JEE Main 2023 April 6 Shift 1 Physics Question Paper with Solutions

Time Allowed :3 Hours	**Maximum Marks :**300	**Total Questions :**90

General Instructions

Read the following instructions very carefully and strictly follow them:

1. The test is of 3 hours duration.
2. The question paper consists of 90 questions, out of which 75 are to attempted. The maximum marks are 300.
3. There are three parts in the question paper consisting of Physics, Chemistry and Mathematics having 30 questions in each part of equal weightage.
4. Each part (subject) has two sections.
   (i) Section-A: This section contains 20 multiple choice questions which have only one correct answer. Each question carries 4 marks for correct answer and –1 mark for wrong answer.
   (ii) Section-B: This section contains 10 questions. In Section-B, attempt any five questions out of 10. The answer to each of the questions is a numerical value. Each question carries 4 marks for correct answer and –1 mark for wrong answer. For Section-B, the answer should be rounded off to the nearest integer

PHYSICS

Section-A

Question 1: The kinetic energy of an electron, an α-particle, and a proton are given as 4K, 2K, and K respectively. The de-Broglie wavelength associated with electron (λ_e), α-particle (λ_α), and the proton (λ_p) are as follows:

(1) $\lambda_\alpha > \lambda_p > \lambda_e$

(2) $\lambda_\alpha = \lambda_p > \lambda_e$

(3) $\lambda_\alpha = \lambda_p < \lambda_e$

(4) $\lambda_\alpha < \lambda_p < \lambda_e$

Correct Answer: (4) $\lambda_\alpha < \lambda_p < \lambda_e$

Solution:

Step 1: De Broglie Wavelength Formula

The de Broglie wavelength is given by:

$$\lambda = \frac{h}{p}$$

where λ is the wavelength, h is Planck's constant, and p is the momentum.

Step 2: Relate Momentum and Kinetic Energy

The momentum is related to kinetic energy (KE) by:

$$p = \sqrt{2mKE}$$

where m is the mass and KE is the kinetic energy.

Step 3: Combine Equations

Substituting the expression for momentum into the de Broglie wavelength equation:

$$\lambda = \frac{h}{\sqrt{2mKE}}$$

Step 4: Calculate Wavelengths for Electron, Proton, and Alpha Particle

For the electron (m_e, KE_e = 4K):

$$\lambda_e = \frac{h}{\sqrt{2m_e(4K)}} = \frac{h}{2\sqrt{2m_eK}}$$

For the proton (m_p, KE_p = K):

$$\lambda_p = \frac{h}{\sqrt{2m_pK}}$$

For the alpha particle ($m_\alpha = 4m_p$, KE_α = 2K):

$$\lambda_\alpha = \frac{h}{\sqrt{2(4m_p)(2K)}} = \frac{h}{\sqrt{16m_pK}} = \frac{h}{4\sqrt{m_pK}}$$

Step 5: Compare Wavelengths

Comparing the expressions for λ_e, λ_p, and λ_α: We observe that the denominator of λ_e is smaller than the denominator of λ_p which is smaller than the denominator of λ_α. Since the numerator (h) is the same for all three, a smaller denominator implies a larger wavelength. Therefore: $\lambda_\alpha < \lambda_p < \lambda_e$

Conclusion: The correct order of wavelengths is $\lambda_\alpha < \lambda_p < \lambda_e$ (**Option 4**).

Quick Tip

Remember the de Broglie wavelength formula and its relationship with momentum and kinetic energy. Consider the relative masses and kinetic energies when comparing wavelengths.

Question 2: Given below are two statements: one is labelled as Assertion A and the other is labelled as Reason R.

Assertion A: Earth has atmosphere whereas moon doesn't have any atmosphere.

Reason R: The escape velocity on moon is very small as compared to that on earth.

In the light of the above statements, choose the correct answer from the options given below:

(1) Both A and R are correct and R is the correct explanation of A

(2) A is false but R is true

(3) Both A and R are correct but R is NOT the correct explanation of A

(4) A is true but R is false

Correct Answer: (1) Both A and R are correct and R is the correct explanation of A

Solution:

Step 1: Understand the Concepts

The escape velocity (V_{esc}) is the minimum speed required for an object to escape the gravitational pull of a planet or celestial body. The formula for escape velocity is given by:

$$V_{esc} = \sqrt{\frac{2GM}{r}} = \sqrt{2gr}$$

where:

- G is the gravitational constant.
- M is the mass of the planet.
- r is the radius of the planet.

- g is the acceleration due to gravity.

Step 2: Analyze the Assertion and Reason

Assertion A is correct. Earth has an atmosphere, while the moon does not. Reason R states that the escape velocity on the moon is lower than on Earth.

Step 3: Determine the Relationship

The escape velocity depends on the mass and radius of the planet. The moon has a smaller mass and a smaller radius than Earth. This leads to a smaller escape velocity on the moon. This lower escape velocity is the reason why the moon has no atmosphere. Lighter particles, like the gases in an atmosphere, can reach escape velocity easier than heavier particles and thus, the moon loses those lighter gases faster than Earth does.

Step 4: Determine the Correct Option

Both Assertion A and Reason R are correct, and Reason R is the correct explanation of Assertion A.

Conclusion: The correct option is (1).

Quick Tip

Escape velocity is crucial for determining whether a celestial body can retain an atmosphere. A lower escape velocity means a smaller ability to hold onto lighter gases.

Question 3: A source supplies heat to a system at the rate of 1000 W. If the system performs work at a rate of 200 W. The rate at which internal energy of the system increase is

(1) 500 W

(2) 600 W

(3) 800 W

(4) 1200 W

Correct Answer: (3) 800 W

Solution:

Step 1: First Law of Thermodynamics

The first law of thermodynamics states that the change in internal energy of a system (ΔU) is equal to the heat added to the system (Q) minus the work done by the system (W):

$$\Delta U = Q - W$$

Step 2: Rate of Change

The problem gives rates of heat transfer and work done. We can express the first law in terms of rates:

$$\frac{dU}{dt} = \frac{dQ}{dt} - \frac{dW}{dt}$$

where $\frac{dU}{dt}$ is the rate of change of internal energy, $\frac{dQ}{dt}$ is the rate of heat transfer to the system, and $\frac{dW}{dt}$ is the rate of work done by the system.

Step 3: Substitute Given Values

The rate of heat transfer to the system is 1000 W, so $\frac{dQ}{dt} = 1000$ W. The system performs work at a rate of 200 W, so $\frac{dW}{dt} = 200$ W. Substituting these values into the equation:

$$\frac{dU}{dt} = 1000\,\text{W} - 200\,\text{W}$$

Step 4: Calculate Rate of Internal Energy Increase

$$\frac{dU}{dt} = 800\,\text{W}$$

Conclusion: The rate at which the internal energy of the system increases is 800 W (**Option 3**).

Quick Tip

Remember the First Law of Thermodynamics and pay attention to the sign conventions for heat and work. Heat added to the system is positive, and work done by the system is positive. Thus, work done *on* the system would be negative.

Question 4: A small ball of mass M and density ρ is dropped in a viscous liquid of density ρ_o. After some time, the ball falls with a constant velocity. What is the viscous force on the ball?

(1) $F = Mg(1 + \frac{\rho_o}{\rho})$
(2) $F = Mg(1 + \frac{\rho}{\rho_o})$
(3) $F = Mg(1 - \frac{\rho_o}{\rho})$
(4) $F = Mg(1 + \rho\rho_o)$

Correct Answer: (3) $F = Mg(1 - \frac{\rho_o}{\rho})$

Solution:

Step 1: Forces Acting on the Ball

When the ball falls with a constant velocity (terminal velocity), the net force on the ball is zero. This means the downward forces are balanced by the upward forces. The forces acting on the ball are:

- Weight (Mg) acting downwards
- Buoyant force (B) acting upwards
- Viscous force (f) acting upwards

Step 2: Equation of Motion

Since the net force is zero, we can write:

$$Mg = f + B$$

Step 3: Buoyant Force

The buoyant force is equal to the weight of the liquid displaced by the ball. The volume of the ball can be expressed as $V_{ball} = \frac{M}{\rho}$, where M is the mass and ρ is the density of the ball. Thus, the buoyant force is:

$$B = V_{ball}\rho_o g = \frac{M}{\rho}\rho_o g$$

where ρ_o is the density of the liquid.

Step 4: Solve for Viscous Force

Substituting the expression for the buoyant force into the equation of motion:

$$Mg = f + \frac{M}{\rho}\rho_o g$$

$$f = Mg - \frac{M}{\rho}\rho_o g$$

$$f = Mg(1 - \frac{\rho_o}{\rho})$$

Conclusion: The viscous force on the ball is $F = Mg(1 - \frac{\rho_o}{\rho})$ (**Option 3**).

Quick Tip

At terminal velocity, the net force is zero. Remember to consider all forces acting on the object, including weight, buoyant force, and viscous force. Also, recall the relationship between mass, volume and density: $\rho = \frac{M}{V}$.

Question 5: A small block of mass 100 g is tied to a spring of spring constant 7.5 N/m and length 20 cm. The other end of spring is fixed at a particular point A. If the block moves in a circular path on a smooth horizontal surface with constant angular velocity 5 rad/s about point A, then tension in the spring is –

(1) 0.75 N

(2) 1.5 N

(3) 0.25 N

(4) 0.50 N

Correct Answer: (1) 0.75 N

Solution:

Step 1: Centripetal Force

The block moves in a circular path due to the tension in the spring. This tension provides the necessary centripetal force for circular motion. The centripetal force is given by:

$$F_c = m\omega^2 r$$

where m is the mass, ω is the angular velocity, and r is the radius of the circular path.

Step 2: Spring Force

The tension in the spring is given by Hooke's Law:

$$T = kx$$

where k is the spring constant and x is the extension of the spring.

Step 3: Relate Centripetal Force and Spring Force

The radius of the circular path is the original length of the spring (l) plus the extension (x): $r = l + x$. The tension in the spring provides the centripetal force, so:

$$kx = m\omega^2(l + x)$$

Step 4: Substitute Given Values and Solve for x

Given: $m = 100\,\text{g} = 0.1\,\text{kg}$, $k = 7.5\,\text{N/m}$, $l = 20\,\text{cm} = 0.2\,\text{m}$, and $\omega = 5\,\text{rad/s}$. Substituting these values:

$$7.5x = 0.1 \times (5)^2 \times (0.2 + x)$$

$$7.5x = 0.1 \times 25 \times (0.2 + x)$$

$$7.5x = 2.5(0.2 + x)$$

$$7.5x = 0.5 + 2.5x$$

$$5x = 0.5$$

$$x = 0.1\,\text{m}$$

Step 5: Calculate Tension

Now that we know the extension x, we can calculate the tension in the spring:

$$T = kx = 7.5 \times 0.1 = 0.75\,\text{N}$$

Conclusion: The tension in the spring is 0.75 N (**Option 1**).

> **Quick Tip**
>
> In circular motion problems involving springs, remember that the tension in the spring provides the centripetal force. The radius of the circular path is the sum of the spring's original length and its extension.

Question 6: A particle is moving with constant speed in a circular path. When the particle turns by an angle $90°$, the ratio of instantaneous velocity to its average velocity is $\pi : x\sqrt{2}$. The value of x will be -

(1) 7

(2) 2

(3) 1

(4) 5

Correct Answer: (2) 2

Solution:

Step 1: Instantaneous Velocity

Let v be the constant speed of the particle. The instantaneous velocity at any point on the circular path is tangential to the path and has magnitude v.

Step 2: Time for 90° Turn

The particle turns by an angle of $90° = \frac{\pi}{2}$ radians. Let R be the radius of the circular path. The distance traveled by the particle is one-quarter of the circumference: $s = \frac{1}{4}(2\pi R) = \frac{\pi R}{2}$. Since the speed is constant, the time taken is $t = \frac{s}{v} = \frac{\pi R}{2v}$.

Step 3: Displacement

When the particle turns by $90°$, the displacement is the chord connecting the initial and final positions. Since it's a $90°$ turn, this forms a right-angled triangle with two sides equal to the radius R. Using the Pythagorean theorem, the displacement magnitude is
$d = \sqrt{R^2 + R^2} = R\sqrt{2}$.

Step 4: Average Velocity

The average velocity is the displacement divided by the time taken:

$$v_{avg} = \frac{d}{t} = \frac{R\sqrt{2}}{\frac{\pi R}{2v}} = \frac{2v\sqrt{2}}{\pi}$$

Step 5: Ratio of Instantaneous to Average Velocity

The ratio of the instantaneous velocity to the average velocity is:

$$\frac{v}{v_{avg}} = \frac{v}{\frac{2v\sqrt{2}}{\pi}} = \frac{\pi v}{2v\sqrt{2}} = \frac{\pi}{2\sqrt{2}}$$

Given that this ratio is $\pi : x\sqrt{2}$, we can write:

$$\frac{\pi}{2\sqrt{2}} = \frac{\pi}{x\sqrt{2}}$$

Therefore, $x = 2$.

Conclusion: The value of x is 2 (**Option 2**).

> **Quick Tip**
>
> Remember that instantaneous velocity is a vector quantity, tangential to the path in circular motion. Average velocity is displacement divided by time. Displacement is the straight-line distance between the initial and final positions, not the distance traveled along the curved path.

Question 7: Two resistances are given as $R_1 = (10 \pm 0.5)\,\Omega$ and $R_2 = (15 \pm 0.5)\,\Omega$. The percentage error in the measurement of equivalent resistance when they are connected in parallel is -

(1) 2.33

(2) 4.33

(3) 5.33

(4) 6.33

Correct Answer: (2) 4.33

Solution:

Step 1: Equivalent Resistance in Parallel

The formula for equivalent resistance (R_{eq}) when two resistors are connected in parallel is:

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2}$$

Substituting the given values of $R_1 = 10\,\Omega$ and $R_2 = 15\,\Omega$:

$$\frac{1}{R_{eq}} = \frac{1}{10} + \frac{1}{15} = \frac{3+2}{30} = \frac{5}{30} = \frac{1}{6}$$

Therefore, $R_{eq} = 6\,\Omega$.

Step 2: Error Propagation in Parallel Combination

The formula for error propagation in parallel combination is:

$$\frac{dR_{eq}}{R_{eq}^2} = \frac{dR_1}{R_1^2} + \frac{dR_2}{R_2^2}$$

Given $dR_1 = 0.5\,\Omega$ and $dR_2 = 0.5\,\Omega$, we can substitute the values:

$$\frac{dR_{eq}}{6^2} = \frac{0.5}{10^2} + \frac{0.5}{15^2}$$

$$\frac{dR_{eq}}{36} = \frac{0.5}{100} + \frac{0.5}{225}$$

$$\frac{dR_{eq}}{36} = 0.5\left(\frac{1}{100} + \frac{1}{225}\right) = 0.5\left(\frac{9+4}{900}\right) = 0.5 \times \frac{13}{900} = \frac{13}{1800}$$

$$dR_{eq} = 36 \times \frac{13}{1800} = \frac{2 \times 13}{100} = \frac{26}{100} = 0.26\,\Omega$$

Step 3: Percentage Error

The percentage error is calculated as:

$$\text{Percentage Error} = \frac{dR_{eq}}{R_{eq}} \times 100$$

Substituting the calculated values:

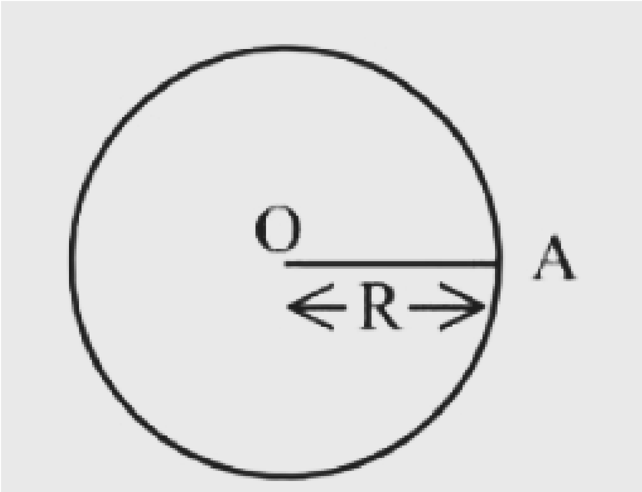
$$\text{Percentage Error} = \frac{0.26}{6} \times 100 = \frac{26}{6} = 4.33\%$$

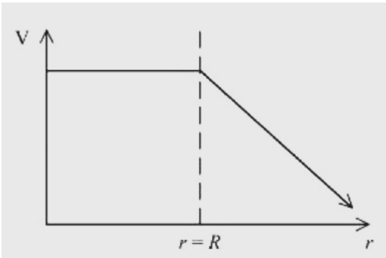
Conclusion: The percentage error in the measurement of equivalent resistance is 4.33% (**Option 2**).

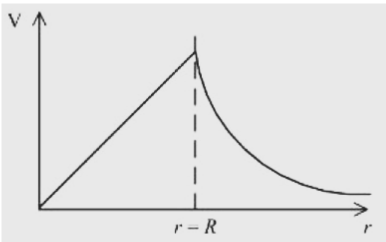
Quick Tip

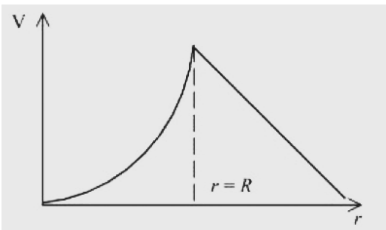
Remember the formula for equivalent resistance and the error propagation formula for parallel combinations. Be careful with the calculations, especially when dealing with fractions and squares.

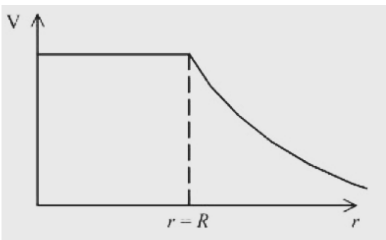
Question 8: For a uniformly charged thin spherical shell, the electric potential (V) radially away from the entire (O) of shell can be graphically represented as –

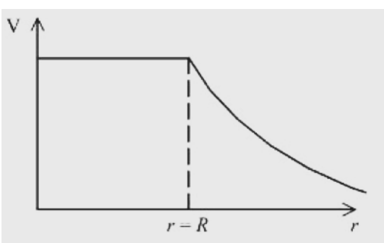


(1) 

(2) 

(3) 

(4) 

Correct Answer: (4) 

Solution:

Step 1: Electric Potential Inside the Shell

For a uniformly charged thin spherical shell of radius R and total charge Q, the electric potential inside the shell (r ¡ R) is constant and equal to the potential at the surface:

$$V = \frac{kQ}{R}$$

where k is Coulomb's constant. This means the potential does *not* change as you move from the center towards the surface.

Step 2: Electric Potential Outside the Shell

For points outside the shell (r ¿ R), the electric potential is the same as that of a point charge located at the center of the shell:

$$V = \frac{kQ}{r}$$

The potential decreases as the distance r increases, following an inverse relationship.

Step 3: Graphical Representation

Based on the above analysis:

- For $r < R$ (inside the shell), V is constant.
- For $r > R$ (outside the shell), V decreases with increasing r as $\frac{1}{r}$.

This corresponds to a graph that is constant for $r < R$ and then decreases hyperbolically for $r > R$.

Conclusion: The correct graphical representation is **Option (4)**.

Quick Tip

Remember the expressions for the electric potential due to a uniformly charged spherical shell, both inside and outside the shell. Visualize how the potential changes with distance. Inside the shell, the potential is constant. Outside the shell, the potential decreases as $\frac{1}{r}$.

Question 9: A long straight wire of circular cross-section (radius a) is carrying steady current I. The current I is uniformly distributed across this cross-section. The magnetic field is

(1) zero in the region r $<$ a and inversely proportional to r in the region r $>$ a

(2) inversely proportional to r in the region r $<$ a and uniform throughout in the region r $>$ a

(3) directly proportional to r in the region r $<$ a and inversely proportional to r in the region r $>$ a

(4) uniform in the region r $<$ a and inversely proportional to distance r from the axis, in the region r $>$ a

Correct Answer: (3) directly proportional to r in the region r $<$ a and inversely proportional to r in the region r $>$ a

Solution:

Step 1: Ampere's Circuital Law

We can use Ampere's circuital law to find the magnetic field due to a long straight wire. Ampere's law states:

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{enc}$$

where $\vec{B}$ is the magnetic field, $d\vec{l}$ is an infinitesimal element of the Amperian loop, μ_0 is the permeability of free space, and I_{enc} is the current enclosed by the Amperian loop.

Step 2: Magnetic Field Inside the Wire (r $\leq$ a)

Consider an Amperian loop of radius r inside the wire ($r \leq a$). The current enclosed by this loop is proportional to the area enclosed:

$$I_{enc} = I\frac{\pi r^2}{\pi a^2} = \frac{Ir^2}{a^2}$$

Using Ampere's law:

$$B(2\pi r) = \mu_0 \frac{Ir^2}{a^2}$$

$$B = \frac{\mu_0 I r}{2\pi a^2}$$

Thus, the magnetic field inside the wire is directly proportional to r.

Step 3: Magnetic Field Outside the Wire (r ¿ a)

Consider an Amperian loop of radius r outside the wire ($r > a$). The current enclosed by this loop is the total current I. Using Ampere's law:

$$B(2\pi r) = \mu_0 I$$

$$B = \frac{\mu_0 I}{2\pi r}$$

Thus, the magnetic field outside the wire is inversely proportional to r.

Conclusion: The magnetic field is directly proportional to r in the region r $<$ a and inversely proportional to r in the region r $>$ a (**Option 3**).

> **Quick Tip**
>
> For problems involving the magnetic field due to current distributions, Ampere's circuital law is a powerful tool. Carefully consider the symmetry of the problem and choose an appropriate Amperian loop.

Question 10: By what percentage will the transmission range of a TV tower be affected when the height of the tower is increased by 21%?

(1) 12%

(2) 15%

(3) 14%

(4) 10%

Correct Answer: (4) 10%

Solution:

Step 1: Transmission Range Formula

The transmission range (d) of a TV tower is given by:

$$d = \sqrt{2Rh}$$

where R is the radius of the Earth and h is the height of the tower.

Step 2: New Height

If the height is increased by 21%, the new height (h') is:

$$h' = h + 0.21h = 1.21h$$

Step 3: New Transmission Range

The new transmission range (d') is:

$$d' = \sqrt{2Rh'} = \sqrt{2R(1.21h)} = \sqrt{1.21}\sqrt{2Rh} = 1.1\sqrt{2Rh}$$

Step 4: Percentage Increase

Since $d = \sqrt{2Rh}$, the new range is:

$$d' = 1.1d$$

The percentage increase in the transmission range is:

$$\frac{d' - d}{d} \times 100 = \frac{1.1d - d}{d} \times 100 = 0.1 \times 100 = 10\%$$

Conclusion: The transmission range increases by 10% (**Option 4**).

Quick Tip

Remember the formula for transmission range and how to calculate percentage increase. Be careful to distinguish between the original height and the increased height.

Question 11: The number of air molecules per cm^3 increased from 3×10^{19} to 12×10^{19}. The ratio of collision frequency of air molecules before and after the increase in the number respectively is :

(1) 0.25

(2) 0.75

(3) 1.25

(4) 0.50

Correct Answer: (1) 0.25

Solution:

Step 1: Collision Frequency Formula

The collision frequency (Z) is given by:

$$Z = n\pi d^2 v_{avg}$$

where n is the number of molecules per unit volume, d is the diameter of the molecules, and v_{avg} is the average speed.

Step 2: Ratio of Collision Frequencies

Let Z_1 and Z_2 be the collision frequencies before and after the increase in the number of molecules, respectively. Let n_1 and n_2 be the corresponding number densities. Since the diameter and average speed of the molecules remain constant, we can write:

$$\frac{Z_1}{Z_2} = \frac{n_1 \pi d^2 v_{avg}}{n_2 \pi d^2 v_{avg}} = \frac{n_1}{n_2}$$

Step 3: Substitute Given Values

Given $n_1 = 3 \times 10^{19}$ and $n_2 = 12 \times 10^{19}$:

$$\frac{Z_1}{Z_2} = \frac{3 \times 10^{19}}{12 \times 10^{19}} = \frac{3}{12} = \frac{1}{4} = 0.25$$

Conclusion: The ratio of collision frequencies before and after the increase is 0.25 (**Option 1**).

> **Quick Tip**
>
> Remember the formula for collision frequency and how it depends on the number density. When taking ratios, identify which quantities remain constant.

