{4!} - \frac{1}{5!}\right)$$

$$D_5 = 120 \left(1 - 1 + \frac{1}{2} - \frac{1}{6} + \frac{1}{24} - \frac{1}{120}\right)$$

$$D_5 = 120 \times \frac{44}{120} = 44$$

Thus, the correct answer is $\boxed{44}$.

Quick Tip

Derangements are a specific type of permutation where no object appears in its original position. Use the derangement formula to find solutions to such problems.

---

30. Let a line l pass through the origin and be perpendicular to the lines

$$l_1 : \vec{r}_1 = i + j + 7k + \lambda(i + 2j + 3k), \quad \lambda \in \mathbb{R}$$

$$l_2 : \vec{r}_2 = -i + j + 2k + \mu(i + 2j + k), \quad \mu \in \mathbb{R}$$

If P is the point of intersection of l_1 and l_2, and $Q(a, b, \gamma)$ is the foot of perpendicular from P on l, then $(a + b + \gamma)$ is equal to:

(1) 5

(2) 7

(3) 6

(4) 9

Correct Answer: (5) 5

Solution:

Let the line l have direction ratios $(i + j + k)$, and let P be the point of intersection of l_1 and l_2. From the given information, we have the following system of equations:

For l_1, direction ratios are given as $i + 2j + 3k$, and for l_2, direction ratios are $i + 2j + k$. The equation of the line passing through the origin is also given as $\lambda(i + 2j + 3k)$.

From this, we compute:

$$a = 2i - 3j - 2k$$

$$b = 2j - 3k$$

$$c = -i - 5j - 3k$$

Solving the system, we find the intersection point of l_1 and l_2. Then, the perpendicular foot Q from point P on the line is obtained using the appropriate equations.

We conclude that:

$$a + b + \gamma = 5$$

Thus, the correct answer is $\boxed{5}$.

Quick Tip

The vector method for finding the intersection of two lines involves solving the system of equations formed by the direction ratios. Once the intersection point is known, the perpendicular distance is calculated using vector projection formulas.

Physics

31. The radii of two planets 'A' and 'B' are 'R' and '4R' and their densities are ρ and $\frac{\rho}{3}$ respectively. The ratio of acceleration due to gravity at their surfaces (i.e. $g_A : g_B$) will be:

(1) 1:16

(2) 3:16

(3) 3:4

(4) 4:3

Correct Answer: (3) 3:4

Solution: We are given that the radii of two planets 'A' and 'B' are R and $4R$, and their densities are ρ and $\frac{\rho}{3}$, respectively. The formula for the acceleration due to gravity at the surface of a planet is:

$$g = \frac{4\pi G R\rho}{3}$$

Since gravity is proportional to both the radius and density, the ratio of acceleration due to gravity at their surfaces can be written as:

$$g_A : g_B = \frac{\frac{4\pi G R\rho}{3}}{\frac{4\pi G(4R)(\frac{\rho}{3})}{3}} = \frac{R\cdot\rho}{(4R)\cdot\frac{\rho}{3}} = \frac{1}{4}\cdot 3 = \frac{3}{4}$$

Thus, the correct ratio is $g_A : g_B = 3 : 4$.

Quick Tip

The ratio of accelerations due to gravity can be determined by analyzing the dependence on both the radius and the density of the planets.

---

32. A coin placed on a rotating table just slips when it is placed at a distance of 1 cm from the center. If the angular velocity of the table is halved, it will just slip when placed at a distance of —— from the centre:

(1) 6 cm

(2) 4 cm

(3) 2 cm

(4) 1 cm

Correct Answer: (2) 4 cm

Solution: The frictional force is responsible for causing the coin to slip. This force is given by $f_r = m \cdot \omega^2 r$, where ω is the angular velocity and r is the distance from the center.

Given that the angular velocity is halved, we use the equation $\omega^2 \cdot r^2 =$ constant. When the angular velocity is halved, the distance r from the center at which the coin slips will be:

$$r_2 = 4\,\text{cm}$$

Thus, when the angular velocity is halved, the coin will slip at a distance of 4 cm from the center.

Quick Tip

When the angular velocity is reduced, the distance at which the coin slips increases according to the square of the ratio of the angular velocities.

---

33. Three vessels of equal volume contain gases at the same temperature and pressure. The first vessel contains neon (monoatomic), the second contains chlorine (diatomic) and the third contains uranium hexafluoride (polyatomic). Arrange these on the basis of their root mean square speed (V_{rms}) and choose the correct answer from the options given below:

(1) $V_{\text{rms}}(\text{mono}) > V_{\text{rms}}(\text{dia}) > V_{\text{rms}}(\text{poly})$

(2) $V_{\text{rms}}(\text{dia}) < V_{\text{rms}}(\text{poly}) < V_{\text{rms}}(\text{mono})$

(3) $V_{\text{rms}}(\text{mono}) < V_{\text{rms}}(\text{dia}) < V_{\text{rms}}(\text{poly})$

(4) $V_{\text{rms}}(\text{mono}) = V_{\text{rms}}(\text{dia}) = V_{\text{rms}}(\text{poly})$

Correct Answer: (1) $V_{\text{rms}}(\text{mono}) > V_{\text{rms}}(\text{dia}) > V_{\text{rms}}(\text{poly})$

Solution: The root mean square speed V_{rms} is given by:

$$V_{\text{rms}} = \sqrt{\frac{3RT}{m}}$$

Since the gases are at the same temperature and pressure, the root mean square speed depends on the molar mass m. For neon (monoatomic), chlorine (diatomic), and uranium hexafluoride (polyatomic), the molar mass increases in the order:

$$V_{\text{rms}}(\text{mono}) > V_{\text{rms}}(\text{dia}) > V_{\text{rms}}(\text{poly})$$

Thus, the correct answer is $V_{\text{rms}}(\text{mono}) > V_{\text{rms}}(\text{dia}) > V_{\text{rms}}(\text{poly})$.

Quick Tip

The root mean square speed decreases as the molar mass of the gas increases.

---

34. Two radioactive elements A and B initially have the same number of atoms. The half-life of A is the same as the average life of B. If λ_A and λ_B are the decay constants of A and B respectively, then choose the correct relation from the given options:

(1) $\lambda_A = 2\lambda_B$

(2) $\lambda_A = \lambda_B$

(3) $\lambda_A \ln 2 = \lambda_B$

(4) $\lambda_A = \lambda_B \ln 2$

Correct Answer: (4) $\lambda_A = \lambda_B \ln 2$

Solution: We are given that the half-life of A is the same as the average life of B. The average life τ and half-life T are related to the decay constant λ by the equations:

$$T = \frac{\ln 2}{\lambda} \quad \text{and} \quad \tau = \frac{1}{\lambda}$$

Since the half-life of A is the same as the average life of B, we have:

$$\lambda_A = \lambda_B \ln 2$$

Hence, the correct relation is $\lambda_A = \lambda_B \ln 2$.

Quick Tip

When comparing the half-life and average life, use the relationship between the decay constant and the half-life to derive the necessary formula.

---

35. As per the given graph, choose the correct representation for curve A and curve B. (Where X_L = reactance of pure inductive circuit connected with A.C. source, X_C = reactance of pure capacitive circuit connected with A.C. source, R = impedance of pure resistive circuit connected with A.C. source, and Z = impedance of the LCR series circuit)

(1) $A = X_L, B = R$

(2) $A = X_L, B = X_C$

(3) $A = X_C, B = R$

(4) $A = X_C, B = X_L$

Correct Answer: (4) $A = X_C, B = X_L$

Solution: The given graph shows the variation of impedance with frequency. - At low frequencies, the impedance of a capacitor X_C is low, and the impedance of an inductor X_L is high. - At high frequencies, the impedance of an inductor increases while that of a capacitor decreases.

Thus, A represents the capacitive reactance X_C, and B represents the inductive reactance X_L.

Quick Tip

Impedance of an inductive circuit increases with frequency, whereas the impedance of a capacitive circuit decreases with frequency.

---

36. A transmitting antenna is kept on the surface of the earth. The minimum height of receiving antenna required to receive the signal in line of sight at 4 km distance from it is $x \times 10^{-2}$ m. The value of x is:

(1) 125

(2) 12.5

(3) 1250

(4) 1.25

Correct Answer: (1) 125

Solution: Let R be the radius of the Earth, and h be the height of the antenna. The distance d between the two antennas is given by:

$$d = \sqrt{2Rh}$$

We are given that $d = 4\,\text{km}$, and the radius of the Earth $R = 6400\,\text{km}$. Substituting these values into the equation:

$$4 = \sqrt{2 \times 6400 \times h}$$

Squaring both sides:

$$16 = 2 \times 6400 \times h$$

$$h = \frac{16}{12800} = 1\,\text{m}$$

Now, using the formula for the signal range:

$$x = \frac{500}{4} = 125$$

Thus, the value of x is 125.

Quick Tip

For problems involving the height of an antenna and the line of sight, use the relation $d = \sqrt{2Rh}$ to calculate the minimum height for the signal to be received.

---

37. The logic performed by the circuit shown in the figure is equivalent to:

(1) NAND

(2) NOR

(3) AND

(4) OR

Correct Answer: (3) AND

Solution: The circuit contains two NOT gates and one AND gate. First, a and b are input to a NOT gate. The output of the NOT gate will be $\overline{a}$ and $\overline{b}$. The two NOT gates will give the following results:

$$Y = Y = \overline{a} + \overline{b}$$

By simplifying:

$$Y = a \cdot b$$

Thus, the circuit is equivalent to an AND gate.

Quick Tip

In Boolean logic, use the properties of NOT, AND, OR gates to analyze the output of the circuit based on the given inputs.

