JEE Main 2023 8 April Shift 2 Mathematics Question Paper

Time Allowed :3 Hours	**Maximum Marks :**300	**Total s :**90

General Instructions

Read the following instructions very carefully and strictly follow them:

(A) The Duration of test is 3 Hours.

(B) This paper consists of 90 s.

(C) There are three parts in the paper consisting of Physics, Chemistry and Mathematics having 30 s in each part of equal weightage.

(D) Each part (subject) has two sections.

(i) Section-A: This section contains 20 multiple choice s which have only one correct answer. Each carries 4 marks for correct answer and –1 mark for wrong answer.

(E) (ii) Section-B: This section contains 10 s. In Section-B, attempt any five out of 10. The answer to each of the s is a numerical value. Each carries 4 marks for correct answer and –1 mark for wrong answer. For Section-B, the answer should be rounded off to the nearest integer.

1. Let $A = \left\{ \theta \in (0, 2\pi) : \frac{1+2i\sin\theta}{1-i\sin\theta} \text{ is purely imaginary} \right\}$. Then the sum of the elements in A is:

(1) π

(2) 3π

(3) 4π

(4) 2π

---

2. Let P be the plane passing through the line

$$\frac{x-1}{1} = \frac{y-2}{-3} = \frac{z+5}{7}$$

and the point $(2, 4, -3)$. If the image of the point $(-1, 3, 4)$ in the plane P is (α, β, γ), then $\alpha + \beta + \gamma$ is equal to:

(A) 12 (B) 9 (C) 10 (D) 11

---

3. If $A = \begin{bmatrix} 1 & 5 \\ \lambda & 10 \end{bmatrix}$, $A^{-1} = \alpha A + \beta I$, and $\alpha + \beta = -2$, then $4\alpha^2 + \beta^2 + \lambda^2$ is equal to:

(1) 14

(2) 12

(3) 19

(4) 10

4: The area of the quadrilateral $ABCD$ with vertices $A(2, 1, 1)$, $B(1, 2, 5)$, $C(-2, -3, 5)$, and $D(1, -6, -7)$ is equal to:

(1) 54

(2) $9\sqrt{38}$

(3) 48

(4) $8\sqrt{38}$

5: $25^{190} - 19^{190} - 8^{190} + 2^{190}$ is divisible by:

(1) 34 but not by 14

(2) 14 but not by 34

(3) Both 14 and 34

(4) Neither 14 nor 34

6: Let O be the origin and OP and OQ be the tangents to the circle

$$x^2 + y^2 - 6x + 4y + 8 = 0$$

at the points P and Q. If the circumcircle of the triangle OPQ passes through the point

$$\left(\alpha, \frac{1}{2}\right),$$

then the value of α is:

(1) $-\frac{1}{2}$

(2) $\frac{5}{2}$

(3) 1

(4) $\frac{3}{2}$

7: Let a_n be the n^{th} term of the series

$$5 + 8 + 14 + 23 + 35 + 50 + \dots$$

and $S_n = \sum_{k=1}^{n} a_k$. Then $S_{30} - a_{40}$ is equal to:

(1) 11260

(2) 11280

(3) 11290

(4) 11310

8: If $\alpha > \beta > 0$ are the roots of the equation $ax^2 + bx + 1 = 0$, and

$$\lim_{x \to \frac{1}{\alpha}} \left[\frac{1 - \cos(x^2 + bx + a)}{2(1 - ax)^2} \right]^{\frac{1}{2}} = \frac{1}{k} \left(\frac{1}{\beta} - \frac{1}{\alpha} \right),$$

then k is equal to:

(1) β

(2) 2α

(3) 2β

(4) α

9: If the number of words, with or without meaning, which can be made using all the letters of the word MATHEMATICS in which C and S do not come together, is $(6! \cdot k)$, k is equal to:

(1) 1890

(2) 945

(3) 2835

(4) 5670

10: Let S be the set of all values of $\theta \in [-\pi, \pi]$ for which the system of linear equations

$$x + y + \sqrt{3}z = 0,$$

$$-x + (\tan\theta)y + \sqrt{7}z = 0,$$

$$x + y + (\tan\theta)z = 0$$

has a non-trivial solution. Then $\frac{120}{\pi}\sum_{\theta \in S}\theta$ is equal to:

(1) 20

(2) 40

(3) 30

(4) 10

11: For $a, b \in \mathbb{Z}$ and $|a - b| \leq 10$, let the angle between the plane $P : ax + y - z = b$ and the line $L : x - 1 = a - y = z + 1$ be $\cos^{-1}\left(\frac{1}{3}\right)$. If the distance of the point $(6, -6, 4)$ from the plane P is $3\sqrt{6}$, then $a^4 + b^2$ is equal to:

(1) 85

(2) 48

(3) 25

(4) 32

12: Let the vectors $\mathbf{u}_1 = \hat{i} + \hat{j} + a\hat{k}, \mathbf{u}_2 = \hat{i} + b\hat{j} + \hat{k}$, and $\mathbf{u}_3 = c\hat{i} + \hat{j} + \hat{k}$ be coplanar. If the vectors $\mathbf{v}_1 = (a+b)\hat{i} + c\hat{j} + c\hat{k}, \mathbf{v}_2 = a\hat{i} + (b+c)\hat{j} + a\hat{k}, \mathbf{v}_3 = b\hat{i} + b\hat{j} + (c+a)\hat{k}$ are also coplanar, then $6(a+b+c)$ is equal to:

(1) 4

(2) 12

(3) 6

(4) 0

13: The absolute difference of the coefficients of x^{10} and x^7 in the expansion of $\left(2x^2 + \frac{1}{2x}\right)^{11}$ is equal to:

(1) $10^3 - 10$

(2) $11^3 - 11$

(3) $12^3 - 12$

(4) $13^3 - 13$

---

14. Let $A = \{1, 2, 3, 4, 5, 6, 7\}$. Then the relation $R = \{(x, y) \in A \times A : x + y = 7\}$ is:

(1) Symmetric but neither reflexive nor transitive

(2) Transitive but neither symmetric nor reflexive

(3) An equivalence relation

(4) Reflexive but neither symmetric nor transitive

---

15. If the probability that the random variable X takes values x is given by $P(X = x) = k(x+1)3^{-x}, x = 0, 1, 2, \ldots$, where k is a constant, then $P(X \geq 2)$ is equal to:

(1) $\frac{7}{27}$

(2) $\frac{11}{18}$

(3) $\frac{7}{18}$

(4) $\frac{20}{27}$

---

16. The integral $\int \left(\frac{x}{2}\right)^x + \left(\frac{2}{x}\right)^x \log x \, dx$ is equal to:

(1) $\left(\frac{x}{2}\right)^x \log_2 \left(\frac{2}{x}\right) + C$

(2) $\left(\frac{x}{2}\right)^x - \left(\frac{2}{x}\right)^x + C$

(3) $\left(\frac{x}{2}\right)^x \log_2 \left(\frac{x}{2}\right) + C$

(4) $\left(\frac{x}{2}\right)^x + \left(\frac{2}{x}\right)^x + C$

---

17. The value of $36\big(4\cos^2 9° - 1\big)\big(4\cos^2 27° - 1\big)\big(4\cos^2 81° - 1\big)\big(4\cos^2 243° - 1\big)$ is:

(1) 27

(2) 54

(3) 18

(4) 36

---

18. Let $A(0,1)$, $B(1,1)$, and $C(1,0)$ be the midpoints of the sides of a triangle with incentre at the point D. If the focus of the parabola $y^2 = 4ax$ passing through D is $(\alpha + \beta\sqrt{3}, 0)$, where α and β are rational numbers, then $\frac{\alpha}{\beta^2}$ is equal to:

(1) 6

(2) 8

(3) $\frac{9}{2}$

(4) 12

---

19. The negation of $(p \wedge (\sim q)) \vee (\sim p)$ is equivalent to:

(1) $p \wedge (\sim q)$

(2) $p \wedge (q \wedge (\sim p))$

(3) $p \vee (q \vee (\sim p))$

(4) $p \wedge q$

20. Let the mean and variance of 12 observations be $\frac{9}{2}$ and 4, respectively. Later on, it was observed that two observations were considered as 9 and 10 instead of 7 and 14 respectively. If the correct variance is $\frac{m}{n}$, where m and n are coprime, then $m+n$ is equal to:

(1) 316

(2) 317

(3) 315

(4) 314

---

21. Let $R = \{a, b, c, d, e\}$ and $S = \{1, 2, 3, 4\}$. Total number of onto functions $f : R \to S$ such that $f(a) \neq 1$, is equal to:

---

22. Let m and n be the numbers of real roots of the quadratic equations $x^2 - 12x + [x] + 31 = 0$ and $x^2 - 5|x+2| - 4 = 0$, respectively, where $[x]$ denotes the greatest integer less than or equal to x. Then $m^2 + mn + n^2$ is equal to ______.

---

23. Let P_1 be the plane $3x - y - 7z = 11$ and P_2 be the plane passing through the points $(2, -1, 0)$, $(2, 0, -1)$, and $(5, 1, 1)$. If the foot of the perpendicular drawn from the point $(7, 4, -1)$ on the line of intersection of the planes P_1 and P_2 is (α, β, γ), then $\alpha + \beta + \gamma$ is equal to _____.

---

24. If the domain of the function

$$f(x) = \log_e\left(\frac{6x^2 + 5x + 1}{2x - 1}\right) + \cos^{-1}\left(\frac{2x^2 - 3x + 4}{3x - 5}\right)$$

is $(\alpha, \beta) \cup (\gamma, \delta)$, then $18(\alpha^2 + \beta^2 + \gamma^2 + \delta^2)$ is equal to _____.

---

25. Let the area enclosed by the lines $x + y = 2$, $y = 0$, $x = 0$, and the curve $f(x) = \min\left\{x^2 + \frac{3}{4}, 1 + [x]\right\}$, where $[x]$ denotes the greatest integer less than or equal to x, be A. Then the value of $12A$ is _____.

26. Let $0 < z < y < x$ be three real numbers such that $\frac{1}{x}, \frac{1}{y}, \frac{1}{z}$ are in an arithmetic progression and $x, \sqrt{2}y, z$ are in a geometric progression. If $xy + yz + zx = \frac{3}{\sqrt{2}}xyz$, then $3(x + y + z)^2$ is equal to ______.

---

27. Let the solution curve $x = x(y)$, $0 < y \leq \frac{\pi}{2}$, of the differential equation

$$(\log_e(\cos y))^2 \cos y \, dx - (1 + 3x \log_e(\cos y)) \sin y \, dy = 0$$

satisfy $x\left(\frac{\pi}{3}\right) = \frac{1}{2\log_e 2}$. If $x\left(\frac{\pi}{6}\right) = \frac{1}{\log_e m - \log_e n}$, where m and n are co-prime integers, then mn is equal to

---

28. Let $[t]$ denote the greatest integer function. If

$$\int_0^{2.4} [x^2] dx = \alpha + \beta\sqrt{2} + \gamma\sqrt{3} + \delta\sqrt{5},$$

then $\alpha + \beta + \gamma + \delta$ is equal to ______.

---

29. The ordinates of the points P and Q on the parabola $y^2 = 12x$ are in the ratio $3 : 1$. If $R(\alpha, \beta)$ is the point of intersection of the tangents to the parabola at P and Q, then $\frac{\beta^2}{\alpha}$ is equal to _____.

---

30. Let k and m be positive real numbers such that the function

$$f(x) = \begin{cases} 3x^2 + \frac{k}{\sqrt{x+1}}, & 0 < x < 1, \\ mx^2 + k^2, & x \geq 1 \end{cases}$$

is differentiable for all $x > 0$. Then $8f'(8)\left(\frac{1}{f(8)}\right)$ is equal to ______.
