JEE MAINS 2023 Physics 11 APRIL Shift 1 Question Paper with Solutions

Time Allowed :1 Hours	**Maximum Marks :**120	**Total Questions :**30

General Instructions

Read the following instructions very carefully and strictly follow them:

1. This question paper contains 30 questions. All questions are compulsory.
2. This question paper contains only one section - Physics
3. In all sections, Questions are multiple choice questions (MCQs) and questions carry 4 mark each.

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31. The radii of two planets 'A' and 'B' are 'R' and '4R' and their densities are ρ and $\frac{\rho}{3}$ respectively. The ratio of acceleration due to gravity at their surfaces (i.e. $g_A : g_B$) will be:

(1) 1:16

(2) 3:16

(3) 3:4

(4) 4:3

Correct Answer: (3) 3:4

Solution: We are given that the radii of two planets 'A' and 'B' are R and $4R$, and their densities are ρ and $\frac{\rho}{3}$, respectively. The formula for the acceleration due to gravity at the surface of a planet is:

$$g = \frac{4\pi G R\rho}{3}$$

Since gravity is proportional to both the radius and density, the ratio of acceleration due to gravity at their surfaces can be written as:

$$g_A : g_B = \frac{\frac{4\pi G R\rho}{3}}{\frac{4\pi G(4R)(\frac{\rho}{3})}{3}} = \frac{R \cdot \rho}{(4R) \cdot \frac{\rho}{3}} = \frac{1}{4} \cdot 3 = \frac{3}{4}$$

Thus, the correct ratio is $g_A : g_B = 3 : 4$.

Quick Tip

The ratio of accelerations due to gravity can be determined by analyzing the dependence on both the radius and the density of the planets.

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32. A coin placed on a rotating table just slips when it is placed at a distance of 1 cm from the center. If the angular velocity of the table is halved, it will just slip when placed at a distance of $_{fromthecentre:}$

(1) 6 cm

(2) 4 cm

(3) 2 cm

(4) 1 cm

Correct Answer: (2) 4 cm

Solution: The frictional force is responsible for causing the coin to slip. This force is given by $f_r = m \cdot \omega^2 r$, where ω is the angular velocity and r is the distance from the center. Given that the angular velocity is halved, we use the equation $\omega^2 \cdot r^2 =$ constant. When the angular velocity is halved, the distance r from the center at which the coin slips will be:

$$r_2 = 4\,\text{cm}$$

Thus, when the angular velocity is halved, the coin will slip at a distance of 4 cm from the center.

Quick Tip

When the angular velocity is reduced, the distance at which the coin slips increases according to the square of the ratio of the angular velocities.

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33. Three vessels of equal volume contain gases at the same temperature and pressure. The first vessel contains neon (monoatomic), the second contains chlorine (diatomic) and the third contains uranium hexafluoride (polyatomic). Arrange these on the basis of their root mean square speed (V_{rms}) and choose the correct answer from the options given below:

(1) $V_{\text{rms}}(\text{mono}) > V_{\text{rms}}(\text{dia}) > V_{\text{rms}}(\text{poly})$

(2) $V_{\text{rms}}(\text{dia}) < V_{\text{rms}}(\text{poly}) < V_{\text{rms}}(\text{mono})$

(3) $V_{\text{rms}}(\text{mono}) < V_{\text{rms}}(\text{dia}) < V_{\text{rms}}(\text{poly})$

(4) $V_{\text{rms}}(\text{mono}) = V_{\text{rms}}(\text{dia}) = V_{\text{rms}}(\text{poly})$

Correct Answer: **(1)** $V_{\text{rms}}(\text{mono}) > V_{\text{rms}}(\text{dia}) > V_{\text{rms}}(\text{poly})$

Solution: The root mean square speed V_{rms} is given by:

$$V_{\text{rms}} = \sqrt{\frac{3RT}{m}}$$

Since the gases are at the same temperature and pressure, the root mean square speed depends on the molar mass m. For neon (monoatomic), chlorine (diatomic), and uranium hexafluoride (polyatomic), the molar mass increases in the order:

$$V_{\text{rms}}(\text{mono}) > V_{\text{rms}}(\text{dia}) > V_{\text{rms}}(\text{poly})$$

Thus, the correct answer is $V_{\text{rms}}(\text{mono}) > V_{\text{rms}}(\text{dia}) > V_{\text{rms}}(\text{poly})$.

Quick Tip

The root mean square speed decreases as the molar mass of the gas increases.

