JEE Main 2023 April 12 Shift 1 Question Paper with Solutions

Time Allowed : 3 Hours	**Maximum Marks :** 300	**Total Questions :** 90

General Instructions

Read the following instructions very carefully and strictly follow them:

1. The test is of 3 hours duration.
2. The question paper consists of 90 questions, out of which 75 are to attempted. The maximum marks are 300.
3. There are three parts in the question paper consisting of Physics, Chemistry and Mathematics having 30 questions in each part of equal weightage.
4. Each part (subject) has two sections.

   (i) Section-A: This section contains 20 multiple choice questions which have only one correct answer. Each question carries 4 marks for correct answer and –1 mark for wrong answer.

   (ii) Section-B: This section contains 10 questions. In Section-B, attempt any five questions out of 10. The answer to each of the questions is a numerical value. Each question carries 4 marks for correct answer and –1 mark for wrong answer. For Section-B, the answer should be rounded off to the nearest integer

PHYSICS

Section-A

31. An ice cube has a bubble inside. When viewed from one side the apparent distance of the bubble is 12 cm. When viewed from the opposite side, the apparent distance of the bubble is observed as 4 cm. If the side of the ice cube is 24 cm, the refractive index of the ice cube is:

(1) $\frac{4}{3}$
(2) $\frac{3}{2}$
(3) $\frac{2}{3}$
(4) $\frac{6}{5}$

Correct Answer: (2) $\frac{3}{2}$

Solution:

Given:

- $d_1' = 12$ cm
- $d_2' = 4$ cm
- $t = 24$ cm

Using the refraction formula:

$$\mu = \frac{\text{real depth}}{\text{apparent depth}}$$

From the first side:

$$\mu = \frac{d_1}{d_1'} \implies \mu = \frac{d_1}{12} \implies d_1 = 12\mu \quad \cdots (1)$$

From the opposite side:

$$\mu = \frac{d_2}{d_2'} \implies \mu = \frac{d_2}{4} \implies d_2 = 4\mu \quad \cdots (2)$$

The sum of the real distances is equal to the side of the ice cube:

$$d_1 + d_2 = t \implies d_1 + d_2 = 24 \quad \cdots (3)$$

Substituting equations (1) and (2) into (3):

$$12\mu + 4\mu = 24$$

$$16\mu = 24$$

$$\mu = \frac{24}{16} = \frac{3}{2}$$

Therefore, the refractive index of the ice cube is $\frac{3}{2}$.

The correct answer is (2) $\frac{3}{2}$.

Quick Tip

For solving problems related to refractive index and apparent distance, remember that the speed of light in a medium is related to the refractive index, and use the appropriate formulas to solve for unknowns.

---

32. Two satellites A and B move round the earth in the same orbit. The mass of A is twice the mass of B. The quantity which is same for the two satellites will be:

(1) Potential energy

(2) Total energy

(3) Kinetic energy

(4) Speed

Correct Answer: (4) Speed

Solution: Step 1: Understanding orbital motion of satellites: The total energy E and kinetic energy $K.E.$ of a satellite moving in a circular orbit around the Earth depend only on the radius of the orbit and the gravitational constant, not on the mass of the satellite.

Step 2: Speed of the satellite: The speed of the satellite v in orbit is given by:

$$v = \sqrt{\frac{GM_p}{R}}$$

Since the radius of orbit and gravitational constant are the same for both satellites, their speed will be the same.

Quick Tip

In orbital motion problems, remember that speed is independent of the mass of the satellite, while potential energy and kinetic energy depend on the mass.

---

33. The amplitude of 15 sin (1000 πt) is modulated by 10 sin ($4\pi t$) signal. The amplitude modulated signal contains frequencies of:

(1) 500 Hz.

(2) 2 Hz.

(3) 250 Hz.

(4) 498 Hz.

(5) 502 Hz.

(1) and (3) only (1) and (4) only (1) and (2) only (1), (4) and (5) only

Correct Answer: (4) (1), (4) and (5) only

Solution: Carrier signal: $15\sin(1000\pi t)$Modulating signal: $10\sin(4\pi t)$

The general form of a sinusoidal wave is A

$\sin(2\pi ft)$, where A is the amplitude, f is the frequency, and t is time.

1. Carrier signal frequency (f_c) :$2\pi f_c = 1000\pi f_c = \frac{1000\pi}{2\pi} = 500$ Hz

2. Modulating signal frequency (f_m) :$2\pi f_m = 4\pi f_m = \frac{4\pi}{2\pi} = 2$ Hz

In amplitude modulation, the modulated signal contains the carrier frequency and two sideband frequenc

- Carrier frequency $(f_c) = 500$ Hz
- Lower sideband frequency $(f_c - f_m) = 500$ Hz $-$ 2 Hz $= 498$ Hz
- Upper sideband frequency $(f_c + f_m) = 500$ Hz $+$ 2 Hz $= 502$ Hz

The frequencies present in the amplitude modulated signal are:

- 500 Hz (1)
- 498 Hz (4)
- 502 Hz (5)

Therefore, the correct answer is (4) (1), (4) and (5) only.

Quick Tip

In amplitude modulation, the frequencies produced are the carrier frequency f_c plus and minus the modulating frequency f_m. Thus, always consider both sum and difference of frequencies.

34. In an n-p-n common emitter (CE) transistor, the collector current changes from 5 mA to 16 mA for the change in base current from 100 A and 200 A, respectively. The current gain of the transistor is:

(1) 110

(2) 0.9

(3) 210

(4) 9

Correct Answer: (1) 110

Solution: The current gain β for a common emitter transistor is given by the formula:

$$\beta = \frac{\Delta I_C}{\Delta I_B}$$

Where:

$\Delta I_C = 16 \text{ mA} - 5 \text{ mA} = 11 \text{ mA}$

$\Delta I_B = 200\mu A - 100\mu A = 100\mu A$

Now substitute the values:

$$\beta = \frac{11 \text{ mA}}{100\mu A} = \frac{11 \times 10^{-3}}{100 \times 10^{-6}} = 110$$

Thus, the current gain is $\boxed{110}$.

Quick Tip

The current gain β in a common emitter transistor is the ratio of change in collector current to change in base current. A higher β indicates better amplification.

35. If the r.m.s. speed of chlorine molecules is 490 m/s at 27°C, the r.m.s. speed of argon molecules at the same temperature will be (Atomic mass of argon = 39.9u, molecular mass of chlorine = 70.9u):

(1) 751.7 m/s

(2) 451.7 m/s

(3) 651.7 m/s

(4) 551.7 m/s

Correct Answer: (3) 651.7 m/s

Solution:

The r.m.s. speed of gas molecules is related to the temperature and molecular mass by the formula:

$$V_{\text{rms}} = \sqrt{\frac{3RT}{M}}$$

Where:

R is the gas constant.

T is the temperature.

M is the molar mass of the gas.

Since both gases are at the same temperature, we can use the ratio of their r.m.s. speeds:

$$\frac{\nu_{\text{Ar}}}{\nu_{\text{Cl}}} = \sqrt{\frac{M_{\text{Cl}}}{M_{\text{Ar}}}}$$

Substitute the given values:

$M_{\text{Cl}} = 70.9\,\text{u}$

$M_{\text{Ar}} = 39.9\,\text{u}$

$\nu_{\text{Cl}} = 490\,\text{m/s}$

$$\frac{\nu_{\text{Ar}}}{490} = \sqrt{\frac{70.9}{39.9}} = 1.33$$

Thus,

$$\nu_{\text{Ar}} = 1.33 \times 490 = 651.7\,\text{m/s}$$

Thus, the r.m.s. speed of argon molecules is $\boxed{651.7}$ m/s.

Quick Tip

The r.m.s. speed of gas molecules is inversely proportional to the square root of their molar mass. Use this relation to compare the speeds of different gases at the same temperature.

36. A proton and an α-particle are accelerated from rest by 2V and 4V potentials, respectively. The ratio of their de-Broglie wavelength is:

(1) 4:1.

(2) 2:1.

(3) 8:1.

(4) 16:1.

Correct Answer: (1) 4:1

Solution: Given:

- $V_p = 2V$
- $V_\alpha = 4V$
- $q_p = e$
- $q_\alpha = 2e$
- $m_p = m_p$
- $m_\alpha = 4m_p$

$\frac{\lambda_p}{\lambda_\alpha} = \sqrt{\frac{4m_p \cdot 2e \cdot 4V}{m_p \cdot e \cdot 2V}}$ $\frac{\lambda_p}{\lambda_\alpha} = \sqrt{\frac{32m_peV}{2m_peV}}$ $\frac{\lambda_p}{\lambda_\alpha} = \sqrt{16}$ $\frac{\lambda_p}{\lambda_\alpha} = 4$

Therefore, the ratio is 4:1.

The correct answer is (1).

Quick Tip

For de-Broglie wavelength problems, remember that the wavelength is inversely proportional to the square root of the kinetic energy. The particle with higher energy has a smaller wavelength.

---

37. Given below are two statements:

Statement I: The diamagnetic property depends on temperature.

Statement II: The included magnetic dipole moment in a diamagnetic sample is always opposite to the magnetizing field.

In light of the given statement, choose the correct answer from the options below:

(1) Statement I is incorrect but Statement II is true.

(2) Both Statement I and Statement II are true.

(3) Both Statement I and Statement II are false.

(4) Statement I is correct but Statement II is false.

Correct Answer: (1) Statement I is incorrect but Statement II is true.

Solution:

Step 1: Analyzing the given statements

Statement I: Diamagnetic properties are independent of temperature. Therefore, Statement I is incorrect.

Statement II: In diamagnetic materials, the induced magnetic dipole moment is opposite to the external magnetic field, which makes Statement II true.

Thus, the correct answer is that Statement I is incorrect, and Statement II is true.

Quick Tip

In diamagnetic materials, the magnetic dipoles always align opposite to the external magnetic field, and the property is independent of temperature.

---

38. A wire of resistance 160 Ω is melted and drawn into a wire of one-fourth of its length. The new resistance of the wire will be:

(1) 10 Ω

(2) 640 Ω

(3) 40 Ω

(4) 16 Ω

Correct Answer: (1) 10 Ω

Solution: Given that the volume of the wire remains constant, we use the relation:

$$A_1 L_1 = A_2 L_2$$

Where: - A_1 and L_1 are the area and length of the original wire, - A_2 and L_2 are the area and length of the new wire.

The volume of the wire is constant, so the area of cross-section A_2 and the length L_2 change according to the new dimensions.

Since the length of the new wire is one-fourth of the original, we have:

$$A_1L_1 = A_2L_2 \quad \Rightarrow \quad A_2 = 4A_1$$

For resistance R, we know:

$$R = \rho\frac{L}{A}$$

Thus, for the new wire:

$$R_2 = \rho\frac{L_2}{A_2} = \rho\frac{L}{4A} = \frac{1}{16}R_1$$

Substituting $R_1 = 160\Omega$:

$$R_2 = \frac{1}{16} \times 160 = 10\Omega$$

Thus, the new resistance is $\boxed{10}\Omega$.

Quick Tip

For problems involving changes in dimensions of a wire, remember that resistance is inversely proportional to the cross-sectional area and directly proportional to the length.

---

39. Match List I with List II:

List I		List II	
A.	Spring constant	I.	(T^{-1})
B.	Angular speed	II.	(MT^{-2})
C.	Angular momentum	III.	(ML^2)
D.	Moment of Inertia	IV.	(ML^2T^{-1})

Choose the correct answer from the options given below:

(1) A-II, B-I, C-IV, D-III

(2) A-IV, B-I, C-III, D-II

(3) A-II, B-III, C-I, D-IV

(4) A-I, B-III, C-II, D-IV

Correct Answer: (1) A-II, B-I, C-IV, D-III

Solution:

Spring constant: The spring constant K is given by:

$$[K] = \frac{[F]}{[x]} = \frac{MLT^{-2}}{L} = MT^{-2}$$

Thus, spring constant has units of $\boxed{MT^{-2}}$, which corresponds to List II.

Angular speed: The angular speed ω has dimensions:

$$[\omega] = \frac{[\theta]}{[t]} = \frac{1}{T} = T^{-1}$$

Thus, angular speed has dimensions $\boxed{T^{-1}}$, corresponding to List I.

Angular momentum: Angular momentum L is given by:

$$[L] = [M][L]^2 = ML^2$$

Thus, angular momentum has dimensions $\boxed{ML^2}$, corresponding to List III.

Moment of inertia: The moment of inertia I is:

$$[I] = [M][L]^2 = ML^2$$

Thus, moment of inertia has dimensions $\boxed{ML^2}$, corresponding to List IV.

Thus, the correct match is $A - II, B - I, C - IV, D - III$.

Quick Tip

When dealing with physical quantities, always express their dimensions and match them with the correct units. This helps in understanding the relationships between different concepts.

---

40. Three forces $F_1 = 10\,\mathbf{N}$, $F_2 = 8\,\mathbf{N}$, $F_3 = 6\,\mathbf{N}$ are acting on a particle of mass 5 kg. The forces F_2 and F_3 are applied perpendicular so that particle remains at rest. If the force F_1 is removed, then the acceleration of the particle is:

(1) 2 ms^{-2}.

(2) 0.5 ms^{-2}.

(3) 4.8 ms^{-2}.

(4) 7 ms^{-2}.

Correct Answer: (1) 2 ms^{-2}

Solution:

Analyzing the situation

The forces F_2 and F_3 are applied perpendicular to each other. The resultant of F_2 and F_3 can be found using the Pythagorean theorem:

$$F_{\text{res}} = \sqrt{F_2^2 + F_3^2} = \sqrt{8^2 + 6^2} = \sqrt{64 + 36} = \sqrt{100} = 10\,\text{N}.$$

The force $F_1 = 10\,\text{N}$ is removed, and the net force acting on the particle is the resultant force of F_2 and F_3.

The acceleration a is given by:

$$a = \frac{F_{\text{res}}}{m} = \frac{10}{5} = 2\,\text{ms}^{-2}.$$

Thus, the acceleration of the particle is $2\,\text{ms}^{-2}$.

Quick Tip

When forces are applied perpendicular to each other, the resultant force is found using the Pythagorean theorem. Then, use $F = ma$ to calculate acceleration.

---

41. A body cools from 80°C to 60°C in 5 minutes. The temperature of the surrounding is 20°C. The time it takes to cool from 60°C to 40°C is:

(1) 500 s.

(2) $\frac{25}{3}$ s.

(3) 450 s.

(4) 420 s.

Correct Answer: (1) 500 s

Solution: We will use Newton's Law of Cooling.

$$\frac{dT}{dt} = -k(T - T_s)$$

where:

- T is the temperature of the object at time t
- T_s is the temperature of the surroundings
- k is a positive constant

Cooling from $80°C$ **to** $60°C$**:**

- Initial temperature $T_1 = 80°C$
- Final temperature $T_2 = 60°C$
- Time $t_1 = 5$ minutes $= 300$ seconds
- Surrounding temperature $T_s = 20°C$

Using the average temperature:

$$\frac{T_1 + T_2}{2} = \frac{80 + 60}{2} = 70°C$$

Applying Newton's Law:

$$\frac{T_2 - T_1}{t_1} = -k\left(\frac{T_1 + T_2}{2} - T_s\right)$$

$$\frac{60 - 80}{300} = -k(70 - 20)$$

$$\frac{-20}{300} = -k(50)$$

$$k = \frac{20}{300 \cdot 50} = \frac{2}{1500} = \frac{1}{750}$$

Cooling from $60°C$ **to** $40°C$**:**

- Initial temperature $T_1 = 60°C$
- Final temperature $T_2 = 40°C$
- Time t_2 (to be found)
- Surrounding temperature $T_s = 20°C$

Using the average temperature:

$$\frac{60 + 40}{2} = 50°C$$

Applying Newton's Law:

$$\frac{40-60}{t_2} = -k(50-20)$$

$$\frac{-20}{t_2} = -k(30)$$

$$t_2 = \frac{20}{k \cdot 30} = \frac{20}{\frac{1}{750} \cdot 30} = \frac{20 \cdot 750}{30} = 20 \cdot 25 = 500 \text{ seconds}$$

Therefore, the time it takes to cool from $60°C$ to $40°C$ is 500 seconds.

The correct answer is (1) 500 s.

Quick Tip

Use the rate of cooling formula and solve using temperature differences. The time to cool down is inversely proportional to the temperature difference.

---

42. An engine operating between the boiling and freezing points of water will have:

(1) efficiency more than 27%.

(2) efficiency less than the efficiency a Carnot engine operating between the same two temperatures.

(3) efficiency equal to 27%.

(4) efficiency less than 27%.

(1) $2, 3$ and 4 only

(2) 2 and 3 only

(3) 2 and 4 only

(4) 1 and 2 only

Correct Answer: (3) 2 and 4 only

Solution: The efficiency η of a Carnot engine is given by the formula:

$$\eta = \left(1 - \frac{T_2}{T_1}\right) \times 100$$

where T_1 and T_2 are the temperatures of the hot and cold reservoirs in Kelvin.

For an engine operating between the freezing point (0°C) and the boiling point (100°C) of water:

$$T_1 = 100 + 273 = 373\,\text{K}, \quad T_2 = 0 + 273 = 273\,\text{K}.$$

Substituting these values into the formula:

$$\eta = \left(1 - \frac{273}{373}\right) \times 100 = 26.8\%.$$

Thus, the efficiency of the engine is less than 27

Quick Tip

The efficiency of a Carnot engine depends on the temperature difference between the hot and cold reservoirs. The higher the difference, the higher the efficiency.

---

43. Given below are two statements:

Statement I: A truck and a car moving with the same kinetic energy are brought to rest by applying brakes which provide equal retarding forces. Both come to rest in equal distance.

Statement II: A car moving towards east takes a turn and moves towards north, the speed remains unchanged. The acceleration of the car is zero.

In light of the given statements, choose the most appropriate answer from the options given below:

(1) Statement I is correct but Statement II is incorrect

(2) Statement I is incorrect but Statement II is correct

(3) Both Statement I is correct but Statement II are incorrect

(4) Both Statement I is correct but Statement II are correct

Correct Answer: (1) Statement I is correct but Statement II is incorrect

Solution:

Statement I:

For a truck and a car moving with the same kinetic energy, the distance to stop under the same retarding force can be determined by using the equation:

$$\text{Work done} = \Delta KE$$

Since the initial kinetic energy is the same for both vehicles, the work done (force times distance) to bring them to rest will be equal. Thus, both vehicles come to rest in the same distance.

Statement II:

In Statement II, when a car changes its direction from east to north, its speed may remain constant, but its velocity is changing because velocity is a vector quantity. Since the direction of velocity is changing, the car has acceleration. Therefore, the acceleration is not zero.

$$\Delta\vec{V} = \vec{V_f} - \vec{V_i}$$

As velocity is changing, acceleration $\vec{a} \neq 0$.

Thus, Statement II is incorrect.

Thus, the correct answer is $\boxed{1}$.

Quick Tip

Always remember that acceleration is a vector quantity. Even if speed remains constant, if the direction of motion changes, acceleration is present.

---

44. A particle is executing Simple Harmonic Motion (SHM). The ratio of potential energy and kinetic energy of the particle when its displacement is half of its amplitude will be:

(1) 1 : 1

(2) 2 : 1

(3) 1 : 4

(4) 1 : 3

Correct Answer: (4) 1 : 3

Solution: In Simple Harmonic Motion, the displacement of the particle can be represented as:

$$x = \frac{A}{2}$$

Where A is the amplitude of the motion.

The potential energy $P.E.$ and kinetic energy $K.E.$ in SHM are given by the equations:

$$P.E. = \frac{1}{2}kx^2$$

$$K.E. = \frac{1}{2}kA^2 - \frac{1}{2}kx^2$$

Substitute $x = \frac{A}{2}$ into the equations:

$$P.E. = \frac{1}{2}k\left(\frac{A}{2}\right)^2 = \frac{A^2k}{8}$$

$$K.E. = \frac{1}{2}kA^2 - \frac{A^2k}{8} = \frac{3A^2k}{8}$$

Now, find the ratio of $P.E.$ to $K.E.$:

$$\frac{P.E.}{K.E.} = \frac{\frac{A^2k}{8}}{\frac{3A^2k}{8}} = \frac{1}{3}$$

Thus, the ratio of potential energy to kinetic energy is $\boxed{1:3}$.

Quick Tip

In SHM, the total mechanical energy is constant, and the potential and kinetic energies are interchanged during the motion. The ratio of potential to kinetic energy depends on the displacement.

---

45. A ball is thrown vertically upward with an initial velocity of 150 m/s. The ratio of velocity after 3 s and 5 s is $\frac{x+1}{x}$. The value of x is:

Take $g = 10$ m/s^2.

(1) 6

(2) 5

(3) -5

(4) 10

Correct Answer: (2) 5

Solution:

The equation for velocity during uniformly accelerated motion is:

$$v = u + at$$

Where:

$u = 150$ m/s (initial velocity),

$a = -10\,\text{m/s}^2$ (acceleration due to gravity),

t is the time.

For $t = 3\,\text{s}$:

$$v(3) = 150 - 10(3) = 150 - 30 = 120\,\text{m/s}$$

For $t = 5\,\text{s}$:

$$v(5) = 150 - 10(5) = 150 - 50 = 100\,\text{m/s}$$

Now, use the given ratio:

$$\frac{120}{100} = \frac{x+1}{x}$$

Simplifying:

$$\frac{6}{5} = \frac{x+1}{x}$$

Cross-multiply:

$$6x = 5(x+1) \quad \Rightarrow \quad 6x = 5x + 5 \quad \Rightarrow \quad x = 5$$

Thus, the value of x is $\boxed{5}$.

Quick Tip

When solving problems involving velocity under uniform acceleration, always use the kinematic equations and substitute values for initial velocity, acceleration, and time to find the required quantity.

---

46. Given below are two statements: one is labelled as Assertion A and the other is labelled as Reason R.

Assertion A: If an electric dipole of dipole moment 30×10^{-5} Cm is enclosed by a closed surface, the net flux coming out of the surface will be zero.

Reason R: Electric dipole consists of two equal and opposite charges.

In the light of above statements, choose the correct answer from the options given below:

(1) Both A and R are true and R is the correct explanation of A

(2) A is true but R is false

(3) Both A and R true but R is NOT the correct explanation of A

(4) A is false but R is true

Correct Answer: (1) Both A and R are true and R is the correct explanation of A

Solution: Using Gauss's Law:

$$\Phi = \frac{Q_{\text{in}}}{\epsilon_0}$$

Since the electric dipole has no net charge (it consists of equal and opposite charges), the net charge $Q_{\text{in}} = 0$. Hence, the net flux $\Phi = 0$.

Thus, Assertion A is true.

Reason R states that an electric dipole consists of two equal and opposite charges, which is true.

Thus, both Assertion A and Reason R are true, and Reason R correctly explains Assertion A.

Therefore, the correct answer is $\boxed{1}$.

Quick Tip

For any closed surface enclosing a dipole, the net flux is zero because the net charge enclosed by the surface is zero.

---

47. Given below are two statements: one is labelled as Assertion A and the other is labelled as Reason R.

Assertion A: EM waves used for optical communication have longer wavelengths than that of microwave, employed in Radar technology.

Reason R: Infrared EM waves are more energetic than microwaves.

In the light of given statements, choose the correct answer from the options given below:

(1) A is false but R is true

(2) A is true but R is false

(3) Both A and R true but R is NOT the correct explanation of A

(4) Both A and R true and R is the correct explanation of A

Correct Answer: (1) A is false but R is true

Solution: Optical communication is performed in the frequency range of 1 THz to 1000 THz (from Microwave to UV). Therefore, the wavelength of EM waves used for optical communication is shorter than that of microwaves used in Radar. Hence, Assertion A is incorrect.

On the other hand, Reason R is true because infrared waves have higher frequency and energy compared to microwaves.

Thus, Assertion A is false but Reason R is true.

Therefore, the correct answer is $\boxed{1}$.

Quick Tip

The energy of EM waves is directly proportional to their frequency. Higher frequency waves (such as infrared) are more energetic than lower frequency waves (such as microwaves).

---

48. A 12.5 eV electron beam is used to bombard gaseous hydrogen at room temperature. The number of spectral lines emitted will be:

(1) 2.

(2) 1.

(3) 3.

(4) 4.

Correct Answer: (3) 3

Solution: According to Bohr's postulates, an electron makes a jump to higher energy orbital if it absorbs a photon of energy equal to the difference between the energies of an excited state and the ground state. Assuming that the collided electron takes energy equal to 10.2 eV or 12.09 eV from the incoming electron beam (some part lost due to collision), the maximum excited state is $n = 3$.

The number of spectral lines is given by:

$$\frac{3(3-1)}{2} = 3.$$

Thus, the number of spectral lines emitted is 3.

Quick Tip

The number of spectral lines corresponds to the possible transitions between different energy levels in the atom. Use the formula $\frac{n(n-1)}{2}$ to calculate the number of spectral lines.

---

49. The ratio of escape velocity of a planet to the escape velocity of Earth will be: Given: Mass of the planet is 16 times the mass of Earth and radius of the planet is 4 times the radius of Earth.

(1) 4:1.

(2) 2:1.

(3) $1:\sqrt{2}$.

(4) $1:4$.

Correct Answer: (2) 2:1

Solution: The escape velocity is given by the formula:

$$v_e = \sqrt{\frac{2GM}{R}}$$

where:

- v_e is the escape velocity
- G is the gravitational constant
- M is the mass of the celestial body
- R is the radius of the celestial body

Let:

- M_p be the mass of the planet
- R_p be the radius of the planet
- M_e be the mass of the Earth

- R_e be the radius of the Earth
- v_{ep} be the escape velocity of the planet
- v_{ee} be the escape velocity of the Earth

Given:

- $M_p = 16M_e$
- $R_p = 4R_e$

Escape velocity of Earth:

$$v_{ee} = \sqrt{\frac{2GM_e}{R_e}}$$

Escape velocity of the planet:

$$v_{ep} = \sqrt{\frac{2GM_p}{R_p}}$$

Ratio:

$$\frac{v_{ep}}{v_{ee}} = \frac{\sqrt{\frac{2GM_p}{R_p}}}{\sqrt{\frac{2GM_e}{R_e}}} = \sqrt{\frac{M_pR_e}{M_eR_p}}$$

Substitute the given values:

$$\frac{v_{ep}}{v_{ee}} = \sqrt{\frac{16M_eR_e}{M_e4R_e}} = \sqrt{\frac{16}{4}} = \sqrt{4} = 2$$

Therefore, the ratio of the escape velocity of the planet to the escape velocity of Earth is 2:1.

The correct answer is (2) 2:1.

Quick Tip

Escape velocity depends on both the mass and radius of the planet. A larger mass and a smaller radius result in a higher escape velocity.

---

50. Given below are two statements:

Statement I: When the frequency of an a.c. source in a series LCR circuit increases, the current in the circuit first increases, attains a maximum value and then decreases.

Statement II: In a series LCR circuit, the value of the power factor at resonance is one.

In light of the given statements, choose the most appropriate answer from the options given below:

(1) Statement I is incorrect but Statement II is true.

(2) Both Statement I and Statement II are false.

(3) Statement I is correct but Statement II is false.

(4) Both Statement I and Statement II are true.

Correct Answer: (4) Both Statement I and Statement II are true.

Solution:

Both statements are correct based on the behavior of a series LCR circuit at resonance.

Statement I: As the frequency of the AC source in a series LCR circuit increases, the current initially increases, reaches a maximum at resonance, and then decreases as the frequency continues to rise. This is due to the changing impedance in the circuit.

Statement II: At resonance, the power factor of a series LCR circuit is one because the impedance is at its minimum, and the current and voltage are in phase.

> Quick Tip
>
> At resonance, the series LCR circuit behaves as a purely resistive circuit, where the power factor is maximum (equal to one).

SECTION-B

51. For a certain organ pipe, the first three resonance frequencies are in the ratio of 1:3:5 respectively. If the frequency of the fifth harmonic is 405 Hz and the speed of sound in air is 324 m/s^{-1}, the length of the organ pipe is ______ m.

Correct Answer: 1 m

Solution: For the 5th harmonic in a closed organ pipe, the relationship between the frequency f, speed of sound v, and the length ℓ of the pipe is given by:

$$f_5 = \frac{5v}{4\ell}$$

Given:

- $f_5 = 405\,\text{Hz}$

- $v = 324\,\text{m/s}^{-1}$

Substitute the given values into the equation:

$$405 = \frac{5 \times 324}{4\ell}$$

Solving for ℓ:

$$405 \times 4\ell = 5 \times 324$$

$$1620\ell = 1620$$

$$\ell = 1\,\text{m}$$

Thus, the length of the organ pipe is $\boxed{1}$ m.

Quick Tip

In a closed organ pipe, the frequencies of the harmonics are related to the length of the pipe. For the 5th harmonic, use the formula $f_5 = \frac{5v}{4\ell}$ to calculate the length.

---

52. For a rolling spherical shell, the ratio of rotational kinetic energy and total kinetic energy is $\frac{x}{5}$. The value of x is:

Correct Answer: 2

Solution: For a rolling spherical shell, the angular velocity $\omega = \frac{v}{R}$. The rotational kinetic energy is given by:

$$K_{\text{rot}} = \frac{1}{2}\left(\frac{2}{3}mR^2\right)\left(\frac{v}{R}\right)^2 = \frac{1}{2}\left(\frac{2}{3}mR^2\right)\left(\frac{v^2}{R^2}\right) = \frac{1}{3}mv^2.$$

The total kinetic energy is:

$$K_{\text{total}} = \frac{1}{2}mv^2 + \frac{1}{2}\left(\frac{2}{3}mR^2\right)\left(\frac{v}{R}\right)^2 = \frac{1}{2}mv^2 + \frac{1}{3}mv^2 = \frac{5}{6}mv^2.$$

The ratio of rotational kinetic energy to total kinetic energy is:

$$\frac{K_{\text{rot}}}{K_{\text{total}}} = \frac{\frac{1}{3}mv^2}{\frac{5}{6}mv^2} = \frac{2}{5}.$$

Thus, $\frac{x}{5} = \frac{2}{5}$, so $x = 2$.

Quick Tip

For rolling objects, the total kinetic energy is the sum of translational and rotational kinetic energy. The ratio of rotational kinetic energy depends on the moment of inertia of the object.

---

53. A compass needle oscillates 20 times per minute at a place where the dip is 30° and 30 times per minute where the dip is 60°. The ratio of total magnetic field due to the earth at two places respectively is $\frac{4}{\sqrt{x}}$. The value of x is:

Correct Answer: 243

Solution:

The frequency of oscillation of a compass needle in a magnetic field is given by:

$$f = \frac{1}{2\pi}\sqrt{\frac{MB_H}{I}}$$

where:

- f is the frequency of oscillation
- M is the magnetic moment of the needle
- B_H is the horizontal component of the Earth's magnetic field
- I is the moment of inertia of the needle

Since M and I are constant, we have:

$$f \propto \sqrt{B_H}$$

$$f^2 \propto B_H$$

Also, $B_H = B\cos\delta$, where B is the total magnetic field and δ is the dip angle.

So, $f^2 \propto B\cos\delta$.

Let $f_1 = 20$ oscillations/minute and $\delta_1 = 30°$. Let $f_2 = 30$ oscillations/minute and $\delta_2 = 60°$.

Then,

$$f_1^2 \propto B_1\cos\delta_1$$

$$f_2^2 \propto B_2\cos\delta_2$$

Therefore,

$$\frac{f_1^2}{f_2^2} = \frac{B_1 \cos\delta_1}{B_2 \cos\delta_2}$$

Substituting the given values:

$$\frac{20^2}{30^2} = \frac{B_1 \cos 30°}{B_2 \cos 60°}$$

$$\frac{400}{900} = \frac{B_1(\sqrt{3}/2)}{B_2(1/2)}$$

$$\frac{4}{9} = \frac{B_1}{B_2}\sqrt{3}$$

$$\frac{B_1}{B_2} = \frac{4}{9\sqrt{3}}$$

We are given that $\frac{B_1}{B_2} = \frac{4}{\sqrt{x}}$.

Therefore,

$$\frac{4}{\sqrt{x}} = \frac{4}{9\sqrt{3}}$$

$$\sqrt{x} = 9\sqrt{3}$$

$$x = (9\sqrt{3})^2 = 81 \cdot 3 = 243$$

Thus, $x = 243$.

The value of x is 243.

Quick Tip

The frequency of oscillation of a compass needle is inversely proportional to the square root of the horizontal component of the magnetic field. Use this relationship to compare magnetic fields at different locations.

---

54. A conducting circular loop is placed in a uniform magnetic field of 0.4 T with its plane perpendicular to the field. Somehow, the radius of the loop starts expanding at a constant rate of 1 mm/s. The magnitude of induced emf in the loop at an instant when the radius of the loop is 2 cm will be __________, μV.

Correct Answer: (1) 50 μV

Solution: Given:

- $B = 0.4$ T

- $\frac{dr}{dt} = 1$ mm/s $= 1 \times 10^{-3}$ m/s
- $r = 2$ cm $= 2 \times 10^{-2}$ m
- $\theta = 0^\circ$ (plane perpendicular to the field)

The area of the loop is $A = \pi r^2$. The magnetic flux is $\Phi = B\pi r^2 \cos 0^\circ = B\pi r^2$.

Induced emf:

$$\varepsilon = -\frac{d(B\pi r^2)}{dt} = -B\pi\frac{d(r^2)}{dt} = -B\pi(2r\frac{dr}{dt}) = -2\pi Br\frac{dr}{dt}$$

Substituting the given values:

$$\varepsilon = -2\pi(0.4)(2 \times 10^{-2})(1 \times 10^{-3}) = -1.6\pi \times 10^{-5} \text{ V}$$

$$\varepsilon = -16\pi \times 10^{-6} \text{ V} = -16\pi\mu\text{V}$$

Magnitude of the induced emf:

$$|\varepsilon| = 16\pi\mu\text{V}$$

$$|\varepsilon| = 16 \times 3.14159\mu\text{V} \approx 50.265\mu\text{V}$$

Therefore, the magnitude of the induced emf in the loop is approximately 50.265 μV.

Answer: 50.265

Quick Tip

The induced emf in a loop is related to the rate of change of magnetic flux through the loop. When the area changes, use $\epsilon = B\frac{dA}{dt}$ to calculate the emf.

---

55. To maintain a speed of 80 km/h by a bus of mass 500 kg on a plane rough road for 4 km distance, the work done by the engine of the bus will be ________ KJ. [The coefficient of friction between tyre of bus and road is 0.04].

Correct Answer: 784 KJ

Solution: For constant speed, work done by the engine $\text{WD}_{\text{engine}}$ + work done by friction $\text{WD}_{\text{friction}} = 0$ (by Work-Energy Theorem).

Thus, we can write:

$$\text{WD}_{\text{engine}} = -\text{WD}_{\text{friction}} = -[\mu mgx]$$

where $\mu = 0.04$, $m = 500\,\text{kg}$, $g = 9.8\,\text{m/s}^2$, and $x = 4\,\text{km} = 4 \times 10^3\,\text{m}$.

So,

$$\text{WD}_{\text{engine}} = -0.04 \times 500 \times 9.8 \times 4 \times 10^3 = 784\,\textbf{KJ}.$$

Thus, the work done by the engine is 784 KJ.

Quick Tip

For constant speed motion, the work done by the engine exactly compensates the work done by friction. The formula for work done by friction is $W = \mu mgx$, where μ is the coefficient of friction.

---

56. A common example of alpha decay is

$$^{238}_{92}U \rightarrow ^{234}_{90}Th + ^{4}_{2}He + Q.$$

Given:

$$^{238}_{92}U = 238.05060\,u, \quad ^{234}_{90}Th = 234.04360\,u, \quad ^{4}_{2}He = 4.00260\,u, \quad 1u = 931.5\,\textbf{MeV/c}^2.$$

The energy released Q during the alpha decay of $^{238}_{92}U$ is $_{MeV.}$

Correct Answer: 4.0986 MeV

Solution: Energy released Q is given by the equation:

$$Q = (\Delta m)_{\text{amu}} \times 931.5\,\text{MeV}.$$

Here, the mass defect Δm is:

$$\Delta m = m_u - m_{\text{Th}} - m_{\text{He}} = 238.05060\,u - 234.04360\,u - 4.00260\,u = 0.0044\,u.$$

Thus,

$$Q = 0.0044 \times 931.5\,\text{MeV} = 4.0986\,\text{MeV}.$$

Therefore, the energy released during the alpha decay of $^{238}_{92}U$ is 4.0986 MeV.

Quick Tip

To calculate the energy released during a nuclear decay, find the mass defect and multiply it by $931.5\,\text{MeV}$ (the energy equivalent of 1 atomic mass unit).

57. The current flowing through a conductor connected across a source is 2A and 1.2 A at 0°C and 100°C respectively. The current flowing through the conductor at 50°C will be _______ $\times 10^2$ mA.

Correct Answer: 15

Solution: Using the formula for the current through the conductor, which relates the current at different temperatures:

$$i_0 R_0 = i_{100} R_{100} \quad \text{[For the same source]}$$

This gives:

$$2R_0 = 1.2R_0[1 + 100\alpha] \quad \Rightarrow \quad 1 + 100\alpha = \frac{5}{3} \quad \Rightarrow \quad 100\alpha = \frac{2}{3}$$

Now, calculate α:

$$50\alpha = \frac{1}{3}$$

Thus, the current at 50°C will be:

$$i_{50} R_{50} = i_0 R_0$$

Substituting values:

$$i_{50} = \frac{i_0 R_0}{R_{50}} = \frac{2R_0}{R_0(1 + 50\alpha)} = \frac{2}{1 + \frac{1}{3}} = \frac{2}{\frac{4}{3}} = 1.5\,\text{A}$$

Thus, the current at 50°C is $1.5\,\text{A} = 15 \times 10^2$ mA.

Therefore, the correct answer is $\boxed{15}$ mA.

Quick Tip

For resistive materials, the temperature dependence of resistance is often linear. Use the relation $i_0 R_0 = i_{100} R_{100}$ to find the current at different temperatures.

58. Two convex lenses of focal length 20 cm each are placed coaxially with a separation of 60 cm between them. The image of the distant object formed by the combination is at _______ cm from the first lens.

Correct Answer: (100)

Solution: Given:

- Focal length of each lens, $f_1 = f_2 = 20\,\text{cm}$
- Separation between the lenses, $d = 60\,\text{cm}$
- The object is at a considerable distance (assumed to be at infinity).

Objective: Find the distance of the final image from the first lens.

Approach:

Step 1: Image Formation by the First Lens For the first lens ($f_1 = 20\,\text{cm}$), the object is at infinity. Using the lens formula:

$$\frac{1}{f} = \frac{1}{v} - \frac{1}{u}$$

where:

- u is the object distance (infinity),
- v is the image distance,
- f is the focal length.

Plugging in the values:

$$\frac{1}{20} = \frac{1}{v} - \frac{1}{\infty} \implies \frac{1}{v} = \frac{1}{20} \implies v = 20\,\text{cm}$$

So, the first lens forms an image at 20 cm from itself.

Step 2: Object for the Second Lens The image formed by the first lens acts as the object for the second lens. The separation between the lenses is 60 cm, and the first image is 20 cm from the first lens. Therefore, the distance of this image (object for the second lens) from the second lens is:

$$u_2 = d - v_1 = 60\,\text{cm} - 20\,\text{cm} = 40\,\text{cm}$$

Since the image is on the same side as the incoming light for the second lens, we consider u_2 as negative in the lens formula (real image for the first lens acts as a virtual object for the second lens).

Step 3: Image Formation by the Second Lens Using the lens formula for the second lens ($f_2 = 20\,\text{cm}$):

$$\frac{1}{f_2} = \frac{1}{v_2} - \frac{1}{u_2}$$

Plugging in the values:

$$\frac{1}{20} = \frac{1}{v_2} - \frac{1}{-40} \implies \frac{1}{20} = \frac{1}{v_2} + \frac{1}{40}$$

Solving for v_2:

$$\frac{1}{v_2} = \frac{1}{20} - \frac{1}{40} = \frac{2-1}{40} = \frac{1}{40} \implies v_2 = 40\,\text{cm}$$

The positive sign indicates that the final image is formed on the opposite side of the second lens from where the light is coming.

Step 4: Distance of the Final Image from the First Lens The final image is 40 cm from the second lens. Since the lenses are 60 cm apart, the distance from the first lens is:

$$\text{Total distance} = d + v_2 = 60\,\text{cm} + 40\,\text{cm} = 100\,\text{cm}$$

Conclusion: The image of the distant object formed by the combination of the two convex lenses is located at $\boxed{100\,\text{cm}}$ from the first lens.

> **Quick Tip**
>
> When multiple lenses are used in combination, treat the image formed by the first lens as the object for the second lens and use the lens formula for each to find the final image position.

---

59. Glycerine of density 1.25×10^3 kg/m^{-3} is flowing through the conical section of pipe. The area of cross-section of the pipe at its ends is 10 cm^2 and 5 cm^2 and pressure drop across its length is 3 Nm^{-2}. The rate of flow of glycerine through the pipe is $x \times 10^{-5}$ m^3 s^{-1}. The value of x is _______ .

Correct Answer: 4

Solution: From the continuity equation:

$$A_1 v_1 = A_2 v_2 \quad (1),$$

where $A_1 = 10\,\text{cm}^2 = 10 \times 10^{-4}\,\text{m}^2$, $A_2 = 5\,\text{cm}^2 = 5 \times 10^{-4}\,\text{m}^2$.

By Bernoulli's equation:

$$P_1 + \frac{1}{2}\rho v_1^2 = P_2 + \frac{1}{2}\rho v_2^2,$$

we have:

$$P_1 - P_2 = \frac{1}{2}\rho(v_2^2 - v_1^2).$$

Given $P_1 - P_2 = 3\,\text{N/m}^2$, we get:

$$3 = \frac{1}{2} \times 1.25 \times 10^3 \times \left(1 - \left(\frac{5}{10}\right)^2\right) v_2^2.$$

Simplifying:

$$3 = \frac{1}{2} \times 1.25 \times 10^3 \times \left(1 - \frac{1}{4}\right) v_2^2,$$

$$3 = \frac{1}{2} \times 1.25 \times 10^3 \times \frac{3}{4} v_2^2,$$

$$v_2^2 = \frac{3 \times 4}{1.25 \times 10^3 \times 3} = 8 \times 10^{-2} \quad v_2 = 8 \times 10^{-2}\,\text{m/s}.$$

Now, using the discharge rate $Q = A_2 v_2$:

$$Q = 5 \times 10^{-4} \times 8 \times 10^{-2} = 4 \times 10^{-5}\,\text{m}^3/\text{s}.$$

Thus, the value of x is 4.

Quick Tip

The continuity equation ensures that the rate of flow is constant in an incompressible fluid. Use Bernoulli's principle to relate the pressure drop to the change in velocity.

---

60. 64 identical drops each charged up to a potential of 10 mV are combined to form a bigger drop. The potential of the bigger drop will be ______ mV.

Correct Answer: (160)

Solution: Let q be the charge on each drop, and the potential of each drop is given by:

$$V = \frac{Kq}{r} \quad \text{(Equation 1)}$$

Now, for the combination of 64 drops, the volume of the total drop is given by:

$$64 \times \frac{4}{3}\pi r^3 = \frac{4}{3}\pi R^3$$

This implies that:

$$R = 4r \quad \text{and} \quad Q = 64q$$

Now, the potential of the bigger drop is:

$$V_{\text{bigger}} = \frac{KQ}{R} = \frac{K \times 64q}{4r} = 16 \times \frac{Kq}{r}$$

Substituting the value of $\frac{Kq}{r} = 10\,\text{mV}$:

$$V_{\text{bigger}} = 16 \times 10\,\text{mV} = 160\,\text{mV}$$

Thus, the potential of the bigger drop is $\boxed{160}$ mV.

Quick Tip

When combining identical drops, the charge adds up while the radius increases by a factor related to the number of drops. Use the formula $V = \frac{Kq}{r}$ to calculate the potential.
