JEE Main 2023 April 11 Shift-2 Question Paper with Solutions

Time Allowed :3 Hours	**Maximum Marks :**300	**Total Questions :**90

General Instructions

Read the following instructions very carefully and strictly follow them:

1. The Duration of test is 3 Hours.
2. This paper consists of 90 Questions.
3. There are three parts in the paper consisting of Physics, Chemistry and Mathematics having 30 questions in each part of equal weightage..
4. Each part (subject) has two sections.
   (i) Section-A: This section contains 20 multiple choice questions which have only one correct answer. Each carries 4 marks for correct answer and –1 mark for wrong answer..
5. (ii) Section-B: This section contains 10 questions. In Section-B, attempt any five questions out of 10. The answer to each of the questions is a numerical value. Each carries 4 marks for correct answer and –1 mark for wrong answer. For Section-B, the answer should be rounded off to the nearest integer.

Mathematics

Section A

1: The angle of elevation of the top P of a tower from the feet of one person standing due South of the tower is 45° and from the feet of another person standing due West of the tower is 30°. If the height of the tower is 5 meters, then the distance (in meters) between the two persons is equal to:

(1) 10

(2) $5\sqrt{5}$

(3) $\frac{5}{2}\sqrt{5}$

(4) 5

Correct Answer: (1) 10

Solution:

Provided that: A tower $AB = 5\,\text{m}$, and angles $\angle APB = 45^\circ$, $\angle PAB = 90^\circ$.

Step 1: Apply trigonometry in $\triangle ABP$.

From the Provided that data, we know:

$$\tan 45^\circ = \frac{AB}{AP}.$$

Since $\tan 45^\circ = 1$, we have:

$$1 = \frac{5}{AP}.$$

Thus, $AP = 5\,\text{m}$.

Step 2:Apply trigonometry in $\triangle ABQ$.

Now, consider $\triangle ABQ$, where $\angle BAQ = 30^\circ$. Using the tangent formula:

$$\tan 30^\circ = \frac{AB}{AQ}.$$

Substitute the known values:

$$\frac{1}{\sqrt{3}} = \frac{5}{AQ}.$$

Rearranging, we get:

$$AQ = 5\sqrt{3}\,\text{m}.$$

Step 3: Calculate the hypotenuse PQ.

Using the Pythagoras theorem in $\triangle APQ$:

$$PQ^2 = AP^2 + AQ^2.$$

Substituting $AP = 5\,\text{m}$ and $AQ = 5\sqrt{3}\,\text{m}$:

$$PQ^2 = 5^2 + (5\sqrt{3})^2.$$

Simplify:

$$PQ^2 = 25 + 75 = 100.$$

Thus, $PQ = \sqrt{100} = 10\,\text{cm}$.

Conclusive Answer: The length of PQ is $10\,\text{cm}$.

Conclusion: The correct option is **(A) 10 cm**.

Quick Tip

For problems involving angles of elevation and distances,Apply trigonometric identities like $\tan$ and the Pythagorean theorem to calculate distances accurately.

2: Let a, b, c, and d be positive real numbers such that $a + b + c + d = 11$. If the maximum value of $a^5b^3c^2d$ is 3750β, then the value of β is:

(1) 55

(2) 108

(3) 90

(4) 110

Correct Answer: (3) 90

Solution:

Provided that

$$a + b + c + d = 11 \quad \text{and } \max(a^5b^3c^2d) = ?$$

$$(a, b, c, d > 0)$$

To maximize the Provided that expression, weApply the Arithmetic Mean (A.M.) and Geometric Mean (G.M.) inequality:

$$\text{A.M.} \geq \text{G.M.}$$

Let us assume the terms as: $\frac{a}{5}, \frac{a}{5}, \frac{a}{5}, \frac{a}{5}, \frac{a}{5}, \frac{b}{3}, \frac{b}{3}, \frac{b}{3}, \frac{c}{2}, \frac{c}{2}, d$

Now applying A.M. $\geq$ G.M., we get:

$$\frac{\frac{a}{5} + \frac{a}{5} + \frac{a}{5} + \frac{a}{5} + \frac{a}{5} + \frac{b}{3} + \frac{b}{3} + \frac{b}{3} + \frac{c}{2} + \frac{c}{2} + d}{11} \geq \left(\frac{a^5 b^3 c^2 d}{5^5 \cdot 3^3 \cdot 2^2 \cdot 1} \right)^{\frac{1}{11}}$$

Simplifying the left-hand side:

$$\frac{11}{11} \geq \left(\frac{a^5 b^3 c^2 d}{5^5 \cdot 3^3 \cdot 2^2 \cdot 1} \right)^{\frac{1}{11}}$$

Thus, we get:

$$a^5 b^3 c^2 d \leq 5^5 \cdot 3^3 \cdot 2^2$$

The maximum value of $a^5 b^3 c^2 d$ is:

$$\max(a^5 b^3 c^2 d) = 5^5 \cdot 3^3 \cdot 2^2 = 337500$$

We can express this as:

$$337500 = 90 \times 3750 \quad \text{where } \beta = 90$$

$$\beta = 90$$

Explanation: To achieve the maximum value, the numbers must be distributed in the ratios consistent with their powers in $a^5 b^3 c^2 d$, ensuring the product is maximized. Using A.M. $\geq$ G.M. is crucial in such optimization problems.

Quick Tip

To maximize expressions of the form $a^m b^n \ldots$, distribute the total sum proportionally to the powers in the expression using the AM-GM inequality.

3: If $f : \mathbb{R} \to \mathbb{R}$ is a continuous function satisfying

$$\int_0^{\frac{\pi}{2}} f(\sin 2x) \sin x \, dx + \alpha \int_0^{\frac{\pi}{4}} f(\cos 2x) \cos x \, dx = 0,$$

then the value of α is:

(1) $-\sqrt{3}$

(2) $\sqrt{3}$

(3) $-\sqrt{2}$

(4) $\sqrt{2}$

Correct Answer: (3) $-\sqrt{2}$

Solution:

$$F : \mathbb{R} \to \mathbb{R}$$

We are Provided that the integral equation:

$$\int_0^{\frac{\pi}{2}} F(\sin 2x) \sin x \, dx + \alpha \int_0^{\frac{\pi}{2}} F(\cos 2x) \cos x \, dx = 0$$

To simplify, we split the range of integration for the first term as follows:

$$\int_0^{\frac{\pi}{2}} F(\sin 2x) \sin x \, dx = \int_0^{\frac{\pi}{4}} F(\sin 2x) \sin x \, dx + \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} F(\sin 2x) \sin x \, dx$$

Rewriting the equation, we have:

$$\int_0^{\frac{\pi}{4}} F(\sin 2x) \sin x \, dx + \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} F(\sin 2x) \sin x \, dx + \alpha \int_0^{\frac{\pi}{4}} F(\cos 2x) \cos x \, dx = 0$$

Using the property of definite integrals:

$$\int_0^{a} F(x) \, dx = \int_a^{0} F(a - x) \, dx,$$

we let $x = t + \frac{\pi}{4}$, and substitute in the integrals. This yields:

$$\int_0^{\frac{\pi}{4}} F(\cos 2x) \sin\left(\frac{\pi}{4} - x\right) dx + \int_0^{\frac{\pi}{4}} F(\cos 2t) \sin\left(t + \frac{\pi}{4}\right) dx + \alpha \int_0^{\frac{\pi}{4}} F(\cos 2x) \cos x \, dx = 0$$

Simplifying the trigonometric terms, we combine them:

$$\int_0^{\frac{\pi}{4}} F(\cos 2x) \left\{\sin\left(\frac{\pi}{4} - x\right) + \sin\left(x + \frac{\pi}{4}\right) + \alpha \cos x\right\} dx = 0$$

Using the addition formulas for sine, we get:

$$\sin\left(\frac{\pi}{4} - x\right) + \sin\left(x + \frac{\pi}{4}\right) = \sqrt{2}\cos x.$$

Thus, the equation becomes:

$$\int_0^{\frac{\pi}{4}} F(\cos 2x)\left(\sqrt{2} + \alpha\right)\cos x\, dx = 0$$

Factoring out the common term, we have:

$$\left(\sqrt{2} + \alpha\right)\int_0^{\frac{\pi}{4}} F(\cos 2x)\cos x\, dx = 0$$

In the interval $\left(0, \frac{\pi}{4}\right)$, both $F(\cos 2x)$ and $\cos x$ are non-zero.

Thus, for the equation to hold, we must have:

$$\sqrt{2} + \alpha = 0$$

$$\Rightarrow \alpha = -\sqrt{2}$$

Conclusive Answer: $\alpha = -\sqrt{2}$.

Quick Tip

For definite integrals involving trigonometric substitutions, carefully transform the limits and simplify the equation step-by-step. Constant functions often simplify such problems effectively.

4: Let f and g be two functions defined by:

$$f(x) = \begin{cases} x + 1, & x < 0 \\ |x - 1|, & x \geq 0 \end{cases}, \quad g(x) = \begin{cases} x + 1, & x < 0 \\ 1, & x \geq 0 \end{cases}.$$

Then $(g \circ f)(x)$ is:

(1) continuous everywhere but not differentiable at $x = 1$

(2) continuous everywhere but not differentiable exactly at one point

(3) differentiable everywhere

(4) not continuous at $x = -1$

Correct Answer: (2) continuous everywhere but not differentiable exactly at one point

Solution:

$$f(x) = \begin{cases} x+1, & x < 0 \\ 1-x, & 0 \le x < 1 \\ x-1, & 1 \le x \end{cases}$$

$$g(x) = \begin{cases} x+1, & x < 0 \\ 1, & x \ge 0 \end{cases}$$

Step 1: Compute the composition $g(f(x))$.

Case 1: When $x < 0$:

For $x < 0$, $f(x) = x + 1$. Substituting into $g(x)$:

$$g(f(x)) = g(x+1) = (x+1) + 1 = x + 2.$$

Case 2: When $0 \le x < 1$:

For $0 \le x < 1$, $f(x) = 1 - x$. Substituting into $g(x)$:

$$g(f(x)) = g(1-x) = 1, \quad \text{(since } g(x) = 1 \text{ for } x \ge 0\text{)}.$$

Case 3: When $x \ge 1$:

For $x \ge 1$, $f(x) = x - 1$. Substituting into $g(x)$:

$$g(f(x)) = g(x-1) = 1, \quad \text{(since } g(x) = 1 \text{ for } x \ge 0\text{)}.$$

Step 2: Final composition of $g(f(x))$.

Combining all cases:

$$g(f(x)) = \begin{cases} x+2, & x < -1 \\ 1, & x \ge -1 \end{cases}$$

Step 3: Analyze continuity and differentiability.

- **Continuity:** $g(f(x))$ is continuous everywhere because there are no jumps or breaks in the function definition.
- **Differentiability:** At $x = -1$:
  - For $x < -1$, the derivative is $\frac{d}{dx}(x + 2) = 1$.
  - For $x \geq -1$, the derivative is $\frac{d}{dx}(1) = 0$.
  - The left-hand derivative (1) and right-hand derivative (0) are not equal at $x = -1$.

  Conclusion: $g(f(x))$ is not differentiable at $x = -1$, but it is differentiable everywhere else.

Final Observations:

- $g(f(x))$ is **continuous everywhere**.
- $g(f(x))$ is **not differentiable at** $x = -1$.
- $g(f(x))$ is **differentiable everywhere else**.

Quick Tip

To analyze composite functions, evaluate their continuity and differentiability case-by-case by substituting into the Provided that functions' definitions.

5: If the radius of the largest circle with centre $(2, 0)$ inscribed in the ellipse $x^2 + 4y^2 = 36$ is r, then $12r^2$ is equal to:

(1) 69

(2) 72

(3) 115

(4) 92

Correct Answer: (4) 92

Solution:

Provided that: Center of the ellipse $C(2, 0)$ and the equation of the ellipse:

$$x^2 + 4y^2 = 36 \implies \frac{x^2}{36} + \frac{y^2}{9} = 1$$

Step 1: Equation of the normal at a point P on the ellipse.

The parametric coordinates of a point P on the ellipse are $P(6\cos\theta, 3\sin\theta)$.

The equation of the normal at P is Provided that by:

$$(6\sec\theta)x - (3\csc\theta)y = 27$$

Step 2: Normal passes through $C(2,0)$.

Substituting $C(2,0)$ into the equation of the normal:

$$(6\sec\theta)(2) - (3\csc\theta)(0) = 27$$

$$\implies \sec\theta = \frac{27}{12} = \frac{9}{4}.$$

Step 3: Determine $\cos\theta$ and $\sin\theta$.

Using the relation $\sec\theta = \frac{1}{\cos\theta}$:

$$\cos\theta = \frac{4}{9}, \quad \sin\theta = \sqrt{1-\cos^2\theta} = \sqrt{1-\left(\frac{4}{9}\right)^2} = \sqrt{\frac{65}{81}} = \frac{\sqrt{65}}{9}.$$

Step 4: Parametric point P.

Using the parametric equations of the ellipse:

$$P\left(6\cos\theta, 3\sin\theta\right) = \left(6\cdot\frac{4}{9}, 3\cdot\frac{\sqrt{65}}{9}\right) = \left(\frac{8}{3}, \frac{\sqrt{65}}{3}\right).$$

Step 5: Distance between P and $C(2,0)$.

The distance γ between $P\left(\frac{8}{3}, \frac{\sqrt{65}}{3}\right)$ and $C(2,0)$ is:

$$\gamma = \sqrt{\left(\frac{8}{3}-2\right)^2 + \left(\frac{\sqrt{65}}{3}-0\right)^2}$$

$$= \sqrt{\left(\frac{8}{3}-\frac{6}{3}\right)^2 + \left(\frac{\sqrt{65}}{3}\right)^2}$$

$$= \sqrt{\left(\frac{2}{3}\right)^2 + \left(\frac{\sqrt{65}}{3}\right)^2}$$

$$= \sqrt{\frac{4}{9}+\frac{65}{9}} = \sqrt{\frac{69}{9}} = \frac{\sqrt{69}}{3}.$$

Step 6: Value of $12\gamma^2$.

The value of $12\gamma^2$ is:

$$12\gamma^2 = 12 \cdot \left(\frac{\sqrt{69}}{3}\right)^2 = 12 \cdot \frac{69}{9} = \frac{12 \cdot 69}{9} = 92.$$

Conclusive Answer: The value of $12\gamma^2$ is 92.

Quick Tip

To determine the radius of a circle inscribed in an ellipse, find the point on the ellipse boundary closest to the circle's center and calculate the radius accordingly.

6: Let the mean of 6 observations $1, 2, 4, 5, x,$ and y and their variance be 10. Then their mean deviation about the mean is equal to:

(1) $\frac{7}{3}$

(2) $\frac{10}{3}$

(3) $\frac{8}{3}$

(4) 3

Correct Answer: (3) $\frac{8}{3}$

Solution:

Provided that: The mean of $1, 2, 4, 5, x, y$ is 5, and the variance is 10.

Step 1: Calculate the mean.

The formula for the mean is:

$$\text{Mean} = \frac{\text{Sum of all values}}{\text{Number of values}}.$$

Substituting the Provided that values:

$$\frac{12 + x + y}{6} = 5.$$

Simplifying:

$$12 + x + y = 30 \implies x + y = 18.$$

Step 2:Apply the formula for variance.

The formula for variance is:

$$\text{Variance} = \frac{\text{Sum of squares of values}}{\text{Number of values}} - (\text{Mean})^2.$$

Substituting the Provided that variance 10 and mean 5:

$$\frac{x^2 + y^2 + 46}{6} - 5^2 = 10.$$

Simplify:

$$\frac{x^2 + y^2 + 46}{6} - 25 = 10 \implies \frac{x^2 + y^2 + 46}{6} = 35.$$

Multiply through by 6:

$$x^2 + y^2 + 46 = 210 \implies x^2 + y^2 = 164.$$

Step 3: Solve for x and y.

From $x + y = 18$ and $x^2 + y^2 = 164$, we solve:

$$(x + y)^2 = x^2 + y^2 + 2xy \implies 18^2 = 164 + 2xy \implies 324 = 164 + 2xy.$$

$$2xy = 160 \implies xy = 80.$$

The quadratic equation for x and y is:

$$t^2 - (x + y)t + xy = 0 \implies t^2 - 18t + 80 = 0.$$

Solving this quadratic equation:

$$t = \frac{-(-18) \pm \sqrt{(-18)^2 - 4(1)(80)}}{2(1)} = \frac{18 \pm \sqrt{324 - 320}}{2} = \frac{18 \pm 2}{2}.$$

$$t = 10 \text{ or } t = 8.$$

Thus, $x = 8$ and $y = 10$ (or vice versa).

Step 4: Calculate the mean deviation.

The formula for mean deviation is:

$$\text{Mean Deviation} = \frac{\sum |x - \bar{x}|}{\text{Number of values}},$$

where $\bar{x}$ is the mean. Substituting the values:

$$\text{Mean Deviation} = \frac{|1 - 5| + |2 - 5| + |4 - 5| + |5 - 5| + |8 - 5| + |10 - 5|}{6}.$$

Simplify:

$$\text{Mean Deviation} = \frac{4+3+1+0+3+5}{6} = \frac{16}{6} = \frac{8}{3}.$$

Conclusive Answer: The mean deviation is $\frac{8}{3}$.

Quick Tip

The mean deviation about the mean is calculated using absolute differences of observations from their mean. It measures dispersion in the data set.

7: Let $A = \{1, 3, 4, 6, 9\}$ and $B = \{2, 4, 5, 8, 10\}$. Let R be a relation defined on $A \times B$ such that $R = \{((a_1, b_1), (a_2, b_2)) : a_1 \leq b_2$ and $b_1 \leq a_2\}$. Then the number of elements in the set R is:

(1) 52

(2) 160

(3) 26

(4) 180

Correct Answer: (2) 160

Solution:

Step 1: Analyze the Provided that conditions.

We are Provided that choices for a_1 and b_2 as follows:

- $a_1 = 1 \Rightarrow 5$ choices of b_2
- $a_1 = 3 \Rightarrow 4$ choices of b_2
- $a_1 = 4 \Rightarrow 4$ choices of b_2
- $a_1 = 6 \Rightarrow 2$ choices of b_2
- $a_1 = 9 \Rightarrow 1$ choice of b_2

The total number of ways for (a_1, b_2) is 16 ways.

Step 2: Analyze choices for b_1 and a_2.

Similarly, we are Provided that choices for b_1 and a_2 as follows:

- $b_1 = 2 \Rightarrow 4$ choices of a_2
- $b_1 = 4 \Rightarrow 3$ choices of a_2
- $b_1 = 5 \Rightarrow 2$ choices of a_2
- $b_1 = 8 \Rightarrow 1$ choice of a_2

Step 3: Calculate the total number of required elements.

Combining the above, the total required elements in R is:

$$\text{Total required elements in } R = 16 \times 10 = 160.$$

Conclusive Answer: The required number of elements in R is 160.

Quick Tip

When calculating the size of a relation involving conditions, break down the constraints and count the valid combinations for each pair systematically.

8: Let P be the plane passing through the points $(5, 3, 0)$, $(13, 3, -2)$, and $(1, 6, 2)$. For $\alpha \in \mathbb{N}$, if the distances of the points $A(3, 4, \alpha)$ and $B(2, \alpha, a)$ from the plane P are 2 and 3 respectively, then the positive value of a is:

(1) 5

(2) 6

(3) 4

(4) 3

Correct Answer: (3) 4

Solution:

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 8 & 0 & -2 \\ 4 & -3 & -2 \end{vmatrix} = \hat{i}(-6) + 8\hat{j} - 24\hat{k}$$

The determinant is computed to find the normal vector of the plane. Expanding along the first row gives the coefficients of $\hat{i}, \hat{j}, \hat{k}$.

$$\text{Normal of the plane } = 3\hat{i} - 4\hat{j} + 12\hat{k}$$

Simplifying the determinant gives the normal vector, which defines the orientation of the plane.

$$\text{Plane: } 3x - 4y + 12z = 3$$

Using the normal vector and a point on the plane, the equation of the plane is determined.

$$\text{Distance from } A(3,4,\alpha)$$

$$\frac{|9 - 16 + 12\alpha - 3|}{13} = 2$$

The distance formula is applied to point $A(3,4,\alpha)$ and equated to 2 as Provided that in the problem.

$$\alpha = 3$$

$$\alpha = -8 \text{ (rejected)}$$

Solving the distance equation gives two possible values for α. $\alpha = -8$ is rejected based on the Provided that conditions or physical constraints.

$$\text{Distance from } B(2,3,a)$$

$$\frac{|6 - 12 + 12a - 3|}{13} = 3$$

Similarly, the distance formula is applied to point $B(2,3,a)$ and equated to 3 as specified.

$$a = 4$$

Solving the equation gives the value of a that satisfies the distance condition.

Quick Tip

To find the distance of a point from a plane, always substitute into the formula for the distance. Be sure to simplify and check for positive solutions when working with natural numbers.

9: If the letters of the word MATHS are permuted and all possible words so formed are arranged as in a dictionary with serial number, then the serial number of the word THAMS is:

(1) 102

(2) 103

(3) 101

(4) 104

Correct Answer: (2) 103

Solution:

To determine the rank of the word **THAMS**, let usApply the factorial method to count the number of permutations before the word **THAMS** appears in alphabetical order.

Step 1: Assign numerical positions to the letters.

The alphabetical order of the letters **T, H, A, M, S** is:

$$A = 1, H = 2, M = 3, S = 4, T = 5$$

Thus, the word **THAMS** corresponds to the sequence: $5, 2, 1, 3, 4$.

Step 2: Count permutations for each letter.

$$\begin{aligned}
\text{For } T: 4! &\quad \text{(4 letters after T in alphabetical order: H, A, M, S)} \\
\text{For } H: 3! &\quad \text{(1 letter after H: A)} \\
\text{For } A: 2! &\quad \text{(0 letters after A)} \\
\text{For } M: 1! &\quad \text{(0 letters after M)} \\
\text{For } S: 0! &\quad \text{(0 letters after S)}
\end{aligned}$$

Step 3: Compute the total permutations before THAMS.

$$\text{Total} = 4 \times 4! + 3! \times 1 + 0 + 0 + 0$$
$$= 4 \times 24 + 6$$
$$= 96 + 6 = 102$$

Step 4: Add 1 for the word itself.

$$\text{Rank of THAMS} = 102 + 1 = 103$$

Quick Tip

To find the serial number of a word in a dictionary-like arrangement, calculate the number of permutations for each prefix and add them step by step until you reach the desired word.

