JEE Main 2023 April 11 Shift-2 Mathematics Question Paper

Time Allowed :3 Hours	Maximum Marks :300	Total Questions :90

General Instructions

Read the following instructions very carefully and strictly follow them:

1. The Duration of test is 3 Hours.
2. This paper consists of 90 Questions.
3. There are three parts in the paper consisting of Physics, Chemistry and Mathematics having 30 questions in each part of equal weightage..
4. Each part (subject) has two sections.
   (i) Section-A: This section contains 20 multiple choice questions which have only one correct answer. Each carries 4 marks for correct answer and –1 mark for wrong answer..
5. (ii) Section-B: This section contains 10 questions. In Section-B, attempt any five questions out of 10. The answer to each of the questions is a numerical value. Each carries 4 marks for correct answer and –1 mark for wrong answer. For Section-B, the answer should be rounded off to the nearest integer.

Mathematics

Section A

1: The angle of elevation of the top P of a tower from the feet of one person standing due South of the tower is $45°$ and from the feet of another person standing due West of the tower is $30°$. If the height of the tower is 5 meters, then the distance (in meters) between the two persons is equal to:

(1) 10

(2) $5\sqrt{5}$

(3) $\frac{5}{2}\sqrt{5}$

(4) 5

2: Let a, b, c, and d be positive real numbers such that $a+b+c+d=11$. If the maximum value of $a^5b^3c^2d$ is 3750β, then the value of β is:

(1) 55

(2) 108

(3) 90

(4) 110

3: If $f:\mathbb{R}\to\mathbb{R}$ is a continuous function satisfying

$$\int_0^{\frac{\pi}{2}} f(\sin 2x)\sin x\,dx + \alpha\int_0^{\frac{\pi}{4}} f(\cos 2x)\cos x\,dx = 0,$$

then the value of α is:

(1) $-\sqrt{3}$

(2) $\sqrt{3}$

(3) $-\sqrt{2}$

(4) $\sqrt{2}$

4: Let f and g be two functions defined by:

$$f(x) = \begin{cases} x+1, & x < 0 \\ |x-1|, & x \geq 0 \end{cases}, \quad g(x) = \begin{cases} x+1, & x < 0 \\ 1, & x \geq 0 \end{cases}.$$

Then $(g \circ f)(x)$ is:

(1) continuous everywhere but not differentiable at $x = 1$

(2) continuous everywhere but not differentiable exactly at one point

(3) differentiable everywhere

(4) not continuous at $x = -1$

5: If the radius of the largest circle with centre $(2, 0)$ inscribed in the ellipse $x^2 + 4y^2 = 36$ is r, then $12r^2$ is equal to:

(1) 69

(2) 72

(3) 115

(4) 92

6: Let the mean of 6 observations $1, 2, 4, 5, x,$ and y and their variance be 10. Then their mean deviation about the mean is equal to:

(1) $\frac{7}{3}$

(2) $\frac{10}{3}$

(3) $\frac{8}{3}$

(4) 3

7: Let $A = \{1, 3, 4, 6, 9\}$ and $B = \{2, 4, 5, 8, 10\}$. Let R be a relation defined on $A \times B$ such that $R = \{((a_1, b_1), (a_2, b_2)) : a_1 \leq b_2$ and $b_1 \leq a_2\}$. Then the number of elements in the set R is:

(1) 52

(2) 160

(3) 26

(4) 180

8: Let P be the plane passing through the points $(5,3,0)$, $(13,3,-2)$, and $(1,6,2)$. For $\alpha \in \mathbb{N}$, if the distances of the points $A(3,4,\alpha)$ and $B(2,\alpha,a)$ from the plane P are 2 and 3 respectively, then the positive value of a is:

(1) 5

(2) 6

(3) 4

(4) 3

9: If the letters of the word MATHS are permuted and all possible words so formed are arranged as in a dictionary with serial number, then the serial number of the word THAMS is:

(1) 102

(2) 103

(3) 101

(4) 104

10: If four distinct points with position vectors $\vec{a}, \vec{b}, \vec{c}, and\ \vec{d}$ are coplanar, then $[\vec{a}\vec{b}\vec{c}]$ is equal to:

(1) $[\vec{d}\vec{c}\vec{a}] + [\vec{b}\vec{d}\vec{a}] + [\vec{c}\vec{d}\vec{b}]$

(2) $[\vec{d}\vec{b}\vec{a}] + [\vec{a}\vec{c}\vec{d}] + [\vec{d}\vec{b}\vec{c}]$

(3) $[\vec{a}\vec{d}\vec{b}] + [\vec{d}\vec{c}\vec{a}] + [\vec{d}\vec{b}\vec{c}]$

(4) $[\vec{b}\vec{c}\vec{d}] + [\vec{d}\vec{a}\vec{c}] + [\vec{d}\vec{b}\vec{a}]$

11: The sum of the coefficients of three consecutive terms in the binomial expansion of $(1+x)^{n+2}$, which are in the ratio $1:3:5$, is equal to:

(1) 63

(2) 92

(3) 25

(4) 41

12: Let $y = y(x)$ be the solution of the differential equation $\frac{dy}{dx} + \frac{5}{x(x^5+1)}y = \frac{(x^5+1)^2}{x^7}$, $x > 0$. If $y(1) = 2$, then $y(2)$ is equal to:

(1) $\frac{693}{128}$

(2) $\frac{637}{128}$

(3) $\frac{697}{128}$

(4) $\frac{679}{128}$

13: The converse of $((\sim p) \wedge q) \Rightarrow r$ is:

(1) $(p \vee (\sim q)) \Rightarrow (\sim r)$

(2) $((\sim p) \vee q) \Rightarrow r$

(3) $(\sim r) \Rightarrow ((\sim p) \wedge q)$

(4) $(\sim r) \Rightarrow (p \wedge q)$

14: If the 1011th term from the end in the binomial expansion of $\left(\frac{4x}{5} - \frac{5}{2x}\right)^{2022}$ is 1024 times the 1011th term from the beginning, the $|x|$ is equal to:

(1) 8

(2) 12

(3) $\frac{5}{16}$

(4) 15

15: If the system of linear equations:

$$7x + 11y + \alpha z = 13, \quad 5x + 4y + 7z = \beta, \quad 175x + 194y + 57z = 361,$$

has infinitely many solutions, then $\alpha + \beta + 2$ is equal to:

(1) 3

(2) 6

(3) 5

(4) 4

16: Let the line passing through the point $P(2,-1,2)$ and $Q(5,3,4)$ meet the plane $x-y+z=4$ at the point T. Then the distance of the point R from the plane $x+2y+3z+2=0$, measured parallel to the line $\frac{x-7}{2}=\frac{y+3}{2}=\frac{z-2}{1}$, is equal to:

(1) 3

(2) $\sqrt{61}$

(3) $\sqrt{31}$

(4) $\sqrt{189}$

17: Let the function $f:[0,2]\to\mathbb{R}$ be defined as:

$$f(x)=\begin{cases} e^{\min\{x^2, x-\lfloor x\rfloor\}}, & x\in[0,1), \\ e^{x-\log_e x}, & x\in[1,2), \end{cases}$$

where $\lfloor t\rfloor$ denotes the greatest integer less than or equal to t.

Then the value of the integral $\int_0^2 x f(x)\,dx$ is:

(1) $(e-1)\left(e^2+\frac{1}{2}\right)$

(2) $1+\frac{3e}{2}$

(3) $2e-\frac{1}{2}$

(4) $2e-1$

18: For $a\in\mathbb{C}$, let:

$$A=\{z\in\mathbb{C}:\text{Re}(a+\overline{z})>\text{Im}(\overline{a}+z)\}, \textbf{and} \quad B=\{z\in\mathbb{C}:\text{Re}(a+\overline{z})<\text{Im}(\overline{a}+z)\}.$$

Among the two statements:

(S1): If $\text{Re}(a), \text{Im}(a)>0$, then the set A contains all the real numbers.

(S2): If $\text{Re}(a), \text{Im}(a) < 0$, then the set B contains all the real numbers.

