JEE Main 2023 April 10 Shift 1 Question Paper with Solutions

Time Allowed :3 Hours	**Maximum Marks :**300	**Total Questions :**90

General Instructions

Read the following instructions very carefully and strictly follow them:

1. The test is of 3 hours duration.
2. The question paper consists of 90 questions, out of which 75 are to attempted. The maximum marks are 300.
3. There are three parts in the question paper consisting of Physics, Chemistry and Mathematics having 30 questions in each part of equal weightage.
4. Each part (subject) has two sections.
   (i) Section-A: This section contains 20 multiple choice questions which have only one correct answer. Each question carries 4 marks for correct answer and –1 mark for wrong answer.
   (ii) Section-B: This section contains 10 questions. In Section-B, attempt any five questions out of 10. The answer to each of the questions is a numerical value. Each question carries 4 marks for correct answer and –1 mark for wrong answer. For Section-B, the answer should be rounded off to the nearest integer

MATHEMATICS

Section-A

Question 1: An arc PQ of a circle subtends a right angle at its centre O. The midpoint of the arc PQ is R. If $\vec{OP} = \vec{u}$, $\vec{OR} = \vec{v}$ and $\vec{OQ} = \alpha\vec{u} + \beta\vec{v}$, then α, β^2 are the roots of the equation:

(1) $3x^2 - 2x - 1 = 0$

(2) $3x^2 + 2x - 1 = 0$

(3) $x^2 - x - 2 = 0$

(4) $x^2 + x - 2 = 0$

Correct Answer: (3) $x^2 - x - 2 = 0$

Solution:

Let $\vec{u} = \hat{i}$ and $\vec{OQ} = \hat{j}$. Since R is the midpoint of the arc PQ, $\vec{OR}$ bisects the right angle POQ. We can express $\vec{v}$ in terms of $\vec{u}$ and $\vec{OQ}$ (which we've set as $\hat{i}$ and $\hat{j}$ respectively).

Since R is the midpoint of the arc PQ, the vector $\vec{OR}$ is given by:

$$\vec{v} = \frac{1}{\sqrt{2}}\hat{i} + \frac{1}{\sqrt{2}}\hat{j}$$

Given that $\vec{OQ} = \alpha\vec{u} + \beta\vec{v}$, we have:

$$\hat{j} = \alpha\hat{i} + \beta\left(\frac{1}{\sqrt{2}}\hat{i} + \frac{1}{\sqrt{2}}\hat{j}\right)$$

$$\hat{j} = \left(\alpha + \frac{\beta}{\sqrt{2}}\right)\hat{i} + \frac{\beta}{\sqrt{2}}\hat{j}$$

Comparing the coefficients of $\hat{i}$ and $\hat{j}$ on both sides, we get:

$\alpha + \frac{\beta}{\sqrt{2}} = 0$ and $\frac{\beta}{\sqrt{2}} = 1$

From the second equation, $\beta = \sqrt{2}$. Substituting this into the first equation gives:

$\alpha + \frac{\sqrt{2}}{\sqrt{2}} = 0$, so $\alpha = -1$.

We are given that α and β^2 are the roots of a quadratic equation.

Since $\alpha = -1$ and $\beta = \sqrt{2}$, then $\beta^2 = 2$.

Let the quadratic equation be $x^2 + bx + c = 0$.

The sum of the roots is $-b = \alpha + \beta^2 = -1 + 2 = 1$, so $b = -1$.

The product of the roots is $c = \alpha \cdot \beta^2 = (-1)(2) = -2$.

Therefore, the quadratic equation is $x^2 - x - 2 = 0$.

Quick Tip

For problems involving vector geometry and quadratic equations, setting up a clear coordinate system and comparing coefficients can help simplify the problem.

Question 2: A square piece of tin of side 30 cm is to be made into a box without top by cutting a square from each corner and folding up the flaps to form a box. If the volume

of the box is maximum, then its surface area (in cm^2) is equal to :

(1) 800

(2) 1025

(3) 900

(4) 675

Correct Answer: (1) 800

Solution:

Let x be the side length of the square cut from each corner.

The dimensions of the open box are:

Length = $30 - 2x$

Breadth = $30 - 2x$

Height = x

The volume V of the box is given by:

$V(x) = (30 - 2x)(30 - 2x)x = (30 - 2x)^2 x = 4x^3 - 120x^2 + 900x$

To maximize the volume, we find the critical points by taking the derivative of $V(x)$ with respect to x and setting it equal to zero:

$V'(x) = \frac{dV}{dx} = 12x^2 - 240x + 900$

$0 = 12x^2 - 240x + 900$

$0 = x^2 - 20x + 75$

$0 = (x - 5)(x - 15)$

So, $x = 5$ or $x = 15$.

If $x = 15$, the length and breadth would be $30 - 2(15) = 0$, which is not possible. Therefore, $x = 5$.

The surface area of the open box (without the top) is:

$S(x) = (30 - 2x)^2 + 4x(30 - 2x)$

Substituting $x = 5$:

$S(5) = (30 - 2(5))^2 + 4(5)(30 - 2(5)) = (20)^2 + 20(20) = 400 + 400 = 800$ cm^2

Quick Tip

When maximizing or minimizing quantities involving three-dimensional shapes, be sure to consider the physical constraints of the problem. In this case, the side of the square cut from the corners cannot be greater than half the side of the original square piece of tin.

Question 3: Let O be the origin and the position vector of the point P be $-\hat{i} - 2\hat{j} + 3\hat{k}$. If the position vectors of the A, B and C are $-2\hat{i} + \hat{j} - 3\hat{k}$, $2\hat{i} + 4\hat{j} - 2\hat{k}$ and $-4\hat{i} + 2\hat{j} - \hat{k}$ respectively, then the projection of the vector $\vec{OP}$ on a vector perpendicular to the vectors $\vec{AB}$ and $\vec{AC}$ is:

(1) $\frac{10}{3}$

(2) $\frac{8}{3}$

(3) $\frac{7}{3}$

(4) 3

Correct Answer: (4) 3

Solution:

Given Points:

$$P(-1, -2, 3), \quad A(-2, 1, -3), \quad B(2, 4, -2), \quad C(-4, 2, -1)$$

To Find: Position vector $\overrightarrow{OP}$

Step 1: Find $\overrightarrow{AB \times AC}$:

$$\overrightarrow{AB} = \langle 4, 3, 1 \rangle, \quad \overrightarrow{AC} = \langle -2, 1, 2 \rangle$$

$$\overrightarrow{AB \times AC} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 4 & 3 & 1 \\ -2 & 1 & 2 \end{vmatrix}$$

Expanding the determinant:

$$\overrightarrow{AB \times AC} = \hat{i}(3 \cdot 2 - 1 \cdot 1) - \hat{j}(4 \cdot 2 - 1 \cdot -2) + \hat{k}(4 \cdot 1 - 3 \cdot -2)$$

$$\overrightarrow{AB \times AC} = \hat{i}(5) - \hat{j}(8+2) + \hat{k}(4+6)$$

$$\overrightarrow{AB \times AC} = 5\hat{i} - 10\hat{j} + 10\hat{k}$$

Step 2: Compute $\overrightarrow{OP}$:

$$\overrightarrow{OP} = \frac{\overrightarrow{AB \times AC}}{|\overrightarrow{AB \times AC}|}$$

Magnitude of $\overrightarrow{AB \times AC}$:

$$|\overrightarrow{AB \times AC}| = \sqrt{5^2 + (-10)^2 + 10^2}$$

$$|\overrightarrow{AB \times AC}| = \sqrt{25 + 100 + 100} = \sqrt{225} = 15$$

Direction Cosine:

$$\overrightarrow{OP} = \frac{5\hat{i} - 10\hat{j} + 10\hat{k}}{15}$$

$$\overrightarrow{OP} = \frac{-\hat{i} - 2\hat{j} + 3\hat{k}}{\sqrt{225}}$$

Simplify:

$$\overrightarrow{OP} = \frac{-5 + 20 + 30}{\sqrt{25 + 100 + 100}}$$

$$= \frac{45}{15} = 3$$

Quick Tip

The cross-product of two vectors results in a vector that is perpendicular to both original vectors. The projection of one vector onto another can be calculated using the dot product and the magnitude.

Question 4: If A is a 3×3 matrix and $|A| = 2$, then $|3\text{adj}(|3A|A^2)|$ is equal to :

(1) $3^{12} \cdot 6^{10}$

(2) $3^{11} \cdot 6^{10}$

(3) $3^{12} \cdot 6^{11}$

(4) $3^{10} \cdot 6^{11}$

Correct Answer: (2) $3^{11} \cdot 6^{10}$

Solution:

Given:

$$|A| = 2$$

Step 1: Calculate $|3A|$:

$$|3A| = 3^3 \cdot |A|$$

$$|3A| = 3^3 \cdot 2 = 27 \cdot 2$$

Step 2: Adj of $|3A|A^2$:

$$\text{Adj.}(|3A|A^2) = \text{Adj.}\{(3^3 \cdot 2)A^2\}$$

$$= (2 \cdot 3^3)^2 \cdot (\text{Adj.}A)^2$$

$$= 2^2 \cdot 3^6 \cdot (\text{Adj.}A)^2$$

Step 3: Calculate $|3\,\textbf{Adj.}(|3A|A^2)|$:

$$|3 \cdot \text{Adj.}(|3A|A^2)| = |2^2 \cdot 3^6 \cdot (\text{Adj.}A)^2|$$

$$= (2^2 \cdot 3^7)^3 \cdot |\text{Adj.}A|^2$$

$$= 2^6 \cdot 3^{21} \cdot (|A|^2)^2$$

$$= 2^6 \cdot 3^{21} \cdot (2^2)^2$$

$$= 2^{10} \cdot 3^{21}$$

Step 4: Final Value:

$$|3 \cdot \text{Adj.}(|3A|A^2)| = 6^{10} \cdot 3^{11}$$

Quick Tip

Review the properties of determinants, especially concerning scalar multiples, powers of matrices, and the adjoint of a matrix. Remember that for a $n \times n$ matrix A, $|kA| = k^n|A|$, $|A^k| = |A|^k$, and $|\text{adj}(A)| = |A|^{n-1}$.

Question 5: Let two vertices of a triangle ABC be $(2, 4, 6)$ and $(0, -2, -5)$, and its centroid be $(2, 1, -1)$. If the image of the third vertex in the plane $x + 2y + 4z = 11$ is (α, β, γ), then $\alpha\beta + \beta\gamma + \gamma\alpha$ is equal to:

(1) 76

(2) 74

(3) 70

(4) 72

Correct Answer: (2) 74

Solution:

Given Two vertices of Triangle A(2, 4, 6) and B(0, -2, -5) and if centroid G(2, 1, -1)

Let Third vertices be (x, y, z)

$$\text{Now } \frac{2+0+x}{3} = 2, \quad \frac{4-2+y}{3} = 1, \quad \frac{6-5+z}{3} = -1$$

$$x = 4, \quad y = 1, \quad z = -4$$

Third vertices C(4, 1, -4)

Now, Image of vertices C(4, 1, -4) in the given plane is Dα, β, γ

$$x + 2y + 4z - 11 = 0$$

$$\frac{\alpha - 4}{1} = \frac{\beta - 1}{2} = \frac{\gamma + 4}{4} = -\frac{4 + 2 - 16 - 11}{1 + 4 + 16}$$

$$\frac{\alpha - 4}{1} = \frac{\beta - 1}{2} = \frac{\gamma + 4}{4} = \frac{-25}{21}$$

$$\alpha = 6, \quad \beta = 5, \quad \gamma = 4$$

$$\text{Then } \alpha\beta + \beta\gamma + \gamma\alpha$$

$$= (6 \times 5) + (5 \times 4) + (4 \times 6)$$

$$= 30 + 20 + 24 = 74$$

Quick Tip

Remember the formula for the centroid of a triangle and the relationship between a point, its image in a plane, and the normal vector to the plane.

Question 6: The negation of the statement $(p \vee q) \wedge (q \vee (\sim r))$ is

(1) $((\sim p) \vee r) \wedge (\sim q)$

(2) $((\sim p) \vee (\sim q)) \wedge (\sim r)$

(3) $((\sim p) \vee (\sim q)) \vee (\sim r)$

(4) $(p \vee r) \wedge (\sim q)$

Correct Answer: (1) $((\sim p) \vee r) \wedge (\sim q)$

Solution:

$$(p \vee q) \wedge (q \vee (\sim r))$$

Taking the negation:

$$\sim [(p \vee q) \wedge (q \vee (\sim r))]$$

Using De Morgan's laws:

$$=\sim (p \vee q) \vee \sim (q \vee (\sim r))$$

Simplifying each term:

$$= (\sim p \wedge \sim q) \vee (\sim q \wedge r)$$

Rewriting using distributive properties:

$$= (\sim p \vee r) \wedge (\sim q)$$

Quick Tip

De Morgan's Laws are crucial for simplifying negations of logical statements. Remember that $\sim (P \wedge Q) = (\sim P) \vee (\sim Q)$ and $\sim (P \vee Q) = (\sim P) \wedge (\sim Q)$. Also, the distributive law states that $A \wedge (B \vee C) = (A \wedge B) \vee (A \wedge C)$, and the commutative law states that $A \wedge B = B \wedge A$.

Question 7: The shortest distance between the lines $\frac{x+2}{1} = \frac{y}{-2} = \frac{z-5}{2}$ and $\frac{x-4}{1} = \frac{y-1}{2} = \frac{z+3}{0}$ is:

(1) 8

(2) 7

(3) 6

(4) 9

Correct Answer: (4) 9

Solution:

Given:

$$\frac{x+2}{1} = \frac{y}{-2} = \frac{z-5}{2} \quad \text{and} \quad \frac{x-4}{1} = \frac{y-1}{2} = \frac{z+3}{0}$$

Shortest Distance Formula:

$$d = \frac{\left\| \begin{vmatrix} a_2 - a_1 & b_2 - b_1 & c_2 - c_1 \\ l_1 & m_1 & n_1 \\ l_2 & m_2 & n_2 \end{vmatrix} \right\|}{\sqrt{l_1^2 + m_1^2 + n_1^2}\sqrt{l_2^2 + m_2^2 + n_2^2}}$$

Given Directions:

$$\overrightarrow{l_1, m_1, n_1} = \langle 1, -2, 2 \rangle, \quad \overrightarrow{l_2, m_2, n_2} = \langle 1, 2, 0 \rangle$$

Position Vectors:

$$(a_2 - a_1, b_2 - b_1, c_2 - c_1) = \langle 4 + 2, 1 - 0, -3 - 5 \rangle = \langle 6, 1, -8 \rangle$$

Determinant Calculation:

$$\begin{vmatrix} 6 & 1 & -8 \\ 1 & -2 & 2 \\ 1 & 2 & 0 \end{vmatrix} = \hat{i}\begin{vmatrix} -2 & 2 \\ 2 & 0 \end{vmatrix} - \hat{j}\begin{vmatrix} 1 & 2 \\ 1 & 0 \end{vmatrix} + \hat{k}\begin{vmatrix} 1 & -2 \\ 1 & 2 \end{vmatrix}$$

Simplify:

$$= \hat{i}((-4) - (4)) - \hat{j}((0) - (2)) + \hat{k}((2) - (-2))$$
$$= -4\hat{i} - 2\hat{j} + 4\hat{k}$$

Magnitude of Determinant:

$$|d| = \frac{|54|}{\sqrt{(-4)^2 + (-2)^2 + (4)^2}}$$
$$= \frac{54}{\sqrt{16 + 4 + 16}} = \frac{54}{6} = 9$$

Final Answer: The shortest distance $d = 9$.

Quick Tip

The shortest distance between two skew lines is the length of the perpendicular common to both lines. The formula is based on the projection of the vector joining any two points on the lines onto the vector perpendicular to both direction vectors.

Question 8: If the coefficient of x^7 in $\left(ax - \frac{1}{bx^2}\right)^{13}$ and the coefficient of x^{-5} in $\left(ax + \frac{1}{bx^2}\right)^{13}$ are equal, then a^4b^4 is equal to:

(1) 22

(2) 44

(3) 11

(4) 33

Correct Answer: (1) 22

Solution:

Given:

$$\left(ax - \frac{1}{bx^2}\right)^{13}$$

General Term:

$$T_{r+1} = \binom{13}{r}(ax)^{13-r}\left(-\frac{1}{bx^2}\right)^r$$

Simplify:

$$T_{r+1} = \binom{13}{r}a^{13-r}\left(-\frac{1}{b}\right)^r x^{13-r}\cdot x^{-2r}$$

$$T_{r+1} = \binom{13}{r}a^{13-r}\left(-\frac{1}{b}\right)^r x^{13-3r} \qquad \cdots(1)$$

Coefficient of x^7:

$$13-3r=7 \quad\Rightarrow\quad r=2$$

Substitute $r=2$ into (1):

$$T_3 = \binom{13}{2}a^{11}\left(-\frac{1}{b}\right)^2 x^7$$

$$T_3 = \binom{13}{2}a^{11}\frac{1}{b^2}x^7$$

Coefficient of x^7:

$$\binom{13}{2}\frac{a^{11}}{b^2}$$

Coefficient of x^{-5}:

$$13-3r=-5 \quad\Rightarrow\quad r=6$$

Substitute $r=6$ into (1):

$$T_7 = \binom{13}{6}a^7\left(-\frac{1}{b}\right)^6 x^{-5}$$

$$T_7 = \binom{13}{6}a^7\frac{1}{b^6}x^{-5}$$

Coefficient of x^{-5}:

$$\binom{13}{6}\frac{a^7}{b^6}$$

Condition: Coefficient of $x^7 = x^{-5}$:

$$\binom{13}{2}\frac{a^{11}}{b^2} = \binom{13}{6}\frac{a^7}{b^6}$$

Simplify:

$$\frac{a^{11}}{b^2} = \frac{a^7}{b^6}$$

$$a^4 \cdot b^4 = \frac{\binom{13}{6}}{\binom{13}{2}}$$

Simplify Binomial Coefficients:

$$\binom{13}{6} = \frac{13 \cdot 12 \cdot 11 \cdot 10 \cdot 9 \cdot 8}{6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}$$

$$\binom{13}{2} = \frac{13 \cdot 12}{2}$$

$$a^4 \cdot b^4 = \frac{\frac{13 \cdot 12 \cdot 11 \cdot 10 \cdot 9 \cdot 8}{720}}{\frac{13 \cdot 12}{2}} = 22$$

Final Answer: $a^4 \cdot b^4 = 22$

Quick Tip

Pay careful attention to the powers of x in the binomial expansions. Make sure the question is accurately transcribed.

Question 9: A line segment AB of length λ moves such that the points A and B remain on the periphery of a circle of radius γ. Then the locus of the point, that divides the line segment AB in the ratio 2 : 3, is a circle of radius :

(1) $\frac{2}{3}\lambda$

(2) $\frac{\sqrt{19}}{7}\lambda$

(3) $\frac{3}{5}\lambda$

(4) $\frac{\sqrt{19}}{5}\lambda$

Correct Answer: (4) $\frac{\sqrt{19}}{5}\lambda$

Solution:

Given: The triangle OAB is an equilateral triangle, and the point P divides AB in the ratio $2:3$.

