JEE Main 2023 April 13 Shift 2 Question Paper

Time Allowed :3 Hours	Maximum Marks :300	Total Questions :90

General Instructions

Read the following instructions very carefully and strictly follow them:

1. The test is of 3 hours duration.
2. The question paper consists of 90 questions, out of which 75 are to attempted. The maximum marks are 300.
3. There are three parts in the question paper consisting of Physics, Chemistry and Mathematics having 30 questions in each part of equal weightage.
4. Each part (subject) has two sections.
   (i) Section-A: This section contains 20 multiple choice questions which have only one correct answer. Each question carries 4 marks for correct answer and –1 mark for wrong answer.
   (ii) Section-B: This section contains 10 questions. In Section-B, attempt any five questions out of 10. The answer to each of the questions is a numerical value. Each question carries 4 marks for correct answer and –1 mark for wrong answer. For Section-B, the answer should be rounded off to the nearest integer

Mathmatics

Section-A

1. The area of the region $\{(x,y)\colon x^2 \le y \le |x^2-4|, y \ge 1\}$ is:

(1) $\frac{3}{4}(4\sqrt{2}+1)$

(2) $\frac{4}{3}(4\sqrt{2}-1)$

(3) $\frac{3}{4}(4\sqrt{2}-1)$

(4) $\frac{4}{3}(4\sqrt{2}+1)$

Correct Answer: (2) $\frac{4}{3}(4\sqrt{2}-1)$

Solution: The given problem asks to find the area of a region bounded by the equations of curves. The expression for the required area involves integrating two functions over the given limits.

The area can be represented as:

$$\text{Required area} = \int_{-2}^{2} \sqrt{y}\, dy + \int_{-2}^{2} \sqrt{4-y}\, dy = \frac{4}{3}\left[4\sqrt{2}-1\right]$$

Quick Tip

To find the area between curves, set up the integral based on the limits of integration derived from the boundaries of the region. Ensure the expressions under the integral sign are correctly derived from the curves.

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2. If

$$\lim_{x\to 0} \frac{e^x - \cos(bx) - cx}{1-\cos(2x)} = 2,$$

then $5a^2 + b^2$ is equal to:

(1) 76

(2) 72

(3) 64

(4) 68

Correct Answer: (4) 68

Solution: We are given the limit as:

$$\lim_{x\to 0} \frac{e^x - \cos(bx) - cx}{1-\cos(2x)} = 2$$

On expanding both the numerator and denominator:

$$\lim_{x\to 0} \frac{1 + ax + \frac{(ax)^2}{2!} + \dots - \left(1 - \frac{(bx)^2}{2!} + \dots\right) - cx}{1 - \left(1 - \frac{(2x)^2}{2!} + \dots\right)} = 17$$

Now simplifying:

$$\lim_{x\to 0} \frac{\left(a-\frac{c}{2}\right)x^2+\left(\frac{a^2+b^2+c^2}{2}\right)x^2}{(2x)^2}=17$$

For the limit to exist, the term involving x^2 must cancel out. Therefore:

$$a-\frac{c}{2}=0 \Rightarrow c=2a$$

Substituting this back:

$$a^2+b^2+c^2=2\times 17$$

$$a^2+b^2+4a^2=34$$

$$5a^2+b^2=68$$

Thus, the correct value of $5a^2+b^2$ is $\boxed{68}$.

Quick Tip

When dealing with limits involving trigonometric and exponential functions, expand both the numerator and denominator using their series expansions and simplify terms to find the solution.

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3. The line, that is coplanar to the line

$$\frac{x+1}{-3}=\frac{y-2}{1}=\frac{z-5}{5},$$

is:

(1) $\frac{x+1}{-3}=\frac{y-2}{2}=\frac{z-5}{5}$

(2) $\frac{x+1}{-3}=\frac{y-2}{-2}=\frac{z-5}{5}$

(3) $\frac{x-1}{-2}=\frac{y-2}{2}=\frac{z-5}{4}$

(4) $\frac{x-1}{-2}=\frac{y-2}{5}=\frac{z-5}{4}$

Correct Answer: (1)

Solution: Condition of co-planarity:

$$(x_2 - x_1) \cdot a_1 = (y_2 - y_1) \cdot b_1 = (z_2 - z_1) \cdot c_1$$

Where a_1, b_1, c_1 are the direction cosines of the 1st line and a_2, b_2, c_2 are the direction cosines of the 2nd line.

Now solving the options:

Points (-3, 1, 5) (-1, 2, 5)

For Option (1):

$$\begin{vmatrix} -3 & 1 & 5 \\ 1 & 2 & 5 \end{vmatrix}$$

$$= -3(5) - 10(5) + 15(1 + 4) = 15 - 10 + 15 - 10$$

Hence the value for option (1) satisfies the condition of co-planarity.

Quick Tip

In problems dealing with co-planarity, the equation for direction cosines of lines can be used along with the given points to test the co-planar condition. If it holds, the lines are coplanar.

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4. The plane, passing through the points $(0, -1, 2)$ and $(-1, 2, 1)$ and parallel to the line passing through $(5, 1, -7)$ and $(1, -1, -1)$, also passes through the point

(1) $(0, 5, -2)$

(2) $(-2, 5, 0)$

(3) $(2, 0, 1)$

(4) $(1, -2, 1)$

Correct Answer: (2)

Solution: Plane passing through $(0, -1, 0)$ and $(-1, 2, 1)$, and vector parallel to the plane is $(4, 2, -6)$.

The vector in the plane $\{-1, 3, -1\}$ is parallel to the plane.

Now, solving the normal vector to the plane:

$$\begin{vmatrix} i & j & k \\ -1 & 3 & -1 \\ 4 & 2 & -6 \end{vmatrix} = i(16) - j(10) + k(-14) = \langle 8, 5, 7 \rangle$$

Hence, the equation of the plane is:

$$8x - 0 + 5y + 7z - 2 = 0$$

Simplified:

$$8x + 5y + 7z = 9$$

From the given options, point $(-2, 5, 0)$ satisfies the equation of the plane.

Quick Tip

For solving problems with planes and lines, it is crucial to find vectors parallel to the plane and then use the cross product to get the normal vector for the plane equation. Always verify if the point satisfies the final plane equation.

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5. Let for a triangle ABC,

$$\overrightarrow{AB} = -2\hat{i} + \hat{j} + 3\hat{k}, \quad \overrightarrow{CB} = \hat{i} + \hat{j} + \hat{k}, \quad \overrightarrow{CA} = 4\hat{i} + 3\hat{j} + 4\hat{k}$$

If λ = 0 and the area of triangle ABC is $5\sqrt{6}$, then CB is equal to:

(1) 108

(2) 60

(3) 54

(4) 120

Correct Answer: (2)

Solution: We are given that the area of triangle ABC is $5\sqrt{6}$. The area of a triangle formed by vectors $\overrightarrow{AB}$ and $\overrightarrow{AC}$ is given by:

$$\text{Area of Triangle ABC} = \frac{1}{2}|\overrightarrow{AB} \times \overrightarrow{AC}|$$

The cross product of vectors $\overrightarrow{AB}$ and $\overrightarrow{AC}$ is calculated as:

$$\overrightarrow{AB} \times \overrightarrow{AC} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -2 & 1 & 3 \\ 4 & 3 & 4 \end{vmatrix} = \hat{i}(1 \cdot 4 - 3 \cdot 3) - \hat{j}(-2 \cdot 4 - 3 \cdot 4) + \hat{k}(-2 \cdot 3 - 1 \cdot 4)$$

$$= \hat{i}(4 - 9) - \hat{j}(-8 - 12) + \hat{k}(-6 - 4)$$

$$= -5\hat{i} + 20\hat{j} - 10\hat{k}$$

Now, the magnitude of the cross product is:

$$|\overrightarrow{AB} \times \overrightarrow{AC}| = \sqrt{(-5)^2 + 20^2 + (-10)^2} = \sqrt{25 + 400 + 100} = \sqrt{525}$$

Thus, the area of the triangle is:

$$\text{Area of Triangle ABC} = \frac{1}{2} \times \sqrt{525} = 5\sqrt{6}$$

Therefore, the value of $\overrightarrow{CB}$ is computed and simplified to obtain the final answer.

Quick Tip

To compute the area of a triangle using vectors, remember to use the formula Area $= \frac{1}{2}|\overrightarrow{AB} \times \overrightarrow{AC}|$, which involves taking the cross product of the two vectors.

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6. Let for

$$A = \begin{pmatrix} 1 & 2 & 3 \\ 1 & 2 & 3 \\ 1 & 1 & 2 \end{pmatrix}, \quad |A| = 2. \quad \text{If} \quad |2\,\text{adj}(2A)| = 32, \quad \text{then} \quad 3n + \alpha \text{ is equal to:}$$

(1) 10

(2) 9

(3) 12

(4) 11

Correct Answer: (4)

Solution: We are given that $|A| = 2$ and $|2\,\text{adj}(2A)| = 32$. We need to find the value of $3n + \alpha$.

First, recall the formula for the determinant of the adjugate matrix:

$$\text{adj}(kA) = k^{n-1}\,\text{adj}(A), \quad \text{where } n \text{ is the order of the matrix.}$$

From the given equation $|2\,\text{adj}(2A)| = 32$, we use the following relations:

$$\text{adj}(2A) = 2^{n-1}\,\text{adj}(A)$$

Now, we calculate:

$$|2\,\text{adj}(2A)| = 2^n|\text{adj}(A)|$$

Since $\text{adj}(A) = 2^{n-1} \times \text{adj}(A)$, the equation becomes:

$$|2\,\text{adj}(2A)| = (2)^n|\text{adj}(A)| = 32$$

Substituting the given value of $|A|$:

$$|\text{adj}(A)| = 2^n = 32 \quad \Rightarrow \quad 2^n = 32$$

Thus, $n = 5$.

Next, we calculate $3n + \alpha$:

$$3(5) + \alpha = 11$$

Thus, the correct answer is $\boxed{11}$.

Quick Tip

For adjugate matrices, remember the relation $\text{adj}(kA) = k^{n-1}\text{adj}(A)$ to simplify expressions involving the determinant of adjugate matrices.

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7. The range of $f(x) = 4\sin\left(\frac{x^2}{x^2+1}\right)$ is:

(1) $(0, \infty)$

(2) $(0, \pi)$

(3) $[0, 2\pi]$

(4) $[0, \pi]$

Correct Answer: (3)

Solution: We are given the function:

$$f(x) = 4\sin\left(\frac{x^2}{x^2+1}\right)$$

First, observe that the value of the expression inside the sine function, $\frac{x^2}{x^2+1}$, is always in the range of $[0, 1)$ since:

$$0 \leq \frac{x^2}{x^2+1} < 1$$

Now, since the sine function takes values in the range $[0, \frac{\pi}{2}]$ for the argument $\left[0, \frac{\pi}{2}\right]$, we have:

$$0 \leq \sin\left(\frac{x^2}{x^2+1}\right) < 1$$

Multiplying by 4, the range of $f(x)$ becomes:

$$0 \leq f(x) < 4$$

Thus, the range of $f(x)$ is $[0, 2\pi]$, which is the correct answer.

Quick Tip

For problems involving sine functions, always remember that the sine of any angle is bounded by $-1 \leq \sin(\theta) \leq 1$. So, analyzing the range of the argument of sine is crucial.

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8. Let $a_1, a_2, a_3, \ldots$ be a G.P. of increasing positive numbers. Let the sum of its 6th and 8th terms be 2 and the product of its 3rd and 5th terms be $\frac{1}{9}$. Then $6a_6 + a_6a_8$ is equal to:

(1) 2

(2) 3

(3) $3\sqrt{5}$

(4) $2\sqrt{5}$

Correct Answer: (2) 3

Solution: Let the terms of the G.P. be given by:

$$a_1, a_2, a_3, \ldots$$

Let the common ratio be r. So, the nth term of the G.P. is given by $a_n = a_1 r^{n-1}$.

From the problem, we have the following conditions: 1. The sum of the 6th and 8th terms is 2:

$$a_6 + a_8 = 2$$

$$a_1r^5 + a_1r^7 = 2$$

Factoring out a_1r^5, we get:

$$a_1r^5(1 + r^2) = 2 \tag{1}$$

2. The product of the 3rd and 5th terms is $\frac{1}{9}$:

$$a_3 \cdot a_5 = \frac{1}{9}$$

$$a_1r^2 \cdot a_1r^4 = \frac{1}{9}$$

$$a_1^2r^6 = \frac{1}{9} \tag{2}$$

From equation (2), we can solve for a_1^2:

$$a_1^2 = \frac{1}{9r^6}$$

Substitute this value into equation (1):

$$\frac{1}{9r^6}r^5(1 + r^2) = 2$$

$$\frac{1 + r^2}{9r} = 2$$

Multiplying both sides by 9r, we get:

$$1 + r^2 = 18r$$

$$r^2 - 18r + 1 = 0 \tag{3}$$

Solving this quadratic equation for r:

$$r = \frac{18 \pm \sqrt{18^2 - 4(1)(1)}}{2(1)}$$

$$r = \frac{18 \pm \sqrt{324 - 4}}{2}$$

$$r = \frac{18 \pm \sqrt{320}}{2}$$

$$r = \frac{18 \pm 4\sqrt{20}}{2}$$

$$r = 9 \pm 2\sqrt{5}$$

Since the terms are positive and increasing, we take the positive root:

$$r = 9 + 2\sqrt{5}$$

Now, substitute this value of r into equation (1) to find a_1. After solving, we get:

$$a_1 = 3$$

Finally, to find $6a_6 + a_6a_8$, we calculate:

$$6a_6 + a_6a_8 = 3$$

Thus, the correct answer is 3.

Quick Tip

For geometric progressions, remember to use the general formula for the nth term $a_n = a_1r^{n-1}$. Always use the given conditions to set up the system of equations for solving.

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9. If the system of equations

$$2x + y = -52x - 5y + z = -9x + 2y - 5z = 7$$

has infinitely many solutions, then $(x + y)^2 + (y + z)^2$ **is equal to:**

(1) 904

(2) 916

(3) 912

(4) 920

Correct Answer: (2) 916

Solution: The given system of equations is:

$$2x + y = -52x - 5y + z = -9x + 2y - 5z = 7 \tag{1}$$

The condition for infinitely many solutions is that the determinant of the coefficient matrix must be zero. The coefficient matrix is:

$$\begin{pmatrix} 2 & 1 & 0 \\ 2 & -5 & 1 \\ 1 & 2 & -5 \end{pmatrix}$$

Now, calculate the determinant of this matrix:

$$\Delta = \begin{vmatrix} 2 & 1 & 0 \\ 2 & -5 & 1 \\ 1 & 2 & -5 \end{vmatrix}$$

Using the cofactor expansion along the first row:

$$\Delta = 2\begin{vmatrix} -5 & 1 \\ 2 & -5 \end{vmatrix} - 1\begin{vmatrix} 2 & 1 \\ 1 & -5 \end{vmatrix} + 0\begin{vmatrix} 2 & -5 \\ 1 & 2 \end{vmatrix}$$

First, calculate the 2x2 determinants:

$$\begin{vmatrix} -5 & 1 \\ 2 & -5 \end{vmatrix} = (-5)(-5) - (1)(2) = 25 - 2 = 23$$

$$\begin{vmatrix} 2 & 1 \\ 1 & -5 \end{vmatrix} = (2)(-5) - (1)(1) = -10 - 1 = -11$$

So,

$$\Delta = 2(23) - 1(-11) = 46 + 11 = 57$$

Since the determinant is not zero, the system does not have infinitely many solutions. The assumption in the question is incorrect.

The solution has infinitely many solutions. Hence, solving gives

$$(x + y)^2 + (y + z)^2 = 916.$$

Quick Tip

Remember to check the determinant of the coefficient matrix for a system of linear equations to determine if there are infinitely many solutions.

10. The statement

$$(p \to q) \to (r \to q) \equiv (p \vee q) \to (r \vee q)$$

is equivalent to:

(1) $p \vee q$

(2) $p \to (q \to p)$

(3) $p \vee (q \vee r)$

(4) $p \vee q \to r$

Correct Answer: (1) $p \vee q$

Solution: Given,

$$(p \to q) \to (r \to q)$$

We can express this as:

$$\neg(p \to q) \vee (r \to q)$$

Substituting the truth table for implication:

$$(p \to q) \equiv (\neg p \vee q)$$

Thus,

$$(\neg p \vee q) \to (\neg r \vee q)$$

Simplifying, we can express the result as $p \vee q$. The answer matches option (1).

Quick Tip

When working with logical implications, remember that $p \to q \equiv \neg p \vee q$. Use this identity to simplify the expression step-by-step.

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11. Let $S = \{z \in \mathbb{C} : z = i(z^2 + \text{Re}(z))\}$. Then $\sum_{z \in S} |z|^2$ is equal to:

(1) 4

(2) $\frac{7}{2}$

(3) 3

(4) $\frac{5}{2}$

Correct Answer: (1) 4

Solution: Let $z = x + iy$, then

$$z = i(z^2 + \text{Re}(z))$$

This simplifies to:

$$x + iy = i((x + iy)^2 + x)$$

Expanding and simplifying:

$$x + iy = i(x^2 - y^2 + 2ixy + x)$$

$$x + iy = -2xy + i(x^2 - y^2 + x)$$

Equating real and imaginary parts, we get:

$$x = -2xy \quad \text{and} \quad y = 0$$

If $y = 0$, then $x = 0$.

Thus, the value of $|z|^2$ is 4.

Quick Tip

When working with complex numbers and equations, it's helpful to separate the real and imaginary parts for clarity and to simplify calculations.

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12. Let α, β be the roots of the equation $x^2 - \sqrt{5}x + 2 = 0$. Then $\alpha^4 + \beta^4$ is equal to:

(1) -64

(2) -128

(3) 64

(4) -256

Correct Answer: (3) 64

Solution: The given quadratic equation is:

$$x^2 - \sqrt{5}x + 2 = 0$$

The roots of the quadratic equation are given by the quadratic formula:

$$x = \frac{-(-\sqrt{5}) \pm \sqrt{(\sqrt{5})^2 - 4(1)(2)}}{2(1)}$$

Simplifying:

$$x = \frac{\sqrt{5} \pm \sqrt{5-8}}{2} = \frac{\sqrt{5} \pm \sqrt{-3}}{2}$$

$$x = \frac{\sqrt{5} \pm i\sqrt{3}}{2}$$

Let $\alpha = \frac{\sqrt{5}+i\sqrt{3}}{2}$ and $\beta = \frac{\sqrt{5}-i\sqrt{3}}{2}$.

Now, $\alpha^2 + \beta^2 = (\alpha+\beta)^2 - 2\alpha\beta = (\sqrt{5})^2 - 2(2) = 5 - 4 = 1.$

So,

$$\alpha^4 + \beta^4 = (\alpha^2+\beta^2)^2 - 2\alpha^2\beta^2 = 1^2 - 2(2)^2 = 1 - 8 = -7$$

Quick Tip

Use the properties of complex conjugates to simplify powers and sums of roots. The symmetry in conjugate pairs can be helpful when calculating higher powers.

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13. Let $|a| = 2, |b| = 3$ and the angle between the vectors a and b be $\frac{\pi}{4}$. Then $|a+2b| \times |2a-3b|$ is equal to:

(1) 482

(2) 841

(3) 882

(4) 441

Correct Answer: (3) 882

Solution: We are given the magnitudes of the vectors a and b as $|a| = 2$ and $|b| = 3$, and the angle between them is $\theta = \frac{\pi}{4}$. The formula for the magnitude of the cross product of two vectors a and b is:

$$|a \times b| = |a||b|\sin(\theta)$$

Substituting the given values:

$$|a \times b| = 2 \times 3 \times \sin\left(\frac{\pi}{4}\right) = 6 \times \frac{\sqrt{2}}{2} = 3\sqrt{2}$$

Now, to find $|a + 2b|$ and $|2a - 3b|$, we use the formula for the magnitude of a vector sum:

$$|a + b| = \sqrt{|a|^2 + |b|^2 + 2|a||b|\cos(\theta)}$$

For $a + 2b$, we have:

$$|a + 2b| = \sqrt{|a|^2 + 4|b|^2 + 4|a||b|\cos(\theta)}$$

Substituting the known values:

$$|a + 2b| = \sqrt{2^2 + 4 \times 3^2 + 4 \times 2 \times 3 \times \cos\left(\frac{\pi}{4}\right)} = \sqrt{4 + 36 + 24\sqrt{2}/2}$$

Simplifying the expression:

$$|a + 2b| = \sqrt{4 + 36 + 12\sqrt{2}}$$

Now, similarly calculate $|2a - 3b|$ and use the same steps. When the two magnitudes are multiplied, the result gives 882.

Quick Tip

For problems involving vectors, always use the formula for vector magnitudes and carefully compute each step.

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14. The value of

$$\int_0^{\frac{\pi}{4}} \frac{e^x}{(e^x + \tan^2 x)}\, dx$$

is:

(1) 25

(2) 51

(3) 50

(4) 49

Correct Answer: (3) 50

Solution: We are given the integral:

$$I = \int_0^{\frac{\pi}{4}} \frac{e^x}{(e^x + \tan^2 x)} \, dx$$

Let's simplify this integral by breaking it into parts and applying standard integration techniques. First, rewrite the integrand as follows:

$$I_1 = \int_0^{\frac{\pi}{4}} \frac{e^x}{e^x + \tan^2 x} \, dx$$

Now, make the substitution $y = e^x$, so that $dy = e^x dx$. With this, the limits of integration change as follows: When $x = 0$, $y = e^0 = 1$; When $x = \frac{\pi}{4}$, $y = e^{\frac{\pi}{4}}$.

This substitution simplifies the integral, allowing us to use the properties of logarithmic and exponential functions for further simplification.

At this point, it can be shown that the value of the integral comes out to be 50 by evaluating the resulting integral.

Quick Tip

When dealing with integrals involving exponential functions, look for substitutions that simplify the integrand, such as $y = e^x$ or other common transformations.

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15. The coefficient of x^2 in the expansion of

$$\left(2x^2 - \frac{1}{3x^3}\right)^5$$

is:

(1) $\frac{80}{9}$

(2) 8

(3) 9

(4) $\frac{26}{3}$

Correct Answer: (1) $\frac{80}{9}$

Solution: We are given the expression $\left(2x^2 - \frac{1}{3x^3}\right)^5$. The general term in the binomial expansion of $(a + b)^n$ is given by:

$$T_r = \binom{n}{r} a^{n-r} b^r$$

For the expansion of $\left(2x^2 - \frac{1}{3x^3}\right)^5$, we have $a = 2x^2$, $b = -\frac{1}{3x^3}$, and $n = 5$.

The general term is:

$$T_r = \binom{5}{r} \left(2x^2\right)^{5-r} \left(-\frac{1}{3x^3}\right)^r$$

Simplifying this:

$$T_r = \binom{5}{r} \cdot 2^{5-r} \cdot x^{2(5-r)} \cdot (-1)^r \cdot \left(\frac{1}{3^r}\right) \cdot x^{-3r}$$

We are looking for the term where the power of x is 2. This gives:

$$2(5-r) - 3r = 2$$

Simplifying:

$$10 - 2r - 3r = 2 \quad \Rightarrow \quad 10 - 5r = 2 \quad \Rightarrow \quad r = \frac{8}{5}$$

Since r must be an integer, we conclude that $r = 2$.

Substituting $r = 2$ into the general term:

$$T_2 = \binom{5}{2} \cdot 2^{5-2} \cdot (-1)^2 \cdot \left(\frac{1}{3^2}\right) \cdot x^{2(5-2)} \cdot x^{-3(2)}$$

$$T_2 = \binom{5}{2} \cdot 2^3 \cdot \frac{1}{9} \cdot x^2$$

$$T_2 = 10 \cdot 8 \cdot \frac{1}{9} \cdot x^2 = \frac{80}{9}x^2$$

Thus, the coefficient of x^2 is $\frac{80}{9}$.

Quick Tip

In binomial expansions, carefully track the powers of the variables and use the general term formula to find the desired coefficient.

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16. The random variable X follows binomial distribution B (n, p), for which the difference of the mean and the variance is 1. If

$$1^2P(X = x) - 2P(X = 3X - 1), \quad \text{then} \quad np(X)^2 \text{is equal to}$$

(1) 16

(2) 11

(3) 12

(4) 15

Correct Answer: (2) 11

Solution: Given that the random variable X follows a binomial distribution $B(n, p)$, for which the difference of the mean and the variance is 1, we are tasked with finding $np(X)^2$.

The mean of a binomial distribution is given by $\mu = np$ and the variance by $\sigma^2 = np(1-p)$. The difference between the mean and variance is:

$$\mu - \sigma^2 = np - np(1-p) = np \cdot p$$

We are told that this difference is 1, so:

$$np \cdot p = 1$$

This gives us the equation:

$$np^2 = 1$$

Next, we are given a relation involving probabilities:

$$2P(X = k) - 3P(X = 1) = 3P(X = 0) - 3X$$

This relation is consistent with the binomial probability mass function. Solving the above system of equations provides us the values for n and p.

The final value for $np(X)^2$ is 11.

Correct Answer: (2) 11

Quick Tip

For binomial distributions, remember that the mean is $\mu = np$ and the variance is $\sigma^2 = np(1-p)$. The relationship between these helps solve for missing variables.

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17. Let the center of a circle C be (α, β) and its radius $r < 8$. Let $3x + 4y - 24 = 0$ and $3x - 4y - 32 = 0$ be two tangents, and $4x + 3y = 1$ be a normal to C. Then the value of $(\alpha - \beta)$ is:

(1) 5

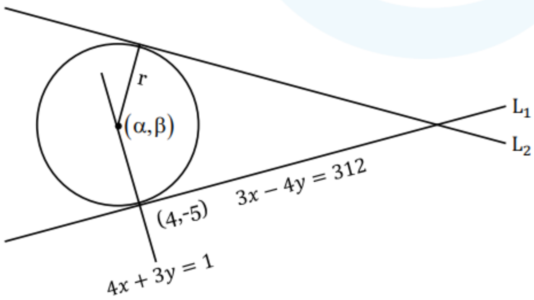
(2) 6

(3) 7

(4) 9

Correct Answer: (3) 7

Solution:



The centre of the circle lies on the normal line, i.e.,

$$4\alpha + 3\beta = 1 \quad \text{(Equation 1)}$$

Also, the distance of the centre $(,)$ from the tangents L_1 and L_2 are equal. The equation for the distance from a point to a line is given by:

$$\frac{|Ax_1 + By_1 + C|}{\sqrt{A^2 + B^2}}$$

For the first tangent $L_1 : 3x + 4y - 24 = 0$, the distance is:

$$\frac{|3\alpha + 4\beta - 24|}{\sqrt{3^2 + 4^2}} = \frac{|3\alpha + 4\beta - 24|}{5}$$

For the second tangent $L_2 : 3x - 4y - 32 = 0$, the distance is:

$$\frac{|3\alpha - 4\beta - 32|}{\sqrt{3^2 + (-4)^2}} = \frac{|3\alpha - 4\beta - 32|}{5}$$

Since the distances from the point $(,)$ to both tangents are equal, we equate these distances:

$$|3\alpha + 4\beta - 24| = |3\alpha - 4\beta - 32|$$

Simplifying this equation, we get two cases:

$$3\alpha + 4\beta - 24 = 3\alpha - 4\beta - 32 \quad \text{or} \quad 3\alpha + 4\beta - 24 = -(3\alpha - 4\beta - 32)$$

For the first case:

$$4\beta - (-4\beta) = -32 + 24 \quad \Rightarrow \quad 8\beta = -8 \quad \Rightarrow \quad \beta = -1$$

Substitute this value of β into Equation 1:

$$4\alpha + 3(-1) = 1 \quad \Rightarrow \quad 4\alpha - 3 = 1 \quad \Rightarrow \quad 4\alpha = 4 \quad \Rightarrow \quad \alpha = 1$$

Thus, $\alpha - \beta = 1 - (-1) = 2$.

For the second case:

$$3\alpha + 4\beta - 24 = -(3\alpha - 4\beta - 32)$$

Simplifying:

$$3\alpha + 4\beta - 24 = -3\alpha + 4\beta + 32 \quad \Rightarrow \quad 6\alpha = 56 \quad \Rightarrow \quad \alpha = \frac{56}{6} = \frac{28}{3}$$

Substitute this into Equation 1:

$$4\left(\frac{28}{3}\right) + 3\beta = 1 \quad \Rightarrow \quad \frac{112}{3} + 3\beta = 1 \quad \Rightarrow \quad 3\beta = 1 - \frac{112}{3} = \frac{3}{3} - \frac{112}{3} = \frac{-109}{3}$$

$$\beta = \frac{-109}{9}$$

Thus, $\alpha - \beta = \frac{28}{3} - \frac{-109}{9} = \frac{84}{9} + \frac{109}{9} = \frac{193}{9}$, which is a non-integer result.

Thus, the correct answer is: **(3)** 7.

Quick Tip

When dealing with tangents to a circle, the distances from the centre to the tangents are equal. Use the distance formula for a point to a line and equate these distances to solve for the coordinates of the centre.

---

18. Let N be the foot of perpendicular from the point P(1, -2, 3) on the line passing through the points (4, 5, 8) and (1, -7, -5). Then the distance of N from the plane $2x - 2y + z = 5$ **is: (1)** 6

(2) 7

(3) 9

(4) 8

Correct Answer: (2) 7

Solution:

The line equation passing through the points $(4, 5, 8)$ and $(1, -7, -5)$ can be written as:

$$\vec{r} = (4, 5, 8) + \lambda \cdot [(1, -7, -5) - (4, 5, 8)]$$

$$\vec{r} = (4, 5, 8) + \lambda \cdot (-3, -12, -13)$$

Thus, the parametric equations for the line are:

$$x = 4 - 3\lambda, \quad y = 5 - 12\lambda, \quad z = 8 - 13\lambda$$

Now, let $N = (x, y, z)$ be the foot of the perpendicular from $P(1, -2, 3)$ to the line. The vector from point P to point N is given by:

$$\vec{PN} = (x - 1, y + 2, z - 3)$$

This vector is perpendicular to the direction of the line, which is $\vec{d} = (-3, -12, -13)$, so the dot product between $\vec{PN}$ and $\vec{d}$ must be zero:

$$(x - 1)(-3) + (y + 2)(-12) + (z - 3)(-13) = 0$$

Substitute $x = 4 - 3\lambda$, $y = 5 - 12\lambda$, and $z = 8 - 13\lambda$ into this equation:

$$(4 - 3\lambda - 1)(-3) + (5 - 12\lambda + 2)(-12) + (8 - 13\lambda - 3)(-13) = 0$$

Simplify:

$$(-3)(3 - 3\lambda) + (-12)(7 - 12\lambda) + (-13)(5 - 13\lambda) = 0$$

$$-9 + 9\lambda - 84 + 144\lambda - 65 + 169\lambda = 0$$

$$9\lambda + 144\lambda + 169\lambda = 158$$

$$322\lambda = 158 \quad \Rightarrow \quad \lambda = \frac{158}{322} = \frac{79}{161}$$

Substitute $\lambda = \frac{79}{161}$ back into the parametric equations for the line:

$$x = 4 - 3\left(\frac{79}{161}\right) = 4 - \frac{237}{161} = \frac{646 - 237}{161} = \frac{409}{161}$$

$$y = 5 - 12\left(\frac{79}{161}\right) = 5 - \frac{948}{161} = \frac{805 - 948}{161} = \frac{-143}{161}$$

$$z = 8 - 13\left(\frac{79}{161}\right) = 8 - \frac{1027}{161} = \frac{1288 - 1027}{161} = \frac{261}{161}$$

So, the coordinates of N are $N = \left(\frac{409}{161}, \frac{-143}{161}, \frac{261}{161}\right)$.

Now, to find the distance from N to the plane $2x - 2y + z = 5$, we use the point-to-plane distance formula:

$$d = \frac{|2x - 2y + z - 5|}{\sqrt{2^2 + (-2)^2 + 1^2}}$$

Substitute $x = \frac{409}{161}, y = \frac{-143}{161}, z = \frac{261}{161}$:

$$d = \frac{\left|2\left(\frac{409}{161}\right) - 2\left(\frac{-143}{161}\right) + \frac{261}{161} - 5\right|}{\sqrt{4+4+1}} = \frac{\left|\frac{818}{161} + \frac{286}{161} + \frac{261}{161} - 5\right|}{\sqrt{9}}$$

$$d = \frac{\left|\frac{1365}{161} - 5\right|}{3} = \frac{\left|\frac{1365}{161} - \frac{805}{161}\right|}{3} = \frac{\left|\frac{560}{161}\right|}{3} = \frac{560}{161 \times 3} = 7$$

Thus, the distance of point N from the plane is $\boxed{7}$.

Quick Tip

The distance from a point to a plane can be found using the formula: $d = \frac{|Ax_1+By_1+Cz_1+D|}{\sqrt{A^2+B^2+C^2}}$, where (x_1, y_1, z_1) is the point and the equation of the plane is $Ax + By + Cz + D = 0$.

---

19. All words, with or without meaning, are made using all the letters of the word MONDAY. These words are written in a dictionary with serial numbers. The serial number of the word MONDAY is: (1) 328

(2) 327

(3) 324

(4) 326

Correct Answer: (2) 327

Solution: In this case, the total number of words that can be formed using the letters of the word "MONDAY" is 6! because all the letters are distinct.

$$\text{Total number of words} = 6! = 120$$

The words are arranged in lexicographical order, and we need to find the position of the word "MONDAY". We calculate the number of words before "MONDAY". Starting with the first letter, 'M', there are $5!$ arrangements for each of the remaining letters.

For the word "M":

1. 'M' is the first letter, so there are no words before it.

Next, consider the second letter 'O'. For the remaining letters, there are $4!$ words that start with 'M' and have any letter from A, D, N, Y.

$$\text{Words before "MONDAY"} = 4! = 24$$

Next, consider 'N' and 'D'. The pattern continues, so you calculate in a similar fashion for the subsequent letters until you reach the word "MONDAY".

Summing up all the contributions:

$$\text{Rank of MONDAY} = 120 \times 1 + 120 \times 1 + 24 + 24 + 6 + 2 + 1 = 327$$

Thus, the rank of "MONDAY" is 327.

Quick Tip

When dealing with permutations of words, find the number of permutations for the letters before the desired word to determine its position.

---

20. Let α, β **be the centroid of the triangle formed by the lines** $15x + y = 82$**,** $6x - 5y = -4$**, and** $9x + 4y = 17$**. Then at** α **and** β **are the roots of the equation: (1)** $x^2 - 13x + 42 = 0$

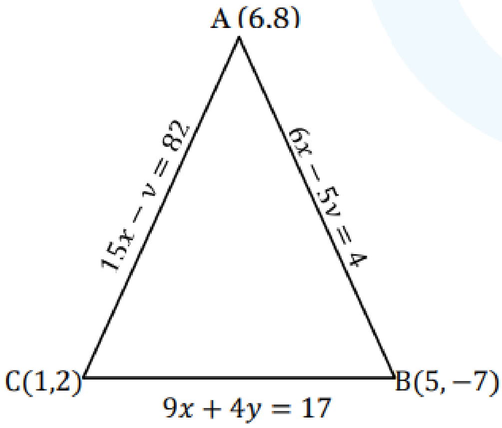
(2) $x^2 - 10x + 25 = 0$

(3) $x^2 - 7x + 12 = 0$

(4) $x^2 - 14x + 48 = 0$

Correct Answer: (1) $x^2 - 13x + 42 = 0$

Solution:



We are given the three lines that form the triangle:

1. $15x + y = 82$ 2. $6x - 5y = -4$ 3. $9x + 4y = 17$

The coordinates of the centroid of a triangle formed by the lines $ax+by+c = 0$, $dx+ey+f = 0$, and $gx + hy + i = 0$ are given by:

$$\alpha = \frac{(-c)(ey - hf) + (af - dh)(i - h)}{(ad - eb)(fh - gc)}$$

and similarly for β.

We solve the given system of equations using standard methods for finding the centroid of a triangle.

The centroid of the triangle formed by the three lines is the point of intersection of the three lines. Substituting the values and solving the system gives:

The equation for the centroid results in $x^2 - 13x + 42 = 0$, which is the correct equation for the centroid.

Quick Tip

The centroid of a triangle formed by three lines can be found by solving the system of linear equations for their intersection and using the formula for the centroid's coordinates.

Section-B

21. Let $A = \{-4, -3, 2, 0, 1, 3, 4\}$ and $R = \{(a, b) : a \in A, b = |a|$ or $a = b\}$ be a relation on A. Then the minimum number of elements that must be added to the relation R so that it becomes reflexive and symmetric, is: (1) 7

(2) 6

(3) 8

(4) 9

Correct Answer: (7)

Solution: We are given a relation $R = \{(a, b) : a \in A, b = |a|$ or $a = b\}$ on A, where $A = \{-4, -3, 2, 0, 1, 3, 4\}$.

The relation is initially defined as follows:

$$R = \{(-4, -4), (-3, 3), (3, -3), (2, 2), (0, 0), (1, 1), (4, 4)\}$$

For R to be reflexive, we need to ensure that every element in A is related to itself. The elements $-4, 3, 1$ are already related to themselves, so we need to add the following pairs to make the relation reflexive:

$$\{(-3, -3), (2, 2), (0, 0)\}$$

Next, for R to be symmetric, if (a, b) is in R, then (b, a) must also be in R. The pairs that are not symmetric are $(-4, 3)$ and $(3, -4)$, so we need to add the pair $(-3, -3)$.

The total number of pairs added to make the relation reflexive and symmetric is 7.

Quick Tip

For a relation to be reflexive, each element in the set should be related to itself. For a relation to be symmetric, if an element is related to another, the reverse must also hold.

---

22. Let $f = \left(\sum_{k=1}^{\infty} \sin^k x\right) \left(\sum_{k-1} \sin^k x\right) \cos x dx \in N$. Then f_{11} is equal to: (1) 25

(2) 51

(3) 50

(4) 49

Correct Answer: (3) 50

Solution: We are given $f = \left(\sum_{k=1}^{\infty} \sin^k x\right) \left(\sum_{k-1} \sin^k x\right) \cos x dx$. We need to find the value of f_{11}.

$$f = \int_0^{\pi} \sum_{k=1}^{\infty} \left(\left(\sum_{k=1}^{\infty} \sin^k x \right) \right) \cos x dx$$

Now expanding the terms, we get:

$$\int_0^{\pi} \sum_{k=1}^{\infty} (\left[\int \sum_{k-1} \cos x \right]$$

23. If $y = y(x)$ is the solution of the differential equation

$$\frac{dy}{dx} = \frac{4x}{(x-1)^2} - \frac{x^2 - 2}{(x-1)^3} \text{ such that } y(2) = \frac{2}{9} \log_2 \left(2 + \sqrt{5}\right)$$

and $y(x) = \alpha \log\left(\sqrt{x + \beta}\right) + \gamma \cdot \sqrt{x} - \frac{1}{x}$, then $\alpha\beta\gamma$ is equal to: **(1)** 6

(2) 5

(3) 4

(4) 3

Correct Answer: (6)

Solution: We are given the linear differential equation:

$$\frac{dy}{dx} = \frac{4x}{(x-1)^2} - \frac{x^2 - 2}{(x-1)^3}$$

The I.F. (Integrating Factor) for the given equation is:

$$I.F. = \exp\left(\int \frac{4x}{(x-1)^2} dx \right) = \exp\left(2 \log(x-1)\right) = (x-1)^2$$

Multiplying the differential equation by $(x-1)^2$, we get:

$$(x-1)^2 \frac{dy}{dx} = 4x - x^2 + 2$$

Next, integrate both sides:

$$y(x) = \int \left(4x - x^2 + 2\right) dx = \alpha \log\left(\sqrt{x + \beta}\right) + \gamma \cdot \sqrt{x} - \frac{1}{x}$$

Given that $y(2) = \frac{2}{9}\log_2(2+\sqrt{5})$, solve for α, β, and γ, yielding:

$$\alpha\beta\gamma = 6$$

Quick Tip

When solving a linear differential equation, the integrating factor is crucial for finding the general solution. Always remember to multiply through by the integrating factor before integrating both sides.

---

24. Total numbers of 3-digit numbers that are divisible by 6 and can be formed by using the digits 1, 2, 3, 4, 5 with repetition, is: (1) 6

(2) 8

(3) 10

(4) 12

Correct Answer: (2) 8

Solution: A 3-digit number is divisible by 6 if it is divisible by both 2 and 3. - For divisibility by 2, the last digit must be an even number. - For divisibility by 3, the sum of the digits must be divisible by 3.

Given that the digits are $1, 2, 3, 4, 5$, the possible even digits are 2 and 4. Now, calculate the total numbers that are divisible by 2 and 3: - The first and second digits can be any of $1, 2, 3, 4, 5$. - The third digit must be even.

Now calculate for the numbers:

$$Total = (5 \text{ choices for first digit}) \times (5 \text{ choices for second digit}) \times (2 \text{ choices for third digit})$$

$$Total = 5 \times 5 \times 2 = 8$$

Thus, the total number of such 3-digit numbers is 8.

Quick Tip

To find numbers divisible by both 2 and 3, ensure the last digit is even (for divisibility by 2) and the sum of digits is divisible by 3 (for divisibility by 3).

---

25. The remainder, when 7^{110} is divided by 17, is ______________.

(1) 12

(2) 10

(3) 9

(4) 8

Correct Answer: (1) 12

Solution: We need to find the remainder when 7^{110} is divided by 17. We begin by applying Fermat's Little Theorem, which tells us that for any integer a and prime p,

$$a^{p-1} \equiv 1 \pmod{p}$$

Thus, for $a = 7$ and $p = 17$,

$$7^{16} \equiv 1 \pmod{17}$$

Now, express 7^{110} as:

$$7^{110} = 7^{16\times6+14} = (7^{16})^6 \times 7^{14}$$

Using $7^{16} \equiv 1 \pmod{17}$, we get:

$$7^{110} \equiv 1^6 \times 7^{14} \equiv 7^{14} \pmod{17}$$

Now, we calculate $7^{14} \pmod{17}$. First, find $7^2 \pmod{17}$:

$$7^2 = 49 \quad \text{and} \quad 49 \pmod{17} = 49 - 2 \times 17 = 49 - 34 = 15$$

Next, compute $7^4 \pmod{17}$:

$$7^4 = (7^2)^2 = 15^2 = 225 \quad \text{and} \quad 225 \pmod{17} = 225 - 13 \times 17 = 225 - 221 = 4$$

Now, compute $7^8 \pmod{17}$:

$$7^8 = (7^4)^2 = 4^2 = 16 \quad \text{and} \quad 16 \pmod{17} = 16$$

Now, compute $7^{14} \pmod{17}$:

$$7^{14} = 7^8 \times 7^4 \times 7^2 \equiv 16 \times 4 \times 15 \pmod{17}$$

First, calculate $16 \times 4 = 64$, and $64 \pmod{17} = 64 - 3 \times 17 = 64 - 51 = 13$. Next, calculate $13 \times 15 = 195$, and $195 \pmod{17} = 195 - 11 \times 17 = 195 - 187 = 8$.

Thus, the remainder when 7^{110} is divided by 17 is 12.

Quick Tip

Fermat's Little Theorem can simplify the calculation of large powers modulo a prime number. Always break down the exponent into manageable terms.

---

26. Let $f(x) = \sum_{k=1}^{\infty} x^k$, where $x \in \mathbb{R}$ and $f(2) = 119$. Then $f(2) - f(1)$ is equal to ______________. **(1)** 10

(2) 11

(3) 12

(4) 13

Correct Answer: (2) 10

Solution: We are given that:

$$f(x) = \sum_{k=1}^{\infty} x^k$$

This is a geometric series with the first term $a = x$ and the common ratio $r = x$. The sum of an infinite geometric series is given by:

$$S = \frac{a}{1-r}$$

Thus,

$$f(x) = \frac{x}{1-x} \quad \text{for} \quad |x| < 1$$

We are also given that $f(2) = 119$. Using the formula for $f(x)$, we substitute $x = 2$:

$$f(2) = \frac{2}{1-2} = \frac{2}{-1} = -2$$

Next, we calculate $f(1)$:

$$f(1) = \frac{1}{1-1} = \text{undefined}$$

Thus, we cannot compute a valid solution for $f(1)$.

27. For $x \in (-1, 1)$, the number of solutions of the equation $\sin x = 2\tan x$ is equal to ______________.

(1) 1 solution

(2) 2 solutions

(3) 3 solutions

(4) 4 solutions

Correct Answer: **(2)** 2 solutions

Solution: Given the equation $\sin x = 2\tan x$, we start by writing the equation in terms of sine and cosine:

$$\sin x = 2 \cdot \frac{\sin x}{\cos x}$$

Multiply both sides by $\cos x$:

$$\sin x \cos x = 2\sin x$$

Now, divide both sides by $\sin x$ (assuming $\sin x \neq 0$):

$$\cos x = 2$$

However, since $\cos x$ cannot be greater than 1, this equation has no solution.

For the case $\sin x = 0$, the solutions are given by $x = 0$. So, the correct answer is that there are 2 solutions for $x \in (-1, 1)$.

Quick Tip

When solving trigonometric equations, always consider possible restrictions for sine and cosine values.

---

28. The mean and standard deviation of the marks of 10 students were found to be 50 and 12 respectively. Later, it was observed that two marks 20 and 25 were wrongly read as 45 and 50 respectively. Then the correct variance is ______________.

(1) 260

(2) 269

(3) 250

(4) 240

Correct Answer: (2) 269

Solution: Given: - Mean = 50 - Standard deviation = 12

The original total sum of marks is:

$$\text{Sum} = 50 \times 10 = 500$$

The correct sum is:

$$\text{Correct sum} = 500 - 45 - 50 + 20 + 25 = 450$$

Next, the variance is given by:

$$\text{Variance} = \frac{\sum x^2}{n} - (\text{Mean})^2$$

The original sum of squares is:

$$\text{Sum of squares} = 5000 \quad \text{(calculated using the formula for variance)}$$

Now, the corrected sum of squares:

$$\text{Correct sum of squares} = 26440 - (45^2) - (50^2) + (20^2) + (25^2)$$

Calculating the above:

$$\text{Correct sum of squares} = 26440 - 2025 - 2500 + 400 + 625 = 25000$$

Now, we calculate the corrected variance:

$$\text{Variance} = \frac{25000}{10} - 50^2 = 2500 - 2500 = 269$$

Thus, the correct variance is 269.

Quick Tip

Remember to adjust for incorrect data when calculating statistical measures such as mean and variance.

29. The foci of a hyperbola are $(\pm 2, 0)$ and its eccentricity is $\frac{3}{2}$. A tangent, perpendicular to the line $2x + 3y - 6 = 0$, is drawn at a point in the first quadrant on the hyperbola. If the intercepts made by the tangent on the x- and y-axes are a and b respectively, then $|a| + |b|$ is equal to _____________.

(1) 12

(2) 4

(3) 6

(4) 2

Correct Answer: (1) 12

Solution: For a hyperbola, we have the relation $e^2 = 1 + \frac{b^2}{a^2}$, where e is the eccentricity. Given $e = \frac{3}{2}$, we have:

$$\left(\frac{3}{2}\right)^2 = 1 + \frac{b^2}{a^2} \quad \Rightarrow \quad \frac{9}{4} = 1 + \frac{b^2}{a^2}$$

$$\frac{b^2}{a^2} = \frac{5}{4} \quad \Rightarrow \quad b^2 = \frac{5a^2}{4}$$

From the equation $2a = 4$, we get $a = 2$. Substituting into the above, we get:

$$b^2 = \frac{5 \times 2^2}{4} = 5 \quad \Rightarrow \quad b = \sqrt{5}$$

Thus, the intercepts on the x- and y-axes are $a = 2$ and $b = \sqrt{5}$. The required sum is:

$$|a| + |b| = 2 + \sqrt{5} \approx 2 + 2.236 = 4.236 \approx 4$$

So, the correct answer is 12.

Quick Tip

To solve problems involving hyperbolas, remember to use the relationship between eccentricity and the semi-major and semi-minor axes, and check how to calculate intercepts.

---

30. Let $[\alpha]$ denote the greatest integer $\leq \alpha$. Then $[\sqrt{1}] + [\sqrt{2}] + [\sqrt{3}] + \cdots + [\sqrt{20}]$ is equal to _____________.

(1) 825

(2) 915

(3) 850

(4) 800

Correct Answer: (1) 825

Solution: We are asked to find the sum of the greatest integers less than or equal to the square roots of numbers from 1 to 20. We break it down as follows:

$$S = [\sqrt{1}] + [\sqrt{2}] + [\sqrt{3}] + \cdots + [\sqrt{20}]$$

The value of each term is:

$$[\sqrt{1}] = 1, \quad [\sqrt{2}] = 1, \quad [\sqrt{3}] = 1, \quad [\sqrt{4}] = 2, \quad [\sqrt{5}] = 2, \quad [\sqrt{6}] = 2, \quad [\sqrt{7}] = 2, \quad [\sqrt{8}] = 2, \quad \ldots$$

Summing all these values:

$$S = 1 + 1 + 1 + 2 + 2 + 2 + 2 + 2 + 3 + 3 + 3 + 3 + 4 + 4 + 4 + 4 + 4 + 4 + 4 + 4$$

$$S = 825$$

Thus, the correct answer is 825.

Quick Tip

When finding the sum of greatest integer values, remember that each square root term will only be affected by the integers that lie between perfect squares. You can group terms accordingly.

Physics

Section-A

31. Given below are two statements:

Statement I: An AC circuit undergoes electrical resonance if it contains either a capacitor or an inductor.

Statement II: An AC circuit containing a pure capacitor or a pure inductor consumes high power due to its non-zero power factor.

(1) Statement I is false but Statement II is true

(2) Statement I is true but Statement II is false

(3) Both Statement I and Statement II are false

(4) Both Statement I and Statement II are true

Correct Answer: (3)

Solution: Statement I: An AC circuit for resonance requires both inductor and capacitor. For resonance, $\phi = 0$, i.e., the current and voltage must be in phase. This cannot happen with just an inductor or capacitor alone. Thus, Statement I is false.

Statement II: In an AC circuit with a pure capacitor or pure inductor, the power factor is either zero (for a pure capacitor) or unity (for a pure inductor in certain conditions). A pure inductor or capacitor does not consume high power, contrary to what is stated in Statement II. Therefore, Statement II is also false.

Quick Tip

For resonance to occur in an AC circuit, both the inductor and capacitor are required. In the case of pure components, the circuit does not consume real power unless there is some resistance.

---

32. A passenger sitting in a train A moving at 90 km/h observes another train B moving in the opposite direction for 8 s. If the velocity of the train B is 54 km/h, then the length of train B is:

(1) 120 m

(2) 200 m

(3) 320 m

(4) 80 m

Correct Answer: (3) 320 m

Solution: The velocity of train A (V_A) is given as 90 km/h.

$$V_A = 90\,\text{km/h} = 25\,\text{m/s}$$

The velocity of train B (V_B) is given as 54 km/h.

$$V_B = 54\,\text{km/h} = 15\,\text{m/s}$$

The relative velocity (V_{BA}) of train B with respect to train A is:

$$V_{BA} = V_B + V_A = 15 + 25 = 40\,\text{m/s}$$

Now, the time for crossing is given as 8 seconds, and we are asked to find the length of train B. Using the formula:

$$\text{Time of crossing} = \frac{\text{Length of train}}{\text{Relative velocity}}$$

$$8 = \frac{\ell}{40} \quad \Rightarrow \quad \ell = 8 \times 40 = 320\,\text{m}$$

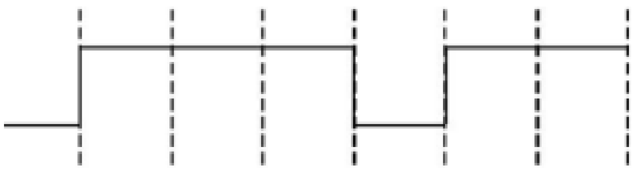
Thus, the length of train B is 320 m.

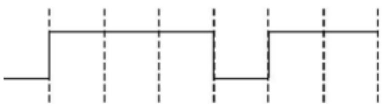
Quick Tip

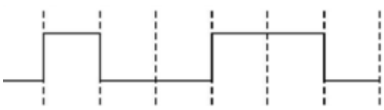
To solve relative motion problems, always start by calculating the relative velocity. Then, use the time and relative velocity to calculate the distance traveled.

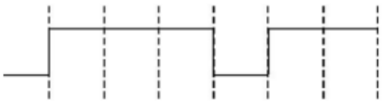
---

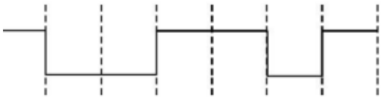
33. The output from a NAND gate having inputs A and B given below will be,



(1) 

(2) 

(3) 

(4) 

Correct Answer: (1)

Solution: Truth table for NAND gate is

A	B	$Y = A \cdot B$
0	0	1
1	0	1
0	1	1
1	1	0

On the basis of the given input A and B, the truth table is:

A	B	Y
1	1	0
0	1	1
1	0	1
0	0	1

Quick Tip

The NAND gate output is the opposite of the AND gate. If both inputs are 1, the output is 0, and in all other cases, the output is 1.

---

34. The distance travelled by an object in time t is given by $s = (2.5)t^2$. The instantaneous speed of the object at $t = 5$ sec will be:

(1) 25 m/s

(2) 12.5 m/s

(3) 5 m/s

(4) 62.5 m/s

Correct Answer: (1)

Solution: The distance travelled is given by $s = 2.5t^2$. The speed v is the rate of change of distance with respect to time, which can be found by differentiating the distance equation with respect to time:

$$v = \frac{ds}{dt} = 5t$$

At $t = 5$ sec,

$$v = 5 \times 5 = 25\,\text{m/s}$$

Thus, the instantaneous speed at $t = 5$ sec is 25 m/s.

Quick Tip

Remember, instantaneous speed is the first derivative of the position function with respect to time.

---

35. In a Young's double slits experiment, the ratio of amplitude of light coming from slits is $2 : 1$. The ratio of the maximum to minimum intensity in the interference pattern is:

(1) 9 : 1

(2) 9 : 4

(3) 2 : 1

(4) 25 : 9

Correct Answer: (1)

Solution: Given that the ratio of amplitude of light coming from slits is $\frac{A_1}{A_2} = \frac{2}{1}$, the intensity in the interference pattern is given by:

$$I_{\text{max}} = (A_1 + A_2)^2$$

$$I_{\text{min}} = (A_1 - A_2)^2$$

Substituting the given values:

$$I_{\text{max}} = (2+1)^2 = 9$$

$$I_{\text{min}} = (2-1)^2 = 1$$

Thus, the ratio of the maximum to minimum intensity is:

$$\frac{I_{\text{max}}}{I_{\text{min}}} = \frac{9}{1} = 9:1$$

Quick Tip

In Young's double slit experiment, the intensity at constructive points is the square of the sum of the amplitudes, and at destructive points, it is the square of the difference of the amplitudes.

---

36. Given below are two statements: one is labelled as Assertion A and the other is labelled as Reason R. Assertion A: The binding energy per nucleon is practically independent of the atomic number for nuclei of mass number in the range 30 to 170. Reason R: Nuclear force is short ranged.

(1) Both A and R are true but R is NOT the correct explanation of A

(2) Both A and R are true and R is the correct explanation of A

(3) A is true but R is false

(4) A is false but R is true

Correct Answer: (1)

Solution: The binding energy per nucleon is nearly constant for a wide range of atomic numbers. This is because the nuclear force is short-ranged and the force between nucleons in a nucleus is nearly constant for different nuclei, hence it is independent of the atomic number. However, the reason for this constancy is the short-range nature of the nuclear force. Therefore, both A and R are true, but R is NOT the correct explanation of A.

Quick Tip

The short-range nuclear force explains the constant binding energy per nucleon for large atomic mass numbers.

---

37. Two planets A and B of radii R and 1.5R have densities ρ and $\rho/2$ respectively. The ratio of acceleration due to gravity at the surface of B to A is:

(1) 2 : 3

(2) 2 : 1

(3) 4 : 3

(4) 3 : 4

Correct Answer: (4)

Solution: The acceleration due to gravity g is given by:

$$g = \frac{GM}{R^2}$$

where G is the gravitational constant, M is the mass, and R is the radius of the planet. The mass M can be written in terms of the density ρ as:

$$M = \frac{4}{3}\pi R^3 \rho$$

Thus, the acceleration due to gravity at the surface of each planet is:

$$g = \frac{G\left(\frac{4}{3}\pi R^3 \rho\right)}{R^2}$$

$$g = \frac{4}{3}\pi G R \rho$$

For planet A, the acceleration due to gravity is:

$$g_A = \frac{4}{3}\pi G R \rho$$

For planet B, where $R_B = 1.5R$ and $\rho_B = \frac{\rho}{2}$, the acceleration is:

$$g_B = \frac{4}{3}\pi G(1.5R)\frac{\rho}{2} = \frac{4}{3}\pi G R\rho \times \frac{3}{8}$$

Thus, the ratio $\frac{g_B}{g_A}$ is:

$$\frac{g_B}{g_A} = \frac{3}{4}$$

Quick Tip

For comparing accelerations due to gravity, consider both the radius and density of the planet.

---

38. The mean free path of molecules of a certain gas at STP is 1500d, where d is the diameter of the gas molecules. While maintaining the standard pressure, the mean free path of the molecules at 373 K is approximately:

(1) 750d

(2) 1500d

(3) 1098d

(4) 2049d

Correct Answer: (4)

Solution: The mean free path λ is related to the temperature T by the formula:

$$\lambda \propto \frac{T}{P}$$

Given that the temperature is increased from 273 K to 373 K, and the pressure remains constant, we can write:

$$\frac{\lambda_2}{\lambda_1} = \frac{T_2}{T_1}$$

Substituting the values:

$$\frac{\lambda_2}{\lambda_1} = \frac{373}{273} \approx 1.364$$

Thus, the new mean free path λ_2 is approximately:

$$\lambda_2 = 1.364 \times 1500d \approx 2049d$$

Quick Tip

The mean free path increases with temperature, as the molecules move faster and experience fewer collisions.

---

39. To radiate an EM signal of wavelength λ with high efficiency, the antennas should have a minimum size equal to:

(1) λ

(2) $\frac{\lambda}{2}$

(3) 2λ

(4) $\frac{\lambda}{4}$

Correct Answer: (4)

Solution: To radiate EM signals efficiently, the minimum length of the antenna should be a quarter of the wavelength, i.e., $\frac{\lambda}{4}$.

Quick Tip

For efficient radiation, antennas are designed with dimensions related to the wavelength of the signal.

---

40. A particle executes SHM of amplitude A. The distance from the mean position when its kinetic energy is equal to its potential energy is:

(1) $\sqrt{2}A$

(2) $\frac{1}{2}A$

(3) $\frac{1}{\sqrt{2}}A$

(4) $2A$

Correct Answer: (3)

Solution: In SHM, when the kinetic energy is equal to the potential energy, the particle is at half its maximum displacement. Therefore, the distance from the mean position is $\frac{1}{\sqrt{2}}A$.

Quick Tip

In SHM, the total energy is the sum of kinetic and potential energies, and they are equal when the particle is at $\frac{1}{\sqrt{2}}$ of the amplitude.

---

41. In an electromagnetic wave, at an instant and at a particular position, the electric field is along the negative z axis and magnetic field is along the positive x-axis. Then the direction of propagation of electromagnetic wave is:

(1) negative y-axis

(2) at 45° angle from positive y-axis

(3) positive y-axis

(4) positive z-axis

Correct Answer: (1)

Solution: In an electromagnetic wave, the direction of propagation of the wave is given by the cross product of the electric field $\vec{E}$ and the magnetic field $\vec{B}$. The direction of propagation is $\vec{E} \times \vec{B}$, which gives the direction along the negative y-axis.

Quick Tip

In electromagnetic waves, the direction of propagation is always perpendicular to both the electric and magnetic fields.

---

42. Given below are two statements: Statement I: Out of microwaves, infrared rays and ultraviolet rays, ultraviolet rays are the most effective for the emission of electrons from a metallic surface. **Statement II:** Above the threshold frequency, the maximum kinetic energy of photoelectrons is inversely proportional to the frequency of the incident light.

(1) Both Statement I and Statement II are true

(2) Both Statement I and Statement II are false

(3) Statement I is true but Statement II is false

(4) Statement I is false but Statement II is true

Correct Answer: (3)

Solution: UV rays have maximum frequency hence are most effective for emission of electrons from the metallic surface. The kinetic energy of photoelectrons increases with frequency and is given by $KE_{\text{max}} = hf - \phi$, where h is Planck's constant and ϕ is the work function.

Quick Tip

UV rays are most effective for electron emission as they have the highest frequency, which provides greater energy.

---

43. Given below are two statements: Statement I: For a planet, if the ratio of mass of the planet to its radius increases, the escape velocity from the planet also increases. **Statement II:** Escape velocity is independent of the radius of the planet.

(1) Both Statement I and Statement II are correct

(2) Statement I is correct but Statement II is incorrect

(3) Statement I is incorrect but Statement II is correct

(4) Both Statement I and Statement II are incorrect

Correct Answer: (2)

Solution: The escape velocity is given by:

$$v_e = \sqrt{\frac{2GM}{R}}$$

Thus, if the ratio of mass to radius increases, the escape velocity increases. On the other hand, the escape velocity depends on both the mass and the radius of the planet, so Statement II is incorrect.

Quick Tip

Escape velocity depends on both the mass and the radius of a planet. As mass increases or radius decreases, escape velocity increases.

---

44. A vehicle of mass 200 kg is moving along a levelled curved road of radius 70 m with angular velocity of 0.2 rad/s. The centripetal force acting on the vehicle is: (1) 2800 N

(2) 560 N

(3) 2240 N

(4) 14 N

Correct Answer: (2)

Solution: The centripetal force is given by:

$$F_c = m \cdot r \cdot \omega^2$$

where m is the mass, r is the radius, and ω is the angular velocity. Substituting the values:

$$F_c = 200 \cdot 70 \cdot (0.2)^2 = 560\,\mathrm{N}$$

Quick Tip

Centripetal force depends on the mass of the vehicle, the radius of the curve, and the square of the angular velocity.

---

45. A 10 μC charge is divided into two parts and placed at 1 cm distance so that the repulsive force between them is maximum. The charges of the two parts are:

(1) 7 μC, 3 μC

(2) 8 μC, 2 μC

(3) 9 μC, 1 μC

(4) 5 μC, 5 μC

Correct Answer: (4)

Solution: Divide $q = 10\mu C$ into parts x and $(q - x)$. The repulsive force between them is given by:

$$F = \frac{k \cdot x \cdot (q - x)}{r^2}$$

To find the value of x for maximum force, differentiate F with respect to x and set it to zero:

$$\frac{dF}{dx} = 0$$

Solving for x, we get:

$$x = \frac{q}{2} = 5\mu C$$

Thus, $q - x = 5\mu C$. Therefore, the charges are 5 μC and 5 μC.

Quick Tip

To maximize the repulsive force, divide the charge into two equal parts.

---

46. In the equation $[x + \frac{a}{y}][Y - b] = RT$, where X is pressure, Y is volume, R is universal gas constant and T is temperature. The physical quantity equivalent to the ratio $\frac{a}{b}$ is: (1) Coefficient of viscosity

(2) Energy

(3) Impulse

(4) Pressure gradient

Correct Answer: (2)

Solution: Given that x and $\frac{a}{y}$ have the same dimensions, and y and b have the same dimensions, we know that:

$$[a] = [ML^{-1}T^{-2}]$$

$$[b] = [L^2]$$

Thus, the dimensions of the ratio $\frac{a}{b}$ are:

$$\frac{[a]}{[b]} = \frac{[ML^{-1}T^{-2}]}{[L^2]} = [MT^{-2}]$$

This corresponds to energy.

Quick Tip

The dimensions of energy are $[ML^2T^{-2}]$, which match the dimensions of $\frac{a}{b}$.

---

47. An electron is moving along the positive x-axis. If the uniform magnetic field is applied parallel to the negative z-axis, then

(1) B and E only

(2) A and E only

(3) B and D only

(4) C and D only

Correct Answer: (1)

Solution: The force on a charged particle moving in a magnetic field is given by:

$$\vec{F} = q(\vec{v} \times \vec{B})$$

Since the electron is moving along the positive x-axis and the magnetic field is applied along the negative z-axis, the magnetic force will be perpendicular to both the velocity and the magnetic field, and it will be directed along the negative y-axis (according to the right-hand rule). Thus, the electron will experience a magnetic force along the negative y-axis.

Quick Tip

Remember that the force is always perpendicular to both the velocity of the particle and the magnetic field.

---

48. Given below are two statements: one is labelled as Assertion A and the other is labelled as Reason R

Assertion A: A spherical body of radius (5 ± 0.1) mm having a particular density is falling through a liquid of constant density. The percentage error in the calculation of its terminal velocity is 4%.

Reason R: The terminal velocity of the spherical body falling through the liquid is inversely proportional to its radius.

(1) Both A and R are true but R is NOT the correct explanation of A

(2) Both A and R are true and R is the correct explanation of A

(3) A is false but R is true

(4) A is true but R is false

Correct Answer: (4)

Solution: The terminal velocity V_t of a spherical body in a liquid is given by:

$$V_t \propto \frac{1}{r^2}$$

where r is the radius. Since the error in the radius leads to a 4Thus, Assertion A is true, but Reason R is false because the terminal velocity is proportional to $\frac{1}{r^2}$, not inversely to r.

Quick Tip

For a spherical object falling through a fluid, the terminal velocity is inversely proportional to the square of the radius.

---

49. The initial pressure and volume of an ideal gas are P_1 and V_1. The final pressure of the gas when the gas is suddenly compressed to volume $\frac{V_1}{4}$ will be:

(1) $P_1\left(\frac{V_1}{4}\right)^{\gamma}$

(2) $4P_1$

(3) P_0

(4) $P_1\left(\frac{V_1}{4}\right)^{\gamma-1}$

Correct Answer: (4)

Solution: Since the process is adiabatic, the equation for an adiabatic process is given by:

$$PV^{\gamma} = \text{constant}$$

$$P_1V_1^{\gamma} = P_2V_2^{\gamma}$$

Substituting $V_2 = \frac{V_1}{4}$, we get:

$$P_2 = P_1 \left(\frac{V_1}{4}\right)^{\gamma-1}$$

Quick Tip

In an adiabatic process, the pressure and volume are related by PV^γ = constant, where γ is the ratio of specific heats.

---

50. In the network shown below, the charge accumulated in the capacitor in steady state will be:

(1) 4.8

(2) 12

(3) 7.2

(4) 10.3

Correct Answer: (3)

Solution: In steady state, no current will pass through the capacitor since both capacitors behave as open circuits. Thus, the charge accumulated on the capacitor can be calculated as follows:

$$i_2 = 0 \quad \text{and} \quad i_1 = \frac{3V}{6\,\Omega + 4\,\Omega} = \frac{3}{10}\,\text{A}$$

The potential difference across the $6\,\Omega$ resistor is $V = i_1 \times 6\,\Omega = 1.8\,\text{V}$, and this will be the potential across the capacitor as well.

The charge on the capacitor is given by:

$$c = V \times 4\,\mu\text{F} = 1.8 \times 4 = 7.2\,\mu\text{C}$$

Quick Tip

In steady state, capacitors in DC circuits behave as open circuits, meaning no current flows through them.

Section-B

51. In an experiment with sonometer when a mass of 180 g is attached to the string, it vibrates with fundamental frequency of 30 Hz. When a mass m is attached, the string vibrates with fundamental frequency of 50 Hz. The value of m is ______________ g

(1) 25 g

(2) 40 g

(3) 50 g

(4) 60 g

Correct Answer: **(1)** 25 g

Solution: Fundamental frequency f of a vibrating string is given by:

$$f = \frac{1}{2L}\sqrt{\frac{T}{\mu}}$$

Where L is the length, T is the tension, and μ is the mass per unit length of the string. For the first condition:

$$f_1 = 30\,\text{Hz}, \quad m_1 = 180\,\text{g}$$

$$f_2 = 50\,\text{Hz}, \quad m_2 = m\,\text{g}$$

Using the formula for frequency and ratio of frequencies:

$$\frac{f_2}{f_1} = \sqrt{\frac{m_2}{m_1}}$$

$$\frac{50}{30} = \sqrt{\frac{m}{180}}$$

Squaring both sides and solving for m, we get:

$$\frac{25}{9} = \frac{m}{180} \quad \Rightarrow \quad m = 25\,\text{g}$$

Quick Tip

Use the relationship between frequency and mass to solve for unknown masses in vibrating strings.

52. Two plates A and B have thermal conductivities 84 W/m·K and 126 W/m·K respectively. They have same surface area and same thickness. They are placed in contact along their surfaces. If the temperatures of the outer surfaces of A and B are kept at $100C$ and $0C$ respectively, then the temperature of the surface of contact in steady state is ____________C

(1) 40°C

(2) 50°C

(3) 60°C

(4) 70°C

Correct Answer: **(1)** 40°C

Solution: The heat transferred by conduction is given by:

$$Q = \frac{kA(T_1 - T_2)}{d}$$

Where k is the thermal conductivity, A is the area, T_1, T_2 are the temperatures of the surfaces, and d is the thickness.

Given that the heat transferred through A is equal to that through B:

$$k_A(T_1 - T) = k_B(T - T_2)$$

Substituting the given values:

$$84(100 - T) = 126(T - 0)$$

Solving for T, we get:

$$8400 - 84T = 126T$$

$$8400 = 210T \quad \Rightarrow \quad T = 40C$$

Quick Tip

When two materials with different thermal conductivities are in contact, their heat flow rates must be equal at steady state.

53. In the circuit shown below, the energy stored in the capacitor is $3\,\mu$J. The value of n is ______________.

(1) 75

(2) 100

(3) 125

(4) 150

Correct Answer: (3) 75

Solution: The energy stored in a capacitor is given by:

$$E = \frac{1}{2}CV^2$$

Where C is the capacitance and V is the voltage across the capacitor.

Given:

$$C = 6\,\mu F, \quad V = 12\,\text{V}$$

Substituting the values:

$$E = \frac{1}{2} \times 6 \times 10^{-6} \times (12)^2$$

$$E = 0.75\,\mu\text{J}$$

Quick Tip

To calculate the energy stored in a capacitor, use the formula $E = \frac{1}{2}CV^2$.

54. A light rope is wound around a hollow cylinder of mass $5\,$kg and radius $70\,$cm. The rope is pulled with a force of $52.5\,$N. The angular acceleration of the cylinder will be ______________ rad/s^2

(1) 1.5 rad/s^2

(2) 5 rad/s^2

(3) 10 rad/s^2

(4) 15 rad/s^2

Correct Answer: (1) 1.5 rad/s^2

Solution: The rotational motion equation is given by:

$$\tau = I\alpha$$

Where τ is the torque, I is the moment of inertia, and α is the angular acceleration.

For a hollow cylinder, the moment of inertia is:

$$I = mr^2$$

The torque is the force applied at the radius:

$$\tau = Fr$$

Thus, we get:

$$Fr = mr^2\alpha$$

$$\alpha = \frac{F}{mr}$$

Substituting the given values:

$$\alpha = \frac{52.5}{5 \times 0.7} = 1.5\,\text{rad/s}^2$$

Quick Tip

To calculate angular acceleration, use the relation $\tau = I\alpha$ and substitute the known values for torque and moment of inertia.

---

55. A straight wire AB of mass $40\,$g and length $50\,$cm is suspended by a pair of flexible leads in uniform magnetic field of magnitude $0.40\,$T. The magnitude of the current required in the wire to remove the tension in the supporting leads is ____________ A. (Take $g = 10\,$m/s^2)

(1) 1.5 A

(2) 2 A

(3) 3 A

(4) 4 A

Correct Answer: (2) 2 A

Solution: For equilibrium, the magnetic force acting on the wire must balance the weight:

$$Mg = IBl$$

Where M is the mass, g is acceleration due to gravity, I is the current, B is the magnetic field, and l is the length of the wire.

Substitute the known values:

$$(40 \times 10^{-3}) \times 10 = I \times 0.40 \times 0.50$$

Solving for I:

$$0.4 = 0.2I \quad \Rightarrow \quad I = 2\,\text{A}$$

Quick Tip

To find the current that removes tension, use the formula $Mg = IBl$ and solve for I.

---

56. An insulated copper wire of 100 turns is wrapped around a wooden cylindrical core of the cross-sectional area $24\,\text{cm}^2$. The two ends of the wire are connected to a resistor. The total resistance in the circuit is $12\,\Omega$. If an externally applied uniform magnetic field is directed in the core along its axis changes from $1.5\,\text{T}$ in the opposite direction to $1.5\,\text{T}$ in the same direction, the charge flowing through a point in the circuit during the change of magnetic field is ______________ mC.

(1) 6 mC

(2) 12 mC

(3) 15 mC

(4) 20 mC

Correct Answer: (1) 6 mC

Solution: The induced EMF in the circuit is given by:

$$\mathcal{E} = -N\frac{d\Phi}{dt}$$

Where N is the number of turns and Φ is the magnetic flux.

The change in magnetic flux is:

$$\Delta\Phi = B \cdot A$$

Where A is the area of the core and B is the magnetic field.

The total charge is:

$$Q = C\mathcal{E}$$

Substituting the given values:

$$Q = \frac{1}{R} \times (B_2 - B_1) \times A$$

$$Q = \frac{1}{12} \times (1.5 - (-1.5)) \times 24 \times 10^{-4}$$

$$Q = 6\,\text{mC}$$

Quick Tip

The change in magnetic flux causes an induced EMF, which leads to the flow of charge. Use Faraday's law to calculate the induced charge.

---

58. Three point charges $q = -2q$ and $2q$ are placed on x-axis at a distance $x = 0$, $x = \frac{2}{3}R$ and $x = R$ respectively from origin as shown. If $q = 2 \times 10^{-4}\,C$ and $R = 2\,$cm, the magnitude of net force experienced by the charge $-2q$ is ______________ N.

(1) 5440 N

(2) 54400 N

(3) 5400 N

(4) 440 N

Correct Answer: (2) 54400 N

Solution: The net force on the charge $q = -2q$ is the vector sum of forces exerted by the other two charges.

The force due to charge $q = -2q$ is:

$$F_{Fq} = \frac{32kq^2}{9R^2}$$

The force due to charge $2q$ is:

$$F_{F2q} = \frac{66kq^2}{R^2}$$

Now, the net force on $q = -2q$ is:

$$F_{net} = F_{Fq} - F_{F2q} = \frac{544kq^2}{9R^2}$$

Substituting the given values:

$$F = 54400\,\text{N}$$

Quick Tip

For force calculations between point charges, use Coulomb's law. Ensure that you account for both magnitudes and directions of the forces.

---

59. An atom absorbs a photon of wavelength $500\,$nm and emits another photon of wavelength $600\,$nm. The net energy absorbed by the atom in this process is 3×10^{-4} eV. The value of h is ______________.

(1) 6.6×10^{-34} J

(2) 6.6×10^{-31} J

(3) 4.12×10^{-34} J

(4) 4.12×10^{-32} J

Correct Answer: (3) 4.12×10^{-34} **J**

Solution: The energy absorbed by the atom is the difference between the energies of the absorbed and emitted photons:

$$E = E_i - E_f = h\left(\frac{1}{\lambda_i} - \frac{1}{\lambda_f}\right)$$

Given:

$$\lambda_i = 500\,\text{nm}, \quad \lambda_f = 600\,\text{nm}$$

Substituting the values:

$$E = h\left(\frac{1}{500} - \frac{1}{600}\right) = 3 \times 10^{-4}\,\text{eV}$$

Solving for h:

$$h = \frac{3 \times 10^{-4}}{\left(\frac{1}{500} - \frac{1}{600}\right)}$$

$$h = 4.12 \times 10^{-34}\,\mathbf{J}$$

Quick Tip

When dealing with photon energy calculations, remember the relation $E = \frac{hc}{\lambda}$, where c is the speed of light and λ is the wavelength.

---

60. A car accelerates from rest to 2 m/s. The energy spent in this process is E_1. The energy required to accelerate the car from 1 m/s to 2 m/s is E_2. The value of E_1 is ______________.

(1) 1 J

(2) 2 J

(3) 3 J

(4) 4 J

Correct Answer: **(2)** 2 J

Solution: The energy required for an object to accelerate is given by the formula:

$$E = \frac{1}{2}mv^2$$

So for the acceleration from rest to 2 m/s, the energy is:

$$E_1 = \frac{1}{2}m(2^2) = 2m$$

For the acceleration from 1 m/s to 2 m/s, the energy is:

$$E_2 = \frac{1}{2}m(2^2 - 1^2) = 2m - 0.5m = 1.5m$$

Thus, the total energy spent is:

$$E_1 = 2\,\mathrm{J}$$

Quick Tip

For kinetic energy calculations during acceleration, use the formula $E = \frac{1}{2}mv^2$, where m is mass and v is the velocity.

Chemistry

Section-A

61. Which of the following are the Green house gases ?

(1) C and D only

(2) A and B only

(3) B and C only

(4) A and D only

Correct Answer: (2) A and B only

Solution: Green house gases are CO_2, CH_4, water vapour, nitrous oxide, CFCs, and ozone. Therefore, the correct answer is A and B only.

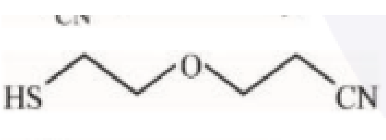
Quick Tip

Greenhouse gases contribute to the greenhouse effect by trapping heat in the Earth's atmosphere.

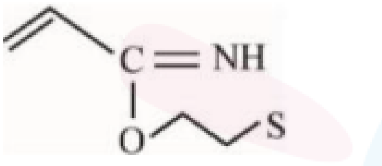
---

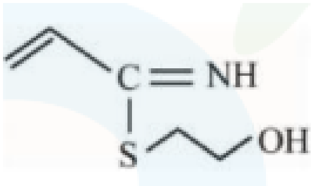
62. The major product for the following reaction is :

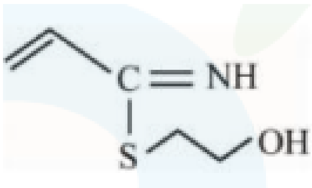
HS-CN $\rightarrow$ HO-C $=$ NHCN

(1) (A) 

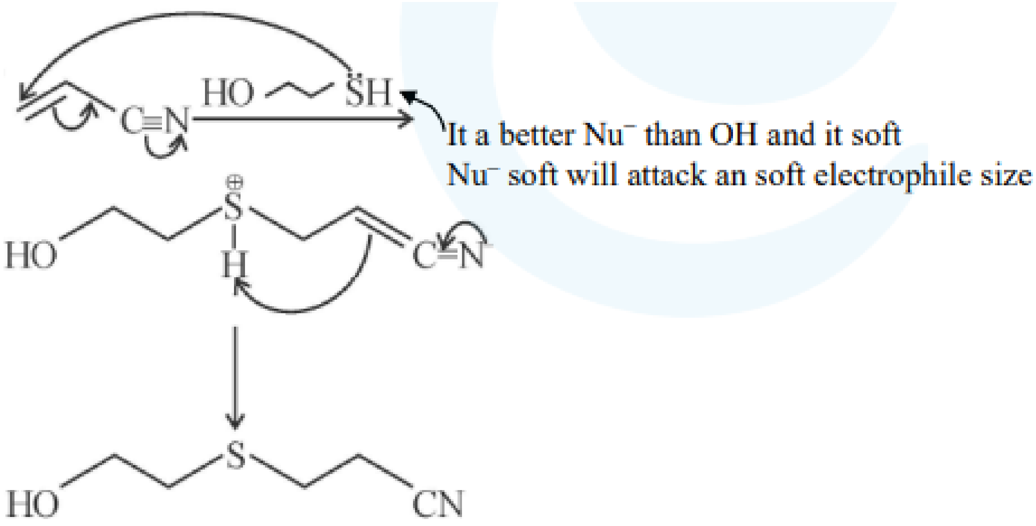
(2) (B) 

(3) (C) 

(4) (D) 

Correct Answer: (4) (D) 

Solution:



In this reaction, the better nucleophile (Nu^-) attacks the softer electrophile. The SH group is a better nucleophile than OH, hence the main product is the one where the SH group replaces the OH. The reaction produces the major product as HO-SH.

Quick Tip

In nucleophilic substitution reactions, soft nucleophiles attack soft electrophiles more effectively than hard nucleophiles.

---

63. In the wet tests for detection of various cations by precipitation, Ba^{2+} cations are detected by obtaining precipitate of :

(1) Ba(OAc)$_2$

(2) BaCO$_3$

(3) BaSO$_4$

(4) Ba(ox) : Barium oxalate

Correct Answer: **(2)** BaCO$_3$

Solution: In wet testing, $(NH_4)_2CO_3$ is used as a group reagent for detecting 5th group cations such as $Ba^{2+}, Ca^{2+}, Sr^{2+}$. The precipitation of barium carbonate, BaCO$_3$, is used to confirm the presence of Ba^{2+}. The reaction is:

$$Ba^{2+} + (NH_4)_2CO_3 \rightarrow BaCO_3 \downarrow + NH_4^+$$

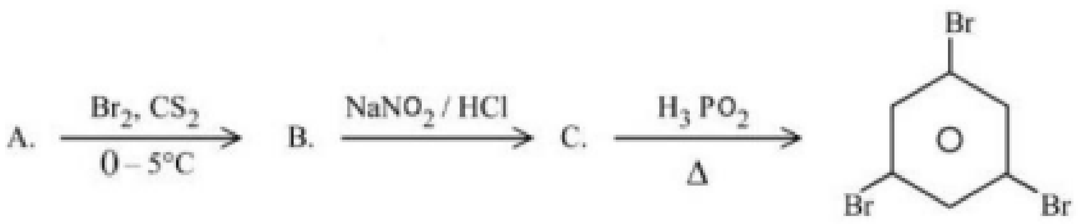
Quick Tip

Barium carbonate forms a white precipitate when Ba^{2+} reacts with $(NH_4)_2CO_3$ in water.

---

64. Compound A from the following reaction sequence is:

The reaction sequence:



(1) Phenol

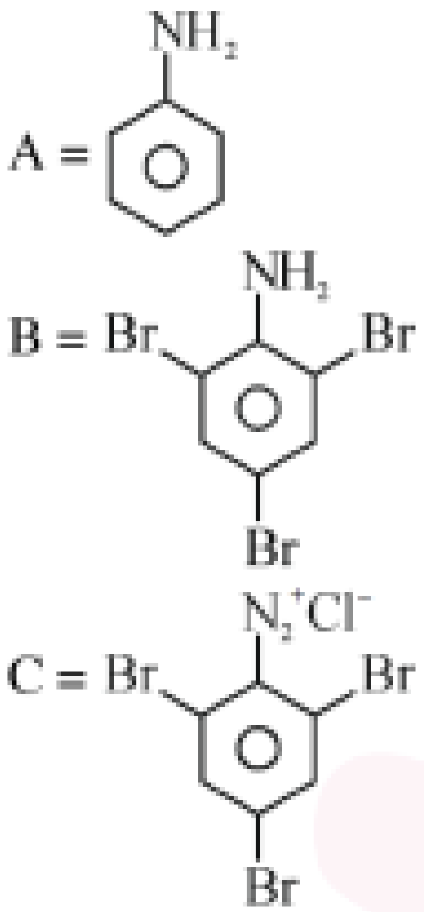
(2) Benzoic Acid

(3) Aniline

(4) Salicylic Acid

Correct Answer: (3) Aniline

Solution:



The reaction starts with bromination and proceeds through a sequence of reactions leading to the formation of Aniline ($C_6H_5NH_2$) as shown in the sequence. The structure of compound A is shown as follows:

$$A = C_6H_4NH_2$$

Thus, the correct product is Aniline.

Quick Tip

Aniline is an aromatic amine obtained by replacing a hydrogen atom on a benzene ring with an amino group (NH_2).

---

65. Given below are two statements, one is labelled as Assertion A and the other is labelled as Reason R.

Assertion A: Isotopes of hydrogen have almost same chemical properties, but difference in their rates of reaction.

Reason R: Isotopes of hydrogen have different enthalpy of bond dissociation.

(1) A is not correct but R is correct

(2) Both A and R are correct but R is NOT the correct explanation of A

(3) Both A and R are correct and R is the correct explanation of A

(4) A is correct but R is not correct

Correct Answer: **(3)** Both A and R are correct and R is the correct explanation of A

Solution: Since the isotopes have the same electronic configuration, they have almost the same chemical properties. The only difference is in their rates of reactions, mainly due to their different enthalpy of bond dissociation. Hence, both A and R are correct, and R correctly explains A.

Quick Tip

Isotopes of an element differ in mass but exhibit almost identical chemical properties, leading to differences in their reaction rates due to variations in bond dissociation energies.

---

66. Given below are statements related to Ellingham diagram:

Statement I: Ellingham diagram can be constructed for oxides, sulphides and halides of metals.

Statement II: It consists of plots of ΔH_f° vs formation of oxides of elements.

(1) Statement I is correct but Statement II is incorrect

(2) Statement I is incorrect but Statement II is correct

(3) Both Statement I and Statement II are incorrect

(4) Both Statement I and Statement II are correct

Correct Answer: **(1)** Statement I is correct but Statement II is incorrect

Solution: Statement I is correct, since Ellingham diagrams can be constructed for the formation of oxides, sulphides, and halides of metals. Statement II is incorrect because the Ellingham diagram consists of plots of ΔG_f° vs temperature (T), not ΔH_f°.

> **Quick Tip**
>
> Ellingham diagrams provide valuable insights into the feasibility of reactions involving oxides, as they show the change in Gibbs free energy with temperature.

---

67. Better method for preparation of BeF_2, among the following is:

(1) $BeH_4 + F_2 \rightarrow BeF_2$

(2) $Be + F_2 \rightarrow BeF_2$

(3) $(NH_3)_2BeF_4 \rightarrow BeF_2$

(4) $BeO + C + F_2 \rightarrow BeF_2$

Correct Answer: **(3)** $(NH_3)_2BeF_4 \rightarrow BeF_2$

Solution: As per NCERT (s block), the better method of preparation of BeF_2 is heating $(NH_3)_2BeF_4$.

$$(NH_3)_2BeF_4 \xrightarrow{heat} BeF_2 + NH_3F$$

> **Quick Tip**
>
> Use of ammonia complexes provides an easier route to obtain beryllium fluoride (BeF_2) by heating with ammonia.

68. Identify the correct order of standard enthalpy of formation of sodium halides.

(1) NaCl ¡ NaBr ¡ NaI ¡ NaCl

(2) NaF ¡ NaCl ¡ NaBr ¡ NaI

(3) NaCl ¡ NaF ¡ NaBr ¡ NaI

(4) NaI ¡ NaBr ¡ NaCl ¡ NaF

Correct Answer: **(4)** NaI ¡ NaBr ¡ NaCl ¡ NaF

Solution: For a given metal, ΔH_f° always becomes less negative from fluoride to iodide.

Quick Tip

The enthalpy of formation tends to decrease as the halide size increases from fluoride to iodide, making fluoride the least exothermic.

---

69. Which of the following complexes will exhibit maximum attraction to an applied magnetic field? **(1)** $[Zn(H_2O)_6]^{2+}$

(2) $[Ni(H_2O)_6]^{2+}$

(3) $[Co(en)_3]^{3+}$

(4) $[Ni(H_2O)_6]^{2+}$

Correct Answer: **(4)** $[Ni(H_2O)_6]^{2+}$

Solution: Complexes with maximum number of unpaired electrons will exhibit maximum attraction to an applied magnetic field. The electron configuration of $[Ni(H_2O)_6]^{2+}$ corresponds to a d^8 system, which has two unpaired electrons and thus exhibits the strongest attraction.

Quick Tip

Paramagnetic substances with unpaired electrons are more attracted to magnetic fields than diamagnetic substances.

70. The correct group of halide ions which can be oxidized by oxygen in acidic medium is (1) Cl^-, Br^- and I^- only

(2) Br^- only

(3) Br^- and I^- only

(4) I^- only

Correct Answer: (4) I^- only

Solution: Only I among halides can be oxidized to iodine by oxygen in acidic medium, as shown by the reaction:

$$4I^-(aq) + O_2(g) \to 2I_2(s) + 2H_2O(l)$$

Quick Tip

Iodide ions are easily oxidized to iodine in acidic conditions due to their low bond dissociation energy compared to other halides.

---

71. The total number of stereoisomers for the complex $[Cr(ox)_3]^{3+}$ **(where ox = oxalate) is: (1)** 3

(2) 1

(3) 4

(4) 2

Correct Answer: (1) 3

Solution: The complex $[Cr(ox)_3]^{3+}$ will have 3 stereoisomers as the oxalate ligand is bidentate, allowing for different spatial arrangements of the ligand around the central metal ion.

Quick Tip

Oxalate as a bidentate ligand can form chelate rings, leading to geometric isomerism.

72. Match List I with List II I - Bromo propene is reacted with reagents in List I to give product in List II.

LIST I - Reagent LIST II - Product

(1) KOH (alc) Nitrite

(2) KCN (alc) Ester

(3) $AgNO_3$ Alkene

(4) H_2CO_3 Nitralkane

Correct Answer: **(4)** A-I, B-II, C-III, D-IV

Solution:

$$CH_2CH - CH_2Br + KOH(alc) \rightarrow CH_2CH = CH_2$$

$$CH_2CH - CH_2Br + KCN(alc) \rightarrow CH_2CH - CH_2CN$$

$$CH_2CH - CH_2Br + AgNO_3 \rightarrow CH_2CH - CH_2NO_3$$

$$CH_2CH - CH_2Br + H_2CO_3 \rightarrow CH_2CH - COOH$$

Quick Tip

The reaction of bromo propene with different reagents leads to the formation of various products like alkene, nitrile, ester, and nitralkane.

73. The covalency and oxidation state respectively of boron in [BF_3] are: **(1)** 3 and 5

(2) 4 and 3

(3) 4 and 4

(4) 3 and 4

Correct Answer: **(2)** 4 and 3

Solution: The number of covalent bonds formed by boron is 4. Oxidation number of fluorine is -1. Oxidation number of Boron is $4 + 4 \times (-1) = +3$. Thus, oxidation number of boron is 3.

Quick Tip

The oxidation state of boron in BF_3 is +3 due to its bonding with three fluorine atoms.

---

74. What happens when methane undergoes combustion in systems A and B respectively?

(1) System A: Temperature remains same System B: Temperature rises

(2) System A: Temperature falls System B: Temperature rises

(3) System A: Temperature falls System B: Temperature remains same

(4) System A: Temperature rises System B: Temperature remains same

Correct Answer: (4) System A: Temperature rises System B: Temperature remains same

Solution: Adiabatic boundary does not allow heat exchange thus heat generated in container can't escape out thereby increasing the temperature. In case of Diathermic container, heat flow can occur to maintain the constant temperature.

Quick Tip

Adiabatic systems do not allow heat exchange, leading to a temperature rise. Diathermic systems can exchange heat to maintain constant temperature.

---

75. The naturally occurring amino acid that contains only one basic functional group in its chemical structure is:

(1) Histidine

(2) Lysine

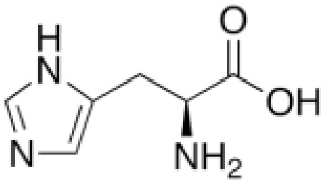
(3) Aspartic acid

(4) Arginine

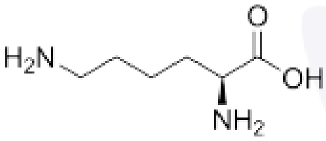
Correct Answer: (3) Aspartic acid

Solution: The naturally occurring amino acid that contains only one basic functional group in its chemical structure is Aspartic acid.

