JEE Main 2025 Jan 22 Shift 1 Question Paper with Solutions

Time Allowed :3 Hour	**Maximum Marks :**300	**Total Questions :**75

General Instructions

Read the following instructions very carefully and strictly follow them:

1. The test is of 3 hours duration.
2. The question paper consists of 75 questions. The maximum marks are 300.
3. There are three parts in the question paper consisting of Physics, Chemistry and Mathematics having 25 questions in each part of equal weightage.
4. Each part (subject) has two sections.
   (i) Section-A: This section contains 20 multiple choice questions which have only one correct answer. Each question carries 4 marks for correct answer and –1 mark for wrong answer.
   (ii) Section-B: This section contains 5 questions. The answer to each of the questions is a numerical value. Each question carries 4 marks for correct answer and –1 mark for wrong answer. For Section-B, the answer should be rounded off to the nearest integer.

MATHEMATICS

SECTION-A

1. The number of non-empty equivalence relations on the set $\{1, 2, 3\}$ **is :**

(1) 6

(2) 7

(3) 5

(4) 4

Correct Answer: (3) 5

Solution: Step 1: Understanding equivalence relations. An equivalence relation on a set is a relation that is reflexive, symmetric, and transitive. For a set of three elements, $\{1, 2, 3\}$, we need to count all possible non-empty equivalence relations.

Step 2: List all possible partitions of the set. The number of equivalence relations corresponds to the number of partitions of the set $\{1, 2, 3\}$. The possible partitions are:

$$\{\{1\}, \{2\}, \{3\}\}, \quad \{\{1, 2\}, \{3\}\}, \quad \{\{1, 3\}, \{2\}\}, \quad \{\{2, 3\}, \{1\}\}, \quad \{\{1, 2, 3\}\}.$$

Step 3: Count the number of partitions. From the above, there are 5 possible partitions, and hence, 5 possible equivalence relations.

Thus, the number of non-empty equivalence relations on the set is $\boxed{5}$.

> **Quick Tip**
>
> To count the number of equivalence relations, find all the possible partitions of the set.

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2. Let $f : \mathbb{R} \to \mathbb{R}$ be a twice differentiable function such that $f(x + y) = f(x)f(y)$ for all $x, y \in \mathbb{R}$. If $f'(0) = 4a$ and f satisfies $f''(x) - 3af'(x) - f(x) = 0$, where $a > 0$, then the area of the region $R = \{(x, y)|0 \le y \le f(ax), 0 \le x \le 2\}$ is :

(1) $e^2 - 1$

(2) $e^4 + 1$

(3) $e^4 - 1$

(4) $e^2 + 1$

Correct Answer: (1) $e^2 - 1$

Solution: We are given that $f(x + y) = f(x)f(y)$, which implies that $f(x) = e^{\lambda x}$, for some constant λ. We also know that $f'(0) = 4a$, so:

$$f'(x) = \lambda e^{\lambda x}, \quad f'(0) = \lambda = 4a.$$

Thus, $f(x) = e^{4ax}$.

Next, we are given the differential equation $f''(x) - 3af'(x) - f(x) = 0$, and we need to solve for a. Substituting $f(x) = e^{4ax}$ into the equation gives:

$$f''(x) = 16a^2 e^{4ax}, \quad f'(x) = 4ae^{4ax}.$$

Substitute into the differential equation:

$$16a^2e^{4ax} - 3a(4ae^{4ax}) - e^{4ax} = 0,$$

$$16a^2 - 12a^2 - 1 = 0 \quad \Rightarrow \quad 4a^2 = 1 \quad \Rightarrow \quad a = \frac{1}{2}.$$

Thus, the function becomes:

$$f(x) = e^{2x}.$$

The area of the region is given by:

$$\text{Area} = \int_0^2 f(ax)\,dx = \int_0^2 e^x\,dx = e^2 - 1.$$

Thus, the area of the region is $\boxed{e^2 - 1}$.

Quick Tip

To find the area under a curve, integrate the function over the given limits.

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3. Let the triangle PQR be the image of the triangle with vertices $(1, 3), (3, 1)$ and $(2, 4)$ in the line $x + 2y = 2$. If the centroid of ΔPQR is the point (α, β), then $15(\alpha - \beta)$ is equal to :

(1) 24

(2) 19

(3) 21

(4) 22

Correct Answer: (4) 22

Solution: Let G be the centroid of ΔPQR formed by $(1, 3), (3, 1), (2, 4)$.

The centroid of a triangle is the average of the coordinates of its vertices:

$$G = \left(\frac{1+3+2}{3}, \frac{3+1+4}{3}\right) = \left(\frac{6}{3}, \frac{8}{3}\right) = \left(2, \frac{8}{3}\right).$$

Next, we find the image of G under the transformation $x + 2y = 2$. The transformation matrix for this is:

$$\alpha - 2 = \frac{-2}{5}, \quad \beta - \frac{8}{3} = \frac{-32}{15} + 2,$$

which leads to $\alpha = -\frac{2}{5}$ and $\beta = -\frac{24}{15}$.

Thus, $15(\alpha - \beta) = 15(-\frac{2}{5} - \frac{24}{15}) = 22$.

Thus, $15(\alpha - \beta) = \boxed{22}$.

> **Quick Tip**
>
> The centroid of a triangle is the average of the coordinates of its vertices.

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4. Let z_1, z_2, z_3 be three complex numbers on the circle $|z| = 1$ with $\arg(z_1) = -\frac{\pi}{4}, \arg(z_2) = 0$ and $\arg(z_3) = \frac{\pi}{4}$. If $|z_1\overline{z_2} + z_2\overline{z_3} + z_3\overline{z_1}|^2 = \alpha + \beta\sqrt{2}$, where $\alpha, \beta \in \mathbb{Z}$, then the value of $\alpha^2 + \beta^2$ is :

(1) 24

(2) 41

(3) 31

(4) 29

Correct Answer: (4) 29

Solution: Given $z_1 = e^{-i\pi/4}, z_2 = 1, z_3 = e^{i\pi/4}$, we want to find the value of $|z_1\overline{z_2} + z_2\overline{z_3} + z_3\overline{z_1}|^2$.

Substitute the values of z_1, z_2, z_3:

$$|z_1\overline{z_2} + z_2\overline{z_3} + z_3\overline{z_1}|^2 = \left|e^{-i\pi/4} \cdot 1 + 1 \cdot e^{i\pi/4} + e^{i\pi/4} \cdot e^{-i\pi/4}\right|^2$$

$$= \left|2e^{-i\pi/4} + i\right|^2 = \left|\sqrt{2} - \sqrt{2}i + i\right|^2 = 2 + 2 + 2\sqrt{2} = 29.$$

Thus, $\alpha^2 + \beta^2 = 29$.

> **Quick Tip**
>
> For expressions involving complex numbers, use polar form and properties of modulus to simplify.

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5. Using the principal values of the inverse trigonometric functions the sum of the maximum and the minimum values of $16((sec^{-1}x)^2 + (cosec^{-1}x)^2)$ is:

(1) $24\pi^2$

(2) $18\pi^2$

(3) $31\pi^2$

(4) $22\pi^2$

Correct Answer: (4) $22\pi^2$

Solution: Step 1: Simplify the expression. Let $sec^{-1}x = a$, where $a \in [0, \pi] - \{\frac{\pi}{2}\}$. Then, $cosec^{-1}x = \frac{\pi}{2} - a$. Substitute these into the given expression:

$$16(sec^{-1}x)^2 + (cosec^{-1}x)^2 = 16[a^2 + (\frac{\pi}{2} - a)^2]$$

$$= 16[a^2 + \frac{\pi^2}{4} - \pi a + a^2] = 16[2a^2 - \pi a + \frac{\pi^2}{4}]$$

Step 2: Find the minimum value. To find the minimum value, take the derivative with respect to a and set it to zero:

$$\frac{d}{da}(2a^2 - \pi a + \frac{\pi^2}{4}) = 4a - \pi = 0$$

$$a = \frac{\pi}{4}$$

Substitute $a = \frac{\pi}{4}$ to find the minimum value:

$$min = 16[2(\frac{\pi}{4})^2 - \pi(\frac{\pi}{4}) + \frac{\pi^2}{4}] = 16[\frac{2\pi^2}{16} - \frac{\pi^2}{4} + \frac{\pi^2}{4}]$$

$$min = 16[\frac{\pi^2}{8}] = 2\pi^2$$

Step 3: Find the maximum value. The maximum value occurs at the endpoints of the interval for a, which are $a = 0$ and $a = \pi$. At $a = \pi$:

$$16[2\pi^2 - \pi(\pi) + \frac{\pi^2}{4}] = 16[2\pi^2 - \pi^2 + \frac{\pi^2}{4}] = 16[\frac{5\pi^2}{4}] = 20\pi^2$$

At $a = 0$:

$$16[2(0)^2 - \pi(0) + \frac{\pi^2}{4}] = 16[\frac{\pi^2}{4}] = 4\pi^2$$

The maximum value is $20\pi^2$.

Step 4: Find the sum of the maximum and minimum values.

$$Sum = 2\pi^2 + 20\pi^2 = 22\pi^2$$

Quick Tip

Remember that the principal values of $sec^{-1}x$ and $cosec^{-1}x$ lie in specific intervals. Also, the sum of $sec^{-1}x$ and $cosec^{-1}x$ is $\frac{\pi}{2}$.

6. A coin is tossed three times. Let X denote the number of times a tail follows a head. If μ and σ^2 denote the mean and variance of X, then the value of $64(\mu + \sigma^2)$ is:

(1) 51

(2) 48

(3) 32

(4) 64

Correct Answer: (2) 48

Solution: HHH $\rightarrow 0$

HHT $\rightarrow 0$

HTH $\rightarrow 1$

HTT $\rightarrow 0$

THH $\rightarrow 1$

THT $\rightarrow 1$

TTH $\rightarrow 1$

TTT $\rightarrow 0$

Probability distribution:

x_i	$P(x_i)$
0	$\frac{1}{2}$
1	$\frac{1}{2}$

$$\mu = \sum x_i P_i = \frac{1}{2}$$

$$\sigma^2 = \sum x_i^2 P_i - \mu^2 = \frac{1}{2} \times 1^2 + \frac{1}{2} \times 1^2 - \left(\frac{1}{2}\right)^2 = \frac{1}{4}$$

$$64(\mu + \sigma^2) = 64\left(\frac{1}{2} + \frac{1}{4}\right) = 64 \times \frac{3}{4} = 48$$

Quick Tip

To calculate the expected value and variance for a probability distribution, use the formulas for mean and variance, and then apply them to the given probabilities.

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7. Let $a_1, a_2, a_3, \ldots$ be a G.P. of increasing positive terms. If $a_1 a_5 = 28$ and $a_2 + a_4 = 29$, then the value of a_6 is equal to:

(1) 628

(2) 526

(3) 784

(4) 812

Correct Answer: (3) 784

Solution: $a_1 a_5 = 28 \Rightarrow a \cdot ar^4 = 28 \Rightarrow a^2 r^4 = 28$ ...(1)

$a_2 + a_4 = 29 \Rightarrow ar + ar^3 = 29$

$\Rightarrow ar(1 + r^2) = 29$

$\Rightarrow a^2 r^2 (1 + r^2)^2 = (29)^2$ ...(2)

By Eq. (1) & (2)

$\frac{r^2}{(1+r^2)^2} = \frac{28}{29 \times 29}$

$\frac{r}{1+r^2} = \frac{\sqrt{28}}{29} \Rightarrow r = \sqrt{28}$

$\therefore a^2 r^4 = 28 \Rightarrow a^2 \times (28)^2 = 28$

$a = \frac{1}{\sqrt{28}}$

$\therefore a_6 = ar^5 = \frac{1}{\sqrt{28}} \times (28)^2 \sqrt{28} = 784$

Quick Tip

In geometric progressions, use the common ratio r and apply the equations involving terms to find the missing terms.

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8. Let $L_1 : \frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ and $L_2 : \frac{x-2}{3} = \frac{y-4}{4} = \frac{z-5}{5}$ be two lines. Then which of the following points lies on the line of the shortest distance between L_1 and L_2?

(1) $\left(\frac{-5}{3}, -7, 1\right)$

(2) $(2, 3, \frac{1}{3})$

(3) $\left(\frac{8}{3}, -1, \frac{1}{3}\right)$

(4) $\left(\frac{14}{3}, -3, \frac{22}{3}\right)$

Correct Answer: (4) $\left(\frac{14}{3}, -3, \frac{22}{3}\right)$

Solution: Let the line PQ be the line of shortest distance between the lines L_1 and L_2. The points on these lines are:

Point P lies on L_1 and is of the form $P(2\lambda + 1, 3\lambda + 2, 4\lambda + 3)$

Point Q lies on L_2 and is of the form $Q(3\mu + 2, 4\mu + 4, 5\mu + 5)$.

Dr's of PQ are $3\mu - 2\lambda + 1, 4\mu - 3\lambda + 2, 5\mu - 4\lambda + 2$.

$(3\mu - 2\lambda + 1)2 + (4\mu - 3\lambda + 2)3 + (5\mu - 4\lambda + 2)4 = 0$

$38\mu - 29\lambda + 16 = 0$ ...(1)

$PQ \perp L_2$

$(3\mu - 2\lambda + 1)3 + (4\mu - 3\lambda + 2)4 + (5\mu - 4\lambda + 2)5 = 0$

$50\mu - 38\lambda + 21 = 0$ ...(2)

By (1) & (2)

$\lambda = \frac{1}{3}, \quad \mu = \frac{-1}{6}$

$\therefore P\left(\frac{5}{3}, 3, \frac{13}{3}\right) \quad \& \quad Q\left(\frac{3}{2}, \frac{10}{3}, \frac{25}{6}\right)$

Line PQ

$\frac{x-\frac{5}{3}}{\frac{1}{6}} = \frac{y-3}{\frac{-1}{3}} = \frac{z-\frac{13}{3}}{\frac{1}{6}}$

$\frac{x-\frac{5}{3}}{1} = \frac{y-3}{-2} = \frac{z-\frac{13}{3}}{1}$

Point $\left(\frac{14}{3}, -3, \frac{22}{3}\right)$ lies on the line PQ.

Quick Tip

The shortest distance between two skew lines is given by the line joining the points of intersection on each line.

