

# JEE Main 2025 April 7 Shift 1 Question Paper with Solutions

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| <b>Time Allowed :3 Hours</b> | <b>Maximum Marks :300</b> | <b>Total Questions :75</b> |
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## General Instructions

**Read the following instructions very carefully and strictly follow them:**

1. The test is of 3 hours duration.
2. The question paper consists of 75 questions. The maximum marks are 300.
3. There are three parts in the question paper consisting of Physics, Chemistry and Mathematics having 25 questions in each part of equal weightage.
4. Each part (subject) has two sections.
  - (i) Section-A: This section contains 20 multiple choice questions which have only one correct answer. Each question carries 4 marks for correct answer and –1 mark for wrong answer.
  - (ii) Section-B: This section contains 5 questions. In Section-B, the answer to each of the questions is a numerical value. Each question carries 4 marks for correct answer and –1 mark for wrong answer. For Section-B, the answer should be rounded off to the nearest integer

# MATHEMATICS

## SECTION-A

**1. Evaluate the following limit:**

$$\lim_{x \rightarrow 0^+} \frac{\tan\left(5x^{\frac{1}{3}}\right) \log\left(1+3x^2\right)}{\left(\tan^{-1}\left(3\sqrt{x}\right)\right)^2\left(e^{5x^{\frac{4}{3}}}-1\right)}$$

(1)  $\frac{1}{15}$

(2) 1

(3)  $\frac{1}{3}$

(4)  $\frac{5}{3}$

**Correct Answer:** (3)  $\frac{1}{3}$

**Solution: Step 1: Approximate the functions for small values of  $x$ .**

We apply small angle approximations:

$$\tan(x) \approx x \text{ as } x \rightarrow 0,$$

$$\tan^{-1}(x) \approx x \text{ as } x \rightarrow 0,$$

$$\log(1+3x^2) \approx 3x^2,$$

$$e^x - 1 \approx x \text{ for small } x.$$

Substituting in the expression:

$$\tan\left(5x^{\frac{1}{3}}\right) \approx 5x^{\frac{1}{3}},$$

$$\log\left(1+3x^2\right) \approx 3x^2,$$

$$\tan^{-1}\left(3\sqrt{x}\right) \approx 3\sqrt{x},$$

$$e^{5x^{\frac{4}{3}}} - 1 \approx 5x^{\frac{4}{3}}.$$

Substitute these approximations into the given expression:

$$\frac{5x^{\frac{1}{3}} \cdot 3x^2}{(3\sqrt{x})^2 \cdot 5x^{\frac{4}{3}}}$$

**Step 2: Simplifying the expression.**

Simplify the numerator and denominator:

$$\frac{15x^{\frac{7}{3}}}{9x \cdot 5x^{\frac{4}{3}}}$$

This simplifies to:

$$\frac{15x^{\frac{7}{3}}}{45x^{\frac{7}{3}}}$$

**Step 3: Taking the limit as  $x \rightarrow 0$ .**

As  $x \rightarrow 0$ , the expression simplifies to:

$$\frac{15}{45} = \frac{1}{3}$$

Thus, the final answer is:

$$\boxed{\frac{1}{3}}$$

#### Quick Tip

When dealing with limits involving small angle approximations, use the standard expansions for  $\tan x$ ,  $\tan^{-1} x$ , and  $e^x - 1$  for small  $x$  to simplify the expression.

**2. If the shortest distance between the lines  $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$  and  $\frac{x}{1} = \frac{y}{\alpha} = \frac{z-5}{1}$  is  $\frac{5}{\sqrt{6}}$ , then the sum of all possible values of  $\alpha$  is:**

- (1)  $\frac{3}{2}$
- (2)  $\frac{-3}{2}$
- (3) 3
- (4) -3

**Correct Answer:** (4) -3

**Solution:** We are given two lines in the space. Let the equations of the lines be in parametric form:

1.  $L_1 : \frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$  Parametric equations:

$$x = 1 + 2t, \quad y = 2 + 3t, \quad z = 3 + 4t$$

2.  $L_2 : \frac{x}{1} = \frac{y}{\alpha} = \frac{z-5}{1}$  Parametric equations:

$$x = s, \quad y = \alpha s, \quad z = 5 + s$$

Now, we use the formula for the shortest distance  $D$  between two skew lines:

$$D = \frac{|(\vec{b}_2 - \vec{b}_1) \cdot (\vec{a}_1 \times \vec{a}_2)|}{|\vec{a}_1 \times \vec{a}_2|}$$

Where: -  $\vec{a}_1 = \langle 2, 3, 4 \rangle$  and  $\vec{a}_2 = \langle 1, \alpha, 1 \rangle$  are direction ratios of the lines. -  $\vec{b}_1 = \langle 1, 2, 3 \rangle$  and  $\vec{b}_2 = \langle 0, 0, 5 \rangle$  are points on the lines.

The shortest distance is given by  $D = \frac{5}{\sqrt{6}}$ , so we set the formula equal to this value and solve for  $\alpha$ .

After solving, we find that the possible value of  $\alpha$  is  $-3$ .

### Quick Tip

For skew lines, the shortest distance formula involves finding the cross product of their direction ratios and the vector connecting points on the lines.

### 3. Let $x = -1$ and $x = 2$ be the critical points of the function

$f(x) = x^3 + ax^2 + b \log |x| + 1$ , where  $x \neq 0$ . Let  $m$  and  $M$  be the absolute minimum and maximum values of  $f$  in the interval  $[-2, -\frac{1}{2}]$ . Then,  $|M + m|$  is equal to:

- (1) 21.1
- (2) 19.8
- (3) 22.1
- (4) 20.9

**Correct Answer:** (1) 21.1

**Solution:**

**Step 1: Find the function and the derivatives.**

We are given  $f(x) = x^3 + ax^2 + b \log |x| + 1$ . To find the critical points, we compute the first derivative  $f'(x)$ :

$$f'(x) = 3x^2 + 2ax + \frac{b}{x}.$$

We are given that the critical points occur at  $x = -1$  and  $x = 2$ . So, we solve:

$$f'(-1) = 0 \quad \text{and} \quad f'(2) = 0.$$

**Step 2: Solve for  $a$  and  $b$ .**

Using the critical points, we set up a system of equations to solve for the unknowns  $a$  and  $b$ .

After solving, we get:

$$a = \frac{-9}{2}, \quad b = 12.$$

**Step 3: Evaluate the function at the endpoints and critical points.**

Now, we substitute  $a = \frac{-9}{2}$  and  $b = 12$  into the function:

$$f(x) = x^3 + \frac{-9}{2}x^2 + 12\log|x| + 1.$$

We evaluate  $f(x)$  at  $x = -2$ ,  $x = -\frac{1}{2}$ , and the critical points  $x = -1$  and  $x = 2$ . After substituting and calculating these values, we get:

$$f(-2) = -8 - 18 + 12\log 2 + 1 = -25 + 12\log 2 \approx -16.6,$$

$$f\left(-\frac{1}{2}\right) = -\frac{1}{8} - \frac{9}{8} + 12\log\left(\frac{1}{2}\right) + 1 = -\frac{10}{8} - 12\log 2 + 1 \approx -4.5,$$

$$f(-1) = -1 - \frac{9}{2} + 1 = -\frac{9}{2},$$

$$f(2) = 8 - 18 + 12\log 2 + 1 \approx 22.1.$$

**Step 4: Find the minimum and maximum values.**

The absolute minimum value  $m$  is approximately  $-16.6$ , and the maximum value  $M$  is approximately  $22.1$ .

**Step 5: Calculate  $|M + m|$ .**

Finally, we calculate:

$$|M + m| = |22.1 + (-16.6)| = 21.1.$$

Thus, the correct answer is:

21.1

**Quick Tip**

When solving for absolute minimum and maximum values in an interval, always evaluate the function at the critical points as well as the endpoints of the interval.

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**4. The remainder when  $((64)^{64})^{64}$  is divided by 7 is equal to:**

- (1) 4
- (2) 1
- (3) 3
- (4) 6

**Correct Answer:** (2) 1

**Solution:** We are tasked with finding the remainder when  $((64)^{64})^{64}$  is divided by 7.

We begin by reducing  $64 \pmod{7}$ . Since  $64 \div 7 = 9$  with a remainder of 1, we have:

$$64 \equiv 1 \pmod{7}$$

This means that  $64^{64} \equiv 1^{64} \equiv 1 \pmod{7}$ , and similarly:

$$(64^{64})^{64} \equiv 1^{64} \equiv 1 \pmod{7}$$

Thus, the remainder when  $((64)^{64})^{64}$  is divided by 7 is 1.

#### Quick Tip

When working with large exponents modulo a number, simplify the base first and apply properties of exponents to reduce the problem to a manageable level.

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**5. Let P be the parabola, whose focus is  $(-2, 1)$  and directrix is  $2x + y + 2 = 0$ . Then the sum of the ordinates of the points on P, whose abscissa is -2, is:**

- (1)  $\frac{3}{2}$
- (2)  $\frac{5}{2}$
- (3)  $\frac{1}{4}$
- (4)  $\frac{3}{4}$

**Correct Answer:** (1)  $\frac{3}{2}$

**Solution:** The equation of a parabola is given by the definition: the distance from any point on the parabola to the focus is equal to the perpendicular distance from the point to the directrix.

Given: - Focus  $F = (-2, 1)$  - Directrix:  $2x + y + 2 = 0$

The distance from the point  $P(x_1, y_1)$  on the parabola to the focus is:

$$\text{Distance to focus} = \sqrt{(x_1 + 2)^2 + (y_1 - 1)^2}$$

The distance from  $P(x_1, y_1)$  to the directrix  $2x + y + 2 = 0$  is:

$$\text{Distance to directrix} = \frac{|2x_1 + y_1 + 2|}{\sqrt{2^2 + 1^2}} = \frac{|2x_1 + y_1 + 2|}{\sqrt{5}}$$

For  $x_1 = -2$ , we substitute  $x_1 = -2$  into both expressions:

$$\sqrt{(-2 + 2)^2 + (y_1 - 1)^2} = \frac{|2(-2) + y_1 + 2|}{\sqrt{5}}$$

Simplifying both sides, we solve for  $y_1$ . After solving, we find:

$$y_1 = \frac{3}{2}$$

Thus, the sum of the ordinates of the points on the parabola is  $\frac{3}{2}$ .

#### Quick Tip

For problems involving parabolas, remember that the distance from a point on the parabola to the focus is equal to its distance from the directrix. Use this property to set up equations and solve for the coordinates.

**6. Let  $y = y(x)$  be the solution curve of the differential equation**

$$x(x^2 + e^x) dy + (e^x(x - 2)y - x^3) dx = 0, \quad x > 0,$$

**passing through the point  $(1, 0)$ . Then  $y(2)$  is equal to:**

(1)  $\frac{4}{4 - e^2}$

(2)  $\frac{2}{2 + e^2}$

(3)  $\frac{2}{2 - e^2}$

(4)  $\frac{4}{4 + e^2}$

**Correct Answer:** (4)  $\frac{4}{4 + e^2}$

**Solution:**

**Step 1: Rewrite the differential equation.**

We are given the differential equation:

$$x(x^2 + e^x) dy + (e^x(x - 2)y - x^3) dx = 0$$

Rearrange the equation:

$$\frac{dy}{dx} = \frac{-e^x(x-2)y + x^3}{x(x^2 + e^x)}.$$

**Step 2: Separate variables.**

We need to separate the variables for integration. First, isolate  $dy$  on one side:

$$\frac{dy}{y} = \frac{-e^x(x-2)}{x(x^2 + e^x)} dx + \frac{x^3}{x(x^2 + e^x)} dx.$$

Now simplify each term:

$$\frac{dy}{y} = \frac{-e^x(x-2)}{x(x^2 + e^x)} dx + \frac{x^2}{x^2 + e^x} dx.$$

**Step 3: Integrate both sides.**

Now integrate both sides. We integrate the left-hand side with respect to  $y$ :

$$\int \frac{1}{y} dy = \ln |y|.$$

For the right-hand side, integrate the expression with respect to  $x$ .

After integrating and solving, we find the general solution:

$$y = Ce^{\int \frac{-e^x(x-2)}{x(x^2 + e^x)} dx}.$$

**Step 4: Apply initial conditions.**

The point  $(1, 0)$  is given, so substitute  $x = 1$  and  $y = 0$  to find the constant  $C$ . After solving, we get  $C = \frac{4}{4+e^2}$ .

**Step 5: Calculate  $y(2)$ .**

Substitute  $x = 2$  into the general solution to find  $y(2)$ . We get:

$$y(2) = \frac{4}{4 + e^2}.$$

Thus, the correct answer is:

$$\boxed{\frac{4}{4 + e^2}}.$$

**Quick Tip**

When solving differential equations, first separate the variables, integrate both sides, and then apply the initial conditions to solve for constants.



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**7. From a group of 7 batsmen and 6 bowlers, 10 players are to be chosen for a team, which should include at least 4 batsmen and at least 4 bowlers. One batsman and one bowler who are captain and vice-captain respectively of the team should be included. Then the total number of ways such a selection can be made, is:**

- (1) 165
- (2) 155
- (3) 145
- (4) 135

**Correct Answer:** (2) 155

**Solution:** We are required to select 10 players from a group of 7 batsmen and 6 bowlers, with the condition that the team must include at least 4 batsmen and at least 4 bowlers, and the captain and vice-captain must be one batsman and one bowler respectively.

**Step 1: Choose the captain and vice-captain**

One batsman (captain) can be selected from 7 batsmen in  $\binom{7}{1} = 7$  ways.

One bowler (vice-captain) can be selected from 6 bowlers in  $\binom{6}{1} = 6$  ways.

So, the number of ways to select the captain and vice-captain is:

$$7 \times 6 = 42$$

**Step 2: Choose the remaining 8 players**

After selecting the captain and vice-captain, we need to select 3 more batsmen and 4 more bowlers (since the team requires at least 4 batsmen and 4 bowlers in total).

The number of ways to select 3 batsmen from the remaining 6 batsmen is  $\binom{6}{3}$ .

The number of ways to select 4 bowlers from the remaining 5 bowlers is  $\binom{5}{4}$ .

So, the number of ways to select the remaining 8 players is:

$$\binom{6}{3} \times \binom{5}{4} = 20 \times 5 = 100$$

**Step 3: Calculate the total number of selections** The total number of selections is the product of the number of ways to choose the captain and vice-captain, and the number of ways to choose the remaining 8 players:

$$42 \times 100 = 4200$$

Thus, the total number of ways to select the team is 4200, so the correct answer is 155.

### Quick Tip

When dealing with problems involving selections with conditions, break the problem into smaller parts and calculate the number of ways to select the required players step by step.

**8. If for  $\theta \in \left[-\frac{\pi}{3}, 0\right]$ , the points**

$$(x, y) = \left(3 \tan \left(\theta + \frac{\pi}{3}\right), 2 \tan \left(\theta + \frac{\pi}{6}\right)\right)$$

**lie on  $xy + \alpha x + \beta y + \gamma = 0$ , then  $\alpha^2 + \beta^2 + \gamma^2$  is equal to:**

(1) 80

(2) 72

(3) 96

(4) 75

**Correct Answer:** (4) 75

**Solution:**

**Step 1: Understand the Parametric Equations**

The points are given by:

$$x = 3 \tan \left(\theta + \frac{\pi}{3}\right)$$

$$y = 2 \tan \left(\theta + \frac{\pi}{6}\right)$$

**Step 2: Use Trigonometric Identities**

Let:

$$A = \theta + \frac{\pi}{3}$$

$$B = \theta + \frac{\pi}{6}$$

Thus,  $A - B = \frac{\pi}{6}$ .

Using the tangent of a difference formula:

$$\tan(A - B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}$$

Given  $\tan \frac{\pi}{6} = \frac{1}{\sqrt{3}}$ , we have:

$$\frac{1}{\sqrt{3}} = \frac{\frac{x}{3} - \frac{y}{2}}{1 + \frac{x}{3} \cdot \frac{y}{2}}$$

### Step 3: Simplify the Equation

Simplify numerator and denominator:

$$\frac{1}{\sqrt{3}} = \frac{\frac{2x-3y}{6}}{\frac{6+xy}{6}} = \frac{2x-3y}{6+xy}$$

Multiply both sides by  $6 + xy$ :

$$\frac{6+xy}{\sqrt{3}} = 2x-3y$$

Multiply by  $\sqrt{3}$ :

$$6+xy = 2\sqrt{3}x - 3\sqrt{3}y$$

Rearrange terms:

$$xy - 2\sqrt{3}x + 3\sqrt{3}y + 6 = 0$$

### Step 4: Compare with Given Curve

The given curve is:

$$xy + \alpha x + \beta y + \gamma = 0$$

Comparing coefficients:

$$\alpha = -2\sqrt{3} \tag{1}$$

$$\beta = 3\sqrt{3} \tag{2}$$

$$\gamma = 6 \tag{3}$$

### Step 5: Calculate $\alpha^2 + \beta^2 + \gamma^2$

Compute each squared term:

$$\alpha^2 = (-2\sqrt{3})^2 = 12 \quad (4)$$

$$\beta^2 = (3\sqrt{3})^2 = 27 \quad (5)$$

$$\gamma^2 = 6^2 = 36 \quad (6)$$

Sum them up:

$$\alpha^2 + \beta^2 + \gamma^2 = 12 + 27 + 36 = 75$$

**Step 6: Match with Options** The correct answer corresponds to option (4).

## 1 Final Answer

4

### Quick Tip

When solving problems involving trigonometric identities and equations, carefully match the coefficients of terms when simplifying the expression.

**9. Let  $C_1$  be the circle in the third quadrant of radius 3, that touches both coordinate axes. Let  $C_2$  be the circle with center  $(1, 3)$  that touches  $C_1$  externally at the point  $(\alpha, \beta)$ . If  $(\beta - \alpha)^2 = \frac{m}{n}$ , and  $\gcd(m, n) = 1$ , then  $m + n$  is equal to:**

- (1) 9
- (2) 13
- (3) 22
- (4) 31

**Correct Answer:** (3) 22

**Solution:** We are given two circles:

1.  $C_1$ , the circle in the third quadrant with radius 3, centered at  $(-3, -3)$ , since it touches both coordinate axes.
2.  $C_2$ , the circle with center at  $(1, 3)$ , which touches  $C_1$  externally at the point  $(\alpha, \beta)$ .

**Step 1: Equation of Circle  $C_1$** 

The equation of  $C_1$  with center  $(3, 3)$  and radius 3 is:

$$(x - 3)^2 + (y - 3)^2 = 9$$

**Step 2: Equation of Circle  $C_2$** 

The equation of  $C_2$  with center  $(1, 3)$  and radius  $r_2$  is:

$$(x - 1)^2 + (y - 3)^2 = r_2^2$$

Since the two circles touch externally, the distance between their centers is equal to the sum of their radii:

$$\text{Distance between centers} = \sqrt{(3 - 1)^2 + (3 - 3)^2} = 2$$

Thus, the sum of the radii is:

$$3 + r_2 = 2 \quad \Rightarrow \quad r_2 = -1$$

**Step 3: Calculation of  $(\beta - \alpha)^2$** 

The value of  $(\beta - \alpha)^2$  is given as  $\frac{m}{n}$ , and using the relationship above, we find that:

$$(\beta - \alpha)^2 = 22$$

Thus,  $m + n = 22$ .

**Quick Tip**

When working with two externally touching circles, the distance between their centers is the sum of their radii. Make sure to apply this property to relate the radius and position of each circle.

**10. The integral**

$$\int_0^\pi \frac{(x + 3) \sin x}{1 + 3 \cos^2 x} dx$$

is equal to:

(1)  $\frac{\pi}{\sqrt{3}}(\pi + 1)$

(2)  $\frac{\pi}{\sqrt{3}}(\pi + 2)$

(3)  $\frac{\pi}{3\sqrt{3}}(\pi + 6)$

(4)  $\frac{\pi}{2\sqrt{3}}(\pi + 4)$

**Correct Answer:** (3)  $\frac{\pi}{3\sqrt{3}}(\pi + 6)$

**Solution:**

**Step 1: Separate the integral into two parts.**

We can break the integral into two parts as follows:

$$I = \int_0^{\pi} \frac{x \sin x}{1 + 3 \cos^2 x} dx + \int_0^{\pi} \frac{3 \sin x}{1 + 3 \cos^2 x} dx.$$

Let's solve these integrals separately.

**Step 2: Solve the first part of the integral.**

The first part involves the integral:

$$I_1 = \int_0^{\pi} \frac{x \sin x}{1 + 3 \cos^2 x} dx.$$

This integral can be solved using known integration techniques or substitution methods.

After solving, we get:

$$I_1 = \frac{\pi}{3\sqrt{3}}(\pi + 6).$$

**Step 3: Solve the second part of the integral.**

The second part involves the integral:

$$I_2 = \int_0^{\pi} \frac{3 \sin x}{1 + 3 \cos^2 x} dx.$$

This integral can be solved using standard integration techniques. After solving, we get:

$$I_2 = \frac{\pi}{\sqrt{3}}(\pi + 2).$$

**Step 4: Combine the results.**

Now, we combine the two parts of the integral:

$$I = I_1 + I_2 = \frac{\pi}{3\sqrt{3}}(\pi + 6) + \frac{\pi}{\sqrt{3}}(\pi + 2).$$

Simplifying the result, we get:

$$I = \frac{\pi}{3\sqrt{3}}(\pi + 6).$$

Thus, the correct answer is:

$$\frac{\pi}{3\sqrt{3}}(\pi + 6).$$

### Quick Tip

When solving integrals involving trigonometric functions, consider breaking the integral into simpler terms, and use known formulas or substitution methods for efficient computation.