Step 1: Geometry of the Equilateral Triangle:

$$\angle OAP = 60^\circ, \quad \text{and} \quad AP = \frac{2\lambda}{5}, \quad OA = OB = \lambda$$

Step 2: Using the Cosine Rule:

$$\cos 60^\circ = \frac{OA^2 + AP^2 - OP^2}{2 \cdot OA \cdot AP}$$

Substitute $\cos 60^\circ = \frac{1}{2}$:

$$\frac{1}{2} = \frac{\lambda^2 + \frac{4\lambda^2}{25} - OP^2}{2\lambda \cdot \frac{2\lambda}{5}}$$

Simplify the denominator:

$$\frac{1}{2} = \frac{\lambda^2 + \frac{4\lambda^2}{25} - OP^2}{\frac{4\lambda^2}{5}}$$

Multiply through by $\frac{4\lambda^2}{5}$:

$$\frac{2\lambda^2}{5} = \lambda^2 + \frac{4\lambda^2}{25} - OP^2$$

Step 3: Simplify for OP^2:

$$OP^2 = \lambda^2 + \frac{4\lambda^2}{25} - \frac{2\lambda^2}{5}$$

Express all terms with a common denominator:

$$OP^2 = \frac{25\lambda^2}{25} + \frac{4\lambda^2}{25} - \frac{10\lambda^2}{25}$$

$$OP^2 = \frac{19\lambda^2}{25}$$

Step 4: Solve for OP:

$$OP = \sqrt{\frac{19\lambda^2}{25}} = \frac{\sqrt{19}}{5}\lambda$$

Final Answer:

$$OP = \frac{\sqrt{19}}{5}\lambda$$

Quick Tip

Using vectors simplifies the calculations significantly in problems involving locus.

Question 10: For the system of linear equations

$$2x - y + 3z = 5$$

$$3x + 2y - z = 7$$

$$4x + 5y + \alpha z = \beta,$$

which of the following is NOT correct ?

(1) The system in inconsistent for $\alpha = -5$ and $\beta = 8$

(2) The system has infinitely many solutions for $\alpha = -6$ and $\beta = 9$

(3) The system has a unique solution for $\alpha \neq -5$ and $\beta = 8$

(4) The system has infinitely many solutions for $\alpha = -5$ and $\beta = 9$

Correct Answer: (2) The system has infinitely many solutions for $\alpha = -6$ and $\beta = 9$

Solution:

Given:

$$2x - y + 3z = 5, \quad 3x + 2y - z = 7, \quad 4x + 5y + \alpha z = \beta$$

Step 1: Calculate Determinant Δ:

$$\Delta = \begin{vmatrix} 2 & -1 & 3 \\ 3 & 2 & -1 \\ 4 & 5 & \alpha \end{vmatrix}$$

Expanding:

$$\Delta = 7(\alpha + 5)$$

Condition for Unique Solution:

$$\Delta \neq 0 \quad \Rightarrow \quad \alpha \neq -5$$

Condition for Infinite or Inconsistent Solution:

$$\Delta = 0 \quad \Rightarrow \quad \alpha = -5$$

Step 2: Calculate Sub-determinants:

$$\Delta_1 = \begin{vmatrix} 5 & -1 & 3 \\ 7 & 2 & -1 \\ \beta & 5 & -5 \end{vmatrix} = -5(\beta - 9)$$

$$\Delta_2 = \begin{vmatrix} 2 & 5 & 3 \\ 3 & 7 & -1 \\ 4 & \beta & -5 \end{vmatrix} = 11(\beta - 9)$$

$$\Delta_3 = \begin{vmatrix} 2 & -1 & 5 \\ 3 & 2 & 7 \\ 4 & 5 & \beta \end{vmatrix} = 7(\beta - 9)$$

Step 3: Analyze the System:

For an Inconsistent System: At least one of Δ_1, Δ_2, $\Delta_3 \neq 0$:

$$\alpha = -5, \quad \beta = 8 \quad \text{(Option A: True).}$$

For Infinite Solutions:

$$\Delta_1 = \Delta_2 = \Delta_3 = 0 \quad \Rightarrow \quad \beta = 9$$

$$\alpha = -5 \quad \text{(Option D: True).}$$

For a Unique Solution:

$$\alpha \neq -5, \quad \beta = 8 \quad \text{(Option C: True).}$$

Option B: False.

Final Conclusion:

- $\alpha = -5$, $\beta = 8$: Option A (True)
- $\alpha = -5$, $\beta = 9$: Option D (True)
- $\alpha \neq -5$, $\beta = 8$: Option C (True)
- Option B: False

Quick Tip

Using the augmented matrix and row reduction is a systematic way to determine the consistency and the number of solutions of a system of linear equations.

Question 11: Let the first term a and the common ratio r of a geometric progression be positive integers. If the sum of squares of its first three is 33033, then the sum of these terms is equal to:

(1) 210

(2) 220

(3) 231

(4) 241

Correct Answer: (3) 231

Solution:

Let a, ar, and ar^2 be three terms of a geometric progression (GP).

Given:

$$a^2 + (ar)^2 + (ar^2)^2 = 33033$$

Simplify:

$$a^2(1 + r^2 + r^4) = 33033$$

$$33033 = 11^2 \cdot 3 \cdot 7 \cdot 13 \quad \Rightarrow \quad a^2 = 11^2 \quad \Rightarrow \quad a = 11$$

Substitute $a = 11$:

$$1 + r^2 + r^4 = 3 \cdot 7 \cdot 13$$

$$1 + r^2 + r^4 = 273$$

$$r^2(1 + r^2) = 273 - 1 = 272$$

Factorize:

$$r^2(r^2 + 1) = 272 = 16 \cdot 17$$

$$r^2 = 16 \quad \Rightarrow \quad r = 4 \quad (\text{since } r > 0)$$

Sum of the Three Terms:

$$a + ar + ar^2 = a(1 + r + r^2)$$

$$= 11(1 + 4 + 16)$$

$$= 11 \cdot 21 = 231$$

Final Answer: The sum of the three terms is 231.

Quick Tip

When dealing with geometric progressions, remember the formulas for the sum of terms and the sum of squares of terms. Factorization is key in solving these types of problems.

Question 12: Let P be the point of intersection of the line $\frac{x+3}{3} = \frac{y+2}{1} = \frac{1-z}{2}$ and the plane $x + y + z = 2$. If the distance of the point P from the plane $3x - 4y + 12z = 32$ is q, then q and 2q are the roots of the equation:

(1) $x^2 + 18x - 72 = 0$

(2) $x^2 + 18x + 72 = 0$

(3) $x^2 - 18x - 72 = 0$

(4) $x^2 - 18x + 72 = 0$

Correct Answer: (4) $x^2 - 18x + 72 = 0$

Solution:

Given:

$$\frac{x+3}{3} = \frac{y+2}{1} = \frac{1-z}{2} = \lambda$$

Step 1: Parametric Equations of Line

$$x = 3\lambda - 3, \quad y = \lambda - 2, \quad z = 1 - 2\lambda$$

Point $P(3\lambda - 3, \lambda - 2, 1 - 2\lambda)$ will satisfy the equation of the plane $x + y + z = 2$.

Substitute:

$$3\lambda - 3 + \lambda - 2 + 1 - 2\lambda = 2$$

Simplify:

$$2\lambda - 4 = 2 \quad \Rightarrow \quad \lambda = 3$$

Thus:

$$P(6, 1, -5)$$

Step 2: Perpendicular Distance of P from the Plane: Plane equation: $3x - 4y + 12z - 32 = 0$.

Substitute $P(6, 1, -5)$:

$$q = \frac{|3(6) - 4(1) + 12(-5) - 32|}{\sqrt{3^2 + (-4)^2 + 12^2}}$$

Simplify:

$$q = \frac{|18 - 4 - 60 - 32|}{\sqrt{9 + 16 + 144}}$$

$$q = \frac{|-78|}{13} = 6$$

Step 3: Roots of the Quadratic Equation: Given roots $q = 6$ and $2q = 12$.

$$\text{Sum of roots} = 6 + 12 = 18$$

$$\text{Product of roots} = 6 \cdot 12 = 72$$

Quadratic equation:

$$x^2 - (\text{Sum of roots})x + (\text{Product of roots}) = 0$$

Substitute:

$$x^2 - 18x + 72 = 0$$

Final Answer: The quadratic equation is:

$$x^2 - 18x + 72 = 0$$

Quick Tip

Remember the formula for the distance from a point to a plane. Use the properties of roots of a quadratic equation to construct the desired equation.

Question 13: Let f be a differentiable function such that $x^2 f(x) - x = 4\int_0^x t f(t)dt$, $f(1) = \frac{2}{3}$. Then $18f(3)$ is equal to:

(1) 180

(2) 150

(3) 210

(4) 160

Correct Answer: (4) 160

Solution:

Given:

$$x^2 f(x) - x = 4\int_0^x t f(t)\,dt$$

Step 1: Differentiate both sides with respect to x:

$$\frac{d}{dx}\left[x^2 f(x) - x\right] = \frac{d}{dx}\left[4\int_0^x t f(t)\,dt\right]$$

Using Leibniz rule:

$$x^2 f''(x) + 2x f(x) - 1 = 4x f(x)$$

Simplify:

$$x^2 f''(x) + 2x f(x) - 1 = 4x f(x)$$

$$x^2 f''(x) - 2x f(x) - 1 = 0$$

Let $y = f(x)$:

$$x^2 \frac{dy}{dx} - 2xy - 1 = 0$$

Step 2: Rewrite the equation:

$$\frac{dy}{dx} - \frac{2}{x}y = \frac{1}{x^2}$$

This is a first-order linear differential equation. The integrating factor (I.F.) is:

$$\text{I.F.} = e^{\int -\frac{2}{x}\,dx} = e^{-2\ln x} = \frac{1}{x^2}$$

Step 3: Solve the differential equation:

$$\frac{y}{x^2} = \int \frac{1}{x^4}\,dx + C$$

$$\frac{y}{x^2} = -\frac{1}{3x^3} + C$$

Multiply through by x^2:

$$y = -\frac{1}{3x} + Cx^2$$

Step 4: Apply Initial Condition: Given $f(1) = \frac{2}{3}$, substitute $x = 1$ and $y = \frac{2}{3}$:

$$\frac{2}{3} = -\frac{1}{3}(1) + C(1^2)$$

$$\frac{2}{3} = -\frac{1}{3} + C$$

$$C = 1$$

Thus:

$$y = -\frac{1}{3x} + x^2$$

Step 5: Find $f(3)$:

$$f(3) = -\frac{1}{3(3)} + 3^2$$

$$f(3) = -\frac{1}{9} + 9 = \frac{80}{9}$$

Step 6: Find $18f(3)$:

$$18f(3) = 18 \cdot \frac{80}{9} = 160$$

Final Answer: $18f(3) = 160$.

Quick Tip

Remember the Leibniz integral rule for differentiating integrals. Solving first-order linear differential equations often involves integrating factors.

Question 14: Let N denote the sum of the numbers obtained when two dice are rolled. If the probability that $2^N < N!$ is $\frac{m}{n}$, where m and n are coprime, then $4m - 3n$ equal to:

(1) 12

(2) 8

(3) 10

(4) 6

Correct Answer: (2) 8

Solution:

Given: $2^N < N!$ is satisfied for $N \geq 4$.

Required Probability:

$$P(N \geq 4) = 1 - P(N < 4)$$

Step 1: Calculate $P(N < 4)$**:**

- $N = 1$: Not possible.
- $N = 2$: Possible outcomes are $(1, 1)$.

$$P(N = 2) = \frac{1}{36}$$

- $N = 3$: Possible outcomes are $(1, 2)$, $(2, 1)$.

$$P(N = 3) = \frac{2}{36}$$

$$P(N < 4) = P(N = 2) + P(N = 3) = \frac{1}{36} + \frac{2}{36} = \frac{3}{36}$$

Step 2: Calculate $P(N \geq 4)$**:**

$$P(N \geq 4) = 1 - P(N < 4) = 1 - \frac{3}{36} = \frac{33}{36} = \frac{11}{12}$$

Step 3: Express as $\frac{m}{n}$**:**

$$P(N \geq 4) = \frac{11}{12} \quad \Rightarrow \quad m = 11,\, n = 12$$

Step 4: Calculate $4m - 3n$**:**

$$4m - 3n = 4(11) - 3(12) = 44 - 36 = 8$$

Final Answer: $4m - 3n = 8$.

Quick Tip

List all possible outcomes when rolling two dice and find the sum for each outcome. Check the inequality for each possible sum.

Question 15: If $I(x) = \int e^{\sin^2 x} \cos x \sin 2x - \sin x \, dx$ and $I(0) = 1$, then $I\left(\frac{\pi}{3}\right)$ is equal to:

(1) $e^{\frac{3}{4}}$

(2) $-e^{\frac{3}{4}}$

(3) $\frac{1}{2}e^{\frac{3}{4}}$

(4) $-\frac{1}{2}e^{\frac{3}{4}}$

Correct Answer: (3) $\frac{1}{2}e^{\frac{3}{4}}$

Solution:

Given:

$$I = \int e^{\sin^2 x} \sin 2x \cos x \, dx - \int e^{\sin^2 x} \sin x \, dx$$

Step 1: Simplify the Expression:

$$I = \cos x \int e^{\sin^2 x} \sin 2x \, dx - \int \left[(-\sin x) \int e^{\sin^2 x} \sin 2x \, dx \right] dx - \int e^{\sin^2 x} \sin x \, dx$$

Substitute $\sin^2 x = t$, so $\sin 2x \, dx = dt$:

$$I = \cos x \int e^t \, dt + (\sin x \int e^t \, dt) dx - \int e^{\sin^2 x} \sin x \, dx$$

Step 2: Solve the Integration:

$$I = e^{\sin^2 x} \cos x + \int e^{\sin^2 x} \sin x \, dx - \int e^{\sin^2 x} \sin x \, dx$$

Simplify:

$$I = e^{\sin^2 x} \cos x + C$$

Step 3: Apply Initial Condition: Given $I(0) = 1$:

$$I(0) = e^{\sin^2 (0)} \cos(0) + C$$

$$1 = 1 + C \quad \Rightarrow \quad C = 0$$

Thus:

$$I = e^{\sin^2 x} \cos x$$

Step 4: Find $I\left(\frac{\pi}{3}\right)$:

$$\begin{aligned} I\left(\frac{\pi}{3}\right) &= e^{\sin^2\left(\frac{\pi}{3}\right)} \cos\left(\frac{\pi}{3}\right) \\ &= e^{\sin^2\left(\frac{\pi}{3}\right)} \cdot \frac{1}{2} \\ &= e^{\left(\frac{\sqrt{3}}{2}\right)^2} \cdot \frac{1}{2} \\ &= e^{\frac{3}{4}} \cdot \frac{1}{2} = \frac{e^{\frac{3}{4}}}{2} \end{aligned}$$

Final Answer:

$$I\left(\frac{\pi}{3}\right) = \frac{e^{\frac{3}{4}}}{2}$$

Quick Tip

Pay close attention to the trigonometric identities when dealing with integrals involving trigonometric functions. Remember the Leibniz rule for differentiation under the integral sign.

Question 16: $96 \cos \frac{\pi}{33} \cos \frac{2\pi}{33} \cos \frac{4\pi}{33} \cos \frac{8\pi}{33} \cos \frac{16\pi}{33}$ **is equal to :**

(1) 4

(2) 2

(3) 3

(4) 1

Correct Answer: (3) 3

Solution:

$$96 \cos \frac{\pi}{33} \cos \frac{2\pi}{33} \cos \frac{2^2\pi}{33} \cos \frac{2^3\pi}{33} \cos \frac{2^4\pi}{33}$$

General Formula:

$$\cos A \cos 2A \cos 2^2 A \ldots \cos 2^{n-1} A = \frac{\sin(2^n A)}{2^n \sin A}$$

Given:

$$A = \frac{\pi}{33}, \quad n = 5$$

Step 1: Simplify Using the Formula:

$$96 \cos\frac{\pi}{33} \cos\frac{2\pi}{33} \cos\frac{2^2\pi}{33} \cos\frac{2^3\pi}{33} \cos\frac{2^4\pi}{33}$$

$$= \frac{96 \sin\left(2^5 \frac{\pi}{33}\right)}{2^5 \sin\left(\frac{\pi}{33}\right)}$$

Step 2: Evaluate the Terms:

$$= \frac{96 \sin\left(\frac{32\pi}{33}\right)}{32 \sin\left(\frac{\pi}{33}\right)}$$

Using the identity $\sin(\pi - x) = \sin x$:

$$\sin\left(\frac{32\pi}{33}\right) = \sin\left(\pi - \frac{\pi}{33}\right) = \sin\left(\frac{\pi}{33}\right)$$

Step 3: Simplify Further:

$$= \frac{96 \sin\left(\frac{\pi}{33}\right)}{32 \sin\left(\frac{\pi}{33}\right)}$$

$$= \frac{96}{32} = 3$$

Final Answer: 3

Quick Tip

Remember the Leibniz rule for differentiation under the integral sign.

Question 17: Let the complex number $z = x + iy$ be such that $\frac{2z-3i}{2z+i}$ is purely imaginary. If $x + y^2 = 0$, then $y^4 + y^2 - y$ is equal to:

(1) $\frac{3}{2}$

(2) $\frac{2}{3}$

(3) $\frac{4}{3}$

(4) $\frac{3}{4}$

Correct Answer: (4) $\frac{3}{4}$

Solution:

Given:

$$z = x + iy, \quad \frac{2z - 3i}{2z + i} \text{ is purely imaginary.}$$

This implies:

$$\text{Re}\left(\frac{2z - 3i}{2z + i}\right) = 0$$

Step 1: Substitute $z = x + iy$:

$$\frac{2z - 3i}{2z + i} = \frac{2(x + iy) - 3i}{2(x + iy) + i}$$

$$= \frac{2x + 2yi - 3i}{2x + i(2y + 1)}$$

Rationalize the denominator:

$$= \frac{(2x + i(2y - 3))(2x - i(2y + 1))}{(2x + i(2y + 1))(2x - i(2y + 1))}$$

Step 2: Simplify the Numerator and Denominator: The denominator is:

$$(2x)^2 + (2y + 1)^2 = 4x^2 + (2y + 1)^2$$

The numerator is:

$$(2x)^2 + i\left[(2y - 3)(2x) - (2y + 1)(2x)\right]$$

The real part is:

$$\text{Re}\left(\frac{2z - 3i}{2z + i}\right) = \frac{4x^2 + (2y - 3)(2y + 1)}{4x^2 + (2y + 1)^2}$$

Set the real part to 0:

$$4x^2 + (2y - 3)(2y + 1) = 0$$

Step 3: Solve for x and y: Expand:

$$4x^2 + [4y^2 + 2y - 6y - 3] = 0$$

$$4x^2 + 4y^2 - 4y - 3 = 0$$

Substitute $x = -y^2$ (from $x + y^2 = 0$):

$$4(-y^2)^2 + 4y^2 - 4y - 3 = 0$$

$$4y^4 + 4y^2 - 4y - 3 = 0$$

Step 4: Simplify Further:

$$y^4 + y^2 - y - \frac{3}{4} = 0$$

Final Answer: The equation is:

$$y^4 + y^2 - y = \frac{3}{4}$$

and the correct option is (4).