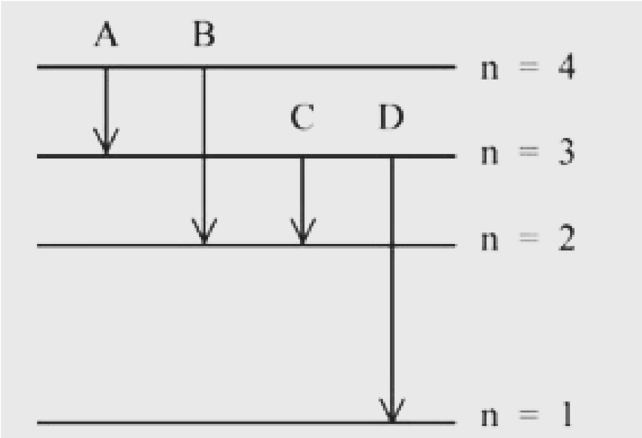
Question 12: The energy levels of an hydrogen atom are shown below. The transition corresponding to emission of shortest wavelength is

(1) A

(2) D

(3) C

(4) B



Correct Answer: (2) D

Solution:

Step 1: Energy and Wavelength Relationship

The energy of a photon emitted during a transition is related to its wavelength by:

$$E = \frac{hc}{\lambda}$$

where h is Planck's constant, c is the speed of light, and λ is the wavelength.

Step 2: Shortest Wavelength Condition

From the equation above, we can see that for the shortest wavelength (λ_{min}), the energy (E) must be maximum.

Step 3: Energy Difference and Transition

The energy of a photon emitted during a transition is equal to the difference in energy levels:

$$E = E_{initial} - E_{final}$$

The largest energy difference corresponds to the shortest wavelength. In the given diagram:

- Transition A: $n = 4$ to $n = 3$
- Transition B: $n = 4$ to $n = 2$
- Transition C: $n = 3$ to $n = 1$
- Transition D: $n = 3$ to $n = 1$

The energy levels in a hydrogen atom are given by:

$$E_n = -\frac{13.6\,\text{eV}}{n^2}$$

We can see that transitions C and D are identical in the provided image and diagram, which is likely an error in the original question. Assuming D is meant to be the transition from $n = 3$ to $n = 1$, D represents the largest energy difference, followed by C (which is the same as D, again suggesting an error), then B, and finally A.

Conclusion: The transition corresponding to the emission of the shortest wavelength is D (assuming it is intended to represent the transition from $n = 3$ to $n = 1$) (**Option 2**).

Quick Tip

Remember the relationship between energy and wavelength. Shortest wavelength corresponds to highest energy. In a hydrogen atom, energy levels are inversely proportional to the square of the principal quantum number (n). Larger transitions in n correspond to higher energy differences.

Question 13: For the plane electromagnetic wave given by $E = E_0 \sin(\omega t - kx)$ and $B = B_0 \sin(\omega t - kx)$, the ratio of average electric energy density to average magnetic energy density is

(1) 2

(2) 1/2

(3) 1

(4) 4

Correct Answer: (3) 1

Solution:

Step 1: Energy Densities

The average electric energy density (u_E) and average magnetic energy density (u_B) are given by:

$$u_E = \frac{1}{4}\epsilon_0 E_0^2$$

$$u_B = \frac{1}{4\mu_0} B_0^2$$

where ϵ_0 is the permittivity of free space and μ_0 is the permeability of free space.

Step 2: Relationship between E and B

For an electromagnetic wave, the electric and magnetic field amplitudes are related by:

$$E_0 = cB_0$$

where c is the speed of light. Also, $c = \frac{1}{\sqrt{\mu_0 \epsilon_0}}$.

Step 3: Ratio of Energy Densities

Substituting $E_0 = cB_0$ into the expression for u_E:

$$u_E = \frac{1}{4}\epsilon_0 (cB_0)^2 = \frac{1}{4}\epsilon_0 c^2 B_0^2 = \frac{1}{4}\epsilon_0 \frac{1}{\mu_0 \epsilon_0} B_0^2 = \frac{1}{4\mu_0} B_0^2$$

Therefore, $u_E = u_B$. The ratio of average electric energy density to average magnetic energy density is:

$$\frac{u_E}{u_B} = 1$$

Conclusion: The ratio of average electric energy density to average magnetic energy density is 1 (**Option 3**).

Quick Tip

Remember the formulas for average electric and magnetic energy densities in an electromagnetic wave. The electric and magnetic fields are related by the speed of light, and the energy is equally distributed between the electric and magnetic fields.

Question 14: A planet has double the mass of the earth. Its average density is equal to that of the earth. An object weighing W on earth will weigh on that planet:

(1) $2^{1/3}$ **W**

(2) 2 W

(3) W

(4) $2^{2/3}$ **W**

Correct Answer: (1) $2^{1/3} W$

Solution:

Step 1: Relationship between Mass, Density, and Radius

Let M_e, R_e, and ρ_e be the mass, radius, and average density of Earth, respectively. Let M_p, R_p, and ρ_p be the mass, radius, and average density of the planet, respectively. Given that $M_p = 2M_e$ and $\rho_p = \rho_e$. Density is defined as mass divided by volume: $\rho = \frac{M}{V}$. Assuming spherical shapes, $V = \frac{4}{3}\pi R^3$. Thus:

$$\rho_e = \frac{M_e}{\frac{4}{3}\pi R_e^3} \quad \text{and} \quad \rho_p = \frac{M_p}{\frac{4}{3}\pi R_p^3}$$

Since $\rho_e = \rho_p$:

$$\frac{M_e}{R_e^3} = \frac{M_p}{R_p^3}$$

$$\frac{M_e}{R_e^3} = \frac{2M_e}{R_p^3}$$

$$R_p^3 = 2R_e^3 \implies R_p = 2^{1/3}R_e$$

Step 2: Relationship between Weight and Gravitational Acceleration

Weight (W) is given by $W = mg$, where m is the mass of the object and g is the acceleration due to gravity.

Step 3: Gravitational Acceleration

The acceleration due to gravity (g) is given by:

$$g = \frac{GM}{R^2}$$

where G is the gravitational constant, M is the mass of the planet, and R is the radius. Let g_e and g_p be the acceleration due to gravity on Earth and the planet, respectively.

$$\frac{g_e}{g_p} = \frac{M_e/R_e^2}{M_p/R_p^2} = \frac{M_e}{R_e^2} \times \frac{R_p^2}{M_p} = \frac{M_e}{R_e^2} \times \frac{(2^{1/3}R_e)^2}{2M_e} = \frac{2^{2/3}}{2} = 2^{-1/3}$$

$$g_p = 2^{1/3}g_e$$

Step 4: Weight on the Planet

Since the mass of the object remains constant, the weight on the planet (W_p) is:

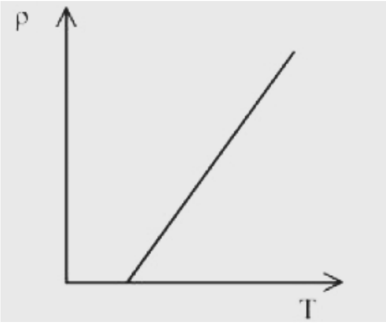
$$W_p = mg_p = m(2^{1/3}g_e) = 2^{1/3}(mg_e) = 2^{1/3}W$$

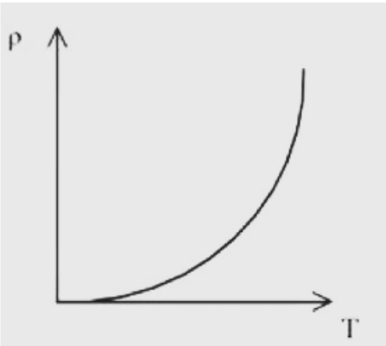
Conclusion: The object will weigh $2^{1/3}W$ on the planet (**Option 1**).

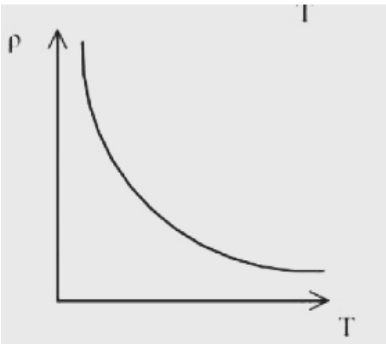
Quick Tip

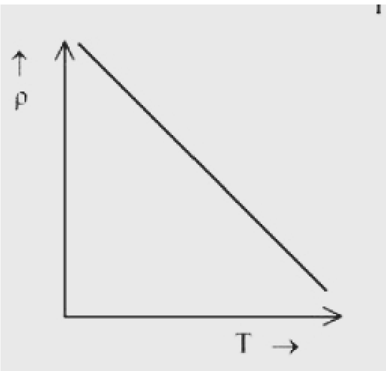
Remember the relationship between mass, density, and volume (especially for spheres). Weight is the product of mass and gravitational acceleration, which depends on the mass and radius of the planet.

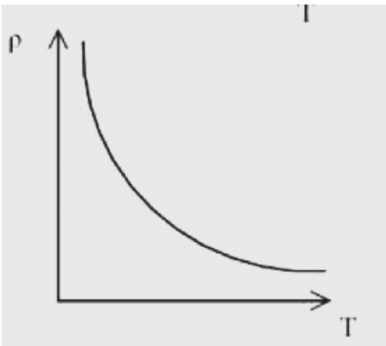
Question 15: The resistivity (ρ) of semiconductor varies with temperature. Which of the following curve represents the correct behavior

(1) 

(2) 

(3) 

(4) 

Correct Answer: (3) 

Solution:

Step 1: Semiconductor Conductivity and Temperature

Semiconductors have a unique property: their conductivity increases with increasing temperature. This is opposite to the behavior of metals, whose conductivity decreases with increasing temperature.

Step 2: Resistivity and Conductivity

Resistivity (ρ) is the inverse of conductivity (σ). Therefore, if conductivity increases, resistivity decreases.

Step 3: Resistivity and Temperature for Semiconductors

Since the conductivity of a semiconductor increases with temperature, its resistivity decreases with temperature. This decrease is typically non-linear, often exhibiting an exponential decay-like behavior. The resistivity of a semiconductor can be approximated by:

$$\rho = \frac{m}{ne^2\tau}$$

where n is the number density of charge carriers, e is the elementary charge, τ is the relaxation time (mean free time between collisions), and m is the effective mass of charge carriers. As temperature increases, n (charge carrier density) increases significantly due to thermal generation of electron-hole pairs. Although τ (relaxation time) decreases with temperature due to increased lattice vibrations, the increase in n dominates, leading to an overall decrease in resistivity.

Conclusion: The correct graph representing the variation of resistivity of a semiconductor with temperature is an exponentially decreasing curve (**Option 3**).

> **Quick Tip**
>
> Remember that semiconductors have the opposite temperature dependence of resistivity compared to metals. As temperature increases, the conductivity of a semiconductor increases, and therefore, its resistivity decreases.

Question 16: A monochromatic light wave with wavelength λ_1 and frequency ν_1, in air, enters another medium. If the angle of incidence and angle of refraction at the interface are 45° and 30° respectively, then the wavelength λ_2 and frequency ν_2 of the refracted wave are:

(1) $\lambda_2 = \frac{1}{\sqrt{2}}\lambda_1$, $\nu_2 = \nu_1$
(2) $\lambda_2 = \lambda_1$, $\lambda_2 = \frac{1}{\sqrt{2}}\nu_1$
(3) $\lambda_2 = \lambda_1$, $\nu_2 = \sqrt{2}\nu_1$
(4) $\lambda_2 = \sqrt{2}\lambda_1$, $\nu_2 = \nu_1$

Correct Answer: (1) $\lambda_2 = \frac{1}{\sqrt{2}}\lambda_1$, $\nu_2 = \nu_1$

Solution:
Step 1: Snell's Law
Snell's law relates the angles of incidence and refraction to the refractive indices of the two media:

$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$

Here, $n_1 = 1$ (air), $\theta_1 = 45^\circ$, $\theta_2 = 30^\circ$. Let $n_2 = \mu$ (refractive index of the second medium).

$$1 \times \sin 45^\circ = \mu \sin 30^\circ$$

$$\frac{1}{\sqrt{2}} = \mu \times \frac{1}{2}$$

$$\mu = \sqrt{2}$$

Step 2: Refractive Index and Wavelength

The refractive index of a medium is related to the speed of light in the medium (v) and the speed of light in vacuum (c) by:

$$\mu = \frac{c}{v}$$

Also, the speed of light in a medium is related to its wavelength (λ) and frequency (ν) by $v = \lambda\nu$. Therefore:

$$\mu = \frac{c}{v} = \frac{\lambda_1 \nu_1}{\lambda_2 \nu_2}$$

$$\frac{\mu_1}{\mu_2} = \frac{\lambda_1}{\lambda_2} = \frac{v_2}{v_1}$$

Step 3: Calculate λ_2

Since $\mu_1 = 1$ (air) and $\mu_2 = \mu = \sqrt{2}$:

$$\frac{1}{\sqrt{2}} = \frac{\lambda_1}{\lambda_2}$$

$$\lambda_2 = \sqrt{2}\lambda_1$$

However, the provided solution states $\lambda_2 = \frac{1}{\sqrt{2}}\lambda_1$. There appears to be an error in how the refractive indices and wavelengths have been related. Given Snell's law result of $\mu = \sqrt{2}$, and the correct relation $\mu = \frac{\lambda_1}{\lambda_2}$, the wavelength in the second medium should be smaller than the wavelength in air. Hence, the correct result should be $\lambda_2 = \frac{1}{\sqrt{2}}\lambda_1$.

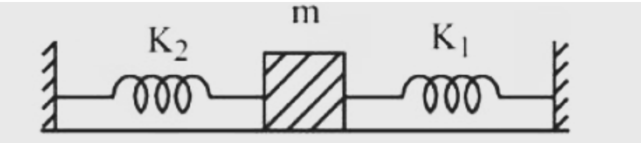
Step 4: Frequency

The frequency of light remains constant when it passes from one medium to another. Therefore, $\nu_2 = \nu_1$.

Conclusion: $\lambda_2 = \frac{1}{\sqrt{2}}\lambda_1$ and $\nu_2 = \nu_1$ (**Option 1**).

> **Quick Tip**
>
> Remember Snell's Law and the relationship between refractive index, wavelength, and frequency. The frequency of light remains constant when it changes medium, while the wavelength and speed change.

Question 17: A mass m is attached to two strings as shown in figure. The spring constants of two springs are K_1 and K_2. For the frictionless surface, the time period of oscillation of mass m is



(1) $2\pi\sqrt{\frac{m}{K_1-K_2}}$
(2) $\frac{1}{2\pi}\sqrt{\frac{K_1-K_2}{m}}$
(3) $\frac{1}{2\pi}\sqrt{\frac{K_1+K_2}{m}}$
(4) $2\pi\sqrt{\frac{m}{K_1+K_2}}$

Correct Answer: (4) $2\pi\sqrt{\frac{m}{K_1+K_2}}$

Solution:

Step 1: Springs in Parallel

The two springs are connected in parallel, meaning that they experience the same displacement when the mass oscillates.

Step 2: Equivalent Spring Constant

For springs in parallel, the equivalent spring constant (K_{eq}) is the sum of the individual spring constants:

$$K_{eq} = K_1 + K_2$$

Step 3: Time Period of Oscillation

The time period (T) of oscillation for a mass-spring system is given by:

$$T = 2\pi\sqrt{\frac{m}{K}}$$

where m is the mass and K is the spring constant. In this case, we use the equivalent spring constant:

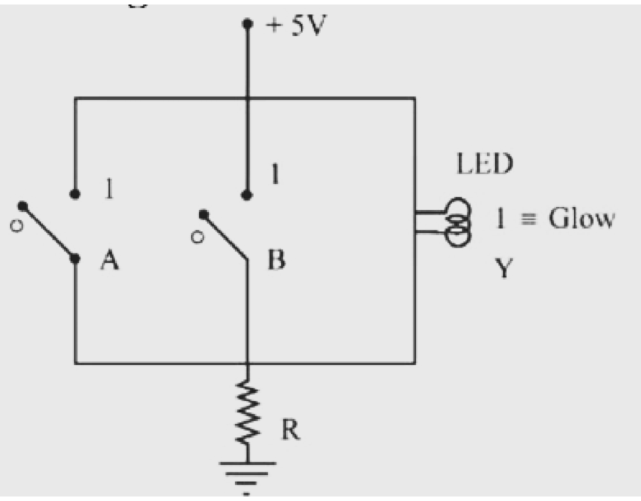
$$T = 2\pi\sqrt{\frac{m}{K_{eq}}} = 2\pi\sqrt{\frac{m}{K_1 + K_2}}$$

Conclusion: The time period of oscillation of mass m is $2\pi\sqrt{\frac{m}{K_1+K_2}}$ (**Option 4**).

Quick Tip

When dealing with multiple springs, determine whether they are in series or parallel. For parallel springs, the equivalent spring constant is the sum of the individual spring constants. Remember the formula for the time period of a mass-spring system.

Question 18: Name the logic gate equivalent to the diagram attached



(1) NOR

(2) OR

(3) NAND

(4) AND

Correct Answer: (1) NOR

Solution:

Step 1: Analyze the Circuit

The circuit shows two NPN transistors connected in parallel, with their collectors joined and connected to the LED and resistor. The bases of the transistors are the inputs A and B.