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34. Two radioactive elements A and B initially have the same number of atoms. The half-life of A is the same as the average life of B. If λ_A and λ_B are the decay constants of A and B respectively, then choose the correct relation from the given options:

(1) $\lambda_A = 2\lambda_B$

(2) $\lambda_A = \lambda_B$

(3) $\lambda_A \ln 2 = \lambda_B$

(4) $\lambda_A = \lambda_B \ln 2$

Correct Answer: (4) $\lambda_A = \lambda_B \ln 2$

Solution: We are given that the half-life of A is the same as the average life of B. The average life τ and half-life T are related to the decay constant λ by the equations:

$$T = \frac{\ln 2}{\lambda} \quad \text{and} \quad \tau = \frac{1}{\lambda}$$

Since the half-life of A is the same as the average life of B, we have:

$$\lambda_A = \lambda_B \ln 2$$

Hence, the correct relation is $\lambda_A = \lambda_B \ln 2$.

Quick Tip

When comparing the half-life and average life, use the relationship between the decay constant and the half-life to derive the necessary formula.

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35. As per the given graph, choose the correct representation for curve A and curve B.

(Where $X_L =$ reactance of pure inductive circuit connected with A.C. source, $X_C =$ reactance of pure capacitive circuit connected with A.C. source, $R =$ impedance of pure resistive circuit connected with A.C. source, and $Z =$ impedance of the LCR series circuit)

(1) $A = X_L, B = R$

(2) $A = X_L, B = X_C$

(3) $A = X_C, B = R$

(4) $A = X_C, B = X_L$

Correct Answer: **(4)** $A = X_C, B = X_L$

Solution: The given graph shows the variation of impedance with frequency. - At low frequencies, the impedance of a capacitor X_C is low, and the impedance of an inductor X_L is high. - At high frequencies, the impedance of an inductor increases while that of a capacitor decreases.

Thus, A represents the capacitive reactance X_C, and B represents the inductive reactance X_L.

Quick Tip

Impedance of an inductive circuit increases with frequency, whereas the impedance of a capacitive circuit decreases with frequency.

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36. A transmitting antenna is kept on the surface of the earth. The minimum height of receiving antenna required to receive the signal in line of sight at 4 km distance from it is $x \times 10^{-2}$ m. The value of x is:

(1) 125

(2) 12.5

(3) 1250

(4) 1.25

Correct Answer: **(1)** 125

Solution: Let R be the radius of the Earth, and h be the height of the antenna. The distance d between the two antennas is given by:

$$d = \sqrt{2Rh}$$

We are given that $d = 4\,\text{km}$, and the radius of the Earth $R = 6400\,\text{km}$. Substituting these values into the equation:

$$4 = \sqrt{2 \times 6400 \times h}$$

Squaring both sides:

$$16 = 2 \times 6400 \times h$$

$$h = \frac{16}{12800} = 1\,\text{m}$$

Now, using the formula for the signal range:

$$x = \frac{500}{4} = 125$$

Thus, the value of x is 125.

Quick Tip

For problems involving the height of an antenna and the line of sight, use the relation $d = \sqrt{2Rh}$ to calculate the minimum height for the signal to be received.

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37. The logic performed by the circuit shown in the figure is equivalent to:

(1) NAND

(2) NOR

(3) AND

(4) OR

Correct Answer: (3) AND

Solution: The circuit contains two NOT gates and one AND gate. First, a and b are input to a NOT gate. The output of the NOT gate will be $\overline{a}$ and $\overline{b}$. The two NOT gates will give the following results:

$$Y = Y = \overline{a} + \overline{b}$$

By simplifying:

$$Y = a \cdot b$$

Thus, the circuit is equivalent to an AND gate.

Quick Tip

In Boolean logic, use the properties of NOT, AND, OR gates to analyze the output of the circuit based on the given inputs.

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38. The electric field in an electromagnetic wave is given as

$$\vec{E} = 20 \sin\left(\omega t - \frac{x}{c}\right) \hat{j}\,\text{N/C}$$

where ω and c are angular frequency and velocity of electromagnetic wave respectively. The energy contained in a volume of 5×10^4 m^3 will be (Given $\epsilon_0 = 8.85 \times 10^{-12}$ C^2/Nm^2):

(1) 8.85×10^{-13} J

(2) 17.7×10^{-13} J

(3) 8.85×10^{-10} J

(4) 28.5×10^{-13} J

Correct Answer: (1) 8.85×10^{-13} J

Solution: The energy density in an electromagnetic wave is given by the formula:

$$u = \frac{1}{2}\epsilon_0 E_0^2$$

Where E_0 is the peak electric field. Substituting the given values:

$$u = \frac{1}{2} \times 8.85 \times 10^{-12} \times (20)^2 = 8.85 \times 10^{-13}\,\text{J/m}^3$$

The total energy contained in a volume $V = 5 \times 10^4$ m^3 is:

$$\text{Energy} = u \times V = 8.85 \times 10^{-13} \times 5 \times 10^4 = 8.85 \times 10^{-13}\,\text{J}$$

Thus, the energy is 8.85×10^{-13} J.