Quick Tip

Remember that a complex number is purely imaginary if its real part is zero.

Question 18: If $f(x) = \frac{(\tan 1^\circ)x + \log_e(123)}{x \log_e(1234) - (\tan 1^\circ)}$, $x > 0$, then the least value of $f(f(x)) + f\left(f\left(\frac{4}{x}\right)\right)$ is:

(1) 2

(2) 4

(3) 8

(4) 0

Correct Answer: (2) 4

Solution:

Given:

$$f(x) = \frac{(\tan 1)x + \log 123}{x \log 1234 - \tan 1}$$

Let:

$$A = \tan 1, \quad B = \log 123, \quad C = \log 1234$$

Rewriting $f(x)$:

$$f(x) = \frac{Ax + B}{xC - A}$$

Step 1: Evaluate $f(f(x))$:

$$f(f(x)) = f\left(\frac{Ax + B}{xC - A}\right)$$

Substitute $f(x)$ into f:

$$f(f(x)) = \frac{A\left(\frac{Ax+B}{xC-A}\right) + B}{C\left(\frac{Ax+B}{xC-A}\right) - A}$$

Simplify:

$$f(f(x)) = \frac{A(Ax+B) + B(xC-A)}{C(Ax+B) - A(xC-A)}$$

Expand numerator and denominator:

$$\text{Numerator: } A^2x + AB + xBC - AB = x(A^2 + BC)$$

$$\text{Denominator: } ACx + BC - ACx + A^2 = A^2 + BC$$

Thus:

$$f(f(x)) = \frac{x(A^2 + BC)}{A^2 + BC} = x$$

Step 2: Evaluate $f(f(4/x))$:

$$f(f(4/x)) = \frac{4}{x}$$

Step 3: Evaluate $f(f(x)) + f(f(4/x))$:

$$f(f(x)) + f(f(4/x)) = x + \frac{4}{x}$$

Step 4: Apply AM-GM Inequality: Using AM-GM:

$$\frac{x + \frac{4}{x}}{2} \geq \sqrt{x \cdot \frac{4}{x}}$$

$$x + \frac{4}{x} \geq 4$$

Final Answer:

$$x + \frac{4}{x} \geq 4$$

Quick Tip

Recognize the form of $f(x)$ and compute $f(f(x))$. Then, calculate $f(\frac{4}{x})$ and $f(f(\frac{4}{x}))$. Finally, use AM-GM inequality to find the minimum value.

Question 19: The slope of tangent at any point (x, y) on a curve $y = y(x)$ is $\frac{x^2+y^2}{2xy}$, $x > 0$. If $y(2) = 0$, then a value of $y(8)$ is:

(1) $4\sqrt{3}$

(2) $-4\sqrt{2}$

(3) $-2\sqrt{3}$

(4) $2\sqrt{3}$

Correct Answer: (1) $4\sqrt{3}$

Solution:

Given:

$$\frac{dy}{dx} = \frac{x^2 + y^2}{2xy}, \quad y = vx$$

Step 1: Substitute $y = vx$:

$$\frac{dy}{dx} = v + x\frac{dv}{dx}$$

$$v + x\frac{dv}{dx} = \frac{x^2 + v^2x^2}{2vx^2}$$

$$v + x\frac{dv}{dx} = \frac{v^2 + 1}{2v}$$

Step 2: Simplify and Separate Variables:

$$x \cdot \frac{dv}{dx} = \frac{v^2 + 1}{2v} - v$$

$$x \cdot \frac{dv}{dx} = \frac{v^2 + 1 - 2v^2}{2v} = \frac{1 - v^2}{2v}$$

$$\frac{2v\,dv}{1 - v^2} = \frac{dx}{x}$$

Step 3: Integrate Both Sides:

$$-\ln|1 - v^2| = \ln|x| + C$$

$$\ln|x| + \ln|1 - v^2| = C$$

$$\ln\left|x(1 - v^2)\right| = C$$

Step 4: Back-Substitute $v = \frac{y}{x}$:

$$\ln\left|\frac{x^2 - y^2}{x}\right| = C$$

$$x^2 - y^2 = cx$$

Step 5: Apply Initial Conditions: At $x = 2$, $y = 0$:

$$2^2 - 0^2 = c(2) \quad \Rightarrow \quad c = 2$$

$$x^2 - y^2 = 2x$$

Step 6: Find $y(8)$: At $x = 8$:

$$8^2 - y^2 = 2(8)$$

$$64 - y^2 = 16 \quad \Rightarrow \quad y^2 = 48 \quad \Rightarrow \quad y = \sqrt{48} = 4\sqrt{3}$$

Final Answer:

$$y(8) = 4\sqrt{3} \quad \text{(Option 1).}$$

Quick Tip

Recognize the given equation as a homogeneous differential equation. Use the substitution $y = vx$ and solve the differential equation. Apply the initial condition to find the constant of integration. Substitute the given value of x to find the corresponding value of y.

Question 20: Let the ellipse E : $x^2 + 9y^2 = 9$ intersect the positive x- and y-axes at the points A and B respectively. Let the major axis of E be a diameter of the circle C. Let the line passing through A and B meet the circle C at the point P. If the area of the triangle with vertices A, P and the origin O is $\frac{m}{n}$, where m and n are coprime, then m - n is equal to :

(1) 16

(2) 15

(3) 18

(4) 17

Correct Answer: (4) 17

Solution:

Given: The circle is defined by the equation:

$$x^2 + y^2 = 9$$

Step 1: Equation of Line AB or AP: The equation of line AB is:

$$\frac{x}{3} + \frac{y}{1} = 1$$

$$x + 3y = 3 \quad \Rightarrow \quad x = 3 - 3y$$

Step 2: Intersection Point of Line AP and the Circle: Substitute $x = 3 - 3y$ into $x^2 + y^2 = 9$:

$$(3 - 3y)^2 + y^2 = 9$$

$$9(1 + y^2 - 2y) + y^2 = 9$$

$$10y^2 - 18y = 0$$

$$y(10y - 18) = 0 \quad \Rightarrow \quad y = \frac{9}{5}$$

Substitute $y = \frac{9}{5}$ into $x = 3 - 3y$:

$$x = 3\left(1 - \frac{9}{5}\right) = 3\left(-\frac{4}{5}\right) = -\frac{12}{5}$$

Thus, the intersection point is:

$$P\left(-\frac{12}{5}, \frac{9}{5}\right)$$

Step 3: Area of $\triangle AOP$: The area of $\triangle AOP$ is given by:

$$\text{Area} = \frac{1}{2} \times \text{Base (OA)} \times \text{Height}$$

Here, Base (OA) $= 3$ and Height $= \frac{9}{5}$:

$$\text{Area} = \frac{1}{2} \times 3 \times \frac{9}{5} = \frac{27}{10}$$

Step 4: Express the Area as $\frac{m}{n}$:

$$\text{Area} = \frac{27}{10}, \quad m = 27, \quad n = 10$$

Step 5: Calculate $m - n$:

$$m - n = 27 - 10 = 17$$

Final Answer: $m - n = 17$, which corresponds to Option D.

Quick Tip

Find the intersection points A and B of the ellipse with the axes. Determine the equation of the circle C using the major axis of the ellipse as the diameter. Find the equation of the line AB and solve the system of equations formed by the line AB and the circle C to find the coordinates of P. Then use the formula for the area of a triangle with vertices at the origin and two other points to find the area of triangle OAP.

Section B

Question 21: Some couples participated in a mixed doubles badminton tournament. If the number of matches played, so that no couple in a match, is 840, then the total numbers of persons, who participated in the tournament, is ______.
Correct Answer: 16

Solution:
Let the number of couples be n.

The total number of ways is given by:

$$\binom{n}{2} \times \binom{n-2}{2} \times 2 = 840$$

Simplify:

$$\frac{n(n-1)}{2} \cdot \frac{(n-2)(n-3)}{2} \cdot 2 = 840$$

$$n(n-1)(n-2)(n-3) = 840 \cdot 4$$

$$n(n-1)(n-2)(n-3) = 3360$$

Factorize 3360:

$$n(n-1)(n-2)(n-3) = 8 \cdot 7 \cdot 6 \cdot 5$$

$$n = 8$$

Number of Persons:

$$\text{Total persons} = 2n = 2(8) = 16$$

Final Answer: The number of persons is 16.

Quick Tip

Consider the total number of ways to choose 4 people out of $2n$. Subtract the number of ways in which a couple can be chosen.

Question 22: The number of elements in the set $\{n \in \mathbb{Z} : |n^2 - 10n + 19| < 6\}$ is ______.

Correct Answer: 6

Solution:

Given:

$$-6 < n^2 - 10n + 19 < 6$$

Step 1: Split the Inequality:

$$n^2 - 10n + 19 - 6 > 0 \quad \text{and} \quad n^2 - 10n + 19 + 6 < 0$$

$$n^2 - 10n + 25 > 0 \quad \text{and} \quad n^2 - 10n + 13 < 0$$

Step 2: Solve $n^2 - 10n + 25 > 0$:

$$(n-5)^2 > 0$$

$$n \in \mathbb{Z} \setminus \{5\} \qquad \cdots (i)$$

Step 3: Solve $n^2 - 10n + 13 < 0$:

$$n^2 - 10n + 13 = 0 \quad \Rightarrow \quad n = 5 \pm 3\sqrt{2}$$

$$5 - 3\sqrt{2} < n < 5 + 3\sqrt{2}$$

Approximate the roots:

$$5 - 3\sqrt{2} \approx 2 \quad \text{and} \quad 5 + 3\sqrt{2} \approx 8$$

$$n \in \{2, 3, 4, 5, 6, 7, 8\} \quad \cdots (ii)$$

Step 4: Combine Results: From $(i) \cap (ii)$:

$$n \in \{2, 3, 4, 5, 6, 8\}$$

Final Answer: The number of values of n is: 6

Quick Tip

Rewrite the absolute value inequality as a compound inequality. Solve each quadratic inequality separately and find the integers that satisfy both inequalities.

Question 23: The number of permutations of the digits 1, 2, 3, ..., 7 without repetition, which neither contain the string 153 nor the string 2467, is ______.

Correct Answer: 4898

Solution:

Given: The numbers are $1, 2, 3, 4, 5, 6, 7$.

Step 1: Numbers having the string (154)**:**

$$\text{Positions: } (154), 2, 3, 6, 7$$

$$\text{Total permutations: } 5! = 120$$

Step 2: Numbers having the string (2467)**:**

$$\text{Positions: } (2467), 1, 3, 5$$

$$\text{Total permutations: } 4! = 24$$

Step 3: Numbers having both strings (154) **and** (2467)**:**

$$\text{Positions: } (154), (2467)$$

$$\text{Total permutations: } 2! = 2$$

Step 4: Apply Inclusion-Exclusion Principle:

$$n((154) \cup (2467)) = 5! + 4! - 2!$$

$$n((154) \cup (2467)) = 120 + 24 - 2 = 142$$

Step 5: Total Numbers:

$$\text{Total permutations: } 7! = 5040$$

Step 6: Numbers having neither (154) nor (2467):

$$\text{Required numbers: } 5040 - 142 = 4898$$

Final Answer:

$$\text{The required numbers are } 4898.$$

Quick Tip

Use the principle of inclusion-exclusion to find the number of permutations containing at least one of the forbidden strings. Subtract this from the total number of permutations.

Question 24: Let $f : (-2, 2) \to \mathbb{R}$ be defined by

$$f(x) = \begin{cases} x\lfloor x \rfloor, & -2 < x < 0 \\ (x-1)\lfloor x \rfloor, & 0 \le x < 2 \end{cases}$$

where $\lfloor x \rfloor$ denotes the greatest integer function. If m and n respectively are the number of points in (-2, 2) at which $y = |f(x)|$ is not continuous and not differentiable, then m + n is equal to ______.

Correct Answer: 4

Solution:

Given:

$$f(x) = \begin{cases} -2x, & -2 < x < -1 \\ -x, & -1 \leq x < 0 \\ 0, & 0 \leq x < 1 \\ x-1, & 1 \leq x < 2 \end{cases}$$

Step 1: Check for Discontinuity:

Clearly, $f(x)$ is discontinuous at $x = -1$ and also non-differentiable.

$$\Rightarrow m = 1$$

Step 2: Check for Differentiability:

Differentiate $f(x)$:

$$f'(x) = \begin{cases} -2, & -2 < x < -1 \\ -1, & -1 < x < 0 \\ 0, & 0 < x < 1 \\ 1, & 1 < x < 2 \end{cases}$$

Clearly, $f(x)$ is non-differentiable at $x = -1, 0, 1$.

Step 3: Check $|f(x)|$:

The absolute value $|f(x)|$ remains the same.

$$\Rightarrow n = 3$$

Step 4: Calculate $m + n$:

$$m + n = 1 + 3 = 4$$

Final Answer:

$$m + n = 4$$

Quick Tip

Rewrite the function piecewise using the definition of the greatest integer function. Then consider the absolute value of the function. Analyze the continuity and differentiability at the points where the definition changes.

Question 25: Let a common tangent to the curves $y^2 = 4x$ and $(x-4)^2 + y^2 = 16$ touch the curves at the points P and Q. Then $(PQ)^2$ is equal to ______.

Correct Answer: 32

Solution:

Given:

$$y^2 = 4x, \quad (x-4)^2 + y^2 = 16$$

Step 1: Equation of the tangent to the parabola: The equation of the tangent to the parabola $y^2 = 4x$ is:

$$y = mx + \frac{1}{m} \quad \cdots (1)$$

This tangent also touches the circle $(x-4)^2 + y^2 = 16$.

Step 2: Condition for tangency: The perpendicular distance from the center of the circle $(4,0)$ to the tangent $y = mx + \frac{1}{m}$ is equal to the radius of the circle:

$$\left| \frac{4m + \frac{1}{m}}{\sqrt{1+m^2}} \right| = 4$$

Step 3: Solve for m: Square both sides:

$$\left(\frac{4m + \frac{1}{m}}{\sqrt{1+m^2}} \right)^2 = 16$$

$$\frac{\left(4m + \frac{1}{m}\right)^2}{1+m^2} = 16$$

$$\left(4m + \frac{1}{m}\right)^2 = 16(1+m^2)$$

$$16m^2 + 8m + \frac{1}{m^2} = 16 + 16m^2$$

$$8m + \frac{1}{m^2} = 16$$

Multiply through by m^2:

$$8m^3 + 1 = 16m^2$$

$$8m^3 - 16m^2 + 1 = 0$$

Solve for distinct m^2:

$$m^2 = \frac{1}{8}$$

Step 4: Point of contact of the parabola: For $m^2 = \frac{1}{8}$, substitute into $y = mx + \frac{1}{m}$:

$$P\left(8, 4\sqrt{2}\right)$$

Step 5: Calculate $(PQ)^2$: The distance between P and Q is given by:

$$PQ = \sqrt{S_1} \quad \Rightarrow \quad (PQ)^2 = S_1$$

$$S_1 = 16 + 32 - 16 = 32$$

Final Answer:

$$(PQ)^2 = 32$$

Quick Tip

Find the equation of the tangent to the parabola in terms of a parameter. Use the distance formula to determine the condition for the line to be tangent to the circle. Solve for the parameter and find the points P and Q. Then calculate $(PQ)^2$.

Question 26: If the mean of the frequency distribution

Class	0-10	10-20	20-30	30-40	40-50
Frequency	2	3	x	5	4

is 28, then its variance is ______.

Correct Answer: 151

Solution:

Given: The data is presented as follows:

C.I.	f	x	$f_i x_i$	x_i^2
$0 - 10$	2	5	10	25
$10 - 20$	3	15	45	225
$20 - 30$	x	25	$25x$	625
$30 - 40$	5	35	175	1225
$40 - 50$	4	45	180	2025

Step 1: Calculate the Mean: The formula for the mean is:

$$\bar{x} = \frac{\sum f_i x_i}{N}$$

Substitute the given data:

$$\bar{x} = \frac{10 + 45 + 25x + 175 + 180}{14 + x}$$

$$28 = \frac{410 + 25x}{14 + x}$$

Solve for x:

$$28(14 + x) = 410 + 25x$$

$$392 + 28x = 410 + 25x$$

$$3x = 410 - 392$$

$$x = \frac{18}{3} = 6$$

Step 2: Variance: The formula for variance is:

$$\text{Variance} = \frac{1}{N} \sum f_i x_i^2 - (\bar{x})^2$$

Substitute the values:

$$\text{Variance} = \frac{1}{20}(18700) - (28)^2$$

$$\text{Variance} = \frac{18700}{20} - 784$$

$$\text{Variance} = 935 - 784 = 151$$

Final Answer:

$$x = 6, \quad \text{Variance} = 151$$

Quick Tip

Use the formula for the mean of a frequency distribution to find the unknown frequency x. Then use the formula for variance to calculate the variance of the distribution.

Question 27: The coefficient of x^7 in $(1 - x + 2x^3)^{10}$ is ______.

Correct Answer: 960

Solution:

Given:

$$(1 - x + 2x^3)^{10}$$

Step 1: General term: The general term T_n in the expansion is given by:

$$T_n = \frac{10!}{a!b!c!}(-x)^b(2x^3)^c$$

$$T_n = \frac{10!}{a!b!c!}(-1)^b x^{b+3c}(2)^c$$

Step 2: Conditions for coefficients: From the power of x, we have:

$$b + 3c = 7 \quad \text{and} \quad a + b + c = 10$$

Step 3: Solve for a, b, and c: Using the above conditions, possible values of (a, b, c) are:

a	b	c
3	7	0
5	4	1
7	1	2

Step 4: Coefficient of x^7: The coefficient of x^7 is:

$$\frac{10!}{3!7!}(-1)^7(1) + \frac{10!}{5!4!1!}(-1)^4(2) + \frac{10!}{7!1!2!}(-1)^1(2)^2$$

Simplify each term:

$$\frac{10!}{3!7!}(-1)^7 = \frac{10 \cdot 9 \cdot 8}{6}(-1) = -120$$

$$\frac{10!}{5!4!1!}(-1)^4(2) = \frac{10 \cdot 9 \cdot 8 \cdot 7 \cdot 6}{120} \cdot 2 = 2520$$
$$\frac{10!}{7!1!2!}(-1)^1(2)^2 = \frac{10 \cdot 9 \cdot 8}{2 \cdot 2}(-1) = -1440$$

Step 5: Combine terms:

$$\text{Coefficient of } x^7 = -120 + 2520 - 1440 = 960$$

Final Answer: The coefficient of x^7 is 960.

Quick Tip

Use the multinomial theorem to find the general term in the expansion. Find combinations of p, q, and r that give x^7. Calculate the coefficients for each combination and sum them up.

Question 28: Let $y = p(x)$ be the parabola passing through the points $(-1, 0)$, $(0, 1)$ and $(1, 0)$. If the area of the region $\{(x, y) : (x + 1)^2 + (y - 1)^2 \leq 1, y \leq p(x)\}$ is A, then $12(\pi - 4A)$ is equal to ______.

Correct Answer: 16

Solution:

Given: The parabola is $x^2 = -4a(y - 1)$, and it passes through the points $(-1, 1)$ and $(1, 0)$.

