JEE Main 2023 25 Jan Shift 2 Physics Question Paper with Solutions

Time Allowed :60 minutes	**Maximum Marks :**100	**Total questions :**30

General Instructions

Read the following instructions very carefully and strictly follow them:

1. The test is of 3 hours duration.
2. The question paper consists of 90 questions. The maximum marks are 300.
3. There are three parts in the question paper consisting of Physics, Chemistry and Mathematics having 30 questions in each part of equal weightage.
4. Each part (subject) has two sections.

   (i) Section-A: This section contains 20 multiple choice questions which have only one correct answer. Each question carries 4 marks for correct answer and –1 mark for wrong answer.

   (ii) Section-B: This section contains 10 questions. In Section-B, attempt any five questions out of 10. The answer to each of the questions is a numerical value. Each question carries 4 marks for correct answer and –1 mark for wrong answer. For Section-B, the answer should be rounded off to the nearest integer.

SECTION-A

1. Match List I with List II:

List I	List II
A. Young's Modulus (Y)	I. $[ML^{-1}T^{-1}]$
B. Co-efficient of Viscosity (η)	II. $[ML^2T^{-1}]$
C. Planck's Constant (h)	III. $[ML^{-1}T^{-2}]$
D. Work Function (Φ)	IV. $[ML^2T^{-2}]$

Choose the correct answer from the options given below:

(1) A-II, B-III, C-IV, D-I

(2) A-III, B-I, C-II, D-IV

(3) A-I, B-III, C-IV, D-II

(4) A-I, B-II, C-III, D-IV

Correct Answer: (2) A-III, B-I, C-II, D-IV.

Solution:

Young's Modulus (Y):

$$Y = \frac{\text{Stress}}{\text{Strain}} = \frac{(F/A)}{(\Delta L/L)} = \frac{[MLT^{-2}]}{[L^2]} = [ML^{-1}T^{-2}].$$

Co-efficient of Viscosity (η):

Applying $F = 6\pi\eta rv$, we have:

$$\eta = \frac{F}{6\pi rv}, \quad [\eta] = \frac{[MLT^{-2}]}{[L][LT^{-1}]} = [ML^{-1}T^{-1}].$$

Planck's Constant (h):

Through $E = h\nu$,

we have:

$$h = \frac{E}{\nu}, \quad [h] = \frac{[ML^2T^{-2}]}{[T^{-1}]} = [ML^2T^{-1}].$$

Work Function (Φ):

Work function has same dimension as that of energy, so

$$[\Phi] = [ML^2T^{-2}].$$

Quick Tip

Determine dimensions from base equations or physical concepts (such as force, energy, etc.) while tackling dimensional analysis problems.

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2. According to the law of equipartition of energy, the molar specific heat of a diatomic gas at constant volume where the molecule has one additional vibrational mode is:

(1) $\frac{9}{2}R$

(2) $\frac{5}{2}R$

(3) $\frac{3}{2}R$

(4) $\frac{7}{2}R$

Correct Answer: (4) $\frac{7}{2}R$.

Solution:

Diatomic gas molecules possess two rotational and three translational degrees of freedom. The molecule adds two more degrees of freedom that correspond to one vibrational mode as it is known that it has one vibrational mode.

Thus, The total degrees of freedom are as follows:

$$f = 3(\text{translational}) + 2(\text{rotational}) + 2(\text{vibrational}) = 7.$$

Applying the formula for molar specific heat at constant volume:

$$C_v = \frac{f}{2}R = \frac{7}{2}R.$$

Final Answer:

$$\boxed{\frac{7}{2}R}$$

Quick Tip

Keep in mind that vibrational modes offer two extra degrees of freedom each mode when dealing with degrees of freedom difficulties.

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3. The light rays from an object have been reflected towards an observer from a standard flat mirror. The image observed by the observer is:

A. Real

B. Erect

C. Smaller in size than the object

D. Laterally inverted

Choose the most appropriate answer from the options given below:

(1) B and D only

(2) B and C only

(3) A and D only

(4) A, C, and D only

Correct Answer: (1) B and D only.

Solution:

An upright, identically sized, laterally inverted, and virtual picture of an actual object is created by a plane mirror. As a result, the image's proper attributes are:

$$\text{Erect (B) and Laterally Inverted (D).}$$

Final Answer:

$$\boxed{\text{B and D only}}$$

Quick Tip

Plane mirrors always produce virtual, erect, same-sized, and laterally inverted images.

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4. For a moving coil galvanometer, the deflection in the coil is 0.05 rad when a current of 10 mA is passed through it. If the torsional constant of the suspension wire is 4.0×10^{-5} Nm/rad, the magnetic field is 0.01 T, and the number of turns in the coil is 200, the area of each turn (in cm^2) is:

(1) 2.0

(2) 1.0

(3) 1.5

(4) 0.5

Correct Answer: (2) 1.0.

Solution:

The following provides the torque operating on the coil:

$$\tau = K\theta.$$

The magnetic torque is:

$$\tau = NiAB.$$

Comparing the two:

$$NiAB = K\theta.$$

Rearranging for A:

$$A = \frac{K\theta}{NiB}.$$

Substituting the given values:

$$A = \frac{(4 \times 10^{-5}) \cdot (0.05)}{200 \cdot (10 \times 10^{-3}) \cdot (0.01)}.$$

Simplifying:

$$A = 10^{-4}\,\text{m}^2 = 1\,\text{cm}^2.$$

Final Answer:

$$\boxed{1.0\,\text{cm}^2}$$

Quick Tip

Use the torque balance equation and carefully replace the provided values for galvanometer-related problems.

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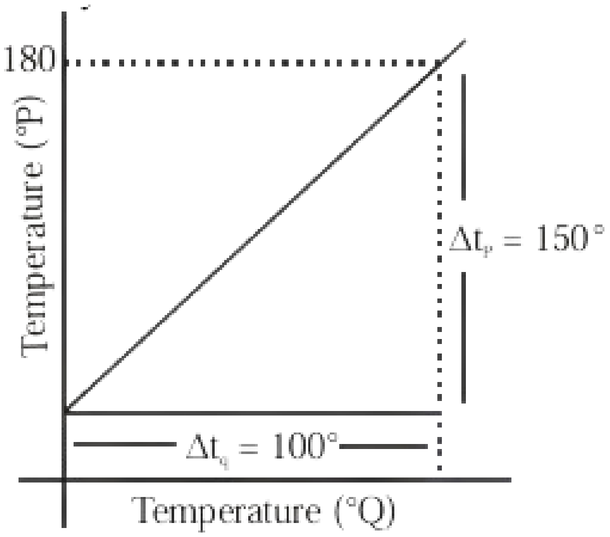
5. The graph between two temperature scales P and Q is shown in the figure. Between the upper fixed point and lower fixed point, there are 150 equal divisions of scale P and 100 divisions on scale Q. The relationship for conversion between the two scales is given by:



(1) $t_q/150 = \frac{t_p - 180}{100}$

(2) $t_q/100 = \frac{t_p - 30}{150}$

(3) $t_p/180 = \frac{t_Q-40}{100}$

(4) $t_p/100 = \frac{t_Q-180}{150}$

Correct Answer: (2).

Solution:

The following describes how temperature scales P and Q relate to one another:

$$\text{Reading on scale - Lower fixed point} = \frac{\text{constant}}{\text{Upper fixed point - Lower fixed point}}.$$

For P and Q, the relationship is:

$$\frac{t_p - 30}{180 - 30} = \frac{t_Q - 30}{100}.$$

Simplifying:

$$t_q = \frac{t_p - 30}{150} \times 100$$

Final Answer:

$$\boxed{t_q = \frac{t_p - 30}{150} \times 100}$$

Quick Tip

For precise computations when using temperature conversion scales, make use of the proportional relationship between scale intervals.

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6. Match List I with List II:

A.	Gauss's Law in Electrostatics	I.	$\oint \vec{E}.d\vec{l} = -\frac{d\phi_B}{dt}$
B.	Faraday's Law	II.	$\oint \vec{B}.d\vec{A} = 0$
C.	Gauss's Law in Magnetism	III.	$\oint \vec{B}.d\vec{l} = \mu_0 i_c + \mu_0 \epsilon_0 \frac{d\phi_E}{dt}$
D.	Ampere-Maxwell Law	IV.	$\oint \vec{E}.d\vec{s} = \frac{q}{\epsilon_0}$

Choose the correct answer from the options given below:

(1) A-IV, B-I, C-II, D-III

(2) A-I, B-II, C-III, D-IV

(3) A-III, B-IV, C-I, D-II

(4) A-II, B-III, C-IV, D-I

Correct Answer: (1). A-IV, B-I, C-II, D-III

Solution:

Applying the definitions of the laws:

- **Gauss's Law in Electrostatics:** $\oint \mathbf{E} \cdot d\mathbf{s} = \frac{q}{\epsilon_0}$.
- **Faraday's Law:** $\oint \mathbf{E} \cdot d\mathbf{l} = -\frac{d\Phi_B}{dt}$.
- **Gauss's Law in Magnetism:** $\oint \mathbf{B} \cdot d\mathbf{A} = 0$.
- **Ampere-Maxwell Law:** $\oint \mathbf{B} \cdot d\mathbf{l} = \mu_0 i_c + \mu_0 \epsilon_0 \frac{d\Phi_E}{dt}$.

Final Answer: A-IV, B-I, C-II, D-III

Quick Tip

Learn how to rapidly link Maxwell's equations with the related physical laws by becoming familiar with their integral versions.

7. Statement I: When a Si sample is doped with Boron, it becomes P type and when doped by Arsenic it becomes N-type semi conductor such that P-type has excess holes and N-type has excess electrons.

Statement II: When such P-type and N-type semi-conductors, are fused to make a junction, a current will automatically flow which can be detected with an externally connected ammeter.

In the light of above statements, choose the most appropriate answer from the options given below.

(1) Both Statement I and statement II are incorrect

(2) Statement I is incorrect but statement II is correct.