Quick Tip

For electromagnetic waves, use the formula for energy density $u = \frac{1}{2}\epsilon_0 E_0^2$ and multiply by volume to find the total energy.

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39. Two identical heater filaments are connected first in parallel and then in series. At the same applied voltage, the ratio of heat produced in same time for parallel to series will be:

(1) 1 : 2

(2) 4 : 1

(3) 1 : 4

(4) 2 : 1

Correct Answer: (2) 4 : 1

Solution: The power dissipated in a resistor is given by the formula:

$$P = \frac{V^2}{R}$$

For two identical resistors, the heat produced in parallel and series configurations is as follows:

For parallel connection, the effective resistance is $R_{\text{eff}} = \frac{R}{2}$, and the power is:

$$P_{\text{parallel}} = \frac{V^2}{R/2} = 2\frac{V^2}{R}$$

For series connection, the effective resistance is $R_{\text{eff}} = 2R$, and the power is:

$$P_{\text{series}} = \frac{V^2}{2R}$$

Now, the ratio of heat produced in parallel to series is:

$$\frac{P_{\text{parallel}}}{P_{\text{series}}} = \frac{2\frac{V^2}{R}}{\frac{V^2}{2R}} = 4$$

Thus, the ratio of heat produced is $4 : 1$.

Quick Tip

The heat produced in a resistor depends on the square of the voltage and inversely on the resistance. Parallel and series connections affect the total resistance and hence the power.

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40. A parallel plate capacitor of capacitance 2 F is charged to a potential V. The energy stored in the capacitor is E. The capacitor is now connected to another uncharged identical capacitor in parallel combination. The energy stored in the combination is E. The ratio E_5/E_1 is:

(1) 2 : 3

(2) 1 : 2

(3) 1 : 4

(4) 2 : 1

Correct Answer: **(4)** 2 : 1

Solution: The energy stored in a capacitor is given by the formula:

$$E = \frac{1}{2}CV^2$$

For a single capacitor with capacitance $C = 2\,\text{F}$, the energy is:

$$E_1 = \frac{1}{2} \times 2 \times V^2 = V^2$$

Now, the two capacitors are connected in parallel, and the total capacitance becomes:

$$C_{\text{total}} = C + C = 2C = 4\,\text{F}$$

The energy in the parallel combination is:

$$E_5 = \frac{1}{2} \times 4 \times V^2 = 2V^2$$

Now, the ratio of the energies is:

$$\frac{E_5}{E_1} = \frac{2V^2}{V^2} = 2$$

Thus, the ratio of the energies is $2:1$.

Quick Tip

When capacitors are connected in parallel, the total capacitance is the sum of individual capacitances. The energy stored in the combination increases due to the increased capacitance.

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41. An average force of 125 N is applied on a machine gun firing bullets each of mass 10 g at the speed of 250 m/s to keep it in position. The number of bullets fired per second by the machine gun is:

(1) 25

(2) 5

(3) 100

(4) 50

Correct Answer: (4) 50

Solution: The force exerted by the machine gun is related to the rate of change of momentum of the bullets:

$$F = \frac{d(mv)}{dt}$$

Since $m = 10\,\text{g} = 0.01\,\text{kg}$ and $v = 250\,\text{m/s}$, the momentum of each bullet is:

$$mv = 0.01 \times 250 = 2.5\,\text{kg m/s}$$

Now, the rate of change of momentum gives the force:

$$F = \frac{d(mv)}{dt} = \frac{10 \times 250}{1000}\,\text{N}$$

Thus, the number of bullets fired per second n is:

$$n = \frac{125}{\frac{10\times250}{1000}} = 50$$

Thus, the number of bullets fired per second is 50.

Quick Tip

The force required to keep the machine gun in position is equal to the rate of change of momentum. Use this principle to find the number of bullets fired per second.