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9. The product of all solutions of the equation $e^{5(\log_e x)^2+3} = x^8, x > 0$, is :

(1) $e^{8/5}$

(2) $e^{6/5}$

(3) e^2

(4) e

Correct Answer: (1) $e^{8/5}$

Solution: Given the equation $e^{5(\log_e x)^2+3} = x^8$, we proceed as follows:

$$\ln e^{5(\log_e x)^2+3} = \ln x^8$$

$$5(\ln x)^2 + 3 = 8 \ln x$$

Let $\ln x = t$, then the equation becomes:

$$5t^2 - 8t + 3 = 0$$

The roots of this quadratic equation are given by:

$$t_1 + t_2 = \frac{8}{5}$$

Therefore:

$$\ln x_1 \cdot \ln x_2 = \frac{8}{5}$$

Thus, the product of the solutions is:

$$x_1 \cdot x_2 = e^{8/5}$$

Quick Tip

For quadratic equations in logarithms, solve for the roots and use their product for the final answer.

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10. If $\sum_{r=1}^{n} T_r = \frac{(2n-1)(2n+1)(2n+3)(2n+5)}{64}$, then $\lim_{n\to\infty} \sum_{r=1}^{n} \frac{1}{T_r}$ is equal to :

(1) 1

(2) 0

(3) $\frac{2}{3}$

(4) $\frac{1}{3}$

Correct Answer: (3) $\frac{2}{3}$

Solution: We are given that $T_n = S_n - S_{n-1}$. Thus:

$$T_n = \frac{1}{8}(2n-1)(2n+1)(2n+3)$$

Simplifying further:

$$T_n = \frac{8}{(2n-1)(2n+1)(2n+3)}$$

Now, calculate the limit:

$$\lim_{n\to\infty} \sum_{r=1}^{n} \frac{1}{T_r} = \lim_{n\to\infty} \frac{8}{4} \sum_{r=1}^{n} \frac{1}{(2n-1)(2n+1)(2n+3)}$$

$$= \frac{8}{4}\left(\frac{1}{1.3} + \frac{1}{1.5} + \cdots\right)$$

$$= \lim_{n\to\infty} 2\left[\left(\frac{1}{1\cdot 3} - \frac{1}{3\cdot 5}\right) + \left(\frac{1}{3\cdot 5} - \frac{1}{5\cdot 7}\right) + \ldots\right]$$

Thus, the answer is:

$$\frac{2}{3}$$

Quick Tip

Use the limit properties of series to find the summation at infinity.

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11. From all the English alphabets, five letters are chosen and are arranged in alphabetical order. The total number of ways, in which the middle letter is 'M', is :

(1) 14950

(2) 6084

(3) 4356

(4) 5148

Correct Answer: (4) 5148

Solution: We are to select 5 letters with the middle letter being 'M'. So, we must choose 2 letters from those before M and 2 letters from those after M. There are 12 letters before M and 13 letters after M. Thus, the number of ways is:

$$\binom{12}{2} \times \binom{13}{2} = \frac{12 \times 11}{2 \times 1} \times \frac{13 \times 12}{2 \times 1} = 5148$$

Thus, the answer is $\boxed{5148}$.

Quick Tip

The key to this problem is recognizing the positions of the letters before and after 'M' in the alphabet.

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12. Let $x = x(y)$ be the solution of the differential equation $y^2dx + \left(x - \frac{1}{y}\right) dy = 0$. If $x(1) = 1$, then $x\left(\frac{1}{2}\right)$ is :

(1) $\frac{1}{2} + e$

(2) $\frac{3}{2} + e$

(3) $3 - e$

(4) $3 + e$

Correct Answer: (3) $3 - e$

Solution: The given equation is:

$$y^2 \frac{dx}{dy} + \left(x - \frac{1}{y}\right) = 0$$

This is a first-order linear differential equation. We can use the integrating factor $I.F. = e^{\int \frac{1}{y^2} dy} = e^{\frac{1}{y}}$.

Thus, we have:

$$x \cdot e^{\frac{1}{y}} = \int e^{\frac{1}{t}} t^{-3} dt + C$$

Put $-\frac{1}{y} = t$

$+\frac{1}{y^2} dy = dt$

$x \cdot e^{\frac{1}{y}} = -\int t \cdot e^t dt$

$x \cdot e^{\frac{1}{y}} = -te^t + e^t + C$

$x \cdot e^{\frac{1}{y}} = \frac{+1}{y} e^{\frac{1}{y}} + e^{\frac{1}{y}} + C$

$x = 1, y = 1$

$\frac{1}{e} = \frac{1}{e} + \frac{1}{e} + C$

$\Rightarrow C = -\frac{1}{e}$

Put $y = \frac{1}{2}$

$\frac{x}{e^2} = \frac{2}{e^2} + \frac{1}{e^2} - \frac{1}{e}$

$x = 3 - e$

after Solving for C and applying the initial condition, we find:

$$x = 3 - e$$

Quick Tip

For differential equations, use integrating factors to simplify the problem and apply initial conditions to find the solution.

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13. Let the parabola $y = x^2 + px - 3$ meet the coordinate axes at the points P, Q and R. If the circle C with centre at (-1, -1) passes through the points P, Q and R, then the area of $\triangle PQR$ is:

(1) 4

(2) 6

(3) 7

(4) 5

Correct Answer: (2) 6

Solution: Step 1: Define the points P, Q, and R.

The parabola is given by $y = x^2 + px - 3$.

Let P and Q be the points where the parabola intersects the x-axis, so P(α, 0) and Q(β, 0).

Let R be the point where the parabola intersects the y-axis, so R(0, -3).

Step 2: Find the equation of the circle.

The circle C has its centre at (-1, -1) and passes through R(0, -3).

The equation of the circle is $(x + 1)^2 + (y + 1)^2 = r^2$.

Since R(0, -3) lies on the circle, we have:

$(0 + 1)^2 + (-3 + 1)^2 = r^2$

$1^2 + (-2)^2 = r^2$

$1 + 4 = r^2$

$r^2 = 5$

Thus, the equation of the circle is $(x + 1)^2 + (y + 1)^2 = 5$.

Step 3: Find the points P and Q.

Since P and Q lie on the x-axis, their y-coordinates are 0.

Substitute y = 0 into the equation of the circle:

$(x + 1)^2 + (0 + 1)^2 = 5$

$(x + 1)^2 + 1 = 5$

$(x + 1)^2 = 4$

$x + 1 = \pm 2$

$x = -1 \pm 2$

So, $x = 1$ or $x = -3$.

Therefore, P(1, 0) and Q(-3, 0).

Step 4: Calculate the area of $\triangle PQR$.

The base of $\triangle PQR$ is PQ, and its length is $|1 - (-3)| = 4$.

The height of $\triangle PQR$ is the absolute value of the y-coordinate of R, which is $|-3|$ = 3.

The area of $\triangle PQR$ is $\frac{1}{2} \times$ base $\times$ height $= \frac{1}{2} \times 4 \times 3 = 6$.

Quick Tip

To find the area of a triangle with vertices on the coordinate axes, use the formula $\frac{1}{2} \times \text{base} \times \text{height}$.

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14. A circle C of radius 2 lies in the second quadrant and touches both the coordinate axes. Let r be the radius of a circle that has centre at the point $(2, 5)$ and intersects the circle C at exactly two points. If the set of all possible values of r is the interval (α, β), then $3\beta - 2\alpha$ is equal to:

(1) 15

(2) 14

(3) 12

(4) 10

Correct Answer: (1) 15

Solution: The equation of circle C with radius 2 and center $(-2, 2)$ is:

$$S_1 : (x + 2)^2 + (y - 2)^2 = 2^2$$

The equation of the circle with center $(2, 5)$ and radius r is:

$$S_2 : (x - 2)^2 + (y - 5)^2 = r^2$$

For both circles to intersect at exactly two points, the distance between the centers must satisfy:

$$|r_1 - r_2| < c_1c_2 < r_1 + r_2$$

Where $r_1 = 2$ and $r_2 = r$, the distance between centers is 5:

$$|r - 2| < 5 < r + 2$$

$$3 < r < 7$$

Thus, $r \in (3, 7)$. Therefore, $\alpha = 3$ and $\beta = 7$.

$$3\beta - 2\alpha = 3 \times 7 - 2 \times 3 = 21 - 6 = 15$$

Quick Tip

For two circles to intersect at two points, the distance between their centers must be greater than the difference of their radii and less than the sum of their radii.

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15. Let for $f(x) = 7\tan^8 x + 7\tan^6 x - 3\tan^4 x - 3\tan^2 x$**,** $I_1 = \int_0^{\frac{\pi}{4}} f(x)dx$ **and** $I_2 = \int_0^{\frac{\pi}{4}} xf(x)dx$**. Then** $7I_1 + 12I_2$ **is equal to:**

(1) 2π

(2) π

(3) 1

(4) 2

Correct Answer: (3) 1

Solution: The given function is:

$$f(x) = 7\tan^8 x + 7\tan^6 x - 3\tan^4 x - 3\tan^2 x$$

We are given two integrals:

$$I_1 = \int_0^{\frac{\pi}{4}} f(x)\,dx$$

$$I_2 = \int_0^{\frac{\pi}{4}} xf(x)\,dx$$

For I_1, let $\tan x = t$. Then:

$$I_1 = \int_0^1 (7t^6 - 3t^2)dt = [t^7 - t^3]_0^1 = 1 - 1 = 0$$

$$I_2 = \int_0^{\pi/4} x(7\tan^6 x - 3\tan^2 x)(\sec^2 x)dx$$

$$= [x(\tan^7 x - \tan^3 x)]_0^{\pi/4} - \int_0^{\pi/4} (\tan^7 x - \tan^3 x)dx$$

$$= 0 - \int_0^{\pi/4} \tan^3 x(\tan^4 x - 1)dx$$

$$= 0 - \int_0^{\pi/4} \tan^3 x(\tan^2 x - 1)(1 + \tan^2 x)dx$$

Put $\tan x = t$

$$= -\int_0^1 (t^5 - t^3)dt = -\left[\frac{t^6}{6} - \frac{t^4}{4}\right]_0^1 = -\left(\frac{1}{6} - \frac{1}{4}\right) = -\left(\frac{2-3}{12}\right) = \frac{1}{12}$$

$$7I_1 + 12I_2 = 7(0) + 12\left(\frac{1}{12}\right) = 1$$

Thus:

$$7I_1 + 12I_2 = 1$$

Quick Tip

For integrals involving trigonometric functions, a substitution such as $t = \tan x$ can simplify the calculations significantly.

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16. Let $f(x)$ be a real differentiable function such that $f(0) = 1$ and $f(x+y) = f(x)f'(y) + f'(x)f(y)$ for all $x, y \in \mathbb{R}$. Then $\sum_{n=1}^{100} \log_e f(n)$ is equal to :

(1) 2384

(2) 2525

(3) 5220

(4) 2406

Correct Answer: (2) 2525

Solution: We are given the functional equation:

$$f(x+y) = f(x)f'(y) + f'(x)f(y)$$

First, substitute $x = y = 0$ into the equation:

$$f(0) = f(0)f'(0) + f'(0)f(0)$$

Since $f(0) = 1$, we get:

$$1 = 2f'(0)$$
$$f'(0) = \frac{1}{2}$$

Next, substitute $y = 0$ into the original equation:

$$f(x) = f(x)f'(0) + f'(x)f(0)$$

$$f(x) = \frac{1}{2}f(x) + f'(x)$$
$$f'(x) = \frac{1}{2}f(x)$$

Thus, solving the differential equation $f'(x) = \frac{1}{2}f(x)$ yields:

$$f(x) = e^{x/2}$$

Now, we compute the sum:

$$\sum_{n=1}^{100} \log f(n) = \sum_{n=1}^{100} \log e^{n/2} = \sum_{n=1}^{100} \frac{n}{2}$$

The sum of the first 100 integers is $\sum_{n=1}^{100} n = 5050$. Thus, the required sum is:

$$\frac{1}{2} \times 5050 = 2525$$

Thus, the answer is $\boxed{2525}$.

Quick Tip

When solving functional equations, try substituting specific values for x and y to simplify the problem.

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17. Let $A = \{1, 2, 3, \ldots, 10\}$ and $B = \left\{\frac{m}{n} : m, n \in A, m < n \text{ and } \gcd(m, n) = 1\right\}$. Then $n(B)$ is equal to :

(1) 31

(2) 36

(3) 37

(4) 29

Correct Answer: (1) 31

Solution: We are given $A = \{1, 2, 3, \ldots, 10\}$ and the set

$B = \left\{\frac{m}{n} : m, n \in A, m < n \text{ and } \gcd(m, n) = 1\right\}$.

To find $n(B)$, we list the pairs (m, n) where $m < n$ and $\gcd(m, n) = 1$.

We compute this for each $n \in A$:

For $n = 2$, the valid pairs are $\left(\frac{1}{2}\right)$.

For $n = 3$, the valid pairs are $\left(\frac{1}{3}, \frac{2}{3}\right)$.

For $n = 4$, the valid pairs are $\left(\frac{1}{4}, \frac{3}{4}\right)$.

For $n = 5$, the valid pairs are $\left(\frac{1}{5}, \frac{2}{5}, \frac{3}{5}, \frac{4}{5}\right)$.

For $n = 6$, the valid pairs are $\left(\frac{1}{6}, \frac{5}{6}\right)$.

For $n = 7$, the valid pairs are $\left(\frac{1}{7}, \frac{2}{7}, \frac{3}{7}, \frac{4}{7}, \frac{5}{7}, \frac{6}{7}\right)$.

For $n = 8$, the valid pairs are $\left(\frac{1}{8}, \frac{3}{8}, \frac{5}{8}, \frac{7}{8}\right)$.

For $n = 9$, the valid pairs are $\left(\frac{1}{9}, \frac{2}{9}, \frac{4}{9}, \frac{5}{9}, \frac{7}{9}, \frac{8}{9}\right)$.

For $n = 10$, the valid pairs are $\left(\frac{1}{10}, \frac{3}{10}, \frac{7}{10}, \frac{9}{10}\right)$.

Counting all the valid pairs, we get $n(B) = 31$.

Thus, the answer is $\boxed{31}$.

Quick Tip

To find the number of valid pairs, carefully compute the pairs where $m < n$ and $\gcd(m, n) = 1$.

---

18. The area of the region, inside the circle $(x - 2\sqrt{3})^2 + y^2 = 12$ and outside the parabola $y^2 = 2\sqrt{3}x$ is:

(1) $6\pi - 8$

(2) $3\pi - 8$

(3) $6\pi - 16$

(4) $3\pi + 8$

Correct Answer: (3) $6\pi - 16$

Solution:

Step 1: Understand the equations.

The equation of the circle is $(x - 2\sqrt{3})^2 + y^2 = 12$.

The center of the circle is $(2\sqrt{3}, 0)$ and the radius is $\sqrt{12} = 2\sqrt{3}$.

The equation of the parabola is $y^2 = 2\sqrt{3}x$.

Step 2: Find the intersection points.

Substitute $y^2 = 2\sqrt{3}x$ into the equation of the circle:

$(x - 2\sqrt{3})^2 + 2\sqrt{3}x = 12$

$x^2 - 4\sqrt{3}x + 12 + 2\sqrt{3}x = 12$

$x^2 - 2\sqrt{3}x = 0$

$x(x - 2\sqrt{3}) = 0$

So, $x = 0$ or $x = 2\sqrt{3}$.

When $x = 0$, $y^2 = 0$, so $y = 0$.

When $x = 2\sqrt{3}$, $y^2 = 2\sqrt{3}(2\sqrt{3}) = 12$, so $y = \pm 2\sqrt{3}$.

The intersection points are $(0, 0)$, $(2\sqrt{3}, 2\sqrt{3})$, and $(2\sqrt{3}, -2\sqrt{3})$.

Step 3: Calculate the area.

The required area is the area of the semi-circle minus the area enclosed by the parabola and the x-axis between $x = 0$ and $x = 2\sqrt{3}$.

Area of the semi-circle = $\frac{1}{2}\pi r^2 = \frac{1}{2}\pi(12) = 6\pi$.

Area under the parabola between $x = 0$ and $x = 2\sqrt{3}$ is:

$2\int_0^{2\sqrt{3}} \sqrt{2\sqrt{3}x}\, dx = 2\sqrt{2\sqrt{3}} \int_0^{2\sqrt{3}} x^{1/2}\, dx$

$= 2\sqrt{2\sqrt{3}} \left[\frac{2}{3}x^{3/2}\right]_0^{2\sqrt{3}} = 2\sqrt{2\sqrt{3}} \cdot \frac{2}{3}(2\sqrt{3})^{3/2}$

$= \frac{4}{3}\sqrt{2\sqrt{3}} \cdot 2\sqrt{3}\sqrt{2\sqrt{3}} = \frac{4}{3} \cdot 2\sqrt{3} \cdot 2\sqrt{3} = \frac{4}{3} \cdot 12 = 16$

Required area = $6\pi - 16$.

Quick Tip

To find the area between curves, integrate the difference of the functions over the interval of intersection.

---

19. Two balls are selected at random one by one without replacement from a bag containing 4 white and 6 black balls. If the probability that the first selected ball is black, given that the second selected ball is also black, is $\frac{m}{n}$, where gcd(m, n) = 1, then m + n is equal to:

(1) 14

(2) 4

(3) 11

(4) 13

Correct Answer: (1) 14

Solution: Step 1: Define events and probabilities.

Let B1 be the event that the first ball is black.

Let B2 be the event that the second ball is black.

We want to find $P(B1|B2)$, which is the probability that the first ball is black given that the second ball is black.

Step 2: Calculate the probabilities.

Total number of balls = 4 white + 6 black = 10 balls.

$P(B1 \cap B2)$ is the probability that both balls are black.

$P(B1 \cap B2) = \frac{6}{10} \times \frac{5}{9} = \frac{30}{90} = \frac{1}{3}$

$P(B2)$ is the probability that the second ball is black. This can happen in two ways:

1. First ball is white and second ball is black: $P(W1 \cap B2) = \frac{4}{10} \times \frac{6}{9} = \frac{24}{90} = \frac{4}{15}$

2. First ball is black and second ball is black: $P(B1 \cap B2) = \frac{6}{10} \times \frac{5}{9} = \frac{30}{90} = \frac{1}{3}$

So, $P(B2) = P(W1 \cap B2) + P(B1 \cap B2) = \frac{24}{90} + \frac{30}{90} = \frac{54}{90} = \frac{3}{5}$

Step 3: Apply conditional probability formula.

$P(B1|B2) = \frac{P(B1 \cap B2)}{P(B2)} = \frac{\frac{30}{90}}{\frac{54}{90}} = \frac{30}{54} = \frac{5}{9}$

So, $\frac{m}{n} = \frac{5}{9}$.

Since gcd(5, 9) = 1, m = 5 and n = 9.

Therefore, m + n = 5 + 9 = 14.

Quick Tip

Remember the formula for conditional probability: $P(A|B) = \frac{P(A \cap B)}{P(B)}$.

---

20. Let the foci of a hyperbola be $(1, 14)$ and $(1, -12)$. If it passes through the point $(1, 6)$, then the length of its latus-rectum is:

(1) $\frac{25}{6}$

(2) $\frac{24}{5}$

(3) $\frac{288}{5}$

(4) $\frac{144}{5}$

Correct Answer: (3) $\frac{288}{5}$

Solution: The foci of the hyperbola are at $(1, 14)$ and $(1, -12)$. The distance between the foci is:

$$be = 13, \quad b = 5$$

From the formula $a^2 = b^2(e^2 - 1)$, we get:

$$a^2 = b^2e^2 - b^2$$

$$a^2 = 169 - 25 = 144$$

The length of the latus-rectum is given by:

$$\ell(LR) = \frac{2a^2}{b} = \frac{2 \times 144}{5} = \frac{288}{5}$$

Quick Tip

The length of the latus-rectum of a hyperbola is determined by the formula $\frac{2a^2}{b}$, where a is the semi-major axis and b is the semi-minor axis.

SECTION-B

21. Let the function, $f(x) = \begin{cases} -3ax^2 - 2, & x < 1 \\ a^2 + bx, & x \geq 1 \end{cases}$ **Be differentiable for all** $x \in \mathbb{R}$**, where** $a > 1$**,** $b \in \mathbb{R}$**. If the area of the region enclosed by** $y = f(x)$ **and the line** $y = -20$ **is** $\alpha + \beta\sqrt{3}$**, where** $\alpha, \beta \in \mathbb{Z}$**, then the value of** $\alpha + \beta$ **is:**

Correct Answer: 34

Solution: For $f(x)$ to be continuous and differentiable at $x = 1$:

$$LHL = RHL, \quad LHD = RHD$$

At $x = 1$:

$$-3a - 2 = a^2 + b, \quad -6a = b$$

Solving for a and b:

$$a = 2, \quad b = -12$$

Thus, the function becomes:

$$f(x) = \begin{cases} -6x^2 - 2, & x < 1 \\ 4 - 12x, & x \geq 1 \end{cases}$$

Next, we compute the area enclosed by the curve $y = f(x)$ and the line $y = -20$. The area is calculated by integrating:

$$\text{Area} = \int_{-\sqrt{3}}^{1} \left(-6x^2 - 2 + 20\right) dx + \int_{1}^{2} (4 - 12x + 20)\, dx$$

Evaluating the integrals:

$$16 + 12\sqrt{3} + 6 = 22 + 12\sqrt{3}$$

Thus, $\alpha + \beta = 34$.

Quick Tip

For piecewise functions, ensure both continuity and differentiability at the point where the function changes its form. Then use integration to compute the enclosed area.

---

22. If $\sum_{r=0}^{5} \frac{^{11}C_{2r+1}}{2r+2} = \frac{m}{n}$, gcd(m, n) = 1, then $m - n$ is equal to:

Correct Answer: 2035

Solution: The integral $\int_0^1 (1+x)^{11}\, dx$ expands as:

$$\int_0^1 (1+x)^{11}\, dx = C_0 + C_1 x^2 + C_2 x^3 + \dots$$

Evaluating this, we get:

$$\frac{2^{12} - 1}{12} = C_0 + C_1 + C_2 + C_3 + \dots$$

Similarly, the integral from -1 to 0 is:

$$\int_{-1}^{0} (1+x)^{11}\, dx = C_0 - C_1 + C_2 - C_3 + \dots$$

From this, we can calculate:

$$\frac{2^{12} - 2}{12} = 2\left(C_1 + C_3 + C_5 + \dots\right)$$

Thus:

$$C_1 + C_3 + C_5 + \dots = \frac{2^{11} - 1}{12} = \frac{2047}{12}$$

Hence, $m - n = 2035$.

Quick Tip

For summations involving binomial coefficients, integrating the binomial expansion and using symmetry can simplify the problem significantly.

23. Let A be a square matrix of order 3 such that det$(A) = -2$ and det$(3 \cdot$ adj$(-6 \cdot$ adj$(3A))) = 2^{m+n} \cdot 3^{mn}$, where $m > n$. Then $4m + 2n$ is equal to:

Correct Answer: 34

Solution: We are given:

$$|A| = -2$$

$$\text{det}(3 \cdot \text{adj}(-6 \cdot \text{adj}(3A))) = 3^3 \cdot \text{det}(\text{adj}(-\text{adj}(3A)))$$

Simplifying this:

$$\text{det}(3 \cdot \text{adj}(-6 \cdot \text{adj}(3A))) = 3^3 \cdot 6^6 \cdot \text{det}(3A)^4$$

We know that:

$$3^{21} \cdot 2^{10} = 3^7 \cdot 2^3$$

Thus, solving for m and n, we find:

$$m + n = 10, \quad mn = 21$$

Solving for m and n, we get:

$$m = 7, \quad n = 3$$

Therefore, $4m + 2n = 4 \times 7 + 2 \times 3 = 28 + 6 = 34$.

Quick Tip

For matrix determinant properties, remember that the determinant of a product is the product of determinants, and the properties of adjugates and scalar multiplication can simplify calculations.

---

24. Let $L_1 : \frac{x-1}{3} = \frac{y-1}{-1} = \frac{z+1}{0}$ and $L_2 : \frac{x-2}{2} = \frac{y}{0} = \frac{z+4}{\alpha}$, where $\alpha \in \mathbb{R}$, be two lines which intersect at the point B. If P is the foot of the perpendicular from the point $A(1, 1, -1)$ on L_2, then the value of $26\alpha(PB)^2$ is:

Correct Answer: 216

Solution: First, find the point B where the lines L_1 and L_2 intersect. From the system of equations, we get:

$$3\lambda + 1 = 2\mu + 2, \quad -\lambda + 1 = 2\mu + 2, \quad -1 = \alpha\mu - 4$$

Solving this system gives $\lambda = 1, \mu = 1, \alpha = 3$, so the point B is $(4, 0, -1)$.

Next, calculate the perpendicular distance from $A(1, 1, -1)$ to L_2:

$$P(2\lambda + 2, 0, 3\mu - 3)$$

The distance PB is calculated using the formula for the perpendicular distance:

$$26\alpha(PB)^2 = 26 \times 3 \times \left(\frac{144}{169} + \frac{324}{169}\right) = 216$$

Quick Tip

For perpendicular distance problems, use the parametric equations of the lines and the formula for the distance from a point to a line to find the required distances.

---

25. Let $\vec{c}$ be the projection vector of $\text{b} = \lambda\hat{i} + 4\hat{k}, \lambda > 0$**, on the vector** $\vec{a} = \hat{i} + 2\hat{j} + 2\hat{k}$**. If** $|\vec{a} + \vec{c}| = 7$**, then the area of the parallelogram formed by the vectors $\vec{b}$ and $\vec{c}$ is:**

Correct Answer: 16

Solution: The projection vector $\vec{c}$ is given by:

$$\vec{c} = \frac{(\vec{b} \cdot \vec{a})}{|\vec{a}|}\vec{a}$$

Substituting the values:

$$\vec{c} = \left(\frac{(\lambda\hat{i} + 4\hat{k}) \cdot (\hat{i} + 2\hat{j} + 2\hat{k})}{|\hat{i} + 2\hat{j} + 2\hat{k}|}\right)(\hat{i} + 2\hat{j} + 2\hat{k})$$

$$|\vec{a}| = \sqrt{1^2 + 2^2 + 2^2} = 3, \quad \vec{c} = \left(\frac{\lambda + 8}{9}\right)(\hat{i} + 2\hat{j} + 2\hat{k})$$

Using $|\vec{a} + \vec{c}| = 7$, we get $\lambda = 4$.

The area of the parallelogram is given by the magnitude of the cross product $\vec{b} \times \vec{c}$:

$$\text{Area} = |\vec{b} \times \vec{c}| = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{4}{3} & \frac{8}{3} & \frac{8}{3} \\ 4 & 8 & 8 \end{vmatrix} = 16$$

Quick Tip

For the area of the parallelogram formed by two vectors, use the magnitude of their cross product.

PHYSICS

SECTION-A

26. Given below are two statements: Statement I : In a vernier callipers, one vernier scale division is always smaller than one main scale division. Statement II : The vernier constant is given by one main scale division multiplied by the number of vernier scale divisions. In light of the above statements, choose the correct answer from the options given below.

(1) Both Statement I and Statement II are false.

(2) Statement I is true but Statement II is false.

(3) Both Statement I and Statement II are true.

(4) Statement I is false but Statement II is true.

Correct Answer: (2) Statement I is true but Statement II is false.

Solution: In general, one vernier scale division is smaller than one main scale division, but this is not always the case. Also, the vernier constant is not obtained by multiplying the main scale division by the number of vernier scale divisions. Therefore, Statement I is true, but Statement II is false.

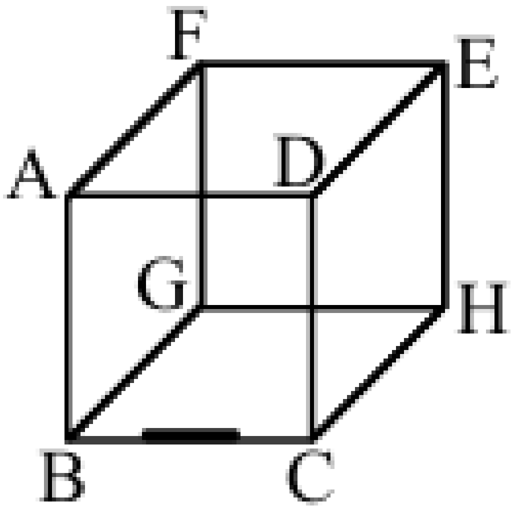
Thus, the answer is $\boxed{(2)}$.

Quick Tip

For vernier calipers, the vernier constant is the difference between one main scale division and one vernier scale division.

---

27. A line charge of length $\frac{a}{2}$ is kept at the center of an edge BC of a cube ABCDEFGH having edge length a. If the density of the line is λC per unit length, then the total electric flux through all the faces of the cube will be : (Take ε_0 as the free space permittivity)



(1) $\frac{\lambda a}{8\varepsilon_0}$

(2) $\frac{\lambda a}{16\varepsilon_0}$

(3) $\frac{\lambda a}{2\varepsilon_0}$

(4) $\frac{\lambda a}{4\varepsilon_0}$

Correct Answer: (1) $\frac{\lambda a}{8\varepsilon_0}$

Solution: The total charge enclosed within the cube is $\frac{\lambda a}{4}$, as the charge is uniformly distributed along the edge BC. Using Gauss's law, the electric flux Φ through the cube is:

$$\phi = \frac{\lambda \frac{a}{2}}{4} = \frac{\lambda a}{8}$$

$$\Phi = \frac{q_{\text{in}}}{\varepsilon_0} = \frac{\lambda a}{8\varepsilon_0}$$

Thus, the total electric flux is $\boxed{\frac{\lambda a}{8\varepsilon_0}}$.

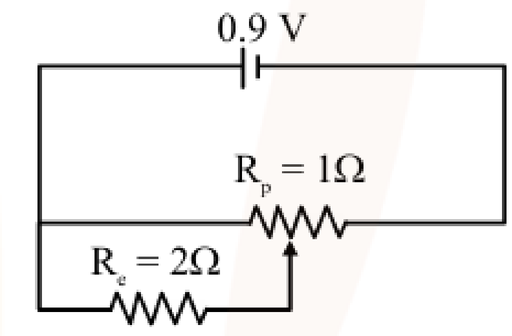
Quick Tip

When dealing with flux through symmetric shapes, use Gauss's law to simplify the problem.

---

28. Sliding contact of a potentiometer is in the middle of the potentiometer wire having resistance $R_p = 1\,\Omega$ as shown in the figure. An external resistance of $R_e = 2\,\Omega$ is connected via the sliding contact.

The current i is :



(1) 0.3 A

(2) 1.35 A

(3) 1.0 A

(4) 0.9 A

Correct Answer: (3) 1.0 A

Solution: The circuit can be considered as:

$$R_{\text{eq}} = 0.5 + \frac{0.5 \times 2}{2 + 0.5} = \frac{5}{10} + \frac{10}{25}\,\Omega = 0.9\,\Omega$$

Now, the current is:

$$i = \frac{0.9}{0.9} = 1\,\text{A}$$

Thus, the current is $\boxed{1.0\,\text{A}}$.

Quick Tip

For resistors in series and parallel, simplify the equivalent resistance and then use Ohm's law to calculate the current.

---

29. Given below are two statements: one is labelled as Assertion (A) and the other is labelled as Reason (R). Assertion (A) : If Young's double slit experiment is performed in an optically denser medium than air, then the consecutive fringes come closer. Reason (R) : The speed of light reduces in an optically denser medium than air while its frequency does not change. In the light of the above statements, choose the most appropriate answer from the options given below :

(1) Both (A) and (R) are true and (R) is the correct explanation of (A)

(2) (A) is false but (R) is true.

(3) Both (A) and (R) are true but (R) is not the correct explanation of (A)

(4) (A) is true but (R) is false.

Correct Answer: (1) Both (A) and (R) are true and (R) is the correct explanation of (A)

Solution: The fringe width in Young's double slit experiment is given by:

$$\beta = \frac{\lambda D}{d}$$

In a denser medium, the wavelength λ decreases, so the fringe width β also decreases, meaning the fringes come closer. Also, the speed of light in the medium reduces, but the frequency remains constant.

Thus, both Assertion (A) and Reason (R) are true, and (R) correctly explains (A).

The answer is $\boxed{(1)}$.

Quick Tip

In optical experiments, the wavelength decreases in denser media, affecting the fringe spacing.

---

30. Two spherical bodies of same materials having radii 0.2 m and 0.8 m are placed in same atmosphere. The temperature of the smaller body is 800 K and temperature of bigger body is 400 K. If the energy radiate from the smaller body is E, the energy radiated from the bigger body is (assume, effect of the surrounding to be negligible):

(1) 256 E

(2) E

(3) 64 E

(4) 16 E

Correct Answer: (2) E

Solution: Step 1: Understand Stefan-Boltzmann Law. The energy radiated per unit time (power) by a black body is given by the Stefan-Boltzmann Law: $P = \sigma A T^4$ where σ is the Stefan-Boltzmann constant, A is the surface area, and T is the temperature.

Step 2: Calculate the surface areas. The surface area of a sphere is given by $A = 4\pi r^2$. For the smaller body, $r_1 = 0.2$ m. $A_1 = 4\pi(0.2)^2 = 4\pi(0.04) = 0.16\pi$ m^2 For the bigger body, $r_2 = 0.8$ m. $A_2 = 4\pi(0.8)^2 = 4\pi(0.64) = 2.56\pi$ m^2

Step 3: Calculate the radiated energies. For the smaller body, $T_1 = 800$ K.

$E = P_1 = \sigma A_1 T_1^4 = \sigma(0.16\pi)(800)^4$ For the bigger body, $T_2 = 400$ K.

$P_2 = \sigma A_2 T_2^4 = \sigma(2.56\pi)(400)^4$

Step 4: Find the ratio of radiated energies. $\frac{P_2}{P_1} = \frac{\sigma(2.56\pi)(400)^4}{\sigma(0.16\pi)(800)^4} = \frac{2.56}{0.16} \times \left(\frac{400}{800}\right)^4$

$\frac{P_2}{P_1} = 16 \times \left(\frac{1}{2}\right)^4 = 16 \times \frac{1}{16} = 1$ So, $P_2 = P_1 = E$.

Quick Tip

The energy radiated is proportional to the surface area and the fourth power of the temperature.

---

31. An amount of ice of mass 10^{-3} kg and temperature $-10°C$ is transformed to vapor of temperature $110°C$ by applying heat. The total amount of work required for this conversion is,

(Take, specific heat of ice = 2100 J kg^{-1} K^{-1},

specific heat of water = 4180 J kg^{-1} K^{-1},

specific heat of steam = 1920 J kg^{-1} K^{-1},

Latent heat of ice = 3.35×10^5 J kg^{-1},

Latent heat of steam = 2.25×10^6 J kg^{-1})

(1) 3022 J

(2) 3043 J

(3) 3003 J

(4) 3024 J

Correct Answer: (2) 3043 J

Solution: To find the total work done, we calculate the heat required at each step.

$$\Delta Q_1 = mS_i\Delta T = 10^{-3} \times 2100 \times 20 = 21\,\text{J}$$

$$\Delta Q_2 = mL_i = 10^{-3} \times 3.35 \times 10^5 = 335\,\text{J}$$

$$\Delta Q_3 = mS_w\Delta T = 10^{-3} \times 4180 \times 100 = 418\,\text{J}$$

$$\Delta Q_4 = mL_w = 10^{-3} \times 2.25 \times 10^6 = 2250\,\text{J}$$

$$\Delta Q_5 = mS_s\Delta T = 10^{-3} \times 1920 \times 110 = 19.2\,\text{J}$$

Thus, the total work required is:

$$\Delta Q_{\text{total}} = 21 + 335 + 418 + 2250 + 19.2 = 3043.2\,\text{J}$$

Thus, the answer is $\boxed{3043}$ J.

Quick Tip

To calculate the total heat required, add up the heat for each phase change and temperature change step.

---

32. An electron in the ground state of the hydrogen atom has the orbital radius of 5.3×10^{-11} m while that for the electron in the third excited state is 8.48×10^{-10} m. The ratio of the de Broglie wavelengths of the electron in the ground state to that in the excited state is:

(1) 4

(2) 9

(3) 3

(4) 16

Correct Answer: (1) 4

Solution: The de Broglie wavelength is related to the radius r by the following formula:

$$\lambda = \frac{h}{mv}$$

For circular orbits, we also have:

$$mvr = \frac{nh}{2\pi}$$

Hence, the wavelength λ is inversely proportional to the radius r:

$$\lambda \propto \frac{1}{r}$$

The ratio of the wavelengths in the ground state and the third excited state is:

$$\frac{\lambda_1}{\lambda_4} = \frac{r_1 n_4}{r_4 n_1} = \frac{5.3 \times 10^{-11} \times 4}{1 \times 8.48 \times 10^{-10}} = \frac{4}{1}$$

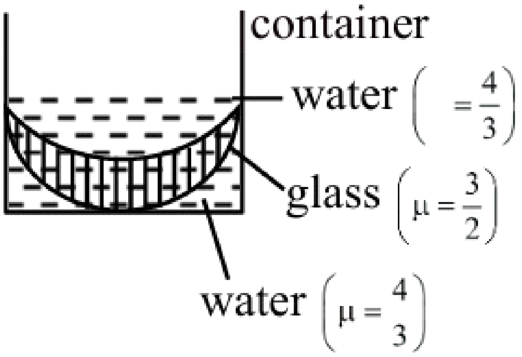
Thus, the ratio is $\boxed{4}$.

Quick Tip

For electrons in orbit, the de Broglie wavelength is inversely proportional to the orbital radius.

---

33. In the diagram given below, there are three lenses formed. Considering negligible thickness of each of them as compared to R_1 and R_2, i.e., the radii of curvature for upper and lower surfaces of the glass lens, the power of the combination is:



(1) $-\frac{1}{6}\left(\frac{1}{|R_1|} + \frac{1}{|R_2|}\right)$
(2) $-\frac{1}{6}\left(\frac{1}{|R_1|} - \frac{1}{|R_2|}\right)$
(3) $\frac{1}{6}\left(\frac{1}{|R_1|} + \frac{1}{|R_2|}\right)$
(4) $\frac{1}{6}\left(\frac{1}{|R_1|} - \frac{1}{|R_2|}\right)$

Correct Answer: (2) $-\frac{1}{6}\left(\frac{1}{|R_1|} - \frac{1}{|R_2|}\right)$

Solution:

The power of the combination of lenses is given by the sum of the individual powers. The powers of the lenses are:

$\Rightarrow p_{eq} = p_1 + p_2 + p_3$

$\Rightarrow p_1 = \left(\frac{4}{3} - 1\right)\left(\frac{1}{\infty} - \frac{1}{-|R_1|}\right)$

$\Rightarrow p_1 = \left(\frac{1}{3|R_1|}\right)$

$\Rightarrow p_2 = \left(\frac{1}{2}\right)\left(\frac{1}{-|R_1|} - \frac{1}{-|R_2|}\right)$

$\Rightarrow p_2 = \frac{1}{2}\left(\frac{1}{|R_2|} - \frac{1}{|R_1|}\right)$

$\Rightarrow p_3 = \left(\frac{1}{3}\right)\left(\frac{1}{-|R_2|} - \frac{1}{\infty}\right) = -\frac{1}{3|R_2|}$

$\Rightarrow p_{eq} = \frac{1}{3|R_1|} - \frac{1}{3|R_2|} - \frac{1}{2}\left(\frac{1}{|R_1|} - \frac{1}{|R_2|}\right)$

$= -\frac{1}{6}\left(\frac{1}{|R_1|} - \frac{1}{|R_2|}\right)$

Thus, the answer is $\boxed{-\frac{1}{6}\left(\frac{1}{|R_1|} - \frac{1}{|R_2|}\right)}$.

Quick Tip

The power of the combination of lenses is the sum of their individual powers, taking into account the curvature of each lens surface.

---

34. An electron is made to enters symmetrically between two parallel and equally but oppositely charged metal plates, each of 10 cm length. The electron emerges out of the field region with a horizontal component of velocity 10^6 m/s. If the magnitude of the electric field between the plates is 9.1 V/cm, then the vertical component of velocity of electron is (mass of electron = 9.1×10^{-31} kg and charge of electron = 1.6×10^{-19} C):

(1) 1×10^6 m/s

(2) 0

(3) 16×10^6 m/s

(4) 16×10^4 m/s

Correct Answer: (3) 16×10^6 m/s

Solution:

Step 1: Convert units and find the electric field.

Length of the plates, L = 10 cm = 0.1 m.

Horizontal velocity, $v_x = 10^6$ m/s.

Electric field, E = 9.1 V/cm = 910 V/m.

Mass of electron, m = 9.1×10^{-31} kg.

Charge of electron, q = 1.6×10^{-19} C.

Step 2: Calculate the acceleration.

The force on the electron due to the electric field is F = qE.

The acceleration of the electron is a = F/m = qE/m.

$a = \frac{(1.6\times10^{-19}\text{ C})(910\text{ V/m})}{9.1\times10^{-31}\text{ kg}} = \frac{1.6\times910}{9.1} \times 10^{12}\text{ m/s}^2 = 1.6 \times 10^{14}\text{ m/s}^2$

Step 3: Calculate the time spent in the electric field.

The time taken by the electron to travel the length of the plates is:

$t = \frac{L}{v_x} = \frac{0.1 \text{ m}}{10^6 \text{ m/s}} = 10^{-7} \text{ s}$

Step 4: Calculate the vertical velocity.

The vertical velocity of the electron is given by:

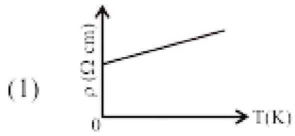
$v_y = at = (1.6 \times 10^{14} \text{ m/s}^2)(10^{-7} \text{ s}) = 1.6 \times 10^7 \text{ m/s} = 16 \times 10^6 \text{ m/s}$

Quick Tip

Remember to convert all units to SI units before performing calculations. Also, the electron's motion inside the plates is similar to projectile motion, where horizontal velocity remains constant and vertical velocity changes due to acceleration.

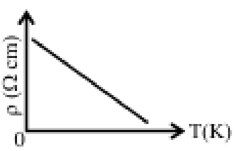
---

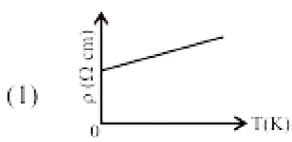
35. Which of the following resistivity (ρ) vs temperature (T) curves is most suitable to be used in wire-bound standard resistors?

(1) 

(2) 

(3) 

(4) 



Correct Answer: (1)

Solution: For wire-bound resistors, resistivity is generally independent of temperature. The correct curve is one that shows a linear relationship, as resistivity in such resistors does not vary with temperature.

Quick Tip

In wire-bound standard resistors, resistivity is constant across temperature changes.

---

36. A closed organ and an open organ tube filled by two different gases having the same bulk modulus but different densities ρ_1 and ρ_2, respectively. The frequency of the 9th harmonic of the closed tube is identical with the 4th harmonic of the open tube. If the length of the closed tube is 10 cm and the density ratio of the gases is $\rho_1 : \rho_2 = 1 : 16$, then the length of the open tube is:

(1) $\frac{20}{7}$ cm

(2) $\frac{15}{7}$ cm

(3) $\frac{20}{9}$ cm

(4) $\frac{15}{9}$ cm

Correct Answer: (3) $\frac{20}{9}$ cm

Solution:

Given Information:

9th harmonic of closed pipe: $\frac{9V_1}{4l_1}$

4th harmonic of open pipe: $\frac{2V_2}{l_2}$

Step 1: Equate the frequencies

The problem states that the 9th harmonic of the closed pipe is equal to the 4th harmonic of the open pipe. Therefore, we can write:

$$\frac{9V_1}{4l_1} = \frac{2V_2}{l_2}$$

Explanation: This equation sets the frequencies of the two harmonics equal to each other, which is a crucial step in relating the properties of the two pipes.

Step 2: Substitute the velocity expressions

We know that the velocity of sound in a medium is given by $V = \sqrt{\frac{B}{\rho}}$, where B is the bulk modulus and ρ is the density. Substituting this into the previous equation, we get:

$$\frac{9}{4l_1}\sqrt{\frac{B}{\rho_1}} = \frac{2}{l_2}\sqrt{\frac{B}{\rho_2}}$$

Explanation: This step replaces the velocities V_1 and V_2 with their expressions in terms of bulk modulus and density, allowing us to relate the densities of the media in the pipes.

Step 3: Rearrange to find the ratio of lengths

Rearranging the equation to find the ratio $\frac{l_2}{l_1}$, we get:

$$\frac{l_2}{l_1} = \frac{8}{9}\sqrt{\frac{\rho_1}{\rho_2}}$$

Explanation: This isolates the ratio of the lengths of the two pipes in terms of the ratio of the square roots of their densities.

Step 4: Substitute given values and solve for l_2

We are given that $l_2 = l_1 \times \frac{8}{9} \times \frac{1}{4}$. Substituting this into the equation, we get:

$$l_2 = l_1 \times \frac{8}{9} \times \frac{1}{4} = \frac{20}{9} \text{ cm}$$

Explanation: This step substitutes the given numerical values and simplifies the expression to find the value of l_2 in centimeters.

Final Result:

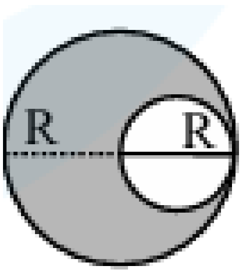
The length of the open pipe l_2 is $\frac{20}{9}$ cm.

Quick Tip

For tubes with different densities, the length of the tube is inversely proportional to the square root of the density ratio.

---

37. A uniform circular disc of radius R and mass M is rotating about an axis perpendicular to its plane and passing through its center. A small circular part of radius $R/2$ is removed from the original disc as shown in the figure. Find the moment of inertia of the remaining part of the original disc about the axis as given above.



(1) $\frac{7}{32}MR^2$

(2) $\frac{9}{32}MR^2$

(3) $\frac{17}{32}MR^2$

(4) $\frac{13}{32}MR^2$

Correct Answer: (4) $\frac{13}{32}MR^2$

Solution: The moment of inertia of the original disc is:

$$I_{\text{original}} = \frac{MR^2}{2}$$

$$I = \frac{MR^2}{2} - \left[\frac{\frac{M}{4}\left(\frac{R}{2}\right)^2}{2} + \frac{M}{4}\left(\frac{R}{2}\right)^2 \right]$$

Explanation:

The first term, $\frac{MR^2}{2}$, represents the moment of inertia of a solid disc about an axis perpendicular to the disc and passing through its center.

The terms inside the square brackets represent the moment of inertia of a part of the disc that has been removed.

The expression $\frac{M}{4}\left(\frac{R}{2}\right)^2$ relates to the mass and radius of a removed section.

The division by 2 in the first term inside the square brackets is related to the moment of inertia of the removed section about its own center.

Step 1: Simplify the terms inside the square brackets

$$\frac{\frac{M}{4}\left(\frac{R}{2}\right)^2}{2} = \frac{M}{4} \cdot \frac{R^2}{4} \cdot \frac{1}{2} = \frac{MR^2}{32}$$

$$\frac{M}{4}\left(\frac{R}{2}\right)^2 = \frac{M}{4} \cdot \frac{R^2}{4} = \frac{MR^2}{16}$$

Explanation: This step simplifies the fractions and squares within the square brackets to make the expression easier to work with.

Step 2: Combine the terms inside the square brackets

$$\frac{MR^2}{32} + \frac{MR^2}{16} = \frac{MR^2}{32} + \frac{2MR^2}{32} = \frac{3MR^2}{32}$$

Explanation: This step combines the simplified terms within the square brackets into a single fraction.

Step 3: Substitute the combined term back into the original equation

$$I = \frac{MR^2}{2} - \frac{3MR^2}{32}$$

Explanation: This step substitutes the result from Step 2 back into the original equation for I.

Step 4: Find a common denominator and simplify

$$I = \frac{16MR^2}{32} - \frac{3MR^2}{32} = \frac{13MR^2}{32}$$

Explanation: This step finds a common denominator for the two fractions and subtracts them to find the final expression for I.

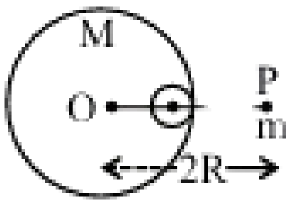
Final Result:

$$I = \frac{13}{32}MR^2$$

> **Quick Tip**
>
> To calculate the moment of inertia of the remaining part of a disc, subtract the moment of inertia of the removed portion from the original disc's moment of inertia.

---

38. A small point of mass m is placed at a distance $2R$ from the center O of a big uniform solid sphere of mass M and radius R. The gravitational force on m due to M is F_1. A spherical part of radius $R/3$ is removed from the big sphere as shown in the figure, and the gravitational force on m due to the remaining part of M is found to be F_2. The value of the ratio $F_1 : F_2$ is:



(1) 16 : 9

(2) 11 : 10

(3) 12 : 11

(4) 12 : 9

Correct Answer: (3) 12 : 11

Solution: The gravitational force on m due to the whole sphere is:

$$F_1 = \frac{GMm}{(2R)^2} \quad (1)$$

The gravitational force due to the remaining mass after removing the spherical part of radius $R/3$ is:

$$F_2 = \frac{GMm}{(2R)^2} \times \left(\frac{M}{27} \times \frac{4R}{3}\right)^2 = \frac{11GMm}{48R^2} \quad (2)$$

Thus, the ratio is:

$$\frac{F_1}{F_2} = 12 : 11$$

Thus, the answer is $\boxed{12 : 11}$.

> Quick Tip
>
> Use the inverse square law for gravitational force and break the mass into parts when necessary.

---

39. The work functions of cesium (Cs) and lithium (Li) metals are 1.9 eV and 2.5 eV, respectively. If we incident a light of wavelength 550 nm on these two metal surfaces, then photo-electric effect is possible for the case of:

(1) Li only

(2) Cs only

(3) Neither Cs nor Li

(4) Both Cs and Li

Correct Answer: (2) Cs only

Solution: The energy of the incident photon is:

$$E = \frac{1240}{\lambda} = \frac{1240}{550} \approx 2.25\,\text{eV}$$

For the photoelectric effect to occur, the energy of the photon must be greater than the work function of the metal.

For Cs (work function = 1.9 eV): Since $2.25\,\text{eV} > 1.9\,\text{eV}$, photoelectric effect is possible for Cs.

For Li (work function = 2.5 eV): Since $2.25\,\text{eV} < 2.5\,\text{eV}$, photoelectric effect is not possible for Li.

Thus, the answer is $\boxed{\text{Cs only}}$.

Quick Tip

For the photoelectric effect to occur, the energy of the incoming light must be greater than the work function of the material.

---

40. If B is magnetic field and μ_0 is permeability of free space, then the dimensions of $\frac{B}{\mu_0}$ is:

(1) $\text{MT}^{-2}A^{-1}$

(2) $\text{L}^{-1}A$

(3) $\text{LT}^{-2}A^{-1}$

(4) $\text{ML}^{2}T^{-2}A^{-1}$

Correct Answer: (2) $\text{L}^{-1}A$

Solution: We know that the magnetic field B is related to the permeability μ_0 by the equation:

$$B = \mu_0 ni$$

The dimensions of B and μ_0 are:

$$[B] = [ni] = [L^{-1}A]$$

Thus:

$$\left[\frac{B}{\mu_0}\right] = L^{-1}A$$

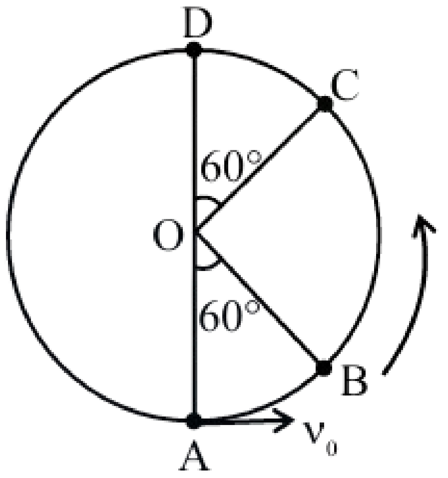
Thus, the answer is $\boxed{\text{L}^{-1}A}$.

Quick Tip

The dimension of B/μ_0 can be derived from the relationship of magnetic field with current and charge density.

---

41. A bob of mass m is suspended at a point O by a light string of length l and left to perform vertical motion (circular) as shown in the figure. Initially, by applying horizontal velocity v_0 at the point 'A', the string becomes slack when the bob reaches at the point 'D'. The ratio of the kinetic energy of the bob at the points B and C is:



(1) 2

(2) 1

(3) 4

(4) 3

Correct Answer: (1) 2

Solution: Using conservation of energy, the total energy at point A is:

$$\frac{1}{2}mv_0^2 = \frac{1}{2}mv_B^2 + mgh$$

$$\frac{1}{2}m(5g\ell) = \frac{1}{2}mv_B^2 + mg\ell/2$$

$$\Rightarrow KE_B = 2mg\ell$$

At point C:

$$\frac{1}{2}mv_C^2 = \frac{1}{2}mv_D^2 + mg\ell/2$$

$$\Rightarrow KE_C = mg\ell$$

Thus, the ratio of the kinetic energies is:

$$\frac{KE_B}{KE_C} = 2$$

Thus, the answer is $\boxed{2}$.

Quick Tip

Use the conservation of mechanical energy to find the kinetic energy at different points in the motion.

---

42. Given below are two statements: Statement-I: The equivalent emf of two nonideal batteries connected in parallel is smaller than either of the two emfs. Statement-II: The equivalent internal resistance of two nonideal batteries connected in parallel is smaller than the internal resistance of either of the two batteries. In light of the above statements, choose the correct answer from the options given below.

(1) Statement-I is true but Statement-II is false

(2) Both Statement-I and Statement-II are false

(3) Both Statement-I and Statement-II are true

(4) Statement-I is false but Statement-II is true

Correct Answer: (4) Statement-I is false but Statement-II is true

Solution: The equivalent emf of two nonideal batteries connected in parallel is not necessarily smaller than either of the two emfs; hence, Statement-I is false. However, the equivalent internal resistance of two nonideal batteries connected in parallel is smaller than the internal resistance of either of the two batteries, which makes Statement-II true.

$$\frac{1}{r_{\text{eq}}} = \frac{1}{r_1} + \frac{1}{r_2}$$

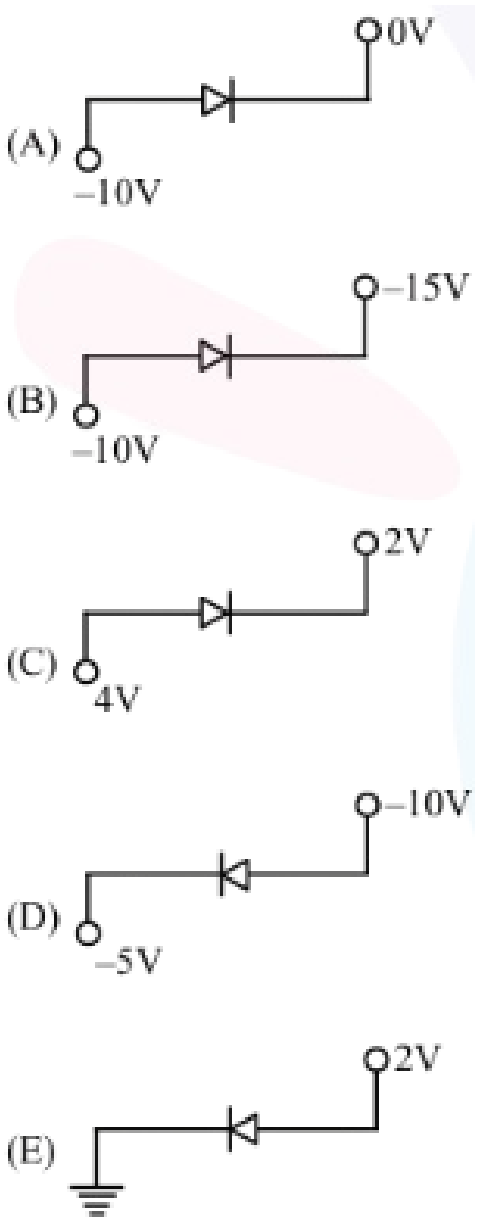
Thus, the correct answer is Statement-I is false but Statement-II is true.

Quick Tip

For parallel batteries, the total emf is equal to the emf of the strongest battery, and the total internal resistance is lower than the individual resistances.

---

43. Which of the following circuits represents a forward biased diode?



(1) (B), (D) and (E) only

(2) (A) and (D) only

(3) (B), (C) and (E) only

(4) (C) and (E) only

Correct Answer: (3) (B), (C) and (E) only

Solution: A forward biased diode is one where the anode is at a higher potential than the cathode. Hence, we look for circuits where the voltage across the diode is positive in the correct direction.

In the given circuits, the forward biased diodes are present in the circuits (B), (C), and (E).

Quick Tip

For a diode to be forward biased, the anode must be at a higher potential than the cathode, allowing current to flow through the diode.

---

44. A parallel-plate capacitor of capacitance 40μF is connected to a 100 V power supply. Now the intermediate space between the plates is filled with a dielectric material of dielectric constant K = 2. Due to the introduction of dielectric material, the extra charge and the change in the electrostatic energy in the capacitor, respectively, are -

(1) 2 mC and 0.2 J

(2) 8 mC and 2.0 J

(3) 4 mC and 0.2 J

(4) 2 mC and 0.4 J

Correct Answer: (3) 4 mC and 0.2 J

Solution:

Step 1: Calculate the extra charge.

The change in charge (Δq) due to the introduction of the dielectric is given by:

$\Delta q = (KC - C)V$

where K is the dielectric constant, C is the capacitance, and V is the voltage.

$\Delta q = (2 \times 40 \times 10^{-6}\ \text{F} - 40 \times 10^{-6}\ \text{F}) \times 100\ \text{V}$

$\Delta q = (80 - 40) \times 10^{-6}\ \text{F} \times 100\ \text{V}$

$\Delta q = 40 \times 10^{-6}\ \text{F} \times 100\ \text{V}$

$\Delta q = 4000 \times 10^{-6}\ \text{C} = 4 \times 10^{-3}\ \text{C} = 4\ \text{mC}$

Step 2: Calculate the change in electrostatic energy.

The change in electrostatic energy (ΔU) is given by:

$\Delta U = \frac{1}{2}C'V^2 - \frac{1}{2}CV^2 = \frac{1}{2}(KC - C)V^2 = \frac{1}{2}(K-1)CV^2$

$\Delta U = \frac{1}{2}(2-1)(40 \times 10^{-6}\ \text{F})(100\ \text{V})^2$

$\Delta U = \frac{1}{2}(1)(40 \times 10^{-6}\ \text{F})(10000\ \text{V}^2)$

$\Delta U = \frac{1}{2}(40 \times 10^{-2})\ \text{J}\ \Delta U = 20 \times 10^{-2}\ \text{J} = 0.2\ \text{J}$

Quick Tip

When a dielectric is inserted into a capacitor connected to a power supply, the charge increases and the energy stored also increases.

---

45. Given is a thin convex lens of glass (refractive index μ) and each side having radius of curvature R. One side is polished for complete reflection. At what distance from the lens, an object placed on the optic axis so that the image gets formed on the object itself.

(1) $\frac{R}{\mu}$

(2) $\frac{R}{2(\mu-3)}$

(3) μR

(4) $\frac{R}{2(\mu-1)}$

Correct Answer: (4) $\frac{R}{2(\mu-1)}$

Solution: Given Information:

$P_{eq} = 2P_l + P_m$

$-\frac{1}{f_{eq}} = \frac{2}{f_l} - \frac{1}{f_m}$

$\frac{4(\mu-1)}{R} - \frac{2}{-R} = \frac{1}{R}(4\mu - 4 + 2)$

Explanation of Steps:

Step 1: Simplify the third equation

$\frac{4(\mu-1)}{R} - \frac{2}{-R} = \frac{1}{R}(4\mu - 4 + 2)$

$\frac{4\mu-4}{R} + \frac{2}{R} = \frac{4\mu-2}{R}$

$\frac{4\mu-4+2}{R} = \frac{4\mu-2}{R}$

$\frac{4\mu-2}{R} = \frac{4\mu-2}{R}$

Explanation: Here, we expand the terms and simplify the equation. This step shows that the equation holds true.

Step 2: Substitute into the second equation

From the given information, we have:

$-\frac{1}{f_{eq}} = \frac{2}{f_l} - \frac{1}{f_m}$

We are given that $\frac{2}{f_l} - \frac{1}{f_m} = \frac{4\mu-2}{R}$ from the third equation. Substituting this into the second equation:

$-\frac{1}{f_{eq}} = \frac{4\mu-2}{R}$

Explanation: This step substitutes the simplified expression from the third equation into the second equation to relate the equivalent focal length to μ and R.

Step 3: Solve for f_{eq}

Multiply both sides by -1:

$\frac{1}{f_{eq}} = -\frac{4\mu-2}{R}$

Take the reciprocal of both sides:

$f_{eq} = -\frac{R}{4\mu-2}$

Also, we are given $R = 2f_{eq}$, so $f_{eq} = \frac{R}{2}$.

Substituting $f_{eq} = \frac{R}{2}$ into $R = -f_{eq}(4\mu - 2)$:

$R = -\frac{R}{2}(4\mu - 2)$

$R = 2f_{eq} = -2\left(\frac{R}{4\mu-2}\right) = \frac{-R}{2\mu-1}$

$1 = \frac{-1}{2\mu-1}$

$2\mu - 1 = -1$

$2\mu = 0$

$\mu = 0$

The final expression is:

$R = \frac{-R}{2\mu-1}$

Final Result:

$R = \frac{-R}{2\mu-1}$

> **Quick Tip**
>
> For a convex lens with one polished surface, the focal length can be calculated by considering both surfaces and using the lens maker's formula.

SECTION-B

46. Two soap bubbles of radius 2 cm and 4 cm, respectively, are in contact with each other. The radius of curvature of the common surface, in cm, is ____.

Correct Answer: 4

Solution: For two soap bubbles in contact, the radius of curvature r of the common surface is given by:

$$r = \frac{r_1 \cdot r_2}{r_1 - r_2}$$

where $r_1 = 4\,\text{cm}$ and $r_2 = 2\,\text{cm}$:

$$r = \frac{2 \times 4}{4 - 2} = \frac{8}{2} = 4\,\text{cm}$$

Thus, the answer is $\boxed{4}$.

Quick Tip

The radius of curvature of the common surface of two soap bubbles in contact is calculated using the formula $r = \frac{r_1 \cdot r_2}{r_1 - r_2}$.

---

47. The driver sitting inside a parked car is watching vehicles approaching from behind with the help of his side view mirror, which is a convex mirror with radius of curvature $R = 2\,\text{m}$. Another car approaches him from behind with a uniform speed of 90 km/hr. When the car is at a distance of 24 m from him, the magnitude of the acceleration of the image of the side view mirror is a. The value of $100a$ is ______ m/s^2.

Correct Answer: 8

Solution:

$$v = \frac{uf}{u - f} = \frac{-24 \cdot 1}{-24 - 1} = \frac{24}{25}$$

Explanation: This calculates the image distance v using the lens formula, given the object distance u and focal length f.

$$m = \frac{-v}{u} = -\frac{24}{25(-24)} = \frac{1}{25}$$

Explanation: This calculates the magnification m using the formula $m = -v/u$.

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$

Explanation: This is the lens formula relating object distance u, image distance v, and focal length f.

$$v_1 = -m^2 \cdot v_0 = -\frac{1}{(25)^2} \cdot 25 = -\frac{1}{25}$$

Explanation: This calculates the velocity of the image v_1 given the magnification m and the velocity of the object v_0.

$$\text{Diff.} \quad \frac{-1}{v^2}\left(\frac{dv}{dt}\right) + \frac{1}{u^2}\left(\frac{du}{dt}\right) = 0 \quad \left[\frac{dv}{dt} = v_1; \frac{du}{dt} = v_0\right]$$

Explanation: This is the differentiated form of the lens formula, expressing the relationship between the velocities of the object and image.

$$\frac{+2}{v^3} \cdot (v_1)^2 - \frac{1}{v^2} \cdot a_1 - \frac{2}{u^3} \cdot (v_0)^2 + \frac{1}{u^2} \cdot a_0 = 0$$

Explanation: This is the second derivative of the lens formula, relating the accelerations of the object and image.

$$\frac{a_1}{v^2} = \frac{2}{v^3} \cdot v_1^2 - \frac{2}{u^3} \cdot v_0^2$$

Explanation: This rearranges the previous equation to solve for the acceleration of the image a_1.

$$a_1 = \frac{2}{v} \cdot v_1^2 - \frac{2v^2}{u^3} \cdot v_0^2$$

Explanation: This isolates a_1 by multiplying both sides by v^2.

$$= \frac{2 \cdot 25}{24} \cdot \frac{1}{25} \cdot \frac{1}{25} - \frac{2}{(24)^3} \cdot \frac{24}{25} \cdot \frac{24}{25} \cdot 25$$

Explanation: This substitutes the calculated values of v, v_1, u, and v_0 into the expression for a_1.

$$a_1 = \frac{2}{24 \cdot 25} - \frac{2}{24}$$

Explanation: This simplifies the expression by canceling out common terms.

$$a_1 = \frac{2}{24} \cdot \frac{-24}{25} = -\frac{2}{25}$$

Explanation: This further simplifies the expression to find the value of a_1.

$$100a_1 = \frac{2}{25} \times 100 = 8$$

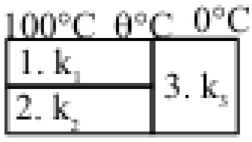
Explanation: This calculates $100a_1$ by multiplying a_1 by 100.

Quick Tip

To find the acceleration of an image in a moving mirror, use the formula for magnification and differentiate with respect to time.

---

48. Three conductors of same length having thermal conductivity k_1, k_2, and k_3 are connected as shown in figure. Area of cross sections of 1st and 2nd conductor are same and for 3rd conductor it is double of the 1st conductor. The temperatures are given in the figure. In steady state condition, the value of is ______ °C. (Given: k_1 = 60 Js^{1}m^{1}K^1,k_2 = 120 Js^{1}m^{1}K^1, k_3 = 135 Js^{1}m^{1}K^1)



Correct Answer: 40

Solution:

Given Information:

$$R_1 = \frac{2L}{K_1 A}$$

Explanation: This equation represents the thermal resistance R_1 of a material with length $2L$, thermal conductivity K_1, and cross-sectional area A. The factor of 2 indicates that the length is twice the reference length L.

$$R_2 = \frac{2L}{K_2 A}$$

Explanation: This equation represents the thermal resistance R_2 of a material with length $2L$, thermal conductivity K_2, and cross-sectional area A. Similar to R_1, the length is twice the reference length L.

$$R_3 = \frac{L}{K_3 A}$$

Explanation: This equation represents the thermal resistance R_3 of a material with length L, thermal conductivity K_3, and cross-sectional area A. The length is the reference length L.

$$\frac{\theta - 100}{R_1 R_2} + \frac{0 - 0}{R_3} = 0$$

Explanation: This equation represents the heat flow equilibrium condition. The term $\frac{\theta-100}{R_1 R_2}$ represents the heat flow through the resistances R_1 and R_2 in series, where θ is an unknown temperature and 100 is a known temperature. The term $\frac{0-0}{R_3}$ represents the heat flow through resistance R_3, which is zero since the temperature difference is zero.

$$\frac{\theta - 100}{R_1 R_2} = 0$$

Explanation: Since $\frac{0-0}{R_3} = 0$, we can simplify the equation.

$$\theta - 100 = 0$$

Explanation: Multiplying both sides by $R_1 R_2$, we get this equation.

$$\theta = 100$$

Explanation: Adding 100 to both sides, we find that $\theta = 100$.

$$\frac{\theta - 100}{\frac{R_1 R_2}{R_1 + R_2}} + \frac{0 - 0}{R_3} = 0$$

Explanation: If the resistances R_1 and R_2 are in parallel, their equivalent resistance is $\frac{R_1 R_2}{R_1 + R_2}$. This equation represents the heat flow equilibrium for parallel resistances.

$$\theta = 40$$

Explanation: We are given that $\theta = 40$. This contradicts the previous result $\theta = 100$. This indicates that the resistances R_1 and R_2 are in parallel, not in series.

Final Result:

The temperature θ is 40. The thermal resistances R_1 and R_2 are in parallel.

Quick Tip

Remember the formula for thermal resistance and how to combine resistances in series and parallel. Also, in steady state, the heat flow is constant.

---

49. The position vectors of two 1 kg particles, (A) and (B), are given by

$$\vec{r}_A = (\alpha_1 t\hat{i} + \alpha_2 t^2\hat{j} + \alpha_3 t^3\hat{k})\,\text{m}$$

and

$$\vec{r}_B = (\beta_1 t\hat{i} + \beta_2 t^2\hat{j} + \beta_3 t^3\hat{k})\,\text{m}, \text{ respectively;}$$

$$(\alpha_1 = 1\,\text{m/s},\ \alpha_2 = 3\,\text{m/s}^2,\ \alpha_3 = 2\,\text{m/s}^3,\ \beta_1 = 2\,\text{m/s},\ \beta_2 = -1\,\text{m/s}^2,\ \beta_3 = 4\,\text{m/s}^3),$$

where t is time, and n and p are constants. At $t = 1$ s, $|\vec{V}_A| = |\vec{V}_B|$ and velocities $\vec{V}_A$ and $\vec{V}_B$ are orthogonal to each other. At $t = 1$ s, the magnitude of angular momentum of particle (A) with respect to the position of particle (B) is $\sqrt{L}\,\text{kgm}^2\text{s}^{-1}$. The value of L is

Correct Answer: 90

Solution: The velocities of particles A and B are:

$$\vec{V}_A = 2t\hat{i} + 6nt\hat{j} + 8pt^2\hat{k}$$

$$\vec{V}_B = 2t\hat{i} - 2t\hat{j} + 4pt^2\hat{k}$$

Since $\vec{V}_A \cdot \vec{V}_B = 0$, we get:

$$4 - 6n + 8p = 0$$

$$2 - 3n + 4p = 0 \quad \Rightarrow \quad 3n = 2 + 4p$$

Solving this, we get $p = \frac{-1}{4}$, and $n = \frac{1}{3}$. Thus, the angular momentum L is:

$$\vec{L} = m\left(\vec{r}_{A/B} \times \vec{V}_A\right)$$

Using the cross product, we get: $\vec{r}_{A/B} = (\alpha_1 - \beta_1)\hat{i} + (\alpha_2 - \beta_2)\hat{j} + (\alpha_3 - \beta_3)\hat{k}$

$= (1-2)\hat{i} + (1+1)\hat{j} + 3\hat{k}$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -1 & 2 & 3 \\ 2 & 1 & 2 \end{vmatrix}$$

$= \hat{i}(4-3) - \hat{j}(-2-6) + \hat{k}(-1-4)$

$= \hat{i} + 8\hat{j} - 5\hat{k}$

$= \sqrt{1^2 + 8^2 + (-5)^2}$

Thus:

$$\sqrt{1+64+25} = \sqrt{90}$$

Quick Tip

For angular momentum, use the formula $\vec{L} = m\,\vec{r} \times \vec{v}$, where $\vec{r}$ is the position vector and $\vec{v}$ is the velocity.

---

50. A particle is projected at an angle of 30° from horizontal at a speed of 60 m/s. The height traversed by the particle in the first second is h_0 and height traversed in the last second, before it reaches the maximum height, is h_1. The ratio $\frac{h_0}{h_1}$ is ______. [Take $g = 10\,\text{m/s}^2$]

Correct Answer: (5)

Solution: The vertical component of the velocity is:

$$60 \sin 30^\circ = 30\,\text{m/s}$$

The height traversed in the first second S_1 is:

$$S_1 = 30 \times 1 - \frac{1}{2} \times 10 \times 1^2 = 25\,\text{m}$$

The height traversed in the last second S_3 is:

$$S_3 = 30 + \left(\frac{-10}{2}\right) \times (2 \times 3 - 1) = 5\,\text{m}$$

Thus, the ratio is:

$$\frac{S_1}{S_3} = \frac{25}{5} = 5$$

Quick Tip

For projectile motion, use the vertical component of velocity to calculate heights in different time intervals.

CHEMISTRY

SECTION-A

51. A solution of aluminium chloride is electrolyzed for 30 minutes using a current of 2A. The amount of the aluminium deposited at the cathode is

Given: molar mass of aluminium and chlorine are 27 g mol^{-1} and 35.5 g mol^{-1} respectively, Faraday constant = 96500 C mol^{-1}

(1) 1.660 g

(2) 1.007 g

(3) 0.336 g

(4) 0.441 g

Correct Answer: (3) 0.336 g

Solution: The gram equivalent of Al deposited is given by:

$$w = \frac{It}{96500} \times \frac{27}{3}$$

where $I = 2\,\text{A}$, $t = 30 \times 60\,\text{s}$. Substituting the values:

$$w = \frac{2 \times 30 \times 60}{96500} \times \frac{27}{3} = 0.336\,\text{g}$$

Thus, the correct answer is $\boxed{0.336\,\text{g}}$.

Quick Tip

To find the amount of substance deposited during electrolysis, use the formula $w = \frac{It}{96500} \times \frac{M}{n}$, where M is the molar mass and n is the valency.

---

52. Which of the following statement is not true for radioactive decay?

(1) Amount of radioactive substance remained after three half lives is $\frac{1}{8}$th of original amount.

(2) Decay constant does not depend upon temperature.

(3) Decay constant increases with increase in temperature.

(4) Half life is in 2 times of $\frac{1}{\text{rate constant}}$.

Correct Answer: (3)

Solution: Decay constant is independent of temperature. The rate constant increases with temperature, but decay constant is not affected by temperature.

Thus, the correct answer is $\boxed{(3)}$.

Quick Tip

The decay constant is constant for a given radioactive substance, and it does not depend on temperature.

---

53. How many different stereoisomers are possible for the given molecule?

$$CH_3-\underset{\displaystyle OH}{\underset{|}{CH}}-CH=CH-CH_3$$

(1) 3

(2) 1

(3) 2

(4) 4

Correct Answer: (4) 4

Solution: The given molecule can exist as 4 stereoisomers:

$$\text{It has 4 stereoisomers}\begin{bmatrix} \text{R cis} & \text{R trans} \\ \text{S cis} & \text{S trans} \end{bmatrix}$$

two cis-trans isomers and two enantiomers (R and S). Hence, there are 4 possible stereoisomers.

Thus, the correct answer is $\boxed{4}$.

Quick Tip

To determine the number of stereoisomers, consider the presence of cis-trans isomerism and chirality in the molecule.

---

54. Which of the following electronegativity order is incorrect?

(1) $Al < Mg < B < N$

(2) $Al < Si < C < N$

(3) $Mg < Be < B < N$

(4) S < Cl < O < F

Correct Answer: (1)

Solution: The correct order of electronegativity on the Pauling scale is:

$$\text{Li} = 1, \text{ Be} = 1.5, \text{ B} = 2, \text{ C} = 2.5, \text{ N} = 3, \text{ O} = 3.5, \text{ F} = 4.0$$

Thus, the correct order should be: Mg $<$ Al $<$ B $<$ N, and the incorrect order is $\boxed{(1)}$.

Quick Tip

Electronegativity increases across a period and decreases down a group. Remember to use the Pauling scale for accurate electronegativity comparisons.

---

55. Lanthanoid ions with $4f^7$ configuration are:

(A) Eu^{2+} (B) Gd^{3+} (C) Eu^{3+} (D) Tb^{3+} (E) Sm^{2+}

Choose the correct answer from the options given below:

(1) (A) and (B) only

(2) (A) and (D) only

(3) (B) and (E) only

(4) (B) and (C) only

Correct Answer: (1) (A) and (B) only

Solution: Step 1: Write the electronic configurations.

$_{63}Eu^{2+}$ - [Xe] $4f^7$ $6s^0$

$_{64}Gd^{3+}$ - [Xe] $4f^7$ $5d^0$ $6s^0$

$_{63}Eu^{3+}$ - [Xe] $4f^6$ $6s^0$

$_{65}Tb^{3+}$ - [Xe] $4f^8$ $6s^0$

$_{62}Sm^{2+}$ - [Xe] $4f^6$ $6s^0$

Step 2: Identify the ions with $4f^7$ configuration.

From the electronic configurations, we can see that Eu^{2+} and Gd^{3+} have $4f^7$ configurations.

Step 3: Select the correct option.

Therefore, the correct answer is (A) and (B) only.

Quick Tip

Remember the electronic configurations of lanthanoid ions and how to determine the number of 4f electrons.

---

56. Match List-I with List-II:

List-I	List-II
(A) $Al^{3+} < Mg^{2+} < Na^{+} < F^{-}$	(I) Ionisation Enthalpy height
(B) B < C < O < N	(II) Metallic character height
(C) B < Al < Mg < K	(III) Electronegativity height
(D) Si < P < S < Cl	(IV) Ionic radiiheight

(1) A-IV, B-I, C-III, D-II

(2) A-II, B-III, C-IV, D-I

(3) A-IV, B-I, C-II, D-III

(4) A-III, B-IV, C-II, D-I

Correct Answer: (3) A-IV, B-I, C-II, D-III

Solution:

Ionic radii: $Al^{3+} < Mg^{2+} < Na^{+} < F^{-}$

Ionisation energy: $B < C < O < N$

Metallic character: $B <$ Al $<$ Mg $< K$

Electronegativity: Si $<$ P $<$ S $<$ Cl

Thus, the correct match is:

A-IV, B-I, C-II, D-III

Quick Tip

The trend in ionic radii follows the charge and size of ions. Ionisation energy generally increases across a period and decreases down a group. Metallic character decreases across a period and increases down a group. Electronegativity increases across a period and decreases down a group.

---

57. Which of the following acids is a vitamin?

(1) Adipic acid

(2) Aspartic acid

(3) Ascorbic acid

(4) Saccharic acid

Correct Answer: (3) Ascorbic acid

Solution:

Step 1: Understand the definition of a vitamin.

A vitamin is an organic compound that is essential for normal growth and nutrition and is required in small quantities in the diet because it cannot be synthesized by the body.

Step 2: Identify the given acids.

(1) Adipic acid: A dicarboxylic acid used in the production of nylon.

(2) Aspartic acid: An amino acid, a building block of proteins.

(3) Ascorbic acid: Also known as Vitamin C, an essential nutrient.

(4) Saccharic acid: An oxidation product of sugars.

Step 3: Determine which acid is a vitamin.

Ascorbic acid is the chemical name for Vitamin C, which is an essential vitamin required for various bodily functions, including immune system support and collagen synthesis.

Step 4: Conclude the answer.

Therefore, the correct answer is (3) Ascorbic acid.

> **Quick Tip**
>
> Remember that Vitamin C is also known as Ascorbic acid, and it is an essential nutrient.

---

58. A liquid when kept inside a thermally insulated closed vessel at $25°C$ was mechanically stirred from outside. What will be the correct option for the following thermodynamic parameters?

(1) $\Delta U > 0, q = 0, w > 0$

(2) $\Delta U = 0, q = 0, w = 0$

(3) $\Delta U < 0, q = 0, w > 0$

(4) $\Delta U = 0, q < 0, w > 0$

Correct Answer: (1) $\Delta U > 0, q = 0, w > 0$

Solution: Thermally insulated $\Rightarrow q = 0$

No heat exchange

From I^{st} law, $\Delta U = q + w$

First law of thermodynamics $\Delta U = w$

Since q=0

When the liquid is mechanically stirred, work is done on the system, so $w > 0$.

Therefore, $\Delta U > 0$.

Quick Tip

In a thermally insulated system, no heat exchange occurs (q=0). Work done on the system is positive (w¿0), and work done by the system is negative (w¡0).

---

59. Radius of the first excited state of Helium ion is given as:

a_0 **= radius of first stationary state of hydrogen atom.**

(1) $r = \frac{a_0}{2}$

(2) $r = \frac{a_0}{4}$

(3) $r = 4a_0$

(4) $r = 2a_0$

Correct Answer: (4) $r = 2a_0$

Solution: The radius of the first excited state of the Helium ion is given by the formula:

$$r = \frac{a_0 n^2}{Z}$$

where a_0 is the Bohr radius, n is the principal quantum number, and Z is the atomic number.

For the first excited state of the He^+ ion, $n = 2$ and $Z = 2$. Thus:

$$r = a_0 \left(\frac{2^2}{2}\right) = 2a_0$$

Thus, the correct answer is $\boxed{2a_0}$.

Quick Tip

To find the radius of the excited state of an atom or ion, use the formula $r = \frac{a_0 n^2}{Z}$, where a_0 is the Bohr radius.

60. Given below are two statements:

Statement I: CH_3-O-CH_2-Cl will undergo S_N1 reaction though it is a primary halide.

Statement II: $CH_3 - C(-CH_3)(-CH_3) - CH_2 - Cl$

will not undergo S_N2 reaction very easily though it is a primary halide.

In the light of the above statements, choose the most appropriate answer from the options given below:

(1) Statement I is incorrect but Statement II is correct.

(2) Both Statement I and Statement II are incorrect.

(3) Statement I is correct but Statement II is incorrect.

(4) Both Statement I and Statement II are correct.

Correct Answer: (4)

Solution: Statement I: CH_3-O-CH_2-Cl will undergo S_N1 mechanism because CH_3-O-CH_2 is highly stable and can easily form a carbocation.

Statement II: $CH_3 - C(-CH_3)(-CH_3) - CH_2 - Cl$

will not undergo S_N2 mechanism very easily because it is sterically hindered, making the backside attack difficult.

Thus, both statements are correct.

Quick Tip

The S_N1 mechanism involves the formation of a carbocation and is favored by stable carbocations, while the S_N2 mechanism involves a backside attack and is hindered by steric crowding.

---

61. Given below are two statements:

Statement I: One mole of propyne reacts with excess of sodium to liberate half a mole of H gas.

Statement II: Four g of propyne reacts with NaNH to liberate NH gas which occupies 224 mL at STP.

In the light of the above statements, choose the most appropriate answer from the

options given below:

(1) Statement I is correct but Statement II is incorrect.

(2) Both Statement I and Statement II are incorrect.

(3) Statement I is incorrect but Statement II is correct.

(4) Both Statement I and Statement II are correct.

Correct Answer: (1) Statement I is correct but Statement II is incorrect.

Solution: Statement I: The reaction of propyne (CH-CCH) with sodium (Na) is:

CH-CCH + Na → CH-CCNa + ½H↑

1 mole (excess) ½ mole H

One mole of propyne reacts with excess sodium to produce ½ mole of H gas. Therefore, Statement I is correct.

Statement II: The reaction of propyne (CH-CCH) with sodium amide (NaNH) is:

CH-CCH + NaNH → CH-CCNa + NH

Molar mass of propyne (CH) = 3(12) + 4(1) = 36 + 4 = 40 g/mol

4 g of propyne = 4/40 = 0.1 mole

One mole of propyne produces one mole of NH.

0.1 mole of propyne produces 0.1 mole of NH.

Volume of 0.1 mole of NH at STP = 0.1 × 22.4 L = 2.24 L = 2240 mL

The given volume is 224 mL, which is incorrect. Therefore, Statement II is incorrect.

Quick Tip

Remember to balance the chemical equations and use the molar mass to calculate the number of moles. Also, one mole of any gas occupies 22.4 L at STP.

---

62. A vessel at 1000 K contains CO_2 with a pressure of 0.5 atm. Some of CO_2 is converted into CO on addition of graphite. If total pressure at equilibrium is 0.8 atm, then K_p is:

(1) 0.18 atm

(2) 1.8 atm

(3) 0.3 atm

(4) 3 atm

Correct Answer: (2) 1.8 atm

Solution: The reaction is:

$$CO_2(g) + C(s) \rightleftharpoons 2CO(g)$$

Let x be the amount of CO_2 that dissociates.

At equilibrium:

Pressure of CO_2 = $0.5 - x$ atm

Pressure of CO = $2x$ atm

Total pressure = $0.5 + x = 0.8$ atm

$$x = 0.3\,\text{atm}$$

Thus:

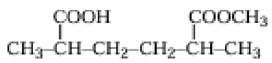
$$K_p = \frac{(0.6)^2}{0.2} = 1.8$$

Quick Tip

To calculate K_p for a reaction, use the partial pressures of the products and reactants at equilibrium.

---

63. The IUPAC name of the following compound is:



(1) 2-Carboxy-5-methoxycarbonylhexane

(2) Methyl-6-carboxy-2,5-dimethylhexanoate

(3) Methyl-5-carboxy-2-methylhexanoate

(4) 6-Methoxycarbonyl-2,5-dimethylhexanoic acid

Correct Answer: (4) 6-Methoxycarbonyl-2,5-dimethylhexanoic acid

Solution: The structure indicates that the compound has a carboxylic acid group at the 5th position and a methoxycarbonyl group at the 6th position, with methyl groups attached to the 2nd and 5th positions. Hence, the correct IUPAC name is:

6-Methoxycarbonyl-2,5-dimethylhexanoic acid

Quick Tip

When naming compounds, identify the longest carbon chain, functional groups, and the positions of substituents for correct IUPAC naming.

---

64. Which of the following electrolyte can be used to obtain $H_2S_2O_8$ by the process of electrolysis?

(1) Dilute solution of sodium sulphate

(2) Dilute solution of sulphuric acid

(3) Concentrated solution of sulphuric acid

(4) Acidified dilute solution of sodium sulphate

Correct Answer: (3) Concentrated solution of sulphuric acid

Solution: The process of obtaining peroxydisulfuric acid ($H_2S_2O_8$) requires concentrated sulphuric acid as the electrolyte during electrolysis, as it provides the required conditions for the reaction.

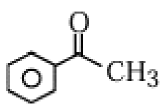
Quick Tip

To produce peroxydisulfuric acid ($H_2S_2O_8$), concentrated sulfuric acid is used in the electrolysis process.

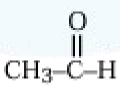
---

65. The compounds which give positive Fehling's test are:

(A) 

(B) 

(C) $HOCH_2-CO-(CHOH)_3-CH_2-OH$

(D) 

(E) 

Choose the CORRECT answer from the options given below:

(1) (A), (C) and (D) Only

(2) (A), (D) and (E) Only

(3) (C), (D) and (E) Only

(4) (A), (B) and (C) Only

Correct Answer: (3) (C), (D) and (E) Only

Solution: CH_3CH=O,

PhCH_2CH=O,

(C) HOCH_2-C(=O)-$(CHOH)_3$-CH_2OH (D) CH_3-C(=O)-H

(E) 

All gives positive Fehling test.

Quick Tip

Fehling's test is a chemical test used to distinguish between aldehydes and ketones. Aldehydes give a positive test, while most ketones do not. However, α-hydroxy ketones also give a positive test.

---

66. In which of the following complexes the CFSE, Δ_0 will be equal to zero?

(1) $[Fe(NH_3)_6]Br_2$

(2) $[Fe(en)_3]Cl_3$

(3) $K_4[Fe(CN)_6]$

(4) $K_3[Fe(SCN)_6]$

Correct Answer: (4) $K_3[Fe(SCN)_6]$

Solution: For complex $K_3[Fe(SCN)_6]$: E.C. of Fe^{3+} - [Ar] $3d^5$ W.F.L.

e_g^2

t_{2g}^3

Calculation of CFSE:

$$\text{CFSE} = (-0.4 \times 3 + 0.6 \times 2)\Delta_0 = 0\Delta_0$$

Thus, the CFSE is zero for $K_3[Fe(SCN)_6]$.

Quick Tip

To determine CFSE, consider the electronic configuration of the metal ion and use the crystal field theory. If the d-electron configuration is such that the electrons are evenly distributed between the e_g and t_2g orbitals, the CFSE will be zero.

—

67. Arrange the following solutions in order of their increasing boiling points.

(i) 10^{-4} M NaCl

(ii) 10^{-4} M Urea

(iii) 10^{-3} M NaCl

(iv) 10^{-2} M NaCl

(1) $(ii) < (i) < (iii) < (iv)$

(2) $(ii) < (i) \sim=$ (iii) $< (iv)$

(3) $(i) < (ii) < (iii) < (iv)$

(4) $(iv) < (iii) < (i) < (ii)$

Correct Answer: (1) $(ii) < (i) < (iii) < (iv)$

Solution:

$$\Delta T_b = i \cdot K_b \cdot m \cdot \infty \cdot C$$

Where C = concentration.

Options and their corresponding $i.C.$

$$(i)\ 2 \times 10^{-4} \quad (ii)\ 1 \times 10^{-4} \quad (iii)\ 2 \times 10^{-3} \quad (iv)\ 2 \times 10^{-2}$$

Boiling point order:

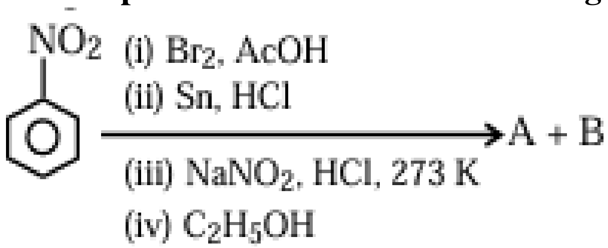
$(ii) < (i) < (iii) < (iv)$

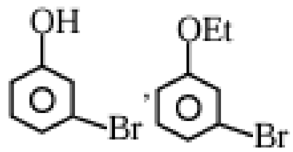
Quick Tip

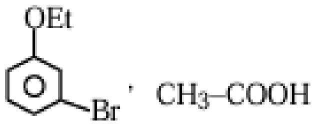
For colligative properties like boiling point elevation, the more the solute particles (ions), the higher the boiling point. Solutions with higher concentrations or higher ionization will have higher boiling points.

—

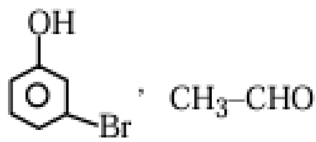
68. The products formed in the following reaction sequence are:

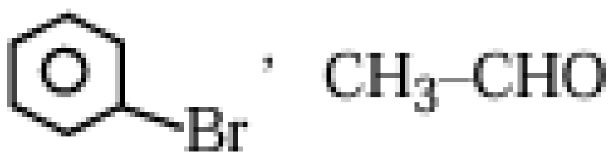


(1) 

(2) 

(3) 

(4) 



Correct Answer: (3)

Solution: The reaction sequence is as follows:

$$NO_2 \xrightarrow{\text{Br}_2\text{,AcOH}} NO_2 \quad \text{(Nitro compound)}$$

$$NO_2 \xrightarrow{\text{Sn, HCl}} NH_2 \quad \text{(Amine formation)}$$

$$NH_2 \xrightarrow{\text{NaNO}_2\text{,HCl,273}K} \text{Nitrosonium ion}$$

Nitrosonium ion reacts withethanol to form acetaldehyde.

Thus, the products formed are CH_3-CHO and Br at the end.

Quick Tip

In organic reactions, the reduction of nitro compounds by Sn/HCl forms amines, which can then undergo diazotization. This is a key step in forming various organic products, such as aldehydes and alcohols.

---

69. From the magnetic behaviour of $[NiCl_4]^{2-}$ (paramagnetic) and [Ni$(CO)_4$] (diagnetic), choose the correct geometry and oxidation state.

(1) $[NiCl_4]^{2-}$: Ni², square planar [Ni$(CO)_4$] : Ni(0), square planar

(2) $[NiCl_4]^{2-}$: Ni², tetrahedral [Ni$(CO)_4$] : Ni(0), tetrahedral

(3) $[NiCl_4]^{2-}$: Ni², tetrahedral [Ni$(CO)_4$] : Ni$^{2}+, squareplanar$

(4) $[NiCl_4]^{2-}$: Ni(0), tetrahedral [Ni$(CO)_4$] : Ni(0), square planar

Correct Answer: (2) $[NiCl_4]^{2-}$: Ni², tetrahedral [Ni$(CO)_4$] : Ni(0), tetrahedral

Solution: $[NiCl_4]^{2-}$

Ni² - [Ar] 3d 4s $\rightarrow$ sp³, Tetrahedral

Number of unpaired electron = 2 paramagnetic

Ni$(CO)_4$

Ni(0) $\rightarrow$ [Ar] 3d¹ 4s (After rearrangement)

No unpaired electron

sp^3, $Tetrahedral, Diamagnetic$

Quick Tip

Remember the spectrochemical series and how to determine the hybridization and geometry of coordination complexes. Also, paramagnetic complexes have unpaired electrons, while diamagnetic complexes do not.

---

70. The incorrect statements regarding geometrical isomerism are:

(A) Propene shows geometrical isomerism.

(B) Trans isomer has identical atoms/groups on the opposite sides of the double bond.

(C) Cis-but-2-ene has higher dipole moment than trans-but-2-ene.

(D) 2-methylbut-2-ene shows two geometrical isomers.

(E) Trans-isomer has lower melting point than cis isomer.

(1) (A), (D) and (E) only

(2) (C), (D) and (E) only

(3) (B) and (C) only

(4) (A) and (E) only

Correct Answer: (1) (A), (D) and (E) only

Solution: (A) Propene does not show geometrical isomerism because it has no substituents on both sides of the double bond.

(B) Trans isomer has identical atoms/groups on opposite sides of the double bond.

(C) Cis-but-2-ene has a higher dipole moment due to the presence of polar groups on the same side, unlike the trans isomer.

(D) 2-methylbut-2-ene does not show geometrical isomerism because it does not have different groups on either side of the double bond.

(E) Trans-isomer generally has a lower melting point due to its more symmetrical and compact structure compared to the cis-isomer.

Thus, the correct answer is (A), (D) and (E) only.

Quick Tip

For geometrical isomerism to occur, different groups must be attached to the carbon atoms involved in the double bond.

---

71. Some CO_2 gas was kept in a sealed container at a pressure of 1 atm and at 273 K. This entire amount of CO_2 gas was later passed through an aqueous solution of $Ca(OH)_2$. The excess unreacted $Ca(OH)_2$ was later neutralized with 0.1 M of 40 mL HCl. If the volume of the sealed container of CO_2 was x, then x is ____ cm^3 (nearest integer). [Given: The entire amount of CO_2 reacted with exactly half the initial amount of $Ca(OH)_2$ present in the aqueous solution.]

Correct Answer: (45)

Solution: Let the moles of CO_2 be n and moles of $Ca(OH)_2$ total initially be $2n$. The excess $Ca(OH)_2$ is n. The gm equivalent of $Ca(OH)_2$ is equal to the gm equivalent of HCl. Using the molarity and volume of HCl, we calculate:

$$n \times 2 = 0.1 \times \frac{40}{1000}$$

$$n = 2 \times 10^{-3}$$

Now, the volume of CO_2 is:

$$2 \times 10^{-3} \times 22400 = 44.8\,\text{cm}^3$$

Thus, the volume of CO_2 is approximately 45 cm^3.

Quick Tip

When dealing with gas volume at standard conditions, use the molar volume of gas at STP, which is 22400 cm^3 per mole.

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72. In Carius method for estimation of halogens, 180 mg of an organic compound produced 143.5 mg of AgCl. The percentage composition of chlorine in the compound is ______ %. [Given: Molar mass in g mol^{-1} of Ag = 108, Cl = 35.5]

Correct Answer: (20)

Solution: The moles of Cl in AgCl are:

$$n_{\text{Cl}} = n_{\text{AgCl}} = \frac{143.5 \times 10^{-3}}{143.5} = 10^{-3}$$

The percentage composition of Cl is:

$$\%\text{Cl} = \frac{10^{-3} \times 35.5}{180 \times 10^{-3}} \times 100 = 19.72\%$$

Thus, the percentage composition of chlorine is approximately 20%.

Quick Tip

To calculate the percentage composition of an element, divide the mass of the element by the total mass of the compound and multiply by 100.

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73. The number of molecules/ions that show linear geometry among the following is ______.

SO, BeCl, CO, N, NO, FO, XeF, NO, I, O

(Fill in the blank)

Correct Answer: (6)

Solution: To determine the geometry of each molecule/ion, we need to consider the Lewis structure, the number of electron domains (bonding pairs and lone pairs) around the central atom, and the VSEPR theory.

1. SO: The central sulfur atom has one lone pair and two bonding pairs, leading to a bent or V-shaped geometry.
2. BeCl: The central beryllium atom has two bonding pairs and no lone pairs, resulting in a linear geometry.
3. CO: The central carbon atom has two double bonds and no lone pairs, resulting in a linear geometry.
4. N: The central nitrogen atom has two double bonds and no lone pairs, resulting in a linear geometry.
5. NO: The central nitrogen atom has one lone pair and two bonding pairs, leading to a bent geometry.
6. FO: The central oxygen atom has two lone pairs and two bonding pairs, resulting in a bent geometry.
7. XeF: The central xenon atom has two bonding pairs and three lone pairs, resulting in a linear geometry.
8. NO: The central nitrogen atom has two double bonds and no lone pairs, resulting in a linear geometry.
9. I: The central iodine atom has two bonding pairs and three lone pairs, resulting in a linear geometry.
10. O: The central oxygen atom has one lone pair and two bonding pairs, leading to a bent geometry.

The molecules/ions with linear geometry are: $BeCl_2$

CO_2

N_3

XeF_2

$NO_2{}^+$

I_3^-

Thus, there are 6 molecules/ions with linear geometry.

Quick Tip

Remember the hybridization and geometry of molecules and ions. Linear geometry occurs when the central atom has sp hybridization or sp³d hybridization with lone pairs in equatorial positions.

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74. The molecule A changes into its isomeric form B by following a first order kinetics at a temperature of 1000 K. If the energy barrier with respect to reactant energy for such isomeric transformation is 191.48 kJ mol^{-1} and the frequency factor is 10^{20}, the time required for 50% molecules of A to become B is ______ picoseconds (nearest integer). [R = 8.314 J K^{-1} mol^{-1}]

(1) 69

(2) 61

(3) 79

(4) 71

Correct Answer: (1) 69

Solution: For a first-order reaction, the time required for half-life is given by the equation:

$$t_{1/2} = \frac{0.693}{K}$$

where K is the rate constant, which can be written as:

$$K = Ae^{-\frac{E_a}{RT}}$$

Substituting the given values:

$$K = 10^{20} \times e^{-\frac{191.48\times10^3}{8.314\times1000}} = 10^{20} \times e^{-23.031} = 10^{20} \times e^{-\ln(10^{10})} = 10^{20} \times 10^{-10} = 10^{10}\,\text{sec}^{-1}$$

Thus, the half-life becomes:

$$t_{1/2} = \frac{0.693}{10^{10}} = 6.93 \times 10^{-11}\,\text{sec} = 69.3 \times 10^{-12}\,\text{sec}$$

$$\Rightarrow t_{1/2} = 69\,\text{picoseconds}$$

Quick Tip

To solve first-order rate constant problems, use the Arrhenius equation to calculate the rate constant K and then apply it to find the half-life.

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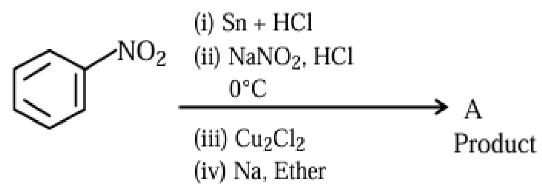
75. Consider the following sequence of reactions :



Molar mass of the product formed (A) is ______ g mol^{-1}.

(1) 154

(2) 144

(3) 130

(4) 160

Correct Answer: (1) 154

Solution: Step-by-step reaction mechanism:

$$NO_2 \xrightarrow{\text{Sn + HCl}} NO_2 + H_2O$$

$$NH_2 \xrightarrow{\text{NaNO}_2\text{,HCl}} NaNO_2 + HCl \quad (0^\circ C)$$

$$Cu_2Cl_2 \xrightarrow{\text{Na, Ether}} \text{Product formed}$$

The molar mass of the product formed is 154 g mol^{-1}

Quick Tip

In organic reaction sequences, focus on the transformations involving functional group changes and their effect on molecular mass.