10: If four distinct points with position vectors $\vec{a}, \vec{b}, \vec{c}, and\ \vec{d}$ are coplanar, then $[\vec{a}\vec{b}\vec{c}]$ is equal to:

(1) $[\vec{d}\vec{c}\vec{a}] + [\vec{b}\vec{d}\vec{a}] + [\vec{c}\vec{d}\vec{b}]$

(2) $[\vec{d}\vec{b}\vec{a}] + [\vec{a}\vec{c}\vec{d}] + [\vec{d}\vec{b}\vec{c}]$

(3) $[\vec{a}\vec{d}\vec{b}] + [\vec{d}\vec{c}\vec{a}] + [\vec{d}\vec{b}\vec{c}]$

(4) $[\vec{b}\vec{c}\vec{d}] + [\vec{d}\vec{a}\vec{c}] + [\vec{d}\vec{b}\vec{a}]$

Correct Answer: (1) $[\vec{d}\vec{c}\vec{a}] + [\vec{b}\vec{d}\vec{a}] + [\vec{c}\vec{d}\vec{b}]$

Solution:

Provided that that the vectors $\vec{a}, \vec{b}, \vec{c}, \vec{d}$ are coplanar, we need to determine the value of $[\vec{a}\,\vec{b}\,\vec{c}]$.

Step 1: Represent the condition of coplanarity.

Since the vectors $\vec{b} - \vec{a}, \vec{c} - \vec{b}, \vec{d} - \vec{c}$ are coplanar, we have:

$$[\vec{b} - \vec{a} \;\; \vec{c} - \vec{b} \;\; \vec{d} - \vec{c}] = 0$$

Step 2: Expand the determinant.

Expanding the determinant using the scalar triple product:

$$(\vec{b} - \vec{a}) \cdot \Big((\vec{c} - \vec{b}) \times (\vec{d} - \vec{c})\Big) = 0$$

Step 3: Simplify the expression.

Using the distributive property:

$$(\vec{b}-\vec{a})\cdot(\vec{c}\times\vec{b}-\vec{c}\times\vec{a}-\vec{a}\times\vec{d})=0$$

Step 4: Represent as scalar triple products.

The scalar triple products can be written as:

$$[\vec{b}\,\vec{c}\,\vec{d}]-[\vec{b}\,\vec{c}\,\vec{a}]-[\vec{b}\,\vec{a}\,\vec{d}]-[\vec{a}\,\vec{c}\,\vec{d}]=0$$

Step 5: Rearrange to find $[\vec{a}\,\vec{b}\,\vec{c}]$.

Finally, rearranging the terms gives:

$$[\vec{a}\,\vec{b}\,\vec{c}]=[\vec{d}\,\vec{c}\,\vec{a}]+[\vec{b}\,\vec{d}\,\vec{a}]+[\vec{c}\,\vec{d}\,\vec{b}]$$

Quick Tip

The scalar triple product isApplyd to determine the volume of a parallelepiped formed by three vectors. If the scalar triple product is zero, the vectors are coplanar.

11: The sum of the coefficients of three consecutive terms in the binomial expansion of $(1+x)^{n+2}$, which are in the ratio $1:3:5$, is equal to:

(1) 63

(2) 92

(3) 25

(4) 41

Correct Answer: (1) 63

Solution:

Provided that the problem:

$$\binom{n+2}{r-1}:\binom{n+2}{r}:\binom{n+2}{r+1}::1:3:5$$

Step 1: Express the first ratio.

$$\frac{\binom{n+2}{r-1}}{\binom{n+2}{r}}=\frac{1}{3}$$

Using the formula for binomial coefficients:

$$\frac{\frac{(n+2)!}{(r-1)!(n+3-r)!}}{\frac{(n+2)!}{r!(n+2-r)!}} = \frac{1}{3}$$

Simplifying:

$$\frac{r}{(n-r+3)} = \frac{1}{3}$$

$$\Rightarrow n - r + 3 = 3r$$

$$\Rightarrow n = 4r - 3 \quad \ldots(1)$$

Step 2: Express the second ratio.

$$\frac{\binom{n+2}{r}}{\binom{n+2}{r+1}} = \frac{3}{5}$$

Using the binomial coefficient formula again:

$$\frac{\frac{(n+2)!}{r!(n+2-r)!}}{\frac{(n+2)!}{(r+1)!(n+1-r)!}} = \frac{3}{5}$$

Simplifying:

$$\frac{r+1}{n+2-r} = \frac{3}{5}$$

Cross-multiplying:

$$5(r+1) = 3(n+2-r)$$

$$5r + 5 = 3n + 6 - 3r$$

$$8r - 1 = 3n \quad \ldots(2)$$

Step 3: Solve equations (1) and (2). Substitute $n = 4r - 3$ from equation (1) into equation (2):

$$8r - 1 = 3(4r - 3)$$

$$8r - 1 = 12r - 9$$

$$4r = 8 \quad \Rightarrow r = 2$$

Step 4: Solve for n. Substitute $r = 2$ into equation (1):

$$n = 4(2) - 3$$

$$n = 5$$

Step 5: Calculate the sum of combinations.

$$\text{Sum} = \binom{7}{1} + \binom{7}{2} + \binom{7}{3}$$
$$= 7 + 21 + 35 = 63$$

Conclusive Answer: The sum of the combinations is 63.

Quick Tip

When coefficients are in a ratio, equate the corresponding binomial coefficients andApply algebra to solve for n and r. Sum the required coefficients for the answer.

12: Let $y = y(x)$ be the solution of the differential equation $\frac{dy}{dx} + \frac{5}{x(x^5+1)}y = \frac{(x^5+1)^2}{x^7}$, $x > 0$. If $y(1) = 2$, then $y(2)$ is equal to:

(1) $\frac{693}{128}$

(2) $\frac{637}{128}$

(3) $\frac{697}{128}$

(4) $\frac{679}{128}$

Correct Answer: (1) $\frac{693}{128}$

Solution:

The Provided that problem involves solving a differential equation using the integrating factor (I.F.) method.

Step 1: Determine the Integrating Factor (I.F.)

$$\text{I.F.} = e^{\int \frac{5\,dx}{x(x^5+1)}} = e^{\int \frac{5x^{-6}\,dx}{(x^{-5}+1)}}$$

Let $1 + x^{-5} = t$, so:

$$-5x^{-6}\,dx = dt$$

Thus:

$$\text{I.F.} = e^{\int \frac{-dt}{t}} = e^{-\ln t} = \frac{1}{t} = \frac{x^5}{1+x^5}$$

Step 2: Solve for y.

$$y \cdot \frac{x^5}{1+x^5} = \int \frac{x^5}{1+x^5} \cdot \frac{(1+x^5)^2}{x^7}\, dx$$

Simplify the integral:

$$y \cdot \frac{x^5}{1+x^5} = \int x^3\, dx + \int x^{-2}\, dx$$

$$y \cdot \frac{x^5}{1+x^5} = \frac{x^4}{4} - \frac{1}{x} + c$$

Step 3: Apply initial conditions. Provided that $x = 1$ and $y = 2$:

$$2 \cdot \frac{1}{2} = \frac{1}{4} - 1 + c$$

$$1 = -\frac{3}{4} + c$$

$$c = \frac{7}{4}$$

Step 4: Final expression for y.

$$y \cdot \frac{x^5}{1+x^5} = \frac{x^4}{4} - \frac{1}{x} + \frac{7}{4}$$

Simplify:

$$y = \frac{x^4}{4} - \frac{1}{x} + \frac{7}{4}$$

Step 5: Compute y for $x = 2$. Substitute $x = 2$:

$$y \cdot \frac{32}{33} = \frac{16}{4} - \frac{1}{2} + \frac{7}{4}$$

$$y \cdot \frac{32}{33} = 4 - 0.5 + 1.75 = \frac{21}{4}$$

$$y = \frac{21}{4} \cdot \frac{33}{32}$$

$$y = \frac{693}{128}$$

Conclusive Answer: $y = \frac{693}{128}$

Quick Tip

To solve a first-order linear differential equation, always calculate the integrating factor first, simplify the integral step-by-step, andApply the initial condition to find the constant.

13: The converse of $((\sim p) \wedge q) \Rightarrow r$ is:

(1) $(p \vee (\sim q)) \Rightarrow (\sim r)$

(2) $((\sim p) \vee q) \Rightarrow r$

(3) $(\sim r) \Rightarrow ((\sim p) \wedge q)$

(4) $(\sim r) \Rightarrow (p \wedge q)$

Correct Answer: (1) $(p \vee (\sim q)) \Rightarrow (\sim r)$

Solution:

Provided that: The statement is $((\sim p) \wedge q) \Rightarrow r$. The converse of a conditional statement $(A \Rightarrow B)$ is defined as $(B \Rightarrow A)$.

Step 1: Identify A and B.

Here:

$$A = ((\sim p) \wedge q), \quad B = r.$$

Thus, the converse is:

$$r \Rightarrow ((\sim p) \wedge q).$$

Step 2: Express the negation of the implication.

The negation of $((\sim p) \wedge q) \Rightarrow r$ is:

$$\sim (((\sim p) \wedge q) \Rightarrow r) = (\sim r) \Rightarrow ((\sim p) \wedge q).$$

Step 3: Derive the logical equivalence.

The converse can also be written equivalently as:

$$r \Rightarrow ((\sim p) \wedge q) \implies (\sim ((\sim p) \wedge q)) \Rightarrow (\sim r).$$

Simplifying further:

$$(p \vee (\sim q)) \Rightarrow (\sim r).$$

Conclusive Answer: The converse is $(p \vee (\sim q)) \Rightarrow (\sim r)$.

Quick Tip

The converse of a conditional statement $(A \Rightarrow B)$ is $(B \Rightarrow A)$. Logical operators like $\wedge$, $\vee$, and $\sim$ follow specific equivalence rules to simplify complex expressions.

14: If the 1011th term from the end in the binomial expansion of $\left(\frac{4x}{5} - \frac{5}{2x}\right)^{2022}$ is 1024 times the 1011th term from the beginning, the $|x|$ is equal to:

(1) 8

(2) 12

(3) $\frac{5}{16}$

(4) 15

Correct Answer: (3) $\frac{5}{16}$

Solution:

We are tasked with evaluating T_{1011} from both the beginning and the end of the expansion.

Step 1: T_{1011} from the beginning.

$$T_{1011} \text{ from beginning } = T_{1010+1} = \binom{2022}{1010}\left(\frac{4x}{5}\right)^{1012}\left(\frac{-5}{2x}\right)^{1010}$$

Step 2: T_{1011} from the end.

$$T_{1011} \text{ from end } = \binom{2022}{1010}\left(\frac{-5}{2x}\right)^{1012}\left(\frac{4x}{5}\right)^{1010}$$

Step 3: Equate T_{1011} from beginning and end.

$$T_{1011} = \binom{2022}{1010}\left(\frac{-5}{2x}\right)^{1012}\left(\frac{4x}{5}\right)^{1010}$$

$$= 2^{10} \cdot \binom{2022}{1010}\left(\frac{-5}{2x}\right)^{1010}\left(\frac{4x}{5}\right)^{1012}$$

Step 4: Simplify the ratio. Using the powers:

$$\left(\frac{-5}{2x}\right)^{2} = 2^{10} \cdot \left(\frac{4x}{5}\right)^{2}$$

Step 5: Solve for x^4. Simplifying the equation:

$$x^4 = \frac{5^4}{2^{16}}$$

$$|x| = \frac{5}{16}$$

Conclusive Answer: $|x| = \frac{5}{16}$

Quick Tip

When working with binomial expansions and terms from the beginning and end,Apply the symmetry property of binomial coefficients. Carefully simplify the algebra to avoid mistakes.

15: If the system of linear equations:

$$7x + 11y + \alpha z = 13, \quad 5x + 4y + 7z = \beta, \quad 175x + 194y + 57z = 361,$$

has infinitely many solutions, then $\alpha + \beta + 2$ is equal to:

(1) 3

(2) 6

(3) 5

(4) 4

Correct Answer: (4) 4

Solution:

We are Provided that the system of equations:

$$7x + 11y + \alpha z = 13 \quad \ldots\text{(i)}$$

$$5x + 4y + 7z = \beta \quad \ldots\text{(ii)}$$

$$175x + 194y + 57z = 361 \quad \ldots\text{(iii)}$$

Step 1: Eliminate x and y.

Multiply equation (i) by 10 and equation (ii) by 21, then subtract from equation (iii):

$$10 \times (7x + 11y + \alpha z) + 21 \times (5x + 4y + 7z) - (175x + 194y + 57z) = 0$$

Simplify:

$$z \cdot (10\alpha + 147 - 57) = 130 + 21\beta - 361$$

Step 2: Solve for α.

Equating coefficients of z:

$$10\alpha + 90 = 0$$

$$\alpha = -9$$

Step 3: Solve for β.

Substitute the known value of α into the equation:

$$130 - 361 + 21\beta = 0$$

$$21\beta = 231$$

$$\beta = 11$$

Step 4: Verify the final condition.

The condition $\alpha + \beta + 2 = 4$ is satisfied:

$$-9 + 11 + 2 = 4$$

Quick Tip

To determine whether a system of linear equations has infinitely many solutions, always check the determinant of the coefficient matrix and ensure that the consistency conditions are satisfied.

16: Let the line passing through the point $P(2, -1, 2)$ and $Q(5, 3, 4)$ meet the plane $x - y + z = 4$ at the point T. Then the distance of the point R from the plane $x + 2y + 3z + 2 = 0$, measured parallel to the line $\frac{x-7}{2} = \frac{y+3}{2} = \frac{z-2}{1}$, is equal to:

(1) 3

(2) $\sqrt{61}$

(3) $\sqrt{31}$

(4) $\sqrt{189}$

Correct Answer: (1) 3

Solution:

We are Provided that two lines:

$$\text{Line 1: } \frac{x-5}{3} = \frac{y-3}{4} = \frac{z-4}{2} = \lambda$$
$$\text{Line 2: } \frac{x+1}{2} = \frac{y+5}{2} = \frac{z-0}{1} = \mu$$

Step 1: Parametric equation of Line 1.

The parametric coordinates of a point R on Line 1 are:

$$R(\lambda) = (3\lambda + 5,\ 4\lambda + 3,\ 2\lambda + 4)$$

The line lies on the plane $3x+5-4y-3+2z+4 = 4$. Substituting the parametric coordinates of R:

$$3(3\lambda + 5) + 5 - 4(4\lambda + 3) - 3 + 2(2\lambda + 4) + 4 = 4$$

Simplify:

$$9\lambda + 15 + 5 - 16\lambda - 12 - 3 + 4\lambda + 8 + 4 = 4$$

$$-3\lambda + 17 = 4$$

$$\lambda = -2$$

Substitute $\lambda = -2$ into the parametric equation of Line 1:

$$R = (3(-2) + 5,\ 4(-2) + 3,\ 2(-2) + 4)$$

$$R = (-1, -5, 0)$$

Step 2: Parametric equation of Line 2.

The parametric coordinates of a point T on Line 2 are:

$$T(\mu) = (2\mu - 1,\ 2\mu - 5,\ \mu)$$

Step 3: Point T lies on the plane.

Substitute $T(\mu)$ into the plane equation:

$$2(2\mu - 1) + 2(2\mu - 5) + 3\mu + 2 = 0$$

Simplify:

$$4\mu - 2 + 4\mu - 10 + 3\mu + 2 = 0$$

$$11\mu - 10 = 0$$

$$\mu = 1$$

Substitute $\mu = 1$ into the parametric equation of Line 2:

$$T = (2(1) - 1,\ 2(1) - 5,\ 1)$$

$$T = (1, -3, 1)$$

Step 4: Find the distance RT.

The coordinates of R are $(-1, -5, 0)$, and the coordinates of T are $(1, -3, 1)$.

The distance RT is Provided that by:

$$RT = \sqrt{(1 - (-1))^2 + (-3 - (-5))^2 + (1 - 0)^2}$$

$$RT = \sqrt{(1 + 1)^2 + (-3 + 5)^2 + (1)^2}$$

$$RT = \sqrt{2^2 + 2^2 + 1^2}$$

$$RT = \sqrt{4 + 4 + 1} = \sqrt{9} = 3$$

Conclusive Answer: The distance $RT = 3$.

Quick Tip

To find the distance of a point from a plane measured parallel to a line, compute the distance using the projection formula and normalize it with the line's direction vector.

17: Let the function $f : [0, 2] \to \mathbb{R}$ be defined as:

$$f(x) = \begin{cases} e^{\min\{x^2, x - \lfloor x \rfloor\}}, & x \in [0, 1), \\ e^{x - \log_e x}, & x \in [1, 2), \end{cases}$$

where $\lfloor t \rfloor$ denotes the greatest integer less than or equal to t.

Then the value of the integral $\int_0^2 x f(x)\, dx$ is:

(1) $(e-1)\left(e^2+\frac{1}{2}\right)$

(2) $1+\frac{3e}{2}$

(3) $2e-\frac{1}{2}$

(4) $2e-1$

Correct Answer: (3) $2e-\frac{1}{2}$

Solution:

We are Provided that the function $F:[0,2]\to\mathbb{R}$ defined as:

$$F(x)=\begin{cases}\min\{x^2,\{x\}\}, & x\in[0,1)\\ [x-\log_e x]=1, & x\in[1,2)\end{cases}$$

For the integral, the value of $F(x)$ is simplified as:

$$F(x)=\begin{cases}e^{x^2}, & x\in[0,1)\\ e, & x\in[1,2)\end{cases}$$

Step 1: Evaluate the integral from 0 to 2.

$$\int_0^2 xF(x)\,dx=\int_0^1 xe^{x^2}\,dx+\int_1^2 xe\,dx$$

Step 2: Solve the first integral.

Let $I_1=\int_0^1 xe^{x^2}\,dx$.

Substitute $u=x^2$, so $du=2x\,dx$.

The limits change as:

$$x=0\Rightarrow u=0,\quad x=1\Rightarrow u=1$$

Thus:

$$I_1=\frac{1}{2}\int_0^1 e^u\,du=\frac{1}{2}[e^u]_0^1=\frac{1}{2}(e-1)$$

Step 3: Solve the second integral.

Let $I_2=\int_1^2 xe\,dx$.

Since e is constant:

$$I_2=e\int_1^2 x\,dx=e\left[\frac{x^2}{2}\right]_1^2$$

$$I_2 = e \cdot \frac{1}{2}\left[4-1\right] = \frac{1}{2}e(4-1) = \frac{3}{2}e$$

Step 4: Combine the results.

$$\int_0^2 xF(x)\,dx = I_1 + I_2 = \frac{1}{2}(e-1) + \frac{3}{2}e$$

$$\int_0^2 xF(x)\,dx = \frac{1}{2}e - \frac{1}{2} + \frac{3}{2}e = 2e - \frac{1}{2}$$

Conclusive Answer: $\int_0^2 xF(x)\,dx = 2e - \frac{1}{2}$

Quick Tip

To evaluate integrals with piecewise-defined functions, break the integral into parts according to the domain of the function, andApply appropriate substitutions to simplify each part.

18: For $a \in \mathbb{C}$, let:

$$A = \{z \in \mathbb{C} : \text{Re}(a+\overline{z}) > \text{Im}(\overline{a}+z)\}, \textbf{and} \quad B = \{z \in \mathbb{C} : \text{Re}(a+\overline{z}) < \text{Im}(\overline{a}+z)\}.$$

Among the two statements:

(S1): If $\text{Re}(a), \text{Im}(a) > 0$, then the set A contains all the real numbers.

(S2): If $\text{Re}(a), \text{Im}(a) < 0$, then the set B contains all the real numbers.

(1) Only (S1) is true

(2) Both are false

(3) Only (S2) is true

(4) Both are true

Correct Answer: (2) Both are false

Solution:

Let $a = x_1 + iy_1$ and $z = x + iy$.

Step 1: Analyze the inequality.

The Provided that inequality is:

$$\text{Re}(a+\overline{z}) > \text{Im}(\overline{a}+z)$$

Simplify both sides:

$$\text{Re}(a + \overline{z}) = x_1 + x, \quad \text{Im}(\overline{a} + z) = -y_1 + y$$

Thus, the inequality becomes:

$$x_1 + x > -y_1 + y$$

Step 2: Check Statement S1.

Provided that values for $S1$:

$$x_1 = 2,\ y_1 = 10,\ x = -12,\ y = 0$$

Substitute these into the inequality:

$$2 - 12 > -10 + 0$$

$$-10 > -10$$

The inequality is not valid. Hence, $S1$ is false.

Step 3: Check Statement S2.

The second inequality is:

$$\text{Re}(a + \overline{z}) < \text{Im}(\overline{a} + z)$$

$$x_1 + x < -y_1 + y$$

Provided that values for $S2$:

$$x_1 = -2,\ y_1 = -10,\ x = 12,\ y = 0$$

Substitute these into the inequality:

$$-2 + 12 < -(-10) + 0$$

$$10 < 10$$

The inequality is not valid. Hence, $S2$ is false.

Conclusive Answer: Both $S1$ and $S2$ are false.

Quick Tip

When working with sets involving real and imaginary parts of complex numbers, substitute expressions for $\text{Re}(z)$ and $\text{Im}(z)$ explicitly to simplify the conditions and verify the set membership.

19: If:

$$\begin{vmatrix} x+1 & x & x \\ x & x+\lambda & x \\ x & x & x+\lambda^2 \end{vmatrix} = \frac{9}{8}(103x+81),$$

then $\lambda, \frac{\lambda}{3}$ are the roots of the equation:

(1) $4x^2 - 24x - 27 = 0$

(2) $4x^2 + 24x + 27 = 0$

(3) $4x^2 - 24x + 27 = 0$

(4) $4x^2 + 24x - 27 = 0$

Correct Answer: (3) $4x^2 - 24x + 27 = 0$

Solution:

We are Provided that the determinant:

$$\begin{vmatrix} x+1 & x & x \\ x & x+d & x \\ x & x & x+d^2 \end{vmatrix} = \frac{9}{8}(103x+81)$$

Step 1: Substitute $x = 0$.