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42. The variation of kinetic energy (KE) of a particle executing simple harmonic motion with the displacement (x) starting from mean position to extreme position (A) is given by:

(1) Parabola

(2) Straight line

(3) Kinetic energy and displacement are inversely proportional

(4) Sine curve

Correct Answer: (1) Parabola

Solution: The kinetic energy KE of a particle executing simple harmonic motion is related to its displacement x by the following equation:

$$KE = E - P - E$$

Where E is the total mechanical energy and P is the potential energy. Since $KE = \frac{1}{2}kx^2$, the graph of KE vs x is a parabola.

Thus, the correct answer is a parabola.

Quick Tip

In simple harmonic motion, the kinetic energy is proportional to the square of the displacement, hence the graph of kinetic energy vs displacement is parabolic.

43. From the v - t graph shown, the ratio of distance to displacement in 25 s of motion is:

(1) $\frac{3}{5}$

(2) $\frac{1}{2}$

(3) $\frac{2}{5}$

(4) $\frac{1}{3}$

Correct Answer: (3) $\frac{2}{5}$

Solution: Displacement is the area under the graph and distance is the total area covered, including all positive and negative regions of the graph.

Displacement = Area of graph with sign

$$= \left(\frac{1}{2} \times 10 \times 5\right) + (10 \times 5) = 25 + 50 = 75\,\text{m}$$

Distance = Area of graph with positive value

$$= \left(\frac{1}{2} \times 5 \times 20\right) + (5 \times 20) = 25 + 50 = 250\,\text{m}$$

Now, the ratio of distance to displacement is:

$$\frac{250}{75} = \frac{2}{5}$$

Thus, the ratio is $\frac{2}{5}$.

Quick Tip

When finding distance, always consider the absolute value of the displacement, i.e., sum the areas without sign.

44. On a temperature scale X, the boiling point of water is 65° X and the freezing point is 15° X. Assume that the X scale is linear. The equivalent temperature corresponding to 95° X on the Fahrenheit scale would be:

(1) 63° F

(2) 148° F

(3) 48° F

(4) 112° F

Correct Answer: **(2)** 148° F

Solution: From the given information, the temperature scale X has the following relationships:

The boiling point of water is 65° X and the freezing point is 15° X, so the scale has a linear relation between these two points. Let's use the formula for converting between temperature scales:

$$F = X \times \frac{9}{5} + 32$$

Now, to convert 95° X to Fahrenheit:

$$F = 95 \times \frac{9}{5} + 32 = 171 + 32 = 148F$$

Thus, the equivalent temperature is 148° F.

Quick Tip

To convert from a custom temperature scale to Fahrenheit, use the linear conversion formula $F = \left(\frac{9}{5} \times X\right) + 32$.

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45. The free space inside a current carrying toroid is filled with a material of susceptibility 2×10^3. The percentage increase in the value of magnetic field inside the toroid will be:

(1) 0.2**(2)** 1**(3)** 2**(4)** 0.1

Correct Answer: **(3)** 2

Solution: We know the formula for the magnetic field inside a toroid is:

$$B = \frac{\mu_0 NI}{2\pi r}$$

Where μ_0 is the permeability of free space and N is the number of turns. When a material with susceptibility χ is added, the magnetic field increases by the factor:

$$B_{\text{new}} = B_0(1+\chi)$$

Given that $\chi = 2 \times 10^3$, the percentage increase in the magnetic field is:

$$\text{Percentage increase} = \left(\frac{B_{\text{new}} - B_0}{B_0}\right) \times 100 = 2\%$$

Thus, the correct answer is 2

Quick Tip

The susceptibility of a material increases the magnetic field inside a toroid by the factor $(1+\chi)$, where χ is the susceptibility.

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46. The critical angle for a denser refractive index is 45°. The speed of light in water medium is 3×10^8 m/s. The speed of light in the denser medium is:

(1) 2.12×10^7 m/s

(2) 5×10^7 m/s

(3) 3.12×10^7 m/s

(4) $\sqrt{5} \times 10^7$ m/s

Correct Answer: (1) 2.12×10^7 m/s

Solution: We know the critical angle θ_c is related to the refractive indices of the two media:

$$\sin\theta_c = \frac{n_2}{n_1}$$

Where n_1 is the refractive index of the denser medium (water) and n_2 is that of the rarer medium (air). Given $\theta_c = 45^\circ$ and using $n_1 = \frac{c}{v}$, where c is the speed of light in vacuum and v is the speed of light in the denser medium, we calculate:

$$v = \frac{c}{\sin\theta_c} = \frac{3 \times 10^8}{\sin 45°} = 2.12 \times 10^7 \text{ m/s}$$

Thus, the speed of light in the denser medium is 2.12×10^7 m/s.