(3) Both Statement I and statement II are correct

(4) Statement I is correct but statement II is incorrect

Correct Answer: (4). Statement I is correct but statement II is incorrect

Solution:

When a P-N junction is formed:

- The electric field formed at the junction due to the diffusion of charge carriers prevents the flow of majority carriers across the junction.
- The current will not flow unless an external voltage is applied to overcome the potential barrier. This is a characteristic feature of the P-N junction under equilibrium conditions.

Final Answer:

$$\boxed{\text{Statement I is correct; Statement II is incorrect.}}$$

Quick Tip

For semiconductor problems, distinguish between intrinsic and external effects (e.g., barrier potential vs. applied voltage).

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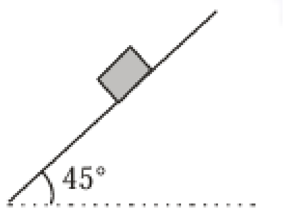
8. Consider a block kept on an inclined plane (inclined at 45°) as shown in the figure. If the force required to just push it up the incline is 2 times the force required to just prevent it from sliding down, the coefficient of friction between the block and inclined plane (μ) is equal to



(1) 0.33

(2) 0.60

(3) 0.25

(4) 0.50

Correct Answer: (1). 0.33

Solution:

Equilibrium equations:

$F_1 = mg\sin 45^\circ + f = mg\sin 45^\circ + \mu N$

$F_1 = \frac{mg}{\sqrt{2}} + \mu mg\cos 45^\circ$

$F_1 = \frac{mg}{\sqrt{2}}(1+\mu)$

$F_2 = mg\sin 45^\circ - f = mg\sin 45^\circ - \mu N$

$= \frac{mg}{\sqrt{2}}(1-\mu)$

$F_1 = 2F_2$

$\frac{mg}{\sqrt{2}}(1+\mu) = 2\frac{mg}{\sqrt{2}}(1-\mu)$

$1+\mu = 2-2\mu$

Final Answer:

$$\boxed{0.33}$$

Quick Tip

In inclined plane problems, analyze forces parallel and perpendicular to the plane, and apply given ratios to find unknowns.

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9. A point charge of 10 µC is placed at the origin. At what location on the X-axis should a point charge of 40µC be placed so that the net electric field is zero at x = 2 cm on the X-axis ?

(1) $x = 6\,\text{cm}$

(2) $x = 4\,\text{cm}$

(3) $x = 8\,\text{cm}$

(4) $x = -4\,\text{cm}$

Correct Answer: (1). $x = 6\,\text{cm}$

Solution:

The net electric field at $x_0 = 2$ cm is:

$$E_P = \frac{K\cdot 10}{2^2} - \frac{K\cdot 40}{(x_0-2)^2}$$

.

Simplifying:

$$\frac{1}{2^2} = \frac{4}{(x_0 - 2)^2}$$

.

Solve for x:

$$x_0 - 2 = 4, \quad x = 6\,\text{cm}.$$

Final Answer: 6 cm

Quick Tip

In electric field problems, equate the magnitudes of opposing fields to find the point of zero net field.

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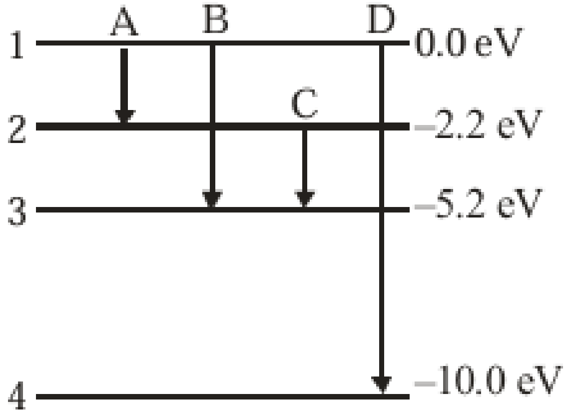
10. The energy levels of an atom is shown in figure. Which one of these transitions will result in the emission of a photon of wavelength 124.1 nm?

Given (h = 6.626×10^{-34}**) JS**



(1) B

(2) A

(3) C

(4) D

Correct Answer: (4) D.

Solution:

Each transition's energy difference is associated with a particular light wavelength. One can compute the wavelength λ of the photon that was released. Using the formula:

$$\lambda = \frac{hc}{\Delta E}$$

where h is Planck's constant,

c is the speed of light,

and ΔE is the energy difference.

Calculating ΔE for each transition:

$\Delta E_A = 2.2$ eV

$\Delta E_B = 5.2$ eV

$\Delta E_C = 3$ eV

$\Delta E_D = 10$ eV

$\lambda_A = \frac{6.62\times10^{-34}\times3\times10^{8}}{2.2\times1.6\times10^{-19}}$

$= \frac{12.41\times10^{-7}}{2.2}$ m

$= \frac{1241}{2.2}$ nm $= 564$ nm

$\lambda_B = \frac{1241}{5.2}$ nm $= 238.65$ nm

$\lambda_C = \frac{1241}{3}$ nm $= 413.66$ nm

$\lambda_D = \frac{1241}{10} = 124.1$ nm

This matches the given wavelength for transition D.

Quick Tip

When calculating photon emissions from energy transitions, ensure that the energy differences are correctly calculated and converted into corresponding wavelengths Applying Planck's equation.

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11. A particle executes simple harmonic motion between $x = -A$ and $x = A$. If the time taken by the particle to go from $x = 0$ to $x = A$ is 2 s, then the time taken by particle in going from $x = -A$ to $x = A/2$ is:

(1) 3 s

(2) $2\,\mathrm{s}$

(3) $1.5\,\mathrm{s}$

(4) $4\,\mathrm{s}$

Correct Answer: (4) $4\,\mathrm{s}$.

Solution:

Let's consider a particle undergoing simple harmonic motion (SHM).

Understanding the Problem:

We are given that the time taken for the particle to travel from its equilibrium position (0) to half of its amplitude (A/2) is denoted as t_1. The time taken for the particle to travel from A/2 to the full amplitude (A) is denoted as t_2.

Key Concepts:

SHM Equation: The displacement of a particle in SHM is often described by the equation $x(t) = A\sin(\omega t)$, where:

$x(t)$ is the displacement at time t

A is the amplitude

ω is the angular frequency

Derivation:

1. Time to reach A/2:

When the particle is at A/2, we have:

$$\frac{A}{2} = A\sin(\omega t_1)$$

$$\frac{1}{2} = \sin(\omega t_1)$$

$$\omega t_1 = \sin^{-1}(\frac{1}{2}) = \frac{\pi}{6}$$

2. Time to reach A:

When the particle is at A, we have:

$$A = A\sin(\omega(t_1 + t_2))$$

$$1 = \sin(\omega(t_1 + t_2))$$

$$\omega(t_1 + t_2) = \sin^{-1}(1) = \frac{\pi}{2}$$

We also know that the time to reach A/2 from 0 is $\omega t_1 = \frac{\pi}{6}$.

The time to reach A from A/2 can be written as:

$$\omega t_2 = \omega(t_1 + t_2) - \omega t_1 = \frac{\pi}{2} - \frac{\pi}{6} = \frac{\pi}{3}$$

3. Ratio of Times:

We can find the ratio of t_1 to t_2:

$$\frac{\omega t_1}{\omega t_2} = \frac{\pi/6}{\pi/3} = \frac{1}{2}$$

$$\frac{t_1}{t_2} = \frac{1}{2}$$

4. Finding t2:

We are given that $t_1 = 2$ seconds.

Using the ratio, we can find t_2:

$$t_2 = 2t_1 = 2 \times 2 = 4 \text{ seconds}$$

Conclusion:

The time taken for the particle to travel from A/2 to A is 4 seconds.

Quick Tip

In simple harmonic motion, the period and fractional parts of the motion are proportional, facilitating calculations for partial oscillations.

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12. Match List I with List II:

List I	List II
A. Isothermal Process	I. Work done by the gas decreases internal energy
B. Adiabatic Process	II. No change in internal energy
C. Isochoric Process	III. The heat absorbed goes partly to increase internal energy and partly to do work
D. Isobaric Process	IV. No work is done on or by the gas

Choose the correct answer from the options given below :

(1) $A-II, B-I, C-III, D-IV$

(2) $A-II, B-I, C-IV, D-III$

(3) $A-I, B-II, C-IV, D-III$

(4) $A-I, B-II, C-III, D-IV$

Correct Answer: (2) $A-II, B-I, C-IV, D-III$.

Solution:

For each thermodynamic process, we match the correct description based on the laws of thermodynamics:

- **Isothermal process:** In an isothermal process, the temperature remains constant, and thus the change in internal energy (ΔU) is zero. Therefore, the correct match is $A \to II$.
- **Adiabatic process:** In an adiabatic process, no heat is exchanged ($Q = 0$). The work done during the process results in a change in internal energy, which gives the match $B \to I$.
- **Isochoric process:** In an isochoric process, the volume remains constant, meaning no work is done ($\Delta W = 0$). The energy change is only due to heat, so the match is $C \to IV$.
- **Isobaric process:** In an isobaric process, the pressure remains constant. Heat added to the system is used both to do work and to increase internal energy, resulting in $D \to III$.

Quick Tip

Understanding the fundamental properties of thermodynamic processes is crucial in correctly associating them with their effects on internal energy, work, and heat exchange.

13. Match List I with List II:

List I	List II
A. Troposphere	I. Approximate 65-75 km over Earth's surface
B. E-Part of Stratosphere	II. Approximate 300 km over Earth's surface
C. F_2-Part of Thermosphere	III. Approximate 10 km over Earth's surface
D. D-Part of Stratosphere	IV. Approximate 100 km over Earth's surface

Choose the correct answer from the options given below :

(1) $A - III, B - IV, C - II, D - I$

(2) $A - I, B - II, C - IV, D - III$

(3) $A - I, B - IV, C - III, D - II$

(4) $A - III, B - II, C - I, D - IV$

Correct Answer: (1) $A - III, B - IV, C - II, D - I$.

Solution:

For each layer of Earth's atmosphere, the typical altitudes are as follows:

- **Troposphere:** The troposphere extends up to approximately 10 km, so the correct match is $A \to III$.
- **E-Part of Stratosphere:** This part of the stratosphere reaches up to about 50 km. Considering the general range, the match is $B \to IV$.
- **F_2-Part of Thermosphere:** The F_2-layer of the thermosphere can extend as high as 300 km, resulting in the match $C \to II$.
- **D-Part of Stratosphere:** The D-part of the stratosphere reaches about 50 km, so the match is $D \to I$.