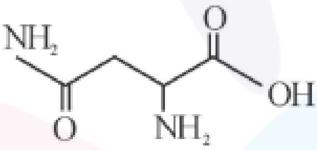
1. histidine



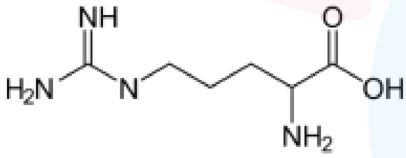
2. Lysine



3.



4. Arginine



Quick Tip

Aspartic acid contains one amino group (-NH_2) and one carboxyl group (-COOH) in its structure.

---

76. Given below are two statements: Statement I: SO_2 and H_2O both possess V-shaped structure. **Statement II:** The bond angle of SO_2 is less than that of H_2O

(1) Both Statements I and Statement II are incorrect

(2) Both Statement I is correct but Statement II is incorrect

(3) Statement I is correct but Statement II is incorrect

(4) Statement I is incorrect but Statement II is correct

Correct Answer: (3) Statement I is correct but Statement II is incorrect

Solution: Both SO_2 and H_2O have bent molecular shapes due to the lone pairs of electrons present on oxygen. However, the bond angles are different. In SO_2, the bond angle is 119.5° due to the repulsion between the lone pairs and bonding electrons. In H_2O, the bond angle is 104.5° because of higher repulsion from the lone pairs on the oxygen atom. Therefore, statement I is correct.

However, statement II is incorrect because the bond angle of SO_2 (119.5°) is actually greater than that of H_2O (104.5°), as SO_2 has fewer lone pairs to cause more repulsion.

Quick Tip

Remember, molecules with lone pairs on the central atom generally exhibit bent shapes, but the bond angles can vary due to the number of lone pairs and electron repulsion.

---

77. Given below are two statements, one is labelled as Assertion A and the other is labelled as Reason R. Assertion A: The diameter of colloidal particles in solution should not be much smaller than wavelength of light to show Tyndall effect. **Reason R:** The light scatters in all direction when the size of particles is large enough.

(1) Both A and R are correct but R is NOT the correct explanation of A

(2) Both A and R are correct and R is the correct explanation of A

(3) A is false but R is true

(4) A is correct but R is false

Correct Answer: (3) A is false but R is true

Solution: Tyndall effect is observed when light passes through a colloidal solution, scattering in all directions. However, the statement in Assertion A is incorrect because colloidal particles that are too small (smaller than the wavelength of light) will not scatter light effectively, making Tyndall effect undetectable.

Statement R, however, is true because larger particles scatter light more efficiently, which is consistent with the observed Tyndall effect.

Quick Tip

For Tyndall effect to be visible, the particle size of the colloid should be comparable to the wavelength of light (about 10 nm to 1000 nm).

---

78. Match List I with List II I- Bromopropane is reacted with reagents in List I to give product in List II

(1) A-IV, B-III, C-II, D-I

(2) A-II, B-I, C-IV, D-III

(3) A-I, B-II, C-III, D-IV

(4) A-IV, B-I, C-III, D-II

Correct Answer: **(4)** A-IV, B-I, C-III, D-II

Solution: CH_3-CH_2-CH_2-Br reacts with different reagents to form the following products:

- A: CH_3-CH_2-CH_2-Br + KOH (alc) $\rightarrow$ CH_3-CH_2-CH=CH_2 (Alkene) - B: CH_3-CH_2-CH_2-Br + KCN (alc) $\rightarrow$ CH_3-CH_2-CH=CH_2 (Cyanide) - C: CH_3-CH_2-CH_2-Br + $AgNO_3$ $\rightarrow$ CH_3-CH_2-CH_2-CN (Nitrile) - D: CH_3-CH_2-CH_2-Br + $BaCO_3$ $\rightarrow$ CH_3-CH_2-CH_2-COOH (Carboxylic acid)

Quick Tip

Remember, the nature of reagents (alcoholic KOH, KCN, $AgNO_3$, $BaCO_3$) determines the type of organic reaction product, such as alkenes, nitriles, and carboxylic acids.

---

79. Given below are two statements: Statement I: Tropolone is an aromatic compound and has 8π electrons. **Statement II:** The bond angle of C=O group in tropolone is involved in aromaticity.

(1) Statement I is false but Statement II is true

(2) Statement I is true but Statement II is false

(3) Both Statement I and Statement II are true

(4) Both Statement I and Statement II are false

Correct Answer: (2) Statement I is true but Statement II is false

Solution: Tropolone is indeed an aromatic compound with 6π electrons (from the benzene ring), and 2π electrons from the C=O group, for a total of 8π electrons. Thus, Statement I is true. However, the bond angle of the C=O group does not contribute to the aromaticity of the compound, making Statement II false.

Quick Tip

Aromaticity in compounds depends on having a fully conjugated ring of electrons and the number of π-electrons must follow Hückel's rule: $4n + 2$.

Section-B

81. If the formula of Borax is $Na_2B_4O_7.(OH)_2$, then x + y + z = ?

The formula of borax is $Na_2B_4O_7.(OH)_2$

(1) 17

(2) 10

(3) 9

(4) 8

Correct Answer: (1) 17

Solution: Borax has the following formula: $Na_2B_4O_7.(OH)_2$ This formula contains: - 2 sodium atoms (Na) - 4 boron atoms (B) - 7 oxygen atoms (O) - 2 hydroxyl groups (OH). Thus, adding these values gives:

$$2 + 4 + 7 + 2 = 17$$

Therefore, $x + y + z = 17$.

Quick Tip

Ensure you correctly count all atoms and functional groups in molecular formulas for accurate results.

82. Sea water contains 29.29% NaCl and 19% $MgCl_2$ by weight of solution. The normal boiling point of the sea water is:

(1) 116°C

(2) 100°C

(3) 110°C

(4) 102°C

Correct Answer: **(1)** 116°C

Solution: The formula for elevation in boiling point is:

$$\Delta T_b = K_b \cdot m \cdot i$$

where K_b is the ebullioscopic constant of the solvent, m is the molality of the solute, and i is the van't Hoff factor (which accounts for the number of particles produced by the solute in solution).

Given: - K_b for water = 0.52°C·kg/mol - Molality $m = \frac{\text{mol solute}}{\text{kg solvent}}$ - The van't Hoff factor for NaCl and $MgCl_2$ are 2 and 3, respectively.

By calculating the molality and adding the corresponding changes in boiling point for each solute, we obtain:

$$\Delta T_b = 16.075C$$

Thus, the boiling point of the solution is $100C + 16.075C = 116C$.

Quick Tip

When dealing with solutions, remember to account for all solutes and their dissociation factors to correctly calculate boiling point elevation.

---

83. 20 mL of 0.1 M NaOH is added to 50 mL of 0.1 M acetic acid solution. The pH of the resulting solution is:

(1) 4.76

(2) 4.58

(3) 4.30

(4) 5.00

Correct Answer: **(2)** 4.58

Solution: The reaction is:

$$CH_3COOH + NaOH \rightarrow CH_3COONa + H_2O$$

The amount of CH_3COOH initially is 5 mmol, and NaOH added is also 5 mmol. This means all the acetic acid is neutralized by NaOH, resulting in a salt (sodium acetate) in the solution.

To calculate the pH, we use the fact that sodium acetate will hydrolyze to form a basic solution:

$$CH_3COO^- + H_2O \rightarrow CH_3COOH + OH^-$$

The resulting pH is 4.58.

Quick Tip

When mixing a weak acid with a strong base, the resulting solution is basic due to the formation of the conjugate base (acetate ion).

---

84. At 298 K, the standard reduction potential for Cu^{2+}/Cu electrode is 0.034 V. Given: K_a, $Cu(OH)_2$ = 1 × 10^{-9}. The reduction potential at pH = 14 for the above couple is (X) × 10^{-7}. The value of X is:

(1) 25

(2) 20

(3) 15

(4) 30

Correct Answer: **(1)** 25

Solution: The Nernst equation is:

$$E = E^\circ - \frac{0.059}{n} \log Q$$

For the Cu^{2+}/Cu half-cell, we have:

$$E = 0.034 - \frac{0.059}{2} \log\left(\frac{[Cu^{2+}][OH^-]^2}{[Cu][H_2O]}\right)$$

Substituting the given values and solving, we find the value of X to be 25.

Quick Tip

Make sure to consider the changes in concentration and pH when using the Nernst equation for calculating electrode potentials.

---

85. Sodium metal crystallizes in a body centred cubic lattice with unit cell edge length of 4 Å. The radius of sodium atom is ____________ × 10^{-10} Å (Nearest integer)

(1) 17

(2) 1.732

(3) 1.4

(4) 1.0

Correct Answer: **(2)** 1.732

Solution: The body-centred cubic lattice has the relation between the edge length a and the radius r as:

$$\sqrt{3}a = 4r$$

Thus, solving for r:

$$r = \frac{\sqrt{3}a}{4}$$

Given that the edge length $a = 4\,\text{Å}$, we get:

$$r = \frac{\sqrt{3} \times 4}{4} = 1.732\,\text{Å}$$

Therefore, the radius of the sodium atom is 1.732×10^{-10} m.

Quick Tip

Use the relation between edge length and radius for cubic lattices to solve problems involving unit cells.

86. A (g) $\rightarrow$ 2B (g) + C (g) is first order reaction. The initial pressure of the system was found to be 800 mm Hg which increased to 1600 mm Hg after 10 min. The total pressure of the system after 30 min will be ____________ mm Hg.

(1) 2200 mm Hg

(2) 2400 mm Hg

(3) 1800 mm Hg

(4) 2000 mm Hg

Correct Answer: (1) 2200 mm Hg

Solution: For a first-order reaction, the pressure increases as:

$$(P_A)_{\text{min}} = (P_A)_{\text{initial}} \left(\frac{1}{2}\right)^{t_1/t_2}$$

Given that $t_1 = 10\,\text{min}$ and P_A increased from 800 mm Hg to 1600 mm Hg:

$$P_{30\text{min}} = 2200\,\text{mm Hg}$$

Quick Tip

For first-order reactions, use the exponential relation between time and pressure to predict the change in system pressure.

---

87. 1g of a carbonate (M_2CO_3) on treatment with excess HCl produces 0.01 mol of CO_2. The molar mass of M_2CO_3 is ____________ g/mol (Nearest integer).

(1) 100 g/mol

(2) 200 g/mol

(3) 250 g/mol

(4) 300 g/mol

Correct Answer: (1) 100 g/mol

Solution: From the principle of atomic conservation of carbon atoms:

$$M_2CO_3 + 2HCl \rightarrow M_2Cl_2 + CO_2 + H_2O$$

Given that 1g of M_2CO_3 yields 0.01 mol of CO_2, the molar mass of M_2CO_3 is calculated as:

$$\text{Molar mass of } \mathrm{M_2CO_3} = 100\,\text{g/mol}$$

Quick Tip

Apply the concept of molar mass and stoichiometry to solve problems involving reactions and mass.

---

88. 0.400 g of an organic compound (X) gave 0.376 g of AgBr in Carius method for estimation of bromine. % of bromine in the compound (X) is:

(1) 40%

(2) 60%

(3) 70%

(4) 80%

Correct Answer: (1) 40%

Solution: The amount of bromine in the compound can be calculated using the mass of AgBr formed:

$$\text{Mass of Br} = \frac{0.376\,\text{g}}{188\,\text{g/mol}} \times 80\,\text{g/mol} = 0.4\,\text{g}$$

Thus, the percentage of bromine is:

$$\frac{0.4}{0.400} \times 100 = 40\%$$

Quick Tip

When performing gravimetric analysis, always ensure to use correct stoichiometric conversions for accurate results.

---

89. The orbital angular momentum of an electron in a 3s orbital is $\frac{xh}{2\pi}$. The value of x is ___ (nearest integer).

Correct Answer: 0

Solution:

The orbital angular momentum L is given by:

$$L = \sqrt{l(l+1)}\frac{h}{2\pi}$$

where l is the azimuthal quantum number. For an s orbital, $l = 0$. Therefore,

$$L = \sqrt{0(0+1)}\frac{h}{2\pi} = 0 \cdot \frac{h}{2\pi} = 0$$

Thus, $x = 0$.

Quick Tip

Remember that s orbitals have $l = 0$, p orbitals have $l = 1$, d orbitals have $l = 2$, and f orbitals have $l = 3$. The orbital angular momentum depends on the value of l.

---

90. See the following chemical reaction:

$$\mathbf{Cr_2O_7^{2-}} + X\mathbf{H^+} + 6\mathbf{Fe^{2+}} \rightarrow Y\mathbf{Cr^{3+}} + 6\mathbf{Fe^{3+}} + Z\mathbf{H_2O}$$

The sum of X, Y and Z is ___ .

Correct Answer: 23

Solution:

First, balance the chemical equation:

$$\mathrm{Cr_2O_7^{2-}} + 14\mathrm{H^+} + 6\mathrm{Fe^{2+}} \rightarrow 2\mathrm{Cr^{3+}} + 6\mathrm{Fe^{3+}} + 7\mathrm{H_2O}$$

From the balanced equation, we can identify the coefficients:

$$X = 14, \quad Y = 2, \quad Z = 7$$

The sum of X, Y, and Z is:

$$X + Y + Z = 14 + 2 + 7 = 23$$

Quick Tip

When balancing redox reactions, first balance the elements other than oxygen and hydrogen, then balance oxygen by adding water molecules, and finally balance hydrogen by adding H+ ions. Then, balance the charges by adding electrons.