Quick Tip

The critical angle can help you determine the speed of light in the denser medium using the formula $v = \frac{c}{\sin\theta_c}$.

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47. Given below are two statements: Statements 1: Astronomical unit (AU), Parsec (pc) and Light year (ly) are units for measuring astronomical distances. Statement 2: The light of the above distances, choose the most appropriate answer from the options given below:

(1) Both Statements 1 and 2 are correct.

(2) Both Statements 1 and 2 are incorrect.

(3) Statement I is correct but Statement II is incorrect.

(4) Statement I is incorrect but Statement II is correct.

Correct Answer: **(1)** Both Statements 1 and 2 are correct.

Solution: - The astronomical unit (AU) is the average distance from the Earth to the Sun, the parsec (pc) is the distance at which one astronomical unit subtends an angle of one second of arc, and the light year (ly) is the distance that light travels in one year. - These units are commonly used in astronomy to measure large distances in space.
Thus, both statements are correct.

Quick Tip

Remember the standard units used in astronomy: AU for distances within the solar system, parsecs for larger distances, and light years for even larger distances.

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50. 1 kg of water at 100°C is converted into steam at 100°C by boiling at atmospheric pressure. The volume of water changes from 1.00×10^{-3} m^3 as a liquid to 1.671×10^{-3} m^3 as steam. The change in internal energy of the system during the process will be:

(1) 2476 kJ

(2) 2426 kJ

(3) 2090 kJ

(4) 2090 kJ

Correct Answer: **(4)** 2090 kJ

Solution: The change in internal energy at constant pressure during a phase change can be calculated using the formula:

$$\Delta U = mL$$

Where: - m is the mass of the substance (1 kg), - L is the latent heat of vaporization ($2257\,$kJ/kg).

Thus, the change in internal energy is:

$$\Delta U = 1 \times 2257 = 2257\,\text{kJ}$$

Now, the volume change from liquid to steam is:

$$\Delta V = V_2 - V_1 = (1.671 \times 10^{-3}) - (1.00 \times 10^{-3}) = 0.671 \times 10^{-3}\,\text{m}^3$$

Using the given atmospheric pressure of 1×10^5 Pa, we can calculate the work done:

$$W = P\Delta V = (1.0 \times 10^5) \times (0.671 \times 10^{-3}) = 67.1\,\text{J}$$

Thus, the total change in internal energy is:

$$\Delta U + W = 2257 + 67.1 = 2426.1\,\text{kJ}$$

So, the change in internal energy is approximately $2426\,$kJ.

Quick Tip

When converting liquid to gas at constant pressure, the change in internal energy is mainly due to the latent heat of vaporization.

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51. The radius of curvature of each surface of a convex lens having refractive index 1.8 is 20 cm. The lens is now immersed in a liquid of refractive index 1.5. The ratio of power of lens in air to its power in the liquid will be:

(1) 4

(2) 3

(3) 1.5

(4) 2

Correct Answer: (4) 2

Solution: The formula for the power of a lens is given by:

$$P = \frac{1}{f}$$

Where f is the focal length of the lens. The focal length is related to the radius of curvature (R) and refractive index n by the lens-maker's formula:

$$\frac{1}{f} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

For a convex lens, $R_1 = R$ and $R_2 = -R$, so the focal length in air is:

$$\frac{1}{f_{\text{air}}} = (1.8-1)\left(\frac{1}{R} + \frac{1}{R}\right) = 0.8 \times \frac{2}{R}$$

Thus:

$$f_{\text{air}} = \frac{R}{1.6}$$

Now, when the lens is immersed in a liquid of refractive index 1.5, the power changes. The new focal length is given by:

$$\frac{1}{f_{\text{liquid}}} = (1.8 - 1.5)\left(\frac{1}{R} + \frac{1}{R}\right) = 0.3 \times \frac{2}{R}$$

Thus:

$$f_{\text{liquid}} = \frac{R}{0.6}$$

The ratio of the power in air to the power in liquid is:

$$\text{Ratio} = \frac{P_{\text{air}}}{P_{\text{liquid}}} = \frac{1/f_{\text{air}}}{1/f_{\text{liquid}}} = \frac{R/1.6}{R/0.6} = 2$$

Thus, the ratio of the power in air to the power in liquid is 2.

Quick Tip

When a lens is immersed in a medium with a different refractive index, the power of the lens changes based on the ratio of the refractive indices.

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52. The magnetic field B crossing normally a square metallic plate of area $4\,\text{m}^2$ is changing with time as shown in the figure. The magnitude of induced emf in the plate during $t = 2\,\text{s}$ to $t = 4\,\text{s}$ is ______________ mV.