Quick Tip

Familiarity with atmospheric layers and their characteristics, such as average altitudes, is essential in atmospheric science and related fields for accurate description and study.

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14. A body of mass m is taken from earth surface to the height equal to twice the radius of earth (R_e), the increase in potential energy will be:

(g = acceleration due to gravity on the surface of Earth)

(1) $3mgR_e$

(2) $\frac{1}{3}mgR_e$

(3) $\frac{2}{3}mgR_e$

(4) $\frac{1}{2}mgR_e$

Correct Answer: (3) $\frac{2}{3}mgR_e$.

Solution:

The potential energy at a distance R from the center of the Earth is given by:

$$U = -\frac{GM_e m}{R}$$

At the surface ($R = R_e$):

$$U_i = -\frac{GM_e m}{R_e}$$

At height $h = 2R_e$ above the surface ($R = 3R_e$):

$$U_f = -\frac{GM_e m}{3R_e}$$

The change in potential energy (ΔU) is:

$$\Delta U = U_f - U_i = \frac{2GM_e m R_e}{3R_e^2} = \frac{2}{3}mgR_e$$

Quick Tip

When calculating changes in gravitational potential energy, consider the total distance from the center of the Earth rather than just the altitude above the surface.

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15. A wire of length 1 m moving with velocity 8 m/s at right angles to a magnetic field of 2 T. The magnitude of induced emf between the ends of wire will be:

(1) 20 V

(2) 8 V

(3) 12 V

(4) 16 V

Correct Answer: (4) 16 V.

Solution:

The induced electromotive force (emf), denoted by ε, in a wire moving perpendicularly through a magnetic field is given by the formula:

$$\varepsilon = Bvl$$

where B represents the magnetic field strength, v is the velocity of the wire, and l is the length of the wire. Substituting the given values:

$$\varepsilon = 2\,\text{T} \times 8\,\text{m/s} \times 1\,\text{m} = 16\,\text{V}$$

Quick Tip

For a conductor moving in a magnetic field, remember that the induced emf is directly proportional to the magnetic field strength, the velocity of the conductor, and its length.

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16. The distance travelled by a particle is related to time t as $x = 4t^2$. The velocity of the particle at $t = 5$ s is:

(1) $40\,\text{m/s}$

(2) $25\,\text{m/s}$

(3) $20\,\text{m/s}$

(4) $8\,\text{m/s}$

Correct Answer: (1) $40\,\text{m/s}$.

Solution:

The velocity v of the particle is the derivative of the position with respect to time. Given the position function:

$$v = \frac{dx}{dt} = \frac{d}{dt}(4t^2) = 8t$$

At $t = 5$ seconds, the velocity is:

$$v = 8 \times 5 = 40\,\text{m/s}$$

Quick Tip

For motion described by a quadratic equation, velocity is linearly proportional to time, indicating constant acceleration.

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17. Two objects are projected with the same velocity u but at different angles α and β with the horizontal. If $\alpha + \beta = 90°$, the ratio of horizontal range of the first object to the 2nd object will be:

(1) $4 : 1$

(2) $2 : 1$

(3) $1 : 2$

(4) $1 : 1$

Correct Answer: (4) $1 : 1$.

Solution:

The range R for a projectile is given by:

$$R = \frac{u^2 \sin 2\theta}{g}$$

Range for projection angle "α"

$R_1 = \frac{u^2 \sin 2\alpha}{g}$

Range for projection angle "β"

$R_2 = \frac{u^2 \sin 2\beta}{g}$

$\alpha + \beta = 90°$ (Given)

$\Rightarrow \beta = 90° - \alpha$

$R_2 = \frac{u^2 \sin 2(90° - \alpha)}{g}$

$R_2 = \frac{u^2 \sin(180° - 2\alpha)}{g}$

$R_2 = \frac{u^2 \sin 2\alpha}{g}$

$\Rightarrow \frac{R_1}{R_2} = \frac{\left(\frac{u^2 \sin 2\alpha}{g}\right)}{\left(\frac{u^2 \sin 2\alpha}{g}\right)} = \frac{1}{1}$

Thus,

$$R_\alpha = R_\beta$$

The ratio $R_\alpha : R_\beta = 1 : 1$.

Quick Tip

In projectile motion, if two angles sum to 90°, their ranges are equal due to the symmetry in the sine function over the interval $[0^\circ, 180^\circ]$.

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18. The resistance of a wire is 5 Ω. If it's stretched to 5 times of its original length, its new resistance will be:

(1) $625\,\Omega$

(2) $5\,\Omega$

(3) $125\,\Omega$

(4) $25\,\Omega$

Correct Answer: (3) $125\,\Omega$.

Solution:

The resistance R of a wire is proportional to its length L and inversely proportional to its cross-sectional area A:

$$R = \rho \frac{L}{A}$$

When the wire is stretched to five times its original length, its new length is $5L$ and its new area A' is $\frac{A}{5}$ (assuming volume conservation):

$$V_i = V_f$$

A$_i l_i = A_f l_f$

Al = A'(5l)

A' = $\frac{A}{5}$

R$_f = \frac{\rho l_f}{A_f} = \frac{\rho(5l)}{\left(\frac{A}{5}\right)}$

= 25$\left(\frac{\rho l}{A}\right)$

= 25 $\times 5 = 125\Omega$

thus, the correct option is (3) $125\,\Omega$.

Quick Tip

Remember that stretching a wire affects both its length and cross-sectional area, impacting resistance significantly due to the squared factor in the new cross-sectional area.

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19. Given below are two statements:

Statement I: Stopping potential in photoelectric effect does not depend on the power of the light source.

Statement II: For a given metal, the maximum kinetic energy of the photoelectron depends on the wavelength of the incident light.

In the light of above statements, choose the most appropriate answer from the options given below.

(1) Statement I is incorrect but statement II is correct

(2) Both Statement I and Statement II are incorrect

(3) Statement I is correct but statement II is incorrect

(4) Both statement I and statement II are correct

Correct Answer: (4) Both statement I and statement II are correct.

Solution:

Statement I: True. The stopping potential in the photoelectric effect depends on the frequency (or wavelength) of the light, rather than its intensity or power.

Statement II: True. The maximum kinetic energy of the emitted photoelectrons is given by the equation:

$$KE_{\text{max}} = \frac{hc}{\lambda} - \phi$$

where λ is the wavelength of the incident light and ϕ is the work function of the metal.

Both statements are accurate and correctly reflect the principles of the photoelectric effect.

Quick Tip

In understanding the photoelectric effect, it's important to distinguish the roles of light intensity and frequency in the emission and energy of photoelectrons.

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20. Every planet revolves around the sun in an elliptical orbit:

Statements:

A. The force acting on a planet is inversely proportional to the square of the distance from the sun.

B. The force acting on a planet is inversely proportional to the product of the masses of the planet and the sun.

C. The centripetal force acting on the planet is directed away from the sun.

D. The square of the time period of the revolution of a planet around the sun is directly proportional to the cube of the semi-major axis of the elliptical orbit.

Options:

(1) A and D only

(2) C and D only

(3) B and C only

(4) A and C only

Correct Answer: (1) A and D only.

Solution:

Statement **A** is correct, as it accurately reflects Newton's law of universal gravitation, which states that the gravitational force between two masses is inversely proportional to the square of the distance between them.

Statement **D** is correct, as it expresses Kepler's third law, which links the square of a planet's orbital period to the cube of its semi-major axis in elliptical orbits.

Statement **B** is incorrect because the gravitational force is directly proportional to the product of the masses, not inversely.

Statement **C** is incorrect because the centripetal force always points towards the center of motion (the sun), not away from it.

Quick Tip

A solid understanding of Newton's laws and Kepler's laws is crucial for correctly interpreting gravitational interactions and orbital mechanics in astronomy.

Section B

21. A capacitor has a capacitance of 5 μF when its parallel plates are separated by an air medium of thickness d. A slab of material with a dielectric constant of 1.5, having an area equal to that of the plates but with thickness $d/2$, is inserted between the plates. The capacitance of the capacitor in the presence of the slab will be ___ μF :

Correct Answer: $6\,\mu F$

Solution:

When a dielectric partially fills the capacitor, the capacitance can be determined by modeling the system as two capacitors in series: one filled with the dielectric and one without. The total capacitance is then given by:

$$\begin{aligned} C_{\text{new}} &= \frac{\epsilon_0 A}{\frac{(\frac{d}{2})(\frac{d}{2})}{1.5} + \frac{(\frac{d}{2})}{1}} \\ &= \frac{\epsilon_0 A}{\frac{d}{3} + \frac{d}{2}} = \frac{6\epsilon_0 A}{5d} \\ &= \frac{6}{5} \times 5\,\mu F = 6\,\mu F \end{aligned}$$

Quick Tip

When dealing with capacitors partially filled with dielectrics, consider simplifying the system into a combination of series capacitors to calculate the total capacitance.

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22. A train blowing a whistle of frequency 320 Hz approaches an observer standing on the platform at a speed of 66 m/s. The frequency observed by the observer will be (given speed of sound = 330 m/s):

Correct Answer: $400\,\text{Hz}$

Solution:

The observed frequency (f') when a source approaches an observer can be determined using the Doppler effect formula:

$$f' = \frac{f}{1 - \frac{v_s}{v_{\text{sound}}}}$$

Substituting the given values:

$$f' = \frac{320\,\text{Hz}}{1 - \frac{66\,\text{m/s}}{330\,\text{m/s}}}$$
$$f' = 400\,\text{Hz}$$

Quick Tip

In Doppler effect problems, carefully consider the velocities of the source and observer relative to the medium, as the sign and magnitude of these velocities are key to determining the correct observed frequency.

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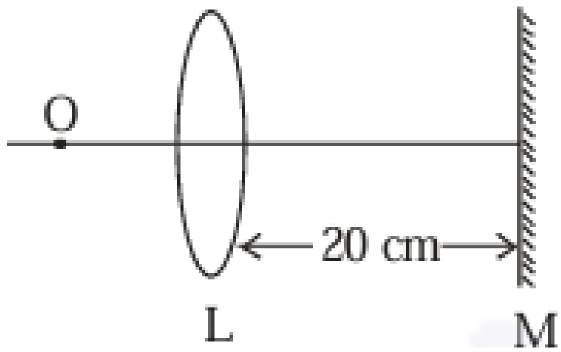
23. An object is placed on the principal axis of a convex lens of focal length 10 cm as shown. A plane mirror is placed on the other side of the lens at a distance of 20 cm. The image produced by the plane mirror is 5 cm inside the mirror. The distance of the object from the lens is:



Correct Answer: 30

Solution:

The system can be analyzed by considering the light passing through the lens, reflecting off the mirror, and then passing back through the lens. We apply the lens formula:

$$\frac{1}{f} = \frac{1}{v} - \frac{1}{u}$$

For the first pass through the lens:

$$\frac{1}{15} - \frac{1}{-u} = \frac{1}{10}$$
$$\Rightarrow \frac{1}{u} = \frac{1}{10} - \frac{1}{15}$$

$u = -30\,\text{cm}$ (The negative sign indicates that the object is on the same side as the incoming light)

To ensure the image formed by the mirror is 5 cm inside, we need to calculate the equivalent object distance after the reflection from the mirror, which results in a distance of 30 cm.

Quick Tip

When dealing with optical systems involving multiple reflections and lenses, analyze each segment of the light's path separately and apply the lens formula, while keeping track of the correct sign conventions.

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24. Two long parallel wires carrying currents 8A and 15A in opposite directions are placed at a distance of 7 cm from each other. A point P is at equidistant from both the wires such that the lines joining the point P to the wires are perpendicular to each other. The magnitude of magnetic field at P is ___ $\times 10^{-6}\,T$.

Correct Answer: $68 \times 10^{-6}\,T$.

Solution:

The magnetic field produced by each wire at point P can be calculated using the Biot-Savart Law:

$$B = \frac{\mu_0 I}{2\pi d}$$

For the 8A current:

$$B_1 = \frac{\mu_0 \times 8}{2\pi \times 7 \times 10^{-2}}$$

For the 15A current:

$$B_2 = \frac{\mu_0 \times 15}{2\pi \times 7 \times 10^{-2}}$$

Since the currents are flowing in opposite directions and point P is equidistant from both, the net magnetic field B_{net} is the vector sum of B_1 and B_2:

$$B_{\text{net}} = \sqrt{B_1^2 + B_2^2} \Rightarrow \frac{\mu_0}{2\pi d}\sqrt{i_1^2 + i_2^2}$$

$$\Rightarrow \frac{4\pi \times 10^{-7}}{2\pi \times \left(\frac{7}{\sqrt{2}}\right) \times 10^{-2}} \times \sqrt{8^2 + 15^2} \quad \left(d = \frac{7}{\sqrt{2}}\,\text{cm}\right)$$

Using the approximation $\sqrt{2} = 1.4$, we find that $B_{\text{net}} = 68 \times 10^{-6}$ T.

Quick Tip

When calculating the net magnetic field from multiple sources, ensure you account for both the magnitude and direction of the individual field components, particularly when currents flow in opposite directions.

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25. A spherical drop of liquid splits into 1000 identical spherical drops. If u_i is the surface energy of the original drop and u_f is the total surface energy of the resulting drops, the ratio $\frac{u_f}{u_i} = \frac{10}{x}$. Then value of x is ___:

Correct Answer: 10.

Solution:

The surface energy of a drop is directly proportional to its surface area. When the original drop is split into many smaller drops, the total surface area—and thus the total surface energy—increases.

Starting with the volume equation:

$$\frac{4}{3}\pi R^3 = 1000 \times \frac{4}{3}\pi r^3$$

$$R = 10r$$

The initial surface energy (u_i) is given by:

$$u_i = T \times 4\pi R^2$$

The final surface energy (u_f) is:

$$u_f = T \times 4\pi r^2 \times 1000$$

Now, we calculate the ratio of final to initial surface energy:

$$\frac{u_f}{u_i} = \frac{1000r^2}{R^2}$$

$$\frac{u_f}{u_i} = \frac{1000r^2}{(10r)^2} = 10$$

So, $x = 10$.

Quick Tip

Remember that when a large drop splits into smaller drops, the increase in surface area (and hence surface energy) demonstrates the non-linear relationship between volume and surface area for spherical objects.

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26. A body of mass 1 kg collides head-on with a stationary body of mass 3 kg. After the collision, the smaller body reverses its direction of motion and moves with a speed of 2 m/s. The initial speed of the smaller body before collision is:

Correct Answer: $4\,m/s$.

Solution:

By applying the principle of conservation of momentum and considering the reversal of direction and speed after the collision, we use the equation of motion:

$$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$$

Using the given masses and velocities, we solve for u_1, which is found to be 4 m/s in the opposite direction.

Quick Tip

When solving collision problems involving direction reversal, it is important to carefully track the sign and magnitude of the velocities while applying conservation of momentum.

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27. A nucleus disintegrates into two smaller parts, which have their velocities in the ratio 3:2. The ratio of their nuclear sizes will be $\left(\frac{x}{3}\right)^{\frac{1}{3}}$. The value of x is:

Correct Answer: 2.

Solution:

Given the velocity ratio and applying the conservation of momentum, the mass ratio is inversely proportional to the velocity ratio. We calculate the size ratio by cubing the mass ratio, as nuclear size is related to volume:

$$\frac{v_1}{v_2} = \frac{3}{2}$$

Using conservation of momentum:

$$m_1 v_1 = m_2 v_2 \Rightarrow \frac{m_1}{m_2} = \frac{2}{3}$$

Since nuclear mass density is constant, we have:

$$\frac{m_1}{\frac{4}{3}\pi r_1^3} = \frac{m_2}{\frac{4}{3}\pi r_2^3}$$

This simplifies to:

$$\left(\frac{r_1}{r_2}\right)^3 = \frac{m_1}{m_2}$$

Taking the cube root:

$$\frac{r_1}{r_2} = \left(\frac{2}{3}\right)^{\frac{1}{3}}$$

So this gives $x = 2$, indicating that the size ratio between the two fragments is 2.

Quick Tip

In problems involving disintegration or explosion, the conservation of momentum is crucial for relating the mass and velocity ratios of the resulting fragments. The size ratio can then be determined by considering the volumetric relationship.

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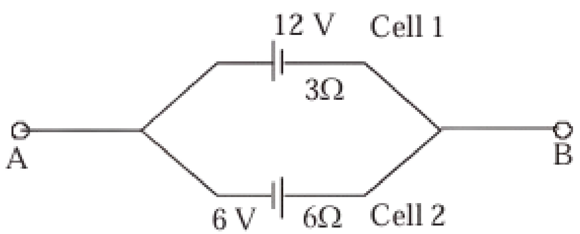
28. Two cells are connected between points A and B as shown. Cell 1 has an emf of 12 V and internal resistance of 3Ω. Cell 2 has an emf of 6 V and internal resistance of 6Ω. An external resistor of 4Ω is connected across A and B. The current flowing through R will be ___ A.



Correct Answer: $1\,A$.

Solution:

The equivalent circuit can be simplified by combining the electromotive forces (emfs) and resistances. First, we calculate the equivalent voltage and resistance, and then apply Ohm's law to find the current through the external resistor:

$$E_{\text{eq}} = \frac{\frac{12}{3} - \frac{6}{6}}{\frac{1}{3} + \frac{1}{6}}$$

$$E_{\text{eq}} = 6\,\text{V}$$

$$r_{\text{eq}} = 2\,\Omega$$

$$R = 4\,\Omega$$

Now, using Ohm's law, the current is calculated as:

$$i = \frac{6}{2+4}$$

Solving these gives the current as $1\,\text{A}$.

Quick Tip

In circuits with multiple sources and resistors, simplifying the circuit into an equivalent single-source circuit makes it much easier to compute the total current or voltage.

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29. A series LCR circuit is connected to an AC source of 220 V, 50 Hz. The circuit contains a resistance $R = 80\,\Omega$, an inductor of inductive reactance $X_L = 70\,\Omega$, and a capacitor of capacitive reactance $X_C = 130\,\Omega$. The power factor of the circuit is $\frac{x}{10}$. The value of x is:

Correct Answer: 8.

Solution:

The power factor $\cos\phi$ of an LCR circuit is given by the formula:

$$\cos\phi = \frac{R}{\sqrt{R^2 + (X_L - X_C)^2}}$$

Substituting the provided values:

$$\cos\phi = \frac{80}{\sqrt{80^2 + (70-130)^2}} = \frac{80}{\sqrt{80^2 + (-60)^2}} = \frac{80}{100} = 0.8$$

From this, $\frac{x}{10} = 0.8$, which gives $x = 8$.

Quick Tip

Understanding the phase relationship between the components of an LCR circuit is key to accurately calculating the power factor, which represents the phase difference between the voltage and current.

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30. If a solid sphere of mass 5 kg and a disc of mass 4 kg have the same radius. Then the ratio of the moment of inertia of the disc about a tangent in its plane to the moment of inertia of the sphere about its tangent will be $\frac{x}{7}$. The value of x is:

Correct Answer: 5.

Solution:

The moment of inertia I_1 of a solid sphere about a tangent to its surface is calculated as:

$$I_1 = I_{\text{CM}} + mR^2 = \frac{2}{5}mR^2 + mR^2 = \frac{7}{5}mR^2$$

For a sphere with mass $m = 5\,\text{kg}$, we get:

$$I_1 = 7R^2$$

The moment of inertia I_2 of the disc about a tangent in its plane is:

$$I_2 = \frac{m_2R^2}{4} + m_2R^2 = \frac{5}{4}m_2R^2$$

For the disc with mass $m = 4\,\text{kg}$, we get:

$$I_2 = 6R^2$$

The ratio $\frac{I_2}{I_1}$ is:

$$\frac{I_2}{I_1} = \frac{5R^2}{7R^2} = \frac{5}{7}$$

Thus, $x = 5$.

Quick Tip

When calculating the moment of inertia about an axis not through the center of mass, remember to apply the parallel axis theorem.