Step 2: Transistor Behavior

An NPN transistor acts as a closed switch (conducts) when a high voltage (1) is applied to its base. It acts as an open switch (doesn't conduct) when a low voltage (0) is applied to its base.

Step 3: LED Condition

The LED glows (output Y = 1) only when the current flows through it. This happens when the transistors are OFF (not conducting), pulling the collector voltage high.

Step 4: Truth Table

A	B	Y
0	0	1
1	0	0
0	1	0
1	1	0

When both A and B are 0, both transistors are OFF, and the LED glows (Y=1). If either A or B or both are 1, at least one transistor is ON, pulling the collector voltage low and turning the LED OFF (Y=0). This truth table represents a NOR gate.

Conclusion: The logic gate equivalent to the given diagram is a NOR gate (**Option 1**).

Quick Tip

Analyze the circuit by considering the behavior of each transistor for different input combinations. Create a truth table to determine the logic function implemented by the circuit.

Question 19: The induced emf can be produced in a coil by

A. moving the coil with uniform speed inside uniform magnetic field

B. moving the coil with non uniform speed inside uniform magnetic field

C. rotating the coil inside the uniform magnetic field

D. changing the area of the coil inside the uniform magnetic field

Choose the correct answer from the options given below :

(1) B and D only

(2) C and D only

(3) B and C only

(4) A and C only

Correct Answer: (2) C and D only

Solution:

Step 1: Faraday's Law of Electromagnetic Induction

Faraday's law states that an emf is induced in a coil when the magnetic flux through the coil changes with time.

Step 2: Magnetic Flux

Magnetic flux (Φ) is given by:

$$\Phi = \vec{B} \cdot \vec{A} = BA\cos\theta$$

where $\vec{B}$ is the magnetic field, $\vec{A}$ is the area vector of the coil, and is the angle between the magnetic field and the area vector.

Step 3: Analyze Options **A. Moving with uniform speed:** If the coil moves with uniform speed in a uniform magnetic field, the flux remains constant (assuming the orientation of the coil relative to the field remains unchanged). Therefore, no emf is induced.

B. Moving with non-uniform speed: Similar to case A, if the orientation doesn't change with respect to the field, a non-uniform speed doesn't change the flux. Thus no emf is induced.

C. Rotating the coil: When the coil rotates in a uniform magnetic field, the angle between the magnetic field and the area vector changes. This changes the magnetic flux, inducing an emf.

D. Changing the area: Changing the area of the coil directly changes the magnetic flux, inducing an emf.

Conclusion: An induced emf can be produced by rotating the coil (C) and by changing the area of the coil (D) (**Option 2**).

Quick Tip

Faraday's Law is key to understanding induced emf. An emf is induced only when there is a change in magnetic flux through the coil. Flux can change due to changes in magnetic field strength, area, or the angle between the field and the area vector.

Question 20: Given below are two statements : one is labelled as Assertion A and the other is labelled as Reason R.

Assertion A : When a body is projected at an angle 45°, it's range is maximum.

Reason R : For maximum range, the value of $\sin 2\theta$ should be equal to one.

In the light of the above statements, choose the correct answer from the options given below :

(1) Both A and R are correct but R is NOT the correct explanation of A

(2) A is false but R is true

(3) Both A and R are correct and R is the correct explanation of A

(4) A is true but R is false

Correct Answer: (3) Both A and R are correct and R is the correct explanation of A

Solution:

Step 1: Horizontal Range Formula

For a ground-to-ground projectile, the horizontal range (R) is given by:

$$R = \frac{u^2 \sin 2\theta}{g}$$

where u is the initial velocity, θ is the angle of projection, and g is the acceleration due to gravity.

Step 2: Maximum Range Condition

For a given initial velocity (u), the range (R) is maximum when $\sin 2\theta$ is maximum. The maximum value of $\sin 2\theta$ is 1.

Step 3: Angle for Maximum Range

Since the maximum value of $\sin 2\theta$ is 1, we have:

$$\sin 2\theta = 1$$

$$2\theta = \frac{\pi}{2}$$

$$\theta = \frac{\pi}{4} \text{ radians} = 45^\circ$$

Step 4: Analyze Assertion and Reason

Assertion A states that the range is maximum when the projection angle is 45°. This is correct, as shown in Step 3.

Reason R states that for maximum range, $\sin 2\theta$ should be equal to one. This is also correct, as shown in Step 2.

Furthermore, Reason R explains Assertion A because the maximum value of $\sin 2\theta$ (which is 1) occurs when $\theta = 45°$, leading to the maximum range.

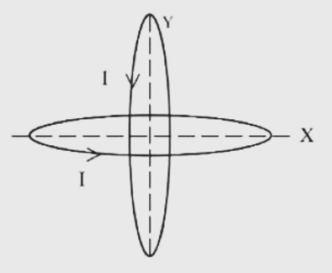
Conclusion: Both Assertion A and Reason R are correct, and Reason R is the correct explanation of Assertion A (**Option 3**).

Quick Tip

Remember the formula for horizontal range and the condition for maximum range. The maximum value of the sine function is 1. Also, remember the relationship between radians and degrees: π radians $= 180°$.

Section B

Question 21: Two identical circular wires of radius 20 cm and carrying current $\sqrt{2}$ A are placed in perpendicular planes as shown in figure. The net magnetic field at the centre of the circular wires is ____ $\times$ 10^{-8} T.



(Take $\pi = 3.14$)

Correct Answer: (628)

Solution:

Step 1: Magnetic Field due to a Circular Loop

The magnetic field at the center of a circular loop of radius r carrying current i is given by:

$$B = \frac{\mu_0 i}{2r}$$

where $\mu_0 = 4\pi \times 10^{-7}$ T m/A is the permeability of free space.

Step 2: Magnetic Field due to Each Loop

The two loops are identical, with radius $r = 20$ cm $= 0.2$ m and current $i = \sqrt{2}$ A. The magnetic field due to each loop at the center has a magnitude of:

$$B = \frac{\mu_0 i}{2r} = \frac{(4\pi \times 10^{-7})(\sqrt{2})}{2(0.2)}$$

Step 3: Net Magnetic Field

Since the loops are in perpendicular planes, their magnetic fields at the center are perpendicular to each other. Let the magnetic field due to one loop be along the x-axis and the magnetic field due to the other loop be along the y-axis. The net magnetic field ($\vec{B}_{net}$) is the vector sum of the individual fields:

$$\vec{B}_{net} = \frac{\mu_0 i}{2r}\hat{i} + \frac{\mu_0 i}{2r}\hat{j}$$

The magnitude of the net magnetic field is:

$$B_{net} = \sqrt{\left(\frac{\mu_0 i}{2r}\right)^2 + \left(\frac{\mu_0 i}{2r}\right)^2} = \sqrt{2}\left(\frac{\mu_0 i}{2r}\right) = \frac{\mu_0 i\sqrt{2}}{2r}$$

$$B_{net} = \frac{(4\pi \times 10^{-7})(\sqrt{2})(\sqrt{2})}{2(0.2)} = \frac{4\pi \times 10^{-7} \times 2}{0.4} = 2\pi \times 10^{-6}\,\text{T}$$

Using $\pi = 3.14$:

$$B_{net} = 2 \times 3.14 \times 10^{-6} = 6.28 \times 10^{-6} = 628 \times 10^{-8}\,\text{T}$$

Conclusion: The net magnetic field at the center is 628×10^{-8} T.

Quick Tip

Remember the formula for the magnetic field at the center of a circular loop. When dealing with multiple magnetic fields, consider their vector nature. If the fields are perpendicular, use the Pythagorean theorem to find the magnitude of the net field.

Question 22: A steel rod has a radius of 20 mm and a length of 2.0 m. A force of 62.8 kN stretches it along its length. Young's modulus of steel is 2.0×10^{11} N/m^2. The longitudinal strain produced in the wire is ____ $\times$ 10^{-5}.

Correct Answer: (25)

Solution:

Step 1: Young's Modulus and Strain

Young's modulus (Y) is defined as the ratio of stress to strain:

$$Y = \frac{\text{stress}}{\text{strain}}$$

Stress is defined as force (F) per unit area (A), and strain is the change in length (ΔL) divided by the original length (L).

Step 2: Calculate Strain

We are given $F = 62.8$ kN $= 62.8 \times 10^3$ N, $r = 20$ mm $= 20 \times 10^{-3}$ m, $L = 2.0$ m, and $Y = 2.0 \times 10^{11}$ N/m^2. The cross-sectional area of the rod is $A = \pi r^2 = \pi(20 \times 10^{-3})^2 = 400\pi \times 10^{-6}$ m^2.