(1) Only (S1) is true

(2) Both are false

(3) Only (S2) is true

(4) Both are true

19: If:

$$\begin{vmatrix} x+1 & x & x \\ x & x+\lambda & x \\ x & x & x+\lambda^2 \end{vmatrix} = \frac{9}{8}(103x+81),$$

then $\lambda, \frac{\lambda}{3}$ are the roots of the equation:

(1) $4x^2 - 24x - 27 = 0$

(2) $4x^2 + 24x + 27 = 0$

(3) $4x^2 - 24x + 27 = 0$

(4) $4x^2 + 24x - 27 = 0$

20: The domain of the function $f(x) = \frac{1}{\sqrt{\lfloor x \rfloor^2 - 3\lfloor x \rfloor - 10}}$, where $\lfloor x \rfloor$ denotes the greatest integer less than or equal to x, is:

(1) $(-\infty, -3] \cup [6, \infty)$

(2) $(-\infty, -2) \cup (5, \infty)$

(3) $(-\infty, -3] \cup (5, \infty)$

(4) $(-\infty, -2) \cup [6, \infty)$

Section B

21: If A is the area in the first quadrant enclosed by the curve $C : 2x^2 - y + 1 = 0$, the tangent to C at the point $(1, 3)$, and the line $x + y = 1$, then the value of $60A$ is _____:

22: Let $A = \{1,2,3,4,5\}$ and $B = \{1,2,3,4,5,6\}$. Then the number of functions $f : A \to B$ satisfying $f(1) + f(2) = f(4) - 1$ is equal to ____.

23: Let the tangent to the parabola $y^2 = 12x$ at the point $(3, \alpha)$ be perpendicular to the line $2x + 2y = 3$. Then the square of the distance of the point $(6, -4)$ from the normal to the hyperbola $\alpha^2 x^2 - 9y^2 = 9\alpha^2$ at its point $(\alpha - 1, \alpha + 2)$ is equal to _____.

24: For $k \in \mathbb{N}$, if the sum of the series $1 + \frac{4}{k} + \frac{8}{k^2} + \frac{13}{k^3} + \frac{19}{k^4} + \ldots$ is 10, then the value of k is _____.

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25. Let the line $\ell : x = \frac{1-y}{-2} = \frac{z-3}{\lambda}$, $\lambda \in \mathbb{R}$ meet the plane $P : x + 2y + 3z = 4$ at the point (α, β, γ). If the angle between the line ℓ and the plane P is $\cos^{-1}\left(\frac{\sqrt{5}}{\sqrt{14}}\right)$, then $\alpha + 2\beta + 6\gamma$ is equal to ____.

26: The number of points where the curve $f(x) = e^{8x} - e^{6x} - 3e^{4x} - e^{2x} + 1, x \in \mathbb{R}$ cuts the x-axis, is equal to ____.

27: If the line $l_1 : 3y - 2x = 3$ is the angular bisector of the line $l_2 : x - y + 1 = 0$ and $l_3 : \alpha x + \beta y + 17 = 0$, then $\alpha^2 + \beta^2 - \alpha - \beta$ is equal to _____.

28: Let the probability of getting a head for a biased coin be $\frac{1}{4}$. It is tossed repeatedly until a head appears. Let N be the number of tosses required. If the probability that the equation $64x^2 + 5Nx + 1 = 0$ has no real root is $\frac{p}{q}$, where p and q are co-prime, then $q - p$ is equal to ____.

29: Let $\vec{a} = \hat{i} + 2\hat{j} + 3\hat{k}$ and $\vec{b} = \hat{i} + \hat{j} - \hat{k}$. If $\vec{c}$ is a vector such that $\vec{a} \cdot \vec{c} = 11$, $\vec{b} \cdot (\vec{a} \times \vec{c}) = 27$ and $\vec{b} \cdot \vec{c} = -\sqrt{3}|\vec{b}|$, then $|\vec{a} \times \vec{c}|^2$ is equal to ____:

30: Let $S = \left\{ z \in \mathbb{C} - \{i, 2i\} : \frac{z^2+8iz-15}{z^2-3iz-2} \in \mathbb{R} \right\}$**. If** $\alpha - \frac{13}{11}i \in S$**,** $\alpha \in \mathbb{R} - \{0\}$**, then** $242\alpha^2$ **is equal to** ________**.**
