

JEE MAINS 2023 Maths 13 APRIL Shift 2 Question Paper

Time Allowed :1 Hours

Maximum Marks :120

Total Questions :30

General Instructions

Read the following instructions very carefully and strictly follow them:

1. This question paper contains 30 questions. All questions are compulsory.
2. This question paper contains only one section - Mathematics
3. In all sections, Questions are multiple choice questions (MCQs) and questions carry 4 mark each.

Section - A

1. The area of the region $\{(x,y): x^2 \leq y \leq |x^2 - 4|, y \geq 1\}$ is:

- (1) $\frac{3}{4}(4\sqrt{2} + 1)$
- (2) $\frac{4}{3}(4\sqrt{2} - 1)$
- (3) $\frac{3}{4}(4\sqrt{2} - 1)$
- (4) $\frac{4}{3}(4\sqrt{2} + 1)$

2. If

$$\lim_{x \rightarrow 0} \frac{e^x - \cos(bx) - cx}{1 - \cos(2x)} = 2,$$

then $5a^2 + b^2$ is equal to:

- (1) 76
- (2) 72
- (3) 64
- (4) 68

3. The line, that is coplanar to the line

$$\frac{x+1}{-3} = \frac{y-2}{1} = \frac{z-5}{5},$$

is:

$$(1) \frac{x+1}{-3} = \frac{y-2}{2} = \frac{z-5}{5}$$

$$(2) \frac{x+1}{-3} = \frac{y-2}{-2} = \frac{z-5}{5}$$

$$(3) \frac{x-1}{-2} = \frac{y-2}{2} = \frac{z-5}{4}$$

$$(4) \frac{x-1}{-2} = \frac{y-2}{5} = \frac{z-5}{4}$$

4. The plane, passing through the points $(0, -1, 2)$ and $(-1, 2, 1)$ and parallel to the line passing through $(5, 1, -7)$ and $(1, -1, -1)$, also passes through the point

(1) $(0, 5, -2)$

(2) $(-2, 5, 0)$

(3) $(2, 0, 1)$

(4) $(1, -2, 1)$

5. Let for a triangle ABC,

$$\overrightarrow{AB} = -2\hat{i} + \hat{j} + 3\hat{k}, \quad \overrightarrow{CB} = \hat{i} + \hat{j} + \hat{k}, \quad \overrightarrow{CA} = 4\hat{i} + 3\hat{j} + 4\hat{k}$$

If $\lambda = 0$ and the area of triangle ABC is $5\sqrt{6}$, then CB is equal to:

(1) 108

(2) 60

(3) 54

(4) 120

6. Let for

$$A = \begin{pmatrix} 1 & 2 & 3 \\ 1 & 2 & 3 \\ 1 & 1 & 2 \end{pmatrix}, \quad |A| = 2. \quad \text{If } |2 \operatorname{adj}(2A)| = 32, \quad \text{then } 3n + \alpha \text{ is equal to:}$$

(1) 10

(2) 9

(3) 12

(4) 11

7. The range of $f(x) = 4 \sin \left(\frac{x^2}{x^2+1} \right)$ is:

- (1) $(0, \infty)$
 - (2) $(0, \pi)$
 - (3) $[0, 2\pi]$
 - (4) $[0, \pi]$
-

8. Let $a_1, a_2, a_3, \dots$ be a G.P. of increasing positive numbers. Let the sum of its 6th and 8th terms be 2 and the product of its 3rd and 5th terms be $\frac{1}{9}$. Then $6a_6 + a_6a_8$ is equal to:

- (1) 2
 - (2) 3
 - (3) $3\sqrt{5}$
 - (4) $2\sqrt{5}$
-

9. If the system of equations

$$2x + y = -5$$

$$2x - 5y + z = -9$$

$$x + 2y - 5z = 7$$

has infinitely many solutions, then $(x + y)^2 + (y + z)^2$ is equal to:

- (1) 904
 - (2) 916
 - (3) 912
 - (4) 920
-

10. The statement

$$(p \rightarrow q) \rightarrow (r \rightarrow q) \equiv (p \vee q) \rightarrow (r \vee q)$$

is equivalent to:

- (1) $p \vee q$
 - (2) $p \rightarrow (q \rightarrow p)$
 - (3) $p \vee (q \vee r)$
 - (4) $p \vee q \rightarrow r$
-

11. Let $S = \{z \in \mathbb{C} : z = i(z^2 + \operatorname{Re}(z))\}$. Then $\sum_{z \in S} |z|^2$ is equal to:

- (1) 4
 - (2) $\frac{7}{2}$
 - (3) 3
 - (4) $\frac{5}{2}$
-

12. Let α, β be the roots of the equation $x^2 - \sqrt{5}x + 2 = 0$. Then $\alpha^4 + \beta^4$ is equal to:

- (1) -64
 - (2) -128
 - (3) 64
 - (4) -256
-

13. Let $|a| = 2, |b| = 3$ and the angle between the vectors a and b be $\frac{\pi}{4}$. Then

$|a + 2b| \times |2a - 3b|$ is equal to:

- (1) 482
 - (2) 841
 - (3) 882
 - (4) 441
-

14. The value of

$$\int_0^{\frac{\pi}{4}} \frac{e^x}{(e^x + \tan^2 x)} dx$$

is:

- (1) 25

- (2) 51
 - (3) 50
 - (4) 49
-

15. The coefficient of x^2 in the expansion of

$$\left(2x^2 - \frac{1}{3x^3}\right)^5$$

is:

- (1) $\frac{80}{9}$
 - (2) 8
 - (3) 9
 - (4) $\frac{26}{3}$
-

16. The random variable X follows binomial distribution B (n, p), for which the difference of the mean and the variance is 1. If

$$1^2P(X = x) - 2P(X = 3X - 1), \quad \text{then } np(X)^2 \text{ is equal to}$$

- (1) 16
 - (2) 11
 - (3) 12
 - (4) 15
-

17. Let the centre of a circle C be (α, β) and its radius $r < 8$. Let $3x + 4y - 24$ and $3x - 4y - 32$ be two tangents and $4x + 3y = 1$ be a normal to C. Then the value of $(\alpha - \beta)$ is equal to :

- (1) 5
 - (2) 6
 - (3) 7
 - (4) 9
-

18. Let N be the foot of perpendicular from the point P(1, -2, 3) on the line passing through the points (4, 5, 8) and (1, -7, -5). Then the distance of N from the plane $2x - 2y + z = 5$ is: (1) 6

(2) 7

(3) 9

(4) 8

19. All words, with or without meaning, are made using all the letters of the word MONDAY. These words are written in a dictionary with serial numbers. The serial number of the word MONDAY is: (1) 328

(2) 327

(3) 324

(4) 326

20. Let α, β be the centroid of the triangle formed by the lines $15x + y = 82, 6x - 5y = -4$, and $9x + 4y = 17$. Then α and β are the roots of the equation: (1) $x^2 - 13x + 42 = 0$

(2) $x^2 - 10x + 25 = 0$

(3) $x^2 - 7x + 12 = 0$

(4) $x^2 - 14x + 48 = 0$

Section - B

21. Let $A = \{-4, -3, 2, 0, 1, 3, 4\}$ and $R = \{(a, b) : a \in A, b = |a| \text{ or } a = b\}$ be a relation on A. Then the minimum number of elements that must be added to the relation R so that it becomes reflexive and symmetric, is:

22. Let $f = \left(\sum_{k=1}^{\infty} \sin^k x\right) \left(\sum_{k=1}^{\infty} \sin^k x\right) \cos x dx \in N$. Then f_{11} is equal to:

23. If $y = y(x)$ is the solution of the differential equation

$$\frac{dy}{dx} = \frac{4x}{(x-1)^2} - \frac{x^2-2}{(x-1)^3} \text{ such that } y(2) = \frac{2}{9} \log_2 (2 + \sqrt{5})$$

and $y(x) = \alpha \log (\sqrt{x+\beta}) + \gamma \cdot \sqrt{x} - \frac{1}{x}$, then $\alpha\beta\gamma$ is equal to:

24. Total numbers of 3-digit numbers that are divisible by 6 and can be formed by using the digits 1, 2, 3, 4, 5 with repetition, is:

25. The remainder, when 7^{110} is divided by 17, is _____.

- (1) 12
 - (2) 10
 - (3) 9
 - (4) 8
-

26. Let $f(x) = \sum_{k=1}^{\infty} x^k$, where $x \in \mathbb{R}$ and $f(2) = 119$. Then $f(2) - f(1)$ is equal to _____.

28. The mean and standard deviation of the marks of 10 students were found to be 50 and 12 respectively. Later, it was observed that two marks 20 and 25 were wrongly read as 45 and 50 respectively. Then the correct variance is _____.

29. The foci of a hyperbola are $(\pm 2, 0)$ and its eccentricity is $\frac{3}{2}$. A tangent, perpendicular to the line $2x + 3y - 6 = 0$, is drawn at a point in the first quadrant on the hyperbola. If the intercepts made by the tangent on the x - and y -axes are a and b respectively, then $|a| + |b|$ is equal to _____.

30. Let $[\alpha]$ denote the greatest integer $\leq \alpha$. Then $[\sqrt{1}] + [\sqrt{2}] + [\sqrt{3}] + \cdots + [\sqrt{20}]$ is equal to _____.