Step 1: Find the value of a:

$$b = -4a(-1) \implies a = \frac{1}{4}$$

Thus, the equation of the parabola becomes:

$$x^2 = -(y - 1)$$

Step 2: Area covered by the parabola: The area covered by the parabola is given by:

$$\int_{-1}^{0} (1 - x^2)dx$$

Integrate:

$$\int_{-1}^{0} (1 - x^2)dx = \left[x - \frac{x^3}{3}\right]_{-1}^{0}$$

Substitute the limits:

$$= (0 - 0) - \left(-1 + \frac{1}{3}\right)$$
$$= \frac{2}{3}$$

Step 3: Required area: The required area is the area of the sector minus the area of the square minus the area covered by the parabola:

$$\text{Required Area} = \frac{\pi}{4} - \left(1 - \frac{2}{3}\right)$$

Simplify:

$$= \frac{\pi}{4} - \frac{1}{3}$$

Step 4: Final calculation:

$$12\,(\pi - 4A) = 12\left[\pi - 4\left(\frac{\pi}{4} - \frac{1}{3}\right)\right]$$

Simplify:

$$= 12\left[\pi - \pi + \frac{4}{3}\right]$$
$$= 12 \times \frac{4}{3} = 16$$

Final Answer:

The required area is 16.

Quick Tip

Determine the equation of the parabola. Find the intersection points of the parabola and circle. Integrate the parabola to find the area under the curve within the specified limits and combine it with the appropriate section of the circle's area.

Question 29: Let a, b, c be three distinct positive real numbers such that $(2a)^{\log_e a} = (bc)^{\log_e b}$ and $b^{\log_e^2 a} = a^{\log_e c}$. Then $6a + 5bc$ is equal to ______.

Correct Answer: Bonus

Solution:

Given:

$$(2a)^{\ln a} = (bc)^{\ln b}, \quad 2a > 0,\ bc > 0$$

Step 1: Taking natural logarithm:

$$\ln a(\ln 2 + \ln a) = \ln b(\ln b + \ln c)$$

Let $\ln 2 = \alpha,\ \ln a = x,\ \ln b = y,\ \ln c = z$, then:

$$\alpha y = xz$$

and

$$x(\alpha + x) = y(y + z)$$

Step 2: Rearranging the equation:

$$\alpha = \frac{xz}{y}, \quad x\left(\frac{xz}{y} + x\right) = y(y + z)$$

$$x^2(z + y) = y^2(y + z)$$

Factorize:

$$y + z = 0 \quad \text{or} \quad x^2 = y^2 \implies x = -y$$

This implies:

$$bc = 1 \quad \text{or} \quad ab = 1$$

Step 3: Case 1 ($bc = 1$**):** If $bc = 1$, then:

$$(2a)^{\ln a} = 1$$

This implies $a = 1$ or $a = \frac{1}{2}$.

For $a = 1$:

$$(a, b, c) = \left(\frac{1}{2}, \lambda, \frac{1}{\lambda}\right),\ \lambda \neq 1, 2, \frac{1}{2}$$

Step 4: Case 2 ($ab = 1$**):** For $ab = 1$, the solutions are:

$$(a, b, c) = \left(\lambda, \frac{1}{\lambda}, \frac{1}{2}\right),\ \lambda \neq 1, 2, \frac{1}{2}$$

Step 5: Summing the values: For the given conditions:

$$6a + 5bc = 3 + 5 = 8$$

Conclusion: In this situation, infinite answers are possible, hence it is a bonus question.

> **Quick Tip**
>
> Take the natural logarithm of both equations. Substitute the second equation into the first to eliminate $\ln c$. Try to solve for the relationship between a, b, and c.

Question 30: The sum of all those terms, of the arithmetic progression 3, 8, 13, ..., 373, which are *not* divisible by 3, is ______.

Correct Answer: 9525

Solution:

Given Arithmetic Progression: $3, 8, 13, \ldots, 373$

Step 1: Finding the number of terms (n):

$$T_n = a + (n-1)d$$

Substitute $T_n = 373$, $a = 3$, $d = 5$:

$$373 = 3 + (n-1)5$$

$$370 = 5(n-1)$$

$$n = \frac{370}{5} = 75$$

Step 2: Sum of the arithmetic progression:

$$\text{Sum} = \frac{n}{2}[a + l]$$

Substitute $n = 75$, $a = 3$, $l = 373$:

$$\text{Sum} = \frac{75}{2}[3 + 373] = 14100$$

Step 3: Finding the sum of terms divisible by 3: Numbers divisible by 3 are $3, 18, 33, \ldots, 363$.

$$363 = 3 + (k-1)15$$

$$360 = (k-1)15$$

$$k - 1 = \frac{360}{15} = 24 \implies k = 25$$

Sum of these terms:

$$\text{Sum} = \frac{k}{2}[a + l]$$

Substitute $k = 25$, $a = 3$, $l = 363$:

$$\text{Sum} = \frac{25}{2}[3 + 363] = 4575$$

Step 4: Required sum:

$$\text{Required Sum} = 14100 - 4575 = 9525$$

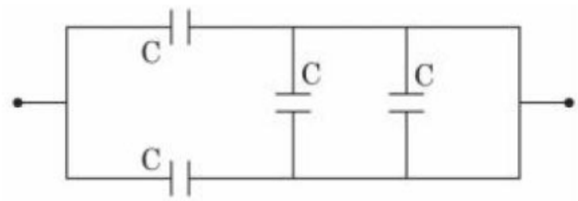
Quick Tip

Find the total number of terms and the sum of all terms in the arithmetic progression. Determine the terms that are divisible by 3 and their sum. Subtract the sum of terms divisible by 3 from the sum of all terms.

Physics

Section-A

Question 31: The equivalent capacitance of the combination shown is



Where $C_1 = C_2 = C$ and $C_3 = C_4 = C$.

(1) 4C

(2) $\frac{5}{3}$C

(3) $\frac{C}{2}$

(4) 2C

Correct Answer: (4) 2C

Solution:

Step 1: Identifying the connections: - Capacitors (3) and (4) are short-circuited due to a direct connection. - This leaves us with capacitors C_1, C_2, and two other capacitors ($C/3$ and C).

Step 2: Parallel combination of C_1 and C_2:

$$C_{\text{eq (parallel)}} = C + C = 2C$$

Step 3: Simplified circuit: After combining C_1 and C_2, the equivalent circuit is shown with a total capacitance of $2C$ in parallel.

Result: The equivalent capacitance of the circuit is $2C$.

Quick Tip

Identify the shorted capacitors, and notice they have no effect on the overall circuit. Then, simplify the circuit by combining parallel capacitances. The formula for parallel capacitance is $C_{eq} = C_1 + C_2 + ...$

Question 32: Match List I with List II :

List I	List II
(A) 3 Translational degrees of freedom	(I) Monoatomic gases
(B) 3 Translational, 2 rotational degrees of freedoms	(II) Polyatomic gases
(C) 3 Translational, 2 rotational and 1 vibrational degrees of freedom	(III) Rigid diatomic gases
(D) 3 Translational, 3 rotational and more than one vibrational degrees of freedom	(IV) Nonrigid diatomic gases

Choose the correct answer from the options given below :

(1) (A)-(I), (B)-(III), (C)-(IV), (D)-(II)

(2) (A)-(I), (B)-(IV), (C)-(III), (D)-(II)

(3) (A)-(IV), (B)-(II), (C)-(I), (D)-(III)

(4) (A)-(IV), (B)-(III), (C)-(II), (D)-(I)

Correct Answer: (1) (A)-(I), (B)-(III), (C)-(IV), (D)-(II)

Solution:

Type of Gas	**No. of Degrees of Freedom**
1. Monoatomic	3 (Translational)
2. Diatomic + rigid	3 (Translational) + 2 (Rotational) = 5
3. Diatomic + non-rigid	3 (Trans) + 2 (Rotational) + 1 (Vibrational)
4. Polyatomic	3 (Trans) + 2 (Rotational) + more than 1 (Vibrational)

Degrees of freedom refer to the number of independent ways a molecule can store energy.

(A) 3 Translational degrees of freedom: This corresponds to movement along the x, y, and z axes. Monoatomic gases, consisting of single atoms, only have these three translational degrees of freedom. Thus, (A) matches with (I).

(B) 3 Translational, 2 rotational degrees of freedom: Diatomic molecules can translate in three directions and rotate about two axes perpendicular to the bond axis. Rigid diatomic molecules do not have vibrational degrees of freedom as the bond length is considered fixed. Thus, (B) matches with (III).

(C) 3 Translational, 2 rotational and 1 vibrational degrees of freedom: Nonrigid diatomic gases have the ability to vibrate along the bond axis, adding one vibrational degree of freedom in addition to the 3 translational and 2 rotational degrees. Thus, (C) matches with (IV).

(D) 3 Translational, 3 rotational and more than one vibrational degrees of freedom: Polyatomic gases, having more than two atoms, can rotate about all three axes. They also have multiple vibrational modes depending on the number of atoms and the molecular structure. Thus, (D) matches with (II).

Quick Tip

Recall the definition of degrees of freedom and how they relate to translational, rotational, and vibrational motions. Monoatomic gases only have translational degrees of freedom. Diatomic molecules can have translational, rotational, and (if nonrigid) vibrational degrees of freedom. Polyatomic molecules generally have all three types.

Question 33: Given below are two statements :

Statements I : If the number of turns in the coil of a moving coil galvanometer is doubled then the current sensitivity becomes double.

Statements II : Increasing current sensitivity of a moving coil galvanometer by only increasing the number of turns in the coil will also increase its voltage sensitivity in the same ratio

In the light of the above statements, choose the correct answer from the options given below :

(1) Both Statement I and Statement II are true

(2) Both Statement I and Statement II are false

(3) Statement I is true but Statement II is false

(4) Statement I is false but Statement II is true

Correct Answer: (3) Statement I is true but Statement II is false

Solution:

Statement I: The current sensitivity of a moving coil galvanometer is defined as the deflection per unit current. It is given by:

$$S_I = \frac{NAB}{k}$$

where N is the number of turns, A is the area of the coil, B is the magnetic field strength, and k is the torsional constant of the suspension.

If the number of turns N is doubled, the current sensitivity S_I also doubles. So, Statement I is true.

Statement II: The voltage sensitivity of a moving coil galvanometer is defined as the deflection per unit voltage. It is given by:

$$S_V = \frac{NAB}{kR} = \frac{S_I}{R}$$

where R is the resistance of the galvanometer coil.

If we increase the number of turns N, the resistance R of the coil also increases. The resistance of a coil is directly proportional to its length, and increasing the number of turns increases the length. Thus, if we only increase N, S_I increases proportionally to N, but R also increases, meaning S_V may not increase in the same ratio.

Specifically, if we double the number of turns, the length of the wire doubles, and therefore, the resistance R also doubles. While S_I doubles, S_V remains the same since both numerator (S_I) and denominator (R) double. So, Statement II is false.

Quick Tip

Remember the formulas for current sensitivity ($S_I = \frac{NAB}{k}$) and voltage sensitivity ($S_V = \frac{NAB}{kR}$). Analyze how changing the number of turns affects both sensitivities. Consider that increasing the number of turns also affects the resistance of the coil.

Question 34: Given below are two statements:

Statement I: Maximum power is dissipated in a circuit containing an inductor, a capacitor and a resistor connected in series with an AC source, when resonance occurs

Statement II: Maximum power is dissipated in a circuit containing pure resistor due to zero phase difference between current and voltage.

In the light of the above statements, choose the correct answer from the options given below:

(1) Statement I is true but Statement II is false

(2) Both Statement I and Statement II are false

(3) Statement I is false but Statement II is true

(4) Both Statement I and Statement II are true

Correct Answer: (4) Both Statement I and Statement II are true

Solution:

Statement I: In a series RLC circuit, resonance occurs when the inductive reactance ($X_L = \omega L$) equals the capacitive reactance ($X_C = \frac{1}{\omega C}$), leading to the impedance being purely resistive ($Z = R$). At resonance, the impedance is minimum, and thus the current is maximum. The average power dissipated in the circuit is given by:

$$P_{avg} = I_{rms}^2 R$$

Since the current is maximum at resonance, the power dissipated is also maximum.
Therefore, Statement I is true.
Statement II: In a purely resistive circuit, the current and voltage are in phase, meaning the phase difference is zero. The average power dissipated in a resistor is given by:

$$P_{avg} = V_{rms} I_{rms} \cos\phi$$

Since $\phi = 0$, $\cos\phi = 1$, so $P_{avg} = V_{rms} I_{rms}$, which is maximum. Therefore, Statement II is true.

Quick Tip

Recall the behavior of RLC circuits at resonance and the power dissipation in resistive circuits. At resonance in an RLC circuit, the impedance is minimum, leading to maximum current and power dissipation. In a purely resistive AC circuit, the current and voltage are in phase, resulting in maximum power dissipation.

Question 35: The range of the projectile projected at an angle of 15° with horizontal is 50 m. If the projectile is projected with same velocity at an angle of 45° with horizontal, then its range will be

(1) $100\sqrt{2}$m

(2) 50 m

(3) 100 m

(4) $50\sqrt{2}$m

Correct Answer: (3) 100 m

Solution:

For a projectile motion, the range R is given by:

$$R = \frac{u^2 \sin(2\theta)}{g}$$

$$\text{Substituting } \theta = 15^\circ:$$
$$R = \frac{u^2 \sin(2 \times 15)}{g} = \frac{u^2}{2g}$$

$$\text{Given that } \frac{u^2}{g} = 100, \text{ we find } \frac{u^2}{2g} = 50.$$

$$\text{Now, for } \theta = 45^\circ, \text{ the range } R' \text{ becomes:}$$
$$R' = \frac{u^2 \sin(2 \times 45)}{g} = \frac{u^2}{g} = 100\,\text{m}.$$

Hence, the maximum range R' for the given value of

$$\frac{u^2}{g} \text{ is } 100\,\text{m}.$$

Quick Tip

The range of a projectile is proportional to $\sin(2\theta)$. Use the given range at 15 degrees to find the relationship between u^2/g and the range. Then, calculate the range at 45 degrees using this relationship.

Question 36: A particle of mass m moving with velocity v collides with a stationary particle of mass 2m. After collision, they stick together and continue to move together with velocity

(1) $\frac{v}{2}$

(2) $\frac{v}{3}$

(3) $\frac{v}{4}$

(4) v

Correct Answer: (2) $\frac{v}{3}$

Solution: Using the principle of momentum conservation:

$$\text{Initial Momentum: } p_i = mv + 2m(0) = mv$$

$$\text{Final Momentum: } p_f = 3m \cdot v'$$

By conservation of momentum: $p_i = p_f$

$$mv = 3m \cdot v' \implies v' = \frac{v}{3}$$

The final velocity of the system (v') is one-third of the initial velocity (v).

Quick Tip

In a perfectly inelastic collision, momentum is conserved. Use the conservation of momentum principle ($m_1u_1 + m_2u_2 = (m_1 + m_2)v$) to solve for the final velocity. Remember that the initial velocity of the stationary particle is 0.

Question 37: Two satellites of masses m and 3m revolve around the earth in circular orbits of radii r & 3r respectively. The ratio of orbital speeds of the satellites respectively is

(1) 3 : 1

(2) 1 : 1

(3) $\sqrt{3} : 1$

(4) 9 : 1

Correct Answer: (3) $\sqrt{3} : 1$

Solution:

The velocity of a satellite in orbit is given by:

$$v = \sqrt{\frac{GM}{r}}$$

where:

- G is the gravitational constant,
- M is the mass of the Earth,
- r is the radius of the orbit.

Rearranging, we see that the velocity is inversely proportional to the square root of the radius:

$$v \propto \frac{1}{\sqrt{r}}$$

Comparing velocities at two different radii, r_1 and r_2:

$$\frac{v_1}{v_2} = \sqrt{\frac{r_2}{r_1}}$$

If $r_2 = 3r_1$, then:

$$\frac{v_1}{v_2} = \sqrt{\frac{3r}{r}} = \sqrt{3}$$

Thus, the velocity at the smaller radius is $\sqrt{3}$ times the velocity at the larger radius.

Quick Tip

The orbital speed of a satellite depends only on the mass of the central body (in this case, Earth) and the orbital radius. The mass of the satellite itself is irrelevant. Use the formula for orbital speed to find the ratio of speeds.

Question 38: Assuming the earth to be a sphere of uniform mass density, the weight of a body at a depth $d = \frac{R}{2}$ from the surface of earth, if its weight on the surface of earth is 200 N, will be:

(1) 500 N

(2) 400 N

(3) 100 N

(4) 300 N

Correct Answer: (3) 100 N

Solution:

The weight of an object at the surface of the Earth is given as:

$$mg = 200\,\text{N}$$

When the object is taken to a depth $d = \frac{R}{2}$ below the surface of the Earth, the effective gravitational acceleration g' is reduced according to the formula:

$$g' = g\left(1 - \frac{d}{R}\right)$$

Substituting $d = \frac{R}{2}$:

$$g' = g\left(1 - \frac{\frac{R}{2}}{R}\right) = g\left(1 - \frac{1}{2}\right) = \frac{g}{2}$$

The weight of the object at this depth becomes:

$$\text{Weight} = mg' = \frac{mg}{2}$$

Substituting $mg = 200\,\text{N}$:

$$\text{Weight} = \frac{200}{2} = 100\,\text{N}$$

Therefore, the weight of the object at a depth of $\frac{R}{2}$ is $100\,\text{N}$.

Quick Tip

Use the formula for the weight of a body at depth d inside a uniform sphere: $W_d = W_s(1 - \frac{d}{R})$. Substitute the given values to calculate the weight at the specified depth.

Question 39: The de Broglie wavelength of a molecule in a gas at room temperature (300 K) is λ_1. If the temperature of the gas is increased to 600 K, then the de Broglie wavelength of the same gas molecule becomes

(1) $2\lambda_1$

(2) $\frac{1}{\sqrt{2}}\lambda_1$

(3) $\sqrt{2}\lambda_1$

(4) $\frac{1}{2}\lambda_1$

Correct Answer: (2) $\frac{1}{\sqrt{2}}\lambda_1$

Solution:

The de Broglie wavelength, λ, of a particle is given by the formula:

$$\lambda = \frac{h}{\sqrt{3mK(T)}}$$

where:

- h is Planck's constant,
- m is the mass of the particle,
- K is Boltzmann's constant, and
- T is the temperature in Kelvin.

To compare the wavelengths λ_1 and λ_2 at two different temperatures T_1 and T_2, we use the ratio:

$$\frac{\lambda_1}{\lambda_2} = \sqrt{\frac{T_2}{T_1}}$$

This equation shows that the wavelength is inversely proportional to the square root of the temperature.

Rearranging for λ_2, we get:

$$\lambda_2 = \lambda_1\sqrt{\frac{T_1}{T_2}}$$

Substituting the values $T_1 = 300$ K and $T_2 = 600$ K:

$$\lambda_2 = \lambda_1\sqrt{\frac{300}{600}}$$

Simplifying further:

$$\lambda_2 = \frac{\lambda_1}{\sqrt{2}}$$

Thus, the wavelength λ_2 at a higher temperature is shorter by a factor of $\sqrt{2}$ compared to λ_1.

