JEE Main 2023 31 Jan Shift 1 Question Paper

Time Allowed :3 Hours	Maximum Marks :300	Total Questions :90

General Instructions

Read the following instructions very carefully and strictly follow them:

1. The test is of 3 hours duration.
2. The question paper consists of 90 questions, out of which 75 are to attempted. The maximum marks are 300.
3. There are three parts in the question paper consisting of Physics, Chemistry and Mathematics having 30 questions in each part of equal weightage.
4. Each part (subject) has two sections.
   (i) Section-A: This section contains 20 multiple choice questions which have only one correct answer. Each question carries 4 marks for correct answer and –1 mark for wrong answer.
   (ii) Section-B: This section contains 10 questions. In Section-B, attempt any five questions out of 10. The answer to each of the questions is a numerical value. Each question carries 4 marks for correct answer and –1 mark for wrong answer. For Section-B, the answer should be rounded off to the nearest integer

PHYSICS

Section-A

1. A bar magnet with a magnetic moment 5.0 Am^2 is placed in parallel position relative to a magnetic field of 0.4 T. The amount of required work done in turning the magnet from parallel to antiparallel position relative to the field direction is:

(1) 4 J

(2) 1 J

(3) 2 J

(4) Zero

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2. If a source of electromagnetic radiation having power 15 kW produces 10^{16} photons per second, the radiation belongs to a part of spectrum is:

(1) Micro waves

(2) Ultraviolet rays

(3) Gamma rays

(4) Radio waves

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3. The amplitude of $15\sin(1000\pi t)$ is modulated by $10\sin(4\pi t)$ signal. The amplitude modulated signal contains frequency(ies) of:

(A) 500 Hz

(B) 2 Hz

(C) 250 Hz

(D) 498 Hz

(E) 502 Hz

Choose the correct answer from the options given below:

(1) A only

(2) A,D and E only

(3) B only

(4) A and B only

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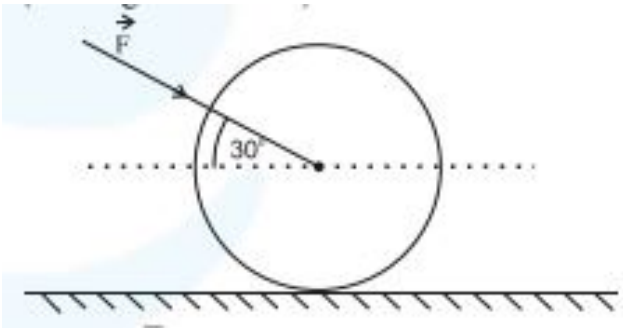
4. As shown in figure, a 70 kg garden roller is pushed with a force of $F = 200$ N at an angle of 30° with horizontal. The normal reaction on the roller is: (Given $g = 10$ m/s^2)



(1) $800\sqrt{2}$ N

(2) 600 N

(3) 800 N

(4) $200\sqrt{3}$ N

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5. The initial speed of a projectile fired from ground is u. At the highest point during its motion, the speed of the projectile is $\frac{\sqrt{5}}{2}u$. The time of flight of the projectile is:

(1) $\frac{u}{2g}$

(2) $\frac{u}{g}$

(3) $\frac{2u}{g}$

(4) $\frac{\sqrt{5}u}{g}$

---

6. Spherical insulating ball and a spherical metallic ball of the same size and mass are dropped from the same height. Choose the correct statement out of the following (Assume negligible air friction):

(1) Time taken by them to reach the earth's surface will be independent of the properties of their materials

(2) Insulating ball will reach the earth's surface earlier than the metal ball

(3) Both will reach the earth's surface simultaneously

(4) Metal ball will reach the earth's surface earlier than the insulating ball

---

7. A free neutron decays into a proton but a free proton does not decay into neutron. This is because:

(1) Neutron is an uncharged particle

(2) Proton is a charged particle

(3) Neutron is a composite particle made of a proton and an electron

(4) Neutron has larger rest mass than proton

---

8. The effect of increase in temperature on the number of electrons in conduction band (n_e) and resistance of a semiconductor will be:

(1) Both n_e and resistance decrease

(2) Both n_e and resistance increase

(3) n_e increases, resistance decreases

(4) n_e decreases, resistance increases

---

9. The maximum potential energy of a block executing simple harmonic motion is 25 J. A is amplitude of oscillation. At $\frac{A}{2}$, the kinetic energy of the block is:

(1) 37.5 J

(2) 9.75 J

(3) 18.75 J

(4) 12.5 J

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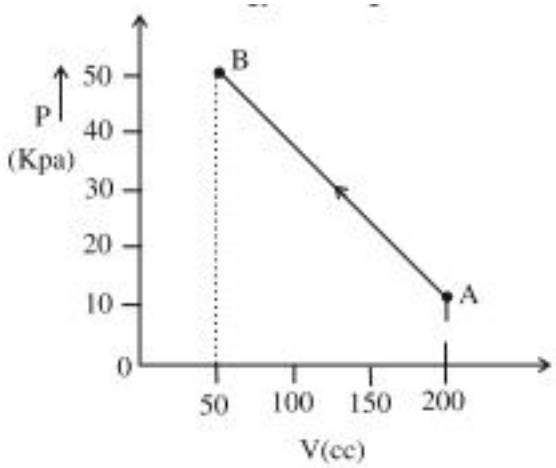
10. The pressure of a gas changes linearly with volume from A to B as shown in the figure. If no heat is supplied to or extracted from the gas, then change in the internal energy of the gas will be:



(1) 6 J

(2) Zero

(3) -4.5 J

(4) 4.5 J

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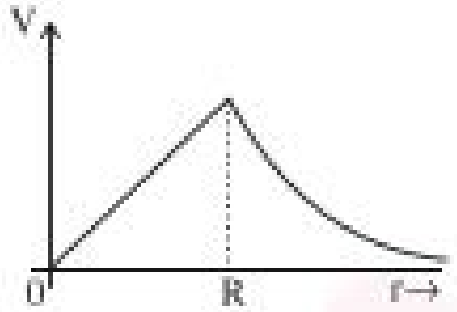
11. Which of the following correctly represents the variation of electric potential (V) of a charged spherical conductor of radius (R) with radial distance (r) from the centre?

(1)



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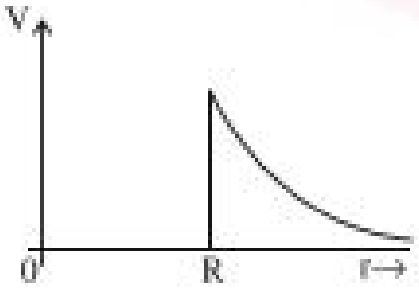

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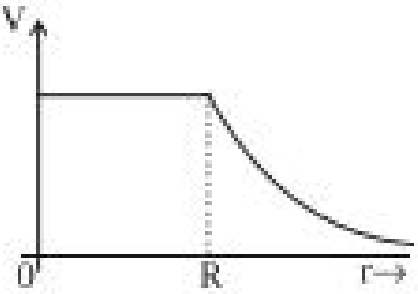

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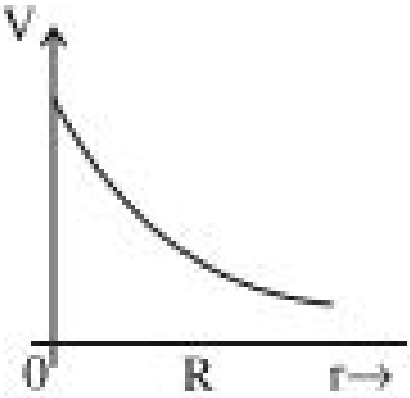

12. Given below are two statements: One is labelled as Assertion A and the other is labelled as Reason R Assertion A: The beam of electrons shows wave nature and exhibits interference and diffraction. Reason R: Davisson Germer experimentally verified the wave nature of electrons. In the light of the above statements, choose the most appropriate answer from the options given below:

(1) A is correct but R is not correct

(2) A is not correct but R is correct

(3) Both A and R are correct but R is not the correct explanation of A

(4) Both A and R are correct and R is the correct explanation of A

---

13. The drift velocity of electrons for a conductor connected in an electrical circuit is V_d. The conductor is now replaced by another conductor with the same material and same length but double the area of cross-section. The applied voltage remains the same. The new drift velocity of electrons will be:

(1) V_d

(2) $\frac{V_d}{2}$

(3) $\frac{V_d}{4}$

(4) $2V_d$

---

14. At a certain depth d below the surface of the Earth, the value of acceleration due to gravity becomes four times that of its value at a height 3R above the Earth's surface. Where R is the radius of Earth (Take $R = 6400$ km). The depth d is equal to:

(1) 5260 km

(2) 640 km

(3) 2560 km

(4) 4800 km

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15. If 1000 droplets of water of surface tension 0.07 N/m, having same radius 1 mm each, combine to form a single drop. In the process, the released surface energy is: (Take $\pi = \frac{22}{7}$)

(1) 7.92×10^{-6} J

(2) 7.92×10^{-4} J

(3) 9.68×10^{-4} J

(4) 8.8×10^{-5} J

---

16. A rod with circular cross-section area 2 cm^2 and length 40 cm is wound uniformly with 400 turns of an insulated wire. If a current of 0.4 A flows in the wire windings, the total magnetic flux produced inside the windings is 4×10^{-6} Wb. The relative permeability of the rod is: (Given: Permeability of vacuum $\mu_0 = 4\pi \times 10^{-7}$ N/A^2)

(1) 12.5

(2) $\frac{32}{5}$

(3) 125

(4) $\frac{5}{16}$

---

17. The correct relation between $\gamma = \frac{C_p}{C_v}$ and temperature T is:

(1) $\gamma \propto \frac{1}{\sqrt{T}}$

(2) $\gamma \propto T^0$

(3) $\gamma \propto \frac{1}{T}$

(4) $\gamma \propto T$

---

18. Two polaroide A and B are placed in such a way that the pass-axis of polaroids are perpendicular to each other. Now, another polaroid C is placed between A and B bisecting the angle between them. If intensity of unpolarised light is I_0, then intensity of transmitted light after passing through polaroid B will be:

(1) $\frac{I_0}{4}$

(2) $\frac{I_0}{2}$

(3) $\frac{I_0}{8}$

(4) Zero

---

19. If R, X_L, and X_C represent resistance, inductive reactance, and capacitive reactance, then which of the following is dimensionless:

(1) RX_LX_C

(2) $\frac{R}{\sqrt{X_LX_C}}$

(3) $\frac{R}{X_LX_C}$

(4) $\frac{X_L}{X_C}$

---

20. 100 balls each of mass m moving with speed v simultaneously strike a wall normally and are reflected back with the same speed, in time t. The total force exerted by the balls on the wall is:

(1) $\frac{100mv}{t}$

(2) $\frac{200mv}{t}$

(3) $\frac{200mvt}{t}$

(4) $\frac{mv}{100t}$

SECTION-B

21. A thin rod having a length of 1 m and area of cross-section 3×10^{-6} m^2 is suspended vertically from one end. The rod is cooled from 210°C to 160°C. After cooling, a mass M is attached at the lower end of the rod such that the length of the rod again becomes 1 m. Young's modulus and the coefficient of linear expansion of the rod are 2×10^{11} N/m^2 and 2×10^{-5} K^{-1}, respectively. The value of M is _____kg. (Take $g = 10$ m/s^2)

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22. The speed of a swimmer is 4 km/h in still water. If the swimmer makes his strokes normal to the flow of the river of width 1 km, he reaches a point 750 m down the stream on the opposite bank. The speed of the river water is ..... km/h.