(1) 8 V

(2) 12 V

(3) 16 V

(4) 10 V

Correct Answer: (1) 8 V

Solution: The induced emf in the plate is given by Faraday's Law of Induction:

$$\text{emf} = -\frac{d\Phi}{dt}$$

Where Φ is the magnetic flux given by:

$$\Phi = B \times A$$

Here, $A = 4\,\text{m}^2$, and $\frac{dB}{dt}$ is the slope of the B-time graph from $t = 2\,\text{s}$ to $t = 4\,\text{s}$.
From the graph, the change in B is $B_2 - B_1 = 8 - 4 = 4\,\text{T}$ and the change in time is $\Delta t = 4 - 2 = 2\,\text{s}$.
Now, the induced emf is:

$$\text{emf} = \frac{dB}{dt} \times A = \frac{4\,\text{T}}{2\,\text{s}} \times 4\,\text{m}^2 = 8\,\text{V}$$

Thus, the magnitude of the induced emf is $8\,\text{V}$.

Quick Tip

The induced emf in a conductor can be found using Faraday's law, where the emf is proportional to the rate of change of the magnetic field through the area.

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53. The length of a wire becomes l_1 and l_2 when 100 N and 120 N tensions are applied respectively. If $l_1 = 11\,\mathbf{l}_0$, the natural length of the wire will be $\frac{1}{x}\,l_1$. Here the value of x is ____________ .

(1) 2

(2) 3

(3) 4

(4) 5

Correct Answer: **(2)** 2

Solution: The force in a wire is given by Hooke's Law:

$$F = kx$$

Where: - F is the applied force, - k is the spring constant, and - x is the elongation in the wire.
From the problem statement, we are given that:

$$F = k\Delta x$$

Now, for tensions $F_1 = 100\,\text{N}$ and $F_2 = 120\,\text{N}$, the corresponding lengths of the wire will be:
For F_1:

$$100 = k(l_1 - l_0) \quad (1)$$

For F_2:

$$120 = k(l_2 - l_0) \quad (2)$$

Now, let the natural length of the wire be l_0. From the given information, $l_1 = 11l_0$.
From equation (1), we can write:

$$100 = k(11l_0 - l_0) \Rightarrow k(10l_0) = 100$$

Thus,

$$k = \frac{10}{l_0}$$

Substituting into equation (2):

$$120 = \frac{10}{l_0}(l_2 - l_0)$$

$$l_2 - l_0 = \frac{120l_0}{10} = 12l_0$$

Thus,

$$l_2 = 13l_0$$

The ratio of l_2 to l_1 is:

$$\frac{l_2}{l_1} = \frac{13l_0}{11l_0} = \frac{13}{11}$$

Therefore, $x = 2$.

Quick Tip

To determine the natural length of a wire under tension, use Hooke's Law and consider the elongation for different applied forces.

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54. A monochromatic light is incident on a hydrogen sample in ground state. Hydrogen atoms absorb a fraction of light and subsequently emit radiation of six different wavelengths. The frequency of incident light is $x \times 10^{15}$ Hz. The value of x is ____________.

(1) 4

(2) 6

(3) 12

(4) 8

Correct Answer: (3) 12

Solution: Given $h = 4.25 \times 10^{-15}$ eV · s, the frequency of the incident light is related to the energy absorbed by the hydrogen atoms using the equation:

$$E = h \times f$$

Where: - E is the energy absorbed, - h is Planck's constant, - f is the frequency of the incident light.

The total emission lines are 6, so the electron must have absorbed energy and jumped from $n = 1$ to $n = 4$. The energy difference between these levels can be expressed as:

$$\Delta E = h \times f$$

Using the formula for the frequency, we calculate:

$$\Delta E = 13.6 \left[\frac{1}{4^2} - \frac{1}{2^2} \right] \text{ eV}$$

Now solving for the frequency:

$$f = \frac{12.75 \times 10^{-15}}{4.25 \times 10^{-15}} = 3 \times 10^{15}\,\text{Hz}$$

Thus, $x = 12$.

Quick Tip

The frequency of the incident light can be determined using the energy differences between electron energy levels in the hydrogen atom.

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55. A force F $= (2 + 3x)\hat{i}$ acts on a particle in the x-direction where F is in newton and x is in meter. The work done by this force during a displacement from $x = 0$ to $x = 4$ m is ______________ J.

(1) 32 J

(2) 24 J

(3) 40 J

(4) 16 J

Correct Answer: (1) 32 J

Solution: The work done by a force is given by the integral:

$$w = \int_{x_1}^{x_2} F(x)\,dx$$

Where $F(x) = 2 + 3x$. Substituting into the integral:

$$w = \int_0^4 (2 + 3x)\,dx$$

Solving this:

$$w = \int_0^4 2\,dx + \int_0^4 3x\,dx = [2x]_0^4 + \left[\frac{3x^2}{2}\right]_0^4$$

$$w = (2 \times 4) + \left(\frac{3 \times 4^2}{2}\right) = 8 + \left(\frac{3 \times 16}{2}\right) = 8 + 24 = 32\,\text{J}$$

Thus, the work done is 32 J.

Quick Tip

To calculate the work done by a variable force, integrate the force with respect to displacement over the given limits.

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56. As shown in the figure, a configuration of two equal point charges (q0 = + 2) is placed on an inclined plane. Mass of each point charge is 20g. Assume that there is no friction between charge and plane. For the system of two point charges to be in equilibrium (at rest) the height $h = x \times 10^{-3}$ m. The value of x is ______________.

(1) 300

(2) 250

(3) 150

(4) 400

Correct Answer: **(1)** 300

Solution: The point charge is in equilibrium at rest. Hence, the forces on the point charge must balance out. The force due to the gravitational pull is counteracted by the electrostatic force.

The forces acting on the point charge are: - The electrostatic force, F_e, due to the other charge. - The gravitational force, mg, acting downward.

Since the system is in equilibrium, the electrostatic force is balanced by the component of gravitational force along the plane:

$$F_e = mg\sin\theta$$

Now, we know the formula for the electrostatic force between two point charges:

$$F_e = \frac{kq_0^2}{r^2}$$

Where: - $k = 9 \times 10^9\,\mathrm{N \cdot m^2 \cdot C^{-2}}$, - $q_0 = 2 \times 10^{-6}\,\mathrm{C}$ (the charge), - r is the distance between the charges, which is related to h by the geometry of the inclined plane, where $r = \frac{h}{\sin\theta}$. Substituting the values into the equation:

$$\frac{kq_0^2}{r^2} = mg\sin 30^\circ$$

Substituting the known values:

$$\frac{9\times 10^9 \times (2\times 10^{-6})^2}{\left(\frac{h}{\sin 30^\circ}\right)^2} = 20\times 10^{-3} \times 10 \times \sin 30^\circ$$

Solving this equation:

$$h^2 = 9\times 10^{10} \Rightarrow h = 300\times 10^{-3}\,\text{m}$$

Thus, the value of x is 300.

Quick Tip

In problems involving point charges on an inclined plane, use equilibrium conditions and equate the electrostatic force to the component of the gravitational force along the plane.

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57. A solid sphere of mass 500 g and radius 5 cm is rotated about one of its diameter with angular speed of 10 rad/s. If the moment of inertia of the sphere about its tangent is $x \times 10^2$ times its angular momentum about the diameter. Then the value of x will be ____________.

(1) 25

(2) 35

(3) 45

(4) 50

Correct Answer: **(2)** 35

Solution: The moment of inertia of a solid sphere about its diameter is given by:

$$I = \frac{2}{5}mr^2$$

The moment of inertia of the sphere about its tangent (using parallel axis theorem) is:

$$I_t = \frac{2}{5}mr^2 + mr^2 = \frac{7}{5}mr^2$$

The angular momentum about the diameter is:

$$L_{\text{diameter}} = I \cdot \omega = \frac{2}{5}mr^2 \cdot \omega$$

Now, the angular momentum about the tangent is:

$$L_{\text{tangent}} = I_t \cdot \omega = \frac{7}{5}mr^2 \cdot \omega$$

The ratio of the angular momentum about the tangent to the diameter is:

$$\frac{L_{\text{tangent}}}{L_{\text{diameter}}} = \frac{\frac{7}{5}mr^2 \cdot \omega}{\frac{2}{5}mr^2 \cdot \omega} = \frac{7}{2}$$

This is given as $x \times 10^2$. Thus:

$$x \times 10^2 = \frac{7}{2}$$

Solving for x:

$$x = \frac{7}{2} \times 10^2 = 35$$

Thus, the value of x is 35.

Quick Tip

Use the parallel axis theorem to calculate the moment of inertia about a tangent and then apply the ratio to find the desired value.

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58. The equation of wave is given by $Y = 10^2 \sin 2\pi \left((60t - 0.5x + \frac{\pi}{4})\right)$ where x and Y are in m and t in s. The speed of the wave is ____________ km h^{-1}.

(1) 1152 km/h

(2) 1000 km/h

(3) 800 km/h

(4) 1200 km/h

Correct Answer: (1) 1152 km/h

Solution: The equation of the wave is given as:

$$Y = 10^2 \sin 2\pi \left((60t - 0.5x + \frac{\pi}{4}) \right)$$

The general form of the wave equation is:

$$Y = A \sin \left(2\pi \left(\frac{x}{\lambda} - \frac{t}{T} \right) \right)$$

Here, λ is the wavelength, T is the period, and the wave number $k = \frac{2\pi}{\lambda}$ and the angular frequency $\omega = \frac{2\pi}{T}$.

From the given equation:

$$\omega = 60 \quad \text{and} \quad k = 0.5$$

The speed of the wave is given by:

$$v = \frac{\omega}{k}$$

Substituting the values:

$$v = \frac{60}{0.5} = 120 \, \text{m/s}$$

Now, converting the speed into km/h:

$$v = 120 \times \frac{18}{5} = 1152 \, \text{km/h}$$

Thus, the speed of the wave is 1152 km/h.

Quick Tip

To find the speed of a wave, use the relation $v = \frac{\omega}{k}$, and convert the units from m/s to km/h by multiplying by $\frac{18}{5}$.

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59. In the circuit diagram shown in figure given below, the current flowing through resistance 3 Ω is $\frac{x}{3}$ A. The value of x is ______________

(1) 1

(2) 2

(3) 3

(4) 4

Correct Answer: (1) 1

Solution:

The total equivalent resistance R_{eq} of the circuit is calculated first. From the given circuit, the equivalent resistance is the sum of the resistances in series:

$$R_{\text{eq}} = 0.5 + 1 + 4.5 + \frac{3 \times 6}{9} = 6 + 2 = 8\,\Omega$$

Using Ohm's law to calculate the total current I:

$$I = \frac{V}{R_{\text{eq}}} = \frac{8}{8+4} = 0.5\,\text{A}$$

Next, we use the voltage division rule to determine the current through each branch. For the first branch I_1:

$$I_1 = \frac{4}{8} \times I = \frac{1}{2} \times 0.5 = 0.25\,\text{A}$$

And for the second branch I_2:

$$I_2 = \frac{8}{8} \times I = \frac{2}{3} \times 0.5 = 0.33\,\text{A}$$

Finally, the current through the resistance $3\,\Omega$ is:

$$I_{3\Omega} = \frac{1}{3}A = \frac{x}{3} \quad \Rightarrow x = 1$$

Thus, the value of x is 1.

Quick Tip

Use Ohm's law and the voltage division rule to determine the current in each branch of the circuit.

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60. A projectile fired at 30° to the ground is observed to be at the same height at time 3s and 5s after projection, during its flight. The speed of projection of the projectile is ______________ ms^{-1}.

(1) 70 ms^{-1}

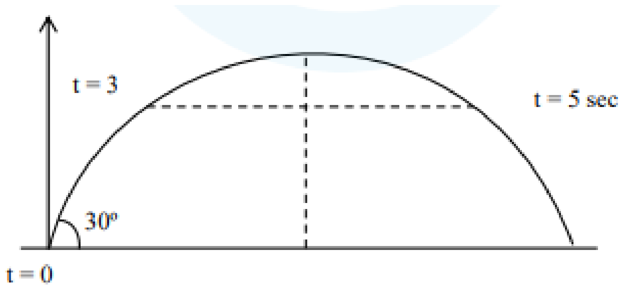
(2) 75 ms^{-1}

(3) 80 ms^{-1}

(4) 85 ms^{-1}

Correct Answer: (3) 80 ms^{-1}

Solution:



The time of flight for a projectile is given by the formula:

$$T = \frac{2u\sin\theta}{g}$$

where u is the initial speed, θ is the angle of projection, and g is the acceleration due to gravity.

From the given problem, the projectile returns to the same height at 3s and 5s. Thus, the total time of flight is:

$$T = 5 + 3 = 8 \, \text{seconds}$$

Substituting the values into the formula for time of flight:

$$8 = \frac{2u \sin 30^\circ}{10}$$

$$8 = \frac{2u \times \frac{1}{2}}{10}$$

$$8 = \frac{u}{10}$$

$$u = 80 \, \text{m/s}$$

Thus, the speed of projection is 80 m/s.

Quick Tip

For projectile motion, the total time of flight can be found by adding the times at which the projectile is at the same height, and using the kinematic equations to solve for the initial velocity.