Quick Tip

The de Broglie wavelength is inversely proportional to the square root of the kinetic energy, which is directly proportional to the temperature. Therefore, if the temperature doubles, the wavelength is reduced by a factor of $\sqrt{2}$.

Question 40: A physical quantity P is given as

$$P = \frac{a^2 b^3}{c\sqrt{d}}$$

The percentage error in the measurement of a, b, c and d are 1%, 2%, 3% and 4% respectively. The percentage error in the measurement of quantity P will be

(1) 14%

(2) 13%

(3) 16%

(4) 12%

Correct Answer: (2) 13%

Solution:

The percentage change in P can be calculated using the formula:

$$\frac{\Delta P}{P} \times 100 = \left(2\frac{\Delta a}{a} + 3\frac{\Delta b}{b} + \frac{\Delta c}{c} + \frac{1}{2}\frac{\Delta d}{d}\right) \times 100$$

Substitute the given values:

$$= 2 \times 1 + 3 \times 2 + 3 + \frac{1}{2} \times 4$$

Simplify each term:

$$= 2 + 6 + 3 + 2$$

Add the results:

$$= 13\%$$

Thus, the percentage change in P is 13%.

Quick Tip

Use the formula for propagation of errors to find the percentage error in P. The percentage error in x^n is n times the percentage error in x. The percentage error in $\sqrt{x}$ is half the percentage error in x. For a quantity given as $P = \frac{a^m b^n}{c^p d^q}$, the percentage error in P is given by $\frac{\Delta P}{P} \times 100\% = (m\frac{\Delta a}{a} + n\frac{\Delta b}{b} + p\frac{\Delta c}{c} + q\frac{\Delta d}{d}) \times 100\%$

Question 41: Consider two containers A and B containing monoatomic gases at the same Pressure (P), Volume (V) and Temperature (T). The gas in A is compressed

isothermally to $\frac{1}{8}$ of its original volume while the gas in B is compressed adiabatically to $\frac{1}{8}$ of its original volume. The ratio of final pressure of gas in B to that of gas in A is

(1) 8

(2) 4

(3) $\frac{1}{8}$

(4) $8^{\frac{3}{2}}$

Correct Answer: (2) 4

Solution:

For part (A), the process is isothermal. Using the isothermal process equation:

$$P_1 V_1 = P_2 V_2$$

Substituting $V_2 = \frac{V}{8}$:

$$PV = P_2 \frac{V}{8}$$

Simplify to find P_2:

$$P_2 = 8P$$

For part (B), the process is adiabatic with $\gamma_{\text{mono}} = \frac{5}{3}$. Using the adiabatic relation:

$$P_1 V_1^{\gamma} = P_2 V_2^{\gamma}$$

Substituting $V_2 = \frac{V}{8}$:

$$PV^{5/3} = P_2 \left(\frac{V}{8}\right)^{5/3}$$

Simplify to find P_2:

$$P_2 = (8)^{5/3} P$$

To calculate the ratio $\frac{P_2}{P_1}$:

$$\frac{P_2}{P_1} = \frac{(8)^{5/3} P}{8P} = (8)^{2/3}$$

Simplify further:

$$(8)^{2/3} = 4$$

Thus, the ratio $\frac{P_2}{P_1}$ is 4.

Quick Tip

For isothermal processes, $PV = constant$. For adiabatic processes, $PV^{\gamma} = constant$, where γ is the adiabatic index (5/3 for a monoatomic gas). Use these relationships to find the final pressures in containers A and B, then compute their ratio.

Question 42: Given below are two statements:

Statements I: Pressure in a reservoir of water is same at all points at the same level of water.

Statements II: The pressure applied to enclosed water is transmitted in all directions equally.

In the light of the above statements, choose the correct answer from the options given below:

(1) Both Statements I and Statements II are false

(2) Both Statements I and Statements II are true

(3) Statements I is true but Statements II is false

(4) Statements I is false but Statements II is true

Correct Answer: (2) Both Statements I and Statements II are true

Solution:

Both Statements I and Statements II are true.

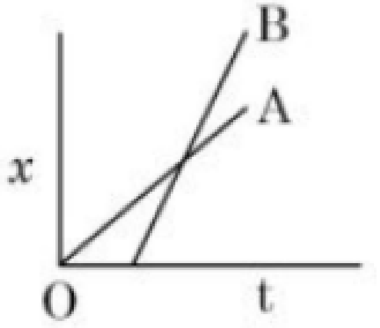
By Theory

By Pascal's law, pressure is equally transmitted to enclosed water in all directions. Statement II directly reflects this. Statement I is also true because at a given depth (level) in a static fluid, the pressure is the same in all directions. This is due to the hydrostatic pressure which depends only on the density of the fluid and depth.

Quick Tip

Pascal's Law and the concept of hydrostatic pressure explain both statements. Pressure in a static fluid at a given depth is independent of direction.

Question 43: The position-time graphs for two students A and B returning from school to their homes are shown in the figure.



(A) A lives closer to the school

(B) B lives closer to the school

(C) A takes lesser time to reach home

(D) A travels faster than B

(E) B travels faster than A

Choose the correct answer from the options given below :

(1) (A) and (E) only

(2) (A), (C) and (E) only

(3) (B) and (E) only

(4) (A), (C) and (D) only

Correct Answer: (1) (A) and (E) only

Solution: Given that we are comparing slopes and their implications:

Step 1: Identify the slopes.

The slope of A is given as:

$$\text{slope of A} = V_A$$

The slope of B is given as:

$$\text{slope of B} = V_B$$

Step 2: Compare the slopes.

Since:

$$(\text{slope})_B > (\text{slope})_A$$

it follows that:

$$V_B > V_A$$

Step 3: Relate velocity to time.

As velocity is inversely related to time (for a given distance), we conclude:

$$t_B < t_A$$

Final Answer: (A) and (E) only.

Quick Tip

In position-time graphs, the slope represents speed, and the y-intercept at a given time represents distance.

Question 44: The energy of an electromagnetic wave contained in a small volume oscillates with

(1) double the frequency of the wave

(2) the frequency of the wave

(3) zero frequency

(4) half the frequency of the wave

Correct Answer: (1) double the frequency of the wave

Solution: To double the frequency of the wave, the electric field is represented as:

$$E = E_0 \sin(\omega t - kx)$$

Step 1: Calculate the energy density.

The energy density of the wave is given by:

$$\text{Energy density} = \frac{1}{2}\varepsilon_0 E_{\text{net}}^2$$

Substitute $E_{\text{net}} = E_0 \sin(\omega t - kx)$:

$$\text{Energy density} = \frac{1}{2}\varepsilon_0 E_0^2 \sin^2(\omega t - kx)$$

Step 2: Use the trigonometric identity.

Using the identity $\sin^2 x = \frac{1}{2}(1 - \cos 2x)$:

$$\text{Energy density} = \frac{1}{4}\varepsilon_0 E_0^2 \left(1 - \cos(2\omega t - 2kx)\right)$$

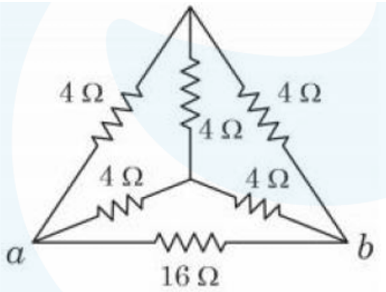
Final Answer: The energy density is:

$$\frac{1}{4}\varepsilon_0 E_0^2 \left(1 - \cos(2\omega t - 2kx)\right)$$

Quick Tip

The energy density of an EM wave is proportional to the square of the electric field. Squaring a sinusoidal function doubles the frequency.

Question 45: The equivalent resistance of the circuit shown below between points a and b is:



(1) $20\,\Omega$

(2) $16\,\Omega$

(3) $24\,\Omega$

(4) $3.2\,\Omega$

Correct Answer: (4) $3.2\,\Omega$

Solution:

Step 1: Write the formula for equivalent resistance.

For a balanced Wheatstone Bridge (WSB), the equivalent resistance between points a and b is given by:

$$\frac{1}{R_{ab}} = \frac{1}{16} + \frac{1}{8} + \frac{1}{8}$$

Step 2: Simplify the equation.

Combine the terms:

$$\frac{1}{R_{ab}} = \frac{1+2+2}{16} = \frac{5}{16}$$

Step 3: Find R_{ab}.

Take the reciprocal:

$$R_{ab} = \frac{16}{5} = 3.2$$

Final Answer: The equivalent resistance R_{ab} is:

$$R_{ab} = 3.2\,\Omega$$

Quick Tip

For complex circuits, use series-parallel combinations or delta-star transformations to simplify the circuit before calculating the equivalent resistance.

Question 46: A carrier wave of amplitude 15 V modulated by a sinusoidal base band signal of amplitude 3 V. The ratio of maximum amplitude to minimum amplitude in an amplitude modulated wave is

(1) 2

(2) 1

(3) 5

(4) $\frac{3}{2}$

Correct Answer: (4) $\frac{3}{2}$

Solution: In amplitude modulation, the maximum amplitude A_{max} and minimum amplitude A_{min} are given by:

$$A_{max} = A_c + A_m$$

$$A_{min} = A_c - A_m$$

where A_c is the carrier amplitude and A_m is the modulating signal amplitude.

Given $A_c = 15\,V$ and $A_m = 3\,V$, we have:

$$A_{max} = 15 + 3 = 18\,V$$

$$A_{min} = 15 - 3 = 12\,V$$

The ratio of maximum amplitude to minimum amplitude is:

$$\frac{A_{max}}{A_{min}} = \frac{18}{12} = \frac{3}{2}$$

Quick Tip

Remember the formulas for maximum and minimum amplitude in AM: $A_{max} = A_c + A_m$ and $A_{min} = A_c - A_m$.

Question 47: A particle executes S.H.M. of amplitude A along x-axis. At t = 0, the position of the particle is $x = -\frac{A}{2}$ and it moves along positive x-axis. The displacement of particle in time t is $x = A\sin(\omega t + \delta)$, then the value δ will be

(1) $\frac{\pi}{4}$

(2) $\frac{\pi}{2}$

(3) $\frac{\pi}{3}$

(4) $\frac{\pi}{6}$

Correct Answer: (4) $\frac{\pi}{6}$

Solution:

Step 1: Write the equation for $\cos\theta$.

Using the given setup:

$$\cos\theta = \frac{A}{2A} = \frac{1}{2}$$

Step 2: Solve for θ.

From the trigonometric identity:

$$\theta = \frac{\pi}{3}$$

Step 3: Calculate the phase difference δ.

The phase difference is given by:

$$\delta = \frac{\pi}{2} - \frac{\pi}{3}$$

Simplify the expression:

$$\delta = \frac{\pi}{6}$$

Final Answer: The values are:

$$\theta = \frac{\pi}{3}, \quad \delta = \frac{\pi}{6}.$$

Quick Tip

The phase constant δ in SHM depends on the initial conditions (position and velocity at t=0). Carefully consider the sign of the sine and cosine of δ to find the correct value.

Question 48: The angular momentum for the electron in Bohr's orbit is L. If the electron is assumed to revolve in second orbit of hydrogen atom, then the change in angular momentum will be

(1) $\frac{L}{2}$

(2) zero

(3) L

(4) 2L

Correct Answer: (3) L

Solution:

Step 1: Write the formula for angular momentum.

The angular momentum is given by:

$$\text{Angular momentum} = \frac{nh}{2\pi}$$

Step 2: Calculate L_1 for $n = 1$.

$$n = 1, \quad L_1 = \frac{h}{2\pi} = L$$

Step 3: Calculate L_2 for $n = 2$.

$$n = 2, \quad L_2 = \frac{2h}{2\pi} = 2L$$

Step 4: Find the change in angular momentum ΔL.

$$\Delta L = L_2 - L_1 = 2L - L = L$$

Final Answer: The change in angular momentum is:

$$\Delta L = L$$

Quick Tip

In Bohr's model, angular momentum is quantized: $L_n = n\hbar$. The change in angular momentum between orbits is simply the difference in their quantized values.

Question 49: An object is placed at a distance of 12 cm in front of a plane mirror. The virtual and erect image is formed by the mirror. Now the mirror is moved by 4 cm towards the stationary object. The distance by which the position of image would be shifted, will be

(1) 4 cm towards mirror

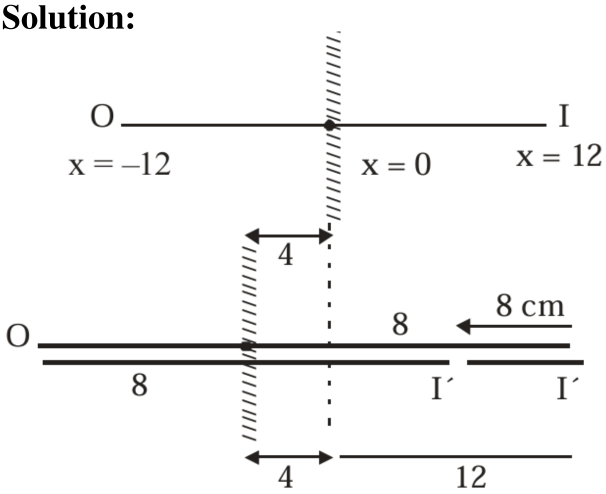
(2) 8 cm away from mirror

(3) 2 cm towards mirror

(4) 8 cm towards mirror

Correct Answer: (4) 8 cm towards mirror

Solution:



Step 1: Initial setup.

The object is placed at $x = -12$ cm, and the mirror is positioned at $x = 0$. The image is initially formed at $x = 12$ cm.

Step 2: Shifting the mirror.

When the mirror is shifted 8 cm towards the object, the new position of the mirror is:

$$x = -8\,\text{cm}.$$

Step 3: Image position relative to the shifted mirror.

The distance of the image from the mirror remains the same as before. Since the mirror is shifted, the image is now formed at:

$$x = -8 + 8 = 0\,\text{cm}.$$

Step 4: Relative shift of the image.

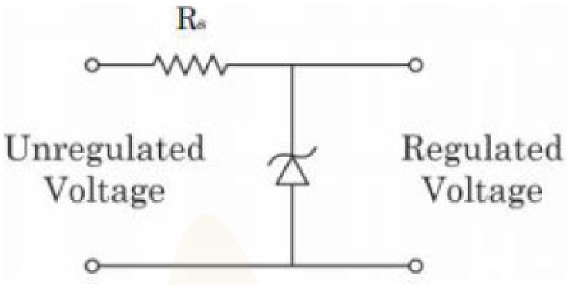
The new image position is 8 cm closer to the mirror compared to its previous position.

Final Answer: The image will be shifted 8 cm towards the mirror.

Quick Tip

In a plane mirror, the image is virtual and the image distance is always equal to the object distance. When the mirror moves, the image shifts by twice the distance the mirror moved.

Question 50: A zener diode of power rating 1.6 W is used as a voltage regulator. If the zener diode has a breakdown voltage of 8 V and it has to regulate voltage fluctuating between 3 V and 10 V, what is the value of resistance R_s for safe operation of the diode?



(1) $13.3\,\Omega$

(2) $13\,\Omega$

(3) $10\,\Omega$

(4) $12\,\Omega$

Correct Answer: (3) $10\,\Omega$

Solution:

Step 1: Calculate the total current I_t.

The total current is given by:

$$I_t = \frac{P_t}{V_t} = \frac{1.6}{8} = 0.2\,\text{A}.$$

Step 2: Use Ohm's Law to find the series resistance R_s.

The voltage drop across R_s is given as $10 - 8 = 2\,\text{V}$. Using Ohm's Law:

$$R_s = \frac{\Delta V}{I} = \frac{2}{0.2}.$$

Step 3: Calculate R_s.

Simplify the expression:

$$R_s = 10\,\Omega.$$

$$\boxed{R_s = 10\,\Omega.}$$

Quick Tip

For Zener diode regulator design, always calculate the resistor value based on the maximum input voltage to ensure the Zener diode doesn't exceed its power rating. Neglecting load current provides a safety margin.

Section B

Question 51: Unpolarised light of intensity 32 Wm^{-2} passes through the combination of three polaroids such that the pass axis of the last polaroid is perpendicular to that of the pass axis of first polaroid. If intensity of emerging light is 3 Wm^{-2}, then the angle between pass axis of first two polaroids is ________.°

Correct Answer: 30 & 60

Solution:

Step 1: Write the expression for I_{net}.

The net intensity is given as:

$$I_{\text{net}} = \frac{I_0}{2}\cos^2\theta\sin^2\theta = \frac{I_0}{8}(\sin 2\theta)^2$$

Step 2: Use the given value for I_{net}.

Substitute $I_{\text{net}} = 3$:

$$3 = \frac{32}{8}(\sin 2\theta)^2$$

Simplify:

$$\sin 2\theta = \frac{\sqrt{3}}{2}.$$

Step 3: Solve for 2θ.

The solution to $\sin 2\theta = \frac{\sqrt{3}}{2}$ gives:

$$2\theta = 60^\circ \quad \text{and} \quad 120^\circ.$$

Step 4: Solve for θ.

Divide by 2 to find θ:

$$\theta = 30^\circ \quad \text{and} \quad 60^\circ.$$

Final Answer: The values of θ are:

$$\theta = 30^\circ \quad \text{and} \quad 60^\circ.$$

Quick Tip

Remember Malus's law: $I = I_0 \cos^2 \theta$, where I_0 is the initial intensity and θ is the angle between the polarizer's axis and the polarization direction of the incident light. For unpolarized light, the initial intensity is halved after the first polarizer.

Question 52: If the earth suddenly shrinks to $\frac{1}{64}$th of its original volume with its mass remaining the same, the period of rotation of earth becomes $\frac{24}{x}$ h. The value of x is ________.

Correct Answer: 16

Solution:

By applying the Angular Momentum Conservation (AMC):

$$\frac{2}{5}MR^2\omega_1^2 = \frac{2}{5}M\left(\frac{R}{4}\right)^2\omega_2^2$$

Step 1: Simplify the equation.

The mass M and constant $\frac{2}{5}$ cancel out, leaving:

$$\frac{\omega_1}{\omega_2} = \left(\frac{R}{R/4}\right)^2 = \frac{1}{16}.$$

Step 2: Relate angular velocity to period.

The relationship between angular velocity and time period is:

$$\frac{\omega_1}{\omega_2} = \frac{T_2}{T_1}.$$

Substituting $\frac{\omega_1}{\omega_2} = \frac{1}{16}$ and $T_1 = 24$:

$$\frac{1}{16} = \frac{T_2}{24}.$$

Step 3: Solve for T_2.

Rearrange to find T_2:

$$T_2 = \frac{24}{16}.$$

Final Answer:

The value of x is:

$$x = 16 \quad \text{Ans.}$$

Quick Tip

Remember the conservation of angular momentum: $I\omega =$ constant. Changes in the moment of inertia (I) directly affect the angular velocity (ω) and thus the period of rotation.

Question 53: Three concentric spherical metallic shells X, Y and Z of radius a, b and c respectively [a $<$ b $<$ c] have surface charge densities σ, $-\sigma$ and σ, respectively. The shells X and Z are at the same potential. If the radii of X & Y are 2 cm and 3 cm, respectively. The radius of shell Z is ________ cm.

Correct Answer: 5

Solution:

The given charges are:

$$q_x = \sigma 4\pi a^2, \quad q_y = -\sigma 4\pi b^2, \quad q_z = \sigma 4\pi c^2.$$

Step 1: Write the potential at point y.

The potential at y is the sum of potentials due to q_x, q_y, and q_z:

$$V_y = \frac{q_x}{4\pi\varepsilon_0 a} + \frac{q_y}{4\pi\varepsilon_0 b} + \frac{q_z}{4\pi\varepsilon_0 c}.$$

Substitute the values of q_x, q_y, and q_z:

$$V_y = \frac{\sigma 4\pi a^2}{4\pi\varepsilon_0 a} - \frac{\sigma 4\pi b^2}{4\pi\varepsilon_0 b} + \frac{\sigma 4\pi c^2}{4\pi\varepsilon_0 c}.$$

Simplify:

$$V_y = \frac{\sigma 4\pi a}{\varepsilon_0} - \frac{\sigma 4\pi b}{\varepsilon_0} + \frac{\sigma 4\pi c}{\varepsilon_0}.$$

Combine terms:

$$V_y = \frac{\sigma 4\pi}{\varepsilon_0 c}\left(a^2 - b^2 + c^2\right).$$

Step 2: Relate c to a and b.

Using the relation:

$$c(a - b + c) = a^2 - b^2 + c^2,$$

expand and simplify:

$$c(a - b) + c^2 = (a + b)(a - b).$$

Factorize:

$$c(a - b) = (a + b)(a - b).$$

Step 3: Solve for c.

If $c = a + b$, substitute $a = 2$ cm and $b = 3$ cm:

$$c = a + b = 2 + 3 = 5\,\text{cm}.$$

Final Answer:

The value of c is:

$$c = 5\,\text{cm}.$$

Quick Tip

For concentric spherical shells, the potential at a given shell is influenced by the charges on all shells with a smaller radius. Carefully consider the superposition of potentials.

Question 54: A transverse harmonic wave on a string is given by

$y(x, t) = 5\sin(6t + 0.003\,x)$

where x and y are in cm and t in sec. The wave velocity is ________ ms^{-1}.

Correct Answer: 20

Solution:

The general equation for a transverse harmonic wave is given by:

$$y(x, t) = A\sin(kx \pm \omega t)$$

where:

- A is the amplitude
- k is the wave number ($k = \frac{2\pi}{\lambda}$)

- ω is the angular frequency ($\omega = 2\pi f$)
- λ is the wavelength
- f is the frequency

Comparing the given equation $y(x, t) = 5\sin(6t + 0.003\,x)$ with the general equation, we have:

- $A = 5\,cm$
- $\omega = 6\,rad/s$
- $k = 0.003\,rad/cm = 0.3\,rad/m$ (Note: we converted cm to m)

The wave velocity (v) is related to the wave number (k) and angular frequency (ω) by:

$$v = \frac{\omega}{k}$$

Substituting the values:

$$v = \frac{6\,rad/s}{0.3\,rad/m} = 20\,m/s$$

Final Answer: The wave velocity is 20 m/s.

Quick Tip

The wave velocity is the ratio of the angular frequency to the wave number: $v = \frac{\omega}{k}$. Make sure your units are consistent (e.g., both in meters or both in centimeters).

Question 55: 10 resistors each of resistance $10\,\Omega$ can be connected in such as to get maximum and minimum equivalent resistance. The ratio of maximum and minimum equivalent resistance will be ________.

Correct Answer: 100

Solution:

Step 1: Calculate R_{max} for resistors in series.

When all 10 resistors are connected in series, the maximum resistance is:

$$R_{\text{max}} = 10R = 10 \times 10 = 100\,\Omega.$$

Step 2: Calculate R_{min} for resistors in parallel.

When all 10 resistors are connected in parallel, the minimum resistance is:

$$R_{\text{min}} = \frac{R}{10} = \frac{10}{10} = 1\,\Omega.$$

Step 3: Find the ratio $\frac{R_{\text{max}}}{R_{\text{min}}}$.

The ratio is:

$$\frac{R_{\text{max}}}{R_{\text{min}}} = \frac{100}{1} = 100.$$

Step 4: Verify R_{min}.

From the above calculations:

$$R_{\text{min}} = \frac{100}{1} = 100\,\Omega.$$

Final Answer:

$$R_{\text{max}} = 100\,\Omega, \quad R_{\text{min}} = 1\,\Omega, \quad \frac{R_{\text{max}}}{R_{\text{min}}} = 100.$$

Quick Tip

For n identical resistors of resistance R:

- Series connection: $R_{eq} = nR$
- Parallel connection: $R_{eq} = \frac{R}{n}$

The ratio is always n^2.

Question 56: The decay constant for a radioactive nuclide is $1.5 \times 10^{-5} s^{-1}$. Atomic weight of the substance is 60 g mole^{-1}. (N$_A$ = 6×10^{23}). The activity of 1.0 µg of the substance is ________ $\times 10^{10}$ Bq.

Correct Answer: 15

Solution:

Step 1: Calculate the number of moles.

The number of moles is given by:

$$\text{No. of moles} = \frac{\text{mass of sample}}{\text{molar mass}} = \frac{1 \times 10^{-6}}{60} = \frac{10^{-7}}{6}.$$

Step 2: Calculate the number of atoms.

Using Avogadro's number ($N_A = 6 \times 10^{23}$):

$$\text{No. of atoms} = n \cdot N_A = \frac{10^{-7}}{6} \times 6 \times 10^{23}.$$

Simplify:

$$\text{No. of atoms} = 10^{16}.$$

Step 3: Calculate the activity at $t = 0$.

The activity at $t = 0$ is given by:

$$A_0 = N_0 \lambda,$$

where $N_0 = 10^{16}$ and $\lambda = 1.5 \times 10^{-5}$.

Substitute the values:

$$A_0 = 10^{16} \times 1.5 \times 10^{-5}.$$

Simplify:

$$A_0 = 15 \times 10^{10}\,\text{Bq}.$$

Final Answer:

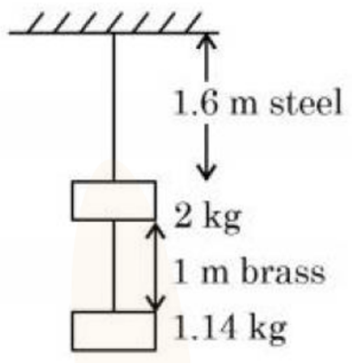
The initial activity is:

$$A_0 = 15 \times 10^{10}\,\text{Bq}.$$

Quick Tip

Activity (A) is the number of decays per unit time. It's calculated as the product of the decay constant (λ) and the number of radioactive atoms (N). Remember to convert units consistently.

Question 57: Two wires each of radius 0.2 cm and negligible mass, one made of steel and the other made of brass are loaded as shown in the figure. The elongation of the steel wire is ________ $\times 10^{-6}$ m. [Young's modulus for steel = $2 \times 10^{11}\ Nm^{-2}$ and g = $10\,ms^{-2}$]



Correct Answer: 20

Solution:

Step 1: Analyze the system.

The forces acting on the system are:

$$T_2 = T_1 + 20 = 20 + 11.4 = 31.4\,\text{N}.$$

Step 2: Calculate the elongation in the steel wire.

The elongation ΔL in the steel wire is given by:

$$\Delta L = \frac{T_2 L}{AY},$$

where:

- $T_2 = 31.4\,\text{N}$,
- $L = 1.6\,\text{m}$ (length of steel wire),
- $A = \pi(0.2 \times 10^{-2})^2\,\text{m}^2$ (cross-sectional area),
- $Y = 2 \times 10^{11}\,\text{Pa}$ (Young's modulus of steel).

Substitute the values:

$$\Delta L = \frac{31.4 \times 1.6}{\pi(0.2 \times 10^{-2})^2 \times 2 \times 10^{11}}.$$

Simplify:

$$\Delta L = 2 \times 10^{-5}\,\text{m}.$$

Convert to micrometers:

$$\Delta L = 20 \times 10^{-6}\,\text{m}.$$

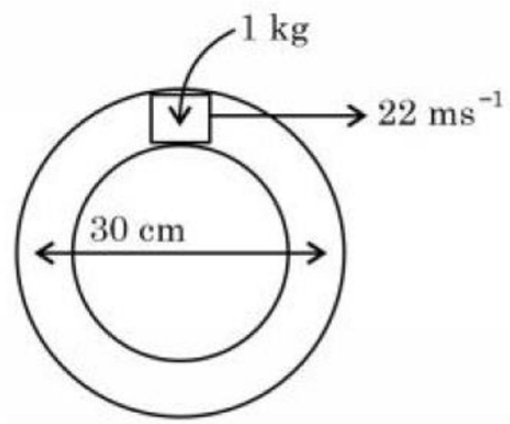
Final Answer: The elongation in the steel wire is:

$$\Delta L = 20 \times 10^{-6}\,\text{m}.$$

Quick Tip

Remember the formula for elongation under tensile stress: $\Delta L = \frac{FL}{AY}$. Ensure consistent units throughout your calculations.

Question 58: A closed circular tube of average radius 15 cm, whose inner walls are rough, is kept in vertical plane. A block of mass 1 kg just fit inside the tube. The speed of block is 22 m/s, when it is introduced at the top of tube. After completing five oscillations, the block stops at the bottom region of tube. The work done by the tube on the block is ________ J. (Given: g = $10\,ms^{-2}$**)**



Correct Answer: 245

Solution:

Step 1: Given data.

The radius of the arc is:

$$R_{\text{arc}} = 15\,\text{cm} = 0.15\,\text{m}.$$

Step 2: Work-energy theorem (WET).

By the work-energy theorem:

$$W_f + W_{\text{gravity}} = \Delta K = K_f - K_i,$$

where:

- W_f is the work done by friction,
- W_{gravity} is the work done by gravity,
- K_f is the final kinetic energy,
- K_i is the initial kinetic energy.

Step 3: Substitute the known values.

The equation becomes:

$$W_f + 10 \times 0.3 = 0 - \frac{1}{2} \times 1 \times (22)^2.$$

Simplify:

$$W_f + 3 = 0 - \frac{1}{2} \times 484.$$

Step 4: Solve for W_f.

$$W_f + 3 = -242.$$

Rearrange:

$$W_f = -245\,\text{J}.$$

Step 5: Work by friction.

The work done by friction is:

$$W_f = -245\,\text{J}.$$

Final Answer:

The work done by friction is:

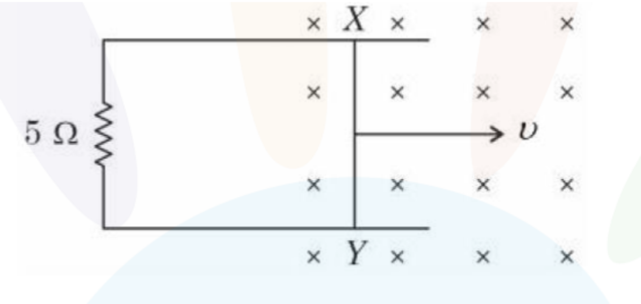
$$W_f = -245\,\text{J}.$$

Quick Tip

The work-energy theorem is crucial here. The work done by non-conservative forces (friction in this case) is equal to the change in mechanical energy.

Question 59: A 1 m long metal rod XY completes the circuit as shown in figure. The plane of the circuit is perpendicular to the magnetic field of flux density 0.15 T. If the resistance of the

circuit is $5\,\omega$, the force needed to move the rod in direction, as indicated, with a constant speed of 4 m/s will be ________ 10^{-3} N.



Correct Answer: 18

Solution:

Step 1: Write the force equation.

The force F is given by:

$$F = I\ell B,$$

where:

- I is the current,
- ℓ is the length of the conductor in the magnetic field,
- B is the magnetic field strength.

Step 2: Substitute I in terms of e, R, and B.

The current I can be expressed as:

$$I = \frac{e}{R}.$$

Substituting this into the force equation:

$$F = \left(\frac{Bv\ell}{R}\right)\ell B = \frac{B^2\ell^2 v}{R}.$$

Step 3: Substitute the known values.

Given:

$$B = 15, \quad \ell = 1, \quad v = 4, \quad R = 5,$$

substitute into the equation:

$$F = \frac{(15)^2 \times (1)^2 \times 4}{5} = 180 \times 10^{-4}.$$

Convert to standard form:

$$F = 18 \times 10^{-3}.$$

Final Answer:

The force is:

$$F = 18\,\text{N}.$$

Quick Tip

Remember the formulas for motional emf ($\mathcal{E} = Blv$) and the force on a current-carrying conductor in a magnetic field ($F = BIl$). Make sure your units are consistent.

Question 60: The current required to be passed through a solenoid of 15 cm length and 60 turns in order to demagnetize a bar magnet of magnetic intensity $2.4 \times 10^3\ Am^{-1}$ **is ________ A.**

Correct Answer: 6

Solution:

The magnetic intensity (H) inside a solenoid is given by:

$$H = \frac{NI}{l}$$

where:

- N is the number of turns (60)
- I is the current (in Amperes)
- l is the length of the solenoid (15 cm = 0.15 m)

We are given that the magnetic intensity required to demagnetize the bar magnet is $H = 2.4 \times 10^3\ Am^{-1}$. We can rearrange the formula to solve for the current (I):

$$I = \frac{Hl}{N}$$

Substituting the values:

$$I = \frac{(2.4 \times 10^3\ Am^{-1})(0.15\,m)}{60} = \frac{360}{60}\,A = 6\,A$$

Final Answer: The current required is 6 A.

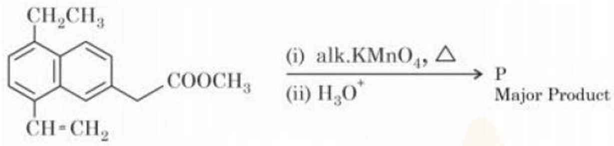
Quick Tip

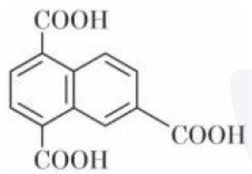
The magnetic field intensity (H) inside a solenoid is directly proportional to the number of turns (N) and the current (I), and inversely proportional to the length (l): $H = \frac{NI}{l}$.

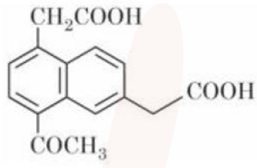
Chemistry

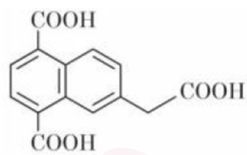
Section-A

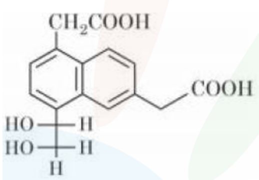
Question 61: The major product 'P' formed in the given reaction is

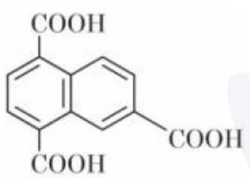


(1) 

(2) 

(3) 

(4) 

Correct Answer: (1) 

Solution:

The given reaction involves the oxidation of an alkyl-substituted naphthalene derivative using alkaline potassium permanganate (alk. $KMnO_4$) followed by acidic workup (H_3O^+). Alkaline $KMnO_4$ is a strong oxidizing agent that can cleave carbon-carbon bonds in alkyl side chains attached to aromatic rings. The alkyl groups are oxidized all the way to carboxylic acid groups (-COOH).

Step 1: Oxidation of the side chains. The two ethyl groups (CH_2CH_3) on the naphthalene ring are oxidized to carboxylic acid groups. The methyl ester group ($COOCH_3$) is also hydrolyzed under the alkaline conditions to a carboxylic acid group.

Step 2: Formation of the product. The final product will be naphthalene-1,2,4-tricarboxylic acid.

Final Answer: The major product P is naphthalene-1,2,4-tricarboxylic acid, which corresponds to option (1).

Quick Tip

Alkaline $KMnO_4$ is a powerful oxidizing agent that converts alkyl side chains on aromatic rings to carboxylic acid groups. Remember that the reaction conditions (alkaline and heat) are crucial for this transformation.

Question 62: Prolonged heating is avoided during the preparation of ferrous ammonium sulphate to

(1) prevent hydrolysis

(2) prevent reduction

(3) prevent breaking

(4) prevent oxidation

Correct Answer: (4) prevent oxidation

Solution:

Ferrous ammonium sulphate, also known as Mohr's salt, is a double salt with the formula $(NH_4)_2Fe(SO_4)_2 \cdot 6H_2O$. It contains iron in the +2 oxidation state (ferrous ion, Fe^{2+}). Ferrous ions are susceptible to oxidation to the +3 oxidation state (ferric ion, Fe^{3+}) in the presence of oxygen from the air, especially at elevated temperatures.

Prolonged heating can accelerate this oxidation process, converting ferrous ions to ferric ions and leading to the formation of ferric compounds as impurities in the Mohr's salt. This changes the composition and properties of the desired product. Therefore, prolonged heating is avoided to minimize the oxidation of Fe^{2+} to Fe^{3+}.

The reaction illustrating this oxidation is:

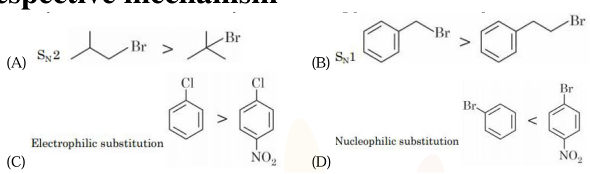
$$4Fe^{2+} + O_2 + 4H^+ \rightarrow 4Fe^{3+} + 2H_2O$$

Final Answer: Prolonged heating is avoided to prevent oxidation.

Quick Tip

Ferrous ions (Fe^{2+}) are easily oxidized to ferric ions (Fe^{3+}) by atmospheric oxygen. Gentle heating and controlled conditions are necessary when preparing and handling ferrous compounds.

Question 63: Identify the correct order of reactivity for the following pairs towards the respective mechanism



(1) (A), (C) and (D) only

(2) (A), (B) and (D) only

(3) (B), (C) and (D) only

(4) (A), (B), (C) and (D)

Correct Answer: (4) (A), (B), (C) and (D)

Solution:

Let's analyze each reaction pair separately:

(A) S_N2 Reaction: In S_N2 reactions, the nucleophile attacks the substrate from the backside, leading to a transition state where the nucleophile and the leaving group are partially bonded to the carbon atom. Steric hindrance plays a significant role in S_N2 reactions. The order of reactivity is: primary (1°) ¿ secondary (2°) ¿ tertiary (3°). The less substituted carbon is more accessible to the nucleophile. The given order is correct.

(B) S$_N$1 Reaction: S$_N$1 reactions proceed through a carbocation intermediate. The rate-determining step is the formation of the carbocation. Therefore, the stability of the carbocation dictates the reactivity. Benzylic carbocations are more stable due to resonance with the aromatic ring. The given order is correct as the benzylic carbocation is more stable.

(C) Electrophilic Substitution: Electrophilic aromatic substitution reactions involve the attack of an electrophile on the aromatic ring. Electron-donating groups (EDG) activate the ring and make it more reactive towards electrophilic substitution by increasing the electron density on the ring. Electron-withdrawing groups (EWG) deactivate the ring. Chlorine (Cl) is weakly deactivating, but nitro (NO_2) is strongly deactivating. Hence, the given order is correct.

(D) Nucleophilic Substitution: Nucleophilic aromatic substitution reactions are favored by electron-withdrawing groups (EWG) at ortho and para positions to the leaving group. EWGs stabilize the negative charge formed in the intermediate. The nitro group is strongly electron-withdrawing compared to bromine and hence activates the ring towards nucleophilic aromatic substitution.

Final Answer: (A), (B), (C) and (D) are all correct.

Quick Tip

For S$_N$2, steric hindrance matters: $1^\circ > 2^\circ > 3^\circ$. For S$_N$1, carbocation stability is key. EDGs activate aromatic rings for electrophilic substitution, while EWGs activate for nucleophilic substitution.

Question 64: Given

(A) $2CO(g) + O_2(g) \rightarrow 2CO_2(g) \qquad \Delta H_1^\circ = -x\,KJ\,mol^{-1}$

(B) $C(graphite) + O_2(g) \rightarrow CO_2(g) \qquad \Delta H_2^\circ = -y\,KJ\,mol^{-1}$

The ΔH° for the reaction

$C(graphite) + \frac{1}{2}O_2(g) \rightarrow CO(g)$ **is**

(1) $\frac{x-2y}{2}$

(2) $\frac{x+2y}{2}$

(3) $\frac{2x-y}{2}$

(4) $2y - x$

Correct Answer: (1) $\frac{x-2y}{2}$

Solution:

We can use Hess's Law to determine the enthalpy change for the target reaction. Hess's Law states that the enthalpy change for a reaction is the same whether it occurs in one step or in a series of steps.

We are given:

1. $2CO(g) + O_2(g) \to 2CO_2(g) \qquad \Delta H_1^\circ = -x$
2. $C(graphite) + O_2(g) \to CO_2(g) \qquad \Delta H_2^\circ = -y$

We want to find the ΔH° for:

$$C(graphite) + \frac{1}{2}O_2(g) \to CO(g)$$

We can manipulate the given equations to obtain the target equation.

Step 1: Reverse equation (1) and divide it by 2:

$$CO_2(g) \to CO(g) + \frac{1}{2}O_2(g) \qquad \Delta H = \frac{x}{2}$$

Step 2: Keep equation (2) as is:

$$C(graphite) + O_2(g) \to CO_2(g) \qquad \Delta H = -y$$

Step 3: Add the modified equations:

$$C(graphite) + O_2(g) + CO_2(g) \to CO_2(g) + CO(g) + \frac{1}{2}O_2(g)$$

Canceling out the CO_2 and simplifying:

$$C(graphite) + \frac{1}{2}O_2(g) \to CO(g)$$

Step 4: Add the enthalpies of the modified equations:

$$\Delta H^\circ = \frac{x}{2} + (-y) = \frac{x}{2} - y = \frac{x-2y}{2}$$

Final Answer: The ΔH° for the reaction is $\frac{x-2y}{2}$.

Quick Tip

Hess's Law is a powerful tool for calculating enthalpy changes. Manipulate the given equations (reverse, multiply/divide) so that when added, they result in the target equation. Remember to adjust the enthalpy changes accordingly.

Question 65: Using column chromatography mixture of two compounds 'A' and 'B' was separated. 'A' eluted first, this indicates 'B' has

(1) high R_f, weaker adsorption

(2) high R_f, stronger adsorption

(3) low R_f, stronger adsorption

(4) low R_f, weaker adsorption

Correct Answer: (3) low R_f, stronger adsorption

Solution:

In column chromatography, compounds are separated based on their differential adsorption to the stationary phase (the adsorbent in the column) and their solubility in the mobile phase (the solvent). Compounds that are less strongly adsorbed to the stationary phase and more soluble in the mobile phase elute faster.

The retention factor (R_f) is a measure of how far a compound travels up the chromatography plate or column relative to the solvent front. It is defined as:

$$R_f = \frac{\text{distance covered by substance from base line}}{\text{total distance covered by solvent from base line}}$$

A higher R_f value indicates that the compound has traveled a greater distance and therefore interacts weakly with the stationary phase (weaker adsorption). Conversely, a lower R_f value indicates that the compound interacted more strongly with the stationary phase (stronger adsorption) and moved a smaller distance.

Since 'A' eluted first, it has a higher R_f value than 'B'. This means that 'B' interacts more strongly with the stationary phase (stronger adsorption) and has a lower R_f value.

Final Answer: 'B' has low R_f, stronger adsorption.

Quick Tip

In chromatography, the first eluted compounds are less polar with higher R_f values while slower moving or retained compounds are typically more polar with lower R_f values indicating stronger interaction with the stationary phase.

Question 66: Lime reacts exothermally with water to give 'A' which has low solubility in water. Aqueous solution of 'A' is often used for the test of CO2. a test in which insoluble B is formed. If B is further reacted with CO2 then soluble compound is formed. 'A' is

(1) Quick lime

(2) Slaked lime

(3) White lime

(4) Lime water

Correct Answer: (2) Slaked lime

Solution:

The given information describes the reaction of lime (calcium oxide, CaO) with water.

Step 1: Reaction of CaO with water (forming 'A'):

$$CaO(s) + H_2O(l) \rightarrow Ca(OH)_2(aq)$$

This reaction forms calcium hydroxide, which is 'A'. The solution of calcium hydroxide is called lime water.

Step 2: Reaction of 'A' with CO_2 (forming 'B'):

$$Ca(OH)_2(aq) + CO_2(g) \rightarrow CaCO_3(s) + H_2O(l)$$

This reaction forms calcium carbonate, which is 'B'. Calcium carbonate is insoluble in water.

Step 3: Reaction of 'B' with more CO_2 (forming soluble compound):

$$CaCO_3(s) + H_2O(l) + CO_2(g) \rightarrow Ca(HCO_3)_2(aq)$$

This reaction forms calcium hydrogencarbonate, which is soluble in water.

Final Answer: Therefore, the substance 'A' is calcium hydroxide (slaked lime).

Quick Tip

Understanding the reactions of lime with water and carbon dioxide is important for many chemistry concepts. The solubility characteristics of the products are key to interpreting the given scenario.

Question 67: Match List I with list II

List I: Industry	List II: Waste Generated
(A) Steel plants	(I) Gypsum
(B) Thermal power plants	(II) Fly ash
(C) Fertilizer industries	(III) Slag
(D) Paper mills	(IV) Bio-degradable wastes

Choose the correct answer from the options given below.

(1) (A)-(III), (B)-(II), (C)-(I), (D)-(IV)

(2) (A)-(IV), (B)-(I), (C)-(II), (D)-(III)

(3) (A)-(II), (B)-(III), (C)-(IV), (D)-(I)

(4) (A)-(III), (B)-(II), (C)-(IV), (D)-(I)

Correct Answer: (4) (A)-(III), (B)-(II), (C)-(I), (D)-(IV)

Solution:

Let's analyze the waste products generated by each industry:

Steel plants (A): Slag (III): Slag is a byproduct of the iron and steel making process. It is a mixture of metal oxides and silicon dioxide.

Thermal power plants (B): Fly ash (II): Fly ash is a fine powder byproduct of burning coal in power plants.

Fertilizer industries (C): Gypsum (I): Gypsum (calcium sulfate dihydrate) is a byproduct in the production of phosphoric acid, a key component of some fertilizers.

Paper mills (D): Bio-degradable wastes (IV): Paper production uses wood pulp and other organic materials, generating biodegradable wastes.

Quick Tip

Remembering the major industrial processes and their byproducts is helpful for this type of question. Steel making produces slag, burning coal generates fly ash, and many fertilizers utilize processes that yield gypsum as a byproduct.

Question 68: Suitable reaction condition for preparation of Methyl phenyl ether is

(1) Benzene, MeBr

(2) $PhO^{\ominus}Na^{\oplus}$, MeOH

(3) Ph-Br, $MeO^{\ominus}Na^{\oplus}$

(4) $PhO^{\ominus}Na^{\oplus}$, MeBr

Correct Answer: (4) $PhO^{\ominus}Na^{\oplus}$, MeBr

Solution:

Methyl phenyl ether (also known as anisole) is prepared using the Williamson ether synthesis. This reaction involves the reaction of an alkoxide ion with a primary alkyl halide (or a less sterically hindered alkyl halide).

The general reaction for Williamson ether synthesis is:

$$R - O^- + R' - X \rightarrow R - O - R' + X^-$$

Where: $R - O^-$ is the alkoxide ion. $R' - X$ is the alkyl halide.

To synthesize methyl phenyl ether (Ph-O-Me), we need:

Phenoxide ion (PhO^-): This is our alkoxide, derived from phenol.

Methyl bromide (MeBr): This is our alkyl halide.

The reaction is:

$$PhO^-Na^+ + MeBr \rightarrow Ph - O - Me + NaBr$$

The phenoxide ion acts as a nucleophile, attacking the methyl carbon of the methyl bromide in an S_N2 reaction. This displaces the bromide ion and forms the ether linkage.

Quick Tip

The Williamson Ether Synthesis is a key reaction for preparing ethers. Remember that it's an S_N2 reaction, so it works best with primary alkyl halides (or those with minimal steric hindrance). If a tertiary alkyl halide is used, an elimination reaction is more likely to occur.

Question 69: The one that does not stabilize 2° and 3° structures of proteins is

(1) H-bonding

(2) –S–S– linkage

(3) van der waals forces

(4) –O–O– linkage

Correct Answer: (4) -O-O- linkage

Solution: The secondary and tertiary structures of proteins are stabilized by various forces:

- Hydrogen bonds
- Disulfide linkages (-S-S-)
- van der Waals forces
- Electrostatic forces of attraction

Peroxide linkages (-O-O-) are not involved in protein structure stabilization.

Quick Tip

Remember the key forces responsible for protein structure: hydrogen bonding, disulfide bridges, van der Waals forces, and electrostatic interactions. Peroxide bonds are not typically found in proteins.

Question 70: The compound which does not exist is

(1) $PbEt_4$

(2) BeH_2

(3) NaO_2

(4) $(NH_4)_2BeF_4$

Correct Answer: (3) NaO_2

Solution: Sodium superoxide (NaO_2) is not a stable compound. While other alkali metal superoxides exist (like KO_2), NaO_2 is highly reactive and tends to disproportionate. The other compounds listed are known and stable.

Quick Tip

Be careful with superoxides. While potassium, rubidium, and cesium form stable superoxides, sodium superoxide is generally not stable under normal conditions.

Question 71: Given below are two reactions, involved in the commercial production of dihydrogen (H_2). The two reactions are carried out at temperature "T_1" and "T_2", respectively

$$C(s) + H_2O(g) \xrightarrow{T_1} CO(g) + H_2(g)$$

$$CO(g) + H_2O(g) \xrightarrow[catalyst]{T_2} CO_2(g) + H_2(g)$$

The temperatures T_1 and T_2 are correctly related as

(1) $T_1 = T_2$

(2) $T_1 < T_2$

(3) $T_1 > T_2$

(4) $T_1 = 100$ K, $T_2 = 1270$ K

Correct Answer: (3) $T_1 > T_2$

Solution: The first reaction, the water-gas reaction, requires high temperatures (around 1270 K) to proceed. The second reaction, the water-gas shift reaction, is typically carried out at lower temperatures (around 673 K) in the presence of a catalyst. Therefore, T_1 ¿ T_2.

Quick Tip

The water-gas reaction needs very high temperatures. The water-gas shift reaction, aided by a catalyst, can operate effectively at lower temperatures.

Question 72: The enthalpy change for the adsorption process and micelle formation respectively are

(1) $\Delta H_{ads} < 0$ and $\Delta H_{mic} < 0$

(2) $\Delta H_{ads} > 0$ and $\Delta H_{mic} < 0$

(3) $\Delta H_{ads} < 0$ and $\Delta H_{mic} > 0$

(4) $\Delta H_{ads} > 0$ and $\Delta H_{mic} > 0$

Correct Answer: (3) $\Delta H_{ads} < 0$ and $\Delta H_{mic} > 0$

Solution: Adsorption: Adsorption is an exothermic process ($\Delta H_{ads} < 0$). Energy is released when molecules adhere to a surface.

Micelle Formation: Micelle formation is an endothermic process ($\Delta H_{mic} > 0$). Energy is required to overcome the repulsion between the hydrophobic tails of the surfactant molecules as they aggregate to form micelles.

Quick Tip

Adsorption is exothermic (releases heat); micelle formation is endothermic (absorbs heat).

Question 73: The pair from the following pairs having both compounds with net non-zero dipole moment is

(1) cis-butene, trans-butene

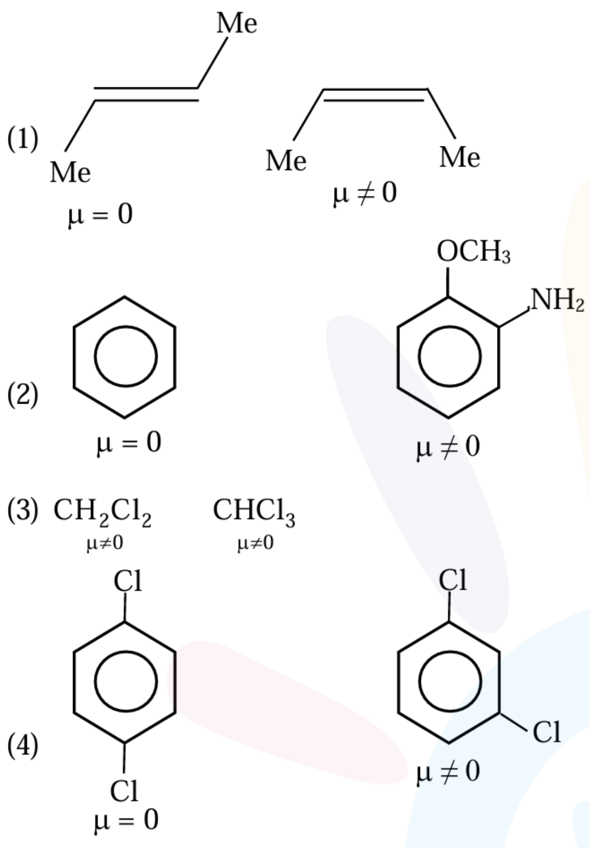
(2) Benzene, anisidine

(3) CH_2Cl_2, $CHCl_3$

(4) 1,4-Dichlorobenzene, 1,3-Dichlorobenzene

Correct Answer: (3) CH_2Cl_2, $CHCl_3$

Solution:



1. **cis-butene, trans-butene:** cis-butene has a net dipole moment due to the asymmetry of the methyl groups on the same side of the double bond. However, trans-butene has a zero dipole moment due to the symmetrical arrangement of the methyl groups on opposite sides, canceling out the individual bond dipoles.

2. **Benzene, anisidine:** Benzene has a zero dipole moment due to its symmetrical structure. Anisidine (methoxybenzene) has a non-zero dipole moment due to the electron-donating nature of the methoxy group (-OCH_3), which creates an uneven distribution of charge density.

3. **CH_2Cl_2, $CHCl_3$:** Both dichloromethane (CH_2Cl_2) and chloroform ($CHCl_3$) have non-zero dipole moments due to the difference in electronegativity between carbon and chlorine, and the asymmetrical arrangement of the atoms.

4. **1,4-Dichlorobenzene, 1,3-Dichlorobenzene:** 1,4-Dichlorobenzene (para-dichlorobenzene) has a zero dipole moment due to its symmetrical structure. The C-Cl bond dipoles cancel each other out. 1,3-Dichlorobenzene (meta-dichlorobenzene) has a non-zero dipole moment because the C-Cl bond dipoles do not cancel each other out completely.

Quick Tip

Molecular symmetry is key to determining dipole moments. Symmetrical molecules often have zero dipole moments because individual bond dipoles cancel each other out. Asymmetrical molecules are more likely to have net dipole moments.

Question 74: Which of the following is used as a stabilizer during the concentration of sulphide ores?

(1) Xanthates

(2) Fatty acids

(3) Pine oils

(4) Cresols

Correct Answer: (4) Cresols

Solution: Cresols are used as stabilizers during froth flotation, a process used for concentrating sulfide ores.

Quick Tip

Cresols are used as froth stabilizers in the concentration of sulfide ores by froth flotation.

Question 75: Which of the following statements are correct?

(A) The M^{3+}/M^{2+} reduction potential for iron is greater than manganese

(B) The higher oxidation states of first row d-block elements get stabilized by oxide ion.

(C) Aqueous solution of Cr^{2+} can liberate hydrogen from dilute acid.

(D) Magnetic moment of V^{2+} is observed between 4.4-5.2 BM.

Choose the correct answer from the options given below:

(1) (C), (D) only

(B) (B), (C) only

(C) (A), (B), (D) only

(D) (A), (B) only

Correct Answer: (2) (B), (C) only

Solution:

(A) **Incorrect:** The M^{3+}/M^{2+} reduction potential for manganese is greater than iron ($E^\circ_{Mn^{3+}/Mn^{2+}}$ = +1.57 V, $E^\circ_{Fe^{3+}/Fe^{2+}}$ = +0.77 V).

(B) **Correct:** Higher oxidation states of first-row d-block elements are stabilized by oxide ions (O^{2-}) due to the formation of strong metal-oxygen bonds.

(C) **Correct:** Cr^{2+} in aqueous solution can liberate hydrogen from dilute acid:

$$Cr^{2+} + H^+ \rightarrow Cr^{3+} + \frac{1}{2}H_2$$

The $E^\circ_{Cr^{3+}/Cr^{2+}}$ = -0.26 V indicates Cr^{2+} can reduce H^+ to H_2.

(D) **Incorrect:** V^{2+} has 3 unpaired electrons, which corresponds to a magnetic moment of approximately 3.87 BM, not within the 4.4-5.2 BM range.

Quick Tip

Review concepts of reduction potentials, stability of oxidation states in d-block elements, and the relationship between unpaired electrons and magnetic moment.

Question 76: Given below are two statements:

Statement I: Aqueous solution of $K_2Cr_2O_7$ is preferred as a primary standard in volumetric analysis over $Na_2Cr_2O_7$ aqueous solution.

Statement II: $K_2Cr_2O_7$ has a higher solubility in water than $Na_2Cr_2O_7$.

In the light of the above statements, choose the correct answer from the options given below:

(1) Statement I is false but Statement II is true

(2) Statement I is true but Statement II is false

(3) Both Statement I and Statement II are true

(4) Both Statement I and Statement II are false

Correct Answer: (2) Statement I is true but Statement II is false

Solution:

Step 1: Understanding the Properties of $K_2Cr_2O_7$ and $Na_2Cr_2O_7$.

- $K_2Cr_2O_7$ is commonly used as a primary standard because it is more stable in aqueous solution compared to $Na_2Cr_2O_7$.

- $Na_2Cr_2O_7$ tends to undergo changes in concentration when dissolved in water, making it less suitable as a primary standard.

Step 2: Solubility Comparison.

- $K_2Cr_2O_7$ is less soluble in water than $Na_2Cr_2O_7$.

- This makes Statement II false.

Final Answer: Statement I is true but Statement II is false.

Quick Tip

$K_2Cr_2O_7$ is preferred as a primary standard in volumetric analysis due to its stability and consistency in aqueous solutions, even though it has lower solubility compared to $Na_2Cr_2O_7$.

Question 77: The octahedral diamagnetic low spin complex among the following is

(1) $[CoF_6]^{3-}$

(2) $[CoCl_6]^{3-}$

(3) $[Co(NH_3)_6]^{3+}$

(4) $[NiCl_4]^{2-}$

Correct Answer: (3) $[Co(NH_3)_6]^{3+}$

Solution:

Step 1: Understanding Coordination Complexes.

- Coordination number (CN) for $[Co(NH_3)_6]^{3+}$ is 6, indicating an octahedral geometry.

- NH_3 is a strong field ligand (SFL) as per the spectrochemical series.

Step 2: Electron Configuration of Co^{3+}.

- Atomic configuration of Co: [Ar] $3d^7$ $4s^2$

- Co^{3+}: [Ar] $3d^6$

Step 3: Effect of Strong Field Ligand.

- NH_3 causes pairing of electrons in the 3d orbitals due to its strong field nature.

- The electronic configuration becomes t_{2g}^6 e_g^0, making the complex diamagnetic (no unpaired electrons).

- The low spin configuration confirms that $[Co(NH_3)_6]^{3+}$ is an octahedral diamagnetic low spin complex.

Final Answer: $[Co(NH_3)_6]^{3+}$ is the correct option.

Quick Tip

In octahedral complexes, the ligand field strength determines whether the complex will be high spin or low spin. Strong field ligands like NH_3 result in low spin configurations, while weak field ligands like F^- and Cl^- favor high spin configurations.

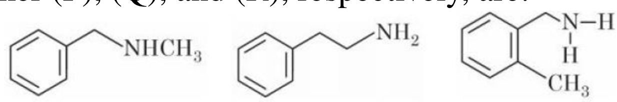
Question 78: Isomeric amines with molecular formula $C_8H_{11}N$ given the following tests:

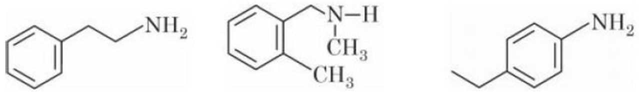
Isomer (P): Can be prepared by Gabriel phthalimide synthesis.

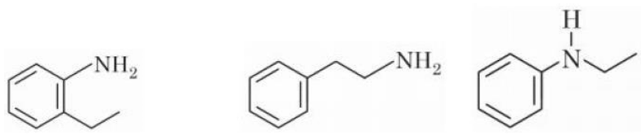
Isomer (Q): Reacts with Hinsberg's reagent to give a solid insoluble in NaOH.

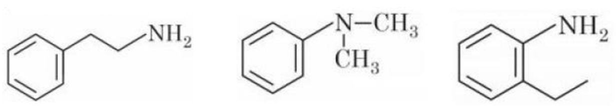
Isomer (R): Reacts with HONO followed by β-naphthol in NaOH to give a red dye.

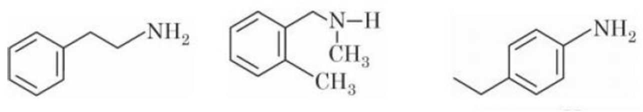
Isomer (P), (Q), and (R), respectively, are:

(1) 

(2) 

(3) 

(4) 

Correct Answer: (2) 

Solution:

Step 1: Analyzing Isomer (P).

- Gabriel phthalimide synthesis is used to prepare primary amines.
- Therefore, (P) must be a primary amine.

Step 2: Analyzing Isomer (Q).

- Hinsberg's reagent reacts with secondary amines to form a solid product that is insoluble in NaOH.
- Thus, (Q) must be a secondary amine.

Step 3: Analyzing Isomer (R).

- Reaction with HONO followed by β-naphthol in NaOH produces a red dye, which is a characteristic test for aromatic primary amines.
- Hence, (R) must be an aromatic primary amine.

Final Answer: (2)

Quick Tip

The Gabriel phthalimide synthesis is specific to primary amines. Hinsberg's reagent helps distinguish between primary, secondary, and tertiary amines based on solubility behavior. The reaction with HONO and β-naphthol confirms the presence of an aromatic primary amine.

Question 79: The number of molecules and moles in 2.8375 litres of O_2 at STP are respectively

(1) 7.527×10^{22} and 0.125 mol

(2) 1.505×10^{23} and 0.250 mol

(3) 7.527×10^{23} and 0.125 mol

(4) 7.527×10^{22} and 0.250 mol

Correct Answer: (1) 7.527×10^{22} and 0.125 mol

Solution:

Step 1: Calculate the number of moles of O_2.

- At STP, 1 mole of a gas occupies 22.7 litres.

- Given volume of O_2: 2.8375 litres.

- Number of moles of O_2 (n_{O_2}) is calculated using:

$$n_{O_2} = \frac{\text{Volume of O}_2}{22.7} = \frac{2.8375}{22.7} = 0.125\,\text{moles}$$

Step 2: Calculate the number of molecules of O_2.

- Number of molecules of O_2 is given by:

$$\text{Molecules of O}_2 = \text{moles} \times N_A$$

Where N_A is Avogadro's number (6.022×10^{23}).

$$\text{Molecules of O}_2 = 0.125 \times 6.022 \times 10^{23} = 7.527 \times 10^{22}\,\text{molecules}$$

Final Answer: The number of molecules is 7.527×10^{22}, and the number of moles is 0.125 mol.

Quick Tip

At STP, use the molar volume of 22.7 litres to calculate the moles of a gas. To find the number of molecules, multiply the moles by Avogadro's number (6.022×10^{23}).

Question 80: Match List I with List II

List I	Polymer	List II
(A)	Nylon-2-Nylon-6	(I) Thermosetting polymer
(B)	Buna-N	(II) Biodegradable polymer
(C)	Urea-Formaldehyde resin	(III) Synthetic rubber
(D)	Dacron	(IV) Polyester

Choose the correct answer from the options given below:

(1) (A)-(IV), (B)-(III), (C)-(I), (D)-(II)

(2) (A)-(II), (B)-(I), (C)-(IV), (D)-(III)

(3) (A)-(IV), (B)-(I), (C)-(III), (D)-(II)

(4) (A)-(II), (B)-(III), (C)-(I), (D)-(IV)

Correct Answer: (4) (A)-(II), (B)-(III), (C)-(I), (D)-(IV)

Solution:

Step 1: Analyze the polymers and their types/classes. - Nylon-2-Nylon-6: A biodegradable polymer. Hence, (A) matches with (II).

- Buna-N: A synthetic rubber. Hence, (B) matches with (III).
- Urea-Formaldehyde resin: A thermosetting polymer. Hence, (C) matches with (I).
- Dacron: A polyester. Hence, (D) matches with (IV).

Final Answer: The correct match is (4) (A)-(II), (B)-(III), (C)-(I), (D)-(IV).

Quick Tip

Understanding the properties and applications of polymers helps in correctly identifying their types. For example, biodegradable polymers like Nylon-2-Nylon-6 decompose naturally, while synthetic rubbers like Buna-N are used for their elasticity.

Section B

Question 81: If the degree of dissociation of an aqueous solution of weak monobasic acid is determined to be 0.3, then the observed freezing point will be ________ % higher than the expected/theoretical freezing point. (Nearest integer)

Correct Answer: 30%

Solution:

Calculation of Percentage Increase in Freezing Point Depression

For a mono basic acid:

$$n = 2$$

The van 't Hoff factor (i) is given by:

$$i = 1 + (n - 1)\alpha$$

Substitute $n = 2$ and $\alpha = 0.3$:

$$i = 1 + (2 - 1) \cdot 0.3$$

$$i = 1.3$$

The percentage increase in freezing point depression is calculated as:

$$\%\text{increase} = \frac{(\Delta T_f)_{\text{obs}} - (\Delta T_f)_{\text{cal}}}{(\Delta T_f)_{\text{cal}}} \times 100$$

Using the formula for freezing point depression:

$$\Delta T_f = \frac{K_f \cdot i \cdot m - K_f \cdot m}{K_f \cdot m} \times 100$$

Simplify:

$$\%\text{increase} = \frac{i - 1}{1} \times 100$$

Substitute $i = 1.3$:

$$\%\text{increase} = \frac{1.3 - 1}{1} \times 100 = 30\%$$

Final Answer: The percentage increase in freezing point depression is 30%.

Quick Tip

The van't Hoff factor (i) accounts for dissociation in solutions. For weak electrolytes, use the formula $i = 1 + (n - 1)\alpha$ to include the degree of dissociation (α) when calculating colligative properties.

Question 82: In the following reactions, the total number of oxygen atoms in X and Y is _______

$$Na_2O + H_2O \rightarrow 2X$$

$$Cl_2O_7 + H_2O \rightarrow 2Y$$

Correct Answer: 5

Solution:

Reactions of Oxides with Water

The reactions of sodium oxide and dichlorine heptoxide with water are as follows:

$$Na_2O + H_2O \rightarrow 2NaOH$$

$$Cl_2O_7 + H_2O \rightarrow 2HClO_4$$

Adding the coefficients from both equations:

$$1 + 4 = 5$$

Quick Tip

In reactions involving oxides and water, carefully balance the equations and count the atoms to ensure accuracy in stoichiometry.

Question 83: The sum of lone pairs present on the central atom of the interhalogens IF_5 and IF_7 is ______

Correct Answer: 1

Solution:

Step 1: Lone pair calculation for IF_5.

- The central atom Iodine (I) in IF_5 has 7 valence electrons.

- Five of these electrons form bonds with Fluorine (F) atoms, leaving 2 electrons as 1 lone pair.

Step 2: Lone pair calculation for IF_7.

- In IF_7, Iodine (I) forms 7 bonds with Fluorine (F) atoms.

- There are no remaining electrons, so there are 0 lone pairs.

Step 3: Add the lone pairs.

- Lone pairs on IF_5: 1

- Lone pairs on IF_7: 0

- Total = 1 + 0 = 1

Final Answer: The sum of lone pairs on the central atom is 1.

Quick Tip

To determine the number of lone pairs, calculate the total valence electrons, subtract those used for bonding, and divide the remaining electrons by 2.

Question 84: The number of bent-shaped molecule(s) from the following is _______

$$N_3^-, NO_2^-, I_3^-, O_3, SO_2$$

Correct Answer: 3

Solution:

Step 1: Analyze the molecular shapes.

- N_3^-: Linear (central atom forms two double bonds, no lone pairs).

- NO_2^-: Bent (due to lone pairs on nitrogen).

- I_3^-: Linear (central iodine atom has three lone pairs).

- O_3: Bent (due to lone pairs on the central oxygen atom).

- SO_2: Bent (due to lone pairs on the central sulfur atom).

Step 2: Count the bent molecules.

- The bent molecules are NO_2^-, O_3, and SO_2.

- Total = 3.

Final Answer: The number of bent-shaped molecules is 3.

Quick Tip

Bent molecular shapes are caused by lone pairs of electrons on the central atom that repel bonding pairs, altering the bond angles.

Question 85: The number of correct statement(s) involving equilibria in physical from the following is _______

(1) Equilibrium is possible only in a closed system at a given temperature.

(2) Both the opposing processes occur at the same rate.

(3) When equilibrium is attained at a given temperature, the value of all its parameters.

(4) For dissolution of solids in liquids, the solubility is constant at a given temperature.

Correct Answer: 3

Solution:

Step 1: Evaluate each statement.

- (A) Equilibrium is possible only in a closed system at a given temperature.
- This is correct as equilibrium requires a closed system to prevent loss of matter.
- (B) Both the opposing processes occur at the same rate.
- This is correct as equilibrium is defined by the equality of forward and backward reaction rates.
- (C) When equilibrium is attained, the value of all parameters becomes constant.
- This is incorrect because the equilibrium values of parameters remain constant but may not be equal.
- (D) For dissolution of solids in liquids, the solubility is constant at a given temperature.
- This is correct as solubility depends on temperature.

Step 2: Count the correct statements.

- Correct statements are (A), (B), and (D).
- Total = 3.

Final Answer: The number of correct statements is 3.

Quick Tip

Equilibrium conditions require a closed system, constant temperature, and equality of rates for forward and reverse processes.

Question 86: At constant temperature, a gas is at a pressure of 940.3 mm Hg. The pressure at which its volume decreases by 40% is _______ mm Hg. (Nearest integer)

Correct Answer: 1567

Solution:

Step 1: Write the given data.

- Initial pressure, $P_{\text{initial}} = 940.3\,\text{mm Hg}$.
- Initial volume, $V_{\text{initial}} = 100\,\text{units (assume)}$.
- Final volume, $V_{\text{final}} = 100 - 40\% = 60\,\text{units}$.
- Final pressure, $P_{\text{final}} = ?$.

Step 2: Apply Boyle's law.

Boyle's law states that:

$$P_{\text{initial}} V_{\text{initial}} = P_{\text{final}} V_{\text{final}}$$

Substitute the values:

$$940.3 \times 100 = P_{\text{final}} \times 60$$

Solve for P_{final}:

$$P_{\text{final}} = \frac{940.3 \times 100}{60} = 1567.16\,\text{mm Hg}$$

Step 3: Round to the nearest integer.

$$P_{\text{final}} = 1567\,\text{mm Hg}.$$

Final Answer: The pressure at which the volume decreases by 40% is 1567 mm Hg.

Quick Tip

Boyle's law states that for a given amount of gas at constant temperature, the product of pressure and volume is constant. Always ensure units are consistent when applying the formula.

Question 87: $\mathbf{FeO_4^{2-}} \xrightarrow{+2.2\,V} \mathbf{Fe^{3+}} \xrightarrow{+0.70\,V} \mathbf{Fe^{2+}} \xrightarrow{-0.45\,V} \mathbf{Fe^{\circ}}$

$E^{\circ}_{\mathbf{FeO_4^{2-}/Fe^{2+}}}$ **is** $x \times 10^{-3}$ **V. The value of** x **is** ________

Correct Answer: 1825

Solution:

Step 1: Combine the half-cell reactions.

- The reduction steps are:

$$FeO_4^{2-} + 3e^- \rightarrow Fe^{3+},\ \Delta G_1$$

$$Fe^{3+} + e^- \rightarrow Fe^{2+},\ \Delta G_2$$

$$FeO_4^{2-} + 4e^- \rightarrow Fe^{2+},\ \Delta G_3$$

Step 2: Use Gibbs free energy relationships.

- Gibbs free energy for the combined reaction:

$$\Delta G_3 = \Delta G_1 + \Delta G_2$$

Substituting $\Delta G = -nFE^{\circ}$:

$$-4FE_3^{\circ} = -3F(2.2) + (-1F)(0.7)$$

Simplify:

$$4E_3^{\circ} = 6.6 + 0.7 = 7.3$$

Step 3: Calculate E_3°.

$$E_3^{\circ} = \frac{7.3}{4} = 1.825\,\text{V}$$

$$E_3^{\circ} = 1.825 \times 10^{-3}\,\text{V}$$

Final Answer: The value of x is 1825.

Quick Tip

To calculate the overall standard reduction potential for a series of redox reactions, use the relationship $\Delta G = -nFE^{\circ}$ and combine the contributions from all steps.

Question 88: A molecule undergoes two independent first-order reactions whose respective half-lives are 12 min and 3 min. If both the reactions are occurring, then the time taken for the 50% consumption of the reactant is ______ min. (Nearest integer)

Correct Answer: 2

Solution:

Step 1: Relationship between rate constants and effective half-life.

For independent first-order reactions:

$$k_{\text{eff}} = k_1 + k_2$$

The effective half-life is given by:

$$\frac{\ln 2}{t_{\text{eff}}} = \frac{\ln 2}{t_1} + \frac{\ln 2}{t_2}$$

Simplify:

$$\frac{1}{t_{\text{eff}}} = \frac{1}{t_1} + \frac{1}{t_2}$$

Step 2: Substitute the given half-lives.

- $t_1 = 12\,\text{min}$, $t_2 = 3\,\text{min}$.

$$\frac{1}{t_{\text{eff}}} = \frac{1}{12} + \frac{1}{3}$$

$$\frac{1}{t_{\text{eff}}} = \frac{1}{12} + \frac{4}{12} = \frac{5}{12}$$

Step 3: Calculate t_{eff}.

$$t_{\text{eff}} = \frac{12}{5} = 2.4\,\text{min}$$

Round to the nearest integer:

$$t_{\text{eff}} = 2\,\text{min}$$

Final Answer: The time taken for 50% consumption of the reactant is 2 minutes.

Quick Tip

For multiple independent first-order reactions, the effective half-life can be calculated using the formula:

$$\frac{1}{t_{\text{eff}}} = \frac{1}{t_1} + \frac{1}{t_2} + \dots$$

Always combine the reciprocal contributions of individual half-lives to find the effective rate.

Question 89: The number of incorrect statement(s) about the black body from the following is

____

(1) Emit or absorb energy in the form of electromagnetic radiation.

(2) Frequency distribution of the emitted radiation depends on temperature.

(3) At a given temperature, intensity vs frequency curve passes through a maximum value.

(4) The maximum of the intensity vs frequency curve is at a higher frequency at higher temperature compared to that at lower temperature.

Correct Answer: 0

Solution:

Step 1: Analyze the given statements.

- (1) A black body emits or absorbs energy in the form of electromagnetic radiation. This is a correct statement based on black body radiation theory.

- (2) The frequency distribution of the emitted radiation depends on temperature. This is a correct statement, as shown by Planck's law.

- (3) At a given temperature, the intensity vs frequency curve passes through a maximum value. This is correct and is a key observation of black body radiation.

- (4) The maximum of the intensity vs frequency curve shifts to a higher frequency at higher temperatures, as per Wien's displacement law. This is also correct.

Step 2: Count the incorrect statements. - All the given statements are correct.

- Number of incorrect statements = 0.

Final Answer: The number of incorrect statements is 0.

Quick Tip

The properties of black body radiation are governed by Planck's law, Wien's displacement law, and Stefan-Boltzmann law. Remember, the peak frequency increases with temperature, and the intensity curve always passes through a maximum.

Question 90: In potassium ferrocyanide, there are ______ pairs of electrons in the t_{2g} set of orbitals.

Correct Answer: 3

Solution:

Step 1: Analyze the electronic configuration of Fe^{2+}.

- Potassium ferrocyanide is represented as K_4[Fe(CN)$_6$].
- In this complex, Fe is in the +2 oxidation state: Fe^{2+}.
- The electronic configuration of Fe^{2+} is:

$$\text{Fe}^{2+} = [\text{Ar}]\,3d^6$$

Step 2: Effect of strong field ligand (CN$^-$).

- CN$^-$ is a strong field ligand as per the spectrochemical series.
- It causes the pairing of electrons in the d-orbitals.
- The t_{2g} set of orbitals (lower energy) gets fully filled with 6 electrons, forming 3 electron pairs.

Step 3: Diagrammatic representation.

$$t_{2g}^6\,e_g^0$$

The t_{2g} set contains 6 electrons, which form 3 pairs.

Final Answer: The number of electron pairs in the t_{2g} set is 3.

Quick Tip

Strong field ligands like CN$^-$ cause maximum pairing of electrons in the lower-energy t_{2g} orbitals, resulting in low-spin configurations in octahedral complexes.