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23. In the figure given below, a block of mass $M = 490$ g is placed on a frictionless table is connected with two springs having the same spring constant ($K = 2$ N/m). If the block is horizontally displaced through X m, then the number of complete oscillations it will make in 14t seconds will be:

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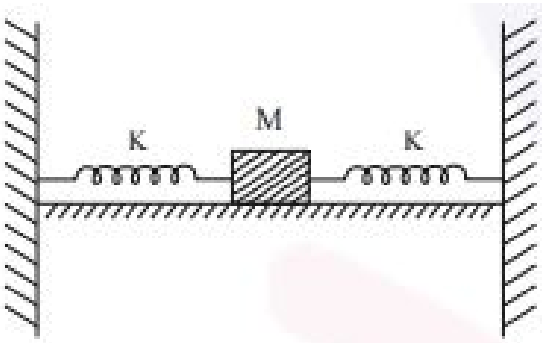
24. In a medium, the speed of light wave decreases to 0.2 times its speed in free space. The ratio of relative permittivity to the refractive index of the medium is $x : 1$. The



value of x is ..... (Given speed of light in free space $c = 3 \times 10^8$ m/s and for the given medium $\mu_r = 1$)

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25. A solid sphere of mass 1 kg rolls without slipping on a plane surface. Its kinetic energy is 7×10^3 J. The speed of the centre of mass of the sphere is ..... cm/s.

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26. An inductor of 0.5 mH, a capacitor of 20 F, and resistance of 20 are connected in series with a 220 V AC source. If the current is in phase with the emf, the amplitude of current of the circuit is $\sqrt{K}$ A. The value of x is:

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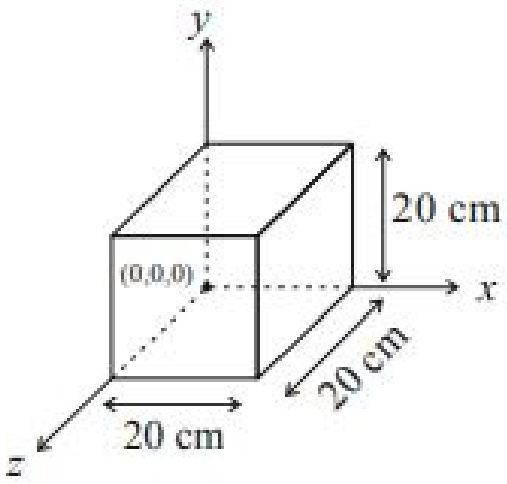
27. Expression for an electric field is given by $\vec{E} = 4000\hat{i}$ V/m. The electric flux through the cube of side 20 cm when placed in the electric field (as shown in the figure) is ..... Vcm.



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28. A lift of mass $M = 500$ kg is descending with a speed of 2 m/s. Its supporting cable

begins to slip, thus allowing it to fall with a constant acceleration of $2\,m/s^2$. The kinetic energy of the lift at the end of fall through a distance of 6 m will be ..... kJ.

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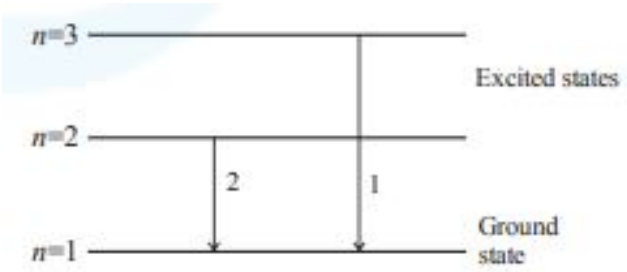
29. For hydrogen atom, λ_1 and λ_2 are the wavelengths corresponding to the transitions 1 and 2, respectively, as shown in the figure. The ratio of λ_1 and λ_2 is $\frac{x}{32}$. The value of x is:



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30. Two identical cells, when connected either in parallel or in series, give the same current in an external resistance of 5. The internal resistance of each cell will be ..... Ω.

CHEMISTRY

Section-A

31. Nd^{2+} = ...

(1) $4f^2 6s^2$

(2) $4f^4$

(3) $4f^3$

(4) $4f^4 6s^2$

---

32. The methods NOT involved in concentration of ore are

(A) Liquation

(B) Leaching

(C) Electrolysis

(D) Hydraulic washing

(E) Froth flotation

Choose the correct answer from the options given below:

(1) B, D and C only

(2) C, D and E only

(3) A and C only

(4) B, D and E only

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33. Consider the following reaction

$$\text{Propanal} + \text{Methanal} \xrightarrow{\text{(i) dil. NaOH}} \xrightarrow{\text{(ii) A}} \xrightarrow{\text{(iii) NaCN}} \xrightarrow{\text{(iv) } H_2O} \text{Product B } (C_5H_8O_3)$$

The correct statement for product B is:

(1) Optically active and adds one mole of bromine

(2) Racemic mixture and is neutral

(3) Racemic mixture and gives a gas with saturated NaHCO solution

(4) Optically active alcohol and is neutral

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34. The correct order of basicity of oxides of vanadium is:

(1) $V_2O_3 > V_2O_4 > V_2O_5$

(2) $V_2O_3 > V_2O_5 > V_2O_4$

(3) $V_2O_5 > V_2O_3 > V_2O_4$

(4) $V_2O_4 > V_2O_3 > V_2O_5$

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35. When Cu^{2+} ion is treated with KI, a white precipitate, X appears in solution. The solution is titrated with sodium thiosulphate, the compound Y is formed. X and Y respectively are:

(1) $X = CuI_2$, $Y = Na_2S_4O_5$

(2) $X = CuI_2$, $Y = Na_2S_4O_6$

(3) $X = CuI$, $Y = Na_2S_4O_3$

(4) $X = CuI$, $Y = Na_2S_4O_6$

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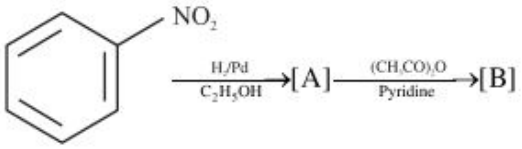
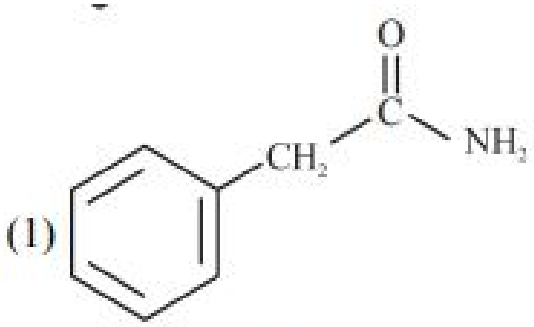
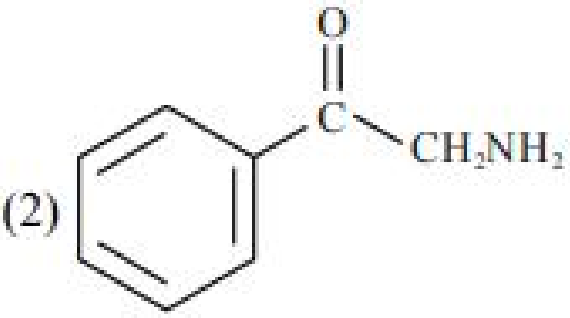
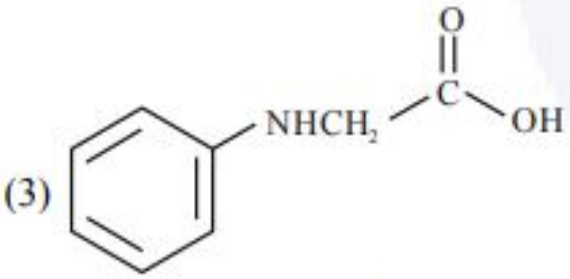
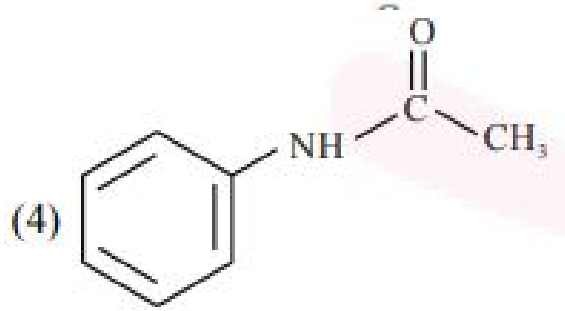
36. The reaction sequence is:

36. 

(1) 

(2) 

(3) 

(4) 

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37. Cobalt chloride when dissolved in water forms pink colored complex X which has octahedral geometry. This solution on treating with cone HCl forms deep blue complex Y which has a Z geometry. X, Y and Z, respectively, are:

(1) $X = [Co(H_2O)_6]^{2+}$, $Y = [CoCl_4]^{2-}$, $Z =$ Tetrahedral

(2) $X = [Co(H_2O)_6]^{3+}$, $Y = [CoCl_3]^{3-}$, $Z =$ Octahedral

(3) $X = [Co(H_2O)_6]^{3+}$, $Y = [CoCl_4]^{2-}$, $Z =$ Octahedral

(4) $X = [Co(H_2O)_6]^{2+}$, $Y = [CoCl_4]^{2-}$, $Z =$ Tetrahedral

---

38. Identify X, Y, and Z in the following reaction. (Equation not balanced)

$$\text{ClO} + \text{NO}_2 \rightarrow X \xrightarrow{H_2O} Y + Z$$

(1) $X = \text{ClONO}_2$, $Y = \text{HOCl}$, $Z = \text{NO}_2$

(2) $X = \text{ClONO}_2$, $Y = \text{HCl}$, $Z = \text{HNO}_3$

(3) $X = \text{ClONO}_2$, $Y = \text{HOCl}$, $Z = \text{HNO}_3$

(4) $X = \text{ClO}_3$, $Y = \text{Cl}_2$, $Z = \text{NO}_2$

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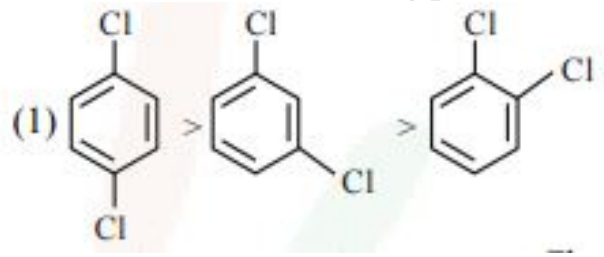
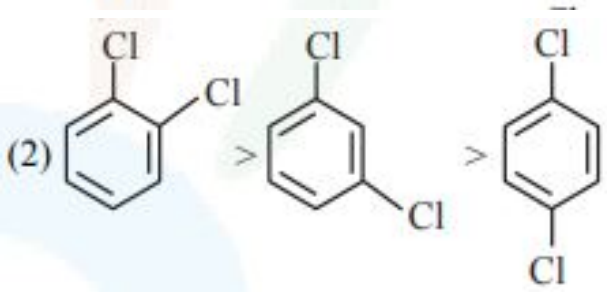
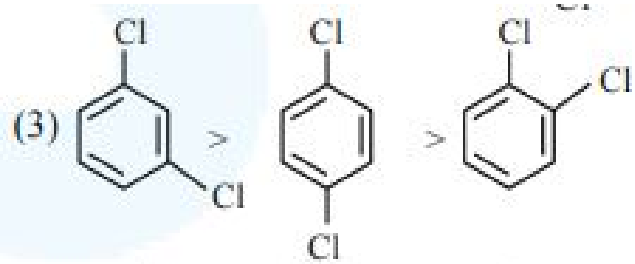
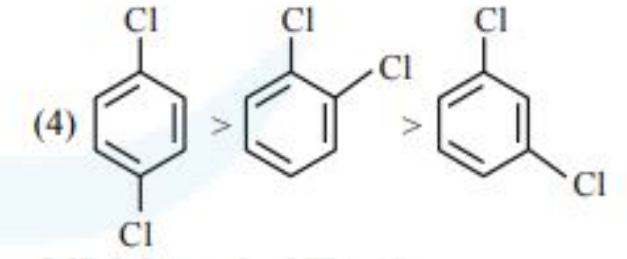
39. The correct order of melting point of dichlorobenzenes is:

(1) 

(2) 

(3) 

(4) 

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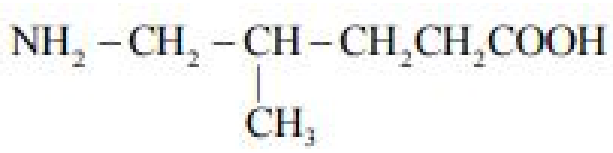
40. A protein 'X' with molecular weight of 70,000 u, on hydrolysis gives amino acids. One of these amino acids is

(1)



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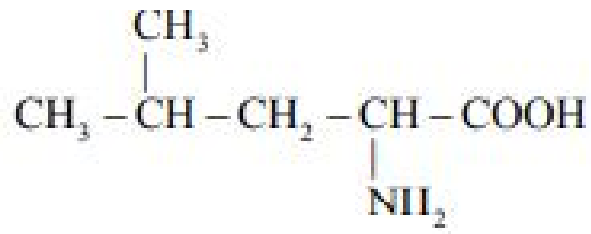

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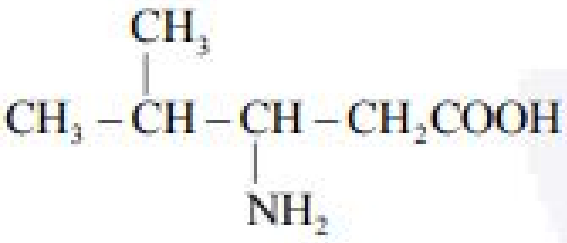

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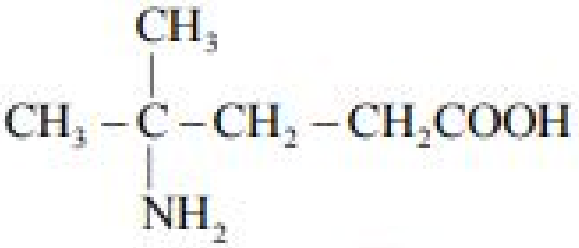

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41. Which transition in the hydrogen spectrum would have the same wavelength as the Balmer type transition from $n = 4$ to $n = 2$ of He^+ spectrum?

(1) $n = 2$ to 1

(2) $n = 1$ to 3

(3) $n = 1$ to 2

(4) $n = 3$ to 4

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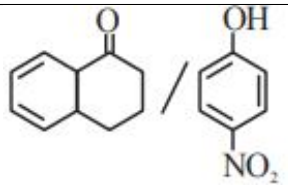
42. Match items of column I and II

Column I (Mixture of compounds)	Column II (Separation Technique)
A. HO/CHCl	i. Crystallization
B. 	ii. Differential solvent extraction
C. Kerosene/Naphthalene	iii. Column chromatography
D. CHO/NaCl	iv. Fractional Distillation

Correct match is:

(1) A-(iii), B-(iv), C-(ii), D-(i)

(2) A-(ii), B-(iii), C-(ii), D-(iv)

(3) A-(ii), B-(iii), C-(iv), D-(i)

(4) A-(ii), B-(iv), C-(i), D-(iii)

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43. The correct increasing order of the ionic radii is:

(1) $Cl^- < Ca^{2+} < K^+ < S^{2-}$

(2) $K^+ < S^{2-} < Ca^{2+} < Cl^-$

(3) $S^{2-} < Cl^- < Ca^{2+} < K^+$

(4) $Ca^{2+} < K^+ < Cl^- < S^{2-}$

---

44. H_2O_2 acts as a reducing agent in:

(1) $2NaOCl + H_2O_2 \rightarrow 2NaCl + H_2O + O_2$

(2) $2Fe^{2+} + 2H_2O_2 \rightarrow 2Fe^{3+} + 2H_2O$

(3) $Mn^{2+} + 2H_2O_2 \rightarrow MnO_2 + 2H_2O$

(4) $Na_2S_4O_6 + 4H_2O_2 \rightarrow Na_2SO_4 + 4H_2O$

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45. Which of the following artificial sweeteners has the highest sweetness value in comparison to cane sugar?

(1) Aspartame

(2) Sucralose

(3) Alitame

(4) Saccharin

46. Match List I with List II

List I	List II
A. XeF_4	I. See-saw
B. SF_4	II. Square planar
C. NH_4^+	III. Bent T-shaped
D. BrF_3	IV. Tetrahedral

Choose the correct answer from the options given below:

(1) A-IV, B-III, C-II, D-I

(2) A-II, B-III, C-III, D-IV

(3) A-I, B-I, C-II, D-III

(4) A-I, B-I, C-IV, D-III

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47. Choose the correct set of reagents for the following conversion

$$\text{trans (Ph-CH=CH-CH}_3\text{)} \rightarrow \text{cis (Ph-CH=CH-CH}_3\text{)}$$

(1) Br_2, alc. KOH, $NaNH_2$, Na(Liq NH_3)

(2) Br_2, aq KOH, $NaNH_2$, H_2 Lindlar Catalyst

(3) Br_2, aq KOH, $NaNH_2$, Na(Liq NH_3)

(4) Br_2, alc. KOH, $NaNH_2$, Lindlar Catalyst

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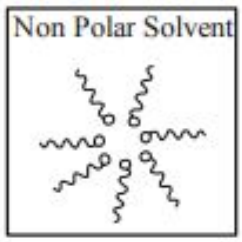
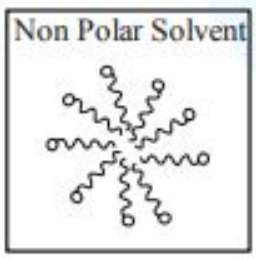
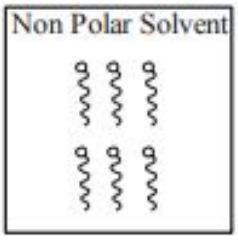
48. Adding surfactants in non-polar solvent, the micelles structure will look like (Surfactant structure):