When $x = 0$, the determinant simplifies to:

$$\begin{vmatrix} 1 & 0 & 0 \\ 0 & d & 0 \\ 0 & 0 & d^2 \end{vmatrix} = \frac{9}{8} \cdot 81$$

Simplify the determinant:

$$1 \cdot d \cdot d^2 = d^3$$

Thus:

$$d^3 = \frac{9}{8} \cdot 81$$
$$d^3 = \frac{729}{8}$$

Step 2: Solve for d.

$$d = \sqrt[3]{\frac{729}{8}} = \frac{\sqrt[3]{729}}{\sqrt[3]{8}} = \frac{9}{2}$$

Step 3: Verify the result.

The eigenvalues of the determinant matrix are:

$$\lambda^3 = \frac{9}{8} \cdot 81$$
$$\lambda^3 = \frac{729}{8}$$
$$\lambda = \sqrt[3]{\frac{729}{8}} = \frac{9}{2}$$

The characteristic equation associated with the eigenvalues is:

$$4x^2 - 24x + 27 = 0$$

This equation has roots:

$$\lambda = \frac{9}{2} \quad \text{and} \quad \lambda = \frac{3}{2}$$

Conclusive Answer: The correct option is **(C)**.

Quick Tip

When solving determinant equations, expand carefully and simplify the minors before substituting back. Equating to the Provided that condition will reveal the desired result.

20: The domain of the function $f(x) = \frac{1}{\sqrt{\lfloor x \rfloor^2 - 3\lfloor x \rfloor - 10}}$, where $\lfloor x \rfloor$ denotes the greatest integer less than or equal to x, is:

(1) $(-\infty, -3] \cup [6, \infty)$

(2) $(-\infty, -2) \cup (5, \infty)$

(3) $(-\infty, -3] \cup (5, \infty)$

(4) $(-\infty, -2) \cup [6, \infty)$

Correct Answer: (4) $(-\infty, -2) \cup [6, \infty)$

Solution:

We are Provided that the function:

$$F(x) = \frac{1}{\sqrt{\lfloor x \rfloor^2 - 3\lfloor x \rfloor - 10}}$$

To ensure that $F(x)$ is defined, the denominator must be positive:

$$\lfloor x \rfloor^2 - 3\lfloor x \rfloor - 10 > 0$$

Step 1: Solve the inequality.

Factorize the quadratic:

$$\lfloor x \rfloor^2 - 3\lfloor x \rfloor - 10 = (\lfloor x \rfloor + 2)(\lfloor x \rfloor - 5)$$

Thus, the inequality becomes:

$$(\lfloor x \rfloor + 2)(\lfloor x \rfloor - 5) > 0$$

Step 2: Analyze the intervals.

The roots of the quadratic are $\lfloor x \rfloor = -2$ and $\lfloor x \rfloor = 5$. Using a sign chart:

$\lfloor x \rfloor \in$	$(-\infty, -2)$	$(-2, 5)$	$(5, \infty)$
Sign of $(\lfloor x \rfloor + 2)(\lfloor x \rfloor - 5)$	$+$	$-$	$+$

The inequality is satisfied when:

$$\lfloor x \rfloor < -2 \quad \text{or} \quad \lfloor x \rfloor > 5$$

Step 3: Refine the solution.

Since $\lfloor x \rfloor$ is the greatest integer less than or equal to x, we must consider:

$$\lfloor x \rfloor \leq -3 \quad \text{or} \quad \lfloor x \rfloor \geq 6$$

Step 4: Write the solution for x.

The values of x satisfying the inequality are:

$$x \in (-\infty, -2) \cup [6, \infty)$$

Conclusive Answer:

$$x \in (-\infty, -2) \cup [6, \infty)$$

Quick Tip

To determine the domain of a function involving $\lfloor x \rfloor$, solve inequalities for $\lfloor x \rfloor$, then map the results back to the corresponding intervals for x.

Section B

21: If A is the area in the first quadrant enclosed by the curve $C : 2x^2 - y + 1 = 0$, the tangent to C at the point $(1, 3)$, and the line $x + y = 1$, then the value of $60A$ is _____:

Correct Answer: 16

Solution:

The equation of the curve is:

$$y = 2x^2 + 1$$

The tangent at the point $P(1, 3)$ is:

$$y = 4x - 1$$

Shaded Area Calculation:

$$A = \int_0^1 (2x^2 + 1)\, dx - \text{area of } \triangle QOT - \text{area of } \triangle PQR + \text{area of } \triangle QRS$$

Step 1: Solve the integral.

$$\int_0^1 (2x^2 + 1)\, dx = \left[\frac{2x^3}{3} + x\right]_0^1 = \frac{2}{3} + 1 = \frac{5}{3}$$

Step 2: Compute the areas of the triangles. 1. $\triangle QOT$:

$$\text{Area} = \frac{1}{2} \cdot \text{base} \cdot \text{height} = \frac{1}{2} \cdot 1 \cdot 1 = \frac{1}{2}$$

2. $\triangle PQR$:

$$\text{Vertices: } P(1, 3), Q(1, 0), R\left(\frac{1}{4}, 0\right)$$

$$\text{Area} = \frac{1}{2}\left|1(0-0) + 1(0-3) + \frac{1}{4}(3-0)\right| = \frac{1}{2}\left|0 - 3 + \frac{3}{4}\right| = \frac{1}{2}\cdot\frac{9}{4} = \frac{9}{8}$$

3. $\triangle QRS$:

$$\text{Vertices: } Q(1,0), R\left(\frac{1}{4},0\right), S\left(\frac{2}{5},\frac{13}{5}\right)$$

$$\text{Area} = \frac{1}{2}\left|1\left(0-\frac{13}{5}\right) + \frac{1}{4}\left(\frac{13}{5}-0\right) + \frac{2}{5}(0-0)\right|$$

$$= \frac{1}{2}\left|-\frac{13}{5} + \frac{13}{20}\right| = \frac{1}{2}\cdot\frac{39}{40} = \frac{9}{40}$$

Step 3: Combine the results.

$$A = \left(\frac{5}{3}\right) - \frac{1}{2} - \frac{9}{8} + \frac{9}{40}$$

Find the LCM and simplify:

$$A = \frac{200}{120} - \frac{60}{120} - \frac{135}{120} + \frac{27}{120} = \frac{200-60-135+27}{120} = \frac{32}{120} = \frac{16}{60}$$

Conclusive Answer: The shaded area is $A = \frac{16}{60}$.

Quick Tip

When finding areas enclosed by curves and lines, ensure the limits of integration match the points of intersection, and carefully simplify the integrand before integrating.

22: Let $A = \{1,2,3,4,5\}$ and $B = \{1,2,3,4,5,6\}$. Then the number of functions $f : A \to B$ satisfying $f(1) + f(2) = f(4) - 1$ is equal to ____.

Correct Answer: 360

Solution:

We are Provided that:

$$f(1) + f(2) + 1 = f(4) \leq 6$$

This implies:

$$f(1) + f(2) \leq 5$$

Step 1: Analyze cases for $f(1)$ and $f(2)$.

The values of $f(1)$ and $f(2)$ are integers such that their sum satisfies the inequality $f(1) + f(2) \leq 5$. We consider the following cases:

Case (i): $f(1) = 1$

$$f(2) = 1, 2, 3, 4 \quad \text{(4 mappings)}$$

Case (ii): $f(1) = 2$

$$f(2) = 1, 2, 3 \quad \text{(3 mappings)}$$

Case (iii): $f(1) = 3$

$$f(2) = 1, 2 \quad \text{(2 mappings)}$$

Case (iv): $f(1) = 4$

$$f(2) = 1 \quad \text{(1 mapping)}$$

Step 2: Count mappings for $f(5)$ and $f(6)$.

Both $f(5)$ and $f(6)$ can each take 6 possible values independently.

Step 3: Calculate the total number of functions.

The total number of functions is:

$$(4 + 3 + 2 + 1) \times 6 \times 6$$

$$= 10 \times 6 \times 6 = 360$$

Conclusive Answer: The total number of functions is 360.

Quick Tip

To solve problems involving functions under constraints, identify the restricted variables and count the number of valid combinations.Apply the principle of counting and account for all other unrestricted variables.

23: Let the tangent to the parabola $y^2 = 12x$ at the point $(3, \alpha)$ be perpendicular to the line $2x + 2y = 3$. Then the square of the distance of the point $(6, -4)$ from the normal to the hyperbola $\alpha^2x^2 - 9y^2 = 9\alpha^2$ at its point $(\alpha - 1, \alpha + 2)$ is equal to _____.

Correct Answer: 116

Solution:

The point $P(3, \alpha)$ lies on the parabola $y^2 = 12x$. Substituting $x = 3$:

$$\alpha^2 = 12 \cdot 3 = 36 \quad \Rightarrow \quad \alpha = \pm 6$$

Step 1: Determine the correct value of α.

The slope of the tangent at $P(3, \alpha)$ is:

$$\frac{dy}{dx} = \frac{6}{\alpha}$$

Provided that that the slope is 1:

$$\frac{6}{\alpha} = 1 \quad \Rightarrow \quad \alpha = 6 \quad (\alpha = -6 \text{ is rejected}).$$

Step 2: Equation of the hyperbola.

The hyperbola is Provided that as:

$$\frac{x^2}{9} - \frac{y^2}{36} = 1$$

The normal at $P(3, 6)$ is:

$$\text{Point } Q(\alpha - 1, \alpha + 2) = (3 - 1, 6 + 2) = (2, 8)$$

Substitute $Q(2, 8)$ into the hyperbola equation:

$$\frac{9x}{5} + \frac{36y}{8} = 45$$

Step 3: Equation of the normal.

The normal passes through $(3, 6)$:

$$2x + 5y - 50 = 0$$

Step 4: Distance of point $(6, -4)$ from the normal.

The distance of $(6, -4)$ from the line $2x + 5y - 50 = 0$ is:

$$\text{Distance} = \frac{|2(6) + 5(-4) - 50|}{\sqrt{2^2 + 5^2}}$$

Simplify:

$$\text{Distance} = \frac{|12 - 20 - 50|}{\sqrt{4 + 25}} = \frac{58}{\sqrt{29}}$$

Step 5: Square of the distance.

The square of the distance is:

$$\left(\frac{58}{\sqrt{29}}\right)^2 = 116$$

Conclusive Answer: The square of the distance is 116.

> **Quick Tip**
>
> To find the square of the distance from a point to a line,Apply the distance formula and square the result. Simplify calculations step-by-step to avoid errors.

24: For $k \in \mathbb{N}$, if the sum of the series $1 + \frac{4}{k} + \frac{8}{k^2} + \frac{13}{k^3} + \frac{19}{k^4} + \ldots$ is 10, then the value of k is _____.

Correct Answer: 2

Solution:

We are Provided that the infinite series:

$$10 = 1 + \frac{4}{k^2} + \frac{8}{k^3} + \frac{13}{k^4} + \frac{19}{k^5} + \ldots \text{ (up to infinity).}$$

Step 1: Simplify the series.

Subtract 1 from both sides:

$$9 = \frac{4}{k^2} + \frac{8}{k^3} + \frac{13}{k^4} + \frac{19}{k^5} + \ldots$$

Multiply through by k:

$$\frac{9}{k} = \frac{4}{k} + \frac{8}{k^2} + \frac{13}{k^3} + \ldots$$

Reorganizing:

$$S = 9\left(1 - \frac{1}{k}\right) = \frac{4}{k} + \frac{4}{k^2} + \frac{5}{k^3} + \frac{6}{k^4} + \ldots$$

Step 2: Rewrite the series.

The series can be expressed as:

$$\frac{S}{k} = \frac{4}{k^2} + \frac{4}{k^3} + \frac{5}{k^4} + \ldots$$

Substituting back:

$$\left(1 - \frac{1}{k}\right) S = \frac{4}{k} + \frac{1}{k^3} + \frac{1}{k^4} + \ldots$$

The infinite geometric series for terms starting from $\frac{1}{k^3}$ is:

$$\sum_{n=3}^{\infty} \frac{1}{k^n} = \frac{\frac{1}{k^3}}{1-\frac{1}{k}}$$

Step 3: Solve for S.

Combine all terms:

$$9\left(1-\frac{1}{k}\right)^2 = \frac{4}{k} + \frac{1}{k^3} \cdot \frac{1}{1-\frac{1}{k}}$$

Simplify further:

$$9\left(1-\frac{1}{k}\right)^3 = 4k\left(1-\frac{1}{k}\right) + 1$$

Step 4: Solve for k.

Equate and solve:

$$9(k-1)^3 = 4k(k-1) + 1$$

Simplify and solve for k:

$$k = 2$$

Conclusive Answer: $k = 2$.

Quick Tip

When solving problems involving series with variable coefficients, first identify the pattern in the numerator or denominator.Apply quadratic or higher-order sequences if necessary.

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25. Let the line $\ell : x = \frac{1-y}{-2} = \frac{z-3}{\lambda}$, $\lambda \in \mathbb{R}$ meet the plane $P : x + 2y + 3z = 4$ at the point (α, β, γ). If the angle between the line ℓ and the plane P is $\cos^{-1}\left(\frac{\sqrt{5}}{\sqrt{14}}\right)$, then $\alpha + 2\beta + 6\gamma$ is equal to ____.

Correct Answer: 11

Solution:

The Provided that line ℓ is:

$$\frac{y-1}{2} = \frac{z-3}{\lambda}, \quad \lambda \in \mathbb{R}.$$

Step 1: Direction ratios of the line.

The direction ratios (Dr's) of the line ℓ are $(1, 2, \lambda)$.

Step 2: Direction ratios of the normal vector of the plane.

The equation of the plane is:

$$x + 2y + 3z = 4.$$

The direction ratios of the normal vector of the plane are $(1, 2, 3)$.

Step 3: Angle between the line and the plane.

The angle between the line ℓ and the plane P is Provided that by:

$$\sin\theta = \frac{|1 + 4 + 3\lambda|}{\sqrt{5 + \lambda^2} \cdot \sqrt{14}}.$$

It is Provided that that:

$$\cos\theta = \sqrt{\frac{5}{14}}, \quad \text{so } \sin\theta = \frac{3}{\sqrt{14}}.$$

Equating:

$$\frac{|1 + 4 + 3\lambda|}{\sqrt{5 + \lambda^2} \cdot \sqrt{14}} = \frac{3}{\sqrt{14}}.$$

Simplify:

$$|1 + 4 + 3\lambda| = 3 \quad \Rightarrow \quad 1 + 4 + 3\lambda = 3.$$

Solve for λ:

$$3\lambda = 2 \quad \Rightarrow \quad \lambda = \frac{2}{3}.$$

Step 4: Variable point on the line.

The parametric equation of the line ℓ is:

$$\left(t, 2t + 1, \frac{2}{3}t + 3\right).$$

Step 5: Find the intersection point with the plane.

The point lies on the plane:

$$t + 2(2t + 1) + 3\left(\frac{2}{3}t + 3\right) = 4.$$

Simplify:

$$t + 4t + 2 + 2t + 9 = 4.$$

$$7t + 11 = 4 \quad \Rightarrow \quad t = -1.$$

Substitute $t = -1$ into the parametric equation of the line:

$$\left(-1, -1, \frac{7}{3}\right) = (\alpha, \beta, \gamma).$$

Step 6: Verify the condition.

The point satisfies:

$$\alpha + 2\beta + 6\gamma = -1 + 2(-1) + 6\left(\frac{7}{3}\right).$$

Simplify:

$$-1 - 2 + 14 = 11.$$

Conclusive Answer: The point is $(-1, -1, \frac{7}{3})$, and the condition $\alpha + 2\beta + 6\gamma = 11$ is satisfied.

Quick Tip

To find the angle between a line and a plane,Apply the sine formula involving the dot product of direction ratios and normal vectors. For parametric equations, substitute carefully into the plane equation to find points of intersection.

26: The number of points where the curve $f(x) = e^{8x} - e^{6x} - 3e^{4x} - e^{2x} + 1, x \in \mathbb{R}$ cuts the x-axis, is equal to ____.

Correct Answer: 2

Solution:

Let:

$$e^{2x} = t$$

The Provided that equation becomes:

$$t^4 - t^3 - 3t^2 - t + 1 = 0$$

Step 1: Transform the equation.

Divide through by t^2:

$$t^2 + \frac{1}{t^2} - \left(t + \frac{1}{t}\right) - 3 = 0$$

Step 2: Substitute $t + \frac{1}{t}$.

Let:

$$t + \frac{1}{t} = u$$

Using the identity:

$$t^2 + \frac{1}{t^2} = u^2 - 2,$$

the equation becomes:

$$u^2 - 2 - u - 3 = 0$$

$$u^2 - u - 5 = 0$$

Step 3: Solve for u.

The quadratic equation $u^2 - u - 5 = 0$ has roots:

$$u = \frac{1 \pm \sqrt{21}}{2}$$

Thus:

$$t + \frac{1}{t} = \frac{1 \pm \sqrt{21}}{2}$$

Step 4: Conclude the solution.

The two real values of t correspond to the two roots of the transformed equation. These are obtained from:

$$t + \frac{1}{t} = \frac{1 + \sqrt{21}}{2} \quad \text{and} \quad t + \frac{1}{t} = \frac{1 - \sqrt{21}}{2}.$$

Conclusive Answer: There are two real values of t.

Quick Tip

When solving exponential equations,Apply substitution to reduce the degree of complexity, and check for valid solutions in the domain of exponential functions.

27: If the line $l_1 : 3y - 2x = 3$ is the angular bisector of the line $l_2 : x - y + 1 = 0$ and $l_3 : \alpha x + \beta y + 17 = 0$, then $\alpha^2 + \beta^2 - \alpha - \beta$ is equal to _____.

Correct Answer: 348

Solution:

Step 1: Find the point of intersection.

The Provided that lines are:

$$\ell_1 : 3y - 2x = 3, \quad \ell_2 : x - y + 1 = 0$$

The point of intersection of ℓ_1 and ℓ_2 is:

$$P \equiv (0, 1).$$

Step 2: Verify the line ℓ_3.

The point $P(0, 1)$ lies on the line:

$$\ell_3 : \alpha x - \beta y + 17 = 0.$$

Substitute $P(0, 1)$ into ℓ_3:

$$\alpha(0) - \beta(1) + 17 = 0 \quad \Rightarrow \quad \beta = -17.$$

Step 3: Find the image of a random point about ℓ_2.

Consider a random point $Q \equiv (-1, 0)$ on $\ell_2 : x - y + 1 = 0$.

The image of $Q(-1, 0)$ about ℓ_2 is:

$$Q' \equiv \left(-\frac{17}{13}, \frac{6}{13}\right).$$

This is calculated using the formula for the reflection of a point about a line.

Step 4: Verify that Q' lies on ℓ_3.

Substitute $Q'\left(-\frac{17}{13}, \frac{6}{13}\right)$ into ℓ_3:

$$\ell_3 : \alpha x - \beta y + 17 = 0.$$

Substitute $\beta = -17$:

$$\alpha\left(-\frac{17}{13}\right) - (-17)\left(\frac{6}{13}\right) + 17 = 0.$$

Simplify:

$$-\frac{17\alpha}{13} + \frac{102}{13} + 17 = 0.$$

Equating coefficients, solve for α:

$$\alpha = 7.$$

Step 5: Final verification.

Now substitute $\alpha = 7$ and $\beta = -17$ into the condition:

$$\alpha^2 + \beta^2 - \alpha - \beta = 348.$$

Quick Tip

When dealing with angular bisectors, equate the perpendicular distances from the point of interest to the Provided that lines, and solve using algebraic methods or substitution.

28: Let the probability of getting a head for a biased coin be $\frac{1}{4}$. It is tossed repeatedly until a head appears. Let N be the number of tosses required. If the probability that the equation $64x^2 + 5Nx + 1 = 0$ has no real root is $\frac{p}{q}$, where p and q are co-prime, then $q - p$ is equal to ____.

Correct Answer: 27

Solution:

We start with the quadratic equation:

$$64x^2 + 5Nx + 1 = 0$$

This is the Provided that equation for analysis.

Step 1: Determine the condition for real roots.

The discriminant of a quadratic equation, D, determines the nature of the roots. Here:

$$D = 25N^2 - 256 < 0$$

For the roots to be non-real, the discriminant must be negative.

Step 2: Solve for N.

Simplify the inequality:

$$N^2 < \frac{256}{25} \Rightarrow N < \frac{16}{5}$$

Since N must be an integer, the possible values of N are:

$$\Rightarrow N = 1, 2, 3$$

Step 3: Calculate the probability.

For each valid N, there are different probabilities. These are calculated as follows:

$$\text{Probability} = \frac{1}{4} + \frac{3}{4} \times \frac{1}{4} + \frac{3}{4} \times \frac{1}{4} \times \frac{1}{4}$$

$\frac{1}{4}$ is the probability of a single event, and the subsequent terms account for conditional probabilities. Simplifying:

$$\text{Probability} = \frac{37}{64}$$

Step 4: Find $q - p$.

Let $q = 37$ and $p = 10$. Then:

$$q - p = 27$$

Conclusive Answer: The value of $q - p$ is 27.

Quick Tip

For problems involving geometric distributions, calculate the probability for each valid value of N and sum them carefully. Always check conditions for integer values of N.

29: Let $\vec{a} = \hat{i} + 2\hat{j} + 3\hat{k}$ and $\vec{b} = \hat{i} + \hat{j} - \hat{k}$. If $\vec{c}$ is a vector such that $\vec{a} \cdot \vec{c} = 11$, $\vec{b} \cdot (\vec{a} \times \vec{c}) = 27$ and $\vec{b} \cdot \vec{c} = -\sqrt{3}|\vec{b}|$, then $|\vec{a} \times \vec{c}|^2$ is equal to ____:

Correct Answer: 285

Solution:

We are Provided that:

$$\vec{a} = \hat{i} + 2\hat{j} + 3\hat{k}, \quad \vec{b} = \hat{i} + \hat{j} - \hat{k}.$$

Step 1: Properties of the vectors. 1. The dot product:

$$\vec{a} \cdot \vec{b} = 0$$

This implies that $\vec{a}$ and $\vec{b}$ are perpendicular.

2. The cross product:

$$\vec{b} \times (\vec{a} \times \vec{c}) = -3\vec{a}.$$

Step 2: Relationship involving angle θ.

Let θ be the angle between $\vec{b}$ and $\vec{a} \times \vec{c}$. The magnitude of their product is:

$$|\vec{b} \cdot (\vec{a} \times \vec{c})| \sin\theta = 3\sqrt{14}.$$

The magnitude of the dot product is:

$$|\vec{b} \cdot (\vec{a} \times \vec{c})| \cos\theta = 27.$$

Step 3: Solve for $\sin\theta$.

Divide the equations to find $\sin\theta$:

$$\sin\theta = \frac{\sqrt{14}}{\sqrt{95}}.$$

Step 4: Find $|\vec{b} \times (\vec{a} \times \vec{c})|$.

From the Provided that relationships:

$$|\vec{b} \times (\vec{a} \times \vec{c})| = 3\sqrt{95}.$$

Step 5: Find $|\vec{a} \times \vec{c}|$.

From the magnitude relationship:

$$|\vec{a} \times \vec{c}| = \sqrt{3} \times \sqrt{95}.$$

Conclusive Answer:

$$|\vec{a} \times \vec{c}|^2 = 3 \times 95 = 285.$$

Quick Tip

Lagrange's identity is helpful for calculating the magnitude of the cross product. Remember the scalar triple product represents the volume of the parallelepiped formed by the three vectors.

30: Let $S = \left\{z \in \mathbb{C} - \{i, 2i\} : \frac{z^2+8iz-15}{z^2-3iz-2} \in \mathbb{R}\right\}$. If $\alpha - \frac{13}{11}i \in S$, $\alpha \in \mathbb{R} - \{0\}$, then $242\alpha^2$ is equal to ________.

Correct Answer: 1680

Solution:

We are tasked with solving the Provided that expression:

$$\frac{z^2 + 8iz - 15}{z^2 - 3iz - 2} \in \mathbb{R}.$$

Step 1: Simplify the condition.

Rewrite the Provided that condition:

$$1 + \frac{(11iz - 13)}{z^2 - 3iz - 2} \in \mathbb{R}.$$

Let:

$$Z = \alpha - \frac{13}{11}i.$$

Step 2: Imaginary part condition.

The imaginary part of the denominator must satisfy:

$$(z^2 - 3iz - 2) \text{ is imaginary}.$$

Substitute $z = x + iy$:

$$z^2 = x^2 - y^2 + 2xyi, \quad -3iz = -3(x + iy)i = -3yi + 3x,$$

so:

$$z^2 - 3iz - 2 = (x^2 - y^2 + 3x - 2) + (2xy - 3y)i.$$

For the expression to be purely imaginary:

$$\text{Re}(z^2 - 3iz - 2) = 0 \quad \Rightarrow \quad x^2 - y^2 + 3x - 2 = 0.$$

Step 3: Solve the real part equation.

Rewriting:

$$x^2 = y^2 - 3y + 2.$$

Factorize:

$$x^2 = (y-1)(y-2).$$

Step 4: Solve for z.

Let:

$$z = \alpha - \frac{13}{11}i,$$

where:

$$x = \alpha, \quad y = -\frac{13}{11}.$$

Substitute $x = \alpha$ and $y = -\frac{13}{11}$ into $x^2 = (y-1)(y-2)$:

$$\alpha^2 = \left(-\frac{13}{11} - 1\right)\left(-\frac{13}{11} - 2\right).$$

Simplify:

$$\alpha^2 = \left(-\frac{24}{11}\right)\left(-\frac{35}{11}\right),$$

$$\alpha^2 = \frac{24 \times 35}{121}.$$

Step 5: Calculate $242\alpha^2$.

Substitute $\alpha^2 = \frac{24\times35}{121}$:

$$242\alpha^2 = 242 \cdot \frac{24 \times 35}{121}.$$

Simplify:

$$242\alpha^2 = 48 \cdot 35 = 1680.$$

Conclusive Answer: $242\alpha^2 = 1680$.

Quick Tip

For a complex number to be purely real, its imaginary part must be zero. For a complex number to be purely imaginary, its real part must be zero.

Physics

Section A

31: Eight equal drops of water are falling through air with a steady speed of 10 cm/s. If the drops collapse, the new velocity is :-

(1) $10 cm/s$

(2) $40 cm/s$

(3) $16 cm/s$

(4) $5 cm/s$

Correct Answer: (2): 40 cm/s

Solution:

Step 1: Volume calculation-

The volume of a sphere is Provided that by:

$$V = \frac{4}{3}\pi r^3.$$

For eight smaller spheres, the total volume is:

$$8 \times \frac{4}{3}\pi r^3 = \frac{4}{3}\pi R^3,$$

where R is the radius of the larger sphere.

Step 2: Radius calculation.

Equating the volumes:

$$R = 2r.$$

Step 3: Velocity calculation.

The terminal velocity of a sphere moving through a fluid is Provided that by:

$$V = \frac{2r^2}{9\eta}(\rho_b - \rho_{\text{air}}),$$

where:

- r is the radius of the sphere,

- η is the fluid's viscosity,

- ρ_b and ρ_{air} are the densities of the sphere and the air, respectively.

From the equation, terminal velocity is proportional to the square of the radius:

$$V \propto r^2.$$

Step 4: Ratio of velocities.

Using the proportionality:

$$\frac{V_1}{V_2} = \left(\frac{r}{R}\right)^2.$$

Substituting $R = 2r$:

$$\frac{V_1}{V_2} = \left(\frac{r}{2r}\right)^2 = \frac{1}{4}.$$

Step 5: Evaluate V_2.

Rewriting the ratio:

$$V_2 = V_1 \times 4.$$

Provided that $V_1 = 10\,\text{km/s}$:

$$V_2 = 10 \times 4 = 40\,\text{km/s}.$$

Conclusive Answer: The velocity V_2 is $40\,\text{km/s}$.

Quick Tip

Terminal velocity is reached when the gravitational force is balanced by the viscous drag and buoyant force. It is proportional to the square of the radius of the drop.

32: Provided that below are two statements : One is labelled as Assertion A and the other is labelled as Reason R.

Assertion A: A bar magnet dropped through a metallic cylindrical pipe takes more time to come down compared to a non-magnetic bar with same geometry and mass.

Reason R: For the magnetic bar, Eddy currents are produced in the metallic pipe which oppose the motion of the magnetic bar.

In the light of the above statements, choose the correct answer from the options Provided that below:

(1) A is true but R is false

(2) Both A and R are true but R is **NOT** the correct explanation of A

(3) A is false but R is true

(4) Both A and R are true and R is the correct explanation of A

Correct Answer: (4): Both A and R are true and R is the correct explanation of

Solution:

If a bar magnet falls through a metallic cylindrical pipe, its motion alters the magnetic flux, inducing eddy currents in the pipe. According to Lenz's law, these eddy currents generate a magnetic field that opposes the magnet's motion, exerting an upward force that slows its descent. As a result, the bar magnet takes longer to reach the bottom compared to a non-magnetic bar of the same shape and mass, which is unaffected by magnetic forces.

In contrast, a non-magnetic bar falling through the same pipe experiences only gravitational force and air resistance. Since there are no eddy currents or opposing magnetic forces, it falls more quickly.

Therefore, both Assertion A and Reason R are correct, and Reason R appropriately explains Assertion A.

Quick Tip

Lenz's Law states that the induced current will flow in such a direction as to create a magnetic field that opposes the change in the original magnetic flux.

33: A space ship of mass 2×10^4 kg is launched into a circular orbit close to the earth surface. The additional velocity to be imparted to the space ship in the orbit to overcome the gravitational pull will be (if $g = 10$ m/s^2 and radius of earth = 6400 km):

(1) $7.9(\sqrt{2} - 1)$ km/s

(2) $7.4(\sqrt{2} - 1)$ km/s

(3) $11.2(\sqrt{2} - 1)$ km/s

(4) $8(\sqrt{2} - 1)$ km/s

Correct Answer: (4): $8(\sqrt{2} - 1)$ km/s

Solution:

Calculating ΔV using the Provided that formula-

Step 1: Formula for ΔV.

The change in velocity is Provided that by:

$$\Delta V = \sqrt{\frac{GM}{R}}\,(\sqrt{2} - 1),$$

where:

- G is the gravitational constant,

- M is the mass of the central body (e.g., Earth),

- R is the radius of the orbit.

Step 2: Evaluate the expression.

Rewrite the term $\sqrt{\frac{GM}{R}}$:

$$\Delta V = \sqrt{\frac{GM}{R}} = \sqrt{\frac{GM}{R^2} \cdot R}.$$

Thus:

$$\Delta V = \sqrt{gR}\,(\sqrt{2} - 1),$$

where $g = \frac{GM}{R^2}$ is the acceleration due to gravity at the surface.

Step 3: Substitute numerical values.

Using the Provided that value for $\sqrt{gR}$:

$$\Delta V = \sqrt{gR}\,(\sqrt{2} - 1) = 8000(\sqrt{2} - 1)\,\text{m/s}.$$

Convert to km/s:

$$\Delta V = 8(\sqrt{2} - 1)\,\text{km/s}.$$

Conclusive Answer: $\Delta V = 8(\sqrt{2} - 1)\,\text{km/s}$.

Quick Tip

The escape velocity is $\sqrt{2}$ times the orbital velocity of a satellite close to the surface of a planet.

34: A projectile is projected at 30° from horizontal with initial velocity 40 ms^{-1}. The velocity of the projectile at $t = 2$ s from the start will be :

(Provided that $g = 10$ m/s^2)

(1) Zero

(2) $20\sqrt{3}$ ms^{-1}

(3) $40\sqrt{3}$ ms^{-1}

(4) 20 ms^{-1}

Correct Answer: (2): $20\sqrt{3}$ ms^{-1}

Solution:

Step 1: Initial velocity components.

The initial velocity of the projectile is $u = 40\,\text{ms}^{-1}$, and the angle of projection is $30°$.

1. Horizontal velocity component (U_x):

$$U_x = u\cos 30° = 40 \cdot \cos 30° = 40 \cdot \frac{\sqrt{3}}{2} = 20\sqrt{3}\,\text{ms}^{-1}.$$

2. Vertical velocity component (U_y):

$$U_y = u\sin 30° = 40 \cdot \sin 30° = 40 \cdot \frac{1}{2} = 20\,\text{ms}^{-1}.$$

Step 2: Velocity after $t = 2\,$s.

1. Horizontal velocity (V_x) remains constant because there is no horizontal acceleration:

$$V_x = U_x = 20\sqrt{3}\,\text{ms}^{-1}.$$

2. Vertical velocity (V_y) is affected by gravity ($g = 10\,\text{ms}^{-2}$):

$$V_y = U_y - gt = 20 - 10 \cdot 2 = 20 - 20 = 0\,\text{ms}^{-1}.$$

Step 3: Resultant velocity (V).

The resultant velocity is the vector sum of V_x and V_y:

$$V = \sqrt{V_x^2 + V_y^2}.$$

Substitute the values:

$$V = \sqrt{(20\sqrt{3})^2 + (0)^2} = \sqrt{400 \cdot 3} = \sqrt{1200} = 20\sqrt{3}\,\text{ms}^{-1}.$$

Conclusive Answer: The velocity of the projectile after $t = 2\,$s is $20\sqrt{3}\,\text{ms}^{-1}$.

Quick Tip

In projectile motion, the horizontal component of velocity remains constant, while the vertical component changes due to gravity.

35: A plane electromagnetic wave of frequency 20 MHz propagates in free space along x-direction. At a particular space and time, $\vec{E} = 6.6\hat{j}$ v/m. What is $\vec{B}$ at this point?

(1) $-2.2 \times 10^{-8}\hat{k}$ T

(2) $-2.2 \times 10^{-8}\hat{i}$ T

(3) $2.2 \times 10^{-8}\hat{k}$ T

(4) $2.2 \times 10^{-8}\hat{i}$ T

Correct Answer: (3): $2.2 \times 10^{-8}\hat{k}$ T

Solution:

Provided that: $\vec{E} = 6.6\hat{j}$ V/m, frequency $f = 20$ MHz, and the wave propagates along the x-direction.

Step 1: Evaluate the magnitude of $\vec{B}$.

The magnitude of the magnetic field $\vec{B}$ is related to the magnitude of the electric field $\vec{E}$ by

$$|\vec{B}| = \frac{|\vec{E}|}{c},$$

where $c = 3 \times 10^8$ m/s is the speed of light in free space.

$$|\vec{B}| = \frac{6.6}{3 \times 10^8} = 2.2 \times 10^{-8} \text{ T}.$$

Step 2: Determine the direction of $\vec{B}$.

For an electromagnetic wave propagating in the direction $\vec{C}$, the electric field $\vec{E}$, magnetic field $\vec{B}$, and propagation direction $\vec{C}$ are mutually perpendicular, and follow the right-hand rule: $\vec{E} \times \vec{B} = \vec{C}$. In this case, $\vec{C}$ is in the x-direction ($\hat{i}$), and $\vec{E}$ is in the y-direction ($\hat{j}$). Thus, $\vec{B}$ must be in the z-direction ($\hat{k}$) for the cross product to be in the x-direction. That is,

$$\hat{j} \times \vec{B} = \hat{i}.$$

Only $\vec{B}$ being in the positive z-direction satisfies this:

$$\hat{j} \times \hat{k} = \hat{i}.$$

Step 3: Combine magnitude and direction. Combining the magnitude and direction, we get

$$\vec{B} = (2.2 \times 10^{-8})\hat{k} \text{ T}.$$

Conclusive Answer: $\vec{B} = 2.2 \times 10^{-8}\hat{k}$ T.

Quick Tip

In an electromagnetic wave, the electric and magnetic fields are perpendicular to each other and to the direction of propagation.

36: A car P travelling at 20 ms^{-1} sounds its horn at a frequency of 400 Hz. Another car Q is travelling behind the first car in the same direction with a velocity 40 ms^{-1}. The frequency heard by the passenger of the car Q is approximately [Take, velocity of sound $= 360$ ms^{-1}]

(1) $471Hz$

(2) $514Hz$

(3) $421Hz$

(4) $485Hz$

Correct Answer: (3): 421 Hz

Solution:

Provided that: Velocity of car P, $V_P = 20$ ms^{-1}, velocity of car Q, $V_Q = 40$ ms^{-1}, frequency of the horn, $f = 400$ Hz, and velocity of sound, $V_s = 360$ ms^{-1}.

Step 1: Apply the Doppler effect formula. The apparent frequency f_{app} heard by the passenger in car Q when both cars are moving in the same direction is Provided that by:

$$f_{\text{app}} = \left[\frac{V_s - (-V_Q)}{V_s - (-V_P)}\right] f = \left[\frac{V_s + V_Q}{V_s + V_P}\right] f.$$

Note that we use the negative of the velocities since both cars are moving in the same direction as the sound wave. If the observer (car Q) is moving towards the source (car P) and the source is moving away from the observer, then we take V_Q positive and V_P negative. In this case, both are positive because both are moving in the same direction.

Step 2: Substitute the Provided that values. Substituting the Provided that values into the Doppler effect formula:

$$f_{\text{app}} = \left[\frac{360 + 40}{360 + 20}\right] \times 400 = \frac{400}{380} \times 400 = \frac{400}{380} \times 400 \approx 421 \text{ Hz}.$$

Conclusive Answer: The frequency heard by the passenger of car Q is approximately 421 Hz.

Quick Tip

When the source and observer are moving in the same direction, the apparent frequency is lower than the actual frequency if the source is moving faster than the observer.

37: A body of mass 500 g moves along x-axis such that its velocity varies with displacement x according to the calculation $v = 10\sqrt{x}$ m/s the force acting on the body is :-

(1) $25N$

(2) $5N$

(3) $166N$

(4) $125N$

Correct Answer: (1): 25 N

Solution:

Provided that: Mass $m = 500$ g $= 0.5$ kg and velocity $v = 10\sqrt{x}$ m/s.

Step 1: Find the acceleration.

Acceleration a is the rate of change of velocity with respect to time, which can be expressed as:

$$a = \frac{dv}{dt} = \frac{dv}{dx} \times \frac{dx}{dt} = v\frac{dv}{dx}.$$

We are Provided that $v = 10\sqrt{x}$. Differentiating v with respect to x:

$$\frac{dv}{dx} = \frac{d}{dx}(10\sqrt{x}) = 10 \times \frac{1}{2\sqrt{x}} = \frac{5}{\sqrt{x}}.$$

So, the acceleration is

$$a = v\frac{dv}{dx} = 10\sqrt{x} \times \frac{5}{\sqrt{x}} = 50 \text{ ms}^{-2}.$$

Step 2: Evaluate the force-

Force F is Provided that by Newton's second law:

$$F = ma.$$

Substituting the Provided that values:

$$F = 0.5 \times 50 = 25 \text{ N}.$$

Conclusive Answer: The force acting on the body is 25 N.

Quick Tip

When velocity is Provided that as a function of displacement, acceleration can be Evaluated using $a = v\frac{dv}{dx}$.

38: The ratio of the de-Broglie wavelengths of proton and electron having same Kinetic energy:

(Assume $m_p = m_e \times 1840$)

(1) 1 : 62

(2) 1 : 30

(3) 1 : 43

(4) 2 : 43

Correct Answer: (3): 1:43

Solution:

Provided that: Proton and electron have the same kinetic energy, K, and $m_p = 1840m_e$.

Step 1: Recall the de-Broglie wavelength formula-

The de-Broglie wavelength λ is Provided that by

$$\lambda = \frac{h}{p},$$

where h is Planck's constant and p is the momentum. Since kinetic energy $K = \frac{p^2}{2m}$, we have $p = \sqrt{2mK}$. Therefore,

$$\lambda = \frac{h}{\sqrt{2mK}}.$$

Step 2: Evaluate the ratio of wavelengths-

Let λ_p and λ_e be the de-Broglie wavelengths of the proton and electron, respectively. Then

$$\frac{\lambda_p}{\lambda_e} = \frac{\frac{h}{\sqrt{2m_pK}}}{\frac{h}{\sqrt{2m_eK}}} = \sqrt{\frac{m_e}{m_p}}.$$

Since $m_p = 1840m_e$, we have

$$\frac{\lambda_p}{\lambda_e} = \sqrt{\frac{m_e}{1840m_e}} = \frac{1}{\sqrt{1840}} \approx \frac{1}{42.9} \approx \frac{1}{43}.$$

Conclusive Answer: The ratio of the de-Broglie wavelengths is approximately 1:43.

Quick Tip

For particles with the same kinetic energy, the de-Broglie wavelength is inversely proportional to the square root of the mass.

39: If force (F), velocity (V) and time (T) are considered as fundamental physical quantity, then dimensional formula of density will be :-

(1) $FV^{-2}T^{2}$

(2) $FV^{4}T^{-6}$

(3) $FV^{-4}T^{-2}$

(4) $F^{2}V^{-2}T^{6}$

Correct Answer: (3): $FV^{-4}T^{-2}$

Solution:

Provided that: Force (F), velocity (V), and time (T) are fundamental quantities.

Step 1: Write the dimensional formula for density-

Let the dimensional formula for density ρ be

$$[\rho] = F^x V^y T^z,$$

where x, y, and z are constants to be determined.

Step 2: Express the dimensions of fundamental quantities in terms of M, L, and T-

The dimensions of force, velocity, and time are:

$$[F] = [MLT^{-2}], \quad [V] = [LT^{-1}], \quad [T] = [T].$$

The dimensions of density are $[\rho] = [ML^{-3}]$.

Step 3: Substitute the dimensions and equate the powers-

Substituting the dimensions into the equation for $[\rho]$:

$$[ML^{-3}] = [MLT^{-2}]^x [LT^{-1}]^y [T]^z = M^x L^{x+y} T^{-2x-y+z}.$$

Equating the powers of M, L, and T on both sides, we get the following system of equations:

$$x = 1$$

$$x + y = -3$$

$$-2x - y + z = 0$$

Step 4: Solve the system of equations-

From the first equation, $x = 1$. Substituting into the second equation:

$$1 + y = -3 \implies y = -4.$$

Substituting $x = 1$ and $y = -4$ into the third equation:

$$-2(1) - (-4) + z = 0 \implies -2 + 4 + z = 0 \implies z = -2.$$

Step 5: Write the dimensional formula for density. Substituting $x = 1$, $y = -4$, and $z = -2$:

$$[\rho] = F^1 V^{-4} T^{-2} = F V^{-4} T^{-2}.$$

Conclusive Answer: The dimensional formula for density is $FV^{-4}T^{-2}$.

Quick Tip

Dimensional analysis is a powerful tool for checking the correctness of equations and deriving calculationships between physical quantities.

40: An electron is allowed to move with constant velocity along the axis of current carrying straight solenoid.

A. The electron will experience magnetic force along the axis of the solenoid.

B. The electron will not experience magnetic force.

C. The electron will continue to move along the axis of the solenoid.

D. The electron will be accelerated along the axis of the solenoid.

E. The electron will follow parabolic path inside the solenoid.

Choose the correct answer from the options Provided that below:

(1) A and D only

(2) B, C and D only

(3) B and E only

(4) B and C only

Correct Answer: (4): B and C only

Solution:

Provided that: An electron moves with constant velocity along the axis of a current-carrying solenoid.

Step 1: Recall the magnetic force formula. The magnetic force $\vec{F}_m$ experienced by a charged particle with charge q moving with velocity $\vec{v}$ in a magnetic field $\vec{B}$ is Provided that by

$$\vec{F}_m = q(\vec{v} \times \vec{B}).$$

Step 2: Analyze the force on the electron. Inside a solenoid, the magnetic field is uniform and directed along the axis of the solenoid. Since the electron is moving along the axis of the solenoid, its velocity $\vec{v}$ is parallel to the magnetic field $\vec{B}$. Therefore, the cross product $\vec{v} \times \vec{B}$ is zero, and the magnetic force on the electron is also zero:

$$\vec{F}_m = 0.$$

Step 3: Determine the motion of the electron. Since the magnetic force on the electron is zero, there is no net force acting on it in the direction perpendicular to its motion. Thus, the electron will continue to move along the axis of the solenoid with constant velocity. It will not be accelerated or follow a parabolic path.

Conclusive Answer: The electron will not experience magnetic force (B), and it will continue to move along the axis of the solenoid (C).

Quick Tip

A charged particle moving parallel or antiparallel to a magnetic field experiences no magnetic force.

41: The Thermodynamic process, in which internal energy of the system remains constant is

(1) Isobaric

(2) Isochoric

(3) Adiabatic

(4) Isothermal

Correct Answer: (4): Isothermal

Solution:

Step 1: Recall the calculationship between internal energy and temperature-

The change in internal energy (ΔU) of a system is related to the change in temperature (ΔT) by

$$\Delta U = nC_v \Delta T,$$

where n is the number of moles and C_v is the molar specific heat at constant volume.

Step 2: Consider the isothermal process-

In an isothermal process, the temperature of the system remains constant. Therefore, $\Delta T = 0$.

Step 3: Evaluate the change in internal energy-

Substituting $\Delta T = 0$ into the equation for ΔU:

$$\Delta U = nC_v(0) = 0.$$

Step 4: Conclude about the internal energy-

Since $\Delta U = 0$, the internal energy of the system remains constant in an isothermal process.

Conclusive Answer: The thermodynamic process in which internal energy remains constant is isothermal.

Quick Tip

In an isothermal process, the temperature remains constant, leading to zero change in internal energy.

42: In satellite communication, the uplink frequency band used is:

(1) $76 - 88$ MHz

(2) $420 - 890$ MHz

(3) $3.7 - 4.2$ GHz

(4) $5.925 - 6.425$ GHz

Correct Answer: (4): 5.925 – 6.425 GHz

Solution:

In satellite communication, the uplink frequency band is used for transmitting signals from an earth station to a satellite. The downlink frequency band is used for transmitting signals from a satellite to an earth station.

Uplink frequencies are generally higher than downlink frequencies to minimize interference.

A common standard for satellite communication uses the following frequency bands:

Uplink-

5.8-6.2 GHz

Downlink -

4-4.2 GHz

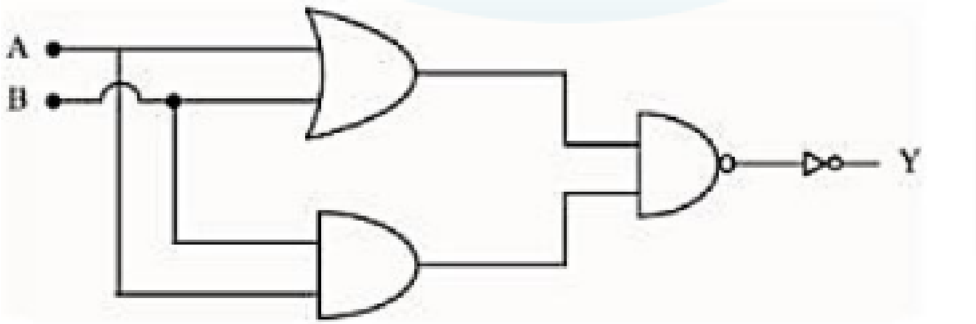
Option (4) falls within the standard uplink frequency band.

Conclusive Answer: The uplink frequency band used in satellite communication is 5.925 – 6.425 GHz.

Quick Tip

Uplink frequencies are higher than downlink frequencies in satellite communication to reduce interference.

43: The logic operations performed by the Provided that digital circuit is equivalent to:



(1) OR

(2) $NAND$

(3) NOR

(4) AND

Correct Answer: (4): AND

Solution:

Step 1: Interpret the first NAND gate-

The output of the first NAND gate (topmost) is $\overline{A \cdot B}$. Since there is a connection bridging across from the inputs A and B to the lower NAND gate this lower NAND gate's output is also $\overline{A \cdot B}$.

Step 2: Interpret the second NAND gate-

The second NAND gate (middle) takes the outputs of the first and third NAND gates as inputs. Let X_1 and X_2 be the outputs of the first and third NAND gates, respectively. Then, the output of the second NAND gate is $\overline{X_1 \cdot X_2}$. The output of the first NAND gate is $\overline{A+B}$ and the output of the second NAND gate is $\overline{AB}$. Hence the output of the second NAND gate becomes $\overline{(\overline{A+B}).(\overline{AB})}$.

Step 3: Interpret the NOT gate-

The NOT gate inverts the output of the second NAND gate. Let X_3 be the output of the second NAND gate. Then the output Y of the NOT gate is $\overline{X_3}$. The output of the NOT gate is $\overline{\overline{(\overline{A+B}).(\overline{AB})}} = (\overline{A+B}).(\overline{AB})$.

Step 4: Evaluate the expression-

Using De Morgan's theorem, we can Evaluate the expression for Y:

$$\begin{aligned} Y &= \overline{\overline{(A+B)} \cdot \overline{AB}} \\ &= \overline{\overline{A+B}} + \overline{\overline{AB}} \\ &= (A+B) + AB \\ &= (A+B)(1+A) + (A+B)(B) \\ &= A + AB + B + AB \\ &= A + AB + AB + B \\ &= AB + AB \\ &= AB \\ &= A \cdot B \end{aligned}$$

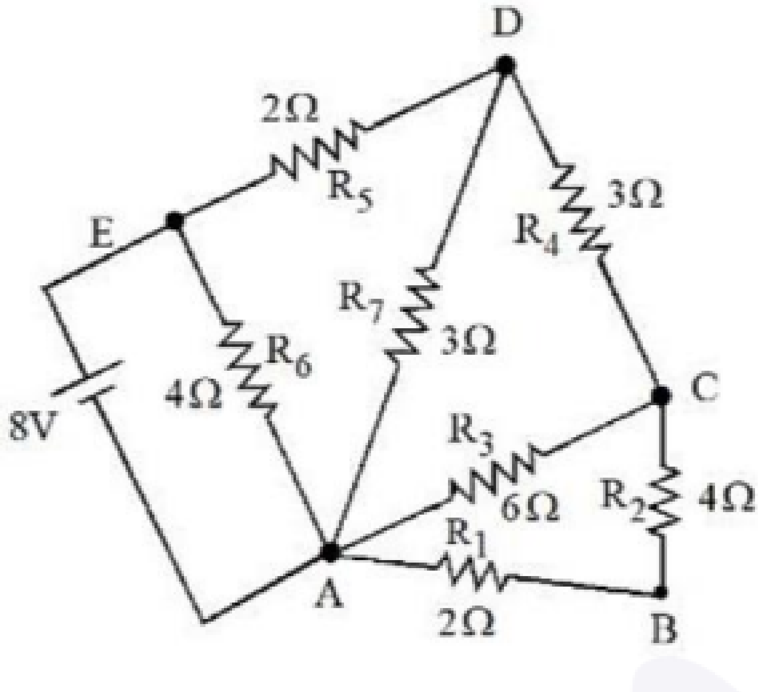
This is the logic for an AND gate.

Conclusive Answer: The Provided that digital circuit is equivalent to an AND gate.

Quick Tip

De Morgan's theorem is very useful in Evaluateing Boolean expressions. Remember to analyze each gate separately and then combine the results.

44: The current flowing through R_2 is:



(1) $\frac{1}{3}$ A

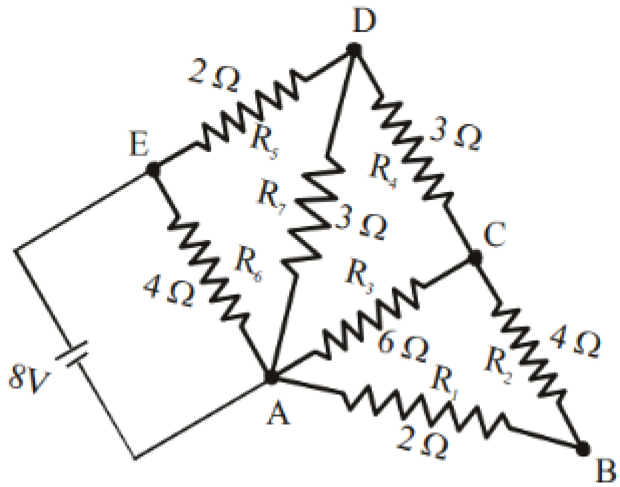
(2) $\frac{1}{4}$ A

(3) $\frac{2}{3}$ A

(4) $\frac{1}{2}$ A

Correct Answer: (1): $\frac{1}{3}$ A

Solution:



Step 1: Determine the equivalent resistance-

The total resistance in the circuit is Provided that as:

$$R_{\text{eq}} = 4\,\Omega.$$

Step 2: Evaluate the total current (i)-

Using Ohm's law:

$$i = \frac{V}{R_{\text{eq}}}.$$

Substitute the Provided that values:

$$i = \frac{8}{4} = 2\,\text{A}.$$

Step 3: Evaluate i_1-

The current i_1 is determined using the current division rule:

$$i_1 = \frac{2 \times 3}{3+6}.$$

Evaluate:

$$i_1 = \frac{6}{9} = \frac{2}{3}\,\text{A}.$$

Step 4: Evaluate i_2-

The current i_2 is Provided that by:

$$i_2 = \frac{i_1}{2}.$$

Substitute $i_1 = \frac{2}{3}$:

$$i_2 = \frac{\frac{2}{3}}{2} = \frac{1}{3}\,\text{A}.$$

Conclusive Answer:

$$i = 2\,\text{A}, \quad i_1 = \frac{2}{3}\,\text{A}, \quad i_2 = \frac{1}{3}\,\text{A}.$$

Quick Tip

Symmetry in circuits can greatly Evaluate calculations. Look for identical resistances and branches to Evaluate complex networks.

45: When vector $\vec{A} = 2\hat{i} + 3\hat{j} + 2\hat{k}$ is subtracted from vector $\vec{B}$, it gives a vector equal to $2\hat{j}$. Then the magnitude of vector $\hat{B}$ will be:

(1) 3

(2) $\sqrt{5}$

(3) $\sqrt{33}$

(4) $\sqrt{6}$

Correct Answer: (3): $\sqrt{33}$

Solution:

Provided that: $\vec{A} = 2\hat{i} + 3\hat{j} + 2\hat{k}$ and $\vec{B} - \vec{A} = 2\hat{j}$.

Step 1: Solve for vector B. We are Provided that

$$\vec{B} - \vec{A} = 2\hat{j}.$$

Adding $\vec{A}$ to both sides, we get

$$\vec{B} = 2\hat{j} + \vec{A} = 2\hat{j} + (2\hat{i} + 3\hat{j} + 2\hat{k}) = 2\hat{i} + 5\hat{j} + 2\hat{k}.$$

Step 2: Evaluate the magnitude of B. The magnitude of vector $\vec{B} = x\hat{i} + y\hat{j} + z\hat{k}$ is Provided that by $|\vec{B}| = \sqrt{x^2 + y^2 + z^2}$. Therefore,

$$|\vec{B}| = \sqrt{2^2 + 5^2 + 2^2} = \sqrt{4 + 25 + 4} = \sqrt{33}.$$

Conclusive Answer: The magnitude of vector B is $\sqrt{33}$.

Quick Tip

Vector addition and subtraction are performed component-wise. The magnitude of a vector is the square root of the sum of the squares of its components.

46: If V is the gravitational potential due to sphere of uniform density on it's surface, then it's value at the center of sphere will be:-

(1) $\frac{4}{3}V$

(2) V

(3) $\frac{3V}{2}$

(4) $\frac{V}{2}$

Correct Answer: (3): $\frac{3V}{2}$

Solution:

Step 1: Recall the gravitational potential at the surface. The gravitational potential V at the surface of a sphere of mass M and radius R is Provided that by

$$V = -\frac{GM}{R}.$$

Step 2: Recall the gravitational potential inside a uniform sphere. The gravitational potential V_{centre} at a distance r from the center of a uniform sphere of mass M and radius R, where $r \leq R$, is Provided that by

$$V(r) = -\frac{GM}{2R^3}(3R^2 - r^2).$$

At the center of the sphere, $r = 0$, so

$$V_{\text{centre}} = -\frac{GM}{2R^3}(3R^2) = -\frac{3GM}{2R}.$$

Step 3: Express the potential at the center in terms of the potential at the surface. We are Provided that that the potential at the surface is $V = -\frac{GM}{R}$. Therefore,

$$V_{\text{centre}} = -\frac{3GM}{2R} = \frac{3}{2}\left(-\frac{GM}{R}\right) = \frac{3}{2}V.$$

Conclusive Answer: The gravitational potential at the center of the sphere is $\frac{3V}{2}$.

Quick Tip

The gravitational potential inside a uniform sphere is not inversely proportional to the distance from the center, unlike the potential outside the sphere.

47: The root mean square speed of molecules of nitrogen gas at $27°$C is approximately : (Provided that mass of a nitrogen molecule $= 4.6 \times 10^{-26}$ kg and take Boltzmann constant $k_B = 1.4 \times 10^{-23}$ JK^{-1})

(1) 1260 m/s

(2) 91 m/s

(3) 523 m/s

(4) 27.4 m/s

Correct Answer: (3): 523 m/s

Solution:

Provided that: Temperature $T = 27°C = 300$ K, mass of a nitrogen molecule $m = 4.6 \times 10^{-26}$ kg, and Boltzmann constant $k_B = 1.4 \times 10^{-23}$ JK^{-1}.

Step 1: Recall the formula for root mean square speed. The root mean square (rms) speed V_{rms} of gas molecules is Provided that by

$$V_{\text{rms}} = \sqrt{\frac{3RT}{M_w}} = \sqrt{\frac{3RT}{mN_A}} = \sqrt{\frac{3k_BT}{m}},$$

where R is the universal gas constant, T is the temperature in Kelvin, M_w is the molar mass, m is the mass of one molecule, N_A is Avogadro's number, and $k_B = \frac{R}{N_A}$ is the Boltzmann constant.

Step 2: Substitute the Provided that values. Substituting the Provided that values into the formula:

$$V_{\text{rms}} = \sqrt{\frac{3 \times 1.4 \times 10^{-23} \times 300}{4.6 \times 10^{-26}}} = \sqrt{\frac{3 \times 1.4 \times 3 \times 10^{-23+2+1}}{4.6 \times 10^{-26}}} = \sqrt{\frac{12.6 \times 10^{-21+27}}{4.6}}$$

$$= \sqrt{\frac{12.6 \times 10^{6}}{4.6}} = \sqrt{2.73 \times 10^5} = \sqrt{27.39 \times 10^4} \approx 5.23 \times 10^2 = 523 \text{ m/s}.$$

Conclusive Answer: The root mean square speed of nitrogen molecules is approximately 523 m/s.

Quick Tip

The root mean square speed is directly proportional to the square root of the temperature and inversely proportional to the square root of the molecular mass.

48: The energy of He^+ ion in its first excited state is, (The ground state energy for the Hydrogen atom is -13.6 eV):

(1) -13.6 eV

(2) -54.4 eV

(3) -27.2 eV

(4) -3.4 eV

Correct Answer: (1): -13.6 eV

Solution:

Solution:

Step 1: Formula for energy levels.

The energy of an electron in a hydrogen-like atom is Provided that by:

$$E = -\left(\frac{13.6Z^2}{n^2}\right)\,\text{eV},$$

where: - Z is the atomic number, - n is the principal quantum number (energy level), - $13.6\,\text{eV}$ is the ground state energy of the hydrogen atom.

Step 2: First excited state ($n = 2$).

For the first excited state, the principal quantum number is:

$$n = 2.$$

Substitute $n = 2$ into the energy formula:

$$E = -\frac{13.6Z^2}{2^2}.$$

Evaluate:

$$E = -\frac{13.6Z^2}{4}.$$

Step 3: For $Z = 1$ (hydrogen atom).

Substitute $Z = 1$ into the equation:

$$E = -\frac{13.6 \cdot 1^2}{4} = -13.6\,\text{eV}.$$

Conclusive Answer:

The energy of the electron in the first excited state is:

$$E = -13.6\,\text{eV}.$$

Quick Tip

The energy levels of hydrogen-like atoms are proportional to Z^2 and inversely proportional to n^2.

49: When one light ray is reflected from a plane mirror with 30° angle of reflection, the angle of deviation of the ray after reflection is:

(1) 140°

(2) 130°

(3) 120°

(4) 110°

Correct Answer: (3): 120°

Solution:

Provided that: Angle of reflection $r = 30^\circ$.

Step 1: Recall the law of reflection. According to the law of reflection, the angle of incidence (i) is equal to the angle of reflection (r). Therefore, $i = r = 30^\circ$.

Step 2: Determine the angle of deviation. The angle of deviation (δ) is the angle between the incident ray and the reflected ray. It is Provided that by:

$$\delta = 180^\circ - 2i = 180^\circ - 2r.$$

Since $r = 30^\circ$, $\delta = 180 - 2(30) = 180 - 60 = 120^\circ$.

Step 3: Evaluate the angle of deviation. Substituting $i = 30^\circ$:

$$\delta = 180^\circ - 2(30^\circ) = 180^\circ - 60^\circ = 120^\circ.$$

Conclusive Answer: The angle of deviation is 120°.

Quick Tip

The angle of deviation for reflection from a plane mirror is twice the glancing angle (complement of the angle of incidence).

50: A capacitor of capacitance C is charge to a potential V. The flux of the electric field through a closed surface enclosing the positive plate of the capacitor is :

(1) Zero

(2) $\frac{CV}{\varepsilon_0}$

(3) $\frac{2CV}{\varepsilon_0}$

(4) $\frac{CV}{2\varepsilon_0}$

Correct Answer: (2): $\frac{CV}{\varepsilon_0}$

Solution:

Provided that: Capacitance C and potential difference V.

Step 1: Recall Gauss's law. Gauss's law states that the electric flux Φ through a closed surface is equal to the enclosed charge q divided by the permittivity of free space ε_0:

$$\Phi = \frac{q}{\varepsilon_0}.$$

Step 2: Relate charge, capacitance, and potential difference. The charge q on a capacitor is related to its capacitance C and the potential difference V across it by

$$q = CV.$$

Step 3: Evaluate the electric flux. Substituting $q = CV$ into Gauss's law:

$$\Phi = \frac{CV}{\varepsilon_0}.$$

Conclusive Answer: The flux of the electric field through the closed surface is $\frac{CV}{\varepsilon_0}$.

Quick Tip

Gauss's law relates the electric flux through a closed surface to the net charge enclosed by the surface. Remember the calculationship between charge, capacitance, and voltage for a capacitor: $Q = CV$.

Section B

51: A coil has an inductance of 2H and resistance of 4Ω. A 10 V is applied across the coil. The energy stored in the magnetic field after the current has built up to its equilibrium value will be ____ $\times 10^{-2}$ J.

Correct Answer: 625

Solution:

Provided that: Inductance $L = 2$ H, resistance $R = 4\Omega$, and applied voltage $V = 10$ V.

Step 1: Analyze the circuit at steady state. At steady state, the inductor acts as a short circuit, meaning it has zero resistance. The circuit effectively becomes a simple resistor connected to the voltage source.

(0,0) to[battery1, l=10V] (0,2) to[L, l=2H] (2,2) to[R, l=4Ω] (4,2) – (4,0) – (0,0) ; $\implies$ at steady state (0,0) to[battery1, l=10V] (0,2) to[R, l=4Ω] (2,2) – (2,0) – (0,0) ;

Step 2: Evaluate the current at steady state. Using Ohm's law, the current I through the resistor at steady state is

$$I = \frac{V}{R} = \frac{10}{4} = \frac{5}{2} = 2.5 \text{ A}.$$

Step 3: Evaluate the energy stored in the magnetic field. The energy E stored in the magnetic field of an inductor is Provided that by

$$E = \frac{1}{2}LI^2.$$

Substituting the Provided that values:

$$E = \frac{1}{2} \times 2 \times \left(\frac{5}{2}\right)^2 = \frac{25}{4} = 6.25 \text{ J} = 625 \times 10^{-2} \text{ J}.$$

Conclusive Answer: The energy stored in the magnetic field is 625×10^{-2} J.

Quick Tip

At steady state, an inductor in a DC circuit acts as a short circuit. The energy stored in an inductor's magnetic field is Provided that by $E = \frac{1}{2}LI^2$.

52: A metallic cube of side 15 cm moving along y-axis at a uniform velocity of 2 ms^{-1}. In a region of uniform magnetic field of magnitude 0.5T directed along z-axis. In equilibrium the potential difference between the faces of higher and lower potential developed because of the motion through the field will be _____ mV.

(0,0,0) – (2,0,0) – (2,2,0) – (0,2,0) – cycle; (0,0,2) – (2,0,2) – (2,2,2) – (0,2,2) – cycle; (0,0,0) – (0,0,2); (2,0,0) – (2,0,2); (2,2,0) – (2,2,2); (0,2,0) – (0,2,2); [-¿] (2,2,0) – (3,2,0) node[anchor=west] x; [-¿] (0,2,0) – (0,3,0) node[anchor=south] y; [-¿] (0,0,0) – (0,0,3) node[anchor=east] z; [-¿] (0.5,2.5,0) – (0.5,3,0) node[anchor=south west] $\vec{v}$; [-¿] (0,0,1.5) – (0.5,0,1.5) node[anchor=north east] $\vec{B}$;

Correct Answer: 150

Solution:

Provided that: Side of the cube $l = 15$ cm $= 0.15$ m, velocity $\vec{v} = 2\hat{j}$ ms^{-1}, and magnetic field $\vec{B} = 0.5\hat{k}$ T.

Step 1: Determine the induced EMF. The induced electromotive force (EMF) or potential difference V across the faces of the cube due to its motion in the magnetic field is Provided that by

$$V = Blv,$$

where B is the magnetic field strength, l is the length of the side perpendicular to both the velocity and magnetic field, and v is the velocity.

Step 2: Substitute the Provided that values. Substituting the Provided that values:

$$V = (0.5\text{ T})(0.15\text{ m})(2\text{ ms}^{-1}) = 0.15\text{ V} = 150\text{ mV}.$$

Conclusive Answer: The potential difference developed between the faces is 150 mV.

Quick Tip

Motional EMF is induced when a conductor moves through a magnetic field. The induced EMF is Provided that by $V = Blv$, where l is the length of the conductor perpendicular to both the velocity and the magnetic field.

53: In the Provided that circuit, $C_1 = 2\ \mu$F, $C_2 = 0.2\ \mu$F, $C_3 = 2\ \mu$F, $C_4 = 4\ \mu$F, $C_5 = 2\ \mu$F, $C_6 = 2\ \mu$F. The charge stored on capacitor C_4 is ____ μC.

(0,0) to[battery1, l=10V] (0,2) to[C, l=C_1] (2,2) to[C, l=C_3] (4,2) to[C, l=C_4] (4,0) to[C, l=C_5] (2,0) to[C, l=C_6] (0,0) (2,0) to[C, l=C_2] (2,2) ;

Correct Answer: 4

Solution:

We are tasked with calculating the equivalent capacitance and charges in the Provided that circuit.