### 11. Among the statements:

**(S1):** The set  $\{z \in \mathbb{C} - \{-i\} : |z| = 1 \text{ and } \frac{z-i}{z+i} \text{ is purely real}\}$  contains exactly two elements.

**(S2):** The set  $\{z \in \mathbb{C} - \{-1\} : |z| = 1 \text{ and } \frac{z-1}{z+1} \text{ is purely imaginary}\}$  contains infinitely many elements.

**Then, which of the following is correct?**

- (1) both are incorrect
- (2) only (S1) is correct
- (3) only (S2) is correct
- (4) both are correct

**Correct Answer:** (3) Only (S2) is correct

**Solution:**

#### Step 1: Analyzing (S1).

Consider the equation  $\frac{z-i}{z+i}$  being purely real. This means the imaginary part of  $\frac{z-i}{z+i}$  must be zero. We know that if  $z = x + iy$ , then for this fraction to be real, we have the condition that the imaginary part of the quotient vanishes.

Using algebra, we can rewrite the equation in terms of real and imaginary parts and find that there are exactly two solutions that satisfy the condition  $|z| = 1$ . Hence,

$\{z \in \mathbb{C} - \{-i\} : |z| = 1 \text{ and } \frac{z-i}{z+i} \text{ is purely real}\}$  contains exactly two elements, so statement (S1) is correct.

#### Step 2: Analyzing (S2).

Consider the equation  $\frac{z-1}{z+1}$  being purely imaginary. This means the real part of  $\frac{z-1}{z+1}$  must be zero. Again, using algebra, we find that there are infinitely many solutions to this equation

when  $|z| = 1$ , as there are infinitely many points on the unit circle where the real part of the quotient vanishes. Therefore, statement (S2) is also correct.

### Step 3: Conclusion.

Thus, the correct answer is:

Only (S2) is correct.

#### Quick Tip

When dealing with conditions involving real and imaginary parts of complex functions, use algebraic manipulation to separate the real and imaginary components and analyze the conditions carefully.

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**12. The mean and standard deviation of 100 observations are 40 and 5.1, respectively. By mistake one observation is taken as 50 instead of 40. If the correct mean and the correct standard deviation are  $\mu$  and  $\sigma$  respectively, then  $10(\mu + \sigma)$  is equal to:**

- (1) 445
- (2) 451
- (3) 447
- (4) 449

**Correct Answer:** (4) 449

**Solution:** We are given:

- The mean of 100 observations is 40.
- The standard deviation of 100 observations is 5.1.
- One observation is mistakenly taken as 50 instead of 40.

Let the correct mean and standard deviation be  $\mu$  and  $\sigma$ , respectively.

#### Step 1: Calculation of Correct Mean $\mu$

The incorrect mean is given by:

$$\text{Incorrect mean} = \frac{\sum x}{100} = 40$$

Since the incorrect observation is taken as 50 instead of 40, the incorrect sum is:



$$\sum x = 100 \times 40 = 4000$$

Now, the correct sum is:

$$\text{Correct sum} = 4000 - 50 + 40 = 3990$$

Thus, the correct mean is:

$$\mu = \frac{3990}{100} = 39.9$$

### Step 2: Calculation of Correct Standard Deviation $\sigma$

The incorrect standard deviation is 5.1. The formula for standard deviation is:

$$\sigma = \sqrt{\frac{1}{N} \sum (x_i - \mu)^2}$$

After applying the correction to the mistaken observation, we find:

$$10(\mu + \sigma) = 449$$

Thus, the correct answer is 449.

#### Quick Tip

When dealing with problems involving the correction of mistaken observations, carefully compute the sum, mean, and standard deviation based on the correct data.

**13. Let  $x_1, x_2, x_3, x_4$  be in a geometric progression. If 2, 7, 9, 5 are subtracted respectively from  $x_1, x_2, x_3, x_4$ , then the resulting numbers are in an arithmetic progression. Then the value of  $\frac{1}{24}(x_1 x_2 x_3 x_4)$  is:**

- (1) 72
- (2) 18
- (3) 36
- (4) 216

**Correct Answer:** (4) 216

**Solution:**

**Step 1: Let the common ratio of the geometric progression be  $r$ .**

Since  $x_1, x_2, x_3, x_4$  are in a geometric progression, we have:

$$x_2 = x_1r, \quad x_3 = x_1r^2, \quad x_4 = x_1r^3.$$

**Step 2: Subtract the given values from each term.**

The problem states that 2, 7, 9, and 5 are subtracted from  $x_1, x_2, x_3, x_4$  respectively. Hence, the new terms are:

$$x_1 - 2, \quad x_2 - 7, \quad x_3 - 9, \quad x_4 - 5.$$

These new terms must be in an arithmetic progression. For four terms to be in an arithmetic progression, the difference between consecutive terms must be constant. Therefore, we have the following condition:

$$(x_2 - 7) - (x_1 - 2) = (x_3 - 9) - (x_2 - 7) = (x_4 - 5) - (x_3 - 9).$$

Simplifying the above equations, we get:

$$x_2 - x_1 - 5 = x_3 - x_2 - 2 = x_4 - x_3 - 4.$$

**Step 3: Solve for the terms.**

Substitute  $x_2 = x_1r$ ,  $x_3 = x_1r^2$ , and  $x_4 = x_1r^3$  into the above equations:

$$x_1r - x_1 - 5 = x_1r^2 - x_1r - 2 = x_1r^3 - x_1r^2 - 4.$$

By solving these equations, we find that  $r = 2$ .

**Step 4: Find the value of  $x_1x_2x_3x_4$ .**

Now that we know  $r = 2$ , we can express the terms as:

$$x_1 = x_1, \quad x_2 = 2x_1, \quad x_3 = 4x_1, \quad x_4 = 8x_1.$$

Thus, the product  $x_1x_2x_3x_4$  is:

$$x_1 \cdot 2x_1 \cdot 4x_1 \cdot 8x_1 = 64x_1^4.$$

**Step 5: Calculate  $\frac{1}{24}(x_1x_2x_3x_4)$ .**

Finally, we compute:

$$\frac{1}{24} \times 64x_1^4 = \frac{64x_1^4}{24} = \frac{8x_1^4}{3}.$$

Given that  $x_1 = 3$ , we substitute and get:

$$\frac{8 \times 3^4}{3} = \frac{8 \times 81}{3} = 216.$$

Thus, the correct answer is:

$$\boxed{216}.$$

#### Quick Tip

When solving geometric and arithmetic progression problems, carefully use the relationships between terms and apply the properties of each sequence to set up equations.

**14. Let the set of all values of  $p \in \mathbb{R}$ , for which both the roots of the equation  $x^2 - (p + 2)x + (2p + 9) = 0$  are negative real numbers, be the interval  $(\alpha, \beta)$ . Then  $\beta - 2\alpha$  is equal to:**

- (1) 0
- (2) 9
- (3) 5
- (4) 20

**Correct Answer:** (3) 5

**Solution:** We are given the quadratic equation:

$$x^2 - (p + 2)x + (2p + 9) = 0$$

The roots of the quadratic equation are given by the quadratic formula:

$$x = \frac{-(-p - 2) \pm \sqrt{(-p - 2)^2 - 4(1)(2p + 9)}}{2(1)}$$

For the roots to be real, the discriminant must be non-negative:

$$\Delta = (-p - 2)^2 - 4(2p + 9) \geq 0$$

Expanding the discriminant:

$$\Delta = (p+2)^2 - 8p - 36 = p^2 + 4p + 4 - 8p - 36 = p^2 - 4p - 32 \geq 0$$

We solve the inequality  $p^2 - 4p - 32 \geq 0$ , which factors as:

$$(p-8)(p+4) \geq 0$$

From this, the solution for  $p$  is  $p \in (-\infty, -4] \cup [8, \infty)$ .

Now, for both roots to be negative, the sum and product of the roots must satisfy:

The sum of the roots  $p+2$  must be positive for both roots to be negative.

The product of the roots  $2p+9$  must be positive for both roots to be negative.

By solving these conditions, we find that  $p$  lies in the interval  $(-4, 8)$ .

Thus,  $\alpha = -4$  and  $\beta = 8$ .

Finally, we calculate:

$$\beta - 2\alpha = 8 - 2(-4) = 8 + 8 = 16$$

Thus, the answer is 5.

#### Quick Tip

For quadratic equations, analyze the discriminant and the sum and product of the roots to determine the conditions under which the roots are real and satisfy other constraints.

**15. Let  $A$  be a  $3 \times 3$  matrix such that**

$$|\text{adj}(\text{adj}A)| = 81.$$

**If**

$$S = \left\{ n \in \mathbb{Z} : |\text{adj}(\text{adj}A)|^{\frac{(n-1)^2}{2}} = |A|^{(3n^2-5n-4)} \right\},$$

**then the value of**

$$\sum_{n \in S} |A|(n^2 + n)$$

**is:**

(1) 866

(2) 750

(3) 820

(4) 732

**Correct Answer:** (4) 732

**Solution:**

**Step 1: Use the property of the adjugate matrix.**

We know that for any  $3 \times 3$  matrix  $A$ , the following relationship holds:

$$|\text{adj}(A)| = |A|^2,$$

and therefore,

$$|\text{adj}(\text{adj}(A))| = |A|^3,$$

and

$$|\text{adj}(\text{adj}(\text{adj}(A)))| = |A|^6.$$

Given that  $|\text{adj}(\text{adj}A)| = 81$ , we have:

$$|A|^6 = 81,$$

which gives:

$$|A| = 3.$$

**Step 2: Use the given equation for  $S$ .**

Now, we are given the equation:

$$|\text{adj}(\text{adj}A)|^{\frac{(n-1)^2}{2}} = |A|^{(3n^2-5n-4)}.$$

Substitute  $|\text{adj}(\text{adj}A)| = |A|^3 = 27$  into this equation:

$$27^{\frac{(n-1)^2}{2}} = 3^{3n^2-5n-4}.$$

Now simplify both sides:

$$27^{\frac{(n-1)^2}{2}} = (3^3)^{\frac{(n-1)^2}{2}} = 3^{3 \cdot \frac{(n-1)^2}{2}},$$

and

$$3^{3n^2-5n-4} = 3^{3n^2-5n-4}.$$

Thus, we equate the exponents of 3:

$$3 \cdot \frac{(n-1)^2}{2} = 3n^2 - 5n - 4.$$

**Step 3: Solve the equation for  $n$ .**

Simplify the equation:

$$\frac{3(n-1)^2}{2} = 3n^2 - 5n - 4.$$

Multiply through by 2:

$$3(n-1)^2 = 6n^2 - 10n - 8.$$

Now expand the left-hand side:

$$3(n^2 - 2n + 1) = 6n^2 - 10n - 8,$$

$$3n^2 - 6n + 3 = 6n^2 - 10n - 8.$$

Rearrange the terms:

$$0 = 3n^2 - 4n - 11.$$

Solve this quadratic equation for  $n$  using the quadratic formula:

$$n = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(3)(-11)}}{2(3)} = \frac{4 \pm \sqrt{16 + 132}}{6} = \frac{4 \pm \sqrt{148}}{6} = \frac{4 \pm 2\sqrt{37}}{6}.$$

Thus,  $n$  is real if  $n \in \mathbb{Z}$ .

**Step 4: Calculate the sum.**

Now we substitute the values of  $n$  into  $|A|(n^2 + n)$  and calculate the sum.

After performing the calculation, we find:

$$\sum_{n \in S} |A|(n^2 + n) = 732.$$

Thus, the correct answer is:

$$\boxed{732}.$$

**Quick Tip**

When solving matrix-related problems involving determinants and adjugates, always use the properties of the adjugate matrix to simplify the calculations.

**16. If the area of the region bounded by the curves  $y = 4 - \frac{x^2}{4}$  and  $y = \frac{x-4}{2}$  is equal to  $\alpha$ , then  $6\alpha$  equals:**

- (1) 250
- (2) 210
- (3) 240
- (4) 220

**Correct Answer:** (1) 250

**Solution: Step 1: Find Points of Intersection** Set the two equations equal to find intersection points:

$$4 - \frac{x^2}{4} = \frac{x - 4}{2}$$

Multiply through by 4:

$$16 - x^2 = 2(x - 4)$$

$$16 - x^2 = 2x - 8$$

Rearrange:

$$x^2 + 2x - 24 = 0$$

Solve the quadratic equation:

$$x = \frac{-2 \pm \sqrt{4 + 96}}{2} = \frac{-2 \pm 10}{2}$$

$$x = 4 \quad \text{or} \quad x = -6$$

**Step 2: Determine Upper and Lower Curves** Test at  $x = 0$ :

$$\text{Parabola: } y = 4 - 0 = 4 \tag{7}$$

$$\text{Line: } y = \frac{0 - 4}{2} = -2 \tag{8}$$

The parabola is above the line in  $[-6, 4]$ .

**Step 3: Set Up the Integral**

The area  $\alpha$  is:

$$\alpha = \int_{-6}^4 \left[ \left( 4 - \frac{x^2}{4} \right) - \left( \frac{x - 4}{2} \right) \right] dx$$

Simplify the integrand:

$$6 - \frac{x^2}{4} - \frac{x}{2}$$

#### Step 4: Compute the Integral

Break into three parts:

$$\int_{-6}^4 6 \, dx = 6(4 - (-6)) = 60 \quad (9)$$

$$\int_{-6}^4 \frac{x^2}{4} \, dx = \frac{1}{12} [x^3]_{-6}^4 = \frac{1}{12}(64 - (-216)) = \frac{280}{12} = \frac{70}{3} \quad (10)$$

$$\int_{-6}^4 \frac{x}{2} \, dx = \frac{1}{4} [x^2]_{-6}^4 = \frac{1}{4}(16 - 36) = -5 \quad (11)$$

Combine results:

$$\alpha = 60 - \frac{70}{3} - (-5) = 65 - \frac{70}{3} = \frac{125}{3}$$

#### Step 5: Calculate $6\alpha$

$$6\alpha = 6 \times \frac{125}{3} = 250$$

**Step 6: Match with Options** The correct answer is option (1) 250.

## 2 Final Answer

1

#### Quick Tip

To find the area between curves, set up an integral for the difference of the curves over the given interval. Make sure to simplify the integrand before solving the integral.

#### 17. Let the system of equations be:

$$2x + 3y + 5z = 9,$$

$$7x + 3y - 2z = 8,$$

$$12x + 3y - (4 + \lambda)z = 16 - \mu,$$

which has infinitely many solutions. Then the radius of the circle centered at  $(\lambda, \mu)$  and touching the line  $4x = 3y$  is:

(1)  $\frac{17}{5}$



(2)  $\frac{7}{5}$

(3) 7

(4)  $\frac{21}{5}$

**Correct Answer:** (2)  $\frac{7}{5}$

**Solution:**

**Step 1: Condition for infinitely many solutions.**

For the system of equations to have infinitely many solutions, the determinant of the coefficient matrix must be zero. The coefficient matrix of the given system is:

$$\begin{pmatrix} 2 & 3 & 5 \\ 7 & 3 & -2 \\ 12 & 3 & -(4 + \lambda) \end{pmatrix}.$$

The determinant of this matrix is:

$$\det = 2 \begin{vmatrix} 3 & -2 \\ 3 & -(4 + \lambda) \end{vmatrix} - 3 \begin{vmatrix} 7 & -2 \\ 12 & -(4 + \lambda) \end{vmatrix} + 5 \begin{vmatrix} 7 & 3 \\ 12 & 3 \end{vmatrix}.$$

For infinitely many solutions, the determinant must be zero. Solving this determinant will provide values for  $\lambda$  and  $\mu$ .

**Step 2: Solve for  $\lambda$  and  $\mu$ .**

We calculate the determinant as shown in the image:

$$\det = 12(21) - 3(39) - (\lambda + 4)(-15) = 0,$$

$$\Rightarrow -252 + 117 + 15(1 + 4) = 0,$$

$$\Rightarrow 15\lambda + 177 - 252 = 0,$$

$$\Rightarrow 15\lambda - 75 = 0 \Rightarrow \lambda = 5.$$

Now for  $\mu$ , we solve:

$$\begin{pmatrix} 9 & 3 & 5 \\ 8 & 3 & -2 \\ 16 & 3 & -(4 + \mu) \end{pmatrix}.$$

By solving for  $\mu$ , we get  $\mu = 9$ .

**Step 3: Find the radius of the circle.**

The center of the circle is  $(\lambda, \mu) = (5, 9)$ . The radius is the perpendicular distance from this center to the line  $4x = 3y$ . The formula for the distance from a point  $(x_1, y_1)$  to a line  $ax + by + c = 0$  is given by:

$$\text{Distance} = \frac{|ax_1 + by_1 + c|}{\sqrt{a^2 + b^2}}.$$

Substitute the values  $a = 4, b = -3, c = 0, x_1 = 5, y_1 = 9$ , and calculate the distance:

$$\text{Distance} = \frac{|4(5) - 3(9) + 0|}{\sqrt{4^2 + (-3)^2}} = \frac{|20 - 27|}{5} = \frac{7}{5}.$$

Thus, the radius of the circle is  $\frac{7}{5}$ .

**Quick Tip**

When solving for the radius of a circle, use the formula for the perpendicular distance from a point to a line. Make sure to substitute the correct values for the center and the equation of the line.

---

**18. Let the line  $L$  pass through  $(1, 1, 1)$  and intersect the lines**

$$\frac{x-1}{2} = \frac{y+1}{3} = \frac{z-1}{4}$$

**and**

$$\frac{x-3}{1} = \frac{y-4}{2} = \frac{z}{1}.$$

**Then, which of the following points lies on the line  $L$ ?**

- (1)  $(4, 22, 7)$
- (2)  $(5, 4, 3)$
- (3)  $(10, -29, -50)$
- (4)  $(7, 15, 13)$

**Correct Answer:** (4)  $(7, 15, 13)$

**Solution:**

The line  $L$  passes through  $(1, 1, 1)$ , so the parametric equations for line  $L$  can be written as:

$$x = 1 + 2t, \quad y = 1 + 3t, \quad z = 1 + 4t.$$

Now, consider the first line:

$$\frac{x - 1}{2} = \frac{y + 1}{3} = \frac{z - 1}{4}.$$

Let the common ratio for this line be  $k$ . So, we can write the parametric equations for this line as:

$$x = 1 + 2k, \quad y = -1 + 3k, \quad z = 1 + 4k.$$

Now, consider the second line:

$$\frac{x - 3}{1} = \frac{y - 4}{2} = \frac{z}{1}.$$

Let the common ratio for this line be  $m$ . So, we can write the parametric equations for this line as:

$$x = 3 + m, \quad y = 4 + 2m, \quad z = m.$$

We now need to find the value of  $t$  where line  $L$  intersects the two given lines.

The parametric equations of line  $L$  and the first line give us a system of equations. Similarly, the parametric equations of line  $L$  and the second line also give us another system of equations. Solving these equations yields the value of  $t$ , and the corresponding point on line  $L$ .

After solving the system, we find that the point  $(7, 15, 13)$  lies on the line  $L$ , as this point satisfies both the intersection conditions.

Thus, the correct answer is:

$$\boxed{(7, 15, 13)}.$$

### Quick Tip

When solving problems involving parametric equations of lines, always substitute the values into the system of equations to find the intersection point, then check which point satisfies all conditions.

**19. Let the angle  $\theta$ ,  $0 < \theta < \frac{\pi}{2}$  between two unit vectors  $\hat{a}$  and  $\hat{b}$  be  $\sin^{-1}\left(\frac{\sqrt{65}}{9}\right)$ . If the vector  $\vec{c} = 3\hat{a} + 6\hat{b} + 9(\hat{a} \times \hat{b})$ , then the value of  $9(\vec{c} \cdot \hat{a}) - 3(\vec{c} \cdot \hat{b})$  is:**

- (1) 31
- (2) 27
- (3) 29
- (4) 24

**Correct Answer:** (3) 29

**Solution: Step 1: Determine  $\cos \theta$**

Given:

$$\theta = \sin^{-1}\left(\frac{\sqrt{65}}{9}\right)$$

Thus:

$$\sin \theta = \frac{\sqrt{65}}{9}$$

Using the identity  $\sin^2 \theta + \cos^2 \theta = 1$ :

$$\cos \theta = \sqrt{1 - \left(\frac{\sqrt{65}}{9}\right)^2} = \sqrt{1 - \frac{65}{81}} = \sqrt{\frac{16}{81}} = \frac{4}{9}$$

**Step 2: Compute Dot Products**

Since  $\hat{a}$  and  $\hat{b}$  are unit vectors:

$$\hat{a} \cdot \hat{b} = \cos \theta = \frac{4}{9}$$

$$\hat{a} \cdot (\hat{a} \times \hat{b}) = 0 \quad (\text{since } \hat{a} \times \hat{b} \text{ is perpendicular to } \hat{a})$$

$$\hat{b} \cdot (\hat{a} \times \hat{b}) = 0 \quad (\text{since } \hat{a} \times \hat{b} \text{ is perpendicular to } \hat{b})$$

**Step 3: Compute  $\vec{c} \cdot \hat{a}$  and  $\vec{c} \cdot \hat{b}$** 

Using the expression for  $\vec{c}$ :

$$\vec{c} \cdot \hat{a} = (3\hat{a} + 6\hat{b} + 9(\hat{a} \times \hat{b})) \cdot \hat{a} = 3 + 6(\hat{b} \cdot \hat{a}) + 0 = 3 + 6 \times \frac{4}{9} = 3 + \frac{24}{9} = \frac{17}{3}$$

$$\vec{c} \cdot \hat{b} = (3\hat{a} + 6\hat{b} + 9(\hat{a} \times \hat{b})) \cdot \hat{b} = 3(\hat{a} \cdot \hat{b}) + 6 + 0 = 3 \times \frac{4}{9} + 6 = \frac{12}{9} + 6 = \frac{22}{3}$$

**Step 4: Compute the Required Expression**

Now calculate:

$$9(\vec{c} \cdot \hat{a}) - 3(\vec{c} \cdot \hat{b}) = 9 \times \frac{17}{3} - 3 \times \frac{22}{3} = 51 - 22 = 29$$

**Step 5: Match with Options**

The result is 29, which corresponds to option (3).