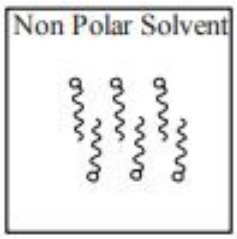
(a)



(b)



(c) 

(d) 

(1) b

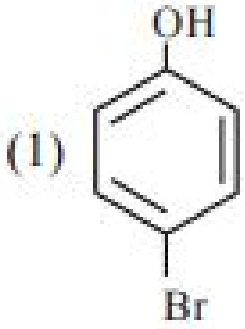
(2) c

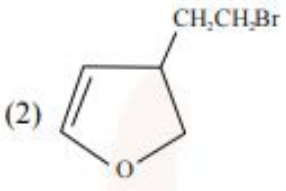
(3) a

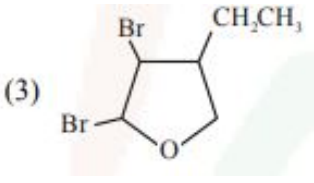
(4) d

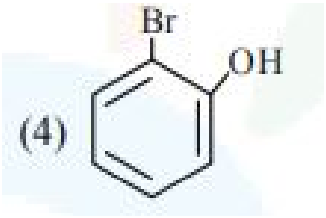
---

49. An organic compound 'A' with empirical formula C_6H_6O gives sooty flame on burning. Its reaction with bromine solution in low polarity solvent results in high yield of B. B is:

(1) 

(2) 

(3) 

(4) 

---

50. Which one of the following statements is correct for electrolysis of brine solution?

(1) Cl_2 is formed at cathode

(2) O_2 is formed at cathode

(3) H_2 is formed at anode

(4) OH^- is formed at cathode

Section-B

51. The logarithm of equilibrium constant for the reaction

$$Pd^{2+}(aq) + 4Cl^-(aq) \rightleftharpoons PdCl_4^{2-}(aq)$$

is:

Given:

$$2.303RT = 0.06\,V, \quad F = 96,485\,C/mol, \quad E^\circ = 0.83\,V, \quad E^\circ = 0.65\,V$$

52.

$$\mathbf{A \rightarrow B}$$

The rate constants of the above reaction at 200 K and 300 K are 0.03 min^{-1} and 0.05 min^{-1} respectively.

The activation energy for the reaction is _____ J (Nearest integer)

Given:

$$\ln 10 = 2.3, \quad R = 8.3\,J\,K^{-1}mol^{-1}, \quad \log 5 = 0.70, \quad \log 3 = 0.48, \quad \log 2 = 0.30$$

---

53. The enthalpy change for the conversion of

$$\frac{1}{2}Cl_2(g) \rightarrow Cl^-(aq) \text{ is } (-) \ldots\ldots \text{ kJ mol}^{-1} \text{ (Nearest integer)}$$

Given:

$\Delta_f H^\circ(Cl_2(g)) = 240\,\text{kJ mol}^{-1}, \quad \Delta_g H^\circ(Cl_2(g)) = -350\,\text{kJ mol}^{-1}, \quad \Delta_{hyd} H^\circ(Cl^-) = -380\,\text{kJ mol}^{-1}$

---

54. On complete combustion, 0.492 g of an organic compound gave 0.792 g of CO_2. The % of carbon in the organic compound is _____ (Nearest integer)

Given:

Weight of carbon in 0.792 g CO_2

---

55. At 27°C, a solution containing 2.5 g of solute in 250.0 mL of solution exerts an osmotic pressure of 400 Pa. The molar mass of the solute is _____

g mol^{-1} (Nearest integer)

Given:

$R = 0.083\,\text{L bar K}^{-1}\text{mol}^{-1}, \quad \pi = \text{osmotic pressure}, \quad \text{T} = 300\,\text{K}, \quad \text{osmotic pressure} = 400\,\text{Pa}, \quad 1\,\text{bar} =$

---

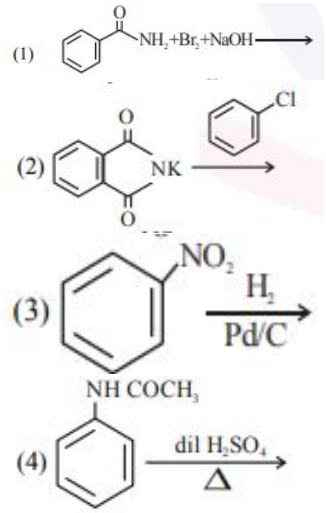
56. Zinc reacts with hydrochloric acid to give hydrogen and zinc chloride. The volume of hydrogen gas produced at STP from the reaction of 11.5 g of zinc with excess HCl is _____ L (Nearest integer)

Given:

Molar mass of Zn $= 65.4\,\text{g mol}^{-1}, \quad$ Molar volume of H_2at STP $= 22.7\,\text{L/mol}$

---

57. How many of the transformations given below would result in aromatic amines?



58. For reaction:

$$SO_2(g) + \frac{1}{2}O_2(g) \rightleftharpoons SO_3(g)$$

$\mathbf{K}_p = 2 \times$

10^{12} at 27°C and 1 atm pressure. The K_c for the same reaction is_____ (Nearest integer)

Given:

$$R = 0.082\,\text{L atm K}^{-1}\text{mol}^{-1}$$

---

59. The oxidation state of phosphorus in hypophosphoric acid is _____

---

60. The total pressure of a mixture of non-reacting gases X (0.6 g) and Y (0.45 g) in a vessel is 740 mm of Hg. The partial pressure of the gas X is _____ mm of Hg (Nearest Integer)

Given:

$$\textbf{Molar mass of X} = 20\,\textbf{g mol}^{-1}, \quad \textbf{Molar mass of Y} = 45\,\textbf{g mol}^{-1}$$

