JEE Main 2023 31 Jan Shift 1 Maths Question Paper

Time Allowed :3 Hours	**Maximum Marks :**300	**Total Questions :**90

General Instructions

Read the following instructions very carefully and strictly follow them:

1. The test is of 3 hours duration.
2. The question paper consists of 90 questions, out of which 75 are to attempted. The maximum marks are 300.
3. There are three parts in the question paper consisting of Physics, Chemistry and Mathematics having 30 questions in each part of equal weightage.
4. Each part (subject) has two sections.
   (i) Section-A: This section contains 20 multiple choice questions which have only one correct answer. Each question carries 4 marks for correct answer and –1 mark for wrong answer.
   (ii) Section-B: This section contains 10 questions. In Section-B, attempt any five questions out of 10. The answer to each of the questions is a numerical value. Each question carries 4 marks for correct answer and –1 mark for wrong answer. For Section-B, the answer should be rounded off to the nearest integer

MATHEMATICS

Section-A

61. If the maximum distance of normal to the ellipse

$\frac{x^2}{4} + \frac{y^2}{b^2} = 1,\ b < 2,$ from the origin is 1, then the eccentricity of the ellipse is:

(1) $\frac{1}{\sqrt{2}}$
(2) $\frac{\sqrt{3}}{2}$
(3) $\frac{5}{4}$

(4) $\frac{\sqrt{5}}{4}$

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62. For all $z \in \mathbb{C}$ on the curve C such that $|z| = 1$, let the locus of the point $z + \frac{1}{z}$ be the curve C_1. Then:

(1) the curves C_1 and C_2 intersect at 4 points

(2) the curves C_1 lies inside C_2

(3) the curves C_1 and C_2 intersect at 2 points

(4) the curves C_2 lies inside C_1

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63. A wire of length 20 m is to be cut into two pieces. A piece of length ℓ_1 is bent to make a square of area A_1, and the other piece of length ℓ_2 is made into a circle of area A_2. If $2A_1 + 3A_2$ is minimum, then $\frac{\ell_1}{\ell_2}$ is equal to:

(1) $6 : 1$

(2) $3 : 1$

(3) $1 : 6$

(4) $4 : 1$

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64. For the system of linear equations

x + y + z = 6

$\alpha x + \beta y + 7z = 3$

x + 2y + 3z = 14

Which of the following is NOT true?

(1) If $\alpha = \beta$, then the system has no solution

(2) If $\alpha = \beta$ and $\alpha \neq 7$, then the system has a unique solution.

(3) There is a unique point (α, β) on the line $x + 2y + 18 = 0$ for which the system has infinitely many solutions.

(4) For every point $(\alpha, \beta) \neq (7, 7)$ on the line $x = 2y + 7$, the system has infinitely many solutions.

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65. Let the shortest distance between the lines

$$L: \frac{x-5}{2} = \frac{y-\lambda}{0} = \frac{z+1}{1}, \quad \lambda \geq 0 \quad \text{and} \quad L_1 : x+1 = y-1 = 4-z = 2\sqrt{6}$$

If (α, β, γ) lies on L, then which of the following is NOT possible?

(1) $\alpha + 2\gamma = 24$

(2) $2\alpha + \gamma = 7$

(3) $2\alpha - \gamma = 9$

(4) $\alpha - 2\gamma = 19$

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66. Let $y = f(x)$ represent a parabola with focus $\left(-\frac{1}{2}, 0\right)$ and directrix $y = -\frac{1}{2}$.

Then

$$S = \left\{ x \in \mathbb{R} : \tan^{-1}\left(\sqrt{f(x)} + \sin\left(\sqrt{f(x)+1}\right)\right) = \frac{\pi}{2} \right\}$$

contains:

(1) Exactly two elements

(2) Exactly one element

(3) An infinite set

(4) An empty set

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67. Let

$$A = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 4 & -1 \\ 0 & 12 & -3 \end{pmatrix}$$

Then the sum of the diagonal elements of the matrix $(A+I)^{11}$ is equal to:

(1) 6144

(2) 4094

(3) 4097

(4) 2050

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68. Let R be a relation on $\mathbb{N} \times \mathbb{N}$ defined by

$$(a, b)\, R\, (c, d) \quad \text{if and only if} \quad ad(b-c) = bc(a-d).$$

Then R is:

(1) Symmetric but neither reflexive nor transitive

(2) Transitive but neither reflexive nor symmetric

(3) Reflexive and symmetric but not transitive

(4) Symmetric and transitive but not reflexive

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69. Let

$$y = f(x) = \sin^3\left(\frac{\pi}{3}\cos\left(\frac{\pi}{3\sqrt{2}}\left(-4x^3+5x^2+1\right)^{\frac{3}{2}}\right)\right)$$

Then, at $x = 1$,

(1) $2y' + 3y = 0$

(2) $2y' + 3y' = 0$

(3) $y' - 3y' = 0$

(4) $y' + 3y' = 0$

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70. If the sum and product of four positive consecutive terms of a G.P. are 126 and 1296, respectively, then the sum of common ratios of all such GPs is:

(1) 7

(2) $\frac{9}{2}$

(3) 3

(4) 14

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71. The number of real roots of the equation

$$\sqrt{x^2-4x+3}+\sqrt{x^2-9}=\sqrt{4x^2-14x+6}$$

is:

(1) 0

(2) 1

(3) 3

(4) 2

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72. Let a differentiable function f satisfy

$$f(x) + \int_3^x f(t)\,dt = \sqrt{x+1}, \quad x \ge 3.$$

Then $f(8)$ is equal to:

(1) 34

(2) 19

(3) 17

(4) 1

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73. If the domain of the function

$f(x) = \dfrac{\lfloor x \rfloor}{1+x^2}$, **where** $\lfloor x \rfloor$ **is the greatest integer less than or equal to** x, **is** $[2,6)$, **then its range is:**

(1) $\left[\frac{5}{26}, \frac{9}{27}\right]$

(2) $\left[\frac{5}{26}, \frac{9}{29}\right]$

(3) $\left[\frac{9}{37}, \frac{29}{109}\right]$

(4) $\left[\frac{5}{37}, \frac{53}{37}\right]$

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74. Let $\mathbf{a} = 2\hat{i} + \hat{j} + \hat{k}$, and $\mathbf{b}$ and $\mathbf{c}$ be two nonzero vectors such that

$|\mathbf{a}+\mathbf{b}+\mathbf{c}| = |\mathbf{a}+\mathbf{b}-\mathbf{c}|$ and $\mathbf{b}\cdot\mathbf{c} = 0$. Consider the following two statements:

(A) $|\mathbf{a}+\lambda\mathbf{c}| \ge |\mathbf{a}|$ for all $\lambda \in \mathbb{R}$.

(B) $\mathbf{a}$ and $\mathbf{c}$ are always parallel.

(1) only (B) is correct

(2) neither (A) nor (B) is correct

(3) only (A) is correct

(4) both (A) and (B) are correct

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75. Let $\alpha \in (0,1)$ and $\beta = \log(1-\alpha)$. Let

$$P_n(x) = x + \frac{x^2}{2} + \frac{x^3}{3} + \cdots + \frac{x^n}{n}, \quad x \in (0,1).$$

Then the integral

$$\int_0^\alpha \frac{1}{1-t}\,dt$$

is equal to:

(1) $\beta - P_0(\alpha)$

(2) $-(\beta + P_0(\alpha))$

(3) $P_0(\alpha) - \beta$

(4) $\beta + P_0(\alpha)$

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76. If $\sin^{-1}\left(\frac{\alpha}{17}\right) + \cos^{-1}\left(\frac{4}{5}\right) - \tan^{-1}\left(\frac{77}{36}\right) = 0, \quad 0 < \alpha < 13$, **then** $\sin^{-1}(\sin\alpha) + \cos^{-1}(\cos\alpha)$ **is equal to:**

(1) π

(2) 16

(3) 0

(4) $16 - 5\pi$

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77. Let a circle C_1 be obtained on rolling the circle

$x^2 + y^2 - 4x - 6y + 11 = 0$ upwards 4 units on the tangent T to it at the point (3, 2).