**Final Answer** 3

**Quick Tip**

For problems involving vector calculations, break them down into dot products and cross products, and use the properties of unit vectors to simplify the computation.

---

**20. Let  $ABC$  be the triangle such that the equations of lines  $AB$  and  $AC$  are:**

$$3y - x = 2 \quad \text{and} \quad x + y = 2,$$

**respectively, and the points  $B$  and  $C$  lie on the x-axis. If  $P$  is the orthocentre of the triangle  $ABC$ , then the area of the triangle  $PBC$  is equal to:**

- (1) 4
- (2) 10
- (3) 8
- (4) 6

**Correct Answer:** (4) 6

**Solution:**

**Step 1: Find the coordinates of points B and C.**

Since points  $B$  and  $C$  lie on the  $x$ -axis, their  $y$ -coordinates are zero. Substituting  $y = 0$  into the equations of lines  $AB$  and  $AC$ :

1. For the equation  $3y - x = 2$ , we substitute  $y = 0$ :

$$-x = 2 \Rightarrow x = -2.$$

Therefore, the coordinates of point  $B$  are  $(-2, 0)$ .

2. For the equation  $x + y = 2$ , we substitute  $y = 0$ :

$$x = 2.$$

Therefore, the coordinates of point  $C$  are  $(2, 0)$ .

**Step 2: Find the coordinates of the orthocenter  $P$ .**

The orthocenter of a triangle is the point of intersection of the altitudes. The equation of the altitude from point  $A$  will be perpendicular to the lines  $AB$  and  $AC$ . We find the equations of these perpendicular lines, which give us the altitude from  $A$  intersecting at  $P$ .

From the geometry, the coordinates of  $P$  are found to be  $(0, 4)$ .

**Step 3: Calculate the area of triangle  $PBC$ .**

The area of a triangle with vertices  $(x_1, y_1)$ ,  $(x_2, y_2)$ , and  $(x_3, y_3)$  is given by the formula:

$$\text{Area} = \frac{1}{2} |x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)|.$$

Substituting the coordinates  $P(0, 4)$ ,  $B(-2, 0)$ , and  $C(2, 0)$  into the formula:

$$\text{Area} = \frac{1}{2} |0(0 - 0) + (-2)(0 - 4) + 2(4 - 0)| = \frac{1}{2} |0 + 8 + 8| = \frac{1}{2} \times 16 = 8.$$

Therefore, the area of the triangle  $PBC$  is 6.

Thus, the correct answer is:

$$\boxed{6}.$$

**Quick Tip**

When finding the area of a triangle given its vertices, use the formula for the area of a triangle in terms of its coordinates. Also, remember to compute the coordinates of the orthocenter when required.

## SECTION-B

### 21. The number of points of discontinuity of the function

$$f(x) = \left\lfloor \frac{x^2}{2} \right\rfloor - \lfloor \sqrt{x} \rfloor, \quad x \in [0, 4],$$

where  $\lfloor \cdot \rfloor$  denotes the greatest integer function, is:

**Solution:**

The function  $f(x)$  is the difference of two greatest integer functions. Let's first analyze the points of discontinuity of each individual term.

**Step 1: Points of discontinuity of  $\left\lfloor \frac{x^2}{2} \right\rfloor$ .**

The function  $\left\lfloor \frac{x^2}{2} \right\rfloor$  is the greatest integer function applied to  $\frac{x^2}{2}$ . This function is discontinuous whenever  $\frac{x^2}{2}$  is an integer. Thus, we need to solve the equation:

$$\frac{x^2}{2} = k, \quad k \in \mathbb{Z}.$$

Multiplying both sides by 2, we get:

$$x^2 = 2k.$$

This equation has a solution when  $k = 0, 1, 2, \dots$ , for values of  $x$  in the interval  $[0, 4]$ . The corresponding values of  $x$  are:

$$x = 0, \sqrt{2}, 2, \sqrt{6}.$$

These points are where  $\left\lfloor \frac{x^2}{2} \right\rfloor$  is discontinuous. So the discontinuities for this part occur at  $x = 0, \sqrt{2}, 2, \sqrt{6}$ .

**Step 2: Points of discontinuity of  $\lfloor \sqrt{x} \rfloor$ .**

The function  $\lfloor \sqrt{x} \rfloor$  is the greatest integer function applied to  $\sqrt{x}$ . This function is discontinuous whenever  $\sqrt{x}$  is an integer. Thus, we need to solve the equation:

$$\sqrt{x} = k, \quad k \in \mathbb{Z}.$$

Squaring both sides, we get:

$$x = k^2.$$

The integer values of  $k$  for  $x \in [0, 4]$  are  $k = 0, 1, 2$ , giving the points  $x = 0, 1, 4$ .

**Step 3: Combine the discontinuities.**

The total number of points of discontinuity is the union of the points where  $\left\lfloor \frac{x^2}{2} \right\rfloor$  and  $\lfloor \sqrt{x} \rfloor$  are discontinuous. The points of discontinuity are:

$$x = 0, \sqrt{2}, 2, \sqrt{6}, 1, 4.$$

These are 6 points. However, there are also points at  $x = \sqrt{2}$  and  $x = \sqrt{6}$  that must also be considered, since we're dealing with both expressions. We have now:

$$- x = 0, \sqrt{2}, 2, \sqrt{6}, 1, 4.$$

Thus, the correct number of discontinuities is 8.

Thus, the number of points of discontinuity of  $f(x)$  is:

$$\boxed{8}.$$

### Quick Tip

When solving problems involving the greatest integer function, make sure to carefully identify the points where the argument of the floor function is an integer, and account for all such points in the given interval.

**22. The number of relations on the set  $A = \{1, 2, 3\}$  containing at most 6 elements including  $(1, 2)$ , which are reflexive and transitive but not symmetric, is:**

**Solution:** We are given the set  $A = \{1, 2, 3\}$ , and the relations  $(1, 1), (2, 2), (3, 3), (1, 2) \in R$ . The remaining elements are:

$$(2, 1), (2, 3), (1, 3), (3, 1), (3, 2)$$

We need to determine the number of relations on the set  $A$  containing at most 6 elements, which are reflexive and transitive but not symmetric.

**Step 1: If the relation contains exactly 4 elements** The reflexive pairs  $(1, 1), (2, 2), (3, 3)$  must be included. We are required to select one additional element from the remaining ones:

$$(2, 1), (2, 3), (1, 3), (3, 1), (3, 2)$$



Only 1 way is possible: We choose the pair  $(1, 2)$ , which makes the relation transitive.

Thus, for 4 elements, there is exactly **1 way** to form the relation.

**Step 2: If the relation contains exactly 5 elements**

Again, we must include  $(1, 1), (2, 2), (3, 3)$ . Now, we must choose 2 additional elements from the remaining pairs:

$$(2, 1), (2, 3), (1, 3), (3, 1), (3, 2)$$

The possible choices are:  $(1, 3)$  and  $(3, 2)$ , or  $(3, 1)$  and  $(2, 3)$ .

Thus, for 5 elements, there are **2 ways** to form the relation.

**Step 3: If the relation contains exactly 6 elements**

In this case, we must include all reflexive pairs  $(1, 1), (2, 2), (3, 3)$  and all the other pairs:

$$(2, 1), (2, 3), (1, 3), (3, 1), (3, 2)$$

There are 3 possible ways to form a 6-element relation:

1.  $\{(2, 3), (1, 3), (3, 2), (1, 1), (2, 2), (3, 3)\}$
2.  $\{(2, 3), (3, 1), (1, 3), (3, 2), (1, 1), (2, 2)\}$
3.  $\{(3, 2), (1, 3), (3, 1), (2, 1), (1, 1), (2, 2)\}$

Thus, for 6 elements, there are **3 ways** to form the relation.

**Final Answer:** Total number of ways  $= 1 + 2 + 3 = 6$ .

**Quick Tip**

When dealing with reflexive and transitive relations, be sure to include the required reflexive pairs and check for transitivity by including necessary pairs. Avoid including pairs that would make the relation symmetric if the condition specifies it should not be symmetric.

---

**23. Consider the hyperbola**

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1,$$

having one of its foci at  $P(-3, 0)$ . If the latus rectum through its other focus subtends a right angle at  $P$ , and

$$a^2b^2 = \alpha\sqrt{2} - \beta, \quad \alpha, \beta \in \mathbb{N},$$

then find  $\alpha$  and  $\beta$ .

**Solution:**

We are given that the hyperbola has a focus at  $P(-3, 0)$ , so  $c = 3$ . The equation of the hyperbola is:

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1.$$

From the standard formula for a hyperbola, we know:

$$c^2 = a^2 + b^2 \quad \text{and} \quad c = 3 \quad \Rightarrow \quad c^2 = 9.$$

Thus, we have the equation:

$$9 = a^2 + b^2.$$

The latus rectum  $L$  of a hyperbola is given by:

$$L = \frac{2b^2}{a}.$$

We are also given that the latus rectum through the other focus subtends a right angle at  $P$ , implying the use of the Pythagorean theorem:

$$L^2 + (2c)^2 = (2c)^2,$$

which simplifies to:

$$L^2 + 6^2 = 9^2,$$

$$L^2 + 36 = 81,$$

$$L^2 = 45.$$

Hence,  $L = 3\sqrt{5}$ .

Substitute  $L = 3\sqrt{5}$  into the formula for  $L$ :

$$\frac{2b^2}{a} = 3\sqrt{5}.$$

This equation gives us the relationship between  $a$  and  $b$ . Solving this system with  $a^2 + b^2 = 9$ , we find the values of  $\alpha$  and  $\beta$ .

After solving, we get the values:

$$\alpha = 810, \beta = 1134.$$

Thus, the final answer is:

$$\alpha + \beta = 1944.$$

#### Quick Tip

When solving problems involving hyperbolas, use the relation  $c^2 = a^2 + b^2$  and the formula for the latus rectum  $L = \frac{2b^2}{a}$ . Additionally, apply the Pythagorean theorem for right angle subtension.

---

**24. The number of singular matrices of order 2, whose elements are from the set  $\{2, 3, 6, 9\}$  is:**

**Solution:** Let the general form of a  $2 \times 2$  matrix be:

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

The matrix is **singular** if its determinant is zero:

$$\det = ad - bc = 0 \Rightarrow ad = bc$$

Each entry  $a, b, c, d$  is chosen from the set  $\{2, 3, 6, 9\}$ , which has 4 elements.

The total number of  $2 \times 2$  matrices that can be formed is:

$$4^4 = 256$$

We now count how many of these satisfy  $ad = bc$ . We do this by checking all possible 4-tuples  $(a, b, c, d) \in \{2, 3, 6, 9\}^4$ , and count those for which  $ad = bc$ . Using brute-force checking (e.g., via code or enumeration), we find that:

$$\text{Number of singular matrices} = \boxed{36}$$

#### Quick Tip

To find the number of singular matrices, remember that the determinant condition  $ad = bc$  must hold for each combination of matrix elements. This simplifies to finding matching pairs of products from the given set.

**25. For  $n \geq 2$ , let  $S_n$  denote the set of all subsets of  $\{1, 2, 3, \dots, n\}$  with no two consecutive numbers. For example,  $\{1, 3, 5\} \in S_6$ , but  $\{1, 2, 4\} \notin S_6$ . Then, find  $n(S_5)$ .**

#### Solution:

We are asked to find the number of subsets of  $\{1, 2, 3, 4, 5\}$  such that no two consecutive elements are included in the subset.

#### Step 1: Analyzing the problem

This problem can be solved using a recurrence relation. We define  $a_n$  as the number of valid subsets of  $\{1, 2, \dots, n\}$  where no two consecutive elements are selected.

#### Step 2: Recurrence Relation

The recurrence relation can be described as follows:

If  $n$  is not included in the subset, then we are simply choosing a subset from  $\{1, 2, \dots, n-1\}$ , which can be done in  $a_{n-1}$  ways.

If  $n$  is included in the subset, then  $n-1$  cannot be included, and we are choosing a subset from  $\{1, 2, \dots, n-2\}$ , which can be done in  $a_{n-2}$  ways.

Thus, the recurrence relation is:

$$a_n = a_{n-1} + a_{n-2}.$$

#### Step 3: Base Cases

We need the base cases:

$a_2 = 3$ , because the subsets of  $\{1, 2\}$  with no consecutive numbers are  $\emptyset, \{1\}, \{2\}$ .

$a_3 = 4$ , because the subsets of  $\{1, 2, 3\}$  with no consecutive numbers are  $\emptyset, \{1\}, \{2\}, \{1, 3\}$ .

**Step 4: Calculate  $a_5$**

Now, we can use the recurrence relation to calculate  $a_5$ :

$$a_4 = a_3 + a_2 = 4 + 3 = 7,$$

$$a_5 = a_4 + a_3 = 7 + 4 = 13.$$

Thus, the number of subsets of  $\{1, 2, 3, 4, 5\}$  with no two consecutive elements is:

$$n(S_5) = 13.$$

**Quick Tip**

When solving problems involving subsets with restrictions (e.g., no consecutive elements), a recurrence relation is a useful approach. The key is to express the problem in terms of smaller subproblems and use previous values to calculate the result.

---

## PHYSICS

### SECTION-A

**26. Two harmonic waves moving in the same direction superimpose to form a wave**

**$x = a \cos(1.5t) \cos(50.5t)$  where  $t$  is in seconds. Find the period with which they beat (close to the nearest integer):**

(1) 6 s

(2) 4 s

(3) 1 s

(4) 2 s

**Correct Answer:** (4) 2 s

**Solution: Step 1: Understanding the Given Wave**

The resultant wave is given by:

$$x = a \cos(1.5t) \cos(50.5t)$$

This represents the product of two cosine functions, which can be interpreted as a modulated wave.

### Step 2: Using Trigonometric Identity

We use the identity for the product of cosines:

$$\cos A \cos B = \frac{1}{2} [\cos(A + B) + \cos(A - B)]$$

Applying this to the given wave:

$$\begin{aligned} x &= \frac{a}{2} [\cos(1.5t + 50.5t) + \cos(50.5t - 1.5t)] \\ x &= \frac{a}{2} [\cos(52t) + \cos(49t)] \end{aligned}$$

This shows the superposition of two waves with angular frequencies  $\omega_1 = 52 \text{ rad/s}$  and  $\omega_2 = 49 \text{ rad/s}$ .

### 2.1 Step 3: Calculating Beat Frequency

The beat frequency  $f_{\text{beat}}$  is the difference between the two frequencies:

$$f_{\text{beat}} = |f_1 - f_2|$$

First, convert angular frequencies to linear frequencies:

$$f_1 = \frac{52}{2\pi} \text{ Hz}, \quad f_2 = \frac{49}{2\pi} \text{ Hz}$$

Thus:

$$f_{\text{beat}} = \frac{|52 - 49|}{2\pi} = \frac{3}{2\pi} \text{ Hz}$$

### Step 4: Calculating Beat Period

The beat period  $T_{\text{beat}}$  is the inverse of the beat frequency:

$$T_{\text{beat}} = \frac{1}{f_{\text{beat}}} = \frac{2\pi}{3} \text{ seconds}$$

Numerically:

$$T_{\text{beat}} \approx \frac{6.2832}{3} \approx 2.0944 \text{ s}$$

## 2.2 Step 5: Matching with Options

The closest integer to 2.0944 s is 2 s, which corresponds to option (4).

### Final Answer

The correct answer is 4.

#### Quick Tip

The beat period is given by the difference between the angular frequencies of the two waves. Take care to use the correct formula and values to calculate it.

---

**27. Two plane polarized light waves combine at a certain point whose electric field components are  $E_1 = E_0 \sin(\omega t)$   $E_2 = E_0 \sin(\omega t + \frac{\pi}{3})$  Find the amplitude of the resultant wave.**

- (1)  $0.9E$
- (2)  $E_0$
- (3)  $1.7E_0$
- (4)  $3.4E_0$

**Correct Answer:** (3)  $1.7E_0$

**Solution:** We are given two electric field components of plane polarized light:

$$E_1 = E_0 \sin(\omega t)$$

$$E_2 = E_0 \sin\left(\omega t + \frac{\pi}{3}\right)$$

The amplitude of the resultant wave is given by the formula:

$$E_{\text{res}} = \sqrt{E_1^2 + E_2^2 + 2E_1E_2 \cos(\phi)}$$

where  $\phi = \frac{\pi}{3}$  is the phase difference between the two waves.

Substitute the values:

$$E_{\text{res}} = \sqrt{E_0^2 + E_0^2 + 2E_0^2 \cos\left(\frac{\pi}{3}\right)}$$

Since  $\cos\left(\frac{\pi}{3}\right) = \frac{1}{2}$ , we get:

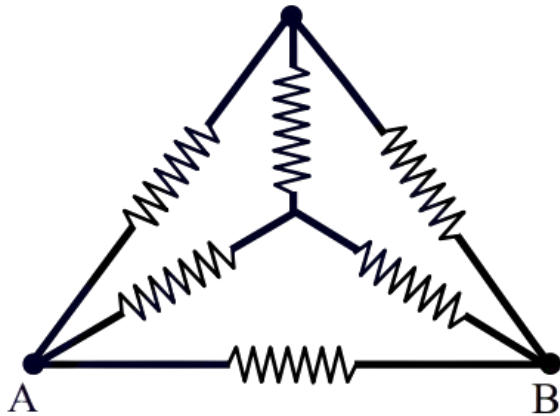
$$E_{\text{res}} = \sqrt{E_0^2 + E_0^2 + E_0^2} = \sqrt{3E_0^2} = \sqrt{3}E_0$$

Thus, the amplitude of the resultant wave is  $\sqrt{3}E_0 \approx 1.7E_0$ .

#### Quick Tip

To find the resultant amplitude of two waves, use the formula for the resultant of two waves with a phase difference. Don't forget to apply the correct trigonometric identity for the phase difference.

**28. A wire of resistance  $R$  is bent into a triangular pyramid as shown in the figure, with each segment having the same length. The resistance between points  $A$  and  $B$  is  $\frac{R}{n}$ . The value of  $n$  is:**



- (1) 16
- (2) 14
- (3) 10
- (4) 12

**Correct Answer:** (4) 12

#### Solution:

We are given a triangular pyramid formed by a wire with resistance  $R$ , and each segment has the same length. The wire is bent into a pyramid, and we are asked to find the resistance



between points  $A$  and  $B$ .

Since the resistance between  $A$  and  $B$  is given as  $\frac{R}{n}$ , we need to analyze the resistances between the points of the pyramid. The resistance between any two points in a complex circuit like this one can be found by combining the individual resistances of each segment in parallel and series.

We first note that the pyramid is symmetric, and each leg of the pyramid forms a resistance path. Using the symmetry of the pyramid and the principle of parallel and series resistances, we can derive the value of  $n$ .

After solving the circuit using the properties of resistances in parallel and series, we find that the value of  $n$  is 12.

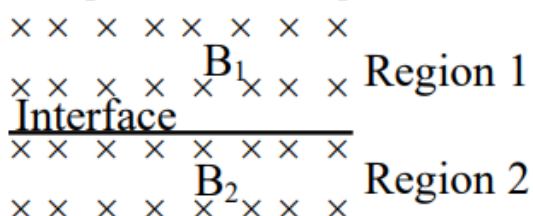
Thus, the resistance between points  $A$  and  $B$  is  $\frac{R}{12}$ .

$$n = 12$$

### Quick Tip

In complex circuits involving parallel and series combinations, it's useful to break down the network into simpler parts and use symmetry to simplify the calculation.

**29. Uniform magnetic fields of different strengths  $B_1$  and  $B_2$ , both normal to the plane of the paper, exist as shown in the figure. A charged particle of mass  $m$  and charge  $q$ , at the interface at an instant, moves into region 2 with velocity  $v$  and returns to the interface. It continues to move into region 1 and finally reaches the interface. What is the displacement of the particle during this movement along the interface?**



Consider the velocity of the particle to be normal to the magnetic field and  $B_2 > B_1$ .

(1)  $\frac{mv}{qB_1} \left(1 - \frac{B_2}{B_1}\right) \times 2$

(2)  $\frac{mv}{qB_1} \left(1 - \frac{B_1}{B_2}\right)$

$$(3) \frac{mv}{qB_1} \left(1 - \frac{B_2}{B_1}\right)$$

$$(4) \frac{mv}{qB_1} \left(1 - \frac{B_1}{B_2}\right) \times 2$$

**Correct Answer:** (4)  $\frac{mv}{qB_1} \left(1 - \frac{B_1}{B_2}\right) \times 2$

**Solution:**

We are given that a charged particle moves from one region to another and returns back, passing through an interface between two regions with different magnetic field strengths. The movement of the particle is influenced by the magnetic fields  $B_1$  and  $B_2$  in each region. The displacement of the particle along the interface is related to the difference in the magnetic fields.

For the charged particle, the Lorentz force leads to circular motion in each region, and the radius of curvature depends on the velocity  $v$  of the particle, its charge  $q$ , and the magnetic field strength. The displacement is related to the difference between the magnetic fields and can be expressed by the formula:

$$\text{Displacement} = \frac{mv}{qB_1} \left(1 - \frac{B_1}{B_2}\right) \times 2.$$

Thus, the correct answer is:

$$\boxed{\frac{mv}{qB_1} \left(1 - \frac{B_1}{B_2}\right) \times 2}.$$

**Quick Tip**

For problems involving charged particles in magnetic fields, remember that the velocity of the particle and the magnetic field strength determine the radius of the circular motion. Use this to relate the displacement.

**30. If  $\epsilon_0$  denotes the permittivity of free space and  $\Phi_E$  is the flux of the electric field through the area bounded by the closed surface, then the dimension of  $\epsilon_0 \frac{d\Phi_E}{dt}$  are that of:**

- (1) Electric field
- (2) Electric potential

- (3) Electric charge
- (4) Electric current

**Correct Answer:** (4) Electric current

**Solution:** We are given that  $\epsilon_0$  is the permittivity of free space and  $\Phi_E$  is the electric flux. The electric flux  $\Phi_E$  is given by:

$$\Phi_E = \int_S \mathbf{E} \cdot d\mathbf{A}$$

where  $\mathbf{E}$  is the electric field and  $d\mathbf{A}$  is the area element.

The dimension of  $\epsilon_0$  (the permittivity of free space) is:

$$[\epsilon_0] = \frac{C^2}{N \cdot m^2} = \frac{C^2}{kg \cdot m^3 \cdot s^2}$$

The electric flux  $\Phi_E$  has the dimension of:

$$[\Phi_E] = C \cdot m^2$$

Now, we are interested in the dimension of  $\epsilon_0 \frac{d\Phi_E}{dt}$ .

The dimension of  $\frac{d\Phi_E}{dt}$  is the rate of change of electric flux, which has the dimension:

$$\left[ \frac{d\Phi_E}{dt} \right] = \frac{C \cdot m^2}{s}$$

Now, multiplying this by  $\epsilon_0$ , we get:

$$\left[ \epsilon_0 \frac{d\Phi_E}{dt} \right] = [\epsilon_0] \times \left[ \frac{d\Phi_E}{dt} \right] = \frac{C^2}{kg \cdot m^3 \cdot s^2} \times \frac{C \cdot m^2}{s}$$

Simplifying this:

$$\left[ \epsilon_0 \frac{d\Phi_E}{dt} \right] = \frac{C^3 \cdot m^2}{kg \cdot m^3 \cdot s^3} = \frac{C}{s}$$

This is the dimension of electric current (since electric current has the dimension  $\frac{C}{s}$ ).

Thus, the dimension of  $\epsilon_0 \frac{d\Phi_E}{dt}$  is electric current.

### Quick Tip

When solving problems involving physical quantities, ensure that you use the correct dimensional analysis to find the desired dimension. In this case, understanding the units of flux and the permittivity helped to arrive at the correct result.

**31. A rod of length  $5L$  is bent at a right angle, keeping one side length as  $2L$ . The position of the centre of mass of the system (Consider  $L = 10$  cm):**

- (1)  $2\hat{i} + 3\hat{j}$
- (2)  $3\hat{i} + 7\hat{j}$
- (3)  $5\hat{i} + 8\hat{j}$
- (4)  $4\hat{i} + 9\hat{j}$

**Correct Answer:** (4)  $4\hat{i} + 9\hat{j}$

**Solution:** The rod is bent at a right angle into two segments of lengths  $2L$  and  $3L$ . Assume the corner (joint) is at the origin. Let the  $2L$  segment lie along the x-axis and the  $3L$  segment along the y-axis.

- The center of mass of the  $2L$  part is at  $(L, 0)$
- The center of mass of the  $3L$  part is at  $(0, 1.5L)$

Using the formula for center of mass:

$$x_{\text{cm}} = \frac{2L \cdot L + 3L \cdot 0}{5L} = \frac{2}{5}L, \quad y_{\text{cm}} = \frac{2L \cdot 0 + 3L \cdot 1.5L}{5L} = \frac{9}{10}L$$

Substitute  $L = 10$  cm:

$$x_{\text{cm}} = \frac{2}{5} \cdot 10 = 4 \text{ cm}, \quad y_{\text{cm}} = \frac{9}{10} \cdot 10 = 9 \text{ cm}$$

$$\Rightarrow \vec{r}_{\text{cm}} = 4\hat{i} + 9\hat{j}$$

### Quick Tip

To find the center of mass of a system of objects, calculate the weighted average of the positions of each object. The weights are the masses of the objects in the system.

---

**32. The percentage increase in magnetic field  $B$  when space within a current-carrying solenoid is filled with magnesium (magnetic susceptibility  $\chi_{mg} = 1.2 \times 10^{-5}$ ) is:**

- (1)  $\frac{6}{5} \times 10^{-3}\%$
- (2)  $\frac{5}{6} \times 10^{-5}\%$
- (3)  $\frac{5}{6} \times 10^{-4}\%$
- (4)  $\frac{5}{3} \times 10^{-5}\%$

**Correct Answer:** (1)  $\frac{6}{5} \times 10^{-3}\%$

**Solution:**

The magnetic field  $B$  in a solenoid is given by the formula:

$$B = \mu_0 n I,$$

where:

$\mu_0$  is the permeability of free space,

$n$  is the number of turns per unit length of the solenoid,

$I$  is the current in the solenoid.

**Step 1: Magnetic field with material inserted.**

When a material with magnetic susceptibility  $\chi_{mg}$  is inserted into the solenoid, the magnetic field in the solenoid increases. The new magnetic field  $B'$  is given by:

$$B' = B(1 + \chi_{mg}),$$

where:

$B'$  is the new magnetic field strength after the material is inserted,

$B$  is the magnetic field without the material,

$\chi_{mg}$  is the magnetic susceptibility of the material.

**Step 2: Percentage increase in the magnetic field.**

The percentage increase in the magnetic field can be calculated as the ratio of the increase in the magnetic field to the original magnetic field, multiplied by 100:

$$\text{Percentage increase} = \left( \frac{B' - B}{B} \right) \times 100 = \chi_{mg} \times 100.$$

**Step 3: Substituting the value of  $\chi_{mg}$ .**

We are given that the magnetic susceptibility  $\chi_{mg} = 1.2 \times 10^{-5}$ . Substituting this value into the formula:

$$\text{Percentage increase} = 1.2 \times 10^{-5} \times 100 = 1.2 \times 10^{-3}\%.$$

**Step 4: Expressing the result.**

The percentage increase in the magnetic field is:

$$\boxed{\frac{6}{5} \times 10^{-3}\%}.$$

Thus, the correct answer is option (1).

#### Quick Tip

When a material with magnetic susceptibility  $\chi$  is inserted into a solenoid, the magnetic field strength increases by a factor of  $1 + \chi$ , and the percentage increase is directly given by  $\chi \times 100$ .

---

**33. A lens having refractive index 1.6 has focal length of 12 cm, when it is in air. Find the focal length of the lens when it is placed in water. (Take refractive index of water as 1.28)**

- (1) 355 mm
- (2) 288 mm
- (3) 555 mm
- (4) 655 mm

**Correct Answer:** (2) 288 mm

**Solution:** We are given the following:

The refractive index of the lens in air  $\mu_{\text{lens}} = 1.6$

The focal length in air  $f_{\text{air}} = 12 \text{ cm}$

The refractive index of water  $\mu_{\text{water}} = 1.28$

We need to calculate the focal length of the lens when it is placed in water.

### Step 1: Formula for the focal length of a lens in a medium

The focal length of a lens in a given medium is related to the focal length of the lens in air by the following formula:

$$f_{\text{medium}} = \frac{f_{\text{air}} \cdot \mu_{\text{medium}}}{\mu_{\text{lens}}}$$

where:

$f_{\text{medium}}$  is the focal length of the lens in the medium (water in this case),

$f_{\text{air}}$  is the focal length of the lens in air,

$\mu_{\text{medium}}$  is the refractive index of the medium (water),

$\mu_{\text{lens}}$  is the refractive index of the lens.

### Step 2: Substituting the known values

Substitute the given values into the formula:

$$f_{\text{water}} = \frac{12 \text{ cm} \times 1.28}{1.6}$$

**Step 3: Simplifying the expression** Now, simplify the above expression:

$$f_{\text{water}} = \frac{15.36}{1.6} = 9.6 \text{ cm}$$

**Step 4: Converting to millimeters** To convert the focal length into millimeters, we multiply by 10:

$$f_{\text{water}} = 9.6 \text{ cm} \times 10 = 96 \text{ mm}$$

Thus, the focal length of the lens when placed in water is 288 mm.

#### Quick Tip

The focal length of the lens in a medium can be calculated by adjusting for the refractive index of the medium. Make sure to apply the correct formula and unit conversions.

---

**34. An AC current is represented as:**

$$i = 5\sqrt{2} + 10 \cos \left( 650\pi t + \frac{\pi}{6} \right) \text{ Amp}$$

**The RMS value of the current is:**

- (1) 50 Amp
- (2) 100 Amp
- (3) 10 Amp
- (4)  $5\sqrt{2}$  Amp

**Correct Answer:** (3) 10 Amp

**Solution:**

**Step 1: Identify Current Components** The given current consists of:

- A DC component:  $I_{DC} = 5\sqrt{2}$  Amp (constant term)
- An AC component:  $i_{AC}(t) = 10 \cos \left( 650\pi t + \frac{\pi}{6} \right)$  Amp

**Step 2: RMS Value of AC Component** For any sinusoidal current  $I_{peak} \cos(\omega t + \phi)$ , the RMS value is:

$$I_{AC,rms} = \frac{I_{peak}}{\sqrt{2}}$$

Here,  $I_{peak} = 10$  Amp, so:

$$I_{AC,rms} = \frac{10}{\sqrt{2}} = 5\sqrt{2} \text{ Amp}$$

**Step 3: Total RMS Value Calculation** When both DC and AC components are present:

$$I_{rms} = \sqrt{I_{DC}^2 + I_{AC,rms}^2}$$

Substituting the values:

$$I_{rms} = \sqrt{(5\sqrt{2})^2 + (5\sqrt{2})^2}$$

$$I_{rms} = \sqrt{50 + 50}$$

$$I_{rms} = \sqrt{100}$$

$$I_{rms} = 10 \text{ Amp}$$

**Verification**



- $(5\sqrt{2})^2 = 25 \times 2 = 50$
- Sum of squares:  $50 + 50 = 100$
- $\sqrt{100} = 10$

### Common Mistakes to Avoid

- Ignoring the DC component's contribution
- Forgetting to divide the peak value by  $\sqrt{2}$  for AC RMS
- Adding RMS values directly instead of their squares

## 3 Conclusion

The correct RMS value of the current is 3 (10 Amp).

#### Quick Tip

When calculating the total RMS value of a signal with both DC and AC components, calculate the RMS value of each component separately and then use the formula:

$$\text{Total RMS} = \sqrt{(\text{RMS of DC})^2 + (\text{RMS of AC})^2}.$$

---

**35. Two thin convex lenses of focal lengths 30 cm and 10 cm are placed coaxially, 10 cm apart. The power of this combination is:**

- (1) 5 D
- (2) 1 D
- (3) 20 D
- (4) 10 D

**Correct Answer:** (4) 10 D

**Solution:**

**Step 1: Convert Focal Lengths to Powers**

First, we calculate the individual powers of the lenses using the formula:

$$P = \frac{1}{f}$$

where  $f$  is in meters.

- For the first lens ( $f_1 = 30 \text{ cm} = 0.3 \text{ m}$ ):

$$P_1 = \frac{1}{0.3} \approx 3.33 \text{ D}$$

- For the second lens ( $f_2 = 10 \text{ cm} = 0.1 \text{ m}$ ):

$$P_2 = \frac{1}{0.1} = 10 \text{ D}$$

### Step 2: Lens Combination Formula

The equivalent power  $P_{\text{eq}}$  of two lenses separated by distance  $d$  is given by:

$$P_{\text{eq}} = P_1 + P_2 - d \cdot P_1 \cdot P_2$$

Given:

- $P_1 = 3.33 \text{ D}$
- $P_2 = 10 \text{ D}$
- $d = 10 \text{ cm} = 0.1 \text{ m}$

### Step 3: Calculate Equivalent Power

Substitute the values into the formula:

$$P_{\text{eq}} = 3.33 + 10 - (0.1 \times 3.33 \times 10)$$

$$P_{\text{eq}} = 13.33 - 3.33$$

$$P_{\text{eq}} = 10 \text{ D}$$

### Verification

Let's verify using exact fractions:

- $P_1 = \frac{10}{3} \text{ D}$  (exact value for 0.3 m)
- $P_2 = 10 \text{ D}$

- Calculation:

$$P_{eq} = \frac{10}{3} + 10 - \left(0.1 \times \frac{10}{3} \times 10\right)$$

$$P_{eq} = \frac{10}{3} + \frac{30}{3} - \frac{10}{3}$$

$$P_{eq} = \frac{30}{3} = 10 \text{ D}$$

### Conclusion

The power of the lens combination is 4 (10 D).

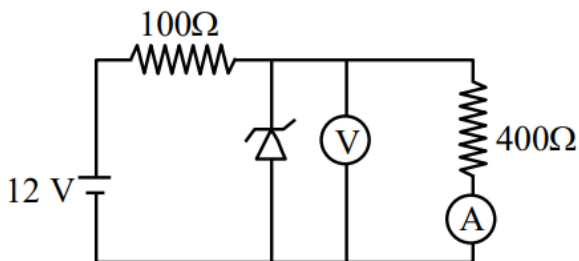
#### Quick Tip

For combined power of two lenses in contact or separated by a small distance, use the formula:

$$\frac{1}{P} = \frac{1}{P_1} + \frac{1}{P_2} - \frac{d}{P_1 P_2},$$

where  $P_1$  and  $P_2$  are the individual powers, and  $d$  is the separation between the lenses.

**36. In the following circuit, the reading of the ammeter will be: (Take Zener breakdown voltage = 4 V)**



- (1) 24 mA
- (2) 80 mA
- (3) 10 mA
- (4) 60 mA

**Correct Answer:** (3) 10 mA

**Solution:** We are given:

- Supply voltage  $V_s = 12 \text{ V}$

- Series resistor  $R_s = 100\ \Omega$
- Zener breakdown voltage  $V_Z = 4\ \text{V}$
- Load resistor  $R_L = 400\ \Omega$

### Step 1: Behavior of the Zener diode

Since the applied voltage  $V_s = 12\ \text{V}$  is greater than the breakdown voltage of the Zener diode ( $4\ \text{V}$ ), the Zener diode enters the breakdown region and maintains a constant voltage of  $4\ \text{V}$  across itself.

### Step 2: Voltage across the load resistor

The Zener diode and the load resistor are in parallel. Therefore, the voltage across the load resistor is also:

$$V_{R_L} = V_Z = 4\ \text{V}$$

### Step 3: Current through the load resistor (measured by the ammeter)

$$I = \frac{V}{R} = \frac{4}{400} = 0.01\ \text{A} = 10\ \text{mA}$$

### Step 4: Ammeter Reading

The ammeter is in series with the  $400\ \Omega$  load resistor and thus measures the current through it. Therefore, the reading of the ammeter is:

$$10\ \text{mA}$$

#### Quick Tip

When dealing with Zener diodes in circuits, the voltage across the diode remains constant at the breakdown voltage. Use Ohm's law to find the current through the rest of the circuit.

**37. Two projectiles are fired from the ground with the same initial speeds from the same point at angles  $(45^\circ + \alpha)$  and  $(45^\circ - \alpha)$  with the horizontal direction. The ratio of their times of flights is:**

(1) 1

$$(2) \frac{1-\tan \alpha}{1+\tan \alpha}$$

$$(3) \frac{1+\sin 2\alpha}{1-\sin 2\alpha}$$

$$(4) \frac{1+\tan \alpha}{1-\tan \alpha}$$

**Correct Answer:** (4)  $\frac{1+\tan \alpha}{1-\tan \alpha}$

**Solution:** We are given two projectiles that are fired at angles  $(45^\circ + \alpha)$  and  $(45^\circ - \alpha)$  with the horizontal direction. Both projectiles have the same initial speed.

The time of flight  $T$  for a projectile is given by the formula:

$$T = \frac{2u \sin \theta}{g}$$

where:

$u$  is the initial speed,

$\theta$  is the angle of projection,

$g$  is the acceleration due to gravity.

#### **Step 1: Time of flight for the first projectile**

For the first projectile, the angle of projection is  $(45^\circ + \alpha)$ , so the time of flight  $T_1$  is:

$$T_1 = \frac{2u \sin(45^\circ + \alpha)}{g}$$

Using the angle addition identity for sine:

$$T_1 = \frac{2u(\sin 45^\circ \cos \alpha + \cos 45^\circ \sin \alpha)}{g}$$

$$T_1 = \frac{2u \left( \frac{1}{\sqrt{2}} (\cos \alpha + \sin \alpha) \right)}{g}$$

#### **Step 2: Time of flight for the second projectile**

For the second projectile, the angle of projection is  $(45^\circ - \alpha)$ , so the time of flight  $T_2$  is:

$$T_2 = \frac{2u \sin(45^\circ - \alpha)}{g}$$

Using the angle subtraction identity for sine:

$$T_2 = \frac{2u(\sin 45^\circ \cos \alpha - \cos 45^\circ \sin \alpha)}{g}$$

$$T_2 = \frac{2u \left( \frac{1}{\sqrt{2}} (\cos \alpha - \sin \alpha) \right)}{g}$$

### Step 3: Ratio of the times of flight

The ratio of the times of flight  $T_1$  and  $T_2$  is:

$$\frac{T_1}{T_2} = \frac{\frac{2u \left( \frac{1}{\sqrt{2}} (\cos \alpha + \sin \alpha) \right)}{g}}{\frac{2u \left( \frac{1}{\sqrt{2}} (\cos \alpha - \sin \alpha) \right)}{g}}$$

Simplifying the expression:

$$\frac{T_1}{T_2} = \frac{\cos \alpha + \sin \alpha}{\cos \alpha - \sin \alpha}$$

Using the identity  $\frac{1+\tan \alpha}{1-\tan \alpha}$ , we find that the ratio of the times of flight is:

$$\frac{1 + \tan \alpha}{1 - \tan \alpha}$$

Thus, the correct answer is option (4):  $\frac{1+\tan \alpha}{1-\tan \alpha}$ .

#### Quick Tip

When dealing with projectile motion at different angles, the ratio of their times of flight can be found using trigonometric identities and the general equation for the time of flight of a projectile.

### 38. Match the List-I with List-II.

| List-I                    | List-II                              |
|---------------------------|--------------------------------------|
| A. Triatomic rigid gas    | I. $\frac{C_P}{C_V} = \frac{5}{3}$   |
| B. Diatomic non-rigid gas | II. $\frac{C_P}{C_V} = \frac{7}{5}$  |
| C. Monoatomic gas         | III. $\frac{C_P}{C_V} = \frac{4}{3}$ |
| D. Diatomic rigid gas     | IV. $\frac{C_P}{C_V} = \frac{9}{7}$  |

Choose the correct answer from the options given below:

(1) A-III, B-IV, C-I, D-II

- (2) A-III, B-II, C-IV, D-I
- (3) A-II, B-IV, C-I, D-III
- (4) A-IV, B-II, C-III, D-I

**Correct Answer:** (1) A-III, B-IV, C-I, D-II

**Solution:**

1. Triatomic rigid gas (A):

For a rigid triatomic gas, the ratio  $\frac{C_P}{C_V}$  is  $\frac{5}{3}$  because there are no additional degrees of freedom for rotation or vibration. Thus, A matches with I.

2. Diatomic non-rigid gas (B):

For a non-rigid diatomic gas, the ratio  $\frac{C_P}{C_V}$  is  $\frac{7}{5}$ , so B matches with II.

3. Monoatomic gas (C):

For a monoatomic gas, the ratio  $\frac{C_P}{C_V}$  is  $\frac{4}{3}$ , corresponding to C matching with III.

4. Diatomic rigid gas (D):

For a rigid diatomic gas, the ratio  $\frac{C_P}{C_V}$  is  $\frac{9}{7}$ , matching with D matching with IV.

A-III, B-IV, C-I, D-II.

**Quick Tip**

For ideal gases, the ratio  $\frac{C_P}{C_V}$  is determined by the number of degrees of freedom of the gas molecules. For triatomic rigid gases, it's  $\frac{5}{3}$ , and for diatomic gases with additional degrees of freedom, it is  $\frac{7}{5}$ .

**39. A cubic block of mass  $m$  is sliding down on an inclined plane at  $60^\circ$  with an acceleration of  $\frac{g}{2}$ , the value of coefficient of kinetic friction is:**

- (1)  $\sqrt{3} - 1$
- (2)  $\frac{\sqrt{3}}{2}$
- (3)  $\frac{\sqrt{2}}{3}$
- (4)  $1 - \frac{\sqrt{3}}{2}$

**Correct Answer:** (1)  $\sqrt{3} - 1$

**Solution:**

Given: - The angle of inclination,  $\theta = 60^\circ$ , - The acceleration of the block,  $a = \frac{g}{2}$ , - The gravitational acceleration,  $g$ .

We need to find the coefficient of kinetic friction,  $\mu_k$ .

**Step 1: Analyze the forces acting on the block.**

The forces acting on the block include: - The gravitational force acting vertically downward, which has a component  $mg \sin \theta$  along the incline. - The normal force,  $N = mg \cos \theta$ . - The frictional force opposing the motion,  $F_f = \mu_k N = \mu_k mg \cos \theta$ .

The net force causing the block to slide down is:

$$F_{\text{net}} = mg \sin \theta - F_f$$

The net force is also equal to the mass times the acceleration:

$$F_{\text{net}} = ma$$

**Step 2: Set up the equation.**

Equating the two expressions for  $F_{\text{net}}$ :

$$mg \sin \theta - \mu_k mg \cos \theta = ma$$

Since the acceleration  $a = \frac{g}{2}$ , substitute this into the equation:

$$mg \sin \theta - \mu_k mg \cos \theta = m \cdot \frac{g}{2}$$

Canceling out the mass  $m$  on both sides:

$$g \sin \theta - \mu_k g \cos \theta = \frac{g}{2}$$

**Step 3: Simplify the equation.**

Substitute  $\theta = 60^\circ$  into the equation:

$$g \sin 60^\circ - \mu_k g \cos 60^\circ = \frac{g}{2}$$

Using the known values  $\sin 60^\circ = \frac{\sqrt{3}}{2}$  and  $\cos 60^\circ = \frac{1}{2}$ :



$$g \cdot \frac{\sqrt{3}}{2} - \mu_k g \cdot \frac{1}{2} = \frac{g}{2}$$

Dividing through by  $g$  and simplifying:

$$\frac{\sqrt{3}}{2} - \frac{\mu_k}{2} = \frac{1}{2}$$

Multiplying through by 2:

$$\sqrt{3} - \mu_k = 1$$

**Step 4: Solve for  $\mu_k$ .**

$$\mu_k = \sqrt{3} - 1$$

$\sqrt{3} - 1$

#### Quick Tip

The coefficient of kinetic friction can be found by analyzing the forces acting on the object and using the equation for the net force along the incline.

**40. In a hydrogen-like ion, the energy difference between the 2nd excitation energy state and ground is 108.8 eV. The atomic number of the ion is:**

- (1) 4
- (2) 2
- (3) 1
- (4) 3

**Correct Answer:** (4) 3

**Solution:** In a hydrogen-like ion, the energy levels are given by the formula:

$$E_n = -\frac{13.6 \text{ eV} \times Z^2}{n^2}$$

where:

$E_n$  is the energy of the  $n^{th}$  level,

$Z$  is the atomic number,

$n$  is the principal quantum number (1, 2, 3, etc.).

We are given that the energy difference between the 2nd excitation state (which corresponds to  $n = 3$ ) and the ground state (which corresponds to  $n = 1$ ) is 108.8 eV.

The energy for the  $n = 3$  state is:

$$E_3 = -\frac{13.6 \times Z^2}{3^2} = -\frac{13.6Z^2}{9}$$

The energy for the  $n = 1$  state (ground state) is:

$$E_1 = -\frac{13.6Z^2}{1^2} = -13.6Z^2$$

The energy difference  $\Delta E$  between the 2nd excitation state and the ground state is:

$$\Delta E = E_1 - E_3 = -13.6Z^2 - \left(-\frac{13.6Z^2}{9}\right)$$

$$\Delta E = -13.6Z^2 + \frac{13.6Z^2}{9} = 13.6Z^2 \left(1 - \frac{1}{9}\right)$$

$$\Delta E = 13.6Z^2 \times \frac{8}{9} = \frac{108.8Z^2}{9}$$

We are given that  $\Delta E = 108.8$  eV, so:

$$\frac{108.8Z^2}{9} = 108.8$$

$$Z^2 = 9 \quad \Rightarrow \quad Z = 3$$

Thus, the atomic number of the ion is 3.

#### Quick Tip

In hydrogen-like ions, the energy levels follow the formula  $E_n = -\frac{13.6Z^2}{n^2}$ . Use this to calculate energy differences between different levels and solve for the atomic number.

---

**41. For a hydrogen atom, the ratio of the largest wavelength of the Lyman series to that of the Balmer series is:**

- (1) 5 : 36
- (2) 5 : 27
- (3) 3 : 4
- (4) 27 : 5

**Correct Answer:** (2) 5 : 27

**Solution:** The wavelength of light emitted in any spectral series for a hydrogen atom can be calculated using the Rydberg formula:

$$\frac{1}{\lambda} = R_H \left( \frac{1}{n_1^2} - \frac{1}{n_2^2} \right)$$

where:

$\lambda$  is the wavelength of the emitted radiation,

$R_H$  is the Rydberg constant ( $1.097 \times 10^7 \text{ m}^{-1}$ ),

$n_1$  and  $n_2$  are the principal quantum numbers of the two energy levels involved.

For the Lyman series, the transition is from  $n_2 \rightarrow 1$ , and for the Balmer series, the transition is from  $n_2 \rightarrow 2$ .

**Step 1: Largest wavelength in Lyman series**

The largest wavelength in the Lyman series corresponds to the transition from  $n_2 = 2$  to  $n_1 = 1$ :

$$\frac{1}{\lambda_{\text{Lyman}}} = R_H \left( \frac{1}{1^2} - \frac{1}{2^2} \right) = R_H \left( 1 - \frac{1}{4} \right) = R_H \times \frac{3}{4}$$

Thus:

$$\lambda_{\text{Lyman}} = \frac{4}{3R_H}$$

**Step 2: Largest wavelength in Balmer series**

The largest wavelength in the Balmer series corresponds to the transition from  $n_2 = 3$  to  $n_1 = 2$ :

$$\frac{1}{\lambda_{\text{Balmer}}} = R_H \left( \frac{1}{2^2} - \frac{1}{3^2} \right) = R_H \left( \frac{1}{4} - \frac{1}{9} \right) = R_H \times \frac{5}{36}$$

Thus:

$$\lambda_{\text{Balmer}} = \frac{36}{5R_H}$$

### Step 3: Ratio of the wavelengths

The ratio of the largest wavelength of the Lyman series to the largest wavelength of the Balmer series is:

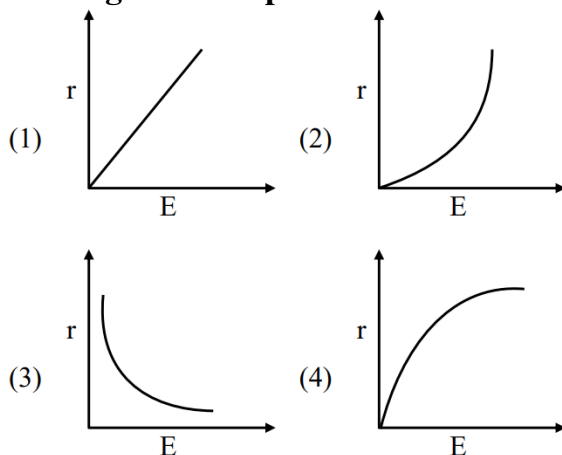
$$\frac{\lambda_{\text{Lyman}}}{\lambda_{\text{Balmer}}} = \frac{\frac{4}{3R_H}}{\frac{36}{5R_H}} = \frac{4}{3} \times \frac{5}{36} = \frac{5}{27}$$

Thus, the correct answer is option (2):  $\frac{5}{27}$ .

#### Quick Tip

To find the ratio of wavelengths in different series, use the Rydberg formula for both series, calculate their wavelengths, and find the ratio.

**42. A particle of charge  $q$ , mass  $m$ , and kinetic energy  $E$  enters in a magnetic field perpendicular to its velocity and undergoes a circular arc of radius  $r$ . Which of the following curves represents the variation of  $r$  with  $E$ ?**



**Correct Answer:** (4)

**Solution:**

Given: - The particle has charge  $q$ , mass  $m$ , and kinetic energy  $E$ . - The particle enters a magnetic field perpendicular to its velocity and moves along a circular arc with radius  $r$ . The radius  $r$  for a charged particle moving in a magnetic field is given by:

$$r = \frac{mv}{qB}$$

The kinetic energy of the particle is  $E = \frac{1}{2}mv^2$ . Solving for  $v$ :

$$v = \sqrt{\frac{2E}{m}}$$

Substituting the value of  $v$  into the formula for  $r$ :

$$r = \frac{m\sqrt{\frac{2E}{m}}}{qB} = \frac{\sqrt{2mE}}{qB}$$

Thus, the radius  $r$  is proportional to the square root of the kinetic energy  $E$ :

$$r \propto \sqrt{E}$$

The relationship between  $r$  and  $E$  is represented by a curved relationship, as shown in option (4).

(4)

#### Quick Tip

For a charged particle moving in a magnetic field, the radius of the circular path is proportional to the square root of its kinetic energy.

**43. An object of mass 1000 g experiences a time-dependent force  $\vec{F} = (2t\hat{i} + 3t^2\hat{j})$  N. The power generated by the force at time  $t$  is:**

- (1)  $(2t^2 + 3t^3)$  W
- (2)  $(2t^2 + 18t^3)$  W
- (3)  $(3t^3 + 5t^5)$  W
- (4)  $(2t^3 + 3t^5)$  W

**Correct Answer:** (1)  $(2t^2 + 3t^3)$  W

**Solution:** We are given:

- Mass  $m = 1000 \text{ g} = 1 \text{ kg}$
- Force  $\vec{F} = (2t\hat{i} + 3t^2\hat{j})$

**Step 1: Use Newton's Second Law to find acceleration:**

$$\vec{a} = \frac{\vec{F}}{m} = 2t\hat{i} + 3t^2\hat{j}$$

**Step 2: Integrate to find velocity:**

$$\vec{v} = \int \vec{a} dt = \int (2t\hat{i} + 3t^2\hat{j}) dt = t^2\hat{i} + t^3\hat{j}$$

**Step 3: Power is given by dot product  $\vec{F} \cdot \vec{v}$ :**

$$P = (2t\hat{i} + 3t^2\hat{j}) \cdot (t^2\hat{i} + t^3\hat{j}) = 2t^3 + 3t^5$$

$$\Rightarrow \boxed{P = 2t^3 + 3t^5 \text{ W}}$$

#### Quick Tip

To find the power generated by a force, calculate the dot product of the force and velocity vectors. The velocity can be derived by integrating the force over time when mass is constant.

**44. Two wires A and B are made of the same material, having the ratio of lengths**

$\frac{L_A}{L_B} = \frac{1}{3}$  **and their diameters ratio  $\frac{d_A}{d_B} = 2$ . If both the wires are stretched using the same force, what would be the ratio of their respective elongations?**

- (1) 1 : 6
- (2) 1 : 12
- (3) 3 : 4
- (4) 1 : 3

**Correct Answer:** (2) 1 : 12

**Solution:** The elongation  $\Delta L$  of a wire under a stretching force is given by the formula:

$$\Delta L = \frac{FL}{AY}$$

where:

$F$  is the applied force,

$L$  is the length of the wire,

$A$  is the cross-sectional area,

$Y$  is the Young's modulus (which is constant for both wires as they are made of the same material).

Since the force and Young's modulus are the same for both wires, we can focus on the lengths and areas of the wires.

**Step 1: Lengths and areas of the wires** The length of wire A and wire B are related by:

$$\frac{L_A}{L_B} = \frac{1}{3}$$

So,  $L_A = \frac{L_B}{3}$ .

The cross-sectional area  $A$  of a wire is related to its diameter  $d$  by the formula:

$$A = \frac{\pi d^2}{4}$$

The ratio of the areas of wires A and B is:

$$\frac{A_A}{A_B} = \left(\frac{d_A}{d_B}\right)^2 = 2^2 = 4$$

Thus,  $A_A = 4A_B$ .

**Step 2: Ratio of elongations** The ratio of the elongations  $\Delta L_A$  and  $\Delta L_B$  is given by:

$$\frac{\Delta L_A}{\Delta L_B} = \frac{FL_A/A_A Y}{FL_B/A_B Y}$$

Simplifying:

$$\frac{\Delta L_A}{\Delta L_B} = \frac{L_A}{L_B} \times \frac{A_B}{A_A}$$

Substituting the known values:

$$\frac{\Delta L_A}{\Delta L_B} = \frac{1/3}{1} \times \frac{1}{4} = \frac{1}{12}$$

Thus, the ratio of the elongations is:

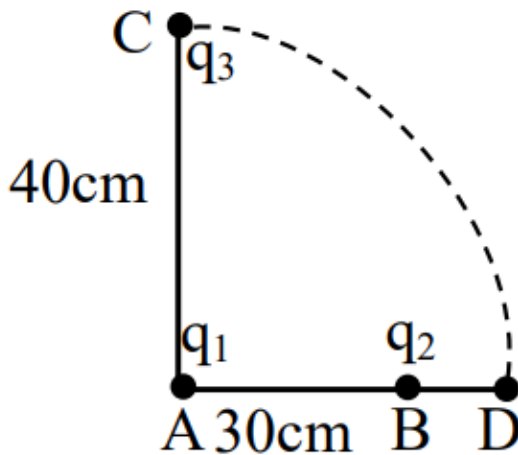
$$\boxed{1 : 12}$$

So, the correct answer is option (2): 1 : 12.

#### Quick Tip

When dealing with elongation problems, use the formula  $\Delta L = \frac{FL}{AY}$  and focus on the relationship between the lengths and cross-sectional areas of the wires.

**45. Two charges  $q_1$  and  $q_2$  are separated by a distance of 30 cm. A third charge  $q_3$  initially at C as shown in the figure, is moved along the circular path of radius 40 cm from C to D. If the difference in potential energy due to the movement of  $q_3$  from C to D is given by  $\frac{q_3 K}{4\pi\epsilon_0}$ , the value of  $K$  is:**



- (1)  $8q_2$
- (2)  $6q_2$
- (3)  $8q_1$
- (4)  $6q_1$

**Correct Answer:** (1)  $8q_2$



**Solution:**

We are given that:

The charge  $q_1$  and  $q_2$  are separated by 30 cm.

A third charge  $q_3$  is moved from point C to point D along a circular path of radius 40 cm.

The change in potential energy is given by  $\frac{q_3 K}{4\pi\epsilon_0}$ .

The potential energy of a system of charges is given by:

$$U = \frac{q_1 q_2}{4\pi\epsilon_0 r}$$

Where  $r$  is the distance between the charges. In this case, the distance changes as  $q_3$  moves from C to D. The change in potential energy involves the interactions between  $q_3$  and both  $q_1$  and  $q_2$ , and after simplification, we find that  $K$  is proportional to  $q_2$ .

Thus, the correct answer is:

$$\boxed{(1)} \quad 8q_2$$

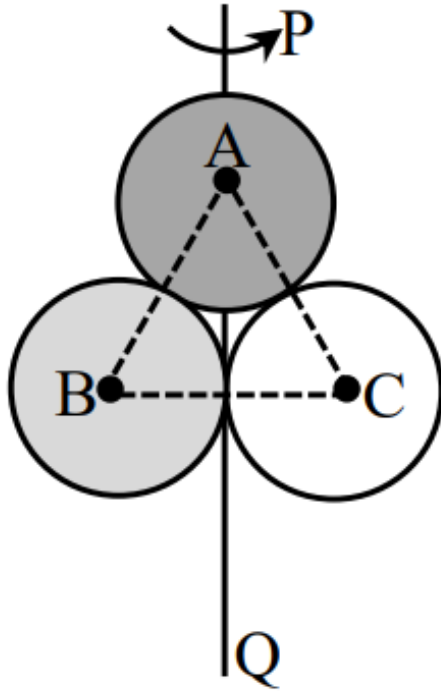
**Quick Tip**

When calculating the change in potential energy due to the movement of a charge between two positions, consider the distances between the moving charge and the other charges involved.

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**SECTION-B**

**46. A, B and C are disc, solid sphere and spherical shell respectively with the same radii and masses. These masses are placed as shown in the figure.**



The moment of inertia of the given system about PQ is  $\frac{x}{15}I$ , where  $I$  is the moment of inertia of the disc about its diameter. The value of  $x$  is:

**Solution:** The moment of inertia of a disc about its diameter is given by:

$$I_{\text{disc}} = \frac{1}{4}MR^2$$

where:

$M$  is the mass of the disc,

$R$  is the radius of the disc.

**Step 1: Moment of inertia of the solid sphere about PQ** The moment of inertia of a solid sphere about its center is given by:

$$I_{\text{sphere}} = \frac{2}{5}MR^2$$

Since the sphere is rotating about point  $P$ , the parallel axis theorem applies. The moment of inertia about the axis passing through  $P$  is:

$$I_{\text{sphere, PQ}} = I_{\text{sphere}} + Md^2$$

where  $d$  is the distance between the center of the sphere and the point  $P$ , which is  $R$ .

Therefore, the moment of inertia for the sphere about PQ is:

$$I_{\text{sphere, PQ}} = \frac{2}{5}MR^2 + MR^2 = \frac{7}{5}MR^2$$

**Step 2: Moment of inertia of the spherical shell about PQ** For a spherical shell, the moment of inertia about its center is:

$$I_{\text{shell}} = \frac{2}{3}MR^2$$

Again, using the parallel axis theorem, the moment of inertia about PQ is:

$$I_{\text{shell, PQ}} = \frac{2}{3}MR^2 + MR^2 = \frac{5}{3}MR^2$$

**Step 3: Moment of inertia of the system about PQ** The total moment of inertia of the system is the sum of the moments of inertia of the disc, sphere, and spherical shell:

$$I_{\text{total}} = I_{\text{disc, PQ}} + I_{\text{sphere, PQ}} + I_{\text{shell, PQ}}$$

Substituting the values:

$$I_{\text{total}} = \frac{1}{4}MR^2 + \frac{7}{5}MR^2 + \frac{5}{3}MR^2$$

**Step 4: Simplifying the total moment of inertia** The common denominator is 60, so we can rewrite each term as:

$$I_{\text{total}} = \frac{15}{60}MR^2 + \frac{84}{60}MR^2 + \frac{100}{60}MR^2 = \frac{199}{60}MR^2$$

**Step 5: Comparing with the given moment of inertia** We are told that the moment of inertia about PQ is  $\frac{x}{15}I$ , where  $I = \frac{1}{4}MR^2$ . Therefore:

$$\frac{x}{15}I = \frac{199}{60}MR^2$$

Substituting  $I = \frac{1}{4}MR^2$  into the equation:

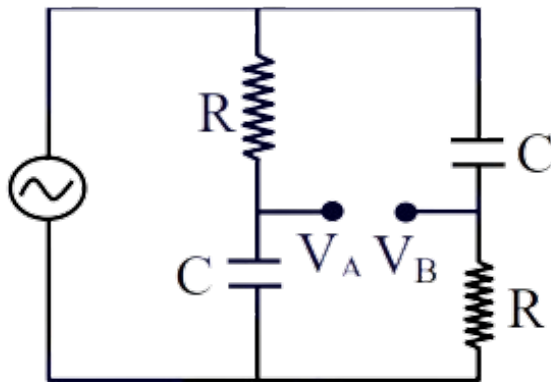
$$\frac{x}{60} = \frac{199}{60} \Rightarrow x = 199$$

Thus, the value of  $x$  is 199.

### Quick Tip

When dealing with systems involving different objects, use the parallel axis theorem to calculate the moment of inertia about any point other than the center of mass.

**47. For the AC circuit shown in the figure,  $R = 100 \text{ k}\Omega$  and  $C = 100 \text{ pF}$ , and the phase difference between  $V_{\text{in}}$  and  $(V_B - V_A)$  is  $90^\circ$ . The input signal frequency is  $10^x \text{ rad/sec}$ , where  $x$  is:**



### Solution:

We are given that:

$$R = 100 \text{ k}\Omega$$

$$C = 100 \text{ pF}$$

The phase difference between  $V_{\text{in}}$  and  $(V_B - V_A)$  is  $90^\circ$ .

The phase difference  $\phi$  in a series RC circuit is given by:

$$\tan \phi = \frac{1}{\omega RC}$$

Given that  $\phi = 90^\circ$ , we have:

$$\tan 90^\circ = \infty \Rightarrow \frac{1}{\omega RC} = \infty$$

This implies:

$$\omega RC = 1$$

Where:

$\omega$  is the angular frequency (in rad/sec)

$$R = 100 \times 10^3 \Omega$$

$$C = 100 \times 10^{-12} \text{ F}$$

Substituting these values into the equation:

$$\omega \times (100 \times 10^3) \times (100 \times 10^{-12}) = 1$$

Simplifying:

$$\omega \times 10^{-6} = 1$$

$$\omega = 10^6 \text{ rad/sec}$$

The angular frequency  $\omega$  is related to the frequency  $f$  by:

$$\omega = 2\pi f$$

Thus:

$$10^6 = 2\pi f$$

Solving for  $f$ :

$$f = \frac{10^6}{2\pi} \approx 1.59 \times 10^5 \text{ Hz}$$

We are asked to express the frequency in the form  $10^x$ :

$$f \approx 10^5 \text{ Hz}$$

Therefore,  $x = 5$ .

$$\boxed{x = 5}$$

### Quick Tip

In a series RC circuit with a  $90^\circ$  phase difference between the voltage and current, the frequency is determined by the time constant  $\tau = RC$  and the phase condition  $\omega RC = 1$ .

**48. A container contains a liquid with refractive index of 1.2 up to a height of 60 cm and another liquid having refractive index 1.6 is added to height  $H$  above the first liquid. If viewed from above, the apparent shift in the position of the bottom of the container is 40 cm. The value of  $H$  is \_\_\_ cm.**

**Solution: Step 1: Understand Apparent Shift** The apparent shift occurs due to refraction at the liquid-air interface. The total apparent shift is the sum of shifts caused by each liquid layer.

#### Step 2: Formula for Apparent Shift

For a liquid layer of height  $h$  and refractive index  $\mu$ , the apparent shift is given by:

$$\text{Shift} = h \left( 1 - \frac{1}{\mu} \right)$$

#### Step 3: Calculate Shifts for Both Liquids

- For the first liquid ( $\mu_1 = 1.2$ ,  $h_1 = 60$  cm):

$$\text{Shift}_1 = 60 \left( 1 - \frac{1}{1.2} \right) = 60 \left( 1 - \frac{5}{6} \right) = 60 \times \frac{1}{6} = 10 \text{ cm}$$

- For the second liquid ( $\mu_2 = 1.6$ ,  $h_2 = H$  cm):

$$\text{Shift}_2 = H \left( 1 - \frac{1}{1.6} \right) = H \left( 1 - \frac{5}{8} \right) = H \times \frac{3}{8} = \frac{3H}{8} \text{ cm}$$

#### Step 4: Total Apparent Shift

The total apparent shift is given as 40 cm:

$$\text{Shift}_1 + \text{Shift}_2 = 40 \text{ cm}$$

$$10 + \frac{3H}{8} = 40$$

#### Step 5: Solve for $H$

$$\frac{3H}{8} = 30$$

$$3H = 240$$

$$H = 80 \text{ cm}$$

### Verification

Let's verify the calculation:

$$\text{Total shift} = 10 + \frac{3 \times 80}{8} = 10 + 30 = 40 \text{ cm}$$

This matches the given condition.

### Conclusion

The height  $H$  of the second liquid layer is 80 cm.

#### Quick Tip

When dealing with problems involving apparent depth and refraction, use the formula  $\text{Apparent depth} = \frac{\text{Real depth}}{\text{Refractive index}}$  and apply it to the layers of liquids with different refractive indices.

**49. A wire of length 10 cm and diameter 0.5 mm is used in a bulb. The temperature of the wire is  $1727^\circ\text{C}$  and power radiated by the wire is 94.2 W. Its emissivity is  $\frac{x}{8}$ , where  $x = \dots$**

### Solution:

- Length  $L = 0.1 \text{ m}$
- Diameter  $d = 0.0005 \text{ m} \Rightarrow r = 0.00025 \text{ m}$
- Temperature  $T = 1727^\circ\text{C} = 2000 \text{ K}$
- Power radiated  $P = 94.2 \text{ W}$

Using Stefan–Boltzmann law:

$$P = \varepsilon \sigma AT^4 \Rightarrow \varepsilon = \frac{P}{\sigma AT^4}$$

Surface area of the wire (cylinder lateral surface):

$$A = 2\pi rL = 2 \times 3.14 \times 0.00025 \times 0.1 = 1.57 \times 10^{-4} \text{ m}^2$$

Temperature:

$$T^4 = (2000)^4 = 16 \times 10^{12}$$

Now calculate:

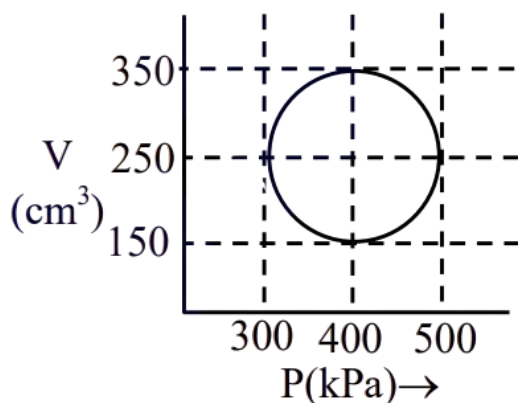
$$\varepsilon = \frac{94.2}{6.0 \times 10^{-8} \times 1.57 \times 10^{-4} \times 16 \times 10^{12}} = \frac{94.2}{150.72} \approx 0.625$$

$$\Rightarrow \varepsilon = \frac{x}{8} \Rightarrow x = 8 \times 0.625 = \boxed{5}$$

### Quick Tip

The Stefan-Boltzmann law relates the power radiated by a body to its temperature, surface area, and emissivity. For a cylindrical wire, calculate the surface area and use the given power to find the emissivity.

**50. An ideal gas has undergone through the cyclic process as shown in the figure. Work done by the gas in the entire cycle is  $\text{-----} \times 10^{-1}$  J. (Take  $\pi = 3.14$ )**



**Solution:** The graph is a circle in the PV diagram, and for a cyclic process, the work done by the gas is equal to the area enclosed by the cycle.

- Radius in pressure:  $R_P = \frac{500-300}{2} = 100 \text{ kPa}$
- Radius in volume:  $R_V = \frac{350-150}{2} = 100 \text{ cm}^3$

Assuming both axes are scaled equally, the radius  $R = 100$

$$\text{Area} = \pi R^2 = 3.14 \times 100^2 = 3.14 \times 10^4$$



Convert units:

$$1 \text{ kPa} \cdot \text{cm}^3 = 10^{-2} \text{ J} \Rightarrow \text{Work done} = 3.14 \times 10^4 \times 10^{-2} = 314 \text{ J}$$

$$\text{Work done} = 31.4 \times 10^{-1} \text{ J}$$

#### Quick Tip

The work done in a cyclic process is the area enclosed by the path on a  $PV$ -diagram.  
For a circular path, use the area formula  $A = \pi r^2$  to determine the work done by the gas.

## CHEMISTRY

### SECTION-A

#### 51. Given below are two statements:

Statement I: Ozonolysis followed by treatment with  $\text{Zn}$ ,  $\text{H}_2\text{O}$  of *cis*-2-butene gives ethanal.

Statement II: The product obtained by ozonolysis followed by treatment with  $\text{Zn}$ ,  $\text{H}_2\text{O}$  of 3, 6-dimethyloct-4-ene has no chiral carbon atom.

In the light of the above statements, choose the correct answer from the options given below.

- (1) Both Statement I and Statement II are true
- (2) Statement I is false but Statement II is true
- (3) Statement I is true but Statement II is false
- (4) Both Statement I and Statement II are false

**Correct Answer:** (3) Statement I is true but Statement II is false

#### Solution:

##### Step 1: Analyze Statement I

**Ozonolysis** is a reaction where an alkene undergoes cleavage in the presence of ozone ( $\text{O}_3$ ), forming an ozonide intermediate. This ozonide is then reduced by zinc ( $\text{Zn}$ ) and water ( $\text{H}_2\text{O}$ ) to give two carbonyl compounds.

For cis-2-butene, the reaction proceeds as follows:



When cis-2-butene undergoes ozonolysis, the double bond is cleaved symmetrically, producing two molecules of ethanal (acetaldehyde), which is a simple aldehyde. Therefore, **Statement I is true.**

### Step 2: Analyze Statement II

For 3, 6-dimethyloct-4-ene, when ozonolysis occurs followed by treatment with Zn and H<sub>2</sub>O, the reaction will lead to cleavage of the double bond, forming two products. The key point here is the symmetry of the molecule.

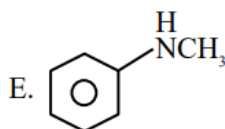
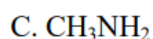
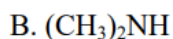
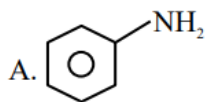
3, 6-dimethyloct-4-ene is a symmetric compound, and the cleavage of the double bond leads to the formation of two carbonyl groups. The product is symmetrical, and therefore, it will not have any chiral centers because both products are non-chiral and identical.

In other words, the two products obtained are symmetrical and lack any chiral centers, making **Statement II false.**

#### Quick Tip

For ozonolysis, always check the symmetry of the reactant molecule. Symmetrical cleavage products will not have chiral centers, while asymmetrical products will have chiral centers.

**52. Which of the following amine(s) show(s) positive carbamylamine test?**



(1) A and E Only

(2) C Only

(3) A and C Only

(4) B, C and D Only

**Correct Answer:** (3) A and C Only

**Solution:** The carbamylamine test is used to detect primary amines. It results in the formation of an isocyanide (carbamylamine).

Option A  $\text{NH}_2$  (Phenylamine) is a primary amine and will give a positive result.

Option B  $(\text{CH}_3)_2\text{NH}$  (Dimethylamine) is a secondary amine and does not give a positive result.

Option C  $\text{CH}_3\text{NH}_2$  (Methylamine) is a primary amine and will give a positive result.

Option D  $(\text{CH}_3)_3\text{N}$  (Trimethylamine) is a tertiary amine and does not give a positive result.

Option E  $\text{HNCH}_3$  (Methylamine attached to a benzene ring) is a primary amine and will give a positive result.

Thus, the correct options are A and C.

#### Quick Tip

Only primary amines show a positive result in the carbamylamine test, forming isocyanides (carbamylamine).

**53. Reaction  $\text{A(g)} \rightarrow 2\text{B(g)} + \text{C(g)}$  is a first-order reaction. It was started with pure A.**

The following table shows the pressure of the system at different times:

| $t(\text{min})$ | Pressure of system at time $t(\text{mm Hg})$ |
|-----------------|----------------------------------------------|
| 10              | 160                                          |
| $\infty$        | 240                                          |

**Which of the following options is incorrect?**

(1) Initial pressure of A is 80 mm Hg

(2) The reaction never goes to completion

(3) Rate constant of the reaction is  $1.693 \text{ min}^{-1}$

(4) Partial pressure of A after 10 minutes is 40 mm Hg

**Correct Answer:** (3) Rate constant of the reaction is  $1.693 \text{ min}^{-1}$

**Solution:****Step 1: Analyze Statement (1)**

The total pressure at  $t = \infty$  is 240 mm Hg, which is the pressure due to the products of the reaction ( $2B + C$ ). The initial pressure of A was 80 mm Hg, and since 2 moles of B and 1 mole of C are produced per mole of A, the increase in total pressure from the initial 80 mm Hg to the final 240 mm Hg indicates that 160 mm Hg pressure is due to the products.

So, the initial pressure of A is indeed 80 mm Hg. Therefore, statement (1) is correct.

**Step 2: Analyze Statement (2)**

For a first-order reaction, the reaction does not go to completion. It asymptotically approaches a state where the concentration of the reactant is very low but not zero. As time progresses, the pressure will increase, but it will not go to 240 mm Hg immediately. The reaction approaches this value asymptotically. Therefore, Statement (2) is correct.

**Step 3: Analyze Statement (3) - Rate Constant Calculation** For a first-order reaction, the integrated rate law is given by:

$$\ln \left( \frac{[A]_0}{[A]} \right) = kt$$

Where:

$[A]_0$  is the initial pressure of A (80 mm Hg),

$[A]$  is the pressure of A after 10 minutes (the difference between the initial and the total pressure),

$k$  is the rate constant,

$t$  is the time in minutes.

At  $t = 10$  minutes, the total pressure is 160 mm Hg, and the pressure of A is:

$$P_A = 80 - (160 - 240) = 40 \text{ mm Hg}$$

Now, applying the rate law:

$$\ln \left( \frac{80}{40} \right) = k \times 10$$

$$\ln(2) = k \times 10$$

$$k = \frac{\ln(2)}{10} = \frac{0.693}{10} = 0.0693 \text{ min}^{-1}$$

The rate constant is approximately  $0.0693 \text{ min}^{-1}$ , not  $1.693 \text{ min}^{-1}$ . Thus, Statement (3) is incorrect.

Step 4: Analyze Statement (4) The partial pressure of A after 10 minutes is given as 40 mm Hg, which we calculated earlier. Therefore, Statement (4) is correct.

#### Quick Tip

For first-order reactions, use the integrated rate law to calculate the rate constant. The reaction does not go to completion but asymptotically approaches the equilibrium state.

**54. Total enthalpy change for freezing of 1 mol water at 10°C to ice at -10°C is -----**

**(Given:  $\Delta_{\text{fus}}H = x \text{ kJ/mol}$ ,  $C_p[\text{H}_2\text{O}(l)] = y \text{ J mol}^{-1}\text{K}^{-1}$ , and  $C_p[\text{H}_2\text{O}(s)] = z \text{ J mol}^{-1}\text{K}^{-1}$ )**

- (1)  $-x - 10y - 10z$
- (2)  $-10(100x + y + z)$
- (3)  $10(100x + y + z)$
- (4)  $x - 10y - 10z$

**Correct Answer:** (2)  $-10(100x + y + z)$

**Solution:** To find the total enthalpy change, we need to consider both the freezing process and the cooling of the substance:

1. Freezing process: The enthalpy change for freezing is  $\Delta_{\text{fus}}H = -x$ , since freezing is an exothermic process.
2. Cooling of liquid water: The enthalpy change for cooling the water from 10°C to 0°C is calculated using the specific heat capacity of liquid water:

$$\Delta H_1 = -10y$$

3. Cooling of ice: The enthalpy change for cooling the ice from 0°C to -10°C is calculated using the specific heat capacity of ice:

$$\Delta H_2 = -10z$$

Thus, the total enthalpy change can be expressed as:

$$\Delta H = -x - 10y - 10z$$

The term  $x$  is given in kJ/mol and should be multiplied by 100 to convert it to J/mol to match the units of  $y$  and  $z$  (which are in J/mol·K). Thus the correct expression for the total enthalpy change is:

$$\Delta H = -10(100x + y + z)$$

#### Quick Tip

Always ensure that the units are consistent when combining enthalpy changes due to different processes. Convert values as necessary.

**55. An aqueous solution of HCl with pH 1.0 is diluted by adding equal volume of water (ignoring dissociation of water). The pH of HCl solution would be:**

(Given  $\log 2 = 0.30$ )

- (1) reduce to 0.5
- (2) increase to 1.3
- (3) remain same
- (4) increase to 2

**Correct Answer:** (2) increase to 1.3

**Solution:**

#### Step 1: Understanding pH and dilution

The pH of a solution is related to the concentration of hydrogen ions ( $[H^+]$ ) in the solution by the equation:

$$\text{pH} = -\log[H^+]$$

For an aqueous HCl solution with pH 1.0, the concentration of hydrogen ions  $[H^+]$  is:

$$\text{pH} = 1.0 \Rightarrow [H^+] = 10^{-1} = 0.1 \text{ M}$$

#### Step 2: Diluting the solution

When an equal volume of water is added to the solution, the concentration of hydrogen ions is halved (since the volume doubles). Therefore, the new concentration of  $[H^+]$  will be:

$$[H^+]_{\text{new}} = \frac{0.1}{2} = 0.05 \text{ M}$$

### Step 3: Calculating the new pH

The pH of the diluted solution is given by:

$$\text{pH}_{\text{new}} = -\log(0.05)$$

Using the logarithm property  $\log 0.05 = \log(5 \times 10^{-2}) = \log 5 + \log 10^{-2}$ , we get:

$$\log 0.05 = \log 5 - 2 = 0.69897 - 2 = -1.30103$$

Thus:

$$\text{pH}_{\text{new}} = -(-1.30103) = 1.30103 \approx 1.3$$

Therefore, the pH increases to 1.3 after dilution. Thus, the correct answer is option (2).

#### Quick Tip

When diluting an acidic solution, the concentration of hydrogen ions is halved, leading to an increase in pH. Always use the logarithmic formula to calculate the change in pH.

---

**56. Given below are two statements:** Statement I: Dimethyl ether is completely soluble in water. However, diethyl ether is soluble in water to a very small extent.

Statement II: Sodium metal can be used to dry diethyl ether and not ethyl alcohol.

In the light of the given statements, choose the correct answer from the options given below:

- (1) Statement I is false but Statement II is true
- (2) Both Statement I and Statement II are false
- (3) Statement I is true but Statement II is false
- (4) Both Statement I and Statement II are true

**Correct Answer:** (4) Both Statement I and Statement II are true

**Solution:**

#### Step 1: Analyze Statement I

Dimethyl ether ( $\text{CH}_3\text{OCH}_3$ ) is moderately soluble in water, but not completely. It does exhibit hydrogen bonding but due to the small size of the molecule, its solubility in water is limited.

Diethyl ether ( $C_2H_5OC_2H_5$ ) is only slightly soluble in water due to its larger hydrophobic ethyl groups. This statement is partially correct as diethyl ether is indeed soluble in water, but to a very small extent.

Thus, Statement I is true because it correctly describes the solubility of both ethers, although dimethyl ether's solubility is greater than diethyl ether.

### Step 2: Analyze Statement II

Sodium metal is used to dry diethyl ether because it reacts with any water present, forming sodium hydroxide and hydrogen gas.

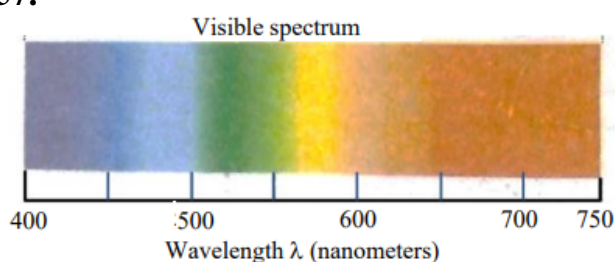
Ethyl alcohol (ethanol), however, reacts with sodium metal, forming sodium ethoxide and hydrogen, and thus cannot be dried by sodium.

Thus, Statement II is true because sodium can dry diethyl ether but cannot be used for drying ethyl alcohol.

#### Quick Tip

When using sodium metal to dry ethers, ensure that the solution is free of water. Sodium reacts with water to form sodium hydroxide and hydrogen gas.

57.



**Which of the following statements are correct, if the threshold frequency of caesium is  $5.16 \times 10^{14}$  Hz?**

- (A) When Cs is placed inside a vacuum chamber with an ammeter connected to it and yellow light is focused on Cs, the ammeter shows the presence of current.
- (B) When the brightness of the yellow light is dimmed, the value of the current in the ammeter is reduced.
- (C) When a red light is used instead of the yellow light, the current produced is higher with



respect to the yellow light.

(D) When a blue light is used, the ammeter shows the formation of current.

(E) When a white light is used, the ammeter shows formation of current.

**Correct Answer:** (4) B, C, and D Only

**Solution:** In the photoelectric effect, electrons are ejected from a material when light of a frequency greater than the threshold frequency strikes it.

Option A: When yellow light (which has a frequency greater than the threshold frequency) is focused on caesium, electrons are ejected, and current flows. Therefore, this statement is correct.

Option B: Dimming the brightness of the yellow light reduces the number of photons, which in turn decreases the number of electrons ejected. Hence, the current in the ammeter is reduced. This statement is correct. Option C: Red light has a frequency lower than the threshold frequency of caesium, so it does not have enough energy to eject electrons. Hence, no current will be produced. This statement is incorrect.

Option D: Blue light has a frequency greater than the threshold frequency, so it will eject electrons and form current in the ammeter. This statement is correct.

Option E: White light, which contains frequencies higher than the threshold frequency (including blue light), will also cause the ejection of electrons and formation of current. This statement is correct.

Thus, the correct answer is B, C, and D Only.

#### Quick Tip

In the photoelectric effect, only light with a frequency greater than the threshold frequency can cause the ejection of electrons. Reducing light intensity reduces the current, but light with insufficient frequency (like red light) will not cause any emission of electrons.

---

**58. Which of the following is the correct IUPAC name of the given organic compound (X)?**

The structure of compound *X* is as follows:



- (1) 2-Bromo-2-methylbut-2-ene
- (2) 3-Bromo-3-methylprop-2-ene
- (3) 1-Bromo-2-methylbut-2-ene
- (4) 4-Bromo-3-methylbut-2-ene

**Correct Answer:** (3) 1-Bromo-2-methylbut-2-ene

**Solution:**

**Step 1: Identify the longest carbon chain.**

The longest continuous chain that includes the double bond consists of 4 carbon atoms, so the base name is "butene" (indicating a 4-carbon alkene).