MATHEMATICS

Section-A

61. If the maximum distance of normal to the ellipse

$\frac{x^2}{4} + \frac{y^2}{b^2} = 1,\ b < 2,$ from the origin is 1, then the eccentricity of the ellipse is:

(1) $\frac{1}{\sqrt{2}}$

(2) $\frac{\sqrt{3}}{2}$

(3) $\frac{5}{4}$

(4) $\frac{\sqrt{5}}{4}$

---

62. For all $z \in \mathbb{C}$ on the curve C such that $|z| = 1$, let the locus of the point $z + \frac{1}{z}$ be the curve C_1. Then:

(1) the curves C_1 and C_2 intersect at 4 points

(2) the curves C_1 lies inside C_2

(3) the curves C_1 and C_2 intersect at 2 points

(4) the curves C_2 lies inside C_1

---

63. A wire of length 20 m is to be cut into two pieces. A piece of length ℓ_1 is bent to make a square of area A_1, and the other piece of length ℓ_2 is made into a circle of area A_2. If $2A_1 + 3A_2$ is minimum, then $\frac{\ell_1}{\ell_2}$ is equal to:

(1) $6 : 1$

(2) $3 : 1$

(3) $1 : 6$

(4) $4 : 1$

---

64. For the system of linear equations

x + y + z = 6

$\alpha x + \beta y + 7z = 3$

x + 2y + 3z = 14

Which of the following is NOT true?

(1) If $\alpha = \beta$, then the system has no solution

(2) If $\alpha = \beta$ and $\alpha \neq 7$, then the system has a unique solution.

(3) There is a unique point (α, β) on the line $x + 2y + 18 = 0$ for which the system has infinitely many solutions.

(4) For every point $(\alpha, \beta) \neq (7, 7)$ on the line $x = 2y + 7$, the system has infinitely many solutions.

---

65. Let the shortest distance between the lines

$$L : \frac{x-5}{2} = \frac{y-\lambda}{0} = \frac{z+1}{1}, \quad \lambda \geq 0 \quad \text{and} \quad L_1 : x + 1 = y - 1 = 4 - z = 2\sqrt{6}$$

If (α, β, γ) lies on L, then which of the following is NOT possible?

(1) $\alpha + 2\gamma = 24$

(2) $2\alpha + \gamma = 7$

(3) $2\alpha - \gamma = 9$

(4) $\alpha - 2\gamma = 19$

---

66. Let $y = f(x)$ represent a parabola with focus $\left(-\frac{1}{2}, 0\right)$ and directrix $y = -\frac{1}{2}$.

Then

$$S = \left\{x \in \mathbb{R} : \tan^{-1}\left(\sqrt{f(x)} + \sin\left(\sqrt{f(x)+1}\right)\right) = \frac{\pi}{2}\right\}$$

contains:

(1) Exactly two elements

(2) Exactly one element

(3) An infinite set

(4) An empty set

---

67. Let

$$A = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 4 & -1 \\ 0 & 12 & -3 \end{pmatrix}$$

Then the sum of the diagonal elements of the matrix $(A + I)^{11}$ is equal to:

(1) 6144

(2) 4094

(3) 4097

(4) 2050

---

68. Let R be a relation on $\mathbb{N} \times \mathbb{N}$ defined by

$$(a, b)\, R\, (c, d) \quad \text{if and only if} \quad ad(b - c) = bc(a - d).$$

Then R is:

(1) Symmetric but neither reflexive nor transitive

(2) Transitive but neither reflexive nor symmetric

(3) Reflexive and symmetric but not transitive

(4) Symmetric and transitive but not reflexive

69. Let

$$y = f(x) = \sin^3\left(\frac{\pi}{3}\cos\left(\frac{\pi}{3\sqrt{2}}\left(-4x^3+5x^2+1\right)^{\frac{3}{2}}\right)\right)$$

Then, at $x = 1$,

(1) $2y' + 3y = 0$

(2) $2y' + 3y' = 0$

(3) $y' - 3y' = 0$

(4) $y' + 3y' = 0$

---

70. If the sum and product of four positive consecutive terms of a G.P. are 126 and 1296, respectively, then the sum of common ratios of all such GPs is:

(1) 7

(2) $\frac{9}{2}$

(3) 3

(4) 14

---

71. The number of real roots of the equation

$$\sqrt{x^2-4x+3} + \sqrt{x^2-9} = \sqrt{4x^2-14x+6}$$

is:

(1) 0

(2) 1

(3) 3

(4) 2

---

72. Let a differentiable function f satisfy

$$f(x) + \int_3^x f(t)\,dt = \sqrt{x+1}, \quad x \geq 3.$$

Then $f(8)$ is equal to:

(1) 34

(2) 19

(3) 17

(4) 1

---

73. If the domain of the function

$f(x) = \frac{\lfloor x \rfloor}{1+x^2}$, **where** $\lfloor x \rfloor$ **is the greatest integer less than or equal to** x, **is** $[2, 6)$, **then its range is:**

(1) $\left[\frac{5}{26}, \frac{9}{27}\right]$

(2) $\left[\frac{5}{26}, \frac{9}{29}\right]$

(3) $\left[\frac{9}{37}, \frac{29}{109}\right]$

(4) $\left[\frac{5}{37}, \frac{53}{37}\right]$

---

74. Let $\mathbf{a} = 2\hat{i} + \hat{j} + \hat{k}$, and $\mathbf{b}$ and $\mathbf{c}$ be two nonzero vectors such that $|\mathbf{a} + \mathbf{b} + \mathbf{c}| = |\mathbf{a} + \mathbf{b} - \mathbf{c}|$ and $\mathbf{b} \cdot \mathbf{c} = 0$. Consider the following two statements:

(A) $|\mathbf{a} + \lambda\mathbf{c}| \geq |\mathbf{a}|$ for all $\lambda \in \mathbb{R}$.