Let C_2 be the image of C_1 in T. Let A and B be the centers of circles C_1 and C_2 respectively, and M and N be respectively the feet of perpendiculars drawn from A and B on the x-axis. Then the area of the trapezium AMNB is:

(1) $2(2 + \sqrt{2})$

(2) $4(1 + \sqrt{2})$

(3) $3 + \sqrt{2}$

(4) $2(1 + \sqrt{2})$

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78. (S1) $(p \Rightarrow q) \vee (p \wedge \neg q)$ **is a tautology (S2)** $(\neg p) \Rightarrow (\neg q) \wedge ((\neg p) \vee q)$ **is a contradiction. Then:**

(1) only (S2) is correct

(2) both (S1) and (S2) are correct

(3) both (S1) and (S2) are wrong

(4) only (S1) is correct

79. The value of

$$\int \frac{(2+3\sin x)}{\sin x(1+\cos x)}\,dx$$

is equal to:

(1) $\frac{7}{2}\sqrt{3}-\log\sqrt{3}$

(2) $2+3\sqrt{3}+\log\sqrt{3}$

(3) $\frac{10}{3}\sqrt{3}-\log\sqrt{3}$

(4) $\sqrt{3}-\log\sqrt{3}$

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80. A bag contains 6 balls. Two balls are drawn from it at random and both are found to be black. The probability that the bag contains at least 5 black balls is:

(1) $\frac{5}{7}$

(2) $\frac{2}{7}$

(3) $\frac{3}{7}$

(4) $\frac{5}{6}$

Section-B

81. Let 5 digit numbers be constructed using the digits

$0, 2, 3, 4, 7, 9$ with repetition allowed, and are arranged in ascending order with serial numbers.

Then the serial number of the number 42923 is:

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82. Let $a_1, a_2, \ldots, a_n$ be in A.P. If $a_5 = 2a_1$ and $a_1 = 18$, then

$$12\left(\frac{1}{\sqrt{a_0}+\sqrt{a_1}}+\frac{1}{\sqrt{a_1}+\sqrt{a_2}}+\cdots+\frac{1}{\sqrt{a_{17}}+\sqrt{a_{18}}}\right)$$

is equal to:

83. Let

θ be the angle between the planes $P_1 : \vec{r}\cdot(\hat{i}+\hat{j}+2\hat{k}) = 9$ and $P_2 : \vec{r}\cdot(2\hat{i}-\hat{j}+\hat{k}) = 15$. **Let**

L be the line that meets P_2 at the point (4, -2, 5) and makes an angle θ with the normal of P_2. If α is the angle between L and P_2, then $(\tan^2\theta)(\cot^2\alpha)$ is equal to:

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84. Let $\alpha > 0$, be the smallest number such that the expansion of

$$\left(\frac{3}{x^3}+\frac{2}{x}\right)^{30} \text{ has a term } \beta x^{-\alpha}, \beta \in \mathbb{N}.$$

Then α is equal to:

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85. Let $\vec{a}$ and $\vec{b}$ be two vectors such that

$$|\vec{a}| = \sqrt{14}, \quad |\vec{b}| = \sqrt{6}, \quad |\vec{a}\times\vec{b}| = \sqrt{48}.$$

Then $(\vec{a}\cdot\vec{b})^2$ is equal to:

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86. Let the line $L : \frac{x-1}{2} = \frac{y+1}{-1} = \frac{z-3}{1}$ intersect the plane

$$2x + y + 3z = 16 \text{ at the point } P.$$

Let the point Q **be the foot of perpendicular from the point** R(1, -1, -3) on the line L. **If α is the area of triangle PQR, then α^2 is equal to:**

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87. The remainder on dividing 5^{99} by 11 is:

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88. If the variance of the frequency distribution

x_i	2	3	4	5	6	7	8
f_i	3	6	16	α	9	5	6

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89. Let for $x \in \mathbb{R}$

$$f(x) = \frac{x+|x|}{2} \quad \text{for} \quad x \geq 0 \quad \text{and} \quad f(x) = \frac{x}{2} \quad \text{for} \quad x < 0$$

$$g(x) = \begin{cases} x^2 & \text{for} \quad x \geq 0, \\ x & \text{for} \quad x < 0 \end{cases}$$

Then the area bounded by the curve $y = f \circ g(x)$ and the lines $y = 0, 2y - x = 15$ is equal to:

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90. Number of 4-digit numbers that are less than or equal to 2800 and either divisible by 3 or by 11, is equal to:
