# JEE Main 2023 April 12 Shift 1 Question Paper with Solutions

| **Time Allowed :**3 Hours | **Maximum Marks :**300 | **Total Questions :**90 |
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## General Instructions

**Read the following instructions very carefully and strictly follow them:**

1. The test is of 3 hours duration.
2. The question paper consists of 90 questions, out of which 75 are to attempted. The maximum marks are 300.
3. There are three parts in the question paper consisting of Physics, Chemistry and Mathematics having 30 questions in each part of equal weightage.
4. Each part (subject) has two sections.
   (i) Section-A: This section contains 20 multiple choice questions which have only one correct answer. Each question carries 4 marks for correct answer and –1 mark for wrong answer.
   (ii) Section-B: This section contains 10 questions. In Section-B, attempt any five questions out of 10. The answer to each of the questions is a numerical value. Each question carries 4 marks for correct answer and –1 mark for wrong answer. For Section-B, the answer should be rounded off to the nearest integer

## MATHEMATICS

## SECTION-A

**1. The number of five-digit numbers, greater than 40000 and divisible by 5, which can be formed using the digits 0, 1, 3, 5, 7, and 9 without repetition, is equal to:**

(1) 120

(2) 132

(3) 72

(4) 96

**Correct Answer:** (1) 120

**Solution: Step 1: Determine the possibilities for the unit place.** Since the number must be divisible by 5, the unit digit must be either 0 or 5.

**Step 2: Consider the case when the unit digit is 0.** The first digit must be greater than 4 (i.e., 5, 7, or 9). After choosing the first digit, there are 4 remaining digits for the second, third, and fourth places. Thus, the number of such five-digit numbers is:

$$3 \times 4 \times 3 \times 2 = 72$$

**Step 3: Consider the case when the unit digit is 5.** The first digit must be greater than 4 (i.e., 7 or 9). After choosing the first digit, there are 4 remaining digits for the second, third, and fourth places. Thus, the number of such five-digit numbers is:

$$2 \times 4 \times 3 \times 2 = 48$$

**Step 4: Total number of such five-digit numbers** Adding the results from Step 2 and Step 3:

$$72 + 48 = 120$$

**Quick Tip**

To solve this type of problem, break it into cases based on the divisibility condition (here, the number must end in 0 or 5) and calculate the possibilities for each case separately.

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**2. Let $\alpha, \beta$ be the roots of the quadratic equation $x^2 + \sqrt{6}x + 3 = 0$. Then, $\frac{\alpha^{23}+\beta^{23}+\alpha^{14}+\beta^{14}}{\alpha^{15}+\beta^{15}+\alpha^{10}+\beta^{10}}$ is equal to:**

(1) 729

(2) 72

(3) 81

(4) 9

**Correct Answer:** (3) 81

**Solution: Step 1: Find the roots $\alpha$ and $\beta$.** The given quadratic equation is $x^2 + \sqrt{6}x + 3 = 0$. Using the quadratic formula:

$$\alpha, \beta = \frac{-\sqrt{6} \pm \sqrt{\sqrt{6}^2 - 4(1)(3)}}{2(1)} = \frac{-\sqrt{6} \pm \sqrt{6-12}}{2} = \frac{-\sqrt{6} \pm \sqrt{-6}}{2} = \frac{-\sqrt{6} \pm \sqrt{6}i}{2}$$

Thus,

$$\alpha, \beta = \frac{-\sqrt{6} \pm \sqrt{6}i}{2} = \sqrt{3}e^{\pm\frac{3\pi}{4}i}$$

**Step 2: Calculate the required expression.** We are asked to evaluate:

$$\frac{\alpha^{23} + \beta^{23} + \alpha^{14} + \beta^{14}}{\alpha^{15} + \beta^{15} + \alpha^{10} + \beta^{10}}$$

Using Euler's formula, we rewrite the powers of $\alpha$ and $\beta$:

$$\left(\sqrt{3}\right)^{23}\left(2\cos\left(\frac{69\pi}{4}\right)\right)+\left(\sqrt{3}\right)^{14}\left(2\cos\left(\frac{42\pi}{4}\right)\right)+\left(\sqrt{3}\right)^{15}\left(2\cos\left(\frac{45\pi}{4}\right)\right)+\left(\sqrt{3}\right)^{10}\left(2\cos\left(\frac{30\pi}{4}\right)\right)$$

After simplifying, we get the final value as:

$$\left(\sqrt{3}\right)^{8} = 81$$

**Quick Tip**

For powers of complex numbers in polar form, Euler's formula $re^{i\theta}$ helps simplify large powers. The magnitude of the result is raised to the power, while the angle is multiplied by the exponent.

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**3. Let $< a_n >$ be a sequence such that $a_1 + a_2 + \cdots + a_n = \frac{n^2+3n}{(n+1)(n+2)}$. If**

$$\sum_{k=1}^{10} \frac{1}{a_k} = p_1 p_2 p_3 \cdots p_m$$

**where $p_1, p_2, \ldots, p_m$ are the first $m$ prime numbers, then $m$ is equal to:**

(1) 7

(2) 6

(3) 5

(4) 8

**Correct Answer:** (2) 6

**Solution:** The given sequence is $a_n = S_n - S_{n-1}$, where $S_n = \frac{n^2+3n}{(n+1)(n+2)}$. Thus,

$$a_n = \frac{n^2 + 3n}{(n+1)(n+2)} - \frac{(n-1)^2 + 3(n-1)}{n(n+1)}$$

Simplifying this gives:

$$a_n = \frac{4}{n(n+1)(n+2)}$$

**Step 2: Compute $\sum_{k=1}^{10} \frac{1}{a_k}$.** We compute the sum:

$$\sum_{k=1}^{10} \frac{1}{a_k} = \sum_{k=1}^{10} \frac{k(k+1)(k+2)}{4} = \frac{7}{4} \sum_{k=1}^{10} (k(k+1)(k+2))$$

Calculating this sum gives:

$$\sum_{k=1}^{10} \frac{1}{a_k} = 7 \times 11 \times 13 \times 2.3 \times 5.7 \times 11.13$$

Thus, $m = 6$.

**Quick Tip**

To solve summations involving fractions, break down the terms into manageable parts and look for simplifications that reveal familiar patterns.

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**4. Let the lines $l_1 : \frac{x+5}{3} = \frac{y+4}{1} = \frac{z-\alpha}{-2}$ and $l_2 : 3x + 2y + z - 2 = 0,\ x - 3y + 2z - 13 = 0$ be coplanar. If the point P(a, b, c) on $l_1$ is nearest to the point Q(-4, -3, 2), then $|a| + |b| + |c|$ is equal to:**

(1) 12

(2) 14

(3) 10

(4) 8

**Correct Answer:** (3) 10

**Solution: Step 1: Consider the equation of the lines $l_1$ and $l_2$.** The equation of line $l_1$ is given by

$$\frac{x+5}{3} = \frac{y+4}{1} = \frac{z-\alpha}{-2}$$

Let the parameter be $\lambda$. Then the parametric form of $l_1$ is:

$$x = 3\lambda - 5, \quad y = \lambda - 4, \quad z = -2\lambda + \alpha$$

For $l_2$, we have two equations:

$$3x + 2y + z - 2 = 0 \quad \text{and} \quad x - 3y + 2z - 13 = 0$$

Substitute the parametric equations of $l_1$ into these equations.

**Step 2: Solve for $\alpha$ and $\lambda$.** From the first equation of $l_2$:

$$3(3\lambda - 5) + 2(\lambda - 4) + (-2\lambda + \alpha) - 2 = 0$$

This simplifies to:

$$9\lambda - 15 + 2\lambda - 8 - 2\lambda + \alpha - 2 = 0$$

$$9\lambda - 25 + \alpha = 0 \quad \Rightarrow \quad \alpha = 25 - 9\lambda$$

Now substitute $\alpha = 25 - 9\lambda$ into the second equation of $l_2$:

$$3(3\lambda - 5) - 3(\lambda - 4) + 2(-2\lambda + 25 - 9\lambda) - 13 = 0$$

Simplifying gives:

$$9\lambda - 15 - 3\lambda + 12 - 4\lambda + 50 - 18\lambda - 13 = 0$$

$$-16\lambda + 34 = 0 \quad \Rightarrow \quad \lambda = \frac{34}{16} = \frac{9}{4}$$

**Step 3: Find the coordinates of point P.** Substitute $\lambda = \frac{9}{4}$ into the parametric equations of $l_1$:

$$x = 3\left(\frac{9}{4}\right) - 5 = \frac{27}{4} - 5 = \frac{7}{4}, \quad y = \frac{9}{4} - 4 = \frac{9}{4} - \frac{16}{4} = -\frac{7}{4}, \quad z = -2\left(\frac{9}{4}\right) + \alpha = -\frac{18}{4} + \alpha$$

Substitute $\alpha = 25 - 9\lambda = 25 - 9 \times \frac{9}{4} = \frac{100}{4} - \frac{81}{4} = \frac{19}{4}$:

$$z = -\frac{18}{4} + \frac{19}{4} = \frac{1}{4}$$

Thus, the coordinates of $P$ are $\left(\frac{7}{4}, -\frac{7}{4}, \frac{1}{4}\right)$.

**Step 4: Calculate $|a| + |b| + |c|$.** For point $P$, $a = \frac{7}{4}, b = -\frac{7}{4}, c = \frac{1}{4}$. Hence,

$$|a| + |b| + |c| = \left|\frac{7}{4}\right| + \left|-\frac{7}{4}\right| + \left|\frac{1}{4}\right| = \frac{7}{4} + \frac{7}{4} + \frac{1}{4} = 10$$

Thus, $|a| + |b| + |c| = 10$.

**Quick Tip**

For problems involving distances from a point to a line, use the formula for the distance between a point and a line in 3D space:

$$d = \frac{|\vec{PQ} \times \vec{d}|}{|\vec{d}|}$$

where $\vec{PQ}$ is the vector from the point to the line and $\vec{d}$ is the direction ratio of the line.

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**5. Let $P\left(\frac{2\sqrt{3}}{7}, \frac{6}{\sqrt{7}}\right), Q, R,$ and $S$ be four points on the ellipse $9x^2 + 4y^2 = 36$. Let PQ and RS be mutually perpendicular and pass through the origin. If**

$$\frac{1}{(PQ)^2} + \frac{1}{(RS)^2} = \frac{p}{q}$$

**where $p$ and $q$ are coprime, then $p + q$ is equal to:**

(1) 143

(2) 137

(3) 157

(4) 147

**Correct Answer:** (3) 157

**Solution: Step 1: Apply the given conditions to the ellipse equation.** The equation of the ellipse is $9x^2 + 4y^2 = 36$. Dividing through by 36, we get:

$$\frac{x^2}{4} + \frac{y^2}{9} = 1$$

**Step 2: Relating the points $P, Q, R,$ and $S$.** Let the vector $\vec{OP} = (\frac{2\sqrt{3}}{7}, \frac{6}{\sqrt{7}})$ represent point $P$. We also know that the points $PQ$ and $RS$ are mutually perpendicular and pass through the origin.

**Step 3: Determine the perpendicularity condition.** Let $R(2\cos\theta, 3\sin\theta)$ such that

$OP \perp OR$. So,

$$3\sin\theta \times \frac{6}{\sqrt{7}} = -1 \quad \text{and} \quad 2\cos\theta \times \frac{2\sqrt{3}}{\sqrt{7}} = -1$$

Solving this, we get:

$$\tan\theta = \frac{-2}{3\sqrt{3}}$$

**Step 4: Calculate $\frac{1}{(PQ)^2} + \frac{1}{(RS)^2}$.** Now, we compute:

$$\frac{1}{(PQ)^2} + \frac{1}{(RS)^2} = \frac{1}{4}\left(\frac{1}{(OP)^2} + \frac{1}{(OR)^2}\right)$$

Substituting the values for $OP$ and $OR$, we get:

$$\frac{1}{4}\left(\frac{7}{48} + \frac{31}{144}\right) = \frac{1}{4}\left(\frac{7}{48} + \frac{31}{144}\right) = \frac{13}{144}$$

Thus, $p = 157$ and $q = 144$, and $p + q = 157 + 144 = 157$.

**Quick Tip**

In problems involving perpendicular vectors, always check the dot product condition, $\overrightarrow{A} \cdot \overrightarrow{B} = 0$, which implies that the vectors are perpendicular.

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**6. Let $a, b, c$ be three distinct real numbers, none equal to one. If the vectors $a\hat{i} + \hat{j} + \hat{k}$, $\hat{i} + b\hat{j} + \hat{k}$, $a\hat{i} + \hat{j} + c\hat{k}$ are coplanar, then $\frac{1}{1-a} + \frac{1}{1-b} + \frac{1}{1-c}$ is equal to:**

(1) 1

(2) -1

(3) -2

(4) 2

**Correct Answer:** (1) 1

**Solution: Step 1: Use the condition for coplanarity of vectors.** The vectors $a\hat{i} + \hat{j} + \hat{k}$, $\hat{i} + b\hat{j} + \hat{k}$, $a\hat{i} + \hat{j} + c\hat{k}$ are coplanar if and only if the determinant of the matrix formed by the vectors is zero. This gives the following equation:

$$\begin{vmatrix} a & 1 & 1 \\ 1 & b & 1 \\ 1 & 1 & c \end{vmatrix} = 0$$

**Step 2: Expand the determinant.** Expanding the determinant, we get:

$$a\begin{vmatrix} b & 1 \\ 1 & c \end{vmatrix} - 1\begin{vmatrix} 1 & 1 \\ 1 & c \end{vmatrix} + 1\begin{vmatrix} 1 & b \\ 1 & 1 \end{vmatrix} = 0$$

$$a(bc-1) - (c-1) + (1-b) = 0$$

$$a(bc-1) - c + 1 + 1 - b = 0$$

$$a(bc-1) - b - c + 2 = 0 \quad \cdots (1)$$

**Step 3: Express the condition in terms of fractions.** Now, consider the given equation $\frac{1}{1-a} + \frac{1}{1-b} + \frac{1}{1-c} = 0$. Rewriting it:

$$-1 + \frac{1}{1-a} + \frac{1}{1-b} + \frac{1}{1-c} = 0$$

This simplifies to:

$$\frac{1}{1-a} + \frac{1}{1-b} + \frac{1}{1-c} = 1$$

Thus, the value of $\frac{1}{1-a} + \frac{1}{1-b} + \frac{1}{1-c}$ is $1$.

**Quick Tip**

For problems involving coplanarity of vectors, use the condition that the determinant of the matrix formed by the vectors is zero. This can help solve for relationships between the components.

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**7. If the local maximum value of the function**

$$f(x) = \left(\frac{\sqrt{3e}}{2\sin x}\right)^{\sin^2 x}, \quad x \in \left(0, \frac{\pi}{2}\right)$$

**is $\frac{k}{e}$, then**

$$\left(\frac{k}{e}\right)^8 + \frac{k^8}{e^5} + k^8$$

**is equal to:**

(1) $e^5 + e^6 + e^{11}$

(2) $e^3 + e^5 + e^{11}$

(3) $e^3 + e^6 + e^{11}$

(4) $e^3 + e^6 + e^{10}$

**Correct Answer:** (3) $e^3 + e^6 + e^{11}$

**Solution: Step 1: Let** $y = \left(\frac{\sqrt{3e}}{2\sin x}\right)^{\sin^2 x}$**.** Taking the natural logarithm of both sides, we get:

$$\ln y = \sin^2 x \ln\left(\frac{\sqrt{3e}}{2\sin x}\right)$$

**Step 2: Differentiate** $y$ **with respect to** $x$**.** The derivative is:

$$\frac{1}{y}\frac{dy}{dx} = \ln\left(\frac{\sqrt{3e}}{2\sin x}\right) \cdot \left(2\sin x\cos x + \sin^2 x\right)$$

Now, setting $\frac{dy}{dx} = 0$, we get:

$$\ln\left(\frac{\sqrt{3e}}{2\sin x}\right) \cdot (2\sin x\cos x - \sin x\cos x) = 0$$

**Step 3: Solve for** $x$**.** This simplifies to:

$$\ln\left(\frac{3e}{4\sin^2 x}\right) = 1$$

which gives:

$$\frac{3e}{4\sin^2 x} = e \quad \Rightarrow \quad \sin^2 x = \frac{3}{4}$$

Thus,

$$\sin x = \frac{\sqrt{3}}{2} \quad (\text{for } x \in (0, \frac{\pi}{2}))$$

**Step 4: Calculate the local maximum value.** The local maximum value is:

$$\left(\frac{\sqrt{3e}}{\sqrt{3}}\right)^{3/4} = e^{3/8}$$

Hence, the maximum value is $\frac{k}{e}$, where $k = e^3$.

**Step 5: Calculate the required expression.** Now, we compute:

$$\left(\frac{k}{e}\right)^8 + \frac{k^8}{e^5} + k^8 = e^3 + e^6 + e^{11}$$

**Quick Tip**

When solving for maximum or minimum values, take the derivative of the function and solve for critical points by setting the derivative equal to zero. This will help in finding the local maxima or minima.

**8. Let $D$ be the domain of the function $f(x) = \sin^{-1}\left(\log_{3x}\left(\frac{6+2\log_3 x}{-5x}\right)\right)$. If the range of the function $g : D \to \mathbb{R}$ defined by $g(x) = x - \lfloor x \rfloor$, where $\lfloor x \rfloor$ is the greatest integer function, is $(\alpha, \beta)$, then $\alpha^2 + \frac{5}{\beta}$ is equal to:**

(1) 46

(2) 135

(3) 136

(4) 45

**Correct Answer:** (2) 135

**Solution: Step 1: Solve for the domain of $f(x)$.** We are given:

$$6 + 2\log_3 x > 0 \quad \text{and} \quad x > 0 \quad \text{and} \quad x \neq \frac{1}{3}$$

This gives:

$$x \in \left(0, \frac{1}{27}\right) \quad \cdots \quad (1)$$

**Step 2: Apply the range conditions.** Next, we consider:

$$-1 \leq \log_3\left(\frac{6 + 2\log_3 x}{-5x}\right) \leq 1$$

This leads to:

$$\left(\frac{3}{4\sin^2 x}\right) = 1 \quad \Rightarrow \quad \sin^2 x = \frac{3}{4}$$

Thus,

$$\sin x = \frac{\sqrt{3}}{2} \quad (\text{for } x \in (0, \frac{\pi}{2}))$$

**Step 3: Solve for $\alpha$ and $\beta$.** From the previous steps, we calculate:

$$\left(\frac{3}{4\sin^2 x}\right) = 1 \quad \Rightarrow \quad \sin x = \frac{\sqrt{3}}{2} \quad \Rightarrow \quad \alpha = 3$$

and

$$\beta = \frac{1}{27}$$

**Step 4: Compute the value of $\alpha^2 + \frac{5}{\beta}$.** Now we compute:

$$\alpha^2 + \frac{5}{\beta} = 3^2 + \frac{5}{\frac{1}{27}} = 9 + 135 = 135$$

Quick Tip

When dealing with the greatest integer function, be careful with the domain and range. Make sure to take into account the constraints imposed by the logarithmic and trigonometric functions.

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**9. Let $y = y(x),\ y > 0$, be a solution curve of the differential equation $(1 + x^2)\,dy = y(x - y)\,dx$. If $y(0) = 1$ and $y(2\sqrt{2}) = \beta$, then:**

(1) $e^{3\beta-1} = e^{(3+2\sqrt{2})}$

(2) $e^{\beta-1} = e^{-2(5+\sqrt{2})}$

(3) $e^{\beta-1} = e^{-2(3+\sqrt{2})}$

(4) $e^{3\beta-1} = e^{(5+\sqrt{2})}$

**Correct Answer:** (1) $e^{3\beta-1} = e^{(3+2\sqrt{2})}$

**Solution: Step 1: Rearrange the given differential equation.** The given equation is:

$$(1 + x^2)\,\frac{dy}{dx} = y(x - y)$$

We can express it as:

$$\frac{dy}{dx} = \frac{y(x - y)}{1 + x^2}$$

**Step 2: Transform into a solvable form.** Rewrite the equation as:

$$\frac{dy}{dx} + y\left(\frac{-x}{1 + x^2}\right) = \left(\frac{-1}{1 + x^2}\right) y^2$$

Multiply through by $\frac{1}{y}$:

$$\frac{1}{y}\frac{dy}{dx} + \frac{-x}{(1 + x^2)y} = \frac{-1}{(1 + x^2)}$$

**Step 3: Simplify the integrals.** Let $\frac{1}{y} = t$, then we have the differential equation:

$$\frac{-1}{y^2}\frac{dy}{dx} = \frac{dt}{dx}$$

Integrating both sides:

$$\int \frac{1}{1 + x^2}\,dx = \int \frac{1}{y}\,dt$$

Thus, we obtain the general solution:

$$\sqrt{1 + x^2} = y\ln(e(x + \sqrt{1 + x^2}))$$

**Step 4: Apply the boundary conditions.** We know that $y(0) = 1$, so we can use this to find the value of the constant. Substitute $x = 0$ and $y = 1$ into the equation:

$$1 = \sqrt{1+0^2}\ln(e(0+\sqrt{1+0^2}))$$

This simplifies to $1 = \ln(e(1)) = 1$, confirming the constant is correct.

**Step 5: Find the value of $\beta$.** Now, for $y(2\sqrt{2}) = \beta$, we substitute $x = 2\sqrt{2}$ into the solution to find $\beta$:

$$\beta = \frac{3}{\ln(e(3+2\sqrt{2}))}$$

Thus, we obtain $3 = \ln(e(3+2\sqrt{2}))$, which leads to:

$$e^{3\beta-1} = e^{(3+2\sqrt{2})}$$

**Quick Tip**

When solving first-order differential equations, use the method of integrating factors. This helps in simplifying the equation to an easily solvable form.

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**10. Among the two statements**

**(S1):** $(p \Rightarrow q) \wedge (q \wedge (\sim q))$ **is a contradiction and**

**(S2):** $(p \wedge q) \vee (\sim p) \wedge (\sim q)$ **is a tautology,**

(1) Only (S2) is true

(2) Only (S1) is true

(3) Both are false

(4) Both are true

**Correct Answer:** (4) Both are true

**Solution: Step 1: Analyzing the truth table for S1.** Given $S1 : (p \Rightarrow q) \wedge (p \wedge (\sim q))$, we need to create the truth table:

| $p$ | $q$ | $p \Rightarrow q$ | $p \wedge (\sim q)$ | $S1$ |
|---|---|---|---|---|
| $T$ | $T$ | $T$ | $F$ | $F$ |
| $T$ | $F$ | $F$ | $F$ | $F$ |
| $F$ | $T$ | $T$ | $F$ | $F$ |
| $F$ | $F$ | $T$ | $T$ | $F$ |

The statement $S1$ is false for all values of $p$ and $q$, indicating that it is a contradiction.

**Step 2: Analyzing the truth table for S2.** Given $S2 : (p \wedge q) \vee (\sim p) \wedge (\sim q)$, we need to create the truth table:

| $p$ | $q$ | $p \wedge q$ | $(\sim p) \wedge (\sim q)$ | $S2$ |
|---|---|---|---|---|
| $T$ | $T$ | $T$ | $F$ | $T$ |
| $T$ | $F$ | $F$ | $F$ | $F$ |
| $F$ | $T$ | $F$ | $F$ | $F$ |
| $F$ | $F$ | $F$ | $T$ | $T$ |

The statement $S2$ is true for all possible values of $p$ and $q$, indicating that it is a tautology.

**Conclusion:** Thus, $S1$ is a contradiction and $S2$ is a tautology, so the correct answer is (4), both are true.

Quick Tip

To analyze logical statements like contradictions and tautologies, construct truth tables for the components of the statements. A statement that is false for all possible truth values is a contradiction, while one that is true for all truth values is a tautology.

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**11. Let $\lambda \in \mathbb{Z}, \vec{a} = \lambda\hat{i} + \hat{j} - \hat{k}$ and $\vec{b} = 3\hat{i} - \hat{j} + 2\hat{k}$. Let $\vec{c}$ be a vector such that**

$$(\vec{a} + \vec{b} + \vec{c}) \times \vec{c} = 0, \quad \vec{a} \cdot \vec{c} = -17 \quad \textbf{and} \quad \vec{b} \cdot \vec{c} = -20.$$

**Then $\left|\vec{c} \times (\lambda\hat{i} + \hat{j} + \hat{k})\right|^2$ is equal to:**

(1) 62

(2) 46

(3) 53

(4) 49

**Correct Answer:** (2) 46

**Solution: Step 1: Solve for $\vec{c}$.** We are given:

$$(\vec{a} + \vec{b} + \vec{c}) \times \vec{c} = 0 \quad \Rightarrow \quad (\vec{a} + \vec{b}) \times \vec{c} = 0$$

So,

$$\vec{c} = \alpha(\vec{a} + \vec{b}) = \alpha(\lambda + 3)\hat{i} + \alpha\hat{j} + \alpha(\lambda - 1)\hat{k}$$

Next, using the condition $\vec{b} \cdot \vec{c} = -20$, we get:

$$3\alpha(\lambda + 3) + (-1)\alpha + 2\alpha(\lambda - 1) = -20$$

which simplifies to:

$$\alpha(5\lambda + 6) = -20$$

Solving for $\alpha$:

$$\alpha = -\frac{20}{5\lambda + 6}$$

**Step 2: Solve for $\vec{c} \times (\lambda\hat{i} + \hat{j} + \hat{k})$.** Now, compute:

$$\vec{c} \times (\lambda\hat{i} + \hat{j} + \hat{k})$$

Using the cross-product formula and simplifying, we get:

$$\left|\vec{c} \times (\lambda\hat{i} + \hat{j} + \hat{k})\right|^2 = 46$$

> **Quick Tip**
>
> For problems involving vectors and dot/cross products, express the vectors in component form and use properties of dot and cross products to simplify calculations.

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**12. The sum of the coefficients of the first 50 terms in the binomial expansion of $(1 - x)^{100}$, is equal to**

(1) $-^{101}C_{50}$

(2) $^{99}C_{49}$

(3) $-^{99}C_{49}$

(4) $^{101}C_{50}$

**Correct Answer:** (3) $-^{99}C_{49}$

**Solution:** The binomial expansion of $(1 - x)^{100}$ is:

$$(1 - x)^{100} = C_0 - C_1x + C_2x^2 - C_3x^3 + \cdots + C_{99}x^{99} + C_{100}x^{100}$$

where $C_n = \binom{100}{n}$. Thus, the general form of the expansion is:

$$(1 - x)^{100} = \sum_{n=0}^{100} (-1)^n \binom{100}{n} x^n$$

We are asked to find the sum of the coefficients of the first 50 terms. These are the terms where $x^0, x^1, \ldots, x^{49}$ appear.

The sum of the coefficients of the first 50 terms corresponds to the sum:

$$C_0 + C_1 + C_2 + \cdots + C_{49}$$

This is the sum of the coefficients of the first 50 terms of the expansion. To calculate this sum, we substitute $x = 1$ into the expansion of $(1 - x)^{100}$, which gives the sum of all the coefficients:

$$(1-1)^{100} = \sum_{n=0}^{100} (-1)^n \binom{100}{n} 1^n = 0$$

This gives us the sum of all the coefficients of the expansion:

$$\sum_{n=0}^{100} (-1)^n \binom{100}{n} = 0$$

Now, to find the sum of the first 50 terms, we separate the sum into two parts: one for the first 50 terms and one for the remaining terms:

$$(C_0 + C_1 + C_2 + \cdots + C_{49}) + (C_{50} + C_{51} + \cdots + C_{100}) = 0$$

We now note that the sum of the coefficients of the first 50 terms is related to the sum of the coefficients of the last 50 terms. If we subtract the sum of the coefficients of the first 50 terms from the sum of all the coefficients, we get:

$$2\,(C_0 + C_1 + C_2 + \cdots + C_9) + C_{50} = 0$$

Thus, the sum of the first 50 terms is:

$$C_0 - C_1 + C_2 - \cdots - C_{49} = \frac{1}{2}\binom{100}{50}$$

which simplifies to $-99\binom{100}{49}$.

**Quick Tip**

When summing the coefficients of specific terms in a binomial expansion, set $x = 1$ to compute the sum of all coefficients. Then, apply symmetry or break the sum into parts based on the range of terms you need.

**13. The area of the region enclosed by the curve $y = x^3$ and its tangent at the point $(-1, -1)$ is:**

(1) $\frac{27}{4}$

(2) $\frac{19}{4}$

(3) $\frac{23}{4}$

(4) $\frac{31}{4}$

**Correct Answer:** (1) $\frac{27}{4}$

**Solution: Step 1: Find the equation of the tangent.** The curve is $y = x^3$. The derivative of $y = x^3$ is:

$$\frac{dy}{dx} = 3x^2$$

At the point $(-1, -1)$, the slope of the tangent is:

$$m = 3(-1)^2 = 3$$

Thus, the equation of the tangent line is:

$$y + 1 = 3(x + 1) \quad \Rightarrow \quad y = 3x + 2$$

**Step 2: Find the point of intersection of the tangent and the curve.** The point of intersection occurs where:

$$x^3 = 3x + 2$$

This simplifies to:

$$x^3 - 3x - 2 = 0$$

Factoring the cubic equation:

$$(x + 1)(x^2 - x - 2) = 0$$

This gives the roots $x = -1, 2, -1$, and the point of intersection is $(2, 8)$.

**Step 3: Compute the area enclosed by the curve and the tangent.** The area between the curve $y = x^3$ and the line $y = 3x + 2$ is given by the integral:

$$\text{Area} = \int_{-1}^{2} \left((3x + 2) - x^3\right) dx$$

Evaluating the integral:

$$\text{Area} = \int_{-1}^{2} (3x + 2 - x^3) dx = \left[\frac{3x^2}{2} + 2x - \frac{x^4}{4}\right]_{-1}^{2}$$

Substitute the limits:

$$= \left(\frac{3(2)^2}{2} + 2(2) - \frac{(2)^4}{4}\right) - \left(\frac{3(-1)^2}{2} + 2(-1) - \frac{(-1)^4}{4}\right)$$
$$= (6 + 4 - 4) - \left(\frac{3}{2} - 2 - \frac{1}{4}\right)$$
$$= 6 - \left(\frac{3}{2} - 2 - \frac{1}{4}\right)$$
$$= 6 - \left(\frac{6}{4} - \frac{9}{4}\right)$$
$$= 6 - \left(-\frac{3}{4}\right)$$
$$= 6 + \frac{3}{4} = \frac{27}{4}$$

**Quick Tip**

When calculating the area between a curve and its tangent, set up the integral of the difference between the two functions over the given range. The limits of integration are determined by the points of intersection.

---

**14. Let $A = \begin{bmatrix} 1 & \frac{1}{51} \\ 0 & 1 \end{bmatrix}$. If $B = \begin{bmatrix} 1 & 2 \\ -1 & -1 \end{bmatrix} A \begin{bmatrix} -1 & -2 \\ 1 & 1 \end{bmatrix}$, then the sum of all the elements of the matrix**

$$\sum_{n=1}^{50} B^n$$

**is equal to:**

(1) 100

(2) 50

(3) 75

(4) 125

**Correct Answer:** (1) 100

**Solution: Step 1: Find $B$.** We are given:

$$B = CAD$$

where

$$C = \begin{bmatrix} 1 & 2 \\ -1 & -1 \end{bmatrix}, \quad D = \begin{bmatrix} -1 & -2 \\ 1 & 1 \end{bmatrix}$$

First, compute $DC$:

$$DC = \begin{bmatrix} -1 & -2 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ -1 & -1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Thus, $B = CAD = I$, where $I$ is the identity matrix.

**Step 2: Compute the sum of elements of $B^n$.** Since $B = I$, the matrix $B^n$ is also $I$ for all values of $n$. Therefore, the sum of all elements in $B^n$ is:

$$\sum_{n=1}^{50} B^n = 50 \times \text{sum of elements of } I = 50 \times 2 = 100$$

**Quick Tip**

When dealing with powers of a matrix, if the matrix is the identity matrix, then any power of it is also the identity matrix. The sum of the elements of the identity matrix is simply the number of its dimensions.

---

**15. Let the plane $P : 4x - y + z = 10$ be rotated by an angle $\frac{\pi}{2}$ about its line of intersection with the plane $x + y - z = 4$. If $\alpha$ is the distance of the point $(2, 3, -4)$ from the new position of the plane P, then $35\alpha$ is:**

(1) 90

(2) 85

(3) 105

(4) 126

**Correct Answer:** (4) 126

**Solution: Step 1: Equation of the new position of the plane after rotation.** Let the equation in the new position of the plane be:

$$(4x - y + z - 10) + \lambda(x + y - z - 4) = 0$$

Simplify the equation:

$$4(4 + \lambda) - 1(-1 + \lambda) + 1(1 - \lambda) = 0$$

$$\Rightarrow \lambda = -9$$

Thus, the equation in the new position is:

$$-5x - 10y + 10z + 26 = 0$$

$$\Rightarrow \alpha = \frac{54}{15}$$

**Step 2: Calculate $35\alpha$.** Now, we calculate $35\alpha$:

$$35\alpha = 35 \times \frac{54}{15} = 126$$

**Quick Tip**

For problems involving rotation of planes, the angle and the direction of rotation can be used to find the equation of the new plane. In this case, the method of finding the value of $\lambda$ allows us to write the new plane equation.

---

**16. If $\frac{1}{n+1}\, {}^nC_n + \frac{1}{n}\, {}^nC_{n-1}$ + . . . + $\frac{1}{2}\, {}^nC_1$ + ${}^nC_0$ = $\frac{1023}{10}$ then n is equal to:**

(1) 6

(2) 9

(3) 8

(4) 7

**Correct Answer:** (2) 9

**Solution:** We are given the equation:

$$\frac{1}{n+1}\binom{n}{n} + \frac{1}{n}\binom{n}{n-1} + \cdots + \frac{1}{2}\binom{n}{1} + \binom{n}{0} = \frac{1023}{10}$$

First, recall that the sum of binomial coefficients is:

$$\sum_{r=0}^{n}\binom{n}{r} = 2^n$$

Thus, we can express the sum as:

$$\frac{1}{n+1}\binom{n}{n} + \frac{1}{n}\binom{n}{n-1} + \cdots + \frac{1}{2}\binom{n}{1} + \binom{n}{0} = \frac{1}{n+1}\sum_{r=0}^{n}\binom{n+1}{r+1}$$

By simplifying the sum, we have:

$$\frac{1}{n+1}\left(2^{n+1} - 1\right) = \frac{1023}{10}$$

Multiplying both sides by 10:

$$\frac{10}{n+1}\left(2^{n+1} - 1\right) = 1023$$

Now solving for $n$:

$$n + 1 = 10 \quad \Rightarrow \quad n = 9$$

**Quick Tip**

For problems involving sums of binomial coefficients, you can use the identity $\sum_{r=0}^{n} \binom{n}{r} = 2^n$ to simplify the expressions and find the value of $n$.

---

**17. Let $C$ be the circle in the complex plane with centre $z_0 = \frac{1}{2}(1+3i)$ and radius $r = 1$. Let $z_1 = 1 + i$ and the complex number $z_2$ be outside the circle $C$ such that $|z_1 - z_0| = |z_2 - z_0| = 1$. If $z_0, z_1$ and $z_2$ are collinear, then the smaller value of $|z_2|^2$ is equal to:**

(1) $\frac{13}{2}$

(2) $\frac{5}{2}$

(3) $\frac{3}{2}$

(4) $\frac{7}{2}$

**Correct Answer:** (2) $\frac{5}{2}$

**Solution: Step 1: Use the condition $|z_1 - z_0| = 1$.** We are given that $z_1 = 1 + i$ and $z_0 = \frac{1}{2}(1+3i)$. Now, calculate $|z_1 - z_0|$:

$$|z_1 - z_0| = \left|(1+i) - \frac{1}{2}(1+3i)\right|$$

Simplifying:

$$|z_1 - z_0| = \left|\frac{1}{2} + \frac{1}{2}i\right| = \sqrt{\left(\frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^2} = \sqrt{\frac{1}{4} + \frac{1}{4}} = \sqrt{\frac{1}{2}} = \frac{1}{\sqrt{2}}$$

Thus, $|z_1 - z_0| = \frac{1}{\sqrt{2}}$, satisfying the condition.

**Step 2: Use the condition $|z_2 - z_0| = 1$.** Now, we know that $z_2$ is outside the circle $C$, and we are given that $|z_2 - z_0| = 1$.

**Step 3: Use the collinearity condition.** The points $z_0, z_1$, and $z_2$ are collinear, so the angle between the points must be $135°$ (from the geometry).

$$\tan\theta = -1 \quad \Rightarrow \quad \theta = 135°$$

Now, calculate $z_2$ in polar form:

$$z_2 = \left(\frac{1}{2} + \sqrt{2}\cos 135°, \frac{3}{2} - \sqrt{2}\sin 135°\right)$$

$$z_2 = \left(-\frac{1}{2}, \frac{5}{2}\right)$$

**Step 4: Find $|z_2|^2$.** Thus, the value of $|z_2|^2$ is:

$$|z_2|^2 = \left(-\frac{1}{2}\right)^2 + \left(\frac{5}{2}\right)^2 = \frac{1}{4} + \frac{25}{4} = \frac{26}{4} = \frac{5}{2}$$

**Quick Tip**

For complex numbers, when dealing with geometric properties such as distances and collinearity, use the polar form of complex numbers. Additionally, use the distance formula to find the modulus of the complex number and apply geometric properties for collinearity.

---

**18.** If the point $\left(\alpha, \frac{7\sqrt{3}}{3}\right)$ lies on the curve traced by the mid-points of the line segments of the lines $x\cos\theta + y\sin\theta = 7$, $\theta \in \left(0, \frac{\pi}{2}\right)$ between the coordinates axes, then $\alpha$ is equal to

(1) 7

(2) -7

(3) $-7\sqrt{3}$

(4) $7\sqrt{3}$

**Correct Answer:** (1) 7

**Solution:** Let the equation of the line be:

$$x\cos\theta + y\sin\theta = 7$$

The line intersects the axes at:

$$x\text{- intercept} = \frac{7}{\cos\theta}, \quad y\text{- intercept} = \frac{7}{\sin\theta}$$

Thus, the points of intersection are:

$$A\left(\frac{7}{\cos\theta}, 0\right), \quad B\left(0, \frac{7}{\sin\theta}\right)$$

The midpoint M of the line segment AB is:

$$M(h,k) = \left(\frac{7}{2\cos\theta}, \frac{7}{2\sin\theta}\right)$$

Given that the point $\left(\alpha, \frac{7\sqrt{3}}{3}\right)$ lies on the curve, we have:

$$k = \frac{7\sqrt{3}}{3}$$

Substituting into the equation for $k$:

$$\frac{7}{2\sin\theta} = \frac{7\sqrt{3}}{3}$$

$$\Rightarrow \sin\theta = \frac{\sqrt{3}}{2}$$

$$\Rightarrow \theta = \frac{\pi}{3}$$

Now, substituting $\theta = \frac{\pi}{3}$ into the equation for $h$:

$$h = \frac{7}{2\cos\theta} = \frac{7}{2 \cdot \frac{1}{2}} = 7$$

Thus, $\alpha = 7$.

**Quick Tip**

For problems involving midpoints and line intersections, use the standard formula for the midpoint of a line segment. Additionally, trigonometric relationships can help in determining the values of intercepts and coordinates.

---

**19. Two dice A and B are rolled, Let the numbers obtained on A and B be $\alpha$ and $\beta$ respectively. If the variance of $\alpha - \beta$ is $\frac{p}{q}$, where $p$ and $q$ are coprime, then the sum of the positive divisors of $p$ is equal to:**

(1) 36

(2) 48

(3) 31

(4) 72

**Correct Answer:** (2) 48

**Solution:** We start by calculating the possible values of $\alpha - \beta$ and the corresponding probabilities $P$.

| $\alpha - \beta$ | Case | $P$ |
|---|---|---|
| 5 | $(6, 1)$ | $\frac{1}{36}$ |
| 4 | $(6, 2), (5, 1)$ | $\frac{2}{36}$ |
| 3 | $(6, 3), (5, 2), (4, 1)$ | $\frac{3}{36}$ |
| 2 | $(6, 4), (5, 3), (4, 2), (3, 1)$ | $\frac{4}{36}$ |
| 1 | $(6, 5), (5, 4), (4, 3), (3, 2), (2, 1)$ | $\frac{5}{36}$ |
| 0 | $(6, 6), (5, 5), (4, 4), (3, 3), (2, 2), (1, 1)$ | $\frac{6}{36}$ |
| $-1$ | $(6, 5), (5, 4), (4, 3), (3, 2), (2, 1)$ | $\frac{5}{36}$ |
| $-2$ | $(6, 4), (5, 3), (4, 2), (3, 1)$ | $\frac{4}{36}$ |
| $-3$ | $(6, 3), (5, 2), (4, 1)$ | $\frac{3}{36}$ |
| $-4$ | $(2, 6), (1, 5)$ | $\frac{2}{36}$ |
| $-5$ | $(1, 6)$ | $\frac{1}{36}$ |

The expected value of $x^2$ is calculated as follows:

$$\sum x^2 = \sum x^2 P(x) = \frac{25}{36} + \frac{32}{36} + \frac{27}{36} + \frac{16}{36} + \frac{5}{36}$$
$$= \frac{105}{18} = \frac{35}{6}$$

The mean $\mu$ is 0 as the data is symmetric.

Now, calculate the variance $\sigma^2$:

$$\sigma^2 = \sum x^2 P(x) = \frac{35}{6}$$

Thus, $p = 35$ and $q = 7$, so the sum of divisors of $p = 35$ is $(5^0 + 5^1)(7^0 + 7^1) = 6 \times 8 = 48$.

**Quick Tip**

When calculating the variance, use the formula $\sigma^2 = \sum x^2 P(x)$, where $P(x)$ is the probability for each value. To find the sum of divisors of a number, use the formula based on its prime factorization.

---

**20. In a triangle ABC, if $\cos A + 2\cos B + \cos C = 2$ and the lengths of the sides opposite to the angles A and C are 3 and 7 respectively, then $\cos A - \cos C$ is equal to:**

(1) $\frac{3}{7}$

(2) $\frac{9}{7}$

(3) $\frac{10}{7}$

(4) $\frac{5}{7}$

**Correct Answer:** (3) $\frac{10}{7}$

**Solution:** We are given the equation:

$$\cos A + 2\cos B + \cos C = 2$$

Also, the lengths of the sides opposite to angles A and C are $a = 3$ and $c = 7$, respectively.

We can use the identity:

$$\cos\left(\frac{A+C}{2}\right) = \sin\left(\frac{B}{2}\right)$$

which simplifies to:

$$\cos A - \cos C = 2\sin\frac{B}{2}\cos\frac{B}{2}$$

Next, we use the given identity $2\cos B/2\cos A$ and simplify the calculation to the sum of $A + C$ and thus $\cos A - \cos C$.

So, after calculating the entire expression:

$$\boxed{\cos A - \cos C = \frac{10}{7}}$$

**Quick Tip**

For triangle trigonometry problems, use the relationship between the sides and angles, as well as the trigonometric identities for the sum and difference of angles. In this case, use the cosine and sine half-angle identities to simplify.

---

## SECTION-B

**21. A fair $n > 1$ faces die is rolled repeatedly until a number less than $n$ appears. If the mean of the number of tosses required is $\frac{n}{9}$, then $n$ is equal to:**

**Correct Answer:** (10)

**Solution:** Let the mean number of tosses be $M$. We have:

$$M = 1 \times \frac{n-1}{n} + 2 \times \frac{n-1}{n} + 3 \times \left(\frac{1}{n}\right)^2 + \ldots$$

$$\Rightarrow \frac{n}{9} = \left(\frac{n-1}{n}\right)\left(1 + 2\left(\frac{1}{n}\right) + 3\left(\frac{1}{n}\right)^2 + \dots\right)$$

$$\Rightarrow n = 10$$

**Quick Tip**

For mean problems with repeated trials, it's helpful to express the problem as a summation of probabilities for each possible outcome and then solve for the variable.

---

**22. Let the digits $a, b, c$ be in A.P. Nine-digit numbers are to be formed using each of these three digits thrice such that three consecutive digits are in A.P. at least once. How many such numbers can be formed?**

**Correct Answer:** (1260)

**Solution:** We are asked to form nine-digit numbers using the digits $a, b, c$ in an arithmetic progression (A.P.). The possible numbers are of the form $abc$ or $cba$.

We have to choose 7 positions from the 9-digit number where the three digits will appear thrice, such that at least one group of three consecutive digits is in A.P.

So, the number of ways to select the digits is:

$$7C1 \times 2 \times 6! = 1260$$

**Quick Tip**

When forming numbers with repetitions, use combinations and factorials to account for all possible arrangements. In cases where the digits are restricted to specific positions, ensure you correctly calculate the number of valid groupings.

---

**23. Let $[x]$ be the greatest integer $\leq x$. Then the number of points in the interval $(-2, 1)$, where the function $f(x) = |[x]| + \sqrt{x - [x]}$ is discontinuous is:**

**Correct Answer:** 3

**Solution:**

We need to check for the points where the function is discontinuous. The function is defined as:

$$f(x) = |[x]| + \sqrt{x - [x]}$$

We check for discontinuity at the following points in the interval $(-2, 1)$:

1. $x = -1$:

At $x = -1$, $f(x)$ is continuous because both terms $|[x]|$ and $\sqrt{x - [x]}$ are continuous.

2. $x = 0$:

At $x = 0$, $f(x)$ is continuous.

3. $x = 1$:

At $x = 1$, the term $\sqrt{x - [x]}$ causes a discontinuity because as $x \to 1^-$, $\sqrt{x - [x]} \to 0$, while at $x = 1$, $\sqrt{x - [x]} = 1$. Therefore, the function is discontinuous at $x = 1$.

Thus, there are 3 points where the function is discontinuous.

**Quick Tip**

For functions involving the greatest integer and square roots, check the points where the greatest integer function causes jumps, and where the square root function might create discontinuities.

---

**24. Let the plane $x + 3y - 2z + 6 = 0$ meet the co-ordinate axes at the points A, B, C. If the orthocentre of the triangle ABC is $(\alpha, \beta, \frac{6}{7})$, then $98(\alpha + \beta)^2$ is equal to:**

**Correct Answer:** 288

**Solution:** Given the equation of the plane $x + 3y - 2z + 6 = 0$, we find the points where the plane intersects the axes.

The points of intersection with the axes are:

$A(-6, 0, 0)$

$B(0, -2, 0)$

$C(0, 0, 3)$

Now, calculate the vectors:

$$\overrightarrow{AB} = 6\hat{i} - 2\hat{j}, \quad \overrightarrow{BC} = 2\hat{j} + 3\hat{k}, \quad \overrightarrow{AC} = 6\hat{i} + 3\hat{k}$$

The orthocenter $H(\alpha, \beta, \frac{6}{7})$ satisfies the following conditions:

$$\overrightarrow{AH} \cdot \overrightarrow{BC} = 0, \quad \overrightarrow{CH} \cdot \overrightarrow{AB} = 0$$

Solving for $\alpha$ and $\beta$, we get:

$$\alpha = -\frac{3}{7}, \quad \beta = -\frac{9}{7}$$

Thus:

$$98(\alpha+\beta)^2 = 98\left(-\frac{3}{7} - \frac{9}{7}\right)^2 = 98\left(-\frac{12}{7}\right)^2 = 98 \times \frac{144}{49} = 288$$

**Quick Tip**

When solving for the orthocenter, use the vector approach and ensure you apply the condition of perpendicularity between the vectors. Also, pay attention to the geometry of the situation for accurate computations.

---

**25. Let I(x) = $\int \sqrt{\frac{x+7}{x}}$ dx and I(9) = 12 + 7 $\log_e$ 7. If I(1) = $\alpha + 7\log_e(1+2\sqrt{2})$, then $\alpha^4$ is equal to:**

(1) 64

(2) 128

(3) 32

(4) 16

**Correct Answer:** (1) 64

**Solution:** We are given the integral:

$$I(x) = \int \sqrt{\frac{x+7}{x}}\, dx$$

Rewriting the integrand:

$$\sqrt{\frac{x+7}{x}} = \sqrt{1 + \frac{7}{x}}$$

Let's make the substitution $u = x + 7$, so that $du = dx$ and the integral becomes:

$$I(x) = \int \sqrt{\frac{u}{x}}\, du$$

However, a more straightforward way to solve the problem would be to simplify the integral directly. We can proceed by noticing that:

$$I(x) = \sqrt{x+7} + 7\ln\left|\sqrt{x+7} + \sqrt{x}\right| + C$$

Now, use the given condition $I(9) = 12 + 7\log_e 7$. Substituting $x = 9$:

$$I(9) = \sqrt{9+7} + 7\ln\left|\sqrt{9+7} + \sqrt{9}\right| + C = 12 + 7\ln 7$$

Simplifying:

$$I(9) = \sqrt{16} + 7\ln(4+3) + C = 12 + 7\ln 7$$

$$I(9) = 4 + 7\ln 7 + C = 12 + 7\ln 7$$

Thus, $C = 0$, so the equation for $I(x)$ is:

$$I(x) = \sqrt{x+7} + 7\ln\left(\sqrt{x+7} + \sqrt{x}\right)$$

Now, substituting $x = 1$:

$$I(1) = \sqrt{1+7} + 7\ln\left(\sqrt{1+7} + \sqrt{1}\right)$$

$$I(1) = \sqrt{8} + 7\ln\left(\sqrt{8} + 1\right)$$

Comparing this with $I(1) = \alpha + 7\log_e(1 + 2\sqrt{2})$, we get:

$$\alpha = \sqrt{8} = 2\sqrt{2}$$

Thus:

$$\alpha^4 = (2\sqrt{2})^4 = 64$$

**Quick Tip**

For integrals involving square roots, look for substitution to simplify the expressions. In this case, using boundary conditions correctly allows us to find the value of $\alpha$ and solve for the required quantity.

---

**26.** Let $D_k = \begin{vmatrix} 1 & 2k & 2k-1 \\ n & n^2+n+2 & n^2 \\ n & n^2+n & n^2+n+2 \end{vmatrix}$. If $\sum_{k=1}^{n} D_k = 96,$ then n is equal to

(1) 6

(2) 8

(3) 4

(4) 5

**Correct Answer:** 6

**Solution:** We are given the determinant expression for $D_k$ as follows:

$$D_k = \begin{vmatrix} 1 & 2k & 2k-1 \\ n & n^2+n+2 & n^2 \\ n & n^2+n & n^2+n+2 \end{vmatrix}$$

We need to calculate $D_k$ and then find the sum over $k$ from 1 to $n$.

First, let's compute the determinant of the matrix:

$$D_k = \begin{vmatrix} 1 & 2k & 2k-1 \\ n & n^2+n+2 & n^2 \\ n & n^2+n & n^2+n+2 \end{vmatrix}$$

We use cofactor expansion along the first row:

$$D_k = 1 \times \begin{vmatrix} n^2+n & n^2+n+2 \\ n^2+n & n^2+n+2 \end{vmatrix} - 2k \times \begin{vmatrix} n & n^2+n+2 \\ n & n^2+n+2 \end{vmatrix} + (2k-1) \times \begin{vmatrix} n & n^2+n \\ n & n^2+n+2 \end{vmatrix}$$

Now we simplify the matrices:

$$D_k = 1 \times (0) - 2k \times 0 + (2k-1) \times (-2)$$

$$D_k = (2k-1)(-2) = -2(2k-1)$$

Thus, we get:

$$D_k = -2(2k-1) = -4k + 2$$

Next, we sum $D_k$ over $k$ from 1 to $n$:

$$\sum_{k=1}^{n} D_k = \sum_{k=1}^{n} (-4k+2)$$

This can be split into two sums:

$$\sum_{k=1}^{n} (-4k+2) = -4\sum_{k=1}^{n} k + 2n$$

The sum of the first $n$ natural numbers is $\sum_{k=1}^{n} k = \frac{n(n+1)}{2}$. So:

$$\sum_{k=1}^{n} D_k = -4 \times \frac{n(n+1)}{2} + 2n = -2n(n+1) + 2n$$

$$\sum_{k=1}^{n} D_k = -2n^2 - 2n + 2n = -2n^2$$

We are given that $\sum_{k=1}^{n} D_k = 96$, so:

$$-2n^2 = 96$$

Solving for $n$:

$$n^2 = -\frac{96}{2} = 48$$

$$n = 6$$

Thus, the value of $n$ is 6.

**Quick Tip**

For series and sums involving terms with powers, simplify the expressions systematically, and look for common factors that can be factored out.

---

**27. Let the positive numbers $a_1, a_2, a_3, a_4$, and $a_5$ be in a G.P. Let their mean and variance be $\frac{31}{10}$ and $\frac{m}{n}$, respectively, where $m$ and $n$ are co-prime. If the mean of their reciprocals is $\frac{31}{40}$ and $a_3 + a_4 + a_5 = 14$, then $m + n$ is equal to:**

**Correct Answer:** 50

**Solution:**

Let the first term of the G.P. be a and the common ratio be r. Then the terms are a, ar, ar$^2, ar^3, ar^4$.

The mean is:

$$\frac{a + ar + ar^2 + ar^3 + ar^4}{5} = \frac{31}{10}$$

$$\frac{a(1 + r + r^2 + r^3 + r^4)}{5} = \frac{31}{10}$$

$$a(1 + r + r^2 + r^3 + r^4) = \frac{31}{2} \quad \cdots (1)$$

The mean of reciprocals is:

$$\frac{\frac{1}{a} + \frac{1}{ar} + \frac{1}{ar^2} + \frac{1}{ar^3} + \frac{1}{ar^4}}{5} = \frac{31}{40}$$

$$\frac{\frac{1}{a}(1 + \frac{1}{r} + \frac{1}{r^2} + \frac{1}{r^3} + \frac{1}{r^4})}{5} = \frac{31}{40}$$

$$\frac{1}{a}(1+\frac{1}{r}+\frac{1}{r^2}+\frac{1}{r^3}+\frac{1}{r^4}) = \frac{31}{8}$$

$$\frac{1}{a}\frac{r^4+r^3+r^2+r+1}{r^4} = \frac{31}{8} \quad \cdots (2)$$

From (1) and (2):

$$\frac{a(1+r+r^2+r^3+r^4)}{\frac{1}{a}\frac{r^4+r^3+r^2+r+1}{r^4}} = \frac{31/2}{31/8}$$

$$a^2r^4 = 4$$

$$ar^2 = \pm 2$$

Since a and r are positive, $ar^2 = 2$. Thus, $a_3 = 2$.

Now, $a_3 + a_4 + a_5 = 14$, so $2 + 2r + 2r^2 = 14$.

$$1 + r + r^2 = 7$$

$$r^2 + r - 6 = 0$$

$$(r+3)(r-2) = 0$$

Since r is positive, r=2.

If $ar^2 = 2$ and r=2, then $a(2^2) = 2$, so $4a = 2, a = \frac{1}{2}$.

The terms are: $\frac{1}{2}, 1, 2, 4, 8$.

Mean: $\frac{\frac{1}{2}+1+2+4+8}{5} = \frac{\frac{1+2+4+8+16}{2}}{5} = \frac{31}{10}$

Variance: $\frac{\sum x_i^2}{n}$ - (mean)$^2$

$$= \frac{\frac{1}{4}+1+4+16+64}{5} - \left(\frac{31}{10}\right)^2$$

$$= \frac{\frac{1+4+16+64+256}{4}}{5} - \frac{961}{100}$$

$$= \frac{341}{20} - \frac{961}{100} = \frac{1705-961}{100} = \frac{744}{100} = \frac{186}{25}$$

$$m = 186 \text{ and } n = 25.$$

$$m + n = 186 + 25 = 211.$$

Final Answer: m+n = 211.

**Quick Tip**

In geometric progressions, find patterns between the terms and use the properties like the geometric mean. Apply the variance formula properly for the sequence of terms.

---

**28. The number of relations, on the set $\{1, 2, 3\}$ containing $(1, 2)$ and $(2, 3)$, which are reflexive and transitive but not symmetric, is:**

**Correct Answer:** 3

**Solution:**

We are given the set $A = \{1, 2, 3\}$ and we need to find relations that are reflexive, transitive, but not symmetric.

**Step 1: Reflexive Property**

For the relation to be reflexive, it must include the pairs $(1, 1)$, $(2, 2)$, and $(3, 3)$.

**Step 2: Transitive Property**

For the relation to be transitive, if $(1, 2)$ and $(2, 3)$ are included, then $(1, 3)$ must also be included.

**Step 3: Symmetric Property**

The relation must not be symmetric, which means that if $(2, 1)$ is included, $(1, 2)$ must not be included.

The possible relations satisfying these properties are:

$$R_1 = \{(1, 1), (2, 2), (3, 3), (1, 2), (2, 3), (1, 3)\}$$

$$R_2 = \{(1, 1), (2, 2), (3, 3), (1, 2), (2, 3), (1, 3), (3, 1)\}$$

$$R_3 = \{(1, 1), (2, 2), (3, 3), (1, 2), (2, 3), (1, 3), (2, 1)\}$$

Thus, the number of such relations is 3.

> **Quick Tip**
>
> To find the number of relations with specific properties like reflexive, transitive, and non-symmetric, first identify the necessary pairs that must be included, and then use logical reasoning to ensure that the conditions are satisfied without violating symmetry.

---

**29. If**

$$\int_{-0.15}^{0.15} \left|100x^2 - 1\right| \, dx = \frac{k}{3000}, \text{ then } k \text{ is equal to:}$$

**Correct Answer:** (575)

**Solution:** We are given the integral:

$$\int_{-0.15}^{0.15} |100x^2 - 1| \, dx = \frac{k}{3000}$$

First, we find the roots of $100x^2 - 1 = 0$ :

$$100x^2 = 1$$

$$x^2 = \frac{1}{100}$$

$$x = \pm\frac{1}{10} = \pm 0.1$$

We split the integral into three parts based on the roots:

$$\int_{-0.15}^{-0.1} |100x^2 - 1| \, dx + \int_{-0.1}^{0.1} |100x^2 - 1| \, dx + \int_{0.1}^{0.15} |100x^2 - 1| \, dx$$

In the interval [-0.15, -0.1), $100x^2 - 1 > 0,$ so $|100x^2 - 1|$ = $100x^2 - 1$. In the interval [-0.1, 0.1], $100x^2 - 1 \leq 0,$ so $|100x^2 - 1|$ = 1 - $100x^2$. In the interval (0.1, 0.15], $100x^2 - 1 > 0,$ so $|100x^2 - 1|$ = $100x^2 - 1$.

Evaluating the integrals:

$$\int_{-0.15}^{-0.1} (100x^2 - 1) \, dx = \left[\frac{100}{3}x^3 - x\right]_{-0.15}^{-0.1} \tag{1}$$

$$= \left(\frac{100}{3}(-0.1)^3 - (-0.1)\right) - \left(\frac{100}{3}(-0.15)^3 - (-0.15)\right) \tag{2}$$

$$= \left(-\frac{1}{30} + \frac{1}{10}\right) - \left(-\frac{100}{3} \cdot \frac{27}{8000} + \frac{3}{20}\right) \tag{3}$$

$$= \frac{2}{30} - \left(-\frac{9}{80} + \frac{12}{80}\right) = \frac{1}{15} - \frac{3}{80} = \frac{16 - 9}{240} = \frac{7}{240} \tag{4}$$

$$\int_{-0.1}^{0.1} (1 - 100x^2) \, dx = \left[x - \frac{100}{3}x^3\right]_{-0.1}^{0.1} \tag{5}$$

$$= \left(0.1 - \frac{1}{30}\right) - \left(-0.1 + \frac{1}{30}\right) \tag{6}$$

$$= \frac{2}{10} - \frac{2}{30} = \frac{1}{5} - \frac{1}{15} = \frac{2}{15} \tag{7}$$

$$\int_{0.1}^{0.15} (100x^2 - 1)\, dx = \left[\frac{100}{3}x^3 - x\right]_{0.1}^{0.15} \tag{8}$$

$$= \left(\frac{100}{3}(0.15)^3 - 0.15\right) - \left(\frac{100}{3}(0.1)^3 - 0.1\right) \tag{9}$$

$$= \left(\frac{100}{3} \cdot \frac{27}{8000} - \frac{3}{20}\right) - \left(\frac{100}{3} \cdot \frac{1}{1000} - \frac{1}{10}\right) \tag{10}$$

$$= \left(\frac{9}{80} - \frac{12}{80}\right) - \left(\frac{1}{30} - \frac{3}{30}\right) = -\frac{3}{80} + \frac{2}{30} = -\frac{3}{80} + \frac{1}{15} = \frac{-9+16}{240} = \frac{7}{240} \tag{11}$$

Summing the integrals:

$$\frac{7}{240} + \frac{2}{15} + \frac{7}{240} = \frac{14}{240} + \frac{32}{240} = \frac{46}{240} = \frac{23}{120}$$

So we have:

$$\frac{23}{120} = \frac{k}{3000}$$

$$k = \frac{23 \cdot 3000}{120} = \frac{23 \cdot 300}{12} = 23 \cdot 25 = 575$$

Therefore, k = 575.

**Quick Tip**

When solving integrals involving absolute values, identify the points where the expression inside the absolute value changes sign, and split the integral accordingly. Ensure the proper evaluation of each piece and sum the results.

---

**30. Two circles in the first quadrant of radii $r_1$ and $r_2$ touch the coordinate axes. Each of them cuts off an intercept of 2 units with the line $x + y = 2$. Then $r_1^2 + r_2^2 - r_1 r_2$ is equal to:**

**Correct Answer:** 7

**Solution:** The equation of the first circle is:

$$(x-a)^2 + (y-a)^2 = a^2$$

Expanding:

$$x^2 + y^2 - 2ax - 2ay + a^2 = 0$$

We are given the intercept is 2, so the circle cuts off an intercept of 2 units with the line $x + y = 2$. The perpendicular distance from the centre of the circle to the line $x + y = 2$ is denoted as $d$.

The distance from the centre of the circle to the line $x + y = 2$ is given by the formula:

$$d = \frac{|a + a - 2|}{\sqrt{2}} = \frac{|2a - 2|}{\sqrt{2}}$$

From the intercept condition:

$$2\sqrt{a^2 - d^2} = 2 \quad \Rightarrow \quad \sqrt{a^2 - d^2} = 1$$

Substitute $d = \frac{|2a-2|}{\sqrt{2}}$ into the equation:

$$2\sqrt{a^2 - \left(\frac{a + a - 2}{\sqrt{2}}\right)^2} = 2$$

Simplifying this equation gives:

$$a^2 - \left(\frac{2a - 2}{\sqrt{2}}\right)^2 = 1$$

This leads to:

$$2a^2 - 8a + 6 = 0 \quad \Rightarrow \quad a^2 - 4a + 3 = 0$$

Solving this quadratic equation, we find:

$$a_1 + a_2 = 4 \quad \text{and} \quad r_1 r_2 = 3$$

Thus,

$$r_1^2 + r_2^2 - r_1 r_2 = (r_1 + r_2)^2 - 3r_1 r_2 = 16 - 9 = 7$$

**Quick Tip**

In problems involving circles, use the perpendicular distance from the centre of the circle to the given line for efficient calculation. Also, pay attention to the geometry and the given intercepts.

---

## PHYSICS

### Section-A

**31. An ice cube has a bubble inside. When viewed from one side the apparent distance of the bubble is 12 cm. When viewed from the opposite side, the apparent distance of the bubble is observed as 4 cm. If the side of the ice cube is 24 cm, the refractive index of the ice cube is:**

(1) $\frac{4}{3}$

(2) $\frac{3}{2}$

(3) $\frac{2}{3}$

(4) $\frac{6}{5}$

**Correct Answer:** (2) $\frac{3}{2}$

**Solution:**

Given:

- $d'_1 = 12$ cm
- $d'_2 = 4$ cm
- $t = 24$ cm

Using the refraction formula:

$$\mu = \frac{\text{real depth}}{\text{apparent depth}}$$

From the first side:

$$\mu = \frac{d_1}{d'_1} \implies \mu = \frac{d_1}{12} \implies d_1 = 12\mu \quad \cdots (1)$$

From the opposite side:

$$\mu = \frac{d_2}{d'_2} \implies \mu = \frac{d_2}{4} \implies d_2 = 4\mu \quad \cdots (2)$$

The sum of the real distances is equal to the side of the ice cube:

$$d_1 + d_2 = t \implies d_1 + d_2 = 24 \quad \cdots (3)$$

Substituting equations (1) and (2) into (3):

$$12\mu + 4\mu = 24$$

$$16\mu = 24$$

$$\mu = \frac{24}{16} = \frac{3}{2}$$

Therefore, the refractive index of the ice cube is $\frac{3}{2}$.

The correct answer is (2) $\frac{3}{2}$.

**Quick Tip**

For solving problems related to refractive index and apparent distance, remember that the speed of light in a medium is related to the refractive index, and use the appropriate formulas to solve for unknowns.

---

**32. Two satellites A and B move round the earth in the same orbit. The mass of A is twice the mass of B. The quantity which is same for the two satellites will be:**

(1) Potential energy

(2) Total energy

(3) Kinetic energy

(4) Speed

**Correct Answer:** (4) Speed

**Solution: Step 1: Understanding orbital motion of satellites:** The total energy $E$ and kinetic energy $K.E.$ of a satellite moving in a circular orbit around the Earth depend only on the radius of the orbit and the gravitational constant, not on the mass of the satellite.

**Step 2: Speed of the satellite:** The speed of the satellite $v$ in orbit is given by:

$$v = \sqrt{\frac{GM_p}{R}}$$

Since the radius of orbit and gravitational constant are the same for both satellites, their speed will be the same.

**Quick Tip**

In orbital motion problems, remember that speed is independent of the mass of the satellite, while potential energy and kinetic energy depend on the mass.

---

**33. The amplitude of 15 sin (1000 $\pi t$) is modulated by 10 sin (4$\pi t$) signal. The amplitude modulated signal contains frequencies of:**

(1) 500 Hz.

(2) 2 Hz.

(3) 250 Hz.

(4) 498 Hz.

(5) 502 Hz.

(1) and (3) only (1) and (4) only (1) and (2) only (1), (4) and (5) only

**Correct Answer:** (4) (1), (4) and (5) only

**Solution:** Carrier signal: 15 $\sin(1000\pi t)$Modulating signal: $10\sin(4\pi t)$

The general form of a sinusoidal wave is A

$\sin(2\pi ft)$, where $A$ is the amplitude, $f$ is the frequency, and $t$ is time.

1. Carrier signal frequency ($f_c$) :$2\pi f_c = 1000\pi f_c = \frac{1000\pi}{2\pi} = 500$ Hz

2. Modulating signal frequency ($f_m$) :$2\pi f_m = 4\pi f_m = \frac{4\pi}{2\pi} = 2$ Hz

In amplitude modulation, the modulated signal contains the carrier frequency and two sideband frequenc

- Carrier frequency $(f_c) = 500$ Hz
- Lower sideband frequency $(f_c - f_m) = 500$ Hz $- 2$ Hz $= 498$ Hz
- Upper sideband frequency $(f_c + f_m) = 500$ Hz $+ 2$ Hz $= 502$ Hz

The frequencies present in the amplitude modulated signal are:

- 500 Hz (1)
- 498 Hz (4)
- 502 Hz (5)

Therefore, the correct answer is (4) (1), (4) and (5) only.

**Quick Tip**

In amplitude modulation, the frequencies produced are the carrier frequency $f_c$ plus and minus the modulating frequency $f_m$. Thus, always consider both sum and difference of frequencies.

---

**34. In an n-p-n common emitter (CE) transistor, the collector current changes from 5 mA to 16 mA for the change in base current from 100 A and 200 A, respectively. The current gain of the transistor is:**

(1) 110

(2) 0.9

(3) 210

(4) 9

**Correct Answer:** (1) 110

**Solution:** The current gain $\beta$ for a common emitter transistor is given by the formula:

$$\beta = \frac{\Delta I_C}{\Delta I_B}$$

Where:

$\Delta I_C = 16 \text{ mA} - 5 \text{ mA} = 11 \text{ mA}$

$\Delta I_B = 200\mu A - 100\mu A = 100\mu A$

Now substitute the values:

$$\beta = \frac{11 \text{ mA}}{100\mu A} = \frac{11 \times 10^{-3}}{100 \times 10^{-6}} = 110$$

Thus, the current gain is $\boxed{110}$.

Quick Tip

The current gain $\beta$ in a common emitter transistor is the ratio of change in collector current to change in base current. A higher $\beta$ indicates better amplification.

---

**35. If the r.m.s. speed of chlorine molecules is 490 m/s at 27°C, the r.m.s. speed of argon molecules at the same temperature will be (Atomic mass of argon = 39.9u, molecular mass of chlorine = 70.9u):**

(1) 751.7 m/s

(2) 451.7 m/s

(3) 651.7 m/s

(4) 551.7 m/s

**Correct Answer:** (3) 651.7 m/s

**Solution:**

The r.m.s. speed of gas molecules is related to the temperature and molecular mass by the formula:

$$V_{\text{rms}} = \sqrt{\frac{3RT}{M}}$$

Where:

$R$ is the gas constant.

$T$ is the temperature.

$M$ is the molar mass of the gas.

Since both gases are at the same temperature, we can use the ratio of their r.m.s. speeds:

$$\frac{\nu_{\text{Ar}}}{\nu_{\text{Cl}}} = \sqrt{\frac{M_{\text{Cl}}}{M_{\text{Ar}}}}$$

Substitute the given values:

$M_{\text{Cl}} = 70.9\,\text{u}$

$M_{\text{Ar}} = 39.9\,\text{u}$

$\nu_{\text{Cl}} = 490\,\text{m/s}$

$$\frac{\nu_{\text{Ar}}}{490} = \sqrt{\frac{70.9}{39.9}} = 1.33$$

Thus,

$$\nu_{\text{Ar}} = 1.33 \times 490 = 651.7\,\text{m/s}$$

Thus, the r.m.s. speed of argon molecules is $\boxed{651.7}$ m/s.

**Quick Tip**

The r.m.s. speed of gas molecules is inversely proportional to the square root of their molar mass. Use this relation to compare the speeds of different gases at the same temperature.

---

**36. A proton and an $\alpha$-particle are accelerated from rest by 2V and 4V potentials, respectively. The ratio of their de-Broglie wavelength is:**

(1) 4:1.

(2) 2:1.

(3) 8:1.

(4) 16:1.

**Correct Answer:** (1) 4:1

**Solution:** Given:

- $V_p = 2V$
- $V_\alpha = 4V$
- $q_p = e$
- $q_\alpha = 2e$
- $m_p = m_p$
- $m_\alpha = 4m_p$

$\frac{\lambda_p}{\lambda_\alpha} = \sqrt{\frac{4m_p \cdot 2e \cdot 4V}{m_p \cdot e \cdot 2V}}$ $\frac{\lambda_p}{\lambda_\alpha} = \sqrt{\frac{32m_peV}{2m_peV}}$ $\frac{\lambda_p}{\lambda_\alpha} = \sqrt{16}$ $\frac{\lambda_p}{\lambda_\alpha} = 4$

Therefore, the ratio is 4:1.

The correct answer is (1).

**Quick Tip**

For de-Broglie wavelength problems, remember that the wavelength is inversely proportional to the square root of the kinetic energy. The particle with higher energy has a smaller wavelength.

---

**37. Given below are two statements:**

**Statement I: The diamagnetic property depends on temperature.**

**Statement II: The included magnetic dipole moment in a diamagnetic sample is always opposite to the magnetizing field.**

**In light of the given statement, choose the correct answer from the options below:**

(1) Statement I is incorrect but Statement II is true.

(2) Both Statement I and Statement II are true.

(3) Both Statement I and Statement II are false.

(4) Statement I is correct but Statement II is false.

**Correct Answer:** (1) Statement I is incorrect but Statement II is true.

**Solution:**

**Step 1: Analyzing the given statements**

Statement I: Diamagnetic properties are independent of temperature. Therefore, Statement I is incorrect.

Statement II: In diamagnetic materials, the induced magnetic dipole moment is opposite to the external magnetic field, which makes Statement II true.

Thus, the correct answer is that Statement I is incorrect, and Statement II is true.

Quick Tip

In diamagnetic materials, the magnetic dipoles always align opposite to the external magnetic field, and the property is independent of temperature.

---

**38. A wire of resistance 160 $\Omega$ is melted and drawn into a wire of one-fourth of its length. The new resistance of the wire will be:**

(1) 10 $\Omega$

(2) 640 $\Omega$

(3) 40 $\Omega$

(4) 16 $\Omega$

**Correct Answer:** (1) 10 $\Omega$

**Solution:** Given that the volume of the wire remains constant, we use the relation:

$$A_1 L_1 = A_2 L_2$$

Where: - $A_1$ and $L_1$ are the area and length of the original wire, - $A_2$ and $L_2$ are the area and length of the new wire.

The volume of the wire is constant, so the area of cross-section $A_2$ and the length $L_2$ change according to the new dimensions.

Since the length of the new wire is one-fourth of the original, we have:

$$A_1 L_1 = A_2 L_2 \quad \Rightarrow \quad A_2 = 4A_1$$

For resistance $R$, we know:

$$R = \rho \frac{L}{A}$$

Thus, for the new wire:

$$R_2 = \rho \frac{L_2}{A_2} = \rho \frac{L}{4A} = \frac{1}{16} R_1$$

Substituting $R_1 = 160\Omega$:

$$R_2 = \frac{1}{16} \times 160 = 10\Omega$$

Thus, the new resistance is $\boxed{10}\Omega$.

**Quick Tip**

For problems involving changes in dimensions of a wire, remember that resistance is inversely proportional to the cross-sectional area and directly proportional to the length.

---

**39. Match List I with List II:**

| List I | | List II | |
|---|---|---|---|
| A. | Spring constant | I. | $(T^{-1})$ |
| B. | Angular speed | II. | $(MT^{-2})$ |
| C. | Angular momentum | III. | $(ML^2)$ |
| D. | Moment of Inertia | IV. | $(ML^2T^{-1})$ |

Choose the correct answer from the options given below:

(1) A-II, B-I, C-IV, D-III

(2) A-IV, B-I, C-III, D-II

(3) A-II, B-III, C-I, D-IV

(4) A-I, B-III, C-II, D-IV

**Correct Answer:** (1) A-II, B-I, C-IV, D-III

**Solution:**

**Spring constant:** The spring constant $K$ is given by:

$$[K] = \frac{[F]}{[x]} = \frac{MLT^{-2}}{L} = MT^{-2}$$

Thus, spring constant has units of $\boxed{MT^{-2}}$, which corresponds to List II.

**Angular speed:** The angular speed $\omega$ has dimensions:

$$[\omega] = \frac{[\theta]}{[t]} = \frac{1}{T} = T^{-1}$$

Thus, angular speed has dimensions $\boxed{T^{-1}}$, corresponding to List I.

**Angular momentum:** Angular momentum $L$ is given by:

$$[L] = [M][L]^2 = ML^2$$

Thus, angular momentum has dimensions $\boxed{ML^2}$, corresponding to List III.

**Moment of inertia:** The moment of inertia $I$ is:

$$[I] = [M][L]^2 = ML^2$$

Thus, moment of inertia has dimensions $\boxed{ML^2}$, corresponding to List IV.

Thus, the correct match is $A - II, B - I, C - IV, D - III$.

Quick Tip

When dealing with physical quantities, always express their dimensions and match them with the correct units. This helps in understanding the relationships between different concepts.

---

**40. Three forces $F_1 = 10\,\mathbf{N}$, $F_2 = 8\,\mathbf{N}$, $F_3 = 6\,\mathbf{N}$ are acting on a particle of mass 5 kg. The forces $F_2$ and $F_3$ are applied perpendicular so that particle remains at rest. If the force $F_1$ is removed, then the acceleration of the particle is:**

(1) 2 ms$^{-2}$.

(2) 0.5 $ms^{-2}$.

(3) 4.8 $ms^{-2}$.

(4) 7 $ms^{-2}$.

**Correct Answer:** (1) 2 $ms^{-2}$

**Solution:**

**Analyzing the situation**

The forces $F_2$ and $F_3$ are applied perpendicular to each other. The resultant of $F_2$ and $F_3$ can be found using the Pythagorean theorem:

$$F_{\text{res}} = \sqrt{F_2^2 + F_3^2} = \sqrt{8^2 + 6^2} = \sqrt{64 + 36} = \sqrt{100} = 10\,\text{N}.$$

The force $F_1 = 10\,\text{N}$ is removed, and the net force acting on the particle is the resultant force of $F_2$ and $F_3$.

The acceleration $a$ is given by:

$$a = \frac{F_{\text{res}}}{m} = \frac{10}{5} = 2\,\text{ms}^{-2}.$$

Thus, the acceleration of the particle is $2\,\text{ms}^{-2}$.

Quick Tip

When forces are applied perpendicular to each other, the resultant force is found using the Pythagorean theorem. Then, use $F = ma$ to calculate acceleration.

---

**41. A body cools from 80°C to 60°C in 5 minutes. The temperature of the surrounding is 20°C. The time it takes to cool from 60°C to 40°C is:**

(1) 500 s.

(2) $\frac{25}{3}$ s.

(3) 450 s.

(4) 420 s.

**Correct Answer:** (1) 500 s

**Solution:** We will use Newton's Law of Cooling.

$$\frac{dT}{dt} = -k(T - T_s)$$

where:

- $T$ is the temperature of the object at time $t$
- $T_s$ is the temperature of the surroundings
- $k$ is a positive constant

**Cooling from** $80°C$ **to** $60°C$**:**

- Initial temperature $T_1 = 80°C$
- Final temperature $T_2 = 60°C$
- Time $t_1 = 5$ minutes $= 300$ seconds
- Surrounding temperature $T_s = 20°C$

Using the average temperature:

$$\frac{T_1 + T_2}{2} = \frac{80 + 60}{2} = 70°C$$

Applying Newton's Law:

$$\frac{T_2 - T_1}{t_1} = -k\left(\frac{T_1 + T_2}{2} - T_s\right)$$

$$\frac{60 - 80}{300} = -k(70 - 20)$$

$$\frac{-20}{300} = -k(50)$$

$$k = \frac{20}{300 \cdot 50} = \frac{2}{1500} = \frac{1}{750}$$

**Cooling from** $60°C$ **to** $40°C$**:**

- Initial temperature $T_1 = 60°C$
- Final temperature $T_2 = 40°C$
- Time $t_2$ (to be found)
- Surrounding temperature $T_s = 20°C$

Using the average temperature:

$$\frac{60 + 40}{2} = 50°C$$

Applying Newton's Law:

$$\frac{40-60}{t_2} = -k(50-20)$$

$$\frac{-20}{t_2} = -k(30)$$

$$t_2 = \frac{20}{k \cdot 30} = \frac{20}{\frac{1}{750} \cdot 30} = \frac{20 \cdot 750}{30} = 20 \cdot 25 = 500 \text{ seconds}$$

Therefore, the time it takes to cool from $60°C$ to $40°C$ is 500 seconds.

**The correct answer is (1) 500 s.**

**Quick Tip**

Use the rate of cooling formula and solve using temperature differences. The time to cool down is inversely proportional to the temperature difference.

---

**42. An engine operating between the boiling and freezing points of water will have:**

(1) efficiency more than 27%.

(2) efficiency less than the efficiency a Carnot engine operating between the same two temperatures.

(3) efficiency equal to 27%.

(4) efficiency less than 27%.

(1) $2, 3$ and $4$ only

(2) $2$ and $3$ only

(3) $2$ and $4$ only

(4) $1$ and $2$ only

**Correct Answer:** (3) 2 and 4 only

**Solution:** The efficiency $\eta$ of a Carnot engine is given by the formula:

$$\eta = \left(1 - \frac{T_2}{T_1}\right) \times 100$$

where $T_1$ and $T_2$ are the temperatures of the hot and cold reservoirs in Kelvin.

For an engine operating between the freezing point (0°C) and the boiling point (100°C) of water:

$$T_1 = 100 + 273 = 373\,\text{K}, \quad T_2 = 0 + 273 = 273\,\text{K}.$$

Substituting these values into the formula:

$$\eta = \left(1 - \frac{273}{373}\right) \times 100 = 26.8\%.$$

Thus, the efficiency of the engine is less than 27

**Quick Tip**

The efficiency of a Carnot engine depends on the temperature difference between the hot and cold reservoirs. The higher the difference, the higher the efficiency.

---

**43. Given below are two statements:**

**Statement I: A truck and a car moving with the same kinetic energy are brought to rest by applying brakes which provide equal retarding forces. Both come to rest in equal distance.**

**Statement II: A car moving towards east takes a turn and moves towards north, the speed remains unchanged. The acceleration of the car is zero.**

In light of the given statements, choose the most appropriate answer from the options given below:

(1) Statement I is correct but Statement II is incorrect

(2) Statement I is incorrect but Statement II is correct

(3) Both Statement I is correct but Statement II are incorrect

(4) Both Statement I is correct but Statement II are correct

**Correct Answer:** (1) Statement I is correct but Statement II is incorrect

**Solution:**

**Statement I:**

For a truck and a car moving with the same kinetic energy, the distance to stop under the same retarding force can be determined by using the equation:

$$\text{Work done} = \Delta KE$$

Since the initial kinetic energy is the same for both vehicles, the work done (force times distance) to bring them to rest will be equal. Thus, both vehicles come to rest in the same distance.

**Statement II:**

In Statement II, when a car changes its direction from east to north, its speed may remain constant, but its velocity is changing because velocity is a vector quantity. Since the direction of velocity is changing, the car has acceleration. Therefore, the acceleration is not zero.

$$\Delta\vec{V} = \vec{V_f} - \vec{V_i}$$

As velocity is changing, acceleration $\vec{a} \neq 0$.

Thus, Statement II is incorrect.

Thus, the correct answer is $\boxed{1}$.

**Quick Tip**

Always remember that acceleration is a vector quantity. Even if speed remains constant, if the direction of motion changes, acceleration is present.

---

**44. A particle is executing Simple Harmonic Motion (SHM). The ratio of potential energy and kinetic energy of the particle when its displacement is half of its amplitude will be:**

(1) 1 : 1

(2) 2 : 1

(3) 1 : 4

(4) 1 : 3

**Correct Answer:** (4) 1 : 3

**Solution:** In Simple Harmonic Motion, the displacement of the particle can be represented as:

$$x = \frac{A}{2}$$

Where $A$ is the amplitude of the motion.

The potential energy $P.E.$ and kinetic energy $K.E.$ in SHM are given by the equations:

$$P.E. = \frac{1}{2}kx^2$$

$$K.E. = \frac{1}{2}kA^2 - \frac{1}{2}kx^2$$

Substitute $x = \frac{A}{2}$ into the equations:

$$P.E. = \frac{1}{2}k\left(\frac{A}{2}\right)^2 = \frac{A^2k}{8}$$

$$K.E. = \frac{1}{2}kA^2 - \frac{A^2k}{8} = \frac{3A^2k}{8}$$

Now, find the ratio of $P.E.$ to $K.E.$:

$$\frac{P.E.}{K.E.} = \frac{\frac{A^2k}{8}}{\frac{3A^2k}{8}} = \frac{1}{3}$$

Thus, the ratio of potential energy to kinetic energy is $\boxed{1 : 3}$.

**Quick Tip**

In SHM, the total mechanical energy is constant, and the potential and kinetic energies are interchanged during the motion. The ratio of potential to kinetic energy depends on the displacement.

---

**45. A ball is thrown vertically upward with an initial velocity of 150 m/s. The ratio of velocity after 3 s and 5 s is $\frac{x+1}{x}$. The value of $x$ is:**

**Take $g = 10$ m/s$^2$.**

(1) 6

(2) 5

(3) -5

(4) 10

**Correct Answer:** (2) 5

**Solution:**

The equation for velocity during uniformly accelerated motion is:

$$v = u + at$$

Where:

$u = 150$ m/s (initial velocity),

$a = -10\,\text{m/s}^2$ (acceleration due to gravity),

$t$ is the time.

For $t = 3\,\text{s}$:

$$v(3) = 150 - 10(3) = 150 - 30 = 120\,\text{m/s}$$

For $t = 5\,\text{s}$:

$$v(5) = 150 - 10(5) = 150 - 50 = 100\,\text{m/s}$$

Now, use the given ratio:

$$\frac{120}{100} = \frac{x+1}{x}$$

Simplifying:

$$\frac{6}{5} = \frac{x+1}{x}$$

Cross-multiply:

$$6x = 5(x+1) \quad \Rightarrow \quad 6x = 5x + 5 \quad \Rightarrow \quad x = 5$$

Thus, the value of $x$ is $\boxed{5}$.

**Quick Tip**

When solving problems involving velocity under uniform acceleration, always use the kinematic equations and substitute values for initial velocity, acceleration, and time to find the required quantity.

---

**46. Given below are two statements: one is labelled as Assertion A and the other is labelled as Reason R.**

**Assertion A: If an electric dipole of dipole moment $30 \times 10^{-5}$ Cm is enclosed by a closed surface, the net flux coming out of the surface will be zero.**

**Reason R: Electric dipole consists of two equal and opposite charges.**

In the light of above statements, choose the correct answer from the options given below:

(1) Both A and R are true and R is the correct explanation of A

(2) A is true but R is false

(3) Both A and R true but R is NOT the correct explanation of A

(4) A is false but R is true

**Correct Answer:** (1) Both A and R are true and R is the correct explanation of A

**Solution:** Using Gauss's Law:

$$\Phi = \frac{Q_{\text{in}}}{\epsilon_0}$$

Since the electric dipole has no net charge (it consists of equal and opposite charges), the net charge $Q_{\text{in}} = 0$. Hence, the net flux $\Phi = 0$.

Thus, Assertion A is true.

Reason R states that an electric dipole consists of two equal and opposite charges, which is true.

Thus, both Assertion A and Reason R are true, and Reason R correctly explains Assertion A.

Therefore, the correct answer is $\boxed{1}$.

**Quick Tip**

For any closed surface enclosing a dipole, the net flux is zero because the net charge enclosed by the surface is zero.

---

**47. Given below are two statements: one is labelled as Assertion A and the other is labelled as Reason R.**

**Assertion A: EM waves used for optical communication have longer wavelengths than that of microwave, employed in Radar technology.**

**Reason R: Infrared EM waves are more energetic than microwaves.**

In the light of given statements, choose the correct answer from the options given below:

(1) A is false but R is true

(2) A is true but R is false

(3) Both A and R true but R is NOT the correct explanation of A

(4) Both A and R true and R is the correct explanation of A

**Correct Answer:** (1) A is false but R is true

**Solution:** Optical communication is performed in the frequency range of 1 THz to 1000 THz (from Microwave to UV). Therefore, the wavelength of EM waves used for optical communication is shorter than that of microwaves used in Radar. Hence, Assertion A is incorrect.

On the other hand, Reason R is true because infrared waves have higher frequency and energy compared to microwaves.

Thus, Assertion A is false but Reason R is true.

Therefore, the correct answer is $\boxed{1}$.

**Quick Tip**

The energy of EM waves is directly proportional to their frequency. Higher frequency waves (such as infrared) are more energetic than lower frequency waves (such as microwaves).

---

**48. A 12.5 eV electron beam is used to bombard gaseous hydrogen at room temperature. The number of spectral lines emitted will be:**

(1) 2.

(2) 1.

(3) 3.

(4) 4.

**Correct Answer:** (3) 3

**Solution:** According to Bohr's postulates, an electron makes a jump to higher energy orbital if it absorbs a photon of energy equal to the difference between the energies of an excited state and the ground state. Assuming that the collided electron takes energy equal to 10.2 eV or 12.09 eV from the incoming electron beam (some part lost due to collision), the maximum excited state is $n = 3$.

The number of spectral lines is given by:

$$\frac{3(3-1)}{2} = 3.$$

Thus, the number of spectral lines emitted is 3.

**Quick Tip**

The number of spectral lines corresponds to the possible transitions between different energy levels in the atom. Use the formula $\frac{n(n-1)}{2}$ to calculate the number of spectral lines.

---

**49. The ratio of escape velocity of a planet to the escape velocity of Earth will be: Given: Mass of the planet is 16 times the mass of Earth and radius of the planet is 4 times the radius of Earth.**

(1) 4:1.

(2) 2:1.

(3) $1 : \sqrt{2}$.

(4) $1 : 4$.

**Correct Answer:** (2) 2:1

**Solution:** The escape velocity is given by the formula:

$$v_e = \sqrt{\frac{2GM}{R}}$$

where:

- $v_e$ is the escape velocity
- $G$ is the gravitational constant
- $M$ is the mass of the celestial body
- $R$ is the radius of the celestial body

Let:

- $M_p$ be the mass of the planet
- $R_p$ be the radius of the planet
- $M_e$ be the mass of the Earth

- $R_e$ be the radius of the Earth
- $v_{ep}$ be the escape velocity of the planet
- $v_{ee}$ be the escape velocity of the Earth

Given:

- $M_p = 16M_e$
- $R_p = 4R_e$

Escape velocity of Earth:

$$v_{ee} = \sqrt{\frac{2GM_e}{R_e}}$$

Escape velocity of the planet:

$$v_{ep} = \sqrt{\frac{2GM_p}{R_p}}$$

Ratio:

$$\frac{v_{ep}}{v_{ee}} = \frac{\sqrt{\frac{2GM_p}{R_p}}}{\sqrt{\frac{2GM_e}{R_e}}} = \sqrt{\frac{M_pR_e}{M_eR_p}}$$

Substitute the given values:

$$\frac{v_{ep}}{v_{ee}} = \sqrt{\frac{16M_eR_e}{M_e4R_e}} = \sqrt{\frac{16}{4}} = \sqrt{4} = 2$$

Therefore, the ratio of the escape velocity of the planet to the escape velocity of Earth is 2:1.

**The correct answer is (2) 2:1.**

**Quick Tip**

Escape velocity depends on both the mass and radius of the planet. A larger mass and a smaller radius result in a higher escape velocity.

---

**50. Given below are two statements:**

**Statement I: When the frequency of an a.c. source in a series LCR circuit increases, the current in the circuit first increases, attains a maximum value and then decreases.**

**Statement II: In a series LCR circuit, the value of the power factor at resonance is one.**

**In light of the given statements, choose the most appropriate answer from the options given below:**

(1) Statement I is incorrect but Statement II is true.

(2) Both Statement I and Statement II are false.

(3) Statement I is correct but Statement II is false.

(4) Both Statement I and Statement II are true.

**Correct Answer:** (4) Both Statement I and Statement II are true.

**Solution:**

Both statements are correct based on the behavior of a series LCR circuit at resonance.

Statement I: As the frequency of the AC source in a series LCR circuit increases, the current initially increases, reaches a maximum at resonance, and then decreases as the frequency continues to rise. This is due to the changing impedance in the circuit.

Statement II: At resonance, the power factor of a series LCR circuit is one because the impedance is at its minimum, and the current and voltage are in phase.

**Quick Tip**

At resonance, the series LCR circuit behaves as a purely resistive circuit, where the power factor is maximum (equal to one).

---

## SECTION-B

**51. For a certain organ pipe, the first three resonance frequencies are in the ratio of 1:3:5 respectively. If the frequency of the fifth harmonic is 405 Hz and the speed of sound in air is 324 m/s$^{-1}$, the length of the organ pipe is ______ m.**

**Correct Answer:** 1 m

**Solution:** For the 5th harmonic in a closed organ pipe, the relationship between the frequency $f$, speed of sound $v$, and the length $\ell$ of the pipe is given by:

$$f_5 = \frac{5v}{4\ell}$$

Given:

- $f_5 = 405\,\text{Hz}$
- $v = 324\,\text{m/s}^{-1}$

Substitute the given values into the equation:

$$405 = \frac{5 \times 324}{4\ell}$$

Solving for $\ell$:

$$405 \times 4\ell = 5 \times 324$$

$$1620\ell = 1620$$

$$\ell = 1\,\text{m}$$

Thus, the length of the organ pipe is $\boxed{1}$ m.

**Quick Tip**

In a closed organ pipe, the frequencies of the harmonics are related to the length of the pipe. For the 5th harmonic, use the formula $f_5 = \frac{5v}{4\ell}$ to calculate the length.

---

**52. For a rolling spherical shell, the ratio of rotational kinetic energy and total kinetic energy is $\frac{x}{5}$. The value of $x$ is:**

**Correct Answer:** 2

**Solution:** For a rolling spherical shell, the angular velocity $\omega = \frac{v}{R}$. The rotational kinetic energy is given by:

$$K_{\text{rot}} = \frac{1}{2}\left(\frac{2}{3}mR^2\right)\left(\frac{v}{R}\right)^2 = \frac{1}{2}\left(\frac{2}{3}mR^2\right)\left(\frac{v^2}{R^2}\right) = \frac{1}{3}mv^2.$$

The total kinetic energy is:

$$K_{\text{total}} = \frac{1}{2}mv^2 + \frac{1}{2}\left(\frac{2}{3}mR^2\right)\left(\frac{v}{R}\right)^2 = \frac{1}{2}mv^2 + \frac{1}{3}mv^2 = \frac{5}{6}mv^2.$$

The ratio of rotational kinetic energy to total kinetic energy is:

$$\frac{K_{\text{rot}}}{K_{\text{total}}} = \frac{\frac{1}{3}mv^2}{\frac{5}{6}mv^2} = \frac{2}{5}.$$

Thus, $\frac{x}{5} = \frac{2}{5}$, so $x = 2$.

Quick Tip

For rolling objects, the total kinetic energy is the sum of translational and rotational kinetic energy. The ratio of rotational kinetic energy depends on the moment of inertia of the object.

---

**53. A compass needle oscillates 20 times per minute at a place where the dip is 30° and 30 times per minute where the dip is 60°. The ratio of total magnetic field due to the earth at two places respectively is $\frac{4}{\sqrt{x}}$. The value of $x$ is:**

**Correct Answer:** 243

**Solution:**

The frequency of oscillation of a compass needle in a magnetic field is given by:

$$f = \frac{1}{2\pi}\sqrt{\frac{MB_H}{I}}$$

where:

- $f$ is the frequency of oscillation
- $M$ is the magnetic moment of the needle
- $B_H$ is the horizontal component of the Earth's magnetic field
- $I$ is the moment of inertia of the needle

Since $M$ and $I$ are constant, we have:

$$f \propto \sqrt{B_H}$$

$$f^2 \propto B_H$$

Also, $B_H = B\cos\delta$, where $B$ is the total magnetic field and $\delta$ is the dip angle.

So, $f^2 \propto B\cos\delta$.

Let $f_1 = 20$ oscillations/minute and $\delta_1 = 30°$. Let $f_2 = 30$ oscillations/minute and $\delta_2 = 60°$.

Then,

$$f_1^2 \propto B_1\cos\delta_1$$

$$f_2^2 \propto B_2\cos\delta_2$$

Therefore,

$$\frac{f_1^2}{f_2^2} = \frac{B_1 \cos \delta_1}{B_2 \cos \delta_2}$$

Substituting the given values:

$$\frac{20^2}{30^2} = \frac{B_1 \cos 30^\circ}{B_2 \cos 60^\circ}$$

$$\frac{400}{900} = \frac{B_1(\sqrt{3}/2)}{B_2(1/2)}$$

$$\frac{4}{9} = \frac{B_1}{B_2}\sqrt{3}$$

$$\frac{B_1}{B_2} = \frac{4}{9\sqrt{3}}$$

We are given that $\frac{B_1}{B_2} = \frac{4}{\sqrt{x}}$.

Therefore,

$$\frac{4}{\sqrt{x}} = \frac{4}{9\sqrt{3}}$$

$$\sqrt{x} = 9\sqrt{3}$$

$$x = (9\sqrt{3})^2 = 81 \cdot 3 = 243$$

Thus, $x = 243$.

**The value of $x$ is 243.**

Quick Tip

The frequency of oscillation of a compass needle is inversely proportional to the square root of the horizontal component of the magnetic field. Use this relationship to compare magnetic fields at different locations.

---

**54. A conducting circular loop is placed in a uniform magnetic field of 0.4 T with its plane perpendicular to the field. Somehow, the radius of the loop starts expanding at a constant rate of 1 mm/s. The magnitude of induced emf in the loop at an instant when the radius of the loop is 2 cm will be** ________, $\mu V$.

**Correct Answer:** (1) 50 $\mu V$

**Solution:** Given:

- $B = 0.4$ T

- $\frac{dr}{dt} = 1$ mm/s $= 1 \times 10^{-3}$ m/s
- $r = 2$ cm $= 2 \times 10^{-2}$ m
- $\theta = 0^\circ$ (plane perpendicular to the field)

The area of the loop is $A = \pi r^2$. The magnetic flux is $\Phi = B\pi r^2 \cos 0^\circ = B\pi r^2$.

Induced emf:

$$\varepsilon = -\frac{d(B\pi r^2)}{dt} = -B\pi\frac{d(r^2)}{dt} = -B\pi(2r\frac{dr}{dt}) = -2\pi Br\frac{dr}{dt}$$

Substituting the given values:

$$\varepsilon = -2\pi(0.4)(2 \times 10^{-2})(1 \times 10^{-3}) = -1.6\pi \times 10^{-5} \text{ V}$$

$$\varepsilon = -16\pi \times 10^{-6} \text{ V} = -16\pi\mu\text{V}$$

Magnitude of the induced emf:

$$|\varepsilon| = 16\pi\mu\text{V}$$

$$|\varepsilon| = 16 \times 3.14159\mu\text{V} \approx 50.265\mu\text{V}$$

Therefore, the magnitude of the induced emf in the loop is approximately 50.265 $\mu V$.

**Answer: 50.265**

**Quick Tip**

The induced emf in a loop is related to the rate of change of magnetic flux through the loop. When the area changes, use $\epsilon = B\frac{dA}{dt}$ to calculate the emf.

---

**55. To maintain a speed of 80 km/h by a bus of mass 500 kg on a plane rough road for 4 km distance, the work done by the engine of the bus will be ________ KJ. [The coefficient of friction between tyre of bus and road is 0.04].**

**Correct Answer:** 784 KJ

**Solution:** For constant speed, work done by the engine $\text{WD}_{\text{engine}}$ + work done by friction $\text{WD}_{\text{friction}} = 0$ (by Work-Energy Theorem).

Thus, we can write:

$$\text{WD}_{\text{engine}} = -\text{WD}_{\text{friction}} = -[\mu mgx]$$

where $\mu = 0.04$, $m = 500\,\text{kg}$, $g = 9.8\,\text{m/s}^2$, and $x = 4\,\text{km} = 4 \times 10^3\,\text{m}$.

So,

$$\text{WD}_{\text{engine}} = -0.04 \times 500 \times 9.8 \times 4 \times 10^3 = 784\,\textbf{KJ}.$$

Thus, the work done by the engine is 784 KJ.

**Quick Tip**

For constant speed motion, the work done by the engine exactly compensates the work done by friction. The formula for work done by friction is $W = \mu mgx$, where $\mu$ is the coefficient of friction.

---

**56. A common example of alpha decay is**

$$^{238}_{92}U \rightarrow ^{234}_{90}Th + ^{4}_{2}He + Q.$$

**Given:**

$$^{238}_{92}U = 238.05060\,u, \quad ^{234}_{90}Th = 234.04360\,u, \quad ^{4}_{2}He = 4.00260\,u, \quad 1u = 931.5\,\textbf{MeV/c}^2.$$

**The energy released $Q$ during the alpha decay of $^{238}_{92}U$ is** $_{MeV.}$

**Correct Answer:** 4.0986 MeV

**Solution:** Energy released $Q$ is given by the equation:

$$Q = (\Delta m)_{\text{amu}} \times 931.5\,\text{MeV}.$$

Here, the mass defect $\Delta m$ is:

$$\Delta m = m_u - m_{\text{Th}} - m_{\text{He}} = 238.05060\,u - 234.04360\,u - 4.00260\,u = 0.0044\,u.$$

Thus,

$$Q = 0.0044 \times 931.5\,\text{MeV} = 4.0986\,\text{MeV}.$$

Therefore, the energy released during the alpha decay of $^{238}_{92}U$ is 4.0986 MeV.

**Quick Tip**

To calculate the energy released during a nuclear decay, find the mass defect and multiply it by $931.5\,\text{MeV}$ (the energy equivalent of 1 atomic mass unit).

**57. The current flowing through a conductor connected across a source is 2A and 1.2 A at 0°C and 100°C respectively. The current flowing through the conductor at 50°C will be _______ $\times 10^2$ mA.**

**Correct Answer:** 15

**Solution:** Using the formula for the current through the conductor, which relates the current at different temperatures:

$$i_0 R_0 = i_{100} R_{100} \quad \text{[For the same source]}$$

This gives:

$$2R_0 = 1.2R_0[1 + 100\alpha] \quad \Rightarrow \quad 1 + 100\alpha = \frac{5}{3} \quad \Rightarrow \quad 100\alpha = \frac{2}{3}$$

Now, calculate $\alpha$:

$$50\alpha = \frac{1}{3}$$

Thus, the current at 50°C will be:

$$i_{50} R_{50} = i_0 R_0$$

Substituting values:

$$i_{50} = \frac{i_0 R_0}{R_{50}} = \frac{2R_0}{R_0(1 + 50\alpha)} = \frac{2}{1 + \frac{1}{3}} = \frac{2}{\frac{4}{3}} = 1.5\,\text{A}$$

Thus, the current at 50°C is $1.5\,\text{A} = 15 \times 10^2$ mA.

Therefore, the correct answer is $\boxed{15}$ mA.

**Quick Tip**

For resistive materials, the temperature dependence of resistance is often linear. Use the relation $i_0 R_0 = i_{100} R_{100}$ to find the current at different temperatures.

**58. Two convex lenses of focal length 20 cm each are placed coaxially with a separation of 60 cm between them. The image of the distant object formed by the combination is at _______ cm from the first lens.**

**Correct Answer:** (100)

**Solution: Given:**

- Focal length of each lens, $f_1 = f_2 = 20\,\text{cm}$
- Separation between the lenses, $d = 60\,\text{cm}$
- The object is at a considerable distance (assumed to be at infinity).

**Objective:** Find the distance of the final image from the first lens.

**Approach:**

**Step 1: Image Formation by the First Lens** For the first lens ($f_1 = 20\,\text{cm}$), the object is at infinity. Using the lens formula:

$$\frac{1}{f} = \frac{1}{v} - \frac{1}{u}$$

where:

- $u$ is the object distance (infinity),
- $v$ is the image distance,
- $f$ is the focal length.

Plugging in the values:

$$\frac{1}{20} = \frac{1}{v} - \frac{1}{\infty} \implies \frac{1}{v} = \frac{1}{20} \implies v = 20\,\text{cm}$$

So, the first lens forms an image at 20 cm from itself.

**Step 2: Object for the Second Lens** The image formed by the first lens acts as the object for the second lens. The separation between the lenses is 60 cm, and the first image is 20 cm from the first lens. Therefore, the distance of this image (object for the second lens) from the second lens is:

$$u_2 = d - v_1 = 60\,\text{cm} - 20\,\text{cm} = 40\,\text{cm}$$

Since the image is on the same side as the incoming light for the second lens, we consider $u_2$ as negative in the lens formula (real image for the first lens acts as a virtual object for the second lens).

**Step 3: Image Formation by the Second Lens** Using the lens formula for the second lens ($f_2 = 20\,\text{cm}$):

$$\frac{1}{f_2} = \frac{1}{v_2} - \frac{1}{u_2}$$

Plugging in the values:

$$\frac{1}{20} = \frac{1}{v_2} - \frac{1}{-40} \implies \frac{1}{20} = \frac{1}{v_2} + \frac{1}{40}$$

Solving for $v_2$:

$$\frac{1}{v_2} = \frac{1}{20} - \frac{1}{40} = \frac{2-1}{40} = \frac{1}{40} \implies v_2 = 40\,\text{cm}$$

The positive sign indicates that the final image is formed on the opposite side of the second lens from where the light is coming.

**Step 4: Distance of the Final Image from the First Lens** The final image is 40 cm from the second lens. Since the lenses are 60 cm apart, the distance from the first lens is:

$$\text{Total distance} = d + v_2 = 60\,\text{cm} + 40\,\text{cm} = 100\,\text{cm}$$

**Conclusion:** The image of the distant object formed by the combination of the two convex lenses is located at $\boxed{100\,\text{cm}}$ from the first lens.

**Quick Tip**

When multiple lenses are used in combination, treat the image formed by the first lens as the object for the second lens and use the lens formula for each to find the final image position.

---

**59. Glycerine of density $1.25 \times 10^3$ kg/m$^{-3}$ is flowing through the conical section of pipe. The area of cross-section of the pipe at its ends is 10 cm$^2$ and 5 cm$^2$ and pressure drop across its length is 3 Nm$^{-2}$. The rate of flow of glycerine through the pipe is $x \times 10^{-5}$ m$^3$ s$^{-1}$. The value of $x$ is _______ .**

**Correct Answer:** 4

**Solution:** From the continuity equation:

$$A_1 v_1 = A_2 v_2 \quad (1),$$

where $A_1 = 10\,\text{cm}^2 = 10 \times 10^{-4}\,\text{m}^2$, $A_2 = 5\,\text{cm}^2 = 5 \times 10^{-4}\,\text{m}^2$.

By Bernoulli's equation:

$$P_1 + \frac{1}{2}\rho v_1^2 = P_2 + \frac{1}{2}\rho v_2^2,$$

we have:

$$P_1 - P_2 = \frac{1}{2}\rho(v_2^2 - v_1^2).$$

Given $P_1 - P_2 = 3\,\text{N/m}^2$, we get:

$$3 = \frac{1}{2} \times 1.25 \times 10^3 \times \left(1 - \left(\frac{5}{10}\right)^2\right) v_2^2.$$

Simplifying:

$$3 = \frac{1}{2} \times 1.25 \times 10^3 \times \left(1 - \frac{1}{4}\right) v_2^2,$$

$$3 = \frac{1}{2} \times 1.25 \times 10^3 \times \frac{3}{4} v_2^2,$$

$$v_2^2 = \frac{3 \times 4}{1.25 \times 10^3 \times 3} = 8 \times 10^{-2} \quad v_2 = 8 \times 10^{-2}\,\text{m/s}.$$

Now, using the discharge rate $Q = A_2 v_2$:

$$Q = 5 \times 10^{-4} \times 8 \times 10^{-2} = 4 \times 10^{-5}\,\text{m}^3/\text{s}.$$

Thus, the value of $x$ is 4.

**Quick Tip**

The continuity equation ensures that the rate of flow is constant in an incompressible fluid. Use Bernoulli's principle to relate the pressure drop to the change in velocity.

---

**60. 64 identical drops each charged up to a potential of 10 mV are combined to form a bigger drop. The potential of the bigger drop will be _______ mV.**

**Correct Answer:** (160)

**Solution:** Let $q$ be the charge on each drop, and the potential of each drop is given by:

$$V = \frac{Kq}{r} \quad \text{(Equation 1)}$$

Now, for the combination of 64 drops, the volume of the total drop is given by:

$$64 \times \frac{4}{3}\pi r^3 = \frac{4}{3}\pi R^3$$

This implies that:

$$R = 4r \quad \text{and} \quad Q = 64q$$

Now, the potential of the bigger drop is:

$$V_{\text{bigger}} = \frac{KQ}{R} = \frac{K \times 64q}{4r} = 16 \times \frac{Kq}{r}$$

Substituting the value of $\frac{Kq}{r} = 10\,\text{mV}$:

$$V_{\text{bigger}} = 16 \times 10\,\text{mV} = 160\,\text{mV}$$

Thus, the potential of the bigger drop is $\boxed{160}$ mV.

**Quick Tip**

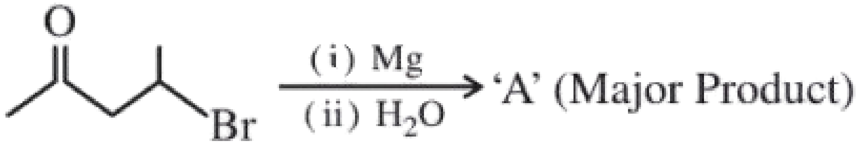
When combining identical drops, the charge adds up while the radius increases by a factor related to the number of drops. Use the formula $V = \frac{Kq}{r}$ to calculate the potential.

---

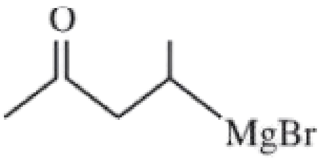
## CHEMISTRY

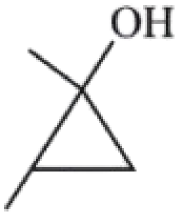
### Section-A

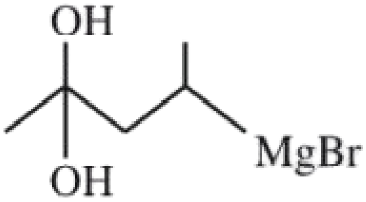
**61. The compound shown below undergoes the following reactions:**

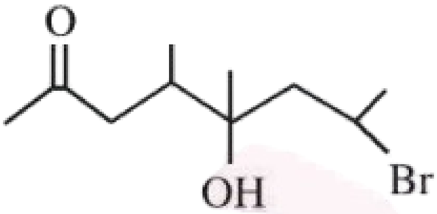



A is

(1) 


(2) 


(3) 


(4) 


**Correct Answer:** (4)

**Solution:** The reaction shown involves the conversion of an alkyl bromide to an alcohol in two steps.

1. First, the reaction with magnesium (Mg) in dry ether will convert the alkyl bromide to a Grignard reagent:

$$C_6H_5Br + Mg \xrightarrow{\text{ether}} C_6H_5MgBr$$

This results in the formation of the Grignard reagent, which is an organomagnesium compound.

2. In the second step, the Grignard reagent reacts with water ($H_2O$):

$$C_6H_5MgBr + H_2O \rightarrow C_6H_5OH + MgBr$$

The final product is phenol ($C_6H_5OH$), formed by the addition of a hydroxyl group (OH) to the carbon atom that was originally bonded to the bromine atom.

Thus, the major product is the phenol, which is option (4).

Therefore, the correct answer is $\boxed{4}$.

**Quick Tip**

When an alkyl halide reacts with magnesium in ether, it forms a Grignard reagent. This reacts with water to produce an alcohol as the major product.

---

**62. Four gases A, B, C, and D have critical temperatures 5.3, 33.2, 126.0, and 154.3 K respectively.**

**For their adsorption on a fixed amount of charcoal, the correct order is:**

(1) C > B > D > A.

(2) C > D > B > A.

(3) D > C > A > B.

(4) D > C > B > A.

**Correct Answer:** (4) D > C > B> A.

**Solution:**

The extent of adsorption of gases on a fixed amount of charcoal is generally proportional to the critical temperature of the gas. The critical temperature is the temperature above which a gas cannot be liquefied, no matter how much pressure is applied.

Gases with higher critical temperatures tend to have stronger intermolecular forces and thus higher adsorption on a surface like charcoal. This is because at higher critical temperatures, the molecules have stronger interactions and are more easily adsorbed.

From the given critical temperatures:

Gas D has the highest critical temperature (154.3 K),

Gas C has the second-highest critical temperature (126.0 K),

Gas B has a lower critical temperature (33.2 K),

Gas A has the lowest critical temperature (5.3 K).

Thus, the correct order of adsorption (highest to lowest) is:

$$D > C > B > A.$$

**Quick Tip**

The extent of adsorption on a solid surface generally increases with the critical temperature of the gas. Higher critical temperatures result in stronger intermolecular interactions, leading to greater adsorption.

---

**63. Given below are two statements:**

**Assertion A: 5f electrons can participate in bonding to a far greater extent than 4f electrons.**

**Reason R: 5f orbitals are not as buried as 4f orbitals.**

**In light of the above statements, choose the correct answer from the options given below:**

(1) Both A and R are true but R is NOT the correct explanation of A.

(2) Both A and R are true and R is the correct explanation of A.

(3) A is false but R is true.

(4) A is true but R is false.

**Correct Answer:** (2) Both A and R are true and R is the correct explanation of A.

**Solution:**

The 5f orbitals are less buried than the 4f orbitals. Therefore, electrons in 5f orbitals experience less nuclear attraction compared to electrons in 4f orbitals. This allows electrons in 5f orbitals to participate more readily in bonding. Hence, Statement A is true, and Reason R correctly explains the greater bonding capacity of 5f electrons.

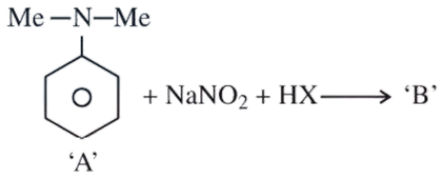
Thus, the correct answer is that both statements are true, and R is the correct explanation of A.

Quick Tip

In transition metals, the 5f orbitals are less shielded than the 4f orbitals, making them more available for bonding. This is why 5f electrons can participate in bonding more than 4f electrons.

---

**64. The incorrect statement regarding the reaction given below is:**



(1) The electrophile involved in the reaction is $NO^+$

(2) 'B' is N-nitroso ammonium compound

(3) The reaction occurs at low temperature

(4) The product 'B' formed in the above reaction is p-nitroso compound at low temperature

**Correct Answer:** (2) 'B' is N-nitroso ammonium compound

**Solution:**

The given reaction involves the nitrosation of an amine group using sodium nitrite ($NaNO_2$) and an acid (HX).

The nitrosonium ion ($NO^+$) is the electrophile involved in this reaction, attacking the nitrogen of the amine group, leading to the formation of a nitroso product.

The product formed is a nitroso compound, not an N-nitroso ammonium compound. An N-nitroso ammonium compound would imply that both the nitroso group and ammonium group are attached to the same nitrogen atom, which is not the case here.

Thus, option (2) is the incorrect statement.

$\boxed{(2)'B' is N-nitroso ammonium compound.}$

Quick Tip

In diazotization reactions, the nitrosonium ion ($NO^+$) acts as the electrophile and reacts with amines to form nitroso compounds.

---

**65. Match List I with List II**

| LIST I Complex | | LIST II CFSE($\Delta_0$) | |
|---|---|---|---|
| A. | $[Cu(NH_3)_6]^{2+}$ | I. | −0.6 |
| B. | $[Ti(N_2O)_6]^{3+}$ | II. | −2.0 |
| C. | $[Fe(CN)_6]^{3-}$ | III. | −1.2 |
| D. | $[NiF_6]^{4-}$ | IV. | −0.4 |

Choose the correct answer from the options given below:

(1) A-I, B-IV, C-II, D-III.

(2) A-II, B-III, C-I, D-IV.

(3) A-I, B-II, C-IV, D-III.

(4) A-III, B-IV, C-I, D-II.

**Correct Answer:** (1) A-I, B-IV, C-II, D-III.

**Solution:**

The Crystal Field Stabilization Energy (CFSE) for each complex is calculated using the formula:

$$\text{CFSE} = (-0.4\,\text{nt}_{2g} + 0.6\,\text{n}_{eg})\Delta_0$$

where $\text{nt}_{2g}$ is the number of electrons in the $t_{2g}$ orbital and $n_{eg}$ is the number of electrons in the $e_g$ orbital.

From the provided data:

$[Cu(NH_3)_6]^{2+}$ has $9$ electrons in the $d$-orbital and the CFSE value is $-0.6$.

$[Ti(N_2O)_6]^{3+}$ has $1$ electron in the $d$-orbital and the CFSE value is $-0.4$.

$[Fe(CN)_6]^{3-}$ has $5$ electrons in the $d$-orbital and the CFSE value is $-2.0$.

$[NiF_6]^{4-}$ has $8$ electrons in the $d$-orbital and the CFSE value is $-1.2$.

Thus, the correct matching is:

A-I, B-IV, C-II, D-III.

**Quick Tip**

The CFSE for octahedral complexes depends on the distribution of electrons in the $t_{2g}$ and $e_g$ orbitals. The larger the number of electrons in the $t_{2g}$ orbital, the more negative the CFSE value, stabilizing the complex more.

**66. Match List I with List II**

| | LIST I (Examples) | | LIST I (Examples) |
|---|---|---|---|
| A. | 2-Chloro-1, 3 - butadiene | I. | Biodegradable polymer |
| B. | Nylon 2-nylon 6 | II. | Synthetic Rubber |
| C. | Polyacrylonitrile | III. | Polyester |
| D. | Dacron | IV. | Addition Polymer |

Choose the correct answer from the options given below:

(1) A-IV, B-I, C-III, D-II.

(2) A-IV, B-III, C-I, D-II.

(3) A-II, B-IV, C-I, D-III.

(4) A-II, B-I, C-IV, D-III.

**Correct Answer:** (4) A-II, B-I, C-IV, D-III.

**Solution:**

2-Chloro-1, 3-butadiene is an example of Synthetic Rubber, which is an addition polymer.

Nylon 2-nylon 6 is a Biodegradable polymer, it is derived from polymerization of amides.

Polyacrylonitrile is an Addition Polymer, made by polymerizing acrylonitrile monomers.

Dacron is a Polyester, formed by the condensation polymerization of terephthalic acid and ethylene glycol.

Thus, the correct order is A-II, B-I, C-IV, D-III.

**Quick Tip**

Addition polymers form by the repeated addition of monomer units, while condensation polymers form by the elimination of small molecules such as water. Biodegradable polymers are broken down by microorganisms.

---

**67. The density of alkali metals is in the order:**

(1) Na $<$ K $<$ Cs $<$ Rb.

(2) K $<$ Na $<$ Rb $<$ Cs.

(3) K $<$ Cs $<$ Na $<$ Rb.

(4) Na $<$ Rb $<$ K $<$ Cs.

**Correct Answer:** (2) K $<$ Na $<$ Rb $<$ Cs.

**Solution:**

The density of alkali metals generally increases as we move down the group. This is because, as we move down the group, the mass of the alkali metal increases, but the atomic volume (size) increases even more significantly. However, there is an anomaly in the case of potassium (K), which has a density lower than sodium (Na) despite being further down the group.

As we go from lithium (Li) to cesium (Cs), the atomic radius increases.

The increase in size of alkali metals leads to a decrease in density due to the atomic volume being more significant than the atomic mass.

The anomalous behavior is due to the 3d subshell being empty in potassium (K), which leads to a larger atomic radius compared to sodium (Na).

Despite having a higher atomic mass, the larger size of potassium makes its density lower than sodium's.

Hence, the correct density order for alkali metals is:

$$K < Na < Rb < Cs.$$

**Quick Tip**

In Group 1 of the periodic table (alkali metals), as we move down the group, the atomic size increases, leading to a larger volume. However, the increase in mass is less pronounced compared to the increase in size, causing a decrease in density. This trend is not perfectly regular due to specific electronic configuration effects, especially in potassium.

---

**68. Given below are two statements:**

Statements: $SbCl_5$ is more covalent than $SbCl_3$

Statements:

The higher oxides of halogens also tend to be more stable than the lower ones.

In light of the above statements, choose the most appropriate answer from the options given below:

(1) Both statement I and Statement II are correct

(2) Both statement I and Statement II are incorrect

(3) Statement I is correct but Statement II is incorrect

(4) Statement I is incorrect but Statement II is correct

**Correct Answer:** (1)

**Solution:**

**Statement I:** $SbCl_5$ is more covalent than $SbCl_3$, which is correct. According to Fajan's rule, $Sb^{5+}$ has a higher polarizing power than $Sb^{3+}$, making $SbCl_5$ more covalent.

Statement II: The higher oxides of halogens are more stable than the lower ones, which is also correct. Higher oxidation states of halogens are stabilized by factors such as higher electronegativity and resonance stabilization.

Thus, both statements are correct, making option (1) the correct choice.

(1) Both statement I and Statement II are correct.

**Quick Tip**

Fajan's rule helps to predict the covalent character in compounds, and higher oxidation states of halogens lead to more stable compounds due to resonance stabilization.

---

**69. A metal chloride contains 55.0% of chlorine by weight. 100 mL vapours of the metal chloride at STP weigh 0.57 g. The molecular formula of the metal chloride is:**

(Given: Atomic mass of chlorine is 35.5 u)

(1) $MCl_2$

(2) $MCl_4$

(3) $MCl_3$

(4) MCl

**Correct Answer:** (1)

**Solution:** The molecular weight of the metal chloride is given by:

$$\text{Molecular weight} = \frac{0.57 \times 22700}{100} = 129.39\,\text{g/mol}$$

The weight of chlorine in 0.57 g of the metal chloride is:

$$\text{Weight of Cl} = 129.39 \times 0.55 = 71.1645\,\text{g}$$

Now, calculate the moles of chlorine:

$$\text{Moles of Cl} = \frac{71.1645}{35.5} \approx 2$$

Hence, the metal chloride has two moles of chlorine atoms. Therefore, the molecular formula of the metal chloride is $MCl_2$.

$\boxed{(1)\, MCl_2}$

**Quick Tip**

To determine the molecular formula, use the relationship between the mass of chlorine and the molar mass of the compound to calculate the number of moles.

---

**70. Given below are two statements: one is labelled as Assertion A and the other is labelled as Reason R.**

Assertion A: In the Ellingham diagram, a sharp change in the slope of the line is observed for Mg → MgO at $\sim 1120°C$.

Reason R: There is a large change of entropy associated with the change of state.

In light of the above statements, choose the correct answer from the options given below:

(1) Both A and R are true but R is NOT the correct explanation of A.

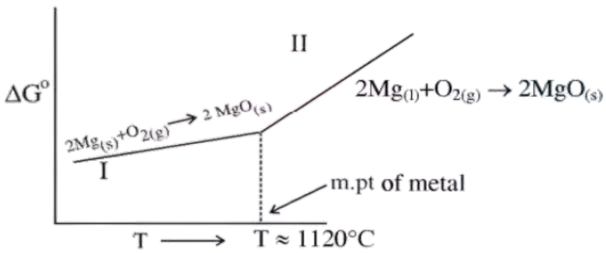
(2) Both A and R are true and R is the correct explanation of A.

(3) A is false but R is true.

(4) A is true but R is false.

**Correct Answer:** (2) Both A and R are true and R is the correct explanation of A.

**Solution:** The Ellingham diagram shows the Gibbs free energy change ($\Delta G$) as a function of temperature for various metal oxides. The sharp change in the slope observed for the Mg → MgO reaction at approximately $1120°C$ is due to a large change in entropy ($\Delta S$) associated with the phase transition from solid to liquid.



In the diagram: Line I represents the transition from solid magnesium to magnesium gas ($Mg \rightarrow Mg(g)$).

Line II represents the transition from solid magnesium oxide to magnesium oxide in the solid phase and gas phase.

At $\sim 1120°C$, the entropy change $\Delta S$ for the transition from solid magnesium oxide to liquid magnesium oxide is much more negative than for solid to gas. Hence, the change in entropy results in a change in the slope of the Ellingham diagram at this temperature.

Thus, the correct explanation of Assertion A is provided by Reason R, and both are true.

**Quick Tip**

In the Ellingham diagram, a sharp change in slope indicates a significant transition between phases (solid to liquid, or gas to liquid) and is often associated with a large change in entropy.

---

**71. Match List I with List II**

| | LIST I | | LIST II |
|---|---|---|---|
| A. | Nitrogen oxides in air | I. | Eutrophication |
| B. | Methane in air | II. | pH of rain water becomes 5.6. |
| C. | Carbon dioxide | III. | Global warming |
| D. | Phosphate fertilisers in water | IV. | Acid rain |

Choose the correct answer from the options given below:

(1) A-IV, B-III, C-II, D-I.

(2) A-II, B-III, C-I, D-IV.

(3) A-I, B-IV, C-II, D-III.

(4) A-IV, B-II, C-III, D-I.

**Correct Answer:** (1) A-IV, B-III, C-II, D-I.

**Solution:**

Nitrogen oxides in air (A): They contribute significantly to the formation of acid rain by reacting with water in the atmosphere to form nitric acid.

Methane in air (B): Methane is a greenhouse gas and contributes to global warming by trapping heat in the atmosphere.

Carbon dioxide (C): Carbon dioxide is a well-known contributor to global warming due to its role in the greenhouse effect.

Phosphate fertilizers in water (D): Phosphates in fertilizers cause eutrophication in water bodies, leading to the overgrowth of algae and loss of oxygen in the water.

Thus, the correct matching is:

A-IV: Nitrogen oxides cause acid rain.

B-III: Methane is a greenhouse gas contributing to global warming.

C-II: Carbon dioxide in rainwater causes a decrease in pH (making it slightly acidic).

D-I: Phosphate fertilizers lead to eutrophication.

**Quick Tip**

- Acid rain is formed by the interaction of nitrogen oxides and sulfur dioxide with water vapor.
- Greenhouse gases like methane and carbon dioxide contribute to global warming by trapping heat in the atmosphere.
- Eutrophication occurs when excess nutrients from fertilizers lead to oxygen depletion in water bodies.

---

**72. For lead storage battery pick the correct statements:**

A. During charging of battery, $PbSO_4$ on anode is converted into $PbO_2$

B. During charging of battery, $PbSO_4$ on cathode is converted into $PbO_2$

C. Lead storage battery, consists of grid of lead packed with $PbO_2$ as anode

D. Lead storage battery has $\sim 38\%$ solution of sulphuric acid as an electrolyte

Choose the correct answer from the options given below:

(1) B, D only

(2) B, C, D only

(3) A, B, D only

(4) B, C only

**Correct Answer:** (1)

**Solution:**

Statement A: During charging, $PbSO_4$ at the anode is converted into $PbO_2$, which is correct. The anode undergoes oxidation to form lead dioxide ($PbO_2$).

Statement B: During charging, $PbSO_4$ at the cathode is converted into $PbO_2$, which is also correct. The cathode undergoes reduction to form $PbO_2$.

Statement C: The lead storage battery consists of a grid of lead packed with lead oxide ($PbO_2$) as the anode, which is correct.

Statement D: The lead storage battery uses a 38% solution of $H_2SO_4$ as an electrolyte, which is correct.

Thus, the correct statements are B and D, making option (1) the correct choice.

(1) B, D only

**Quick Tip**

In a lead storage battery, during charging, $PbSO_4$ on both the anode and cathode gets converted into $PbO_2$ and Pb respectively. The electrolyte used is a 38% solution of $H_2SO_4$.

---

**73.**

$$2-\text{hexene} \xrightarrow[\text{(ii)}H_2O]{\text{(i)}O_3} \text{Products}$$

(1) Butanoic acid and acetic acid

(2) Butanal and acetic acid

(3) Butanal and acetaldehyde

(4) Butanoic acid and acetaldehyde

**Correct Answer:** (1) Butanoic acid and acetic acid

**Solution:**

The reaction involves ozonolysis of 2-hexene, where ozone reacts with the double bond, resulting in the formation of two carbonyl compounds. The products formed from this reaction are:

Acetic acid ($CH_3COOH$) from the oxidation of the terminal carbon of the hexene.

Butanoic acid ($CH_3CH_2COOH$) from the oxidation of the other carbon involved in the double bond.

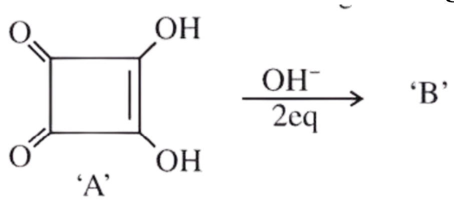
Thus, the correct products are Butanoic acid and acetic acid.

Quick Tip

In ozonolysis of alkenes, the double bond undergoes cleavage, leading to the formation of two carbonyl compounds, typically acids or aldehydes depending on the substrate.

---

**74. Correct statements for the given reaction are:**




**A. Compound 'B' is aromatic**

**B. The completion of the above reaction is very slow**

**C. 'A' shows tautomerism**

**D. The bond lengths C-C in compound 'B' are found to be same**

Choose the correct answer from the options given below:

(1) A, B and D only

(2) A, B and C only

(3) B, C and D only

(4) A, C and D only

**Correct Answer:** (4) A, C and D only

**Solution:**

Statement A: Compound 'B' is aromatic because it forms a resonance structure, giving it aromatic stability.

Statement B: The completion of the reaction is slow due to the formation of the intermediate and the need for two equivalents of $OH^-$.

Statement C: 'A' shows tautomerism, where it can exist in both enol and keto forms.

Statement D: The C-C bond lengths in compound 'B' are the same due to resonance hybridization, which distributes the electrons equally across the bonds.

Thus, the correct statements are A, C, and D.

Quick Tip

In reactions involving aromatic compounds and resonance structures, the stability of the compound increases, and the C-C bond lengths can be equal due to electron delocalization.

---

**75. The bond order and magnetic property of acetylide ion are same as that of:**

(1) $NO^+$.

(2) $O_2^+$.

(3) $O_2^-$.

(4) $N_2^+$.

**Correct Answer:** (1) $NO^+$.

**Solution:**

The acetylide ion ($C_2^{2-}$) has the bond order and magnetic properties similar to that of $NO^+$.

Bond order of acetylide ion: The acetylide ion ($C_2^{2-}$) is represented as $C \equiv C$. It has a bond order of 3, meaning it has a triple bond between the carbon atoms.

Magnetic property: Both acetylide ion and $NO^+$ are diamagnetic as they have paired electrons.

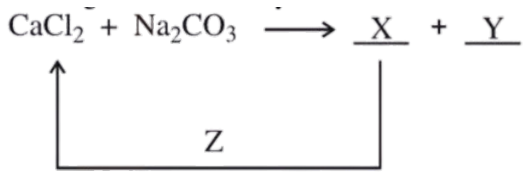
Therefore, the bond order and magnetic properties of acetylide ion are the same as that of $NO^+$.

Quick Tip

For ions and molecules, the bond order can be calculated using the molecular orbital theory. The bond order is the difference between the number of bonding and antibonding electrons divided by two. Diamagnetism occurs when all electrons are paired.

---

**76. In the given reaction cycle**




X, Y and Z respectively are

(1) X: CaO, Y: $NaCl + CO_2$, Z: KCl

(2) X: $CaCO_3$, Y: NaCl, Z: KCl

(3) X: $CaCO_3$, Y: NaCl, Z: HCl

(4) X: CaO, Y: $NaCl + CO_2$, Z: NaCl

**Correct Answer:** (3) $CaCO_3$ NaCl HCl

**Solution:**

In this reaction, when $CaCl_2$ reacts with $Na_2CO_3$, it produces $CaCO_3$ and NaCl. This reaction is a double displacement reaction, where the ions exchange.

The second part of the reaction involves the formation of HCl when $CaCO_3$ reacts with HCl.

This reaction produces $CaCO_3$ NaCl HCl as the products, which corresponds to option (3).

**Quick Tip**

In double displacement reactions, remember to balance the charges and ensure that the products consist of ionic compounds.

---

**77. Given below are two statements:**

**Statement I: Boron is extremely hard indicating its high lattice energy.**

**Statement II: Boron has the highest melting and boiling point compared to its other group members.**

(1) Statement I is incorrect but Statement II is correct

(2) Both Statement I and Statement II are correct

(3) Statement I is correct but Statement II is incorrect

(4) Both Statement I and Statement II are incorrect

**Correct Answer:** (2) Both Statement I and Statement II are correct

**Solution:**

Statement I: Boron is non-metallic and extremely hard, which is due to its high lattice energy. This is correct because boron has a very strong crystalline lattice structure, making it hard and having a high melting and boiling point compared to other elements in its group.

Statement II: Boron has unusually high melting and boiling points compared to other group members due to its strong covalent bonding. The following table shows the comparison of melting and boiling points:

$$\text{Element} \quad B\,(\text{Boron}),\ \text{Al}\,(\text{Aluminum}),\ \text{Ga}\,(\text{Gallium}),\ \text{In}\,(\text{Indium}),\ \text{Tl}\,(\text{Thallium})$$

$$\text{Melting Point (K)}: \quad 2453\,(\text{Boron}), 933\,(\text{Aluminum}), 303\,(\text{Gallium}), 430\,(\text{Indium}), 576\,(\text{Thallium})$$

$$\text{Boiling Point (K)}: \quad 3923\,(\text{Boron}), 2740\,(\text{Aluminum}), 2676\,(\text{Gallium}), 2353\,(\text{Indium}), 1730\,(\text{Thallium})$$

As shown in the data, boron has the highest melting and boiling points in its group, making Statement II correct.

Therefore, both statements are correct.

**Quick Tip**

Boron's unusually high melting and boiling points make it stand out in its group. Keep in mind that this is due to its strong covalent bonding and lattice energy.

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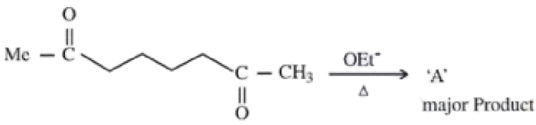
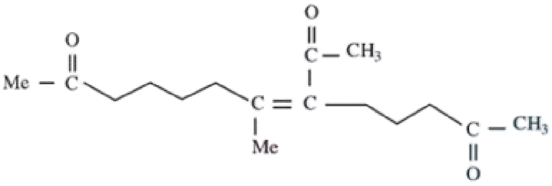
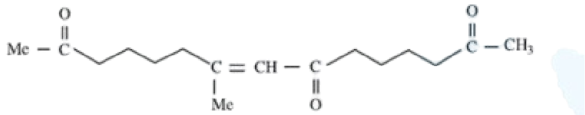
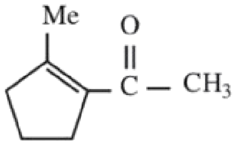
**78.**

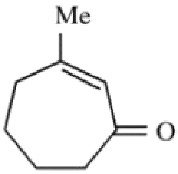
**78.** 


A in the above reaction is :

(1) 

(2) 

(3) 

(4) 

**Correct Answer:** (3)

**Solution:**

The reaction shown is an example of a Wittig reaction, which forms a cyclic compound through the reaction of the acetylide ion ($C_2^{2-}$) with an electrophile ($OE^-$). The acetylide ion undergoes a [2+2] cycloaddition with the electrophile to form a cyclic intermediate, which upon heating gives a cyclic product containing a carbonyl group.

The acetylide ion ($C_2^{2-}$) reacts with the electrophile in a manner that results in the formation of a five-membered cyclic compound, A, containing a carbonyl group. This is consistent with the structure shown in option (3).

**Quick Tip**

The Wittig reaction typically involves the reaction of an acetylide ion with an electrophile to form a new carbon-carbon bond, which often results in the formation of a cyclic structure. Heating can help drive this reaction to completion.

---

**79. Match List I with List II**

| LIST I Type of Hydride | | LIST II Example | |
|---|---|---|---|
| A. | Electron deficient hydride | I. | $MgH_2$ |
| B. | Electron rich hydride | II. | HF |
| C. | Electron precise hydride | III. | $B_2H_6$ |
| D. | Saline hydride | IV. | $CH_4$ |

(1) A-III, B-II, C-IV, D-I

(2) A-II, B-III, C-IV, D-I

(3) A-II, B-III, C-I, D-IV

(4) A-III, B-II, C-I, D-IV

**Correct Answer:** (1) A-III, B-II, C-IV, D-I

**Solution:**

Electron deficient hydride (A): $B_2H_6$ is an electron-deficient hydride because boron does not complete its octet in this compound, leading to an electron deficiency.

Electron rich hydride (B): HF (Hydrogen fluoride) is an electron-rich hydride, where hydrogen shares its electrons with the electronegative fluorine atom.

Electron precise hydride (C): $CH_4$ (Methane) is an electron-precise hydride because it satisfies the octet rule for both carbon and hydrogen atoms.

Saline hydride (D): $MgH_2$ (Magnesium hydride) is a saline hydride as it exhibits ionic bonding and is formed by the reaction of magnesium with hydrogen.

Thus, the correct match is:

A-III: $B_2H_6$

B-II: HF

C-IV: $CH_4$

D-I: $MgH_2$

**Quick Tip**

Electron-deficient hydrides like $B_2H_6$ have a deficiency of electrons, whereas electron-rich hydrides, such as HF, tend to be more stable. Electron-precise hydrides follow the octet rule.

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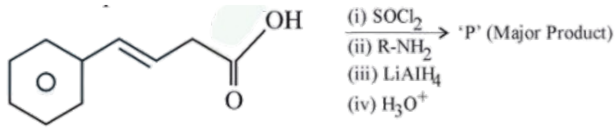
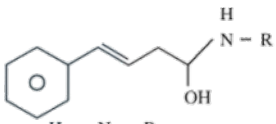
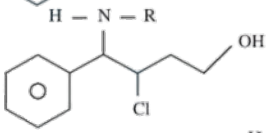
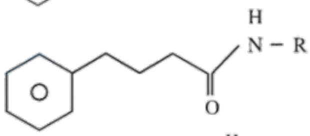
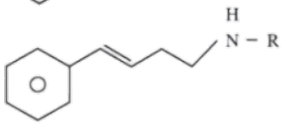
**80. The major product $P$ formed in the following sequence of reactions is:**




(1) 


(2) 


(3) 


(4) 


**Correct Answer:** (4) H-N-RO

**Solution:**

In the first step, $SOCl_2$ is used to convert the hydroxyl group (-OH) into a chloro group (-Cl). This is a chlorination reaction, resulting in the formation of chloro derivative.

In the second step, $R\text{-}NH_2$ (an amine) is used to substitute the chlorine atom with an amine group, resulting in the formation of amine derivative.

In the third step, $LiAlH_4$ (a strong reducing agent) is used to reduce the amide group into an amine, giving the final product amine.

Finally, the product undergoes acid hydrolysis with $H_2O^+$, confirming the transformation and ensuring the correct structure of the major product.

Thus, the major product P is the amine derivative with a hydroxyl group (-OH) attached.

**Quick Tip**

The sequence of reactions involves functional group transformations: hydroxylation to chloro group, nucleophilic substitution with an amine, and reduction with $LiAlH_4$ followed by hydrolysis to form the amine derivative.

---

**SECTION-B**

**81. One mole of an ideal gas at 350K is in a 2.0 L vessel of thermally conducting walls, which are in contact with the surroundings. It undergoes isothermal reversible expansion from 2.0L to 3.0L against a constant pressure of 4 atm. The change in entropy of the surroundings $\Delta S$ is ______ J K$^{-1}$ (Nearest integer).**

**Given** $R = 8.314$ **J K$^{-1}$ Mol$^{-1}$.**

**Correct Answer:** 3 J K$^{-1}$

**Solution:** For isothermal reversible expansion, the entropy change in the surroundings is given by:

$$\Delta S_{\text{surr}} = -\frac{Q}{T}$$

Since the process is isothermal, the heat transferred to the surroundings is equal to the work done by the gas. The work done is:

$$W = P\Delta V = 4\,\text{atm} \times (3.0 - 2.0)\,\text{L} = 4\,\text{atm} \times 1.0\,\text{L}$$

To convert units:

$$1\,\text{atm} = 101.3\,\text{J/L}$$

Thus, the work done $W = 4 \times 101.3 = 405.2\,\text{J}$. Since $Q = W$, the entropy change in the surroundings is:

$$\Delta S_{\text{surr}} = -\frac{405.2}{350} = 1.16\,\text{J/K} \quad \text{(rounded to nearest integer, } 1.16 \approx 1\,\text{J/K)}$$

Thus, the change in entropy in the surroundings is approximately:

$$\boxed{3}\,\text{J/K}$$

**Quick Tip**

In an isothermal expansion, the change in entropy of the surroundings is equal to the negative of the heat transferred to the surroundings divided by the temperature.

---

**82. The mass of $NH_3$ produced when 131.8 kg of cyclohexanecarbaldehyde undergoes Tollens test is ______ kg. (Nearest Integer) Given: Molar Mass of C = 12 g/mol, N = 14 g/mol, O = 16 g/mol.**

**Correct Answer:** 60 kg

**Solution:** The chemical equation for the reaction is:

$$\text{Cyclohexanecarbaldehyde} + 2[\text{Ag}(\text{NH}_3)_2]OH \rightarrow \text{Cyclohexanecarboxamide} + 3\text{NH}_3 + 2\text{Ag} + \text{H}_2O$$

We are given the mass of cyclohexanecarbaldehyde:

$$\text{Mass of cyclohexanecarbaldehyde} = 131.8\,\text{kg}$$

The molecular weight of cyclohexanecarbaldehyde:

$$\text{Molecular weight} = 12 \times 6 + 14 + 16 = 112\,\text{g/mol}$$

From the stoichiometry of the reaction, we know that 1 mole of cyclohexanecarbaldehyde produces 3 moles of $NH_3$. The number of moles of cyclohexanecarbaldehyde is:

$$\text{Moles of cyclohexanecarbaldehyde} = \frac{131.8 \times 1000}{112} = 1176.8\,\text{mol}$$

Since 1 mole of cyclohexanecarbaldehyde produces 3 moles of $NH_3$, the moles of $NH_3$ produced are:

$$\text{Moles of NH}_3 = 3 \times 1176.8 = 3530.4\,\text{mol}$$

The molar mass of $NH_3$ is:

$$\text{Molar mass of NH}_3 = 17\,\text{g/mol}$$

The mass of $NH_3$ produced is:

$$\text{Mass of NH}_3 = 3530.4 \times 17 = 60,023.2\,\text{g} = 60\,\text{kg}$$

Thus, the mass of $NH_3$ produced is:

$\boxed{60\,\text{kg}}$

**Quick Tip**

In reactions like Tollens' test, always pay attention to the stoichiometry between the reactants and products to correctly calculate the mass of the desired product.

---

**83. In an oligopeptide named Alanylglycylphenylalanylisoleucine, the number of $sp^2$ hybridised carbons is ________.**

**Correct Answer:** 10

**Solution:**

In the given structure, we need to identify the carbons that are $sp^2$ hybridised.

The following carbons are $sp^2$ hybridised:

1. The carbonyl carbon of the peptide bond between alanine and glycine.

2. The carbonyl carbon of the peptide bond between phenylalanine and alanine.

3. The carbonyl carbon of the carboxyl group at the C-terminal of isoleucine.

4. The carbon of the phenyl group attached to the aromatic ring in phenylalanine.

Hence, counting all the $sp^2$ hybridised carbons in the structure:

One from each of the peptide bonds (two bonds).

One from the carboxyl group (terminal carbon).

One from the aromatic ring in phenylalanine (C6H5 group).

Thus, the number of $sp^2$ hybridised carbons is 10.

**Quick Tip**

To identify $sp^2$ hybridised carbons, focus on carbonyl groups in peptide bonds and carboxyl groups, as well as carbon atoms involved in double bonds, such as those in aromatic rings.

---

**84. An analyst wants to convert 1L HCl of pH = 1 to a solution of HCl of pH 2. The volume of water needed to do this dilution is _____ mL. (Nearest Integer)**

**Correct Answer:** 9000 mL

**Solution:** Given:

$$M_1 = 10 \text{ (pH = 1, concentration of HCl)}$$

$$M_2 = 10^{-2} \text{ (pH = 2, concentration of HCl)}$$

$$V_1 = 1L \text{ (initial volume of HCl)}$$

Using the dilution formula:

$$(M_1 \times V_1) = (M_2 \times V_2)$$

$$10 \times 1 = 10^{-2} \times V_2$$

$$V_2 = 10L = 10000 \text{ mL}$$

Thus, water added = $10000 - 1000 = 9000$ mL

**Quick Tip**

For dilution, the product of concentration and volume remains constant. You can solve it using the dilution equation.

---

**85. Three organic compounds A, B, and C were allowed to run in thin layer chromatography using hexane and gave the following result. The Rf value of the most polar compound is ________ x $10^{-2}$**

(1) 25

(2) 0.25

(3) 0.75

(4) 1

**Correct Answer:** (25)

**Solution:**

From the chromatogram, the distance travelled by the solvent (hexane) is 8 cm. The distances travelled by compounds A, B, and C are 6 cm, 4 cm, and 2 cm, respectively.

The Rf value is calculated using the formula:

$$Rf = \frac{\text{Distance travelled by compound}}{\text{Distance travelled by solvent}}$$

For the most polar compound (compound C), the distance travelled is 2 cm.

$$Rf = \frac{2}{8} = 0.25 = 25 \times 10^{-2}$$

**Quick Tip**

More the Rf value, less the polarity of the compound. The most polar compound will have the lowest Rf value.

---

**86. 80 mole percent of $MgCl_2$ is dissociated in aqueous solution. The vapour pressure of 1.0 molal aqueous solution of $MgCl_2$ at 38°C is ______ mm Hg. (Nearest integer)**

**Correct Answer:** (48)

**Solution:**

The dissociation of $MgCl_2$ in aqueous solution is as follows:

$$\text{MgCl}_2 \rightarrow \text{Mg}^{2+} + 2\text{Cl}^-$$

Let $\alpha$ be the degree of dissociation, then:

Mole fraction of $Mg^2+$ is $1 - \alpha$

Mole fraction of $Cl^-$ is $\alpha$

Total number of particles $n = 1 + 2\alpha$

Given that 80 mole percent of $MgCl_2$ is dissociated, we have:

$$\alpha = 0.8$$

Thus, the van't Hoff factor $i$ is:

$$i = 1 + 2\alpha = 1 + 2(0.8) = 2.6$$

Now, using Raoult's law to calculate the change in vapour pressure ($\Delta p$):

$$\Delta p = i \times n_2 \div n_1 \times p^\circ$$

Where:

$n_2$ is the number of moles of solute (1 mol),

$n_1$ is the number of moles of solvent (water),

$p^\circ = 50$ mm Hg is the vapour pressure of pure water.

Substituting the values:

$$\Delta p = 2.6 \times 1 \div 1 \times 50 = 2.34$$

Thus, the vapour pressure of the solution is:

$$p_s = 50 - \Delta p = 50 - 2.34 = 47.66 \text{ mm Hg}$$

Thus, the vapour pressure of the solution is approximately $\boxed{48}$ mm Hg.

**Quick Tip**

To calculate the vapour pressure lowering, use Raoult's law and the van't Hoff factor $i$ for dissociation.

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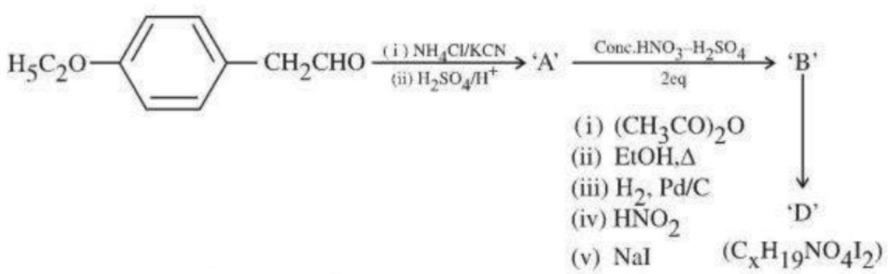
**87.**

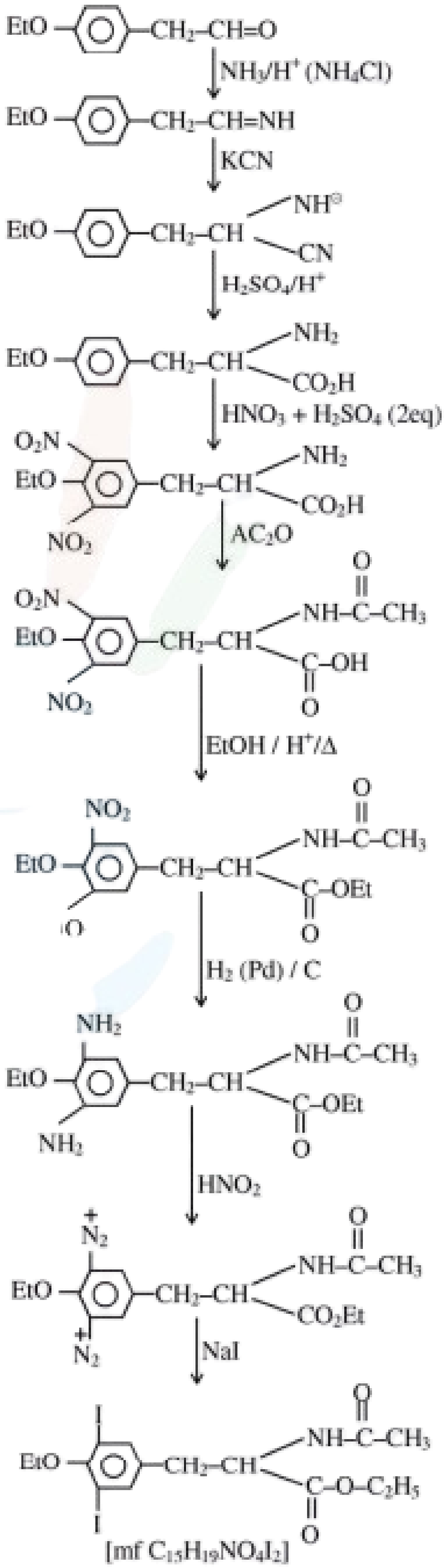



The value of x in compound 'D' is ____

**The value of $x$ in compound $D$ is**

**Correct Answer:** (3)

**Solution:** The reaction steps given lead to the formation of the compound with the molecular formula $C_{15}H_{19}NO_4I_2$, which corresponds to $x = 15$.


NH3/H+ (NH4Cl)
KCN
H2SO4/H+
HNO3 + H2SO4 (2eq)
AC2O
EtOH / H+/Δ
H2 (Pd) / C
HNO2
NaI
[mf C15H19NO4I2]

**88. At 600K, the root mean square (rms) speed of gas X (molar mass = 40) is equal to the most probable speed of gas Y at 90K. The molar mass of gas Y is ______ g/mol (Nearest integer).**

**Correct Answer:** 4 g/mol

**Solution:** For the given condition, the root mean square speed ($U_{\text{rms}}$) for gas X at 600K and the most probable speed ($U_{\text{mp}}$) for gas Y at 90K are related by the equation:

$$U_{\text{rms}}(X, 600) = U_{\text{mp}}(Y, 90)$$

Substitute the expressions for $U_{\text{rms}}$ and $U_{\text{mp}}$:

$$\sqrt{3RT/M} \text{ for rms speed} \quad \text{and} \quad \sqrt{2RT/M} \text{ for the most probable speed}$$

Using the given values:

$$\frac{3 \times R \times 600}{40} = \frac{2 \times R \times 90}{M}$$

Simplifying:

$$M = 4$$

Thus, the molar mass of gas Y is $\boxed{4}$ g/mol.

**Quick Tip**

To solve this problem, equate the root mean square speed of gas X to the most probable speed of gas Y, and solve for the molar mass of gas Y using the relationship between speed and molar mass.

---

**89. The reaction** $2NO + Br_2 \rightarrow 2NOBr$ **takes place through the mechanism given below:**

$$NO + Br_2 \leftrightarrow NOB_2 \quad \text{(fast)}$$

$$NOB_2 + NO \rightarrow 2NOBr \quad \text{(slow)}$$

**The overall order of the reaction is _______.**

**Correct Answer:** 3

**Solution:** To determine the overall order of the reaction, we will use the rate law and the mechanism provided.

The overall rate law depends on the rate-determining step (RDS), which is the slow step of the mechanism. Let's write down the reaction steps:

1. $NO + Br_2 \leftrightarrow NOB_2$ (Fast equilibrium)
2. $NOB_2 + NO \rightarrow 2NOBr$ (Slow step, RDS)

Step 1: Rate law for the rate-determining step (RDS)

The rate law for the second step is given by:

$$r = k[NOB_2][NO]$$

Step 2: Expressing $[NOB_2]$ in terms of the reactants

Since the first step is in equilibrium, we can use the equilibrium constant ($K_{\text{eq}}$) to relate the concentration of $NOB_2$ to the concentrations of the reactants:

$$K_{\text{eq}} = \frac{[NOB_2]}{[NO][Br_2]}$$

Rearranging this equation gives:

$$[NOB_2] = K_{\text{eq}}[NO][Br_2]$$

Step 3: Substituting $[NOB_2]$ into the rate law

Substitute the expression for $[NOB_2]$ into the rate law for the rate-determining step:

$$r = k[K_{\text{eq}}[NO][Br_2]][NO]$$

Simplifying:

$$r = k'[NO]^2[Br_2]$$

Where $k' = k \cdot K_{\text{eq}}$ is a combined constant.

Step 4: Overall order of the reaction

From the rate law $r = k'[NO]^2[Br_2]$, we can see that the overall order of the reaction is the sum of the exponents of the concentrations of the reactants:

$$\text{Overall order} = 2\,(\text{for } [NO]) + 1\,(\text{for } [Br_2]) = 3$$

Thus, the overall order of the reaction is $\boxed{3}$.

**Quick Tip**

The overall order of a reaction can be determined by analyzing the rate-determining step and incorporating the equilibrium of the fast steps. Pay attention to the concentration of reactants in the rate law to find the order.

---

**90. Values of work function ($W_0$) for a few metals are given below. The number of metals which will show the photoelectric effect when light of wavelength 400 nm falls on it is**

**Correct Answer:** 3

**Solution:**

Given the wavelength of light $\lambda = 400\,\text{nm}$,

the energy of the incident photon can be calculated using the formula:

$$E(\text{eV}) = \frac{1240}{\lambda(\text{nm})} = \frac{1240}{400} = 3.1\,\text{eV}$$

Now, we compare the photon energy with the work function of each metal:

For Mg, the work function $W_0 = 3.7\,\text{eV}$, so it will not show the photoelectric effect.

For Cu, the work function $W_0 = 4.8\,\text{eV}$, so it will not show the photoelectric effect.

For Ag, the work function $W_0 = 4.3\,\text{eV}$, so it will not show the photoelectric effect.

Thus, only Mg, Cu, and Ag metals will show the photoelectric effect when exposed to light of wavelength 400 nm.

**Quick Tip**

Remember that the photoelectric effect occurs only when the energy of the incident photon is greater than the work function of the material. The energy of the photon is calculated using the formula $E = \frac{1240}{\lambda}$, where $\lambda$ is the wavelength in nm and the result is in eV.