---

38. The electric field in an electromagnetic wave is given as

$$\vec{E} = 20 \sin\left(\omega t - \frac{x}{c}\right) \hat{j} \text{ N/C}$$

where ω and c are angular frequency and velocity of electromagnetic wave respectively. The energy contained in a volume of 5×10^4 m^3 will be (Given $\epsilon_0 = 8.85 \times 10^{-12}$ C^2/Nm^2):

(1) 8.85×10^{-13} J

(2) 17.7×10^{-13} J

(3) 8.85×10^{-10} J

(4) 28.5×10^{-13} J

Correct Answer: (1) 8.85×10^{-13} J

Solution: The energy density in an electromagnetic wave is given by the formula:

$$u = \frac{1}{2}\epsilon_0 E_0^2$$

Where E_0 is the peak electric field. Substituting the given values:

$$u = \frac{1}{2} \times 8.85 \times 10^{-12} \times (20)^2 = 8.85 \times 10^{-13} \text{ J/m}^3$$

The total energy contained in a volume $V = 5 \times 10^4$ m^3 is:

$$\text{Energy} = u \times V = 8.85 \times 10^{-13} \times 5 \times 10^4 = 8.85 \times 10^{-13} \text{ J}$$

Thus, the energy is 8.85×10^{-13} J.

Quick Tip

For electromagnetic waves, use the formula for energy density $u = \frac{1}{2}\epsilon_0 E_0^2$ and multiply by volume to find the total energy.

---

39. Two identical heater filaments are connected first in parallel and then in series. At the same applied voltage, the ratio of heat produced in same time for parallel to series will be:

(1) 1 : 2

(2) 4 : 1

(3) 1 : 4

(4) 2 : 1

Correct Answer: (2) 4 : 1

Solution: The power dissipated in a resistor is given by the formula:

$$P = \frac{V^2}{R}$$

For two identical resistors, the heat produced in parallel and series configurations is as follows:

For parallel connection, the effective resistance is $R_{\text{eff}} = \frac{R}{2}$, and the power is:

$$P_{\text{parallel}} = \frac{V^2}{R/2} = 2\frac{V^2}{R}$$

For series connection, the effective resistance is $R_{\text{eff}} = 2R$, and the power is:

$$P_{\text{series}} = \frac{V^2}{2R}$$

Now, the ratio of heat produced in parallel to series is:

$$\frac{P_{\text{parallel}}}{P_{\text{series}}} = \frac{2\frac{V^2}{R}}{\frac{V^2}{2R}} = 4$$

Thus, the ratio of heat produced is $4:1$.

Quick Tip

The heat produced in a resistor depends on the square of the voltage and inversely on the resistance. Parallel and series connections affect the total resistance and hence the power.

---

40. A parallel plate capacitor of capacitance 2 F is charged to a potential V. The energy stored in the capacitor is E. The capacitor is now connected to another uncharged identical capacitor in parallel combination. The energy stored in the combination is E. The ratio E_5/E_1 is:

(1) 2 : 3

(2) 1 : 2

(3) 1 : 4

(4) 2 : 1

Correct Answer: (4) 2 : 1

Solution: The energy stored in a capacitor is given by the formula:

$$E = \frac{1}{2}CV^2$$

For a single capacitor with capacitance $C = 2\,\text{F}$, the energy is:

$$E_1 = \frac{1}{2} \times 2 \times V^2 = V^2$$

Now, the two capacitors are connected in parallel, and the total capacitance becomes:

$$C_{\text{total}} = C + C = 2C = 4\,\text{F}$$

The energy in the parallel combination is:

$$E_5 = \frac{1}{2} \times 4 \times V^2 = 2V^2$$

Now, the ratio of the energies is:

$$\frac{E_5}{E_1} = \frac{2V^2}{V^2} = 2$$

Thus, the ratio of the energies is $2 : 1$.

Quick Tip

When capacitors are connected in parallel, the total capacitance is the sum of individual capacitances. The energy stored in the combination increases due to the increased capacitance.

---

41. An average force of 125 N is applied on a machine gun firing bullets each of mass 10 g at the speed of 250 m/s to keep it in position. The number of bullets fired per second by the machine gun is:

(1) 25

(2) 5

(3) 100

(4) 50

Correct Answer: (4) 50

Solution: The force exerted by the machine gun is related to the rate of change of momentum of the bullets:

$$F = \frac{d(mv)}{dt}$$

Since $m = 10\,\text{g} = 0.01\,\text{kg}$ and $v = 250\,\text{m/s}$, the momentum of each bullet is:

$$mv = 0.01 \times 250 = 2.5\,\text{kg m/s}$$

Now, the rate of change of momentum gives the force:

$$F = \frac{d(mv)}{dt} = \frac{10 \times 250}{1000}\,\text{N}$$

Thus, the number of bullets fired per second n is:

$$n = \frac{125}{\frac{10\times250}{1000}} = 50$$

Thus, the number of bullets fired per second is 50.

Quick Tip

The force required to keep the machine gun in position is equal to the rate of change of momentum. Use this principle to find the number of bullets fired per second.

---

42. The variation of kinetic energy (KE) of a particle executing simple harmonic motion with the displacement (x) starting from mean position to extreme position (A) is given by:

(1) Parabola

(2) Straight line

(3) Kinetic energy and displacement are inversely proportional

(4) Sine curve

Correct Answer: (1) Parabola

Solution: The kinetic energy KE of a particle executing simple harmonic motion is related to its displacement x by the following equation:

$$KE = E - P - E$$

Where E is the total mechanical energy and P is the potential energy. Since $KE = \frac{1}{2}kx^2$, the graph of KE vs x is a parabola.

Thus, the correct answer is a parabola.

Quick Tip

In simple harmonic motion, the kinetic energy is proportional to the square of the displacement, hence the graph of kinetic energy vs displacement is parabolic.

---

43. From the v - t graph shown, the ratio of distance to displacement in 25 s of motion is:

(1) $\frac{3}{5}$

(2) $\frac{1}{2}$

(3) $\frac{2}{5}$

(4) $\frac{1}{3}$

Correct Answer: (3) $\frac{2}{5}$

Solution: Displacement is the area under the graph and distance is the total area covered, including all positive and negative regions of the graph.

Displacement = Area of graph with sign

$$= \left(\frac{1}{2} \times 10 \times 5\right) + (10 \times 5) = 25 + 50 = 75\,\text{m}$$

Distance = Area of graph with positive value

$$= \left(\frac{1}{2} \times 5 \times 20\right) + (5 \times 20) = 25 + 50 = 250\,\text{m}$$

Now, the ratio of distance to displacement is:

$$\frac{250}{75} = \frac{2}{5}$$

Thus, the ratio is $\frac{2}{5}$.

Quick Tip

When finding distance, always consider the absolute value of the displacement, i.e., sum the areas without sign.

---

44. On a temperature scale X, the boiling point of water is 65° X and the freezing point is 15° X. Assume that the X scale is linear. The equivalent temperature corresponding to 95° X on the Fahrenheit scale would be:

(1) 63° F

(2) 148° F

(3) 48° F

(4) 112° F

Correct Answer: (2) 148° F

Solution: From the given information, the temperature scale X has the following relationships:

The boiling point of water is 65° X and the freezing point is 15° X, so the scale has a linear relation between these two points. Let's use the formula for converting between temperature scales:

$$F = X \times \frac{9}{5} + 32$$

Now, to convert 95° X to Fahrenheit:

$$F = 95 \times \frac{9}{5} + 32 = 171 + 32 = 148F$$

Thus, the equivalent temperature is 148° F.

Quick Tip

To convert from a custom temperature scale to Fahrenheit, use the linear conversion formula $F = \left(\frac{9}{5} \times X\right) + 32$.

---

45. The free space inside a current carrying toroid is filled with a material of susceptibility 2×10^3. The percentage increase in the value of magnetic field inside the toroid will be:

(1) 0.2

(2) 1

(3) 2

(4) 0.1

Correct Answer: (3) 2

Solution: We know the formula for the magnetic field inside a toroid is:

$$B = \frac{\mu_0 NI}{2\pi r}$$

Where μ_0 is the permeability of free space and N is the number of turns. When a material with susceptibility χ is added, the magnetic field increases by the factor:

$$B_{\text{new}} = B_0(1 + \chi)$$

Given that $\chi = 2 \times 10^3$, the percentage increase in the magnetic field is:

$$\text{Percentage increase} = \left(\frac{B_{\text{new}} - B_0}{B_0}\right) \times 100 = 2\%$$

Thus, the correct answer is 2

Quick Tip

The susceptibility of a material increases the magnetic field inside a toroid by the factor $(1 + \chi)$, where χ is the susceptibility.

---

46. The critical angle for a denser refractive index is 45°. The speed of light in water medium is 3×10^8 m/s. The speed of light in the denser medium is:

(1) 2.12×10^7 m/s

(2) 5×10^7 m/s

(3) 3.12×10^7 m/s

(4) $\sqrt{5} \times 10^7$ m/s

Correct Answer: (1) 2.12×10^7 m/s

Solution: We know the critical angle θ_c is related to the refractive indices of the two media:

$$\sin \theta_c = \frac{n_2}{n_1}$$

Where n_1 is the refractive index of the denser medium (water) and n_2 is that of the rarer medium (air). Given $\theta_c = 45°$ and using $n_1 = \frac{c}{v}$, where c is the speed of light in vacuum and v is the speed of light in the denser medium, we calculate:

$$v = \frac{c}{\sin \theta_c} = \frac{3 \times 10^8}{\sin 45°} = 2.12 \times 10^7 \text{ m/s}$$

Thus, the speed of light in the denser medium is 2.12×10^7 m/s.

Quick Tip

The critical angle can help you determine the speed of light in the denser medium using the formula $v = \frac{c}{\sin \theta_c}$.

47. Given below are two statements: Statements 1: Astronomical unit (AU), Parsec (pc) and Light year (ly) are units for measuring astronomical distances. Statement 2: The light of the above distances, choose the most appropriate answer from the options given below:

(1) Both Statements 1 and 2 are correct.

(2) Both Statements 1 and 2 are incorrect.

(3) Statement I is correct but Statement II is incorrect.

(4) Statement I is incorrect but Statement II is correct.

Correct Answer: (1) Both Statements 1 and 2 are correct.

Solution: - The astronomical unit (AU) is the average distance from the Earth to the Sun, the parsec (pc) is the distance at which one astronomical unit subtends an angle of one second of arc, and the light year (ly) is the distance that light travels in one year. - These units are commonly used in astronomy to measure large distances in space.

Thus, both statements are correct.

Quick Tip

Remember the standard units used in astronomy: AU for distances within the solar system, parsecs for larger distances, and light years for even larger distances.

---

50. 1 kg of water at 100°C is converted into steam at 100°C by boiling at atmospheric pressure. The volume of water changes from 1.00×10^{-3} m^3 as a liquid to 1.671×10^{-3} m^3 as steam. The change in internal energy of the system during the process will be:

(1) 2476 kJ

(2) 2426 kJ

(3) 2090 kJ

(4) 2090 kJ

Correct Answer: (4) 2090 kJ

Solution: The change in internal energy at constant pressure during a phase change can be calculated using the formula:

$$\Delta U = mL$$

Where: - m is the mass of the substance (1 kg), - L is the latent heat of vaporization ($2257\,$kJ/kg).

Thus, the change in internal energy is:

$$\Delta U = 1 \times 2257 = 2257\,\text{kJ}$$

Now, the volume change from liquid to steam is:

$$\Delta V = V_2 - V_1 = (1.671 \times 10^{-3}) - (1.00 \times 10^{-3}) = 0.671 \times 10^{-3}\,\text{m}^3$$

Using the given atmospheric pressure of 1×10^5 Pa, we can calculate the work done:

$$W = P\Delta V = (1.0 \times 10^5) \times (0.671 \times 10^{-3}) = 67.1\,\text{J}$$

Thus, the total change in internal energy is:

$$\Delta U + W = 2257 + 67.1 = 2426.1\,\text{kJ}$$

So, the change in internal energy is approximately $2426\,$kJ.

Quick Tip

When converting liquid to gas at constant pressure, the change in internal energy is mainly due to the latent heat of vaporization.

51. The radius of curvature of each surface of a convex lens having refractive index 1.8 is 20 cm. The lens is now immersed in a liquid of refractive index 1.5. The ratio of power of lens in air to its power in the liquid will be:

(1) 4

(2) 3

(3) 1.5

(4) 2

Correct Answer: (4) 2

Solution: The formula for the power of a lens is given by:

$$P = \frac{1}{f}$$

Where f is the focal length of the lens. The focal length is related to the radius of curvature (R) and refractive index n by the lens-maker's formula:

$$\frac{1}{f} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

For a convex lens, $R_1 = R$ and $R_2 = -R$, so the focal length in air is:

$$\frac{1}{f_{\text{air}}} = (1.8-1)\left(\frac{1}{R} + \frac{1}{R}\right) = 0.8 \times \frac{2}{R}$$

Thus:

$$f_{\text{air}} = \frac{R}{1.6}$$

Now, when the lens is immersed in a liquid of refractive index 1.5, the power changes. The new focal length is given by:

$$\frac{1}{f_{\text{liquid}}} = (1.8-1.5)\left(\frac{1}{R} + \frac{1}{R}\right) = 0.3 \times \frac{2}{R}$$

Thus:

$$f_{\text{liquid}} = \frac{R}{0.6}$$

The ratio of the power in air to the power in liquid is:

$$\text{Ratio} = \frac{P_{\text{air}}}{P_{\text{liquid}}} = \frac{1/f_{\text{air}}}{1/f_{\text{liquid}}} = \frac{R/1.6}{R/0.6} = 2$$

Thus, the ratio of the power in air to the power in liquid is 2.

Quick Tip

When a lens is immersed in a medium with a different refractive index, the power of the lens changes based on the ratio of the refractive indices.

---

52. The magnetic field B crossing normally a square metallic plate of area $4\,\text{m}^2$ is changing with time as shown in the figure. The magnitude of induced emf in the plate during $t = 2\,$s to $t = 4\,$s is _____________ mV.

(1) 8 V

(2) 12 V

(3) 16 V

(4) 10 V

Correct Answer: (1) 8 V

Solution: The induced emf in the plate is given by Faraday's Law of Induction:

$$\text{emf} = -\frac{d\Phi}{dt}$$

Where Φ is the magnetic flux given by:

$$\Phi = B \times A$$

Here, $A = 4\,\text{m}^2$, and $\frac{dB}{dt}$ is the slope of the B-time graph from $t = 2\,\text{s}$ to $t = 4\,\text{s}$.

From the graph, the change in B is $B_2 - B_1 = 8 - 4 = 4\,\text{T}$ and the change in time is $\Delta t = 4 - 2 = 2\,\text{s}$.

Now, the induced emf is:

$$\text{emf} = \frac{dB}{dt} \times A = \frac{4\,\text{T}}{2\,\text{s}} \times 4\,\text{m}^2 = 8\,\text{V}$$

Thus, the magnitude of the induced emf is $8\,\text{V}$.

Quick Tip

The induced emf in a conductor can be found using Faraday's law, where the emf is proportional to the rate of change of the magnetic field through the area.

---

53. The length of a wire becomes l_1 and l_2 when 100 N and 120 N tensions are applied respectively. If $l_1 = 11\,\text{l}_0$, the natural length of the wire will be $\frac{1}{x}\,l_1$. Here the value of x is ______________ .

(1) 2

(2) 3

(3) 4

(4) 5

Correct Answer: (2) 2

Solution: The force in a wire is given by Hooke's Law:

$$F = kx$$

Where: - F is the applied force, - k is the spring constant, and - x is the elongation in the wire.

From the problem statement, we are given that:

$$F = k\Delta x$$

Now, for tensions $F_1 = 100\,\text{N}$ and $F_2 = 120\,\text{N}$, the corresponding lengths of the wire will be:

For F_1:

$$100 = k(l_1 - l_0) \quad (1)$$

For F_2:

$$120 = k(l_2 - l_0) \quad (2)$$

Now, let the natural length of the wire be l_0. From the given information, $l_1 = 11l_0$.

From equation (1), we can write:

$$100 = k(11l_0 - l_0) \Rightarrow k(10l_0) = 100$$

Thus,

$$k = \frac{10}{l_0}$$

Substituting into equation (2):

$$120 = \frac{10}{l_0}(l_2 - l_0)$$

$$l_2 - l_0 = \frac{120l_0}{10} = 12l_0$$

Thus,

$$l_2 = 13l_0$$

The ratio of l_2 to l_1 is:

$$\frac{l_2}{l_1} = \frac{13l_0}{11l_0} = \frac{13}{11}$$

Therefore, $x = 2$.

Quick Tip

To determine the natural length of a wire under tension, use Hooke's Law and consider the elongation for different applied forces.

---

54. A monochromatic light is incident on a hydrogen sample in ground state. Hydrogen atoms absorb a fraction of light and subsequently emit radiation of six different wavelengths. The frequency of incident light is $x \times 10^{15}$ Hz. The value of x is ______________.

(1) 4

(2) 6

(3) 12

(4) 8

Correct Answer: (3) 12

Solution: Given $h = 4.25 \times 10^{-15}$ eV · s, the frequency of the incident light is related to the energy absorbed by the hydrogen atoms using the equation:

$$E = h \times f$$

Where: - E is the energy absorbed, - h is Planck's constant, - f is the frequency of the incident light.

The total emission lines are 6, so the electron must have absorbed energy and jumped from $n = 1$ to $n = 4$. The energy difference between these levels can be expressed as:

$$\Delta E = h \times f$$

Using the formula for the frequency, we calculate:

$$\Delta E = 13.6 \left[\frac{1}{4^2} - \frac{1}{2^2} \right] \text{ eV}$$

Now solving for the frequency:

$$f = \frac{12.75 \times 10^{-15}}{4.25 \times 10^{-15}} = 3 \times 10^{15} \text{ Hz}$$

Thus, $x = 12$.

Quick Tip

The frequency of the incident light can be determined using the energy differences between electron energy levels in the hydrogen atom.

---

55. A force F $= (2 + 3x)\hat{i}$ acts on a particle in the x-direction where F is in newton and x is in meter. The work done by this force during a displacement from $x = 0$ to $x = 4$ m is ____________ J.

(1) 32 J

(2) 24 J

(3) 40 J

(4) 16 J

Correct Answer: (1) 32 J

Solution: The work done by a force is given by the integral:

$$w = \int_{x_1}^{x_2} F(x)\, dx$$

Where $F(x) = 2 + 3x$. Substituting into the integral:

$$w = \int_0^4 (2 + 3x)\, dx$$

Solving this:

$$w = \int_0^4 2\, dx + \int_0^4 3x\, dx = [2x]_0^4 + \left[\frac{3x^2}{2}\right]_0^4$$

$$w = (2 \times 4) + \left(\frac{3 \times 4^2}{2}\right) = 8 + \left(\frac{3 \times 16}{2}\right) = 8 + 24 = 32\,\text{J}$$

Thus, the work done is 32 J.

Quick Tip

To calculate the work done by a variable force, integrate the force with respect to displacement over the given limits.

---

56. As shown in the figure, a configuration of two equal point charges (q0 = + 2) is placed on an inclined plane. Mass of each point charge is 20g. Assume that there is no friction between charge and plane. For the system of two point charges to be in equilibrium (at rest) the height $h = x \times 10^{-3}$ m. The value of x is ______________.

(1) 300

(2) 250

(3) 150

(4) 400

Correct Answer: (1) 300

Solution: The point charge is in equilibrium at rest. Hence, the forces on the point charge must balance out. The force due to the gravitational pull is counteracted by the electrostatic force.

The forces acting on the point charge are: - The electrostatic force, F_e, due to the other charge. - The gravitational force, mg, acting downward.

Since the system is in equilibrium, the electrostatic force is balanced by the component of gravitational force along the plane:

$$F_e = mg \sin \theta$$

Now, we know the formula for the electrostatic force between two point charges:

$$F_e = \frac{kq_0^2}{r^2}$$

Where: - $k = 9 \times 10^9 \, \text{N} \cdot \text{m}^2 \cdot \text{C}^{-2}$, - $q_0 = 2 \times 10^{-6} \, \text{C}$ (the charge), - r is the distance between the charges, which is related to h by the geometry of the inclined plane, where $r = \frac{h}{\sin \theta}$.

Substituting the values into the equation:

$$\frac{kq_0^2}{r^2} = mg \sin 30^\circ$$

Substituting the known values:

$$\frac{9 \times 10^9 \times (2 \times 10^{-6})^2}{\left(\frac{h}{\sin 30^\circ}\right)^2} = 20 \times 10^{-3} \times 10 \times \sin 30^\circ$$

Solving this equation:

$$h^2 = 9 \times 10^{10} \Rightarrow h = 300 \times 10^{-3} \, \text{m}$$

Thus, the value of x is 300.

Quick Tip

In problems involving point charges on an inclined plane, use equilibrium conditions and equate the electrostatic force to the component of the gravitational force along the plane.

57. A solid sphere of mass 500 g and radius 5 cm is rotated about one of its diameter with angular speed of 10 rad/s. If the moment of inertia of the sphere about its tangent is $x \times 10^2$ times its angular momentum about the diameter. Then the value of x will be ______________.

(1) 25

(2) 35

(3) 45

(4) 50

Correct Answer: (2) 35

Solution: The moment of inertia of a solid sphere about its diameter is given by:

$$I = \frac{2}{5}mr^2$$

The moment of inertia of the sphere about its tangent (using parallel axis theorem) is:

$$I_t = \frac{2}{5}mr^2 + mr^2 = \frac{7}{5}mr^2$$

The angular momentum about the diameter is:

$$L_{\text{diameter}} = I \cdot \omega = \frac{2}{5}mr^2 \cdot \omega$$

Now, the angular momentum about the tangent is:

$$L_{\text{tangent}} = I_t \cdot \omega = \frac{7}{5}mr^2 \cdot \omega$$

The ratio of the angular momentum about the tangent to the diameter is:

$$\frac{L_{\text{tangent}}}{L_{\text{diameter}}} = \frac{\frac{7}{5}mr^2 \cdot \omega}{\frac{2}{5}mr^2 \cdot \omega} = \frac{7}{2}$$

This is given as $x \times 10^2$. Thus:

$$x \times 10^2 = \frac{7}{2}$$

Solving for x:

$$x = \frac{7}{2} \times 10^2 = 35$$

Thus, the value of x is 35.

Quick Tip

Use the parallel axis theorem to calculate the moment of inertia about a tangent and then apply the ratio to find the desired value.

---

58. The equation of wave is given by $Y = 10^2 \sin 2\pi \left((60t - 0.5x + \frac{\pi}{4})\right)$ where x and Y are in m and t in s. The speed of the wave is ______________ km h^{-1}.

(1) 1152 km/h

(2) 1000 km/h

(3) 800 km/h

(4) 1200 km/h

Correct Answer: (1) 1152 km/h

Solution: The equation of the wave is given as:

$$Y = 10^2 \sin 2\pi \left((60t - 0.5x + \frac{\pi}{4})\right)$$

The general form of the wave equation is:

$$Y = A \sin \left(2\pi \left(\frac{x}{\lambda} - \frac{t}{T}\right)\right)$$

Here, λ is the wavelength, T is the period, and the wave number $k = \frac{2\pi}{\lambda}$ and the angular frequency $\omega = \frac{2\pi}{T}$.

From the given equation:

$$\omega = 60 \quad \text{and} \quad k = 0.5$$

The speed of the wave is given by:

$$v = \frac{\omega}{k}$$

Substituting the values:

$$v = \frac{60}{0.5} = 120\,\text{m/s}$$

Now, converting the speed into km/h:

$$v = 120 \times \frac{18}{5} = 1152\,\text{km/h}$$

Thus, the speed of the wave is $1152\,\text{km/h}$.

Quick Tip

To find the speed of a wave, use the relation $v = \frac{\omega}{k}$, and convert the units from m/s to km/h by multiplying by $\frac{18}{5}$.

---

59. In the circuit diagram shown in figure given below, the current flowing through resistance 3 Ω is $\frac{x}{3}\,A$. The value of x is ______________

(1) 1

(2) 2

(3) 3

(4) 4

Correct Answer: (1) 1

Solution:

The total equivalent resistance R_{eq} of the circuit is calculated first. From the given circuit, the equivalent resistance is the sum of the resistances in series:

$$R_{\text{eq}} = 0.5 + 1 + 4.5 + \frac{3 \times 6}{9} = 6 + 2 = 8\,\Omega$$

Using Ohm's law to calculate the total current I:

$$I = \frac{V}{R_{\text{eq}}} = \frac{8}{8+4} = 0.5\,\text{A}$$

Next, we use the voltage division rule to determine the current through each branch. For the first branch I_1:

$$I_1 = \frac{4}{8} \times I = \frac{1}{2} \times 0.5 = 0.25\,\text{A}$$

And for the second branch I_2:

$$I_2 = \frac{8}{8} \times I = \frac{2}{3} \times 0.5 = 0.33\,\text{A}$$

Finally, the current through the resistance $3\,\Omega$ is:

$$I_{3\Omega} = \frac{1}{3}A = \frac{x}{3} \quad \Rightarrow x = 1$$

Thus, the value of x is 1.

Quick Tip

Use Ohm's law and the voltage division rule to determine the current in each branch of the circuit.

60. A projectile fired at 30° to the ground is observed to be at the same height at time 3s and 5s after projection, during its flight. The speed of projection of the projectile is ______________ ms^{-1}.

(1) 70 ms^{-1}

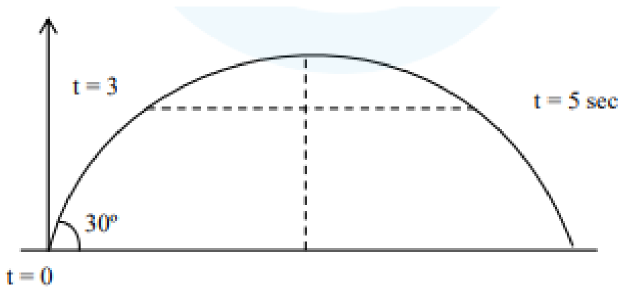
(2) 75 ms^{-1}

(3) 80 ms^{-1}

(4) 85 ms^{-1}

Correct Answer: (3) 80 ms^{-1}

Solution:



The time of flight for a projectile is given by the formula:

$$T = \frac{2u\sin\theta}{g}$$

where u is the initial speed, θ is the angle of projection, and g is the acceleration due to gravity.

From the given problem, the projectile returns to the same height at 3s and 5s. Thus, the total time of flight is:

$$T = 5 + 3 = 8\,\text{seconds}$$

Substituting the values into the formula for time of flight:

$$8 = \frac{2u\sin 30^\circ}{10}$$

$$8 = \frac{2u \times \frac{1}{2}}{10}$$

$$8 = \frac{u}{10}$$

$$u = 80\,\text{m/s}$$

Thus, the speed of projection is 80 m/s.

Quick Tip

For projectile motion, the total time of flight can be found by adding the times at which the projectile is at the same height, and using the kinematic equations to solve for the initial velocity.

Chemistry

61. Which of the following complex has a possibility to exist as meridional isomer?

(1) $[Co(en)_3]Cl_3$

(2) $[Pt(NH_3)_2]Cl_2$

(3) $[Co(en)_3]$

(4) $[Co(NH_3)_3(NO_3)_3]$

Correct Answer: (4) $[Co(NH_3)_3(NO_3)_3]$

Solution: The $[M_3B_3]$ type of compound exists as facial and meridional isomer.

Quick Tip

Facial and meridional isomerism arises in octahedral complexes where the ligands are arranged in such a way that the compound can exist in multiple geometric isomers.

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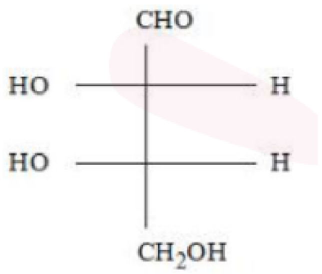
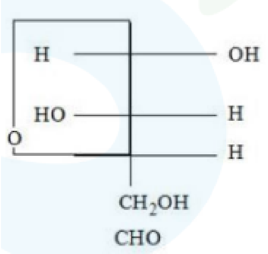
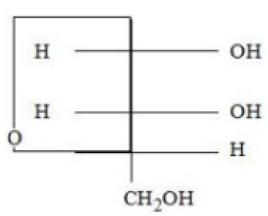
62. L-isomer of tetrose X ($C_4H_8O_4$) gives positive Schiff's test and has two chiral carbons. On acetylation, 'X' yields triacetate. 'X' undergoes following reactions



(1)



(2)



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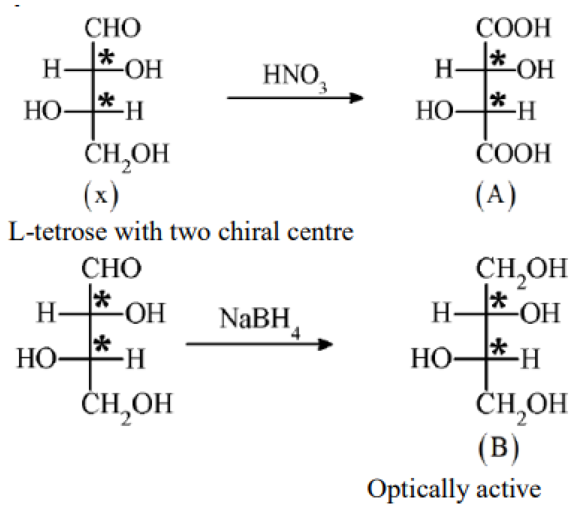
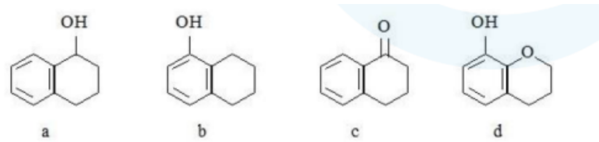
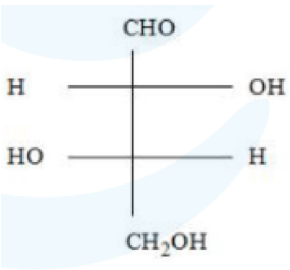
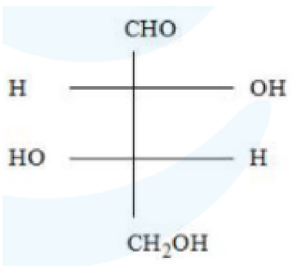




(4) 

Correct Answer: (4) 

Solution: The reaction of L-tetrose with HNO_3 results in the formation of a compound with two chiral centers, and on reduction with $NaBH_4$, a compound (B) is formed which is optically active.

Quick Tip

For carbohydrates, reactions such as Schiff's test and acetylation help identify the presence of aldehyde groups and determine the stereochemistry of the compound.

63. Match list I with list II:

List I	List II
A. KCl	I. Thermoluminescent reactions
B. KCl	II. Fertilizer
C. KOH	III. Sodium potassium pump
D. KOH	IV. Absorber of CO_2

(1) A-III, B-II, C-IV, D-I

(2) A-IV, B-II, C-I, D-III

(3) A-IV, B-III, C-I, D-II

(4) A-V, B-I, C-IV, D-II

Correct Answer: (3) A-IV, B-III, C-I, D-II

Solution: - A. KCl: KCl is used in thermoluminescent reactions, making it related to the light-emitting properties of certain compounds. - B. KCl: KCl is also a fertilizer as it contains potassium and chlorine. - C. KOH: KOH is essential for the sodium-potassium pump, which is crucial for cell function and active transport. - D. KOH: KOH is an absorber of CO_2 because of its basic nature and ability to neutralize CO_2.

Thus, the correct matching is A-IV, B-III, C-I, D-II.

Quick Tip

KCl is often used as a fertilizer because potassium is an essential nutrient for plant growth, and KOH's role in the sodium-potassium pump is vital for maintaining cellular function.

64. For compound having the formula $GaCl_3$, the correct option from the following is:

(1) Ga forms bond with Cl in $GaCl_3$.

(2) Ga is coordinated with Cl in $GaCl_3$.

(3) Ga is more electronegative than Cl and is present as a cationic part of the salt.

(4) Oxidation state of Ga in $GaCl_3$ is +3.

Correct Answer: **(4)** Oxidation state of Ga in $GaCl_3$ is +3.

Solution: - Ga forms bond with Cl: $GaCl_3$ shows ionic bonding, and the bond between Ga and Cl is ionic. - Ga is coordinated with Cl: Ga in $GaCl_3$ is in the +3 oxidation state, which results in covalent bonding with Cl. - Ga is more electronegative than Cl: This statement is incorrect as Ga has lower electronegativity than chlorine. - Oxidation state of Ga: In $GaCl_3$, Ga is in the +3 oxidation state.

Therefore, the correct answer is the oxidation state of Ga in $GaCl_3$ is +3.

Quick Tip

In $GaCl_3$, Ga is in the +3 oxidation state, and this compound forms a covalent bond with chlorine.

---

65. Thin layer chromatography of a mixture shows the following observation:

Given below are two statements: Statement 1: A is more mobile and interacts with the mobile phase more than C, and C is more than B. Statement 2: A is less mobile and interacts with the stationary phase more than C.

(1) A ¿ B ¿ C

(2) C ¿ A ¿ B

(3) A ¿ C ¿ B

(4) B ¿ C ¿ A

Correct Answer: (3) A ¿ C ¿ B

Solution: According to the observation, A is more mobile and interacts with the mobile phase more than C, and C is more than B.

This suggests that A is more mobile, C is less mobile, and B has the least mobility. Hence the correct order is A ¿ C ¿ B.

Quick Tip

Thin layer chromatography helps in understanding the relative mobility of different substances based on their interaction with the mobile and stationary phases.

---

66. When a solution of mixture having two inorganic salts was treated with freshly prepared ferrous sulphate in acidic medium, a dark brown ferric ion was formed when treated with ferric chloride. It gave deep red colour which disappeared on boiling and a brown red ppt was formed. The mixture contains:

(1) CO_3^{2-} NO_3^-

(2) SO_4^{2-} CH_3COO^-

(3) CH_3COO^- $FeCl_3$

(4) SO_4^{2-} CH_3COO^-

Correct Answer: (3) CH_3COO^- $FeCl_3$

Solution: - When a mixture containing the acetates reacts with ferric chloride ($FeCl_3$), it forms a complex and the characteristic brown red precipitate and coloration is formed, which then disappears on boiling. - The reaction suggests the presence of CH_3COO^- and $FeCl_3$ in the mixture, leading to the formation of blood-red and brown colours.

Thus, the correct answer is (3), CH_3COO^- $FeCl_3$.

Quick Tip

When testing for specific ions in inorganic salts, observe changes in colour and precipitate formation which can give clues about the compounds involved.

---

67. The polymer X consists of linear molecules and is closely packed. It is prepared in the presence of triethylaluminium and titanium tetrachloride under low pressure. The polymer X is:

(1) Polyacrylonitrile

(2) Polyterephthalate

(3) Low density polyethylene

(4) High density polyethylene

Correct Answer: (3) Low density polyethylene

Solution: - Low density polyethylene (LDPE) is made by free radical polymerization of ethylene. It is prepared under high pressure and with a catalyst like triethylaluminium and titanium tetrachloride. - LDPE has a high degree of branching, making it less dense and more flexible.

Thus, the correct answer is (3), Low density polyethylene.

Quick Tip

Polymerization under high pressure and specific catalysts leads to the formation of low-density polymers with high flexibility.

---

68. Match list I with list II:

List I Species	List II Geometry/Shape
A. H_2O	I. Tetrahedral
B. Acetylene	II. Linear
C. NH_3	III. Pyramidal
D. ClO_2	IV. Bent

(1) A-I, B-II, C-III, D-IV

(2) A-IV, B-I, C-III, D-II

(3) A-II, B-IV, C-I, D-III

(4) A-III, B-II, C-IV, D-I

Correct Answer: **(1)** A-I, B-II, C-III, D-IV

Solution: - A. H_2O: Water has a bent shape due to the lone pairs on oxygen, making the geometry bent. - B. Acetylene: Acetylene is a linear molecule with a triple bond between the carbons. - C. NH_3: Ammonia has a trigonal pyramidal shape due to the lone pair on nitrogen. - D. ClO_2: Chlorine dioxide is a bent molecule due to the lone pairs on chlorine.

Thus, the correct matching is A-I, B-II, C-III, D-IV.

Quick Tip

Remember that the geometry of molecules is determined by the number of bonding pairs and lone pairs on the central atom, according to the VSEPR theory.

---

69. Given below are two statements:

Statement I: Methane and steam passed over a heated Ni catalyst produces hydrogen gas.
Statement II: Sodium nitrate reacts with NH_4Cl to give H_2O and NaCl.

(1) Both the statement I and II are incorrect

(2) Statement I is incorrect but statement II is correct

(3) Statement I is correct but statement II is incorrect

(4) Both the statements I and II are correct

Correct Answer: (4) Both the statements I and II are correct

Solution: - Statement I is correct. When methane reacts with steam over a nickel catalyst, hydrogen gas is produced in the reaction:

$$CH_4(g) + H_2O(g) \xrightarrow{Ni} CO(g) + 3H_2(g)$$

- Statement II is correct. Sodium nitrate reacts with ammonium chloride to give sodium chloride and water in the reaction:

$$NaNO_3(aq) + NH_4Cl(aq) \rightarrow NaCl(aq) + NaNO_3(aq) + H_2O$$

Thus, the correct answer is (4), Both the statements I and II are correct.

Quick Tip

The reaction of methane and steam over a nickel catalyst is a classic example of a reforming reaction, where hydrogen gas is generated.

---

70. The set which does not have ambidentate ligands (labeled as (i)) is:

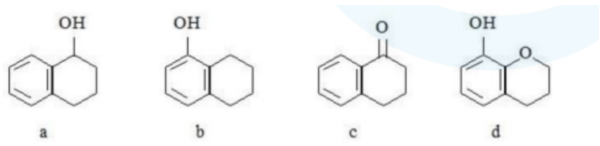
(1) CO_3^{2-}, NO_3^-, NCS^-

(2) $EDTA^-$, NCS^-, CO_2^{2-}

(3) NO^-, CO_3^{2-}, $EDTA^-$

(4) CO_2^{2-}, ethylene diamine, H_2O

Correct Answer: (4) CO_2^{2-}, ethylene diamine, H_2O



Solution: - Ambidentate ligands are ligands that can coordinate through two different donor atoms, such as NO_3^- and NCS^-. - CO_3^{2-}, NO_3^-, and NCS^- can act as ambidentate ligands, but CO_2^{2-} and ethylene diamine do not. - Hence, the correct set that does not have ambidentate ligands is (4).

Thus, the correct answer is (4), CO_2^{2-}, ethylene diamine, H_2O.

Quick Tip

Ambidentate ligands are characterized by their ability to bind metal centers through different atoms within the same ligand.

---

71. Arrange the following compounds in increasing order of rate of aromatic electrophilic substitution reaction:

(1) (a) ¡ (b) ¡ (c)

(2) (b) ¡ (c) ¡ (a)

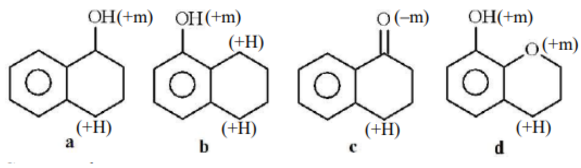
(3) (c) ¡ (a) ¡ (b)

(4) (c) ¡ (b) ¡ (a)

Correct Answer: (1) (a) ¡ (b) ¡ (c)

Solution: - Benzene becomes more reactive towards electrophilic aromatic substitution (EAS) when an electron-donating group (-OH, -NH_2) is attached to the benzene ring.

- Benzene with electron-withdrawing groups (-NO_2, -COOH) decreases the reactivity, as it



deactivates the ring towards electrophilic attack.

Thus, the order is:

(a) ¡ (b) ¡ (c)

Quick Tip

Remember that groups like -OH and -NH_2 activate the aromatic ring by donating electron density, whereas -NO_2 and -COOH deactivates it by withdrawing electron density.

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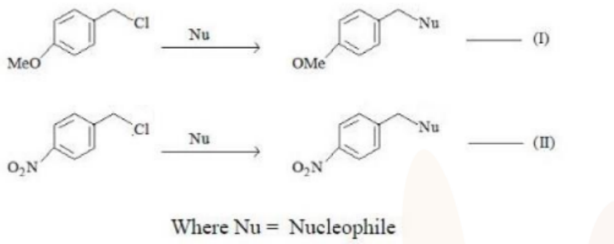
72. Find out the correct statement from the options given below for the above 2 reactions.



(1) Reaction (I) is of 1st order and reaction (II) is of 2nd order

(2) Reaction (I) and (II) both are 2nd order

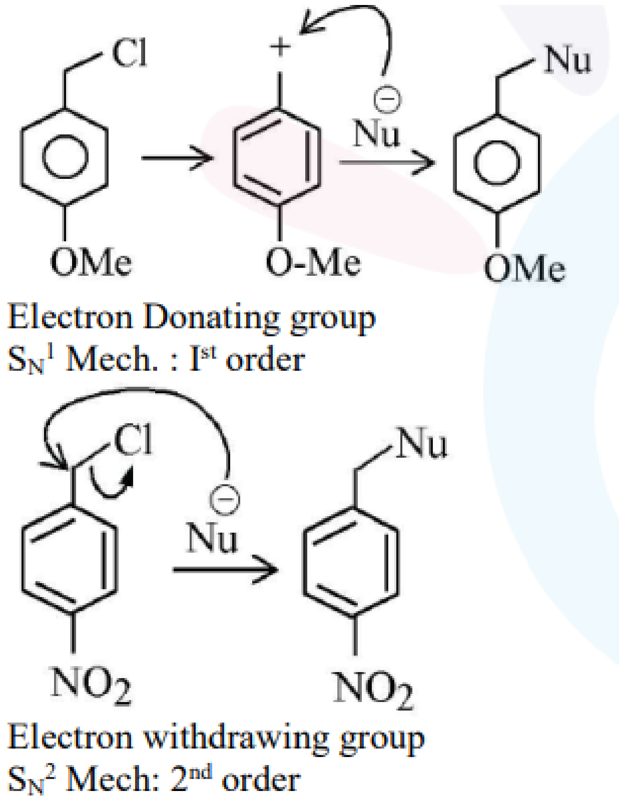
(3) Reaction (I) is of 1st order and reaction (II) is of 1st order

(4) Reaction (I) is of 2nd order and reaction (II) is of 1st order

Correct Answer: (3) Reaction (I) is of 1st order and reaction (II) is of 1st order

Solution:

- In the reaction involving an electron-donating group like $-OCH_3$, the electron density on the benzene ring is increased, leading to a faster nucleophilic attack. This makes the



mechanism of reaction (I) 1st order, as the rate depends on the concentration of the nucleophile only.

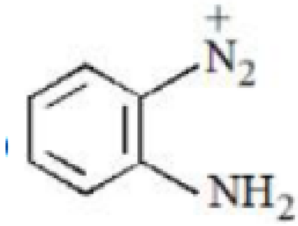
In the reaction involving an electron-withdrawing group like $-NO_2$, the electron density on the benzene ring is decreased, which slows down the nucleophilic substitution and typically follows a 2nd order mechanism, where both the nucleophile and the substrate are involved in the rate-determining step. However, reaction (II) involves a 1st order mechanism because the presence of $-NO_2$ only partially influences the overall rate, which results in an overall first-order reaction.

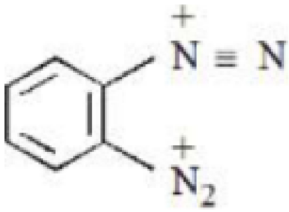
Thus, the correct answer is (3): Reaction (I) is of 1st order and reaction (II) is of 1st order.

Quick Tip

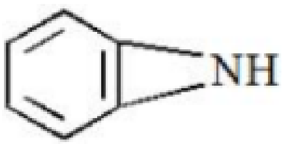
Remember that electron-donating groups (like -OCH3) increase the nucleophilicity of the aromatic ring, leading to a first-order mechanism, while electron-withdrawing groups (like -NO2) decrease the nucleophilicity, making the reaction mechanism slower.

73. o-Phenylenediamine $\xrightarrow{HNO_3} X$ Major Product X is:

(1) 

(2) 

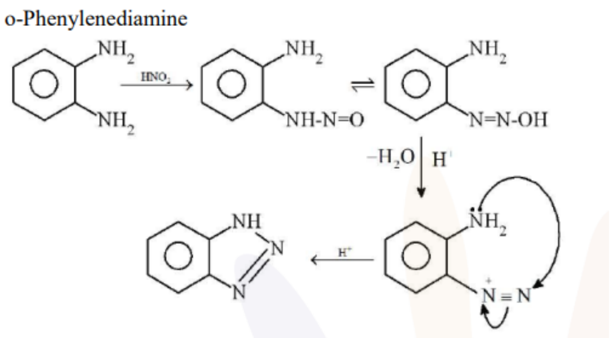
(3) 

(4) 

Correct Answer: (3)

Solution:

When o-phenylenediamine is treated with nitric acid, nitration occurs at the position ortho to the amino group. The nitro group (NO_2) replaces the hydrogen attached to the benzene ring, forming a nitrated product. In this case, the major product formed is an imine ($C = N$)



structure, which is the result of the reaction between the amino group and the nitro group.

Thus, the correct product is a structure with a carbon-nitrogen double bond (C = N).

Quick Tip

When performing reactions involving amines, the functional group often reacts with the reagent to form imines or other derivatives based on the substitution pattern of the aromatic ring.

---

74. For elements B, C, N, Li, Be, O and F, the correct order of first ionization enthalpy is:

(1) B ¡ Li ¡ Be ¡ C ¡ O ¡ N ¡ F

(2) Li ¡ C ¡ B ¡ O ¡ N ¡ F

(3) Li ¡ C ¡ B ¡ O ¡ N ¡ F

(4) Li ¡ B ¡ C ¡ O ¡ N ¡ F

Correct Answer: (3) Li ¡ B ¡ C ¡ O ¡ N ¡ F

Solution: The first ionization energy is the energy required to remove an electron from a neutral atom in its gaseous phase. The general trend for the first ionization energy is that it increases across a period (from left to right) and decreases down a group.

Thus, the order of first ionization enthalpy for the elements provided, starting from the lowest, is:

$$Li < B < C < O < N < F$$

Therefore, the correct order is given by option (3).

Quick Tip

The first ionization enthalpy increases from left to right in a period due to an increase in nuclear charge and decreases down a group due to the increased size of atoms.

---

75. In the extraction process of copper, the product obtained after carrying out the reactions

$$CuO + H_2 \rightarrow Cu + H_2O$$

is called:

(1) Reduced copper

(2) Blister copper

(3) Copper scrap

(4) Copper slag

Correct Answer: (1) Reduced copper

Solution: In the extraction of copper, when copper oxide (CuO) is reduced by hydrogen gas (H_2), the copper is reduced to its metallic form and water vapor is produced as a byproduct. This process produces what is referred to as reduced copper.

Blister copper, on the other hand, refers to copper that is in a nearly pure state but still contains small amounts of other elements such as sulfur, which can create small bubbles (blisters) on its surface during the smelting process.

Thus, the correct product is reduced copper, which is the result of the reduction of CuO with H_2.

Quick Tip

The reduction of metal oxides with hydrogen gas results in the formation of pure metals, while the process in smelting where impurities cause bubbling forms blister copper.

---

76. 25 mL of silver nitrate solution (1M) is added dropwise to 25 mL of potassium iodide (1.05 M) solution. The ions(s) present in very small quantity in the solution is/are:

(1) NO only

(2) Ag and I both

(3) Ag only

(4) I only

Correct Answer: (3) Ag only

Solution: On adding $AgNO_3$ to KI, AgI will form, and the solubility of AgI is very low. As a result, only Ag will remain in very small quantities in the solution. The majority of the I ions will precipitate out as AgI, leaving very few ions in solution.

Thus, the correct answer is Ag ions only.

Quick Tip

When mixing solutions of ionic compounds, check the solubility products (Ksp) to predict whether the ions will precipitate or remain in solution.

---

77. Given below are two statements:

Statement I: If BOD value is 4 ppm and dissolved oxygen is 8 ppm, it is a good quality

water. Statement II: If the concentration of zinc and nitrate is 5 ppm, then it can be used as good quality water.

(1) Statement I is correct but statement II is incorrect

(2) Statement I is incorrect but statement II is correct

(3) Both statements I and II are incorrect

(4) Both statements I and II are correct

Correct Answer: (4) Both statements I and II are correct

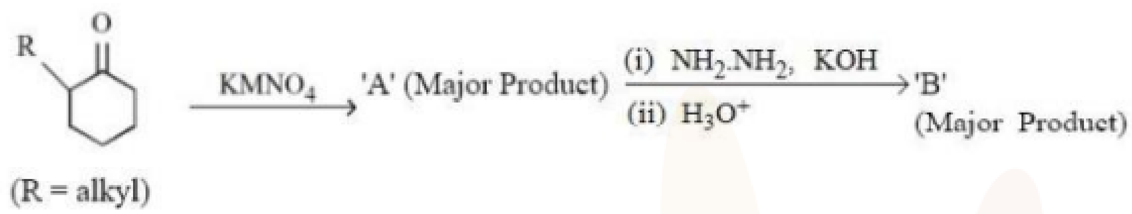
Solution: - **Statement I:** A BOD value of 4 ppm and a dissolved oxygen level of 8 ppm indicate that the water has a good quality because the dissolved oxygen should be higher than the BOD to support aquatic life. - **Statement II:** If the concentration of zinc and nitrate is 5 ppm, this is within permissible limits for drinking water, indicating good quality.

Both statements are correct as they provide accurate interpretations of water quality standards.

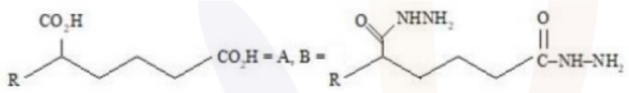
Quick Tip

When evaluating water quality, check for parameters like BOD, dissolved oxygen, and concentrations of hazardous elements such as zinc and nitrates.

78.

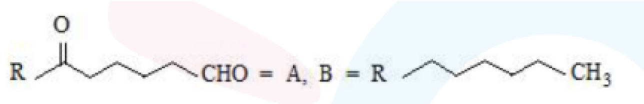


In the above reaction 'A' and 'B' are:

(1) 

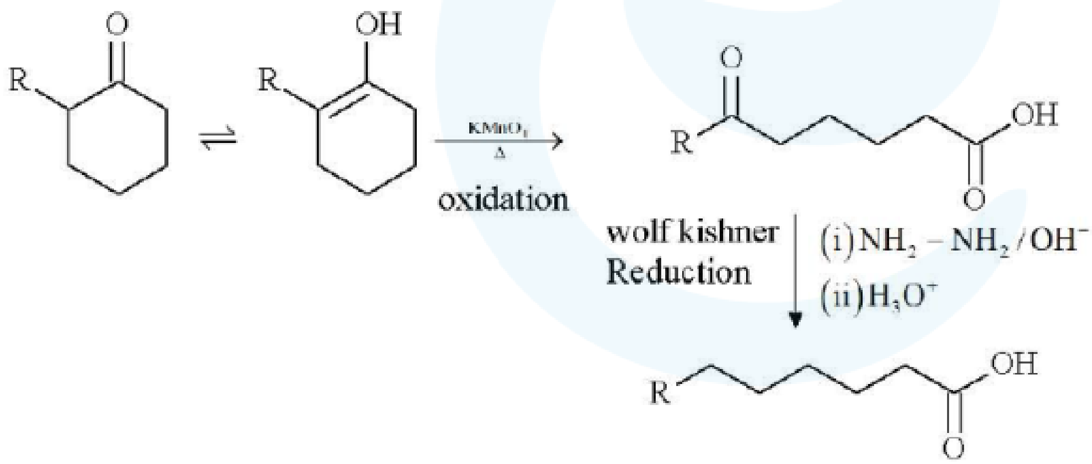
(2) 

(3) 

(4) 

Correct Answer: (3) 

Solution:



- The reaction involves the oxidation of the allyl group to a carboxylic acid group ($COOH$) via $KMnO_4$.

- The second step involves a nucleophilic substitution reaction with NH_3 and $NaOH$, forming the amine group (NH_2).

Thus, the final product is a compound with both $COOH$ and NH_2 groups.

Quick Tip

Oxidation reactions with $KMnO_4$ generally add oxygen to the carbon chain, converting alkyl groups to carboxyl groups.

---

79. Given below are two statements: one is labelled as Assertion A and the other is labelled as Reason R:

Assertion A: In the photoelectric effect electrons are ejected from the metal surface as soon as the beam of light of frequency greater than threshold frequency strikes the surface.

Reason R: When the photon of any energy strikes an electron in the atom transfer of energy from the photon to the electron takes place.

(1) Assertion A is correct but Reason R is not correct

(2) Assertion A is not correct and Reason R is correct

(3) Both A and R are correct and R is the correct explanation of A

(4) Both A and R are correct but R is NOT the correct explanation of A

Correct Answer: (1) Assertion A is correct but Reason R is not correct

Solution: - Assertion A is correct. In the photoelectric effect, electrons are ejected when light of frequency greater than the threshold frequency strikes the surface of a metal. - Reason R is incorrect because the process of energy transfer from the photon to the electron is not the sole factor that leads to the ejection of electrons. The frequency of the light is what matters, not just the energy of the photon.

Thus, the correct option is (1).

Quick Tip

In the photoelectric effect, the energy of the incident light must exceed a certain threshold to eject electrons from the metal surface.

---

80. The complex that dissolves in water is:

(1) Fe[Fe(CN)], Prussian Blue insoluble

(2) Fe[Fe(CN)]

(3) K[Co(CO)]

(4) (NH)[As(MoO)]

Correct Answer: (1) Fe[Fe(CN)], Prussian Blue insoluble

Solution: The compound Fe[Fe(CN)] is known as Prussian Blue, and it is insoluble in water. On the other hand, compounds like K[Co(CO)] are soluble due to the ionic nature of the complex.

Thus, the correct answer is (1), as Prussian Blue (Fe[Fe(CN)]) does not dissolve in water.

> **Quick Tip**
>
> Check the solubility of complexes by analyzing their ionic nature and the solubility product (Ksp).

---

81. Solid fuel used in rocket is a mixture of FeO and Al (in ratio 1 : 2) the heat evolved (KJ) per gram of the mixture is _____________ (Nearest integer) Given $\Delta H_f^\circ(Al_2O_3) = -1700$ **KJ mol**$^{-1}$

$\Delta H_f^\circ(Fe_2O_3) = -840$ **KJ mol**$^{-1}$

(1) 2 KJ

(2) 3 KJ

(3) 4 KJ

(4) 5 KJ

Correct Answer: (4) 4 KJ

Solution: The reaction is:

$$Fe_2O_3 + 2Al \rightarrow Al_2O_3 + 2Fe$$

Now calculate the heat evolved:

$$\Delta H = (\Delta H_f^\circ(Al_2O_3)) - (\Delta H_f^\circ(Fe_2O_3))$$

Substitute the given values:

$$\Delta H = (-1700) - (-840) = -860\,\text{KJ/mol}$$

Moles of Fe_2O_3 and Al are in the ratio 1 : 2. So, 1 mole of Fe_2O_3 weighs 112 g, and 2 moles of Al weigh 54 g.

Total mass of the mixture is:

$$160 + 54 = 214\,\text{g}$$

Now, the heat evolved per gram of the mixture is:

$$\frac{-860\,\text{KJ}}{214} \approx -4.01\,\text{KJ/g}$$

Thus, the heat evolved per gram is approximately 4 KJ. Hence, the correct answer is (4).

Quick Tip

To calculate the heat evolved per gram, divide the total heat by the total mass of the mixture.

---

82. KClO + 6FeSO + 3HSO → KCl + 3Fe(SO) + 3HO The above reaction was studied at 300 K by monitoring the concentration of FeSO, in which initial concentration was 10 M and after half an hour became 8.8 M. The rate of production of Fe(SO) is ______________ × 10 mol L¹ s¹

(1) 333

(2) 334

(3) 335

(4) 336

Correct Answer: (1) 333

Solution: Rate of reaction is given by the change in concentration of FeSO:

$$\frac{-\Delta[FeSO]}{\Delta t}$$

Substitute the given values:

$$\frac{-10 + 8.8}{30 \times 60} = \frac{1.2}{1800} = 6.67 \times 10^{-4}$$

From the given reaction, the rate of production of Fe(SO) is related to the rate of FeSO:

$$\frac{1}{6} \times \frac{-\Delta[FeSO]}{\Delta t}$$

Substitute the value of $\frac{-\Delta[FeSO]}{\Delta t}$:

$$\text{Rate of production of } \mathrm{Fe}_2(SO)_3 = \frac{3}{6} \times 6.67 \times 10^{-4} = 333.33 \times 10^{-6}$$

Thus, the rate of production of Fe(SO) is 333×10^{-6} mol L¹ s¹. Hence, the correct answer is (1).

Quick Tip

The rate of production of a product in a reaction can be calculated by determining the rate of change of concentration of a reactant and using stoichiometric ratios.

---

83. 0.004 M KSO solution is isotonic with 0.01 M glucose solution. Percentage dissociation of KSO is ______________ (Nearest integer)

(1) 70

(2) 75

(3) 80

(4) 85

Correct Answer: (2) 75

Solution: For isotonic solution:

$$i(\text{glucose}) = i(\text{K}_2\text{SO}_4)$$

$$0.01 = i(\text{K}_2\text{SO}_4) \times 0.004$$

$$i(\text{K}_2\text{SO}_4) = \frac{0.01}{0.004} = 2.5$$

Now, for K_2SO_4:

$$i = 1 + (n - 1)$$

$$2.5 = 1 + (n - 1)$$

$$n = 3 \text{ for } K_2SO_4$$

Percentage dissociation:

$$\alpha = \frac{3}{2} = 75\%$$

Thus, the percentage dissociation of K$_2SO_4 is 75\%. The correct answer is (2).$

Quick Tip

For isotonic solutions, use the equation $i = 1 + (n - 1)$ to find the number of particles and dissociation percentage.

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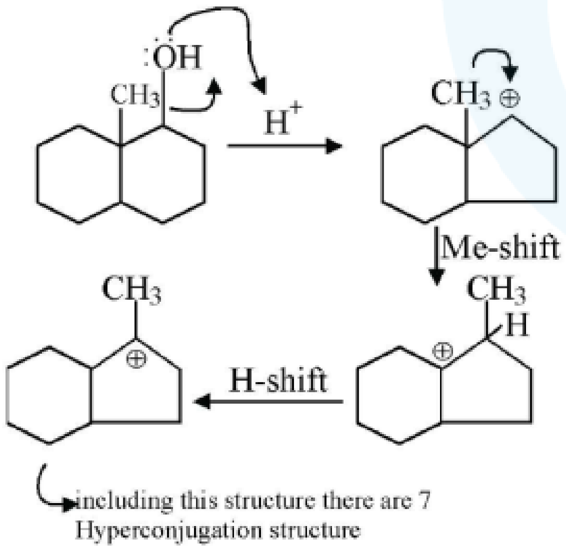
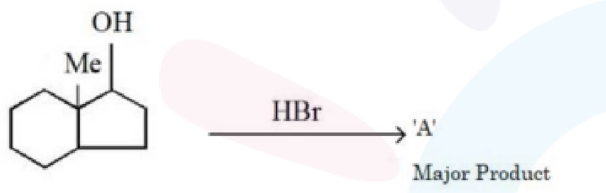
84. The number of hyperconjugation structures involved to stabilize carbocation formed in the below reaction is ______________





(1) 6

(2) 7

(3) 8

(4) 9

Correct Answer: (2) 7

Solution:

In the given reaction, the carbocation formed after the removal of H^+ can undergo several hyperconjugation structures. We see that the carbocation can shift, and the hyperconjugation structures involve the bonding electrons of the adjacent C-H bonds.

After shifting, the carbocation is stabilized by the hyperconjugation effect, and counting all the hyperconjugation structures (including the shift), there are 7 such structures.

Thus, the correct number of hyperconjugation structures is 7. Therefore, the correct answer is (2).

Quick Tip

Hyperconjugation involves the delocalization of electrons from adjacent C-H or C-C bonds to stabilize carbocations.

---

85. A mixture of 1 mole of HO and 1 mole of CO is taken in a 10 liter container and heated to 725 K. At equilibrium, 10 M of water by mass reacts with carbon monoxide according to the equation:

$$CO(g) + H_2O(g) \rightleftharpoons CO_2(g) + H_2(g)$$

The equilibrium constant $K_c \times 10^7$ for the reaction is ____________ (Nearest integer)

(1) 44

(2) 45

(3) 46

(4) 47

Correct Answer: (1) 44

Solution: From the given equation, the change in concentrations is:

$$\text{At equilibrium} \quad 1 - 0.4 = 0.6 \quad 0.4 \quad 0.4$$

$$K_c = \frac{0.4 \times 0.4}{0.6 \times 0.6} = \frac{0.16}{0.36} = 0.444 \approx 44$$

Thus, the equilibrium constant is 44×10^7. Therefore, the correct answer is (1).

Quick Tip

To calculate the equilibrium constant, use the relationship between the concentrations of the products and reactants at equilibrium.

86. An atomic substance A of molar mass 12 g mol^{-1} has a cubic crystal structure with edge length of 300 pm. The no. of atoms present in one unit cell of A is ______________ (Nearest integer)

(1) 3

(2) 4

(3) 5

(4) 6

Correct Answer: (4) 6

Solution: Given: - Molar mass $M = 12$ g mol^{-1} - Density $d = 3.0$ g mL^{-1} - Edge length $a = 300$ pm $= 300 \times 10^{-12}$ m - Avogadro's number $N_A = 6.02 \times 10^{23}$ mol^{-1}

Formula for number of atoms in one unit cell:

$$Z = \frac{N_A \times M}{d \times a^3}$$

Substitute values:

$$Z = \frac{6.02 \times 10^{23} \times 12}{3.0 \times (300 \times 10^{-12})^3}$$

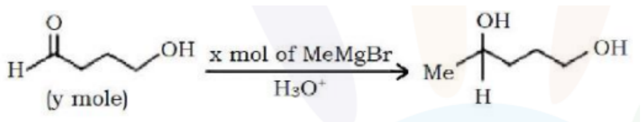
$$Z = 40.635 \times 10^{21} = 6$$

Thus, the number of atoms present in one unit cell is 6. The correct answer is (4).

Quick Tip

The number of atoms in a unit cell can be calculated using the formula $Z = \frac{N_A \times M}{d \times a^3}$.

87.



The ratio x/y on completion of the above reaction is ____________

(1) 1

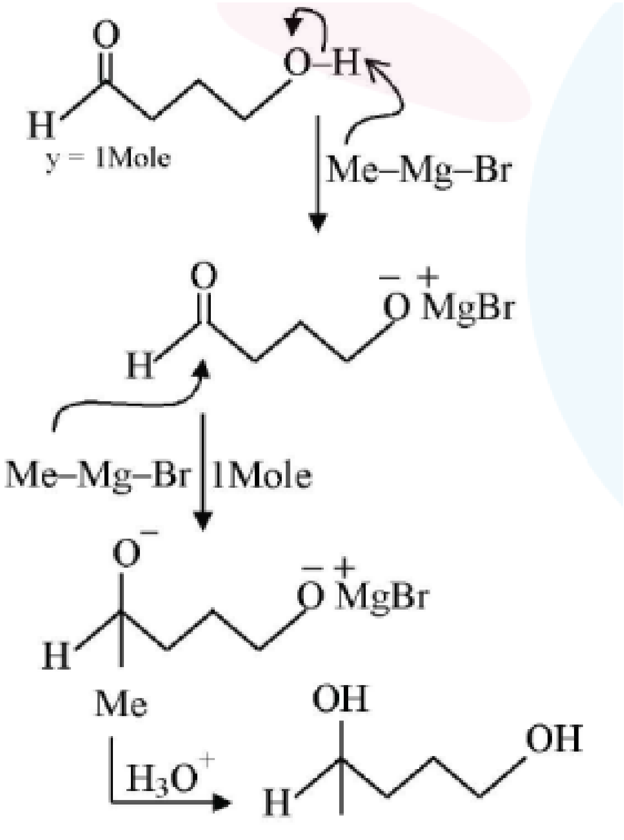
(2) 2

(3) 3

(4) 4

Correct Answer: (2) 2

Solution:



The reaction involves the formation of MeMgBr, with x and y representing the moles of MeMgBr and H, respectively.

From the reaction:

$$\text{H} + \text{MeMgBr} \rightarrow \text{MeMgBr} + \text{OH}^-$$

For the given quantities: - $x = 2$ moles of MeMgBr - $y = 1$ mole of H

Thus, the ratio of x/y is:

$$x/y = 2/1 = 2$$

The correct answer is (2).

Quick Tip

In reactions involving metal organics like MeMgBr, stoichiometry helps find the ratio of reactants and products.

---

88. The ratio of spin-only magnetic moment values $\mu_{\textbf{eff}}[Cr(CN)_6]^{3-}/\mu_{\textbf{eff}}[Cr(H_2O)_6]^{3+}$ is __________

(1) 1

(2) 2

(3) 3

(4) 4

Correct Answer: (1) 1

Solution: The spin-only magnetic moment is calculated using the formula:

$$\mu_{\text{eff}} = \sqrt{n(n+2)}\,\text{BM}$$

For $[Cr(CN)_6]^{3-}$ (d^3):

$$\mu_1 = \sqrt{3(3+2)} = \sqrt{15}\,\text{BM}$$

For $[Cr(H_2O)_6]^{3+}$ (d^3):

$$\mu_2 = \sqrt{3(3+2)} = \sqrt{15}\,\text{BM}$$

Since both have the same electronic configuration, the ratio is:

$$\frac{\mu_1}{\mu_2} = \frac{\sqrt{15}}{\sqrt{15}} = 1$$

Thus, the ratio of magnetic moments is 1. The correct answer is (1).

Quick Tip

For transition metal complexes, the spin-only magnetic moment depends on the number of unpaired electrons.

---

89. In an electrochemical reaction of lead, at standard temperature, if $E^\circ(\text{Pb}^{2+}/\text{Pb}) = m$ volt and $E^\circ(\text{Pb}^{4+}/\text{Pb}^{2+}) = n$ volt, then the value of $E^\circ(\text{Pb}^{4+}/\text{Pb})$ is given by $m - xn$. The value of x is ____________ (Nearest integer)

(1) 1

(2) 2

(3) 3

(4) 4

Correct Answer: (2) 2

Solution: The given reaction is:

$$\text{Pb}^{2+} + 2e^- \rightarrow \text{Pb}$$

$$E^\circ = m \quad \text{and} \quad \Delta G^\circ = -2Fm$$

$$\text{Pb}^{4+} + 4e^- \rightarrow \text{Pb}^{2+}$$

$$E^\circ = n \quad \text{and} \quad \Delta G^\circ = -4Fn$$

Now,

$$\Delta G^\circ = \Delta G_1^\circ - \Delta G_2^\circ$$

$$-2Fm = -4Fn$$

$$2FE = 2Fm + 4Fn \quad \Rightarrow \quad E^\circ = m - 2n$$

Thus, the value of x is 2. The correct answer is (2).

Quick Tip

When dealing with electrochemical reactions, the relationship between the potentials and the standard Gibbs free energies can help calculate the overall cell potential.

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90. A solution of sugar is obtained by mixing 200g of its 25% solution and 500g of its 40% solution (both by mass). The mass percentage of the resulting sugar solution is ____________ (Nearest integer)

(1) 35

(2) 36

(3) 37

(4) 38

Correct Answer: (36)

Solution: Given: - Solution (I): Mass of sugar = 200 g, sugar percentage = 25- Solution (II): Mass of sugar = 500 g, sugar percentage = 40

Mass of sugar in solution (I):

$$\frac{25}{100} \times 200 = 50\,\text{g}$$

Mass of sugar in solution (II):

$$\frac{40}{100} \times 500 = 200\,\text{g}$$

Total mass of solution = 200 + 500 = 700 g Total mass of sugar = 50 + 200 = 250 g

Now, the final percentage of sugar is:

$$\frac{250}{700} \times 100 = 35.71 \approx 36$$

Thus, the mass percentage of sugar is 36

Quick Tip

To calculate the mass percentage of a solution, divide the mass of solute by the total mass of the solution and multiply by 100.