Step 1: Equivalent capacitance of the circuit.

The total equivalent capacitance of the circuit is Provided that as:

$$C_{\text{eq}} = 0.5\,\mu\text{F}.$$

Step 2: Total charge (Q) in the circuit.

The total charge stored in the circuit can be Evaluated using the formula:

$$Q = C_{\text{eq}} \cdot V,$$

where: - C_{eq} is the equivalent capacitance, - $V = 10\,\text{V}$ is the applied voltage.

Substitute the Provided that values:

$$Q = 0.5 \cdot 10 = 5\,\mu\text{C}.$$

Step 3: Charge redistribution (Q').

The charge on a specific branch of the circuit is Evaluated using the charge division formula. For the Provided that branch:

$$Q' = \frac{Q \cdot C_2}{C_2 + C_6}.$$

Substitute $Q = 5\,\mu\text{C}$, $C_2 = 0.8\,\mu\text{F}$, and $C_6 = 0.2\,\mu\text{F}$:

$$Q' = \frac{5 \cdot 0.8}{0.8 + 0.2}.$$

Evaluate:

$$Q' = \frac{4}{1} = 4\,\mu\text{C}.$$

Conclusive Answer:

$$C_{\text{eq}} = 0.5\,\mu\text{F}, \quad Q = 5\,\mu\text{C}, \quad Q' = 4\,\mu\text{C}.$$

Quick Tip

Capacitors in series store the same charge, while capacitors in parallel have the same voltage across them.

54: A nucleus disintegrates into two nuclear parts, in such a way that ratio of their nuclear sizes is $1 : 2^{1/3}$. Their respective speed have a ratio of n : 1. The value of n is ____.

Correct Answer: 2

Solution:

Provided that: Ratio of nuclear sizes is $r_1 : r_2 = 1 : 2^{1/3}$ and ratio of speeds is $V_1 : V_2 = n : 1$.

Step 1: Apply conservation of linear momentum. Since the nucleus disintegrates into two parts, the total momentum before disintegration must be equal to the total momentum after disintegration. Assuming the initial nucleus is at rest, the total initial momentum is zero. Thus, the momenta of the two parts after disintegration must be equal and opposite. Let m_1 and m_2 be the masses of the two parts, and V_1 and V_2 be their respective speeds. Then by the law of conservation of linear momentum (LCM),

$$m_1V_1 = m_2V_2.$$

Thus,

$$\frac{V_1}{V_2} = \frac{m_2}{m_1}.$$

Step 2: Relate mass and radius. Assuming the two nuclear parts have the same density, their masses are proportional to the cubes of their radii:

$$\frac{m_2}{m_1} = \frac{r_2^3}{r_1^3} = \left(\frac{r_2}{r_1}\right)^3.$$

We are Provided that $\frac{r_2}{r_1} = 2^{1/3}$, so

$$\frac{m_2}{m_1} = (2^{1/3})^3 = 2.$$

Step 3: Find the ratio of velocities.

Therefore,

$$\frac{V_1}{V_2} = \frac{m_2}{m_1} = \left(\frac{r_2}{r_1}\right)^3 = \left(\frac{2^{\frac{1}{3}}}{1}\right)^3 = 2.$$

Since $\frac{V_1}{V_2} = \frac{n}{1}$, we have $n = 2$.

Conclusive Answer: The value of n is 2.

Quick Tip

In nuclear disintegration problems, apply conservation of linear momentum and the calcula-tionship between mass and radius (or volume) assuming uniform density.

55: The surface tension of soap solution is 3.5×10^{-2} Nm^{-1}. The amount of work done required to increase the radius of soap bubble from 10 cm to 20 cm is ____ $\times 10^{-4}$ J. (take $\pi = 22/7$)

Correct Answer: 264

Solution:

Provided that: Surface tension $S = 3.5 \times 10^{-2}$ N/m, initial radius $r_i = 10$ cm $= 0.1$ m, and final radius $r_f = 20$ cm $= 0.2$ m.

Step 1: Recall the formula for work done. The work done (W) in increasing the surface area of a soap bubble is Provided that by

$$W = \Delta U = S \times \Delta A,$$

where S is the surface tension and ΔA is the change in surface area.

Step 2: Evaluate the change in surface area. A soap bubble has two surfaces (inner and outer). The initial surface area A_i and final surface area A_f are Provided that by:

$$A_i = 2 \times 4\pi r_i^2 \quad \text{and} \quad A_f = 2 \times 4\pi r_f^2.$$

The change in surface area is

$$\Delta A = A_f - A_i = 2 \times 4\pi(r_f^2 - r_i^2) = 8\pi(r_f^2 - r_i^2).$$

Step 3: Evaluate the work done. Substituting the values into the formula for work done:

$$\begin{aligned} W &= S \times \Delta A = S \times 8\pi(r_f^2 - r_i^2) \\ &= 3.5 \times 10^{-2} \times 8 \times \frac{22}{7} \times (0.2^2 - 0.1^2) \\ &= 3.5 \times 10^{-2} \times 8 \times \frac{22}{7} \times (0.04 - 0.01) \\ &= 3.5 \times 10^{-2} \times 8 \times \frac{22}{7} \times 0.03 \\ &= \frac{3.5 \times 8 \times 22 \times 3 \times 10^{-2} \times 10^{-2}}{7} \\ &= 264 \times 10^{-4} \text{ J}. \end{aligned}$$

Conclusive Answer: The work done is 264×10^{-4} J.

Quick Tip

The work done in changing the surface area of a liquid is Provided that by the product of surface tension and the change in surface area. Remember a soap bubble has two surfaces.

56: A circular plate is rotating horizontal plane, about an axis passing through its center perpendicular to the plate, with an angular velocity ω. A person sits at the center having two

dumbbells in his hands. When he stretches out his hands, the moment of inertia of the system becomes triple. If E be the initial Kinetic energy of the system, then final Kinetic energy will be $\frac{E}{x}$. The value of x is _____.

Correct Answer: 3

Solution:

Provided that: Initial angular velocity $\omega_1 = \omega$, initial moment of inertia $I_1 = I_0$, final moment of inertia $I_2 = 3I_0$, and initial kinetic energy E.

Step 1: Apply conservation of angular momentum. Since there is no external torque acting on the system, the angular momentum is conserved. Thus,

$$I_1\omega_1 = I_2\omega_2,$$

where ω_2 is the final angular velocity. We have $I_0\omega = 3I_0\omega_2$, so

$$\omega_2 = \frac{\omega}{3}.$$

Step 2: Evaluate the initial kinetic energy. The initial kinetic energy E is Provided that by

$$E = \frac{1}{2}I_1\omega_1^2 = \frac{1}{2}I_0\omega^2.$$

Step 3: Evaluate the final kinetic energy. The final kinetic energy E_f is Provided that by

$$E_f = \frac{1}{2}I_2\omega_2^2 = \frac{1}{2}(3I_0)\left(\frac{\omega}{3}\right)^2 = \frac{1}{2} \times 3I_0 \times \frac{\omega^2}{9} = \frac{1}{6}I_0\omega^2 = \frac{1}{3}\left(\frac{1}{2}I_0\omega^2\right) = \frac{E}{3}.$$

Step 4: Find the value of x. We are Provided that that the final kinetic energy is $\frac{E}{x}$. Comparing this with our result $E_f = \frac{E}{3}$, we find $x = 3$.

Conclusive Answer: The value of x is 3.

Quick Tip

When no external torque acts on a rotating system, angular momentum is conserved. Stretching out the arms increases the moment of inertia and decreases the angular velocity.

57: A block of mass 5 kg starting from rest pulled up on a smooth incline plane making an angle of 30° with horizontal with an affective acceleration of 1 ms^{-2}. The power delivered by the pulling force at $t = 10$ s from the starts is ____ W. [use $g = 10$ ms^{-2}] (Evaluate the nearest integer value)

Correct Answer: 300

Solution:

Provided that: Mass $m = 5$ kg, angle of incline $\theta = 30^\circ$, acceleration $a = 1$ m/s^2, time $t = 10$ s, and $g = 10$ m/s^2. Initial velocity $u = 0$ since the block starts from rest.

(0,0) – (4,0); (0,0) – (30:3); (1,0) arc (0:30:1) node[midway,right] 30°; [fill=white] (30:1.5) – ++(0.3,0) – ++(-30:0.3) – ++(-0.3,0) – cycle; [-¿] (0,0) – (30:2) node[midway,above left] $mg \sin 30$;

Step 1: Resolve forces and find pulling force The forces acting on the block along the incline are the pulling force F (upwards), the component of the weight $mg \sin\theta$ (downwards), and the force causing the acceleration ma (upwards). Applying Newton's second law along the incline:

$$F - mg\sin\theta = ma.$$

Substituting the Provided that values:

$$F - 5 \times 10 \times \sin 30^\circ = 5 \times 1.$$

$$F - 5 \times 10 \times \frac{1}{2} = 5 \implies F - 25 = 5.$$

$$F = 30 \text{ N}.$$

Step 2: Evaluate the velocity. Using the first equation of motion:

$$v = u + at,$$

where v is the final velocity, u is the initial velocity, a is the acceleration, and t is the time. Substituting $u = 0$, $a = 1$ m/s^2, and $t = 10$ s:

$$v = 0 + 1 \times 10 = 10 \text{ m/s}.$$

Step 3: Evaluate the power. Power P is the rate at which work is done, which is Provided that by the product of force and velocity:

$$P = Fv.$$

Substituting $F = 30$ N and $v = 10$ m/s:

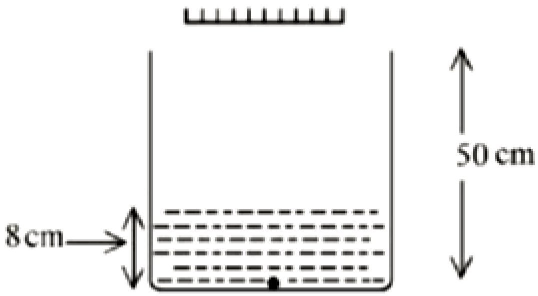
$$P = 30 \times 10 = 300 \text{ W}.$$

Conclusive Answer: The power delivered by the pulling force is 300 W.

Quick Tip

Power is the rate of doing work. For constant force and velocity along the direction of the force, it can be Evaluated as the product of force and velocity.

58: As shown in the figure, a plane mirror is fixed at a height of 50 cm from the bottom of tank containing water $\left(\mu = \frac{4}{3}\right)$. The height of water in the tank is 8 cm. A small bulb is placed at the bottom of the water tank. The distance of image of the bulb formed by mirror from the bottom of the tank is _____ cm.



Correct Answer: 98

Solution:

Step 1: Apparent depth of O.

The calculationship between the apparent depth and real depth in a medium is Provided that by:

$$\text{Apparent depth} = \frac{\text{Real depth}}{\mu},$$

where: - Real depth $= d = 8\,\text{cm}$, - $\mu = 4/3$ is the refractive index of the medium.

Substitute the values:

$$\text{Apparent depth of } O = \frac{d}{\mu} = \frac{8}{4/3} = 6\,\text{cm}.$$

Step 2: Distance between O and I_2.

1. The total real depth of the liquid column is:

$$\text{Real depth} = 42\,\text{cm}.$$

2. The refracted depth:

$$\text{Apparent shift} = \frac{8}{4} \cdot 3 = 6\,\text{cm}.$$

The distance between O and I_2 is Evaluated as:

$$\text{Distance between } O \text{ and } I_2 = 48 + 50 = 98\,\text{cm}.$$

Conclusive Answer:

$$\text{Apparent depth of } O = 6\,\text{cm}, \quad \text{Distance between } O \text{ and } I_2 = 98\,\text{cm}.$$

Quick Tip

When dealing with apparent depth, remember that the object appears closer to the surface than its actual depth. The image formed by a plane mirror is at the same distance behind the mirror as the object is in front of the mirror.

59: Two identical cells each of emf 1.5 V are connected in series across a $10\ \Omega$ resistance. An ideal voltmeter connected across $10\ \Omega$ resistance reads 1.5 V. The internal resistance of each cell is ____ Ω.

Correct Answer: 5

Solution:

Provided that: EMF of each cell $\varepsilon = 1.5$ V, external resistance $R = 10\ \Omega$, and voltmeter reading across the external resistance $V_R = 1.5$ V.

Step 1: Evaluate the current in the circuit. Let r be the internal resistance of each cell. Since the cells are connected in series, the total EMF is $2\varepsilon = 2 \times 1.5 = 3$ V, and the total internal resistance is $2r$. The current I in the circuit is Provided that by

$$I = \frac{2\varepsilon}{R + 2r}.$$

The voltmeter reads the potential difference across the $10\ \Omega$ resistor, which is $V_R = IR = 1.5$ V. Therefore,

$$I = \frac{V_R}{R} = \frac{1.5}{10} = 0.15\ \text{A}.$$

Step 2: Solve for the internal resistance. Substituting $I = 0.15$ A and $2\varepsilon = 3$ V:

$$0.15 = \frac{3}{10 + 2r}.$$

$$10 + 2r = \frac{3}{0.15} = 20.$$

$$2r = 20 - 10 = 10.$$

$$r = \frac{10}{2} = 5\ \Omega.$$

Conclusive Answer: The internal resistance of each cell is 5 Ω.

Quick Tip

When cells are connected in series, their EMFs add up, and so do their internal resistances.

60: A wire of density 8×10^3 kg/m^3 is stretched between two clamps 0.5 m apart. The extension developed in the wire is 3.2×10^{-4} m. If $Y = 8 \times 10^{10}$ N/m^2, the fundamental frequency of vibration in the wire will be _____ Hz.

Correct Answer: 80

Solution:

Step 1: Formula for frequency.

The frequency of a vibrating string is Provided that by:

$$f = \frac{1}{2\ell}\sqrt{\frac{T}{\mu}},$$

where: - ℓ is the length of the string, - T is the tension in the string, - μ is the linear mass density of the string.

Step 2: Substitute tension in terms of Young's modulus.

The tension T can be expressed in terms of Young's modulus Y as:

$$T = Y \cdot A \cdot \frac{\Delta\ell}{\ell},$$

where: - Y is Young's modulus, - A is the cross-sectional area of the string, - $\Delta\ell$ is the extension of the string, - ℓ is the original length of the string.

Substituting for T in the frequency formula:

$$f = \frac{1}{2\ell}\sqrt{\frac{Y \cdot A \cdot \Delta\ell}{\ell \cdot \mu}}.$$

Step 3: Substitute the Provided that values.

Substitute the numerical values:

$$f = \frac{1}{2 \cdot 0.5}\sqrt{\frac{8 \times 10^{10} \cdot 3.2 \times 10^{-4}}{8 \times 10^{3} \cdot 0.5}}.$$

Evaluate the terms:

$$f = \frac{1}{1}\sqrt{\frac{8 \times 10^{10} \cdot 3.2 \times 10^{-4}}{4 \times 10^{3}}}.$$

$$f = \frac{1}{1}\sqrt{6400}.$$

Step 4: Evaluate the frequency.

Evaluate:

$$f = \sqrt{6400} = 80\,\text{Hz}.$$

Conclusive Answer:

The frequency of the vibrating string is:

$$f = 80\,\text{Hz}.$$

Quick Tip

The fundamental frequency of a stretched wire depends on its length, tension, and mass per unit length. Young's modulus relates stress and strain in the wire.

Chemistry

Section A

61: The magnetic moment is measured in Bohr Magneton (BM). Spin only magnetic moment of Fe in $[Fe(H_2O)_6]^{3+}$ and $[Fe(CN)_6]^{3-}$ complexes respectively is:

(1) 3.87 B.M. and 1.732 B.M.

(2) 6.92 B.M. in both

(3) 5.92 B.M. and 1.732 B.M.

(4) 4.89 B.M. and 6.92 B.M.

Correct Answer: (3): 5.92 B.M. and 1.732 B.M.

Solution:

Step 1: Its Provided that electron configuration of Fe^{3+} in $[Fe(H_2O)_6]^{3+}$**-**

In $[Fe(H_2O)_6]^{3+}$, Fe is in the +3 oxidation state.

The electronic configuration of Fe is [Ar] $3d^6$ $4s^2$.

Fe^{3+} has the configuration [Ar] $3d^5$.

Water is a weak field ligand, so it does not cause pairing of electrons.

Number of unpaired electrons, $n = 5$.

Step 2: The spin-only magnetic moment for $[Fe(H_2O)_6]^{3+}$**-**

The spin-only magnetic moment μ is Provided that by

$$\mu = \sqrt{n(n+2)}\ \text{BM},$$

where n is the number of unpaired electrons. Substituting $n = 5$:

$$\mu = \sqrt{5(5+2)} = \sqrt{35} \approx 5.92\ \text{BM}.$$

Step 3: Determine the electron configuration of Fe^{3+} in $[Fe(CN)_6]^{3-}$**-**

In $[Fe(CN)_6]^{3-}$, Fe is also in the +3 oxidation state, so its electronic configuration is [Ar] $3d^5$. However, CN^- is a strong field ligand, causing pairing of electrons.

The five 3d electrons are arranged as follows:

(0,0) rectangle (5,0.5); in 0.5,1.5,2.5,3.5,4.5 (,0) – (,0.5); at (0.25,0.25) ↑↓; at (1.25,0.25) ↑↓; at (2.25,0.25) ↑;

Number of unpaired electrons, $n = 1$.

Step 4: The spin-only magnetic moment for $[Fe(CN)_6]^{3-}$**-**

Substituting $n = 1$ into the formula for magnetic moment:

$$\mu = \sqrt{1(1+2)} = \sqrt{3} \approx 1.732\ \text{BM}.$$

Conclusive Answer: The spin-only magnetic moments are 5.92 B.M. and 1.732 B.M. respectively.

Quick Tip

Strong field ligands cause pairing of electrons in the d-orbitals, while weak field ligands do not.

62: Which one of the following pairs is an example of polar molecular solids?

(1) $SO_2(s), CO_2(s)$

(2) $SO_2(s), NH_3(s)$

(3) $MgO(s), SO_2(s)$

(4) $HCl(s), AlN(s)$

Correct Answer: (2): $SO_2(s), NH_3(s)$

Solution:

Polar molecular solids consist of polar molecules held together by dipole-dipole interactions. Sulfur dioxide (SO) has a bent molecular structure and a net dipole moment, making it a polar molecule. Similarly, ammonia (NH) has a trigonal pyramidal shape with a net dipole moment, classifying it as polar. In contrast, carbon dioxide (CO) has a linear structure and is nonpolar. Magnesium oxide (MgO) is an ionic solid, while hydrogen chloride (HCl) is a polar molecule. Aluminum nitride (AlN) is also an ionic solid.

Conclusive Answer: The pair (SO_2(s), NH_3(s)) is an example of polar molecular solids.

Quick Tip

Polar molecules have a net dipole moment due to differences in electronegativity and asymmetrical molecular geometry.

63: Match List I with List II

	List I Complex		**List II Colour**
A.	$Mg(NH_4)PO_4$	I.	Brown
B.	$K_3[Co(NO_2)_6]$	II.	White
C.	$MnO(OH)_2$	III.	Yellow
D.	$Fe_4[Fe(CN)_6]_3$	IV.	blue

Choose the correct answer from the options Provided that below:

(1) A – II, B – III, C – IV, D – I

(2) A – II, B – IV, C – I, D – III

(3) A – III, B – IV, C – II, D – I

(4) A – II, B – III, C – I, D – IV

Correct Answer: (4): A - II, B - III, C - I, D - IV

Solution:

$Mg(NH_4)PO_4$ is white.

$K_3[Co(NO_2)_6]$ is yellow.

$MnO(OH)_2$ is brown.

$Fe_4[Fe(CN)_6]_3$ is blue (Prussian blue).

Conclusive Answer: A – II, B – III, C – I, D – IV

> Quick Tip
>
> The color of coordination complexes depends on the metal ion, its oxidation state, and the ligands attached to it.

64: A solution is prepared by adding 2 g of "X" to 1 mole of water. Mass percent of "X" in the solution is ____.

(1) 5%

(2) 20%

(3) 2%

(4) 10%

Correct Answer: (4): 10%

Solution:

Provided that: Mass of solute (X) = 2 g and moles of solvent (H_2O) = 1 mole.

Step 1: Evaluate the mass of the solvent-

The molar mass of water is 18 g/mol. Since we have 1 mole of water, the mass of the solvent is:

$$\text{Mass of solvent} = 1 \text{ mol} \times 18 \text{ g/mol} = 18 \text{ g}.$$

Step 2: Evaluate the total mass of the solution-

The total mass of the solution is the sum of the mass of solute and the mass of solvent:

$$\text{Total mass} = \text{Mass of solute} + \text{Mass of solvent} = 2 \text{ g} + 18 \text{ g} = 20 \text{ g}.$$

Step 3: Evaluate the mass percent of X-

The mass percent of X is Provided that by:

$$\% \text{ mass of X} = \frac{\text{Mass of X}}{\text{Total mass}} \times 100 = \frac{2 \text{ g}}{20 \text{ g}} \times 100 = 10\%.$$

Conclusive Answer: The mass percent of ”X” in the solution is 10%.

Quick Tip

Mass percent is the mass of the solute divided by the total mass of the solution, multiplied by 100.

65: If Ni^{2+} is replaced by Pt^{2+} in the complex $[NiCl_2Br_2]^{2-}$, which of the following properties are expected to get changed?

A. Geometry

B. Geometrical isomerism

C. Optical isomerism

D. Magnetic properties

(1) A, B and C

(2) A and D

(3) B and C

(4) A, B and D

Correct Answer: (4): A, B, and D

Solution:

$[NiCl_2Br_2]^{2-}$: This complex has Ni^{2+} with a d^8 configuration.
Since Cl^- and Br^- are weak field ligands, the complex is tetrahedral. Tetrahedral complexes with this formula do not show geometrical isomerism and are optically inactive.

$[PtCl_2Br_2]^{2-}$: This complex has Pt^{2+} with a d^8 configuration. Being a 5d series element, Pt^{2+} forms square planar complexes. Square planar complexes of the type MA_2B_2 can have cis and trans isomers (geometrical isomers), but for these particular ligands there would be no optical isomerism. Also, the change in geometry from tetrahedral to square planar changes the crystal field splitting, which changes the magnetic properties.

Conclusive Answer: Geometry (A), Geometrical isomerism (B), and Magnetic properties (D) are expected to change.

Quick Tip

The geometry and isomerism of coordination complexes are influenced by the metal ion's d-electron configuration and the nature of the ligands.

66: Provided that below are two statements :

Statement I: In the metallurgy process, sulphide ore is converted to oxide before reduction.
Statement II: Oxide ores in general are easier to reduce.

In the light of the above statements, choose the most appropriate answer from the options Provided that below:

(1) Both Statement I and Statement II are correct
(2) Statement I is correct but Statement II is incorrect
(3) Statement I is incorrect but Statement II is correct
(4) Both Statement I and Statement II are incorrect

Correct Answer: (1): Both Statement I and Statement II are correct

Solution:

Statement I: Sulphide ores are converted to oxides before reduction. This is true. The process of converting a sulphide ore to its oxide is called roasting. For example, zinc sulphide is roasted to zinc oxide:

$$2ZnS + 3O_2 \rightarrow 2ZnO + 2SO_2.$$

Statement II: Oxide ores are generally easier to reduce. This statement is also true. Oxides are easier to reduce compared to sulphides. This can be explained by the following:

Carbon reduction of oxides produces CO_2, which is a gas and readily escapes, making the reaction proceed forward more easily. On the other hand, carbon reduction of sulphides would form CS_2 which is less volatile than CO_2. The reaction is:

$$2MS + C \rightarrow 2M + CS_2$$

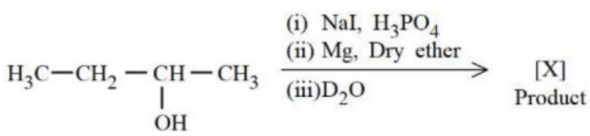
Where M is the metal CO_2 is more volatile compared to CS_2 because of its linear structure, while CS_2 also has a linear structure but has stronger intermolecular forces due to sulfur being larger than oxygen, making CS_2 less volatile, thereby hindering the reaction from being pushed forward.

Conclusive Answer: Both Statement I and Statement II are correct.

Quick Tip

Roasting is the process of converting sulfide ores to oxides by heating them in the presence of air. Oxides are easier to reduce than sulfides because the byproduct (CO_2) is more volatile.

67: Product [X] formed in the above reaction is:



(1) $CH_3 - CH_2 - CH(D) - CH_3$

(2) $CH_3 - CH_2 - CH = CH_2$

(3) $CH_3 - CH = CH - CH_3$

(4) $CH_3 - CH_2 - C(OH)(H) - CH_3$

Correct Answer: (1): $CH_3 - CH_2 - CH(D) - CH_3$

Solution: The reaction sequence proceeds as follows:

Step 1: Reaction with NaI and H_3PO_4-

The alcohol reacts with NaI and H_3PO_4 to form the corresponding iodoalkane via a nucleophilic substitution reaction. H_3PO_4 acts as an acid catalyst, and I^- from NaI acts as the nucleophile. The hydroxyl group is replaced by an iodine atom:

$$CH_3 - CH_2 - CH(OH) - CH_3 \xrightarrow{NaI,\ H_3PO_4} CH_3 - CH_2 - CH(I) - CH_3$$

Step 2: Grignard reagent formation-

The iodoalkane reacts with magnesium in dry ether to form a Grignard reagent. Dry ether is used as a solvent because it stabilizes the Grignard reagent and prevents it from reacting with water.

$$CH_3 - CH_2 - CH(I) - CH_3 \xrightarrow{Mg,\ Dry\ ether} CH_3 - CH_2 - CH(MgI) - CH_3$$

Step 3: Reaction with D_2O-

The Grignard reagent reacts with D_2O (deuterated water) to form the deuterated alkane. The C-MgI bond is replaced by a C-D bond.

$$CH_3 - CH_2 - CH(MgI) - CH_3 \xrightarrow{D_2O} CH_3 - CH_2 - CH(D) - CH_3$$

Conclusive Answer: The product formed is $CH_3 - CH_2 - CH(D) - CH_3$.

Quick Tip

Grignard reagents are formed by the reaction of alkyl halides with magnesium in dry ether. They are strong nucleophiles and strong bases and readily react with compounds containing acidic hydrogen.

68: Provided that below are two statements :

Statement I : Ethene at 333 to 343 K and 6-7 atm pressure in the presence of $AlEt_3$ and $TiCl_4$ undergoes addition polymerization to give LDP.

Statement II : Caprolactam at 533-543 K in H_2O through step growth polymerizes to give Nylon 6.

In the light of the above statements, choose the correct answer from the options Provided that below:

(1) Statement I is true but Statement II is false

(2) Both Statement I and Statement II are true

(3) Statement I is false but Statement II is true

(4) Both Statement I and Statement II are false

Correct Answer: (3): Statement I is false but Statement II is true

Solution:

Statement I: Ethene at 333 to 343 K and 6-7 atm pressure in the presence of $AlEt_3$ and $TiCl_4$ undergoes addition polymerization to give *high-density polyethylene* (HDPE), not low-density polyethylene (LDPE). Ziegler-Natta catalysts (like $AlEt_3$ and $TiCl_4$) are used to produce HDPE. LDPE is produced via free radical polymerization at high temperatures (423-573 K) and very high pressures (1000-2000 atm). Thus, Statement I is false.

Statement II: Caprolactam undergoes ring-opening polymerization in the presence of water at high temperatures (533-543 K) to form Nylon 6. This is a step-growth polymerization. Thus, Statement II is true.

*6(—N(-[:30]O)–) [yshift=3pt]$\xrightarrow[H_2O]{533-543K}$ $[-C(=O)-(CH_2)_5-N(-[:30]H)-]_n$

Conclusive Answer: Statement I is false, but Statement II is true.

Quick Tip

Addition polymerization involves the addition of monomers without loss of atoms, while step-growth polymerization involves the loss of small molecules (like water) during the formation of polymer chains.

69: For a chemical reaction A + B $\rightarrow$ Product, the order is 1 with respect to A and B.

What is the value of x and y?

(1) 80 and 2

(2) 40 and 4

Rate $molL^{-1}S^{-1}$	[A] $molL^{-1}$	[B] $molL^{-1}$
0.10	20	0.5
0.40	x	0.5
0.80	40	Y

(3) 80 and 4

(4) 160 and 4

Correct Answer: (1): 80 and 2

Solution:

Provided that: The reaction A + B $\rightarrow$ Product is first order with respect to both A and B.

Step 1: Recall the rate law-

Since the reaction is first order with respect to both A and B, the rate law is Provided that by:

$$r = k[A][B],$$

where r is the rate, k is the rate constant, and [A] and [B] are the concentrations of A and B, respectively.

Step 2: Set up equations using the provided that data-

We can set up three equations using the provided that data:

$$0.10 = k(20)(0.5) \tag{1}$$

$$0.40 = k(x)(0.5) \tag{2}$$

$$0.80 = k(40)(Y) \tag{3}$$

Step 3: Solve for x-

Dividing the second equation by the first equation:

$$\frac{0.40}{0.10} = \frac{k(x)(0.5)}{k(20)(0.5)}.$$

$$4 = \frac{x}{20} \implies x = 4 \times 20 = 80.$$

Step 4: Solve for Y-

Dividing the third equation by the first equation:

$$\frac{0.80}{0.10} = \frac{k(40)(Y)}{k(20)(0.5)}.$$

$$8 = \frac{40Y}{10} \implies 80 = 40Y \implies Y = \frac{80}{40} = 2.$$

Conclusive Answer: The values of x and Y are 80 and 2, respectively.

Quick Tip

The rate law expresses the relationship between the rate of a reaction and the concentrations of the reactants. The order of a reaction with respect to a particular reactant is the power to which its concentration is raised in the rate law.

70: Which of the following compounds is an example of Freon?

(1) C_2F_4

(2) C_2HF_3

(3) $C_2Cl_2F_2$

(4) $C_2H_2F_2$

Correct Answer: (3): $C_2Cl_2F_2$

Solution:

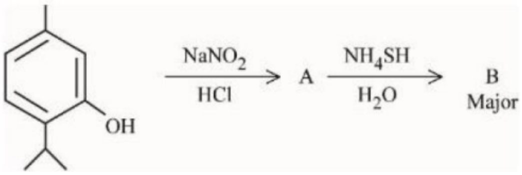
Chlorofluorocarbons (CFCs) and hydrochlorofluorocarbons (HCFCs), collectively known as Freons, are hydrocarbons in which some or all hydrogen atoms are replaced by chlorine and fluorine atoms. Among the given options, only $C_2Cl_2F_2$ (dichlorodifluoroethylene) contains both chlorine and fluorine along with carbon, making it a Freon.

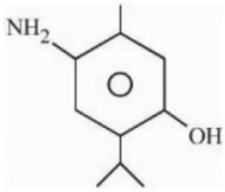
Conclusive Answer: $C_2Cl_2F_2$ is an example of a Freon.

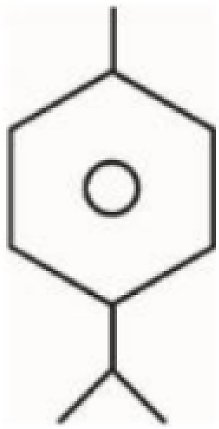
Quick Tip

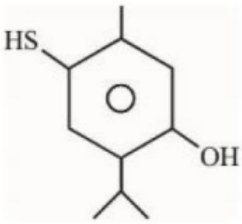
Freons are chlorofluorocarbons (CFCs) which contain chlorine, fluorine, and carbon. They were commonly used as refrigerants but are now being phased out due to their ozone-depleting properties.

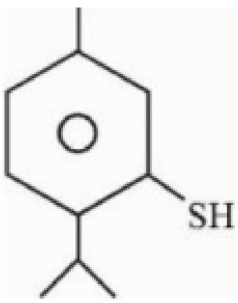
71: Compound 'B' is _____.

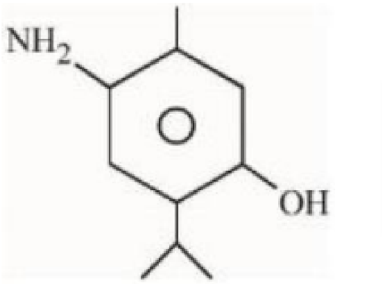


(1) 

(2) 

(3) 

(4) 

Correct Answer: (1) 

Solution: The Provided that reaction sequence is:

**6(——) C(=[2]O)(-)(-[::-60]OH) **6(——) $\xrightarrow[HCl]{NaNO2} A \xrightarrow[H2O]{NH4SH} B$

Step 1: Diazotization-

The first step involves the reaction of the Provided that compound (2-hydroxycyclohexanone) with sodium nitrite ($NaNO_2$) and hydrochloric acid (HCl). This is a diazotization reaction, which converts the amino group (-NH_2) of an aromatic amine to a diazonium salt (-$N_2^+Cl^-$). This reaction doesn't happen with the carbonyl group. Since the starting compound shown is not an amino compound, it needs to have NH2 for diazotization reaction to happen. Hence it would be more logical if the used a starting compound of 2-aminocyclohexanol rather than 2-hydroxycyclohexanone.

Step 2: Reduction of Diazonium salt-

The second step involves the reaction of the diazonium salt (A) with ammonium hydrosulfide (NH_4SH) in water. This reduces the diazonium salt to the corresponding amine, with nitrogen gas as a byproduct.

So the final product B is 2-aminocyclohexanol.

Conclusive Answer: The major product (B) is 2-aminocyclohexanol.

Quick Tip

Diazotization reactions are important for converting aromatic amines into various other functional groups. NH_4SH is a reducing agent that converts diazonium salts to amines.

72: Provided that below are two statements, one is labelled as Assertion A and the other is labelled as Reason R.

Assertion A: C(=[2]O)-C(-[6]Cl) can be subjected to Wolff-Kishner reduction to give $CH_2 - C(-[6]Cl)(-H)(-H)$

Reason R: Wolff-Kishner reduction is used to convert C(=[2]O) into CH_2

In the light of the above statements, choose the correct answer from the options Provided that below:

(1) Both A and R are true and R is the correct explanation of A

(2) A is true but R is false

(3) Both A and R are true but R is NOT the correct explanation of A

(4) A is false but R is true

Correct Answer: (4): A is false but R is true

Solution:

Assertion (A): The Wolff-Kishner reduction converts a carbonyl group (C=O) to a methylene group (CH_2). The Provided that assertion states that 2-chloropropanal (C(=[2]O)-C(-[6]Cl)) can undergo Wolff-Kishner reduction to form 2-chloropropane ($CH_2 - C(-[6]Cl)(-H)(-H)).$
$Thisassertion is False. The reaction proceeds as follows:$
C(=[2]O)-C(-[6]Cl) $\xrightarrow[-OH]{NH2-NH2} CH_2 - C(-[6]Cl)(-H)(-H)$

Reason (R): The Provided that reason states that the Wolff-Kishner reduction is used to convert C(=[2]O) into $CH_2 . This reason is true and is the principle behind the Wolff-Kishner reduction.$

Conclusive Answer: A is false but R is true.

Quick Tip

The Wolff-Kishner reduction is a useful reaction for converting carbonyl groups to methylene groups. Remember that this reaction requires basic conditions.

73: Provided that below are two statements, one is labelled as Assertion A and the other is labelled as Reason R.

Assertion A : $[CoCl(NH_3)_5]^{2+}$ absorbs at lower wavelength of light wrt- $[CoCl(NH_3)_5(H_2O)]^{3+}$
Reason R : It is because the wavelength of the light absorbed depends on the oxidation state of the metal ion.

In the light of the above statements, choose the correct answer from the options Provided that below:

(1) Both A and R are true but R is NOT the correct explanation of A

(2) A is true but R is false

(3) Both A and R are true and R is the correct explanation of A

(4) A is false but R is true

Correct Answer: (4): A is false but R is true

Solution:

Assertion A: The assertion states that $[CoCl(NH_3)_5]^{2+}$ absorbs at a lower wavelength compared to $[CoCl(NH_3)_5(H_2O)]^{3+}$. This assertion is false. The complex with Co^{3+} will have a larger crystal field splitting energy (CFSE) compared to the complex with Co^{2+}, since H_2O is a stronger field ligand than Cl^-, and a higher oxidation state leads to a larger CFSE. A larger CFSE means that higher energy (shorter wavelength) light is absorbed. Thus $[CoCl(NH_3)_5(H_2O)]^{3+}$, should absorb at a lower wavelength (higher energy) than $[CoCl(NH_3)_5]^{2+}$.

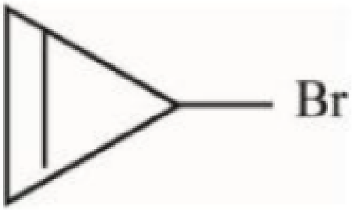
Reason R: The reason states that the wavelength of light absorbed depends on the oxidation state of the metal ion. This reason is true. The oxidation state of the central metal ion affects the crystal field splitting energy (CFSE). A higher oxidation state generally leads to a larger CFSE, resulting in the absorption of light at a lower wavelength (higher energy).

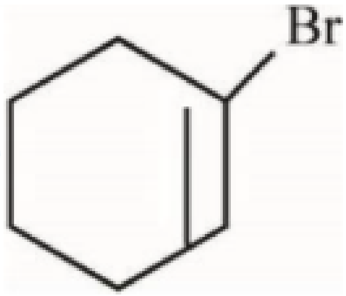
Conclusive Answer: Assertion A is false, but Reason R is true.

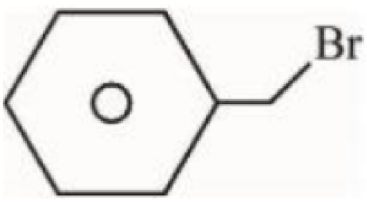
Quick Tip

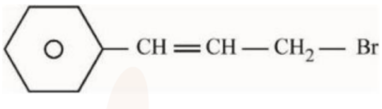
The crystal field splitting energy (CFSE) is the energy difference between the d-orbitals in a coordination complex. It is affected by the metal ion, its oxidation state, and the ligands. A larger CFSE leads to absorption at a lower wavelength (higher energy).

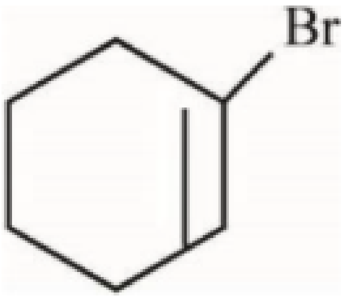
74: Compound from the following that will not produce precipitate on reaction with $AgNO_3$ is:

(1) 

(2) 

(3) 

(4) 

Correct Answer: (2) 

Solution:

The reaction with $AgNO_3$ is used to test for halide ions. A precipitate of AgBr (pale yellow) will form if a bromide ion is available to react with Ag^+.

Compound 1: This is a cyclopropyl bromide. The C-Br bond is not easily broken, so it does not readily form a precipitate with $AgNO_3$. The cation formed is **6(——) $C^+(-[:30,,,,draw=none]1)(-[::60,,,,draw=none]2)(-[::-60,,,,draw=none]3) and is stabilized by aromatic$

Compound 2: This is bromobenzene. The C-Br bond in aryl halides is strong due to resonance and sp^2 hybridization of carbon, thus it does not react with $AgNO_3$ to form a precipitate. The cation **6(——) $C^+(-[:: -60,,,, draw = none]1)(-[: 30,,,, draw = none]2)(-[:: 60,,,, draw = none]3) is not formed because it is highly unstable.$

Compound 3: This is a benzylic bromide. The benzylic carbocation formed after removal of bromide ion is stabilized by resonance with the benzene ring. Hence, it reacts with $AgNO_3$ to form a precipitate. The cation formed is
**6(——) C(=[2]O)(-)(-[::-60]CH_2^+)$**6(------)$ $and is stabilized by benzylic nature.$

Compound 4: This is an allylic bromide. The allylic carbocation formed is stabilized by resonance. Hence, it reacts with $AgNO_3$ to form a precipitate. The cation formed is **6(——) C(=[2]O)(-)(-[::-60]CH=CH-CH_2^+)$**6(------)$ $and is stabilized by allylic nature.$

Conclusive Answer: Bromobenzene will not produce a precipitate with $AgNO_3$.

Quick Tip

Aryl and vinyl halides do not readily react with $AgNO_3$ to form a precipitate due to the stronger C-X bond. Benzylic and allylic halides do form precipitates because they form resonance stabilized carbocations.

75: Provided that below are two statements, one is labelled as Assertion A and the other is labelled as Reason R.

Assertion A : A solution of the product obtained by heating a mole of glycine with a mole of chlorine in presence of red phosphorous generates chiral carbon atom.

Reason R : A molecule with 2 chiral carbons is always optically active.

In the light of the above statements, choose the correct answer from the options Provided that below:

(1) A is false but R is true

(2) Both A and R are true but R is NOT the correct explanation of A

(3) A is true but R is false

(4) Both A and R are true and R is the correct explanation of A

Correct Answer: (3): A is true but R is false

Solution:

Assertion A: Glycine (NH_2CH_2COOH) reacts with chlorine (Cl_2) in the presence of red phosphorus to undergo alpha-halogenation. The product formed is 2-chloroacetic acid:

$$NH_2CH_2COOH \xrightarrow[\text{Red P}]{Cl_2} NH_2CHClCOOH$$

The product, 2-chloroacetic acid or α-chloroacetic acid, which has a chiral carbon (marked with an asterisk):

$H_2N - C^*(-[: -30]Cl)(-[6]COOH)$

This assertion is true.

Reason R: The reason states that a molecule with two chiral carbons is always optically active. This reason is false. A molecule with two chiral centers can be optically inactive if it has an internal plane of symmetry. Such molecules are called meso compounds.

Conclusive Answer: Assertion A is true, but Reason R is false.

Quick Tip

A chiral carbon is a carbon atom bonded to four different groups. A molecule is optically active if it does not have a plane of symmetry, even with chiral carbons. Meso compounds are optically inactive molecules with chiral centers.

76: Alkali metal from the following with least melting point is:

(1) K

(2) Cs

(3) Rb

(4) Na

Correct Answer: (2): Cs

Solution:

Alkali metal's melting points decrease as we move down the group. This trend occurs because metallic bonding, which arises from the electrostatic attraction between positive metal ions and a sea of delocalized electrons, becomes weaker with increasing atomic size. As the atomic radius grows down the group, the outermost electron experiences weaker attraction, resulting in weaker metallic bonding and consequently lower melting points.

Among the Provided that options, Cs is the largest alkali metal and therefore has the weakest metallic bonding. Consequently, Cs has the lowest melting point. The melting points of the Provided that alkali metals are:

Na: 98°C
K: 64°C
Rb: 39°C
Cs: 29°C

Conclusive Answer: Cs has the least melting point.

Quick Tip

Melting points of alkali metals decrease down the group due to weakening metallic bonds caused by increasing atomic size.

77: Which hydride among the following is less stable?

(1) HF

(2) NH_3

(3) BeH_2

(4) LiH

Correct Answer: (3): BeH_2

Solution:

Stability of hydrides is connected with the bond strength between the element and hydrogen. BeH_2 is less stable because it is electron deficient or hypovalent. Be has 2 valence electrons, each forming a covalent bond with hydrogen to give BeH_2. Beryllium needs 4 electrons in its valence shell according to the octet rule to gain stability but only has two electrons bonded to hydrogen, making it electron deficient. This electron deficiency makes BeH_2 less stable and prone to polymerization, forming polymeric $(BeH_2)_n$.

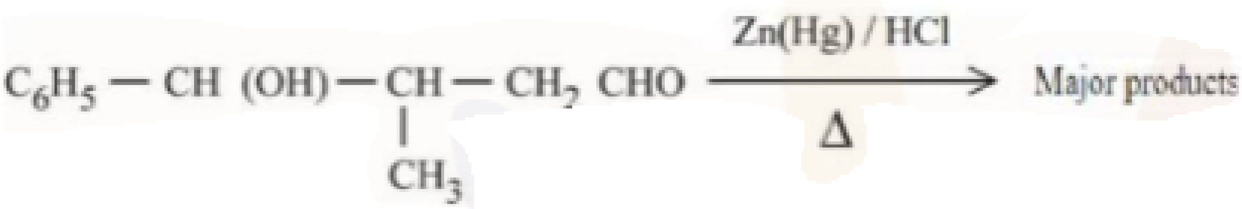
HF, NH_3, and LiH are all stable compounds. HF has a strong polar covalent bond, NH_3 satisfies the octet rule and forms stable covalent bonds, and LiH is an ionic hydride formed between Li^+ and H^-.