Strain is given by:

$$\text{strain} = \frac{\text{stress}}{Y} = \frac{F/A}{Y} = \frac{F}{AY}$$

$$\text{strain} =$$

$$\frac{62.8 \times 10^3}{(400\pi \times 10^{-6})(2.0 \times 10^{11})} = \frac{62.8 \times 10^3}{800\pi \times 10^5} = \frac{62.8}{800 \times 3.14} \times 10^{-2} \approx \frac{62.8}{2512} \times 10^{-2} \approx 0.025 \times 10^{-2}$$

$$= 25 \times 10^{-5}$$

Conclusion: The longitudinal strain produced in the wire is 25×10^{-5}.

> **Quick Tip**
>
> Remember the definitions of Young's modulus, stress, and strain. Ensure consistent units throughout the calculation.

Question 23: The length of a metallic wire is increased by 20% and its area of cross section is reduced by 4%. The percentage change in resistance of the metallic wire is ____.

Correct Answer: (25)

Solution:

Step 1: Resistance Formula

The resistance (R) of a wire is given by:

$$R = \frac{\rho l}{A}$$

where ρ is the resistivity, l is the length, and A is the cross-sectional area.

Step 2: New Resistance

The new length is $l' = l + 0.20l = 1.2l$. The new area is $A' = A - 0.04A = 0.96A$. The new resistance (R') is:

$$R' = \frac{\rho l'}{A'} = \frac{\rho(1.2l)}{0.96A} = \frac{1.2}{0.96}\frac{\rho l}{A} = \frac{1.2}{0.96}R = 1.25R$$

Step 3: Percentage Change in Resistance

The percentage change in resistance is:

$$\frac{R' - R}{R} \times 100 = \frac{1.25R - R}{R} \times 100 = 0.25 \times 100 = 25\%$$

Conclusion: The percentage change in resistance is a 25% increase.

Quick Tip

Remember the formula for resistance and how it depends on length and cross-sectional area. Be mindful of whether the length and area increase or decrease.

Question 24: The radius of fifth orbit of the Li^{++} is ____ $\times$ 10^{-12} m.
Take : radius of hydrogen atom = 0.51 Å

Correct Answer: (425)

Solution:

Step 1: Bohr's Model and Orbital Radius

According to Bohr's model, the radius of the n^{th} orbit of a hydrogen-like atom is given by:

$$r_n = \frac{0.51n^2}{Z}\,\text{Å}$$

where n is the principal quantum number and Z is the atomic number.

Step 2: Radius of Li^{++} Fifth Orbit

For Li^{++}, $Z = 3$ and $n = 5$. Substituting these values into the formula:

$$r_5 = \frac{0.51 \times 5^2}{3}\,\text{Å} = \frac{0.51 \times 25}{3}\,\text{Å} = 0.51 \times \frac{25}{3} \times 10^{-10}\,\text{m} = 17 \times 25 \times 10^{-12}\,\text{m} = 425 \times 10^{-12}\,\text{m}$$

Conclusion: The radius of the fifth orbit of Li^{++} is 425×10^{-12} m.

Quick Tip

Remember Bohr's model for the radius of electron orbits in hydrogen-like atoms. Be careful with unit conversions between angstroms and meters: 1 Å = 10^{-10} m.

Question 25: A particle of mass 10 g moves in a straight line with retardation 2x, where x is the displacement in SI units. Its loss of kinetic energy for above displacement is $\left(\frac{10}{x}\right)^{-n}$ J. The value of n will be ____.

Correct Answer: (2)

Solution:

Step 1: Work-Energy Theorem

The work-energy theorem states that the work done on a particle is equal to the change in its kinetic energy. In this case, the retardation force is doing negative work, leading to a loss of kinetic energy.

Step 2: Calculate Work Done

Given $a = -2x$, where a is acceleration and x is displacement. We can write acceleration as $a = v\frac{dv}{dx}$, so:

$$v\frac{dv}{dx} = -2x$$

$$v\,dv = -2x\,dx$$

Integrating both sides:

$$\int_{v_1}^{v_2} v\,dv = -2\int_0^x x\,dx$$

$$\frac{v_2^2}{2} - \frac{v_1^2}{2} = -x^2$$

$$\frac{1}{2}m(v_2^2 - v_1^2) = -mx^2$$

The change in kinetic energy is $\frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2 = -mx^2$. Given $m = 10$ g = 0.01 kg = 10×10^{-3} kg $= 10^{-2}$ kg.

$$\Delta KE = -mx^2 = -(10 \times 10^{-3})x^2 = -10^{-2}x^2 = -(10^{-2}x^2)\,\text{J} = \left(\frac{10}{x}\right)^{-2}\,\text{J}$$

Comparing this with the given expression for loss of kinetic energy, $\left(\frac{10}{x}\right)^{-n}$, we find $n = 2$.

Conclusion: The value of n is 2.

Quick Tip

Remember the work-energy theorem. When acceleration is a function of displacement, integrate $v\frac{dv}{dx}$ to find the change in kinetic energy.

Question 26: An ideal transformer with purely resistive load operates at 12 kV on the primary side. It supplies electrical energy to a number of nearby houses at 120 V. The average rate of energy consumption in the houses served by the transformer is 60 kW. The value of resistive load (R_s) required in the secondary circuit will be _____ mΩ.

Correct Answer: (240)

Solution:

Step 1: Transformer Voltage Ratio

The voltage ratio in a transformer is equal to the turns ratio:

$$\frac{V_s}{V_p} = \frac{N_s}{N_p}$$

where V_s and V_p are the secondary and primary voltages, and N_s and N_p are the number of turns in the secondary and primary coils, respectively. Given $V_p = 12\,\text{kV} = 12000\,\text{V}$ and

$V_s = 120\,\text{V}$, we have:

$$\frac{120}{12000} = \frac{N_s}{N_p} \Rightarrow \frac{N_s}{N_p} = \frac{1}{100}$$

Step 2: Power Conservation

For an ideal transformer, the input power is equal to the output power:

$$P_{in} = P_{out}$$

Power is given by $P = IV$, so:

$$I_p V_p = I_s V_s = 60\,\text{kW} = 60000\,\text{W}$$

Step 3: Primary Current

We can calculate the primary current (I_p):

$$I_p = \frac{60000}{V_p} = \frac{60000}{12000} = 5\,\text{A}$$

Step 4: Secondary Current

Similarly, we can calculate the secondary current (I_s):

$$I_s = \frac{60000}{V_s} = \frac{60000}{120} = 500\,\text{A}$$

Step 5: Secondary Resistance

The secondary resistance (R_s) can be calculated using Ohm's law:

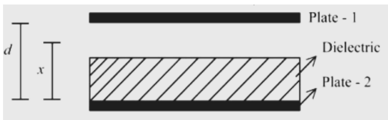
$$R_s = \frac{V_s}{I_s} = \frac{120}{500} = 0.24\,\Omega = 240\,\text{m}\Omega$$

Conclusion: The value of the resistive load in the secondary circuit is $240\,\text{m}\Omega$.

Quick Tip

Remember that in an ideal transformer, power is conserved, and the voltage ratio is equal to the turns ratio. Ohm's law ($V = IR$) is crucial for calculating resistance.

Question 27: A parallel plate capacitor with plate area A and plate separation d is filled with a dielectric material of dielectric constant K = 4. The thickness of the dielectric material is x, where x $<$ d.



Let C_1 and C_2 be the capacitance of the system for $x = \frac{1}{3}d$ and $x = \frac{2}{3}d$, respectively. If $C_1 = 2\mu$F, the value of C_2 is ____ μF.