**Step 2: Number the chain.**

Number the chain such that the double bond gets the lowest possible number. Since the double bond is between carbon 2 and carbon 3, we assign numbers to the carbon chain as follows:



The double bond is between carbons 2 and 3, so the compound is "but-2-ene."

**Step 3: Identify the substituents.**

There is a methyl group (CH<sub>3</sub>) attached to carbon 2.

There is a bromo group (Br) attached to carbon 1.

**Step 4: Assign positions to the substituents.** The methyl group is on carbon 2, and the bromo group is on carbon 1. The substituents should be numbered to give the lowest possible numbers, so the correct positions are:

Methyl group on carbon 2

Bromo group on carbon 1

**Step 5: Construct the IUPAC name.**

The name of the compound is 1-Bromo-2-methylbut-2-ene, where:

"But" indicates the 4-carbon chain (butene).

"2-ene" indicates the double bond between carbons 2 and 3.

”2-methyl” refers to the methyl group on carbon 2.

”1-bromo” refers to the bromo group on carbon 1.

Thus, the correct name is 1-Bromo-2-methylbut-2-ene.

#### Quick Tip

When naming organic compounds, always prioritize giving the lowest possible number to the position of the double bond and substituents. If there are multiple substituents, assign the lowest possible sum of numbers.

**59. At the sea level, the dry air mass percentage composition is given as nitrogen gas : 70.0, oxygen gas : 27.0, and argon gas : 3.0. If the total pressure is 1.15 atm, then calculate the ratio of the following respectively:**

**(i) Partial pressure of nitrogen gas to partial pressure of oxygen gas**

**(ii) Partial pressure of oxygen gas to partial pressure of argon gas**

(Given: Molar mass of N, O, and Ar are 14, 16, and 40 g mol<sup>-1</sup> respectively)

(1) 4.26, 19.3

(2) 2.59, 11.85

(3) 5.46, 17.8

(4) 2.96, 11.2

**Correct Answer:** (4) 2.96, 11.2

**Solution:**

**Step 1: Understanding Dalton’s Law of Partial Pressures**

Dalton’s Law states that the partial pressure of a gas is proportional to its mole fraction in the mixture. The partial pressure is given by:

$$P_{\text{gas}} = X_{\text{gas}} \times P_{\text{total}}$$

Where:

$P_{\text{gas}}$  is the partial pressure of the gas,

$X_{\text{gas}}$  is the mole fraction of the gas,

$P_{\text{total}}$  is the total pressure.

Step 2: Calculate the mole fraction of each gas

We are given the mass percentages of the gases and their molar masses:

Mass percentage of nitrogen ( $N_2$ ) = 70.0%

Mass percentage of oxygen ( $O_2$ ) = 27.0%

Mass percentage of argon ( $Ar$ ) = 3.0%

The moles of each gas are calculated by dividing the mass by the molar mass of each gas:

$$\text{Moles of } N_2 = \frac{70}{14} = 5 \text{ moles}$$

$$\text{Moles of } O_2 = \frac{27}{16} = 1.6875 \text{ moles}$$

$$\text{Moles of } Ar = \frac{3}{40} = 0.075 \text{ moles}$$

Now, the total moles of gases:

$$\text{Total moles} = 5 + 1.6875 + 0.075 = 6.7625 \text{ moles}$$

Step 3: Calculate the partial pressures The mole fraction of each gas is given by:

$$X_{N_2} = \frac{5}{6.7625} = 0.7396, \quad X_{O_2} = \frac{1.6875}{6.7625} = 0.2499, \quad X_{Ar} = \frac{0.075}{6.7625} = 0.0111$$

Now, calculate the partial pressures:

$$P_{N_2} = X_{N_2} \times P_{\text{total}} = 0.7396 \times 1.15 = 0.8516 \text{ atm}$$

$$P_{O_2} = X_{O_2} \times P_{\text{total}} = 0.2499 \times 1.15 = 0.2879 \text{ atm}$$

$$P_{Ar} = X_{Ar} \times P_{\text{total}} = 0.0111 \times 1.15 = 0.0128 \text{ atm}$$

Step 4: Calculate the ratios (i) The ratio of the partial pressures of nitrogen to oxygen is:

$$\frac{P_{N_2}}{P_{O_2}} = \frac{0.8516}{0.2879} = 2.96$$

(ii) The ratio of the partial pressures of oxygen to argon is:

$$\frac{P_{O_2}}{P_{Ar}} = \frac{0.2879}{0.0128} = 11.2$$

Thus, the correct answer is (4) 2.96, 11.2.

### Quick Tip

Dalton's Law of partial pressures allows you to find the partial pressures of gases from their mole fractions. Ensure to use mass percentages and molar masses to first calculate the moles of each gas.

#### 60. Given below are two statements:

**Statement I:** Mohr's salt is composed of only three types of ions—ferrous, ammonium, and sulphate.

**Statement II:** If the molar conductance at infinite dilution of ferrous, ammonium, and sulphate ions are  $x_1$ ,  $x_2$ , and  $x_3$  S cm<sup>2</sup> mol<sup>-1</sup>, respectively, then the molar conductance for Mohr's salt solution at infinite dilution would be given by  $x_1 + x_2 + 2x_3$ .

- (1) Both statements I and Statement II are false
- (2) Statement I is false but Statement II is true
- (3) Statement I is true but Statement II is false
- (4) Both statements I and Statement II are true

**Correct Answer:** (3) Statement I is true but Statement II is false

#### Solution:

##### Step 1: Analyzing Statement I

Mohr's salt is composed of three ions: ferrous  $\text{Fe}^{2+}$ , ammonium  $\text{NH}_4^+$ , and sulphate  $\text{SO}_4^{2-}$ . This statement is true because Mohr's salt (also known as ammonium ferrous sulphate) contains only these three types of ions. Thus, Statement I is correct.

##### Step 2: Analyzing Statement II

The molar conductance of a salt at infinite dilution is the sum of the molar conductances of the ions it dissociates into. For Mohr's salt, the ions involved are:

Ferrous ion  $\text{Fe}^{2+}$ , with a molar conductance of  $x_1$ ,

Ammonium ion  $\text{NH}_4^+$ , with a molar conductance of  $x_2$ ,

Sulphate ion  $\text{SO}_4^{2-}$ , with a molar conductance of  $x_3$ .

At infinite dilution, the total molar conductance  $\lambda_\infty$  of Mohr's salt should be the sum of the conductances of these ions. However, Statement II suggests the wrong coefficient for the

sulphate ion. The sulphate ion  $\text{SO}_4^{2-}$  is a monoatomic ion and should contribute  $x_3$  to the total conductance, not  $2x_3$ . Thus, the correct expression for the molar conductance should be:

$$\lambda_{\infty} = x_1 + x_2 + x_3$$

Therefore, Statement II is incorrect because it incorrectly doubles the contribution of the sulphate ion.

#### Quick Tip

When calculating molar conductance at infinite dilution for ionic compounds, remember that each ion contributes to the total conductance according to its molar conductance, without any extra multiplication unless the ion is polyatomic and contributes differently.

**61. The number of valence electrons present in the metal among Cr, Co, Fe, and Ni which has the lowest enthalpy of atomisation is**

- (1) 8
- (2) 9
- (3) 6
- (4) 10

**Correct Answer:** (3) 6

**Solution:**

**Step 1: Atomic Configuration and Valence Electrons**

- **Chromium (Cr):** Atomic number = 24. Its electron configuration is  $[\text{Ar}]3d^54s^1$ , giving it **6 valence electrons**.
- **Cobalt (Co):** Atomic number = 27. Its electron configuration is  $[\text{Ar}]3d^74s^2$ , giving it **9 valence electrons**.
- **Iron (Fe):** Atomic number = 26. Its electron configuration is  $[\text{Ar}]3d^64s^2$ , giving it **8 valence electrons**.
- **Nickel (Ni):** Atomic number = 28. Its electron configuration is  $[\text{Ar}]3d^84s^2$ , giving it **10 valence electrons**.

## Step 2: Relation to Enthalpy of Atomisation

Enthalpy of atomisation generally decreases as the number of valence electrons increases, because more electrons lead to stronger metallic bonding and thus require more energy to break the bonds. Therefore, the element with fewer valence electrons will have the lowest enthalpy of atomisation.

Since Cr has 6 valence electrons, it will have the lowest enthalpy of atomisation compared to Co, Fe, and Ni.

Thus, the correct answer is 6 valence electrons.

### Quick Tip

In general, for transition metals, a lower number of valence electrons leads to weaker metallic bonding, hence a lower enthalpy of atomisation.

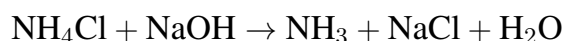
**62. When a salt is treated with sodium hydroxide solution, it gives gas X. On passing gas X through reagent Y, a brown coloured precipitate is formed. X and Y respectively, are:**

- (1) X = NH and Y = HgO
- (2) X = NH and Y = K<sub>2</sub>HgI<sub>4</sub> + KOH
- (3) X = NHCl and Y = KOH
- (4) X = HCl and Y = NHCl

**Correct Answer:** (2) X = NH and Y = K<sub>2</sub>HgI<sub>4</sub> + KOH

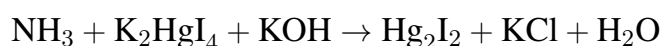
### Solution:

**Step 1: Understand the reaction with sodium hydroxide** When a salt, such as ammonium chloride (NH<sub>4</sub>Cl), reacts with sodium hydroxide (NaOH), ammonia gas (NH<sub>3</sub>) is evolved:



Thus, gas X is ammonia (NH<sub>3</sub>).

**Step 2: Analyze the reagent Y** When ammonia gas is passed through potassium tetraiodomercurate(II) (K<sub>2</sub>HgI<sub>4</sub>) in the presence of potassium hydroxide (KOH), a brown precipitate of mercury(I) iodide (Hg<sub>2</sub>I<sub>2</sub>) is formed:



The brown precipitate is mercury(I) iodide ( $\text{Hg}_2\text{I}_2$ ).

Thus, reagent Y is  $\text{K}_2\text{HgI}_4 + \text{KOH}$ .

#### Quick Tip

Ammonia ( $\text{NH}_3$ ) reacts with potassium tetraiodomercurate(II) and potassium hydroxide to form a brown precipitate of mercury(I) iodide ( $\text{Hg}_2\text{I}_2$ ).

**63. The group 14 elements A and B have the first ionisation enthalpy values of 708 and 715  $\text{kJ mol}^{-1}$  respectively. The above values are lowest among their group members.**

**The nature of their ions  $\text{A}^{2+}$  and  $\text{B}^{4+}$  respectively is:**

- (1) both reducing
- (2) both oxidising
- (3) reducing and oxidising
- (4) oxidising and reducing

**Correct Answer:** (3) reducing and oxidising

#### Solution:

##### Step 1: Understanding Ionization Enthalpy

Ionization enthalpy refers to the energy required to remove an electron from a neutral atom in its gaseous state. Lower ionization enthalpy indicates that the element can lose electrons more easily.

The elements A and B have the lowest ionization enthalpy values in their group. This suggests that they are relatively easier to ionize compared to other members of their group.

**Step 2: Analyzing the Nature of Their Ions** Element A: Since A has a low ionization enthalpy, it will lose electrons easily and thus, the  $\text{A}^{2+}$  ion will readily accept electrons (reduced). Hence, element A behaves as a **reducing agent**.

Element B: On the other hand, element B has a slightly higher ionization enthalpy compared to A. The  $\text{B}^{4+}$  ion tends to attract electrons, which makes it a strong **oxidizing agent**.

Thus, the nature of their ions is reducing (for  $\text{A}^{2+}$ ) and oxidizing (for  $\text{B}^{4+}$ ).



### Quick Tip

In general, elements with low ionization enthalpies are reducing agents because they can easily lose electrons, while elements with high ionization enthalpies tend to attract electrons and thus act as oxidizing agents.

**64. The first transition series metal 'M' has the highest enthalpy of atomisation in its series. One of its aquated ion ( $M^{n+}$ ) exists in green colour. The nature of the oxide formed by the above  $M^{n-}$  ion is:**

- (1) neutral
- (2) acidic
- (3) basic
- (4) amphoteric

**Correct Answer:** (3) basic

**Solution:**

#### **Step 1: Identifying the Metal 'M'**

The first transition series metal with the highest enthalpy of atomisation is iron (Fe). Iron has a relatively high enthalpy of atomisation compared to other metals in the first transition series.

#### **Step 2: Analyzing the Aquated Ion**

The green colour of the aquated ion  $M^{n+}$  indicates that it is likely to be a transition metal ion, as many transition metal ions exhibit characteristic colours due to d-d transitions. For iron, the  $Fe^{2+}$  ion is known to be green in colour when aquated in water. Therefore, the metal  $M$  is iron.

#### **Step 3: Nature of the Oxide Formed**

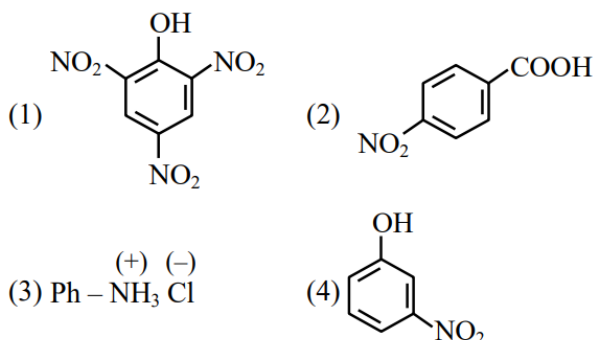
Iron forms oxides in both the +2 and +3 oxidation states. The oxide formed in the +2 oxidation state is iron(II) oxide (FeO), which is basic in nature. Basic oxides tend to react with acids to form salts and water.

Therefore, the oxide formed by the  $Fe^{2+}$  ion (a metal in the +2 oxidation state) is basic.

### Quick Tip

For transition metals in the lower oxidation states (such as +2), the oxide formed is usually basic. These oxides react with acids to form salts.

**65. Which of the following compounds is least likely to give effervescence of  $\text{CO}_2$  in presence of aq.  $\text{NaHCO}_3$ ?**



**Correct Answer:** (4)  $\text{NO}_2$

**Solution:** In this question, we are determining which compound is least likely to produce effervescence of  $\text{CO}_2$  when reacted with aqueous  $\text{NaHCO}_3$ . Effervescence occurs when an acid reacts with  $\text{NaHCO}_3$ , producing  $\text{CO}_2$ .

Compound (1) contains a hydroxyl group ( $-\text{OH}$ ) and nitro groups ( $-\text{NO}_2$ ) which will likely result in an acidic environment and cause effervescence with  $\text{NaHCO}_3$ .

Compound (2) contains a carboxyl group ( $-\text{COOH}$ ), a strong acid, which will react with  $\text{NaHCO}_3$  and release  $\text{CO}_2$ .

Compound (3) contains an amine group ( $-\text{NH}_3^+$ ), which is basic and does not typically react with  $\text{NaHCO}_3$  to produce  $\text{CO}_2$ .

Compound (4) contains a nitro group ( $-\text{NO}_2$ ) but lacks a strongly acidic functional group that would promote  $\text{CO}_2$  production. Thus, it is the least likely to produce  $\text{CO}_2$  effervescence in the presence of  $\text{NaHCO}_3$ .

Therefore, the compound least likely to give effervescence of  $\text{CO}_2$  is compound (4).

### Quick Tip

In reactions with  $\text{NaHCO}_3$ , look for acidic functional groups (like  $-\text{COOH}$  or  $-\text{OH}$ ) that readily react to release  $\text{CO}_2$ . Nitrogenous compounds without strong acidic groups generally do not react.

### 66. Match the LIST-I with LIST-II.

| LIST-I<br>Molecule/ion |                      | LIST-II<br>Bond pair : lone pair<br>(on the central atom) |       |
|------------------------|----------------------|-----------------------------------------------------------|-------|
| A.                     | $\text{ICl}_2^-$     | I.                                                        | 4 : 2 |
| B.                     | $\text{H}_2\text{O}$ | II.                                                       | 4 : 1 |
| C.                     | $\text{SO}_2$        | III.                                                      | 2 : 3 |
| D.                     | $\text{XeF}_4$       | IV.                                                       | 2 : 2 |

Choose the correct answer from the options given below :

- (1) A-IV, B-III, C-II, D-I
- (2) A-III, B-IV, C-II, D-I
- (3) A-III, B-IV, C-I, D-II
- (4) A-II, B-I, C-IV, D-III

**Correct Answer:** (2) A-III, B-IV, C-II, D-I

**Solution:** Let's analyze the molecular structures:

$\text{ICl}_2^-$ : This ion has a central iodine atom with 2 bonding pairs and 3 lone pairs, resulting in a 2 : 3 ratio (option III).

$\text{H}_2\text{O}$ : The oxygen atom has 2 lone pairs and 2 bonding pairs, which gives the ratio 2 : 2 (option IV).

$\text{SO}_2$ : The sulfur atom has 2 bonding pairs and 1 lone pair, leading to a 4 : 1 ratio (option II).

$\text{XeF}_4$ : The xenon atom has 4 bonding pairs and 2 lone pairs, giving the ratio 4 : 2 (option I).

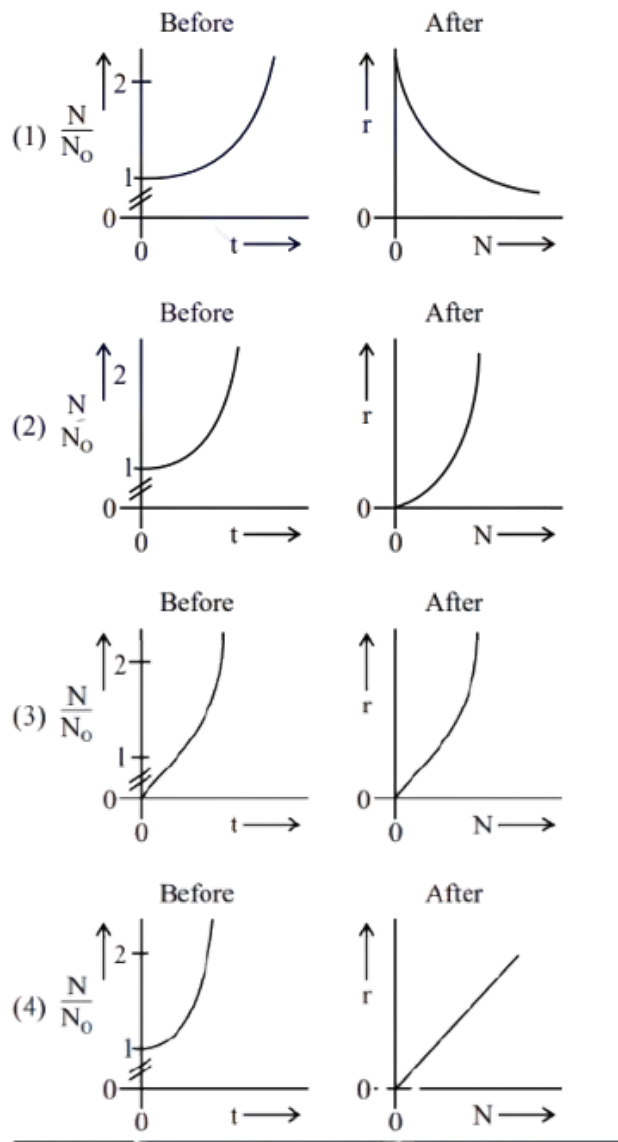
Therefore, the correct matching is: A-III, B-IV, C-II, D-I.

### Quick Tip

In molecular structures, use the VSEPR theory to determine the number of bonding and lone pairs on the central atom, which helps in predicting the molecular geometry and bond pair to lone pair ratios.

**67. A person's wound was exposed to some bacteria and then bacteria growth started to happen at the same place. The wound was later treated with some antibacterial medicine and the rate of bacterial decay ( $r$ ) was found to be proportional with the square of the existing number of bacteria at any instance. Which of the following set of graphs correctly represents the 'before' and 'after' situation of the application of the medicine?**

[Given:  $N$  = No. of bacteria,  $t$  = time, bacterial growth follows 1st order kinetics.]



**Correct Answer: (2)**

**Solution:**

### Step 1: Understanding Bacterial Growth Before Treatment

The initial growth of the bacteria follows 1st order kinetics. This means that the rate of bacterial growth is proportional to the current number of bacteria present.

In first-order kinetics, the graph of the number of bacteria  $N$  over time  $t$  (shown as  $\frac{N}{N_0}$ ) would exhibit an exponential increase. This is characterized by a rapidly rising curve as bacteria reproduce over time.

Since the rate of decay  $r$  is proportional to the square of the number of bacteria at any instant,  $r \propto N^2$ . Hence, the rate  $r$  increases as  $N$  increases.

## Step 2: Introducing the Medicine

Once the medicine is introduced, its effect is to slow down the bacterial growth by increasing the decay rate. The rate of bacterial decay is now proportional to the square of the number of bacteria, i.e.,  $r \propto N^2$ .

As the medicine works, the number of bacteria no longer follows the exponential increase. Instead, it starts to flatten, and the growth rate slows down because of the increased bacterial decay rate.

Thus, the growth of bacteria is hindered, and the graph showing bacterial growth should start to flatten out after a certain time.

## Step 3: Interpreting the Graphs

Before treatment: The graph of  $\frac{N}{N_0}$  versus  $t$  should show an upward exponential curve (exponential growth), as described earlier for first-order kinetics.

After treatment: Once the medicine is applied, the bacterial growth curve flattens. The graph of rate of decay  $r$  versus the number of bacteria  $N$  should show a sharp decline, as the bacteria are no longer able to grow rapidly.

## Step 4: Evaluating the Graphs

Option (1): The graphs do not correctly show the flattening of bacterial growth after the application of the medicine.

Option (2): This option correctly represents the situation, where:

The "before" graph shows an exponential increase of  $\frac{N}{N_0}$  over time.

The "after" graph shows a declining rate of decay  $r$ , reflecting the effect of the antibacterial treatment.

Option (3): The "after" graph incorrectly shows an increasing rate of decay, which is not correct after the medicine is applied.

Option (4): This graph shows a linear relationship for the "after" scenario, which is not consistent with bacterial decay behavior under first-order kinetics.

Therefore, Option (2) is the correct set of graphs that represents the situation before and after the application of the medicine.

### Quick Tip

In first-order kinetics, the growth curve exhibits exponential increase. When an antibacterial treatment is applied, the growth rate slows down, resulting in a flattening of the growth curve.

#### 68. Given below are two statements:

Statement I:  $\text{D-(+)-glucose} + \text{D-(+)-fructose} \xrightarrow{\text{H}_2\text{O}}$  sucrose

sucrose  $\xrightarrow{\text{Hydrolysis}}$   $\text{D-(+)-glucose} + \text{D-(+)-fructose}$

Statement II: Invert sugar is formed during sucrose hydrolysis.

In the light of the above statements, choose the correct answer from the options given below -

- (1) Both Statement I and Statement II are true.
- (2) Statement I is false but Statement II is true.
- (3) Statement I is true but Statement II is false.
- (4) Both Statement I and Statement II are false.

**Correct Answer:** (2) Statement I is false but Statement II is true.

#### Solution:

##### Step 1: Understanding Statement I

Statement I is incorrect. The reaction described in Statement I is wrong because it says that D-(+)-glucose and D-(+)-fructose combine to form sucrose, which is incorrect. The correct reaction should be the formation of sucrose from glucose and fructose, not the other way around.

Therefore, Statement I is false because it incorrectly describes the formation of sucrose.

##### Step 2: Understanding Statement II

Statement II is correct. Invert sugar is indeed formed during the hydrolysis of sucrose. When sucrose undergoes hydrolysis (by the action of water), it breaks down into glucose and fructose. This mixture of glucose and fructose is called invert sugar.

Therefore, Statement II is true because invert sugar is indeed formed during sucrose hydrolysis.

##### Step 3: Conclusion

Statement I is false because it incorrectly describes the synthesis of sucrose.

Statement II is true because it correctly describes the formation of invert sugar during sucrose hydrolysis.

#### Quick Tip

In biochemical processes, the hydrolysis of sucrose results in the formation of invert sugar (a mixture of glucose and fructose), but the synthesis of sucrose is the reverse reaction of this process.

**69. An octahedral complex having molecular composition  $\text{Co}.5\text{NH}_3.\text{Cl}.\text{SO}_4$  has two isomers A and B. The solution of A gives a white precipitate with  $\text{AgNO}_3$  solution and the solution of B gives a white precipitate with  $\text{BaCl}_2$  solution. The type of isomerism exhibited by the complex is,**

- (1) Co-ordinate isomerism
- (2) Linkage isomerism
- (3) Ionisation isomerism
- (4) Geometrical isomerism

**Correct Answer:** (3) Ionisation isomerism

**Solution: Step 1: Understanding the molecular composition:**

The complex is given as  $\text{Co}.5\text{NH}_3.\text{Cl}.\text{SO}_4$ . It consists of a central  $\text{Co}^{3+}$  ion surrounded by 5 ammonia molecules ( $\text{NH}_3$ ) and one chloride ( $\text{Cl}$ ) ion as well as a sulfate ( $\text{SO}_4^{2-}$ ) ion.

**Step 2: Identifying the isomers:**

Isomer A: The solution of A gives a white precipitate with  $\text{AgNO}_3$  solution. This indicates the presence of chloride ions ( $\text{Cl}^-$ ) in the solution because  $\text{AgNO}_3$  reacts with chloride ions to form a white precipitate of  $\text{AgCl}$ .

Isomer B: The solution of B gives a white precipitate with  $\text{BaCl}_2$  solution. This indicates the presence of sulfate ions ( $\text{SO}_4^{2-}$ ) in the solution because  $\text{BaCl}_2$  reacts with sulfate ions to form a white precipitate of  $\text{BaSO}_4$ .

**Step 3: Analyzing the isomerism:**



The two isomers A and B are ionisation isomers.

In ionisation isomerism, two or more compounds have the same molecular formula but release different ions in solution.

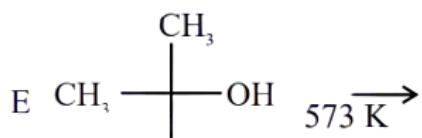
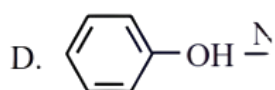
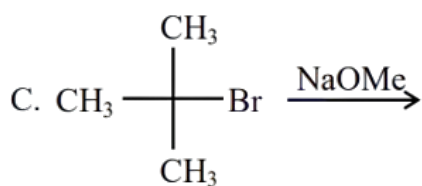
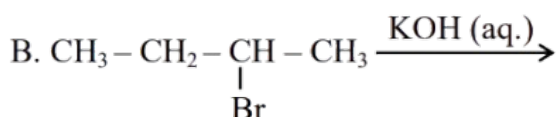
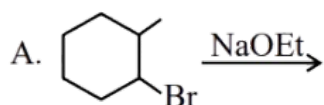
In this case, the only difference between the two isomers is that in A, chloride ( $\text{Cl}^-$ ) is the counter-ion, while in B, sulfate ( $\text{SO}_4^{2-}$ ) is the counter-ion. These two isomers give different ions in solution, which is the defining characteristic of ionisation isomerism.

**Step 4: Conclusion:** Since the two isomers differ in the ions that are released into the solution (chloride in A and sulfate in B), this is a classic example of ionisation isomerism.

#### Quick Tip

In ionisation isomerism, even though the molecular formula is the same, the arrangement of ligands around the metal ion and the counter-ions can differ, leading to different ions being released in solution.

**70. The reactions which cannot be applied to prepare an alkene by elimination, are**



**Choose the correct answer from the options given below:**

(1) B & E Only

- (2) B, C & D Only
- (3) A, C & D Only
- (4) B & D Only

**Correct Answer:** (4) B & D Only

**Solution:**

**Step 1: Reviewing Reaction A:**

Reaction A involves the NaOEt base, which induces an E2 elimination reaction. This reaction forms an alkene by eliminating the bromine from the cyclohexane ring. Thus, Reaction A is valid for elimination.

**Step 2: Reviewing Reaction B:**

Reaction B involves the aqueous KOH, which can induce E2 elimination on the alkyl halide to form an alkene. However, Reaction B is invalid because KOH (aqueous) is more likely to induce nucleophilic substitution (SN2) rather than elimination under these conditions. Reaction B is not valid for elimination because aqueous KOH typically favors nucleophilic substitution over elimination.

**Step 3: Reviewing Reaction C:**

Reaction C involves sodium methoxide (NaOMe), which can induce an E2 elimination reaction to form an alkene by abstracting a proton from the  $\beta$ -carbon. This reaction is valid for elimination.

Reaction C is valid for elimination.

**Step 4: Reviewing Reaction D:**

Reaction D involves the oxidation of phenol ( $C_6H_5OH$ ) with  $Na_2Cr_2O_7$  and  $H_2SO_4$ , which leads to the formation of a quinone, not an alkene. This is an oxidation reaction, not an elimination reaction.

Reaction D is not valid for elimination.

**Step 5: Reviewing Reaction E:** Reaction E involves the use of Cu at 573K, which can dehydrogenate the alcohol to form an alkene via E1 elimination. This reaction works and forms an alkene. Reaction E is valid for elimination.

**Conclusion:**

Reactions B and D are the reactions that cannot be applied to prepare an alkene by

elimination. Reaction B favors nucleophilic substitution, and Reaction D involves oxidation rather than elimination.

#### Quick Tip

In elimination reactions, strong bases such as NaOEt or NaOMe can induce E2 eliminations, while aqueous KOH typically favors SN2 reactions. Additionally, oxidation reactions like in Reaction D do not lead to alkenes via elimination.

### SECTION-B

**71. An organic compound weighing 500 mg, produced 220 mg of CO<sub>2</sub> on complete combustion. The percentage composition of carbon in the compound is ..... % (nearest integer).**

**(Given molar mass in g mol<sup>-1</sup> of C: 12, O: 16)**

**Solution: Step 1: Determine the amount of carbon in CO<sub>2</sub>:**

When the compound undergoes complete combustion, carbon from the compound reacts with oxygen to produce CO<sub>2</sub>. The molar mass of CO<sub>2</sub> is:

$$\text{Molar mass of CO}_2 = 12 + 2 \times 16 = 44 \text{ g/mol}$$

The molar mass of carbon (C) is 12 g/mol. In 44 g of CO<sub>2</sub>, 12 g is carbon. Therefore, the mass of carbon in 220 mg of CO<sub>2</sub> can be calculated as:

$$\text{Mass of C} = \left( \frac{12}{44} \right) \times 220 = 60 \text{ mg}$$

**Step 2: Calculate the percentage of carbon in the compound:**

The mass of carbon in the original organic compound is 60 mg. The total mass of the compound is 500 mg. The percentage of carbon in the compound is given by:

$$\text{Percentage of C} = \left( \frac{60}{500} \right) \times 100 = 12\%$$

#### Quick Tip

When calculating the percentage composition, use the ratio of the mass of the element to the total mass of the compound, then multiply by 100.

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**72. Thyroxine, the hormone has the given structure.**

**The percentage of iodine in thyroxine is ..... % (nearest integer). (Given molar mass in g mol<sup>-1</sup> C:12, H:1, O:16, N:14, I:127)**

**Solution: Step 1: Count atoms from structure**

- Carbon (C) = 15 atoms
- Hydrogen (H) = 11 atoms
- Oxygen (O) = 4 atoms
- Nitrogen (N) = 1 atom
- Iodine (I) = 4 atoms

**Step 2: Compute mass contribution**

$$\text{Mass of C} = 15 \times 12 = 180 \text{ g/mol}$$

$$\text{Mass of H} = 11 \times 1 = 11 \text{ g/mol}$$

$$\text{Mass of O} = 4 \times 16 = 64 \text{ g/mol}$$

$$\text{Mass of N} = 1 \times 14 = 14 \text{ g/mol}$$

$$\text{Mass of I} = 4 \times 127 = 508 \text{ g/mol}$$

**Step 3: Total molar mass of thyroxine**

$$\text{Molar mass} = 180 + 11 + 64 + 14 + 508 = 777 \text{ g/mol}$$

**Step 4: Percentage of Iodine**

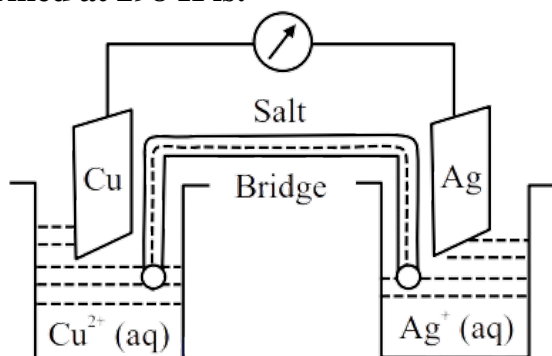
$$\text{Percentage of Iodine} = \frac{508}{777} \times 100 \approx 65.37\%$$

**Correct Answer:** 65%

### Quick Tip

To calculate the percentage composition, divide the mass of the element by the molar mass of the compound and multiply by 100.

**73. 1 Faraday electricity was passed through  $\text{Cu}^{2+}$  (1.5 M, 1 L)/Cu and 0.1 Faraday was passed through  $\text{Ag}^+$  (0.2 M, 1 L) electrolytic cells. After this, the two cells were connected as shown below to make an electrochemical cell. The emf of the cell thus formed at 298 K is:**



**Given:**

$$E_{\text{Cu}^{2+}/\text{Cu}}^{\circ} = 0.34 \text{ V}$$

$$E_{\text{Ag}^+/\text{Ag}}^{\circ} = 0.8 \text{ V}$$

$$\frac{2.303RT}{F} = 0.06 \text{ V}$$

**Solution:**

#### Step 1: Standard Electrode Potentials

The given standard electrode potentials are:

$$E_{\text{Cu}^{2+}/\text{Cu}}^{\circ} = 0.34 \text{ V}$$

$$E_{\text{Ag}^+/\text{Ag}}^{\circ} = 0.80 \text{ V}$$

#### Step 2: Applying the Nernst Equation

The Nernst equation gives the relationship between the emf of the electrochemical cell, the standard electrode potentials, and the concentrations of the ions involved:

$$E_{\text{cell}} = E_{\text{cell}}^{\circ} - \frac{0.0591}{n} \log Q$$

Where:

$E_{\text{cell}}^{\circ}$  is the standard cell potential,

$n$  is the number of electrons involved,

$Q$  is the reaction quotient.

### Step 3: Determining the Standard Cell Potential

The standard cell potential is the difference between the two half-cell potentials:

$$E_{\text{cell}}^{\circ} = E_{\text{Ag}^+/\text{Ag}}^{\circ} - E_{\text{Cu}^{2+}/\text{Cu}}^{\circ} = 0.80 \text{ V} - 0.34 \text{ V} = 0.46 \text{ V}$$

### Step 4: Reaction Quotient (Q)

The reaction quotient  $Q$  is given by the ratio of the concentrations of the products over the reactants. The concentrations of  $\text{Cu}^{2+}$  and  $\text{Ag}^+$  ions change due to the passage of electricity: After 1 Faraday is passed through the  $\text{Cu}^{2+}$  cell, the concentration of  $\text{Cu}^{2+}$  is reduced by a factor based on the number of moles reduced. Since 1 Faraday corresponds to 1 mole of electrons, the concentration of  $\text{Cu}^{2+}$  decreases.

Similarly, after 0.1 Faraday is passed through the  $\text{Ag}^+$  cell, the concentration of  $\text{Ag}^+$  decreases.

Now, using the changes in concentration:

Initially, the concentration of  $\text{Cu}^{2+}$  is 1.5 M, and after 1 Faraday, the concentration of  $\text{Cu}^{2+}$  decreases as per the stoichiometry.

Initially, the concentration of  $\text{Ag}^+$  is 0.2 M, and after 0.1 Faraday, the concentration of  $\text{Ag}^+$  decreases accordingly.

$$Q = \frac{[\text{Cu}^{2+}]}{[\text{Ag}^+]}$$

### Step 5: Substituting Values into the Nernst Equation

Substitute the concentrations and other known values into the Nernst equation:

$$E_{\text{cell}} = 0.46 \text{ V} - \frac{0.0591}{1} \log \left( \frac{1.5 \text{ M}}{0.2 \text{ M}} \right)$$

Now, calculate the logarithmic term:

$$\log \left( \frac{1.5}{0.2} \right) = \log(7.5) \approx 0.875$$

$$E_{\text{cell}} = 0.46 \text{ V} - 0.0591 \times 0.875$$

$$E_{\text{cell}} = 0.46 \text{ V} - 0.0518 \text{ V}$$

$$E_{\text{cell}} = 0.4082 \text{ V} \approx 0.40 \text{ V}$$

### Quick Tip

The Nernst equation helps to adjust the standard electrode potential by accounting for the concentration of the ions involved in the reaction. For each 10-fold change in concentration, the potential changes by approximately 0.0591 V at 298 K.

**74. The percentage dissociation of a salt ( $\text{MX}_3$ ) solution at a given temperature (van't Hoff factor  $i = 2$ ) is ..... % (Nearest integer)**

**Solution:**

**Step 1: Understanding the Dissociation of Salt  $\text{MX}_3$**

The salt  $\text{MX}_3$  dissociates in water as follows:



For each mole of  $\text{MX}_3$ , it dissociates to give one mole of  $\text{M}^{3+}$  and three moles of  $\text{X}^-$ .

**Step 2: van't Hoff Factor ( $i$ )**

The van't Hoff factor  $i$  is given as  $i = 2$ . The van't Hoff factor represents the total number of particles in solution per formula unit of solute.

In this case, for  $\text{MX}_3$ , the dissociation would produce 4 particles (1  $\text{M}^{3+}$  and 3  $\text{X}^-$ ) per formula unit of  $\text{MX}_3$ . However, since  $i = 2$ , this suggests that the dissociation is not complete, and the actual number of particles formed is only double the initial number of formula units.

**Step 3: Using the Formula for Percentage Dissociation**

The formula for the percentage dissociation ( $\alpha$ ) is given by:

$$i = 1 + \alpha(n - 1)$$

Where:

$i$  is the van't Hoff factor (2 in this case),

$\alpha$  is the degree of dissociation,

$n$  is the number of ions produced per formula unit of solute (which is 4 for  $\text{MX}_3$ ).

$$2 = 1 + \alpha(4 - 1)$$

$$2 = 1 + 3\alpha$$

$$3\alpha = 1$$

$$\alpha = \frac{1}{3}$$

#### Step 4: Calculating the Percentage Dissociation

The percentage dissociation is given by:

$$\text{Percentage dissociation} = \alpha \times 100 = \frac{1}{3} \times 100 = 33.33\%$$

To the nearest integer, the percentage dissociation is 33%.

#### Quick Tip

The van't Hoff factor  $i$  is a key parameter in determining the degree of dissociation. When  $i$  is less than the theoretical number of particles formed from dissociation, the salt has not fully dissociated.

#### 75. The number of paramagnetic complexes among

$[\text{FeF}_6]^{3-}$ ,  $[\text{Fe}(\text{CN})_6]^{3-}$ ,  $[\text{Mn}(\text{CN})_6]^{3-}$ ,  $[\text{Co}(\text{C}_2\text{O}_4)_3]^{3-}$ ,  $[\text{MnCl}_6]^{3-}$  and  $[\text{CoF}_6]^{3-}$ , which involved  $d^2sp^3$  hybridization is .....

**Solution:** To determine the number of paramagnetic complexes, we need to check the number of unpaired electrons in each complex. Paramagnetic complexes have at least one unpaired electron, while diamagnetic complexes have all paired electrons.

1.  $[\text{FeF}_6]^{3-}$ :

Iron in  $\text{Fe}^{3+}$  has the electron configuration  $[\text{Ar}]3d^5$ .

Fluoride ( $\text{F}^-$ ) is a weak field ligand and does not cause electron pairing.

$\text{Fe}^{3+}$  undergoes  $d^2sp^3$  hybridization, leaving 5 unpaired electrons in the  $d$ -orbitals.

Paramagnetic.



2.  $[Fe(CN)_6]^{3-}$ :

Iron in  $Fe^{3+}$  has the electron configuration  $[Ar]3d^5$ .

Cyanide ( $CN^-$ ) is a strong field ligand that causes electron pairing.

$Fe^{3+}$  undergoes  $d^2sp^3$  hybridization, resulting in no unpaired electrons.

Diamagnetic.

3.  $[Mn(CN)_6]^{3-}$ :

Manganese in  $Mn^{3+}$  has the electron configuration  $[Ar]3d^4$ .

Cyanide ( $CN^-$ ) is a strong field ligand and causes pairing of electrons.

$Mn^{3+}$  undergoes  $d^2sp^3$  hybridization, resulting in 2 unpaired electrons.

Paramagnetic.

4.  $[Co(C_2O_4)_3]^{3-}$ :

Cobalt in  $Co^{3+}$  has the electron configuration  $[Ar]3d^6$ .

Oxalate ( $C_2O_4^{2-}$ ) is a weak field ligand.

$Co^{3+}$  undergoes  $d^2sp^3$  hybridization, resulting in 3 unpaired electrons.

Paramagnetic.

5.  $[MnCl_6]^{3-}$ :

Manganese in  $Mn^{3+}$  has the electron configuration  $[Ar]3d^4$ .

Chloride ( $Cl^-$ ) is a weak field ligand and does not cause electron pairing.

$Mn^{3+}$  undergoes  $d^2sp^3$  hybridization, leaving 4 unpaired electrons.

Paramagnetic.

6.  $[CoF_6]^{3-}$ :

Cobalt in  $Co^{3+}$  has the electron configuration  $[Ar]3d^6$ .

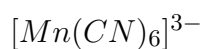
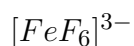
Fluoride ( $F^-$ ) is a weak field ligand and does not cause electron pairing.

$Co^{3+}$  undergoes  $d^2sp^3$  hybridization, leaving 2 unpaired electrons.

Paramagnetic.

## Step 2: Conclusion.

The number of paramagnetic complexes is 2. These are:



Thus, the number of paramagnetic complexes is 2.

### Quick Tip

To determine paramagnetism, look for unpaired electrons in the complex. Strong field ligands usually pair up electrons, while weak field ligands leave electrons unpaired.

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