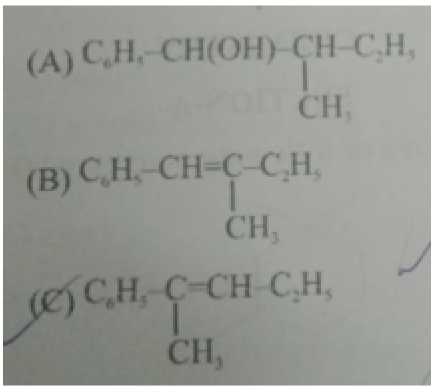
Conclusive Answer: BeH_2 is less stable among the Provided that hydrides.

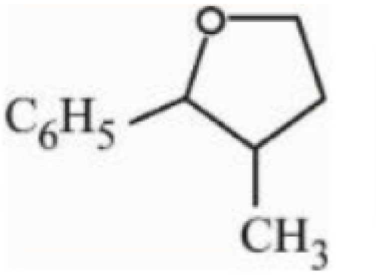
Quick Tip

Electron-deficient or hypovalent compounds are less stable due to their incomplete valence shells. They tend to react with other molecules or polymerize to achieve a more stable configuration.

78: The major product formed in the following reaction is _____.





D. 

Choose the correct answer from the options Provided that below:

(1) C only

(2) A only

(3) B only

(4) D only

Correct Answer: (3): B only

Solution:

$$\mathrm{Ph-\underset{\displaystyle OH}{\underset{|}{CH}}-\underset{\displaystyle CH_3}{\underset{|}{CH}}-CH_2-CHO \xrightarrow[\Delta]{Zn(Hg)/HCl} Ph-CH=\underset{\displaystyle CH_3}{\underset{|}{C}}-CH_2-CH_3}$$

Quick Tip

The Clemmensen reduction is a useful reaction for reducing aldehydes and ketones to alkanes. It is typically carried out under acidic conditions using zinc amalgam and hydrochloric acid.

79: One mole of P_4 reacts with 8 moles $SOCl_2$ to give 4 moles of A, x mole of SO_2 and 2 moles of B. A, B and x respectively are _____.

(1) $POCl_3$, S_2Cl_2 and 4

(2) $POCl_3$, S_2Cl_2 and 2

(3) PCl_3, S_2Cl_2 and 4

(4) PCl_3, S_2Cl_2 and 2

Correct Answer: (1): $POCl_3$, S_2Cl_2, and 4

Solution:

Provided that: 1 mole of P_4 reacts with 8 moles of $SOCl_2$ to give 4 moles of A, x moles of SO_2, and 2 moles of B.

Step 1: The balanced chemical equation-

The balanced chemical equation for the reaction of P_4 with $SOCl_2$ is:

$$P_4 + 8SOCl_2 \rightarrow 4PCl_3 + 2S_2Cl_2 + 4SO_2.$$

In this reaction phosphorus is being oxidised from 0 to +3 state and sulfur is being reduced from +4 to +2 as well as +4 to +1 oxidation state.

Step 2: Identify A, B, and x-

Comparing the balanced equation with the Provided that information, we have:

- A = PCl_3
- B = S_2Cl_2
- x = 4

The correct balanced reaction is $P_4 + 8SOCl_2 \rightarrow 4POCl_3 + 2S_2Cl_2 + 4SO_2$

Conclusive Answer: A = $POCl_3$, B = S_2Cl_2, and x = 4.

Quick Tip

Balancing chemical equations is crucial for stoichiometric calculations. Make sure the number of atoms of each element is equal on both sides of the equation.

80: What weight of glucose must be dissolved in 100 g of water to lower the vapour pressure by 0.20 mmHg? (Assume dilute solution is being formed)

Provided that : Vapour pressure of pure water is 54.2 mmHg at room temperature. Molar mass of glucose is 180g mol^{-1}

(1) 2.59 g

(2) 3.59 g

(3) 3.69 g

(4) 4.69 g

Correct Answer: (3): 3.69 g

Solution:

Step 1: Apply the formula for relative lowering of vapor pressure-

For a dilute solution, the relative lowering of vapor pressure is Provided that by:

$$\frac{P^0 - P_s}{P^0} = \frac{n}{N},$$

where:

- P^0 is the vapor pressure of the pure solvent,

- P_s is the vapor pressure of the solution,

- n is the number of moles of solute,

- N is the number of moles of solvent.

Step 2: Substitute the provided values-

Provided that:

$$\frac{P^0 - P_s}{P^0} = \frac{0.2}{54.2}, \quad N = \frac{100}{18} \text{ moles.}$$

Substitute these values into the equation:

$$\frac{0.2}{54.2} = \frac{n \cdot 18}{100}.$$

Step 3: Solve for n-

Rearranging:

$$n = \frac{100 \cdot 0.2}{54.2 \cdot 18}.$$

Simplify:

$$n = \frac{100}{271 \cdot 18}.$$

Step 4: Evaluate the mass of solute (w)-

The mass of the solute is Provided that by:

$$w = n \cdot M,$$

where $M = 180\,\text{g/mol}$ is the molar mass of the solute. Substituting:

$$w = \frac{100 \cdot 180}{271 \cdot 18}.$$

Simplify:

$$w = 3.69\,\text{g}.$$

Conclusive Answer:

The mass of the solute is:

$$w = 3.69\,\text{g}.$$

Quick Tip

Raoult's law states that the vapor pressure of a solution is directly proportional to the mole fraction of the solvent. For dilute solutions, the relative lowering of vapor pressure is approximately equal to the mole fraction of the solute.

Section B

81: The total number of intensive properties from the following is _____.

Mole, Volume, Molar heat capacity, Molarity, E° cell, Gibbs free energy change, Molar mass, Mole

Correct Answer: 4

Solution:

Intensive properties are properties that do not depend on the amount of substance. Extensive properties are properties that do depend on the amount of substance.

From the Provided that list:

Extensive: Mole, Volume, Gibbs free energy change

Intensive: Molar mass, Molar heat capacity, Molarity, E° cell

There are four intensive properties in the list.

Conclusive Answer: The total number of intensive properties is 4.

Quick Tip

Intensive properties are independent of the amount of substance, while extensive properties depend on the amount of substance.

82: The volume of hydrogen liberated at STP by treating 2.4 g of magnesium with excess of hydrochloric acid is _____ $\times 10^{-2}$ L.

Provided that: Molar volume of gas is 22.4 L at STP. Molar mass of magnesium is 24 g mol^{-1}

Correct Answer: 224

Solution:

Provided that: Mass of magnesium $w = 2.4$ g, molar mass of magnesium $M = 24$ g/mol, and molar volume of gas at STP = 22.4 L.

Step 1: Write the balanced chemical equation-

The balanced chemical equation for the reaction of magnesium with hydrochloric acid is:

$$Mg + 2HCl \rightarrow MgCl_2 + H_2.$$

Step 2: Evaluate the moles of magnesium-

The number of moles n is Provided that by:

$$n = \frac{w}{M} = \frac{2.4 \text{ g}}{24 \text{ g/mol}} = 0.1 \text{ mol}.$$

Step 3: Evaluate the moles of hydrogen gas produced-

From the balanced equation, 1 mole of Mg reacts to produce 1 mole of H_2. Thus, 0.1 mole of Mg will produce 0.1 mole of H_2.

Step 4: Evaluate the volume of hydrogen gas at STP-

The volume of 1 mole of any gas at STP is 22.4 L. Therefore, the volume of 0.1 mole of hydrogen gas at STP is

$$V = n \times 22.4 \text{ L/mol} = 0.1 \text{ mol} \times 22.4 \text{ L/mol} = 2.24 \text{ L} = 224 \times 10^{-2} \text{ L}.$$

Conclusive Answer: The volume of hydrogen liberated at STP is 224×10^{-2} L.

Quick Tip

At STP (Standard Temperature and Pressure), one mole of any ideal gas occupies a volume of 22.4 liters.

83: The number of correct statements about modern adsorption theory of heterogeneous catalysis from the following is ____.

A. The catalyst is diffused over the surface of reactants.
B. Reactants are adsorbed on the surface of the catalyst.
C. Occurrence of chemical reaction on the catalyst's surface through formation of an intermediate.
D. It is a combination of intermediate compound formation theory and the old adsorption theory.
E. It explains the action of the catalyst as well as those of catalytic promoters and poisons.
Correct Answer: 3

Solution:

Let's have an emphasis o each statement:

A. The catalyst is diffused over the surface of reactants. This statement is incorrect. In heterogeneous catalysis, the catalyst is in a different phase than the reactants. The reactants are adsorbed onto the surface of the catalyst, not the other way around.

B. Reactants are adsorbed on the surface of the catalyst. This statement is correct. Adsorption of reactants on the catalyst surface is a crucial step in heterogeneous catalysis.

C. Occurrence of chemical reaction on the catalyst's surface through formation of an intermediate. This statement is correct. The adsorbed reactants form an intermediate compound with the catalyst, which then decomposes to form the products.

D. It is a combination of intermediate compound formation theory and the old adsorption theory. This statement is correct. The modern adsorption theory incorporates the concept of intermediate formation, which was not present in the older adsorption theory.

E. It explains the action of the catalyst as well as those of catalytic promoters and poisons. This statement is incorrect. The modern adsorption theory can explain the role of promoters, which enhance the activity of the catalyst, and poisons, which decrease or inhibit catalytic activity, by influencing adsorption on the catalyst surface.

Conclusive Answer: The number of correct statements is 4 (B, C, D Only).

Quick Tip

In heterogeneous catalysis, the catalyst and reactants are in different phases. The modern adsorption theory explains the process by adsorption of reactants, formation of intermediates on the catalyst surface, and the influence of promoters and poisons.

84: The number of correct statements from the following is _____.

A. For 1 s orbital, the probability density is maximum at the nucleus

B. For 2 s orbital, the probability density first increases to maximum and then decreases sharply to zero.

C. Boundary surface diagrams of the orbitals encloses a region of 100% probability of finding the electron.

D. p and d-orbitals have 1 and 2 angular nodes respectively

E. probability density of p-orbital is zero at the nucleus

Correct Answer: 3

Solution:

A. For 1 s orbital, the probability density is maximum at the nucleus. This statement is **correct**. The 1s orbital has a spherically symmetrical distribution, and the probability density is highest at the nucleus, decreasing as the distance from the nucleus increases.

B. For 2 s orbital, the probability density first increases to maximum and then decreases sharply to zero. This statement is **incorrect**. The probability density for a 2s orbital is maximum at the nucleus, then decreases to zero at the radial node, then increases again to a smaller maximum, and finally decreases asymptotically to zero. It does not increase to a maximum before decreasing. The probability density versus radius plot for the 2s orbital is shown below.

[scale=1] [-¿] (0,0) – (3,0) node[below] r; [-¿] (0,0) – (0,3) node[left] ψ^2; [thick] (0.2,2.8) to[out=-80, in=120] (1,0) to[out=0,in=180] (2,1) to[out=0,in=150] (2.8,0.2);

C. Boundary surface diagrams of the orbitals encloses a region of 100% probability of finding the electron. This statement is **incorrect**. Boundary surface diagrams enclose a region where there is a high (typically 90%) probability of finding the electron, but not 100%. There is always a small, but non-zero probability to find the electron outside that region as well.

D. p and d-orbitals have 1 and 2 angular nodes respectively. This statement is **correct**. The number of angular nodes is equal to the azimuthal quantum number (ℓ). For p orbitals, $\ell = 1$, and for d orbitals, $\ell = 2$.

E. probability density of p-orbital is zero at the nucleus. This statement is **correct**. p-orbitals have a nodal plane passing through the nucleus, so the probability density at the nucleus is zero.

Conclusive Answer: The number of correct statements is 3 (A, D, and E).

Quick Tip

Probability density (ψ^2) represents the probability of finding an electron at a Provided that point in space. Nodes are regions where the probability density is zero.

85: The number of correct statements from the following is _____.

A. E_{cell} is an intensive parameter

B. A negative E° means that the redox couple is a stronger reducing agent than the H^+/H_2 couple.

C. The amount of electricity required for oxidation or reduction depends on the stoichiometry of the electrode reaction.

D. The amount of chemical reaction which occurs at any electrode during electrolysis by a current is proportional to the quantity of electricity passed through the electrolyte.

Correct Answer: 4

Solution:

Let's examine each statement:

A. E_{cell} is an intensive parameter. This statement is **correct**. Intensive properties are independent of the amount of substance present. The cell potential depends on the nature of the redox reaction, concentrations, and temperature, not on the size of the electrochemical cell.

B. A negative E° means that the redox couple is a stronger reducing agent than the H^+/H_2 couple. This statement is **correct**. A negative standard electrode potential (E°) indicates that the redox couple has a greater tendency to lose electrons (be oxidized) compared to the standard hydrogen electrode (SHE), which has E° = 0 V. Thus, the redox couple with negative E° is a stronger reducing agent.

C. The amount of electricity required for oxidation or reduction depends on the stoichiometry of the electrode reaction. This statement is **correct**. The amount of electricity required for a redox reaction is directly proportional to the number of electrons transferred in the balanced half-reaction. This is described by Faraday's laws of electrolysis.

D. The amount of chemical reaction which occurs at any electrode during electrolysis by a current is proportional to the quantity of electricity passed through the electrolyte. This statement is **correct**. This is a direct statement of Faraday's first law of electrolysis.

Conclusive Answer: All four statements (A, B, C, and D) are correct.

Quick Tip

Electrode potential (E°) is an intensive property. A more negative E° indicates a stronger reducing agent. Faraday's laws relate the amount of substance produced or consumed during electrolysis to the quantity of electric charge passed.

86: $Mg(NO_3)_2{\cdot}XH_2O$ and $Ba(NO_3)_2{\cdot}YH_2O$, represent formula of the crystalline forms of nitrate salts. Sum of X and Y is _____.

Correct Answer: 6

Solution:

The Provided that formulas represent hydrated magnesium nitrate and hydrated barium nitrate. The formulas for these hydrated salts are:

Magnesium nitrate hexahydrate: $Mg(NO_3)_2{\cdot}6H_2O$
Barium nitrate: $Ba(NO_3)_2$ (anhydrous)

Therefore, $X = 6$ and $Y = 0$. The sum of X and Y is:

$$X + Y = 6 + 0 = 6.$$

Conclusive Answer: The sum of X and Y is 6.

Quick Tip

Hydrated salts contain water molecules within their crystal structure, represented by the formula $\cdot n\mathrm{H_2O}$, where n is the number of water molecules per formula unit. Anhydrous salts do not contain water molecules in their crystal structure.

87: The number of possible isomeric products formed when 3-chloro-1-butene reacts with HCl through carbocation formation is _____.

Correct Answer: 4

Solution:

Provided that: 3-chloro-1-butene reacts with HCl.

Step 1: Protonation of the alkene-

The reaction starts with the protonation of the double bond in 3-chloro-1-butene by HCl. The proton adds to the carbon with more hydrogen atoms (Markovnikov's rule), leading to the formation of a secondary carbocation.

Step 2: Carbocation rearrangement (hydride shift)-

The initially formed secondary carbocation can undergo a 1,2-hydride shift to form a more stable tertiary carbocation.

Step 3: Nucleophilic attack by chloride ion-

The chloride ion (Cl^-) from HCl can now attack either the secondary or the tertiary carbocation, leading to the formation of different isomeric products.

This forms 1,3-dichlorobutane.

Attack on the tertiary carbocation:

C(-[:60]Cl)(-[:120]C$^+$)$(-[:180]CH_3)(-[0]CH_3)\xrightarrow{Cl^-}C(-[:60]Cl)(-[:120]C(-[:90]Cl))(-[:180]CH_3)$
This forms 2,3-dichlorobutane which has a chiral carbon hence has two stereoisomers. So in total three isomers are possible including one structural isomer and 2 stereoisomers. Attack of Cl^- on secondary carbocation:
$CH_3 - CHCl - CH_2 - CH_2Cl (1 product) Attack of Cl^-$ on tertiary carbocation: $CH_3 - CHCl - CHCl - CH_3 (3 isomers - a pair of enantiomers and a meso compound) Total possible products : 1 + 3 = 4$

Conclusive Answer: The total number of possible isomeric products is 4.

Quick Tip

Carbocation rearrangements can occur through hydride or methyl shifts to form more stable carbocations. Markovnikov's rule states that in the addition of HX to an alkene, the hydrogen atom adds to the carbon atom that already has the most hydrogen atoms.

88: 4.5 moles each of hydrogen and iodine is heated in a sealed ten litre vessel. At equilibrium, 3 moles of HI were found. The equilibrium constant for $H_2(g) + I_2(g) \rightleftharpoons 2HI(g)$ is _____.

Correct Answer: 1

Solution:

Provided that: Initial moles of H_2 = 4.5, initial moles of I_2 = 4.5, moles of HI at equilibrium = 3, and volume of the vessel = 10 L.

Step 1: Write the balanced chemical equation and the ICE table. The balanced chemical equation is:

$$H_2(g) + I_2(g) \rightleftharpoons 2HI(g).$$

The ICE (Initial, Change, Equilibrium) table is:

	H_2	I_2	2HI
Initial (t=0)	4.5	4.5	0
Change	-x	-x	+2x
Equilibrium (t_{eq})	4.5 - x	4.5 - x	2x

We are Provided that that at equilibrium, there are 3 moles of HI. Therefore,

$$2x = 3 \implies x = \frac{3}{2} = 1.5.$$

At equilibrium:

- Moles of H_2 = 4.5 - 1.5 = 3
- Moles of I_2 = 4.5 - 1.5 = 3
- Moles of HI = 3

Step 2: Evaluate the equilibrium concentrations. The equilibrium concentrations are obtained by dividing the moles by the volume (10 L):

- $[H_2] = \frac{3}{10} = 0.3$ M
- $[I_2] = \frac{3}{10} = 0.3$ M
- $[HI] = \frac{3}{10} = 0.3$ M

Step 3: Evaluate the equilibrium constant. The equilibrium constant K_c is Provided that by

$$K_c = \frac{[HI]^2}{[H_2][I_2]}.$$

Substituting the equilibrium concentrations:

$$K_c = \frac{(0.3)^2}{(0.3)(0.3)} = \frac{0.09}{0.09} = 1.$$

Conclusive Answer: The equilibrium constant is 1.

Quick Tip

The equilibrium constant (K_c) is the ratio of the product of the concentrations of the products raised to their stoichiometric coefficients to the product of the concentrations of the reactants raised to their stoichiometric coefficients at equilibrium.

89: Number of compounds from the following which will not produce orange red precipitate with Benedict solution is ______.

Glucose, maltose, sucrose, ribose, 2-deoxyribose, amylose, lactose

Correct Answer: 2

Solution:

Benedict's solution is used to test for the presence of reducing sugars. Reducing sugars are carbohydrates that have a free aldehyde or ketone group that can reduce Cu^{2+} ions in Benedict's solution to Cu^{+} ions, resulting in the formation of an orange-red precipitate of copper(I) oxide (Cu_2O).

The Provided that compounds and their behavior with Benedict's test are:

- Glucose: Reducing sugar (✓)
- Maltose: Reducing sugar (✓)
- Sucrose: Non-reducing sugar ($\times$)
- Ribose: Reducing sugar (✓)
- 2-deoxyribose: Reducing sugar (✓)
- Amylose: Non-reducing sugar ($\times$)
- Lactose: Reducing sugar (✓)

Sucrose and amylose are the two compounds that do not give a positive Benedict's test and will not produce an orange-red precipitate. Sucrose consists of glucose and fructose linked by their anomeric carbons, resulting in no free aldehyde or ketone group. Amylose also does not have a free aldehyde group to react with Benedict's reagent, which is why a precipitate is not formed.

Conclusive Answer: Two compounds (sucrose and amylose) will not produce an orange-red precipitate with Benedict's solution.

Quick Tip

Benedict's test is used to detect reducing sugars, which have a free aldehyde or ketone group. Non-reducing sugars, like sucrose, do not react with Benedict's solution.

90: The maximum number of lone pairs of electrons on the central atom from the following species is _____.

ClO_3^-, XeF_4, SF_4, and I_3^-

Correct Answer: 3

Solution:

To determine the number of lone pairs on the central atom, we need to draw the Lewis structures of each species:

1. ClO_3^- (Chlorate ion): Chlorine (Cl) is the central atom. It has 7 valence electrons. Each oxygen (O) atom contributes 6 valence electrons, and the negative charge adds 1 more electron. Total valence electrons = 7 + (3 × 6) + 1 = 26. The Lewis structure is:

Cl(=[:0]O)(=[:120]O)(-[::-60]O$^-$) The central Cl atom has 1 lone pair.

2. XeF_4 (Xenon tetrafluoride): Xenon (Xe) is the central atom. It has 8 valence electrons. Each fluorine (F) atom contributes 7 valence electrons. Total valence electrons = 8 + (4 × 7) = 36. The Lewis structure is:

Xe(-[:90]F)(-[:180]F)(-[:0]F)(-[:270]F)

The central Xe atom has 2 lone pairs.

3. SF_4 (Sulfur tetrafluoride): Sulfur (S) is the central atom. It has 6 valence electrons. Each fluorine (F) atom contributes 7 valence electrons. Total valence electrons = 6 + (4 × 7) = 34. The Lewis structure is:

S(-[:45]F)(-[:135]F)(-[:225]F)(-[:315]F)

The central S atom has 1 lone pair.

4. I_3^- (Triiodide ion): Iodine (I) is the central atom. It has 7 valence electrons. Each other iodine atom contributes 7 valence electrons, and the negative charge adds 1 more electron. Total valence electrons = 7 + 7 + 7 + 1 = 22. The Lewis structure is:

$I^-(-[::+40]I)(-[::-40]I)$ The central I atom has 3 lone pairs.

Conclusive Answer: The maximum number of lone pairs on the central atom is 3 (in I_3^-).

Quick Tip

Lone pairs are electron pairs that are not involved in bonding. To determine the number of lone pairs on the central atom, draw the Lewis structure of the molecule or ion.