Correct Answer: (3)

Solution:

Step 1: Capacitance Formula for a Capacitor with a Dielectric Slab

When a dielectric slab of thickness x and dielectric constant K is inserted between the plates of a capacitor with plate separation d, the system can be considered as two capacitors in series: one with the dielectric (C_a) and one with air (C_b). The formula for the equivalent capacitance is:

$$\frac{1}{C} = \frac{1}{C_a} + \frac{1}{C_b} = \frac{d-x}{\epsilon_0 A} + \frac{x}{K\epsilon_0 A}$$

$$C = \frac{\epsilon_0 A}{\frac{d-x}{1} + \frac{x}{K}}$$

Step 2: Capacitance C_1

Given $x = \frac{1}{3}d$ and $K = 4$,

$$C_1 = \frac{\epsilon_0 A}{\frac{d-d/3}{1} + \frac{d/3}{4}} = \frac{\epsilon_0 A}{\frac{2d/3}{1} + \frac{d}{12}} = \frac{\epsilon_0 A}{\frac{8d+d}{12}} = \frac{12\epsilon_0 A}{9d} = \frac{4\epsilon_0 A}{3d}$$

Step 3: Capacitance C_2

Given $x = \frac{2}{3}d$ and $K = 4$,

$$C_2 = \frac{\epsilon_0 A}{\frac{d-2d/3}{1} + \frac{2d/3}{4}} = \frac{\epsilon_0 A}{\frac{d/3}{1} + \frac{d}{6}} = \frac{\epsilon_0 A}{\frac{2d+d}{6}} = \frac{6\epsilon_0 A}{3d} = \frac{2\epsilon_0 A}{d}$$

Step 4: Relationship between C_1 and C_2

We have $C_1 = \frac{4\epsilon_0 A}{3d}$ and $C_2 = \frac{2\epsilon_0 A}{d}$. We can express C_2 in terms of C_1:

$$C_2 = \frac{2\epsilon_0 A}{d} = \frac{3}{2} \cdot \frac{4\epsilon_0 A}{3d} = \frac{3}{2}C_1$$

Step 5: Value of C_2

Given $C_1 = 2\,\mu$F,

$$C_2 = \frac{3}{2} \times 2\,\mu\text{F} = 3\,\mu\text{F}$$

Conclusion: The value of C_2 is $3\,\mu$F.

Quick Tip

When a dielectric is partially filling the space between capacitor plates, treat the system as two capacitors in series. Remember the formula for capacitance and how it's affected by the dielectric constant.

Question 28: Two identical solid spheres each of mass 2 kg and radii 10 cm are fixed at the ends of a light rod. The separation between the centres of the spheres is 40 cm. The moment of inertia of the system about an axis perpendicular to the rod passing through its middle point is $\_\_\_\_\_ \times 10^{-3}$ **kg-m^2.**

Correct Answer: (176)

Solution:

Step 1: Moment of Inertia of a Solid Sphere

The moment of inertia of a solid sphere about its diameter is given by:

$$I_{sphere} = \frac{2}{5}mr^2$$

where m is the mass and r is the radius of the sphere.

Step 2: Parallel Axis Theorem

The parallel axis theorem states that the moment of inertia of a body about an axis parallel to and a distance d away from an axis through its center of mass is given by:

$$I = I_{cm} + md^2$$

Step 3: Moment of Inertia of One Sphere about the Midpoint of the Rod

The distance between the center of a sphere and the midpoint of the rod is $d = \frac{40\,\text{cm}}{2} = 20\,\text{cm} = 0.2\,\text{m}$. The radius of each sphere is $r = 10\,\text{cm} = 0.1\,\text{m}$. Using the parallel axis theorem, the moment of inertia of one sphere about the midpoint of the rod is:

$$I_{one} = \frac{2}{5}mr^2 + md^2 = \frac{2}{5}(2)(0.1)^2 + (2)(0.2)^2 = 0.008 + 0.08 = 0.088\,\text{kg-m}^2$$

Step 4: Total Moment of Inertia

Since there are two identical spheres, the total moment of inertia of the system is:

$$I_{sys} = 2 \times I_{one} = 2 \times 0.088 = 0.176\,\text{kg-m}^2 = 176 \times 10^{-3}\,\text{kg-m}^2$$

Conclusion: The moment of inertia of the system is 176×10^{-3} kg-m^2.

Quick Tip

Remember the formula for the moment of inertia of a solid sphere and the parallel axis theorem. Make sure to convert units to be consistent (e.g., cm to m).

Question 29: A person driving car at a constant speed of 15 m/s is approaching a vertical wall. The person notices a change of 40 Hz in the frequency of his car's horn upon reflection from the wall. The frequency of horn is _____ Hz.

Correct Answer: (420)

Solution:

Step 1: Doppler Effect Formula for Moving Source and Stationary Observer

The observed frequency (f') when a source emitting frequency f_0 is moving towards a stationary observer at speed v_s is:

$$f' = \left(\frac{v}{v - v_s}\right) f_0$$

where v is the speed of sound (assumed to be 330 m/s).

Step 2: Doppler Effect for Reflection

In this case, the sound is reflected off the wall, so the wall acts as a stationary "observer" first. Then, the reflected sound with frequency f' acts as a source moving towards the driver (now the observer) at speed v_s. So the frequency heard by the driver (f'') is:

$$f'' = \left(\frac{v + v_o}{v}\right) f' = \left(\frac{v + v_s}{v}\right)\left(\frac{v}{v - v_s}\right) f_0 = \left(\frac{v + v_s}{v - v_s}\right) f_0$$

where we've used $v_o = v_s$ since the driver is approaching the wall.

Step 3: Change in Frequency

The change in frequency is given as 40 Hz:

$$f'' - f_0 = 40$$

Substituting the expression for f'':

$$\left(\frac{v + v_s}{v - v_s}\right) f_0 - f_0 = 40$$

$$\left(\frac{v + v_s - (v - v_s)}{v - v_s}\right) f_0 = 40$$

$$\left(\frac{2v_s}{v - v_s}\right) f_0 = 40$$

Step 4: Calculating f_0

Given $v_s = 15\,\text{m/s}$ and $v = 330\,\text{m/s}$:

$$\left(\frac{2 \times 15}{330 - 15}\right) f_0 = 40$$

$$\left(\frac{30}{315}\right) f_0 = 40$$

$$f_0 = 40 \times \frac{315}{30} = 420\,\text{Hz}$$

Conclusion: The frequency of the horn is 420 Hz.

Quick Tip

For reflection off a stationary object, consider the Doppler effect twice: once for the sound traveling to the object, and once for the reflected sound traveling back to the observer.

Question 30: A pole is vertically submerged in swimming pool, such that it gives a length of shadow 2.15 m within water when sunlight is incident at an angle of 30° with the surface of water. If swimming pool is filled to a height of 1.5 m, then the height of the pole above the water surface in centimeters is (n_w = 4/3) ____.

Correct Answer: (50)

Solution:

Step 1: Snell's Law

Snell's Law relates the angle of incidence (i) and angle of refraction (r) for light passing through two different media with refractive indices n_1 and n_2:

$$n_1 \sin i = n_2 \sin r$$

In this case, $n_1 = 1$ (air), $i = 60^\circ$ (since the angle with the surface is 30°), and $n_2 = n_w = \frac{4}{3}$ (water).

Step 2: Calculate the Angle of Refraction

Using Snell's law:

$$\sin 60^\circ = \frac{4}{3} \sin r$$

$$\sin r = \frac{3}{4} \sin 60^\circ = \frac{3}{4} \times \frac{\sqrt{3}}{2} = \frac{3\sqrt{3}}{8}$$

Step 3: Calculate $\tan r$

We first find $\cos r$:

$$\cos r = \sqrt{1 - \sin^2 r} = \sqrt{1 - \left(\frac{3\sqrt{3}}{8}\right)^2} = \sqrt{1 - \frac{27}{64}} = \sqrt{\frac{37}{64}} = \frac{\sqrt{37}}{8}$$

Now we calculate $\tan r$:

$$\tan r = \frac{\sin r}{\cos r} = \frac{3\sqrt{3}/8}{\sqrt{37}/8} = \frac{3\sqrt{3}}{\sqrt{37}} \approx 0.85$$

Step 4: Calculate the Length of the Pole Submerged in Water

Let x be the horizontal length of the shadow. We have $\tan r = \frac{x}{1.5\,\text{m}}$, where 1.5 m is the depth of the water.

$$x = 1.5 \tan r = 1.5 \times 0.85 = 1.275\,\text{m}$$

Since the total length of the shadow is 2.15 m, the horizontal distance from the pole to the point where light enters the water is $2.15 - 1.275 = 0.875$ m.

Step 5: Calculate the Height of the Pole Above Water

Let y be the height of the pole above the water. Since the angle of incidence is 30° with the water surface, we have:

$$\tan 30^\circ = \frac{y}{0.875}$$

$$y = 0.875 \tan 30^\circ = 0.875 \times \frac{1}{\sqrt{3}} \approx 0.875 \times 0.577 \approx 0.50\,\text{m} = 50\,\text{cm}$$

Conclusion: The height of the pole above the water surface is 50 cm.

Quick Tip

Remember Snell's law and the basic trigonometric relationships in a right-angled triangle. Clear diagrams are very helpful for these types of problems.