(B) $\mathbf{a}$ and $\mathbf{c}$ are always parallel.

(1) only (B) is correct

(2) neither (A) nor (B) is correct

(3) only (A) is correct

(4) both (A) and (B) are correct

---

75. Let $\alpha \in (0, 1)$ and $\beta = \log(1 - \alpha)$. Let

$$P_n(x) = x + \frac{x^2}{2} + \frac{x^3}{3} + \cdots + \frac{x^n}{n}, \quad x \in (0, 1).$$

Then the integral

$$\int_0^{\alpha} \frac{1}{1-t}\, dt$$

is equal to:

(1) $\beta - P_0(\alpha)$

(2) $-(\beta + P_0(\alpha))$

(3) $P_0(\alpha) - \beta$

(4) $\beta + P_0(\alpha)$

---

76. If $\sin^{-1}\left(\frac{\alpha}{17}\right) + \cos^{-1}\left(\frac{4}{5}\right) - \tan^{-1}\left(\frac{77}{36}\right) = 0, \quad 0 < \alpha < 13$, **then** $\sin^{-1}(\sin\alpha) + \cos^{-1}(\cos\alpha)$ **is equal to:**

(1) π

(2) 16

(3) 0

(4) $16 - 5\pi$

---

77. Let a circle C_1 be obtained on rolling the circle

$x^2 + y^2 - 4x - 6y + 11 = 0$ upwards 4 units on the tangent T to it at the point (3, 2).

Let C_2 be the image of C_1 in T. Let A and B be the centers of circles C_1 and C_2 respectively, and M and N be respectively the feet of perpendiculars drawn from A and B on the x-axis. Then the area of the trapezium AMNB is:

(1) $2(2 + \sqrt{2})$

(2) $4(1 + \sqrt{2})$

(3) $3 + \sqrt{2}$

(4) $2(1 + \sqrt{2})$

---

78. (S1) $(p \Rightarrow q) \vee (p \wedge \neg q)$ **is a tautology (S2)** $(\neg p) \Rightarrow (\neg q) \wedge ((\neg p) \vee q)$ **is a contradiction. Then:**

(1) only (S2) is correct

(2) both (S1) and (S2) are correct

(3) both (S1) and (S2) are wrong

(4) only (S1) is correct

---

79. The value of

$$\int \frac{(2 + 3\sin x)}{\sin x(1 + \cos x)}\, dx$$

is equal to:

(1) $\frac{7}{2}\sqrt{3} - \log\sqrt{3}$

(2) $2 + 3\sqrt{3} + \log\sqrt{3}$

(3) $\frac{10}{3}\sqrt{3} - \log\sqrt{3}$

(4) $\sqrt{3} - \log\sqrt{3}$

---

80. A bag contains 6 balls. Two balls are drawn from it at random and both are found to be black. The probability that the bag contains at least 5 black balls is:

(1) $\frac{5}{7}$

(2) $\frac{2}{7}$

(3) $\frac{3}{7}$

(4) $\frac{5}{6}$

Section-B

81. Let 5 digit numbers be constructed using the digits

$0, 2, 3, 4, 7, 9$ with repetition allowed, and are arranged in ascending order with serial numbers.

Then the serial number of the number 42923 is:

---

82. Let $a_1, a_2, \ldots, a_n$ be in A.P. If $a_5 = 2a_1$ and $a_1 = 18$, then

$$12\left(\frac{1}{\sqrt{a_0}+\sqrt{a_1}} + \frac{1}{\sqrt{a_1}+\sqrt{a_2}} + \cdots + \frac{1}{\sqrt{a_{17}}+\sqrt{a_{18}}}\right)$$

is equal to:

83. Let

θ be the angle between the planes $P_1 : \vec{r}\cdot(\hat{i}+\hat{j}+2\hat{k}) = 9$ and $P_2 : \vec{r}\cdot(2\hat{i}-\hat{j}+\hat{k}) = 15$. **Let L be the line that meets P_2 at the point** (4, -2, 5) and makes an angle θ with the normal of P_2. If α is the angle between L and P_2, then $(\tan^2\theta)(\cot^2\alpha)$ is equal to:

---

84. Let $\alpha > 0$, be the smallest number such that the expansion of

$\left(\frac{3}{x^3} + \frac{2}{x}\right)^{30}$ has a term $\beta x^{-\alpha}, \beta \in \mathbb{N}$.

Then α is equal to:

---

85. Let $\vec{a}$ **and** $\vec{b}$ **be two vectors such that**

$$|\vec{a}| = \sqrt{14}, \quad |\vec{b}| = \sqrt{6}, \quad |\vec{a} \times \vec{b}| = \sqrt{48}.$$

Then $(\vec{a} \cdot \vec{b})^2$ **is equal to:**

---

86. Let the line $L : \frac{x-1}{2} = \frac{y+1}{-1} = \frac{z-3}{1}$ **intersect the plane**

$$2x + y + 3z = 16 \text{ at the point } P.$$

Let the point Q **be the foot of perpendicular from the point** R(1, -1, -3) on the line L. **If** α **is the area of triangle PQR, then** α^2 **is equal to:**

---

87. The remainder on dividing 5^{99} **by 11 is:**

---

88. If the variance of the frequency distribution

x_i	2	3	4	5	6	7	8
f_i	3	6	16	α	9	5	6

---

89. Let for $x \in \mathbb{R}$

$$f(x) = \frac{x + |x|}{2} \quad \text{for} \quad x \geq 0 \quad \text{and} \quad f(x) = \frac{x}{2} \quad \text{for} \quad x < 0$$

$$g(x) = \begin{cases} x^2 & \text{for} \quad x \geq 0, \\ x & \text{for} \quad x < 0 \end{cases}$$

Then the area bounded by the curve $y = f \circ g(x)$ and the lines $y = 0, 2y - x = 15$ is equal to:

---

90. Number of 4-digit numbers that are less than or equal to 2800 and either divisible by 3 or by 11, is equal to:
