JEE Main 2023 Feb 1 Shift 1 Question Paper with Solutions

Time Allowed :3 Hours	**Maximum Marks :**300	**Total Questions :**90

General Instructions

Read the following instructions very carefully and strictly follow them:

1. The test is of 3 hours duration.
2. The question paper consists of 90 questions, out of which 75 are to attempted. The maximum marks are 300.
3. There are three parts in the question paper consisting of Physics, Chemistry and Mathematics having 30 questions in each part of equal weightage.
4. Each part (subject) has two sections.
   (i) Section-A: This section contains 20 multiple choice questions which have only one correct answer. Each question carries 4 marks for correct answer and –1 mark for wrong answer.
   (ii) Section-B: This section contains 10 questions. In Section-B, attempt any five questions out of 10. The answer to each of the questions is a numerical value. Each question carries 4 marks for correct answer and –1 mark for wrong answer. For Section-B, the answer should be rounded off to the nearest integer

MATHEMATICS

Section-A

1. If

$$\lim_{n\to\infty}\left(\frac{1}{1+n}+\frac{1}{2+n}+\frac{1}{3+n}+\cdots+\frac{1}{2n}\right)$$

is equal to:

(1) 0

(2) $\log_e 2$

(3) $\log_e\left(\frac{3}{2}\right)$

(4) $\log_e\left(\frac{2}{3}\right)$

Correct Answer: (2) $\log_e 2$

Solution:

$$\lim_{n\to\infty}\sum_{r=1}^{n}\frac{1}{r+r}=\lim_{n\to\infty}\sum_{r=1}^{n}\frac{1}{1+r}$$

Using integration approximation:

$$\int_1^n\frac{dx}{1+x}=[\ln(1+x)]_1^n=\ln(1+n)-\ln(2)\approx\log 2$$

Quick Tip

For infinite series, integral approximation is a useful tool to estimate the limit.

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2. The negation of the expression

$$q\vee(\neg q\wedge p)$$

is equivalent to:

(1) $(\neg p)\wedge(\neg q)$

(2) $p\wedge(\neg q)$

(3) $(\neg p)\vee(\neg q)$

(4) $(\neg p)\vee q$

Correct Answer: (1) $(\neg p) \wedge (\neg q)$

Solution:

$$\neg(q \vee (\neg q \wedge p))$$

$$= \neg q \wedge \neg(\neg q \wedge p)$$

$$= \neg q \wedge (\neg\neg q \vee \neg p) = \neg q \wedge (q \vee \neg p)$$

$$= (\neg q \wedge q) \vee (\neg q \wedge \neg p) = 0 \vee (\neg q \wedge \neg p)$$

$$= (\neg p) \wedge (\neg q)$$

Quick Tip

For logical expressions, use De Morgan's laws and distributive properties to simplify.

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3. In a binomial distribution $B(n, p)$, the sum and product of the mean and variance are 5 and 6, respectively. Then $6(n + p - q)$ is equal to:

(1) 51

(2) 52

(3) 53

(4) 50

Correct Answer: (2) 52

Solution:

Step 1: Using the formulas for mean and variance in a binomial distribution:

$$\text{Mean} = np, \quad \text{Variance} = npq$$

Given:

$$np + npq = 5, \quad npq = 6$$

$$np(1 + q) = 5, \quad 6(np^2 q) = 6$$

$$(1 + q)^2 = 25 \Rightarrow q^2 + 12q + 6 = 25q$$

Solving for q, we get:

$$q = \frac{2}{3}$$

$$p = \frac{1}{3}$$

$$n = 9$$

Now, computing $6(n + p - q)$:

$$6(9 + \frac{1}{3} - \frac{2}{3}) = 52$$

Quick Tip

For binomial distributions, use the given sum and product conditions to find individual values for n, p, q.

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4. The sum to 10 terms of the series

$$\frac{1}{1 + 1^2 + 1^4} + \frac{2}{1 + 2^2 + 2^4} + \frac{3}{1 + 3^2 + 3^4} + \dots$$

is:

(1) $\frac{59}{111}$

(2) $\frac{55}{111}$

(3) $\frac{56}{111}$

(4) $\frac{58}{111}$

Correct Answer: (2) $\frac{55}{111}$

Solution:

Using partial fractions:

$$T_r = \frac{x^2 + r + 1 - (x^2 - r + 1)}{2(r^2 + r + 1)}$$

$$T_1 = \frac{1}{2}\left[\frac{1}{1} - \frac{1}{3}\right]$$

$$T_2 = \frac{1}{2}\left[\frac{1}{2} - \frac{1}{7}\right]$$

$$T_{10} = \frac{1}{2}\left[\frac{1}{29} - \frac{1}{111}\right]$$

Summing over 10 terms:

$$\sum_{r=1}^{10} T_r = \frac{55}{111}$$

Quick Tip

For summation of series, express the terms in partial fractions to simplify calculations.

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5. The value of

$$\frac{1}{1!50!} + \frac{1}{3!48!} + \frac{1}{5!46!} + \cdots + \frac{1}{49!2!} + \frac{1}{51!1!}$$

is:

(1) $\frac{2^{50}}{50!}$

(2) $\frac{2^{50}}{51!}$

(3) $\frac{2^{51}}{51!}$

(4) $\frac{2^{51}}{50!}$

Correct Answer: (2) $\frac{2^{50}}{51!}$

Solution:

$$\sum_{r=1}^{26} \frac{1}{(2r-1)!(51-(2r-1))!} = \sum_{r=1}^{26} \binom{51}{2r-1} \frac{1}{51!}$$

$$= \frac{1}{51!} \sum_{r=1}^{26} \binom{51}{2r-1} = \frac{1}{51!}(2^{50})$$

Quick Tip

For binomial sums involving factorials, expressing terms using combinations simplifies evaluation.

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6. If the orthocentre of a triangle, whose vertices are $(1, 2)$, $(2, 3)$, and $(3, 1)$ **is** (α, β), **then the quadratic equation whose roots are** $\alpha + 4\beta$ **and** $4\alpha + \beta$ **is:**

(1) $x^2 - 19x + 90 = 0$

(2) $x^2 - 18x + 80 = 0$

(3) $x^2 - 22x + 120 = 0$

(4) $x^2 - 20x + 99 = 0$

Correct Answer: (4) $x^2 - 20x + 99 = 0$

Solution:

Step 1: Finding slopes of perpendicular altitudes:

$$m = \frac{-1}{2} = -\frac{1}{2}$$

Here $m_{BH} \times m_{AC} = -1$

$$\beta - 3 = 2\alpha - 4$$

$$\beta = 2\alpha - 1$$

Step 2: Using midpoint formula and relations, we find:

$$\alpha = \frac{5}{3}, \quad \beta = \frac{7}{3}$$

Step 3: Now, computing roots:

$$\alpha + 4\beta = 11, \quad 4\alpha + \beta = 9$$

$$x^2 - (11 + 9)x + 11 \times 9 = 0$$

$$x^2 - 20x + 99 = 0$$

Quick Tip

For orthocenter problems, use perpendicular slopes and midpoint relations effectively.

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1. For a triangle ABC, the value of

$$\cos 2A + \cos 2B + \cos 2C$$

is least. If its inradius is 3 and incentre is M, then which of the following is NOT correct?

(1) Perimeter of $\triangle ABC$ is $18\sqrt{3}$

(2) $\sin 2A + \sin 2B + \sin 2C = \sin A + \sin B + \sin C$

(3) $\overline{MA} \cdot \overline{MB} = -18$

(4) Area of $\triangle ABC$ is $\frac{27\sqrt{3}}{2}$

Correct Answer: (4) $\frac{27\sqrt{3}}{2}$

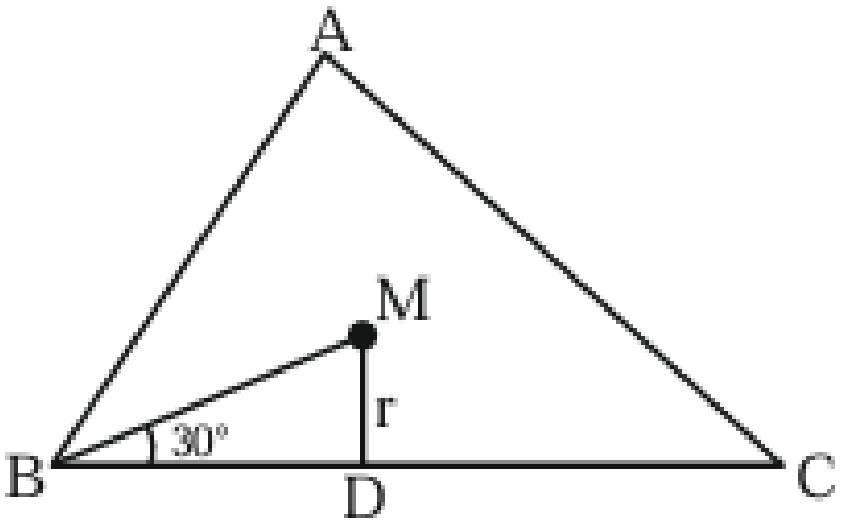
Solution:

If $\cos 2A + \cos 2B + \cos 2C$ is minimum, then

$$A = 60^\circ, \quad B = C = 60^\circ$$

$$\Rightarrow \triangle ABC \text{ is equilateral}$$

$$\text{In-radius} = r = 3$$



$$\text{Area} = \frac{\sqrt{3}}{4}a^2 = 27\sqrt{3}$$

$$\text{Thus, area is incorrectly given as } \frac{27\sqrt{3}}{2}$$

Quick Tip

For equilateral triangles, the area formula is $\frac{\sqrt{3}}{4}a^2$.

8. The combined equation of the two lines

$$ax + by + c = 0 \quad \text{and} \quad a'x + b'y + c' = 0$$

can be written as

$$(ax + by + c)(a'x + b'y + c') = 0$$

The equation of the angle bisectors of the lines represented by the equation

$$2x^2 + xy - 3y^2 = 0$$

is:

(1) $3x^2 + 5xy + 2y^2 = 0$

(2) $x^2 - y^2 + 10xy = 0$

(3) $3x^2 + xy - 2y^2 = 0$

(4) $x^2 - y^2 - 10xy = 0$

Correct Answer: (4) $x^2 - y^2 - 10xy = 0$

Solution:

Step 1: Equation of the pair of angle bisectors for the homogeneous equation

$$ax^2 + 2hxy + by^2 = 0$$

is given by:

$$\frac{x^2 - y^2}{a - b} = \frac{xy}{h}$$

Step 2: Here,

$$a = 2, \quad h = \frac{1}{2}, \quad b = -3$$

Thus, equation becomes:

$$\frac{x^2 - y^2}{2 - (-3)} = \frac{xy}{\frac{1}{2}}$$

$$x^2 - y^2 - 10xy = 0$$

Quick Tip

For angle bisectors of conic sections, use the standard formula and compare coefficients.

9. The shortest distance between the lines

$$\frac{x-5}{1}=\frac{y-2}{2}=\frac{z-4}{-3}, \quad \frac{x+3}{1}=\frac{y+5}{4}=\frac{z-1}{-5}$$

is:

(1) $7\sqrt{3}$

(2) $5\sqrt{3}$

(3) $6\sqrt{3}$

(4) $4\sqrt{3}$

Correct Answer: (3) $6\sqrt{3}$

Solution:

Step 1: Shortest distance formula between skew lines:

$$d=\frac{|(a_1b_2-a_2b_1)+(a_2b_3-a_3b_2)+(a_3b_1-a_1b_3)|}{\sqrt{(a_1-a_2)^2+(b_1-b_2)^2+(c_1-c_2)^2}}$$

Step 2: Substituting values,

$$d=\frac{8(-10+12)-7(-5+3)+3(4-2)}{\sqrt{4+4+4}}$$
$$=\frac{16+14+6}{\sqrt{12}}=\frac{36}{\sqrt{12}}=\frac{36}{2\sqrt{3}}$$
$$=\frac{18}{\sqrt{3}}=6\sqrt{3}$$

Quick Tip

For shortest distance problems, identify direction ratios and apply vector formulas systematically.

10. Let S denote the set of all real values of λ such that the system of equations

$$\lambda x+y+z=1$$

$$x+\lambda y+z=1$$

$$x + y + \lambda z = 1$$

is inconsistent. Then

$$\sum_{\lambda \in S} (|\lambda|^2 + |\lambda|)$$

is equal to:

(1) 2

(2) 12

(3) 4

(4) 6

Correct Answer: (4) 6

Solution:

Step 1: Solving determinant:

$$\begin{vmatrix} \lambda & 1 & 1 \\ 1 & \lambda & 1 \\ 1 & 1 & \lambda \end{vmatrix} = 0$$

Expanding:

$$(\lambda + 2)(\lambda - 1)^2 - 1(\lambda - 1) + (1 - \lambda) = 0$$

Step 2: Finding roots,

$$\lambda = -2, \quad \lambda = 1$$

$$\sum_{\lambda \in S} (|\lambda|^2 + |\lambda|) = 6$$

Quick Tip

For system consistency, solve determinant equations and analyze root conditions.

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11. Let

$$S = \left\{ x : x \in \mathbb{R} \text{ and } (\sqrt{3} + \sqrt{2})^{x-4} + (\sqrt{3} - \sqrt{2})^{x-4} = 10 \right\}$$

Then $|S|$ is equal to:

(1) 2

(2) 4

(3) 6

(4) 0

Correct Answer: (2) 4

Solution:

Let $(\sqrt{3}+\sqrt{2})^{x-4} = t$, then

$$t + \frac{1}{t} = 10$$

Solving for t, we get

$$t = 5 + 2\sqrt{6}, \quad t = 5 - 2\sqrt{6}$$

Thus,

$$(\sqrt{3}+\sqrt{2})^{x-4} = 5 + 2\sqrt{6}, \quad 5 - 2\sqrt{6}$$

Squaring both sides,

$$x^2 - 4 = 2, -2 \Rightarrow x^2 = 6, 2$$

$$x = \pm\sqrt{2}, \pm\sqrt{6}$$

Quick Tip

When solving exponential equations, try using substitutions to simplify the exponent expressions.

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12. Let S be the set of all solutions of the equation

$$\cos^{-1}(2x) - 2\cos^{-1}(\sqrt{1-x^2}) = \pi, \quad x \in \left[-\frac{1}{2}, \frac{1}{2}\right].$$

Then

$$\sum_{x \in S} 2\sin^{-1}(x^2 - 1)$$

is equal to:

(1) 0

(2) $\frac{-2\pi}{3}$

(3) $\pi - \sin^{-1}\left(\frac{\sqrt{3}}{4}\right)$

(4) $\pi - 2\sin^{-1}\left(\frac{\sqrt{3}}{4}\right)$

Correct Answer: (1) 0

Solution:

Rewriting the equation,

$$\cos^{-1}(2x) = \pi + 2\cos^{-1}(\sqrt{1-x^2})$$

Since the LHS belongs to the interval $[0, \pi]$, the equation must be meaningful. Solving,

$$\cos^{-1}(2x) = \pi, \quad \cos^{-1}(\sqrt{1-x^2}) = 0$$

which gives

$$x = -\frac{1}{2}, \quad x = 0$$

Now sum over the set:

$$\sum_{x \in S} 2\sin^{-1}(x^2 - 1) = 0$$

Quick Tip

For inverse trigonometric functions, always check the domain constraints before solving.

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13. If the center and radius of the circle

$$\left|\frac{z-2}{z-3}\right| = 2$$

are respectively (α, β) **and** γ**, then**

$$3(\alpha + \beta + \gamma)$$

is equal to:

(1) 11

(2) 9

(3) 10

(4) 12

Correct Answer: (4) 12

Solution:

Using transformation:

$$\sqrt{(x-2)^2+y^2} = 2\sqrt{(x-3)^2+y^2}$$

Squaring both sides and simplifying,

$$x^2 + y^2 - 20x + 32 = 0$$

Comparing with standard form,

$$\alpha = \frac{10}{3}, \quad \beta = 0$$

$$\gamma = \sqrt{\frac{100}{9} + \frac{32}{3}} = \frac{2}{3}$$

$$3(\alpha+\beta+\gamma) = 3\left(\frac{10}{3} + 0 + \frac{2}{3}\right) = 12$$

Quick Tip

For modulus-based circle problems, use transformation equations and algebraic manipulation.

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14. If $y = y(x)$ is the solution curve of the differential equation

$$\frac{dy}{dx} + y\tan x = x\sec x, \quad 0 \le x \le \frac{\pi}{3}$$

and $y(0) = 1$, then $y\left(\frac{\pi}{6}\right)$ is equal to:

(1) $\frac{\pi}{12} + \frac{\sqrt{3}}{2}\log_e\left(\frac{2}{e\sqrt{3}}\right)$

(2) $\frac{\pi}{12} + \frac{\sqrt{3}}{2}\log_e\left(\frac{2\sqrt{3}}{e}\right)$

(3) $\frac{\pi}{12} - \frac{\sqrt{3}}{2}\log_e\left(\frac{2\sqrt{3}}{e}\right)$

(4) $\frac{\pi}{12} + \frac{\sqrt{3}}{2}\log_e\left(\frac{2}{e\sqrt{3}}\right)$

Correct Answer: (1)

Solution:

The integrating factor (I.F.) for this differential equation is:

$$I.F. = \sec x$$

Multiplying throughout by $\sec x$, we get:

$$\frac{d}{dx}(y \sec x) = x$$

Integrating both sides:

$$y \sec x = x \tan x - \ln(\sec x) + C$$

Using the given initial condition $y(0) = 1$, we determine $C = 1$.

$$y(\sec x) = x \tan x - \ln(\sec x) + 1$$

Substituting $x = \frac{\pi}{6}$:

$$y = \frac{\pi}{12} + \frac{\sqrt{3}}{2} \log_e \left(\frac{2}{e\sqrt{3}} \right)$$

Quick Tip

For first-order linear differential equations, always compute the integrating factor and solve step-by-step.

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15. Let R be a relation on $\mathbb{R}$, given by

$$R = \{(a, b) : 3a - 3b + \sqrt{7} \text{ is an irrational number}\}$$

Then R is:

(1) Reflexive but neither symmetric nor transitive

(2) Reflexive and transitive but not symmetric

(3) Reflexive and symmetric but not transitive

(4) An equivalence relation

Correct Answer: (1) Reflexive but neither symmetric nor transitive

Solution:

Check for reflexivity:

Since

$$3(a-a)+\sqrt{7}=\sqrt{7}$$

which is irrational, the relation is reflexive.

Check for symmetry:

Let $a=\frac{\sqrt{7}}{3}, b=0$. Then $(a,b)\in R$ but $(b,a)\notin R$.

Since

$$3(b-a)+\sqrt{7}=0$$

which is rational, the relation is not symmetric.

Check for transitivity:

Let $(a,b)=\left(1,\frac{2\sqrt{7}}{3}\right)$ and $(b,c)=\left(1,\frac{2\sqrt{7}}{3}\right)$. Then, $(a,b)\in R$ and $(b,c)\in R$, but $(a,c)\notin R$, so the relation is not transitive.

Quick Tip

To check for equivalence relations, verify reflexivity, symmetry, and transitivity systematically.

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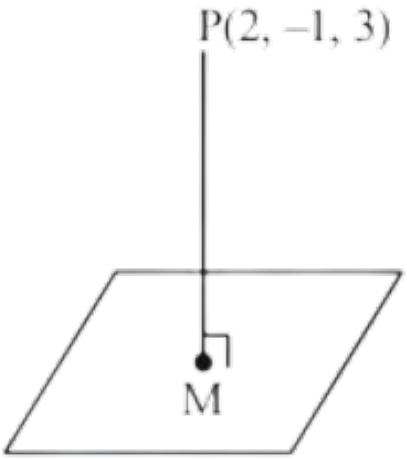
16. Let the image of the point $P(2,-1,3)$ in the plane

$$x+2y-z=0$$

be Q. Then the distance of the plane

$$3x+2y+z+29=0$$

from the point Q is:



(1) $\frac{22\sqrt{2}}{7}$

(2) $\frac{24\sqrt{2}}{7}$

(3) $2\sqrt{14}$

(4) $3\sqrt{14}$

Correct Answer: (4) $3\sqrt{14}$

Solution:

Equation of line PM:

$$\frac{x-2}{1} = \frac{y+1}{2} = \frac{z-3}{-1} = \lambda$$

Any point on this line is:

$$(\lambda + 2, 2\lambda - 1, -\lambda + 3)$$

Solving for λ, we get $\lambda = \frac{1}{2}$. Thus, the midpoint M is:

$$\left(\frac{5}{2}, 0, \frac{5}{2}\right)$$

For the image $Q(\alpha, \beta, \gamma)$, we compute:

$$\alpha + 2 = \frac{5}{2}, \quad \beta - 1 = 0, \quad \gamma + 3 = \frac{5}{2}$$

$$\alpha = \frac{1}{2}, \quad \beta = 1, \quad \gamma = -\frac{1}{2}$$

Using the distance formula:

$$d = \frac{|3(3) + 2(1) + (-1)(-1) + 29|}{\sqrt{3^2 + 2^2 + (-1)^2}}$$

$$= \frac{42}{\sqrt{14}} = 3\sqrt{14}$$

Quick Tip

For reflections in a plane, find the midpoint of the perpendicular segment and use distance formulas.

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17. Let

$$f(x) = \begin{bmatrix} 1+\sin^2 x & \cos^2 x & \sin 2x \\ \sin^2 x & 1+\cos^2 x & \sin 2x \\ \sin^2 x & \cos^2 x & 1+\sin 2x \end{bmatrix}$$

where $x \in \left[\frac{\pi}{6}, \frac{\pi}{3}\right]$. **If** α, β **are respectively the maximum and the minimum values of** f, **then**

(1) $\beta^2 - 2\sqrt{\alpha} = \frac{19}{4}$

(2) $\beta^2 + 2\sqrt{\alpha} = \frac{19}{4}$

(3) $\alpha^2 - \beta^2 = 4\sqrt{3}$

(4) $\alpha^2 + \beta^2 = \frac{9}{2}$

Correct Answer: (1) $\beta^2 - 2\sqrt{\alpha} = \frac{19}{4}$

Solution:

Applying column operations:

$$C_1 \to C_1 + C_2 + C_3$$

$$f(x) = \begin{bmatrix} 2+\sin 2x & \cos^2 x & \sin 2x \\ 2+\sin 2x & 1+\cos^2 x & \sin 2x \\ 2+\sin 2x & \cos^2 x & 1+\sin 2x \end{bmatrix}$$

Subtracting rows:

$$R_2 \to R_2 - R_1, \quad R_3 \to R_3 - R_1$$

$$f(x) = (2+\sin 2x) \begin{bmatrix} 1 & \cos^2 x & \sin 2x \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= (2 + \sin 2x)(1)$$

$$= 2 + \sin 2x$$

For given range $x \in \left[\frac{\pi}{6}, \frac{\pi}{3}\right]$:

$$\sin 2x \in \left[\frac{\sqrt{3}}{2}, 1\right]$$

Thus,

$$2 + \sin 2x \in \left[2 + \frac{\sqrt{3}}{2}, 3\right]$$

$$\alpha = 3, \quad \beta = 2 + \frac{\sqrt{3}}{2}$$

$$\beta^2 - 2\sqrt{\alpha} = \frac{19}{4}$$

Quick Tip

For determinant problems involving matrices, applying column/row operations simplifies the evaluation.

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18. Let

$$f(x) = 2x + \tan^{-1} x, \quad g(x) = \log_e(\sqrt{1 + x^2} + x)$$

where $x \in [0, 3]$. **Then:**

(1) There exists $x \in [0, 3]$ such that $f'(x) < g'(x)$

(2) $\max f(x) > \max g(x)$

(3) There exist $0 < x_1 < x_2 < 3$ such that $f(x) < g(x)$, $\forall x \in (x_1, x_2)$

(4) $\min f'(x) = 1 + \max g'(x)$

Correct Answer: (2)

Solution:

Computing derivatives:

$$g'(x) = \frac{1}{\sqrt{1 + x^2} + x}$$

Since $g(x)$ is always increasing in $[0, 3]$,

$$\min g'(x) = \frac{1}{\sqrt{10} + 3}, \quad \max g'(x) = \frac{1}{\sqrt{1} + 0} = 1$$

For $f(x)$,

$$f(x) = 2x + \tan^{-1} x$$

Since $f(x)$ is increasing,

$$\max f(x) = 6 + \tan^{-1} 3$$

For $g(x)$,

$$\max g(x) = \ln(3 + \sqrt{10})$$

$$6 + \tan^{-1} 3 > \ln(3 + \sqrt{10})$$

Quick Tip

Compare maximum values by checking function growth using derivatives.

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19. The mean and variance of 5 observations are 5 and 8, respectively. If 3 observations are 1, 3, 5, then the sum of cubes of the remaining two observations is:

(1) 1072

(2) 1792

(3) 1216

(4) 1456

Correct Answer: (1) 1072

Solution:

Using the mean formula:

$$\frac{1 + 3 + 5 + a + b}{5} = 5$$

$$a + b = 16$$

Using variance formula:

$$\sigma^2 = \frac{\sum x_i^2}{5} - \left(\frac{\sum x_i}{5}\right)^2$$

$$8 = \frac{1^2 + 3^2 + 5^2 + a^2 + b^2}{5} - 25$$

$$a^2 + b^2 = 130$$

Solving equations:

$$a = 7, b = 9 \quad \text{or} \quad a = 9, b = 7$$

Computing sum of cubes:

$$a^3 + b^3 = 7^3 + 9^3 = 1072$$

Quick Tip

For mean and variance problems, use algebraic equations to find missing values and their statistical properties.

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20. The area enclosed by the closed curve C given by the differential equation

$$\frac{dy}{dx} + \frac{x + a}{y - 2} = 0, \quad y(1) = 0$$

is 4π. **Let P and Q be the points of intersection of the curve C and the y-axis. If normals at P and Q on the curve C intersect the x-axis at points R and S respectively, then the length of the line segment RS is:**

(1) $2\sqrt{3}$

(2) $\frac{2\sqrt{3}}{3}$

(3) 2

(4) $\frac{4\sqrt{3}}{3}$

Correct Answer: (4) $\frac{4\sqrt{3}}{3}$

Solution:

Rewriting the given differential equation:

$$\frac{dy}{dx} = \frac{x + a}{2 - y}$$

Separating variables and integrating,

$$(2 - y)dy = (x + a)dx$$

$$2y - \frac{y^2}{2} = \frac{x^2}{2} + ax + C$$

Given $y(1) = 0$, solving for C,

$$a + C = -\frac{1}{2}$$

For the enclosed area,

$$X^2 + y^2 + 2ax - 4y - 1 - 2a = 0$$

Using standard circle form,

$$\pi r^2 = 4\pi$$

$$r^2 = 4$$

Solving for a,

$$4 = \sqrt{a^2 + 4 + 1 + 2a}$$

$$(a + 1)^2 = 0$$

Quick Tip

For area-related differential equations, solving for constants and using circle properties helps find enclosed regions.

Section-B

21. Let $a_1 = 8, a_2, a_3, \ldots a_n$ be an A.P. If the sum of its first four terms is 50 and the sum of its last four terms is 170, then the product of its middle two terms is:

Solution:

Using the sum formula for A.P.,

$$a_1 + a_2 + a_3 + a_4 = 50$$

$$8 + (8 + d) + (8 + 2d) + (8 + 3d) = 50$$

$$32 + 6d = 50 \Rightarrow d = 3$$

For the last four terms,

$$a_{n-3} + a_{n-2} + a_{n-1} + a_n = 170$$

$$32 + (4n - 10) \cdot 3 = 170$$

$$n = 14$$

Middle terms are:

$$a_7 = 26, \quad a_8 = 29$$

$$\Rightarrow a_7 \cdot a_8 = 754$$

Quick Tip

For A.P. problems, use the sum formula systematically and relate terms for efficient solving.

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22. A(2, 6, 2), B(-4, 0, λ), C(2, 3, -1) and D(4, 5, 0), where $|\lambda| \leq 5$ are the vertices of a quadrilateral ABCD. If its area is 18 square units, then

$5 - 6\lambda$ is equal to:

Solution:

Using the area formula for a quadrilateral in 3D:

$$\text{Area} = \frac{1}{2}\left|\overrightarrow{BD} \times \overrightarrow{AC}\right|$$

Computing cross product:

$$\overrightarrow{AC} \times \overrightarrow{BD} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & -3 & -3 \\ 8 & 5 & -\lambda \end{vmatrix}$$

Expanding determinant:

$$= (3\lambda + 15)\hat{i} - (-24)\hat{j} + (-24)\hat{k}$$

$$= (3\lambda + 15)\hat{i} + 24\hat{j} - 24\hat{k}$$

$$\left|(3\lambda + 15)^2 + 24^2 + (-24)^2\right| = 36$$

$$\lambda^2 + 10\lambda + 9 = 0$$

Solving for λ:

$$\lambda = -1, -9$$

Since $|\lambda| \leq 5$, we take $\lambda = -1$,

$$5 - 6(-1) = 11$$

Quick Tip

For area problems in 3D, use determinant expansion carefully for accurate cross-product computations.

---

23. The number of 3-digit numbers that are divisible by either 2 or 3 but not divisible by 7 is:

Solution:

Total numbers divisible by:

$$\text{By } 2 = 450, \quad \text{By } 3 = 300, \quad \text{By } 7 = 128$$

$$\text{By 2 and 7} = 64, \quad \text{By 3 and 7} = 43, \quad \text{By 2, 3, and 7} = 21$$

Applying the inclusion-exclusion principle,

$$450 + 300 - 150 - 64 - 43 + 21 = 514$$

Quick Tip

For counting problems involving multiple divisibility rules, use the inclusion-exclusion principle.

---

24. The remainder when $19^{200} + 23^{200}$ is divided by 49, is:

Solution:

Using modular arithmetic,

$$(21 + 2)^{200} + (21 - 2)^{200}$$

Applying binomial expansion,

$$2 \sum_{k=0}^{100} \binom{200}{2k} 21^{198-2k} 2^{2k}$$

$$\Rightarrow 2[49I_1 + 2^{200}]$$

Since $2^{200} = 49L + 470$,

$$\text{Remainder} = 49L + 470 \mod 49 = 29$$

Quick Tip

For large power modular problems, express numbers in terms of mod bases and apply binomial theorem.

---

25. If

$$\int_0^1 (x^{21} + x^4 + x^7)(2x^{14} + 3x^7 + 6)^{1/7} dx = \frac{1}{7}(11)^{m/n}$$

where $l, m, n \in \mathbb{N}$, **and** m, n **are coprime, then**

$$l + m + n \text{ is equal to:}$$

Solution:

Applying integral transformation:

$$\int_0^1 x^l (2x^{14} + 3x^7 + 6)^{1/7} dx$$

Setting $t = 42(x^{20} + x^{13} + x^6)dx$,

$$\frac{1}{42}\int_0^{11} t^7 dt$$

$$= \frac{1}{48}(11^{8/7})$$

$$l = 48, \quad m = 8, \quad n = 7$$

$$l + m + n = 63$$

Quick Tip

For power integrals, use substitution and exponent transformation techniques.

---

26. If

$$f(x) = x^2 + g'(1)x + g''(2)$$

and

$$g(x) = f(1)x^2 + xf'(x) + f''(x),$$

then the value of $f(4) - g(4)$ **is equal to:**

Solution:

Computing derivatives:

$$f'(x) = 2x + g'(1), \quad f''(x) = 2$$

Solving for $g(x)$,

$$g(x) = f(1)x^2 + x[2x + g'(1)] + 2$$

$$g'(x) = 2f(1)x + 4x + g'(1)$$

$$g''(x) = 2f(1) + 4$$

Setting boundary conditions:

$$f(1) = -2, \quad g'(1) = -3$$

$$f(x) = x^2 - 3x$$

$$g(x) = -3x + 2$$

$$f(4) - g(4) = 14$$

Quick Tip

For function transformations, differentiate and equate known conditions for solving unknowns.

---

27. Let

$$\vec{v} = \alpha\hat{i} + 2\hat{j} - 3\hat{k}, \quad \vec{w} = 2\alpha\hat{i} + \hat{j} - \hat{k}, \quad \text{and } \hat{u} \text{ be a vector such that } |\hat{u}| = \alpha > 0.$$

If the minimum value of the scalar triple product

$$[\vec{u} \quad \vec{v} \quad \vec{w}]$$

is $-\alpha\sqrt{3401}$, and

$$|\hat{u} \cdot \hat{i}|^2 = \frac{m}{n}$$

where m and n are coprime natural numbers, then

$m + n$ is equal to:

Solution:

Computing the scalar triple product:

$$[\vec{u} \quad \vec{v} \quad \vec{w}] = \hat{u} \cdot (\vec{v} \times \vec{w})$$

Given that the minimum value is $-\alpha\sqrt{3401}$,

$$\cos\theta = -1$$

$$|\hat{u}| = \alpha$$

Computing cross product:

$$\vec{v} \times \vec{w} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \alpha & 2 & -3 \\ 2\alpha & 1 & -1 \end{vmatrix}$$

Expanding the determinant:

$$\vec{v} \times \vec{w} = \hat{i}(-2+3) - \hat{j}(\alpha + 6\alpha) + \hat{k}(\alpha - 4\alpha)$$

$$= \hat{i} - 5\alpha\hat{j} - 3\alpha\hat{k}$$

Computing magnitude:

$$|\vec{v} \times \vec{w}| = \sqrt{1 + 25\alpha^2 + 9\alpha^2} = \sqrt{3401}$$

Equating for α^2,

$$34\alpha^2 = 3400$$

$$\alpha^2 = 100 \Rightarrow \alpha = 10$$

Thus,

$$\hat{u} = \lambda(\hat{i} - 5\alpha\hat{j} - 3\alpha\hat{k})$$

Computing:

$$|\hat{u} \cdot \hat{i}|^2 = \frac{100}{3401} = \frac{m}{n}$$

Since $m = 100, n = 3401$,

$$m + n = 3501$$

Quick Tip

For vector triple products, always compute the cross product first and then take the dot product with the given vector.

---

28. The number of words, with or without meaning, that can be formed using all the letters of the word ASSASSINATION so that the vowels occur together, is:

Solution:

Grouping all vowels together as a single unit:

Vowels: A, A, A, I, I, O

Consonants: S, S, S, S, N, N, T

Total number of ways vowels occur together:

$$= \frac{8!}{4!2!} \times \frac{6!}{3!2!} = 50400$$

Quick Tip

For permutation problems with identical objects, use the formula $\frac{n!}{p!q!r!}$ where p, q, r are repeated elements.

---

29. Let A be the area bounded by the curve

$$y = x|x - 3|$$

the x-axis, and the ordinates $x = -1$ and $x = 2$. Then $12A$ is equal to:

Solution:

Computing the area:

$$A = \int_{-1}^{0} (x^2 - 3x)dx + \int_{0}^{2} (3x - x^2)dx$$

$$A = \left[\frac{x^3}{3} - \frac{3x^2}{2}\right]_{-1}^{0} + \left[\frac{3x^2}{2} - \frac{x^3}{3}\right]_{0}^{2}$$

$$A = \frac{11}{6} + \frac{10}{3} - \frac{31}{6}$$

$$\Rightarrow 12A = 62$$

Quick Tip

For area calculation using integrals, split the function where it changes form and integrate separately.

---

30. Let $f : \mathbb{R} \to \mathbb{R}$ be a differentiable function such that

$$f'(x) + f(x) = \int_{0}^{2} f(t)dt.$$

If $f(0) = e^{-2}$, then

$2f(0) - f(2)$ is equal to:

Solution:

Given the differential equation:

$$\frac{dy}{dx} + y = k$$

Multiplying by the integrating factor e^x:

$$ye^x = k \cdot e^x + c$$

Using initial condition $f(0) = e^{-2}$:

$$c = e^{-2} - k$$

Thus, the general solution is:

$$y = k + (e^{-2} - k)e^{-x}$$

Integrating from 0 to 2:

$$k = \int_0^2 \left(k + (e^{-2} - k)e^{-x}\right) dx$$

Solving,

$$k = e^{-2} - 1$$

$$y = (e^{-2} - 1) + e^{-x}$$

Evaluating at $x = 2$:

$$f(2) = 2e^{-2} - 1, \quad f(0) = e^{-2}$$

$$2f(0) - f(2) = 1$$

Quick Tip

For first-order linear differential equations, use the integrating factor method and apply initial conditions to find constants.

---

31. Match the List I with List II:

List I	List II
A. Intrinsic Semiconductor	I. Fermi-level near conduction band
B. n-type Semiconductor	II. Fermi-level at middle
C. p-type Semiconductor	III. Fermi-level near valence band
D. Metals	IV. Fermi-level inside conduction band

Choose the correct answer from the options given below:

(1) (A) $\rightarrow$ I, (B) $\rightarrow$ II, (C) $\rightarrow$ III, (D) $\rightarrow$ IV

(2) (A) $\rightarrow$ II, (B) $\rightarrow$ I, (C) $\rightarrow$ III, (D) $\rightarrow$ IV

(3) (A) $\rightarrow$ II, (B) $\rightarrow$ III, (C) $\rightarrow$ I, (D) $\rightarrow$ IV

(4) (A) $\rightarrow$ III, (B) $\rightarrow$ I, (C) $\rightarrow$ II, (D) $\rightarrow$ IV

Correct Answer: (3)

Solution:

Based on semiconductor theory, the correct matching is determined by analyzing the Fermi level positions in different types of materials.

Quick Tip

Understanding the position of the Fermi level is key to distinguishing different types of semiconductors and metals.

---

32. An object moves with speed v_1, v_2, v_3 along a line segment AB, BC, and CD respectively as shown in the figure. Where AB = BC and AD = 3AB, then the average speed of the object will be:

(1) $\frac{(v_1+v_2+v_3)}{3}$

(2) $\frac{v_1v_2v_3}{3(v_1v_2+v_2v_3+v_3v_1)}$

(3) $\frac{3v_1v_2v_3}{v_1v_2+v_2v_3+v_3v_1}$

(4) $\frac{(v_1+v_2+v_3)}{3v_1v_2v_3}$

Correct Answer: (3)

Solution:

Given: $AB = x$, $BC = x$, $AD = 3x$

$$\text{Total Distance} = 3x$$

$$\text{Total Time} = \frac{x}{v_1} + \frac{x}{v_2} + \frac{x}{v_3}$$

$$\text{Average Speed} = \frac{3x}{\frac{x}{v_1} + \frac{x}{v_2} + \frac{x}{v_3}} = \frac{3v_1v_2v_3}{v_1v_2 + v_2v_3 + v_3v_1}$$

Quick Tip

For average speed, use the formula:

$$\text{Average Speed} = \frac{\text{Total Distance}}{\text{Total Time}}$$

---

33. Given below are two statements:

Statement-I: Acceleration due to gravity is different at different places on the surface of Earth.

Statement-II: Acceleration due to gravity increases as we go down below the Earth's surface.

Choose the correct answer from the options given below:

(1) Both Statement I and Statement II are true

(2) Both Statement I and Statement II are false

(3) Statement I is true but Statement II is false

(4) Statement I is false but Statement II is true

Correct Answer: (3)

Solution:

Acceleration due to gravity is given by:

$$g_{\text{eff}} = g - \omega^2 R \sin^2 \theta$$

which depends on latitude.

Inside the Earth, gravity decreases with depth as:

$$g_{\text{eff}} = g\left(1 - \frac{d}{R}\right), \quad d \text{ is depth.}$$

Thus, Statement I is true but Statement II is false.

Quick Tip

Gravity varies with altitude, latitude, and depth.

---

34. Match the List I with List II:

List I	List II
A. AC generator	I. Presence of both L and C
B. Transformer	II. Electromagnetic Induction
C. Resonance Phenomenon	III. Quality Factor
D. Sharpness of Resonance	IV. Mutual Inductance

Choose the correct answer from the options given below:

(1) A-IV, B-II, C-I, D-III

(2) A-II, B-I, C-III, D-IV

(3) A-II, B-IV, C-I, D-III

(4) A-IV, B-III, C-I, D-II

Correct Answer: (3)

Solution:

Based on fundamental electrical engineering concepts:

- An AC generator operates on Electromagnetic Induction (II).
- A Transformer operates based on Mutual Inductance (IV).
- Resonance phenomenon occurs due to Presence of both L and C (I).
- Sharpness of Resonance is linked to Quality Factor(III).

Quick Tip

For matching questions, analyze core principles and working mechanisms.

---

35. Match the List-I with List-II:

List I	List II
A. Microwaves	I. Radioactive decay of the nucleus
B. Gamma rays	II. Rapid acceleration and deceleration of electrons in aerials
C. Radio waves	III. Inner shell electrons
D. X-rays	IV. Klystron valve

Choose the correct answer from the options given below:

(1) A-I, B-II, C-III, D-IV

(2) A-IV, B-I, C-II, D-III

(3) A-I, B-III, C-IV, D-II

(4) A-IV, B-III, C-II, D-I

Correct Answer: (2)

Solution:

Based on electromagnetic wave theory, the correct mapping is determined by the characteristics of each wave type.

Quick Tip

Gamma rays are linked to radioactive decay, whereas microwaves are generated using a Klystron valve.

---

36. If Earth has a mass nine times and radius twice that of a planet P, then

$$\frac{V_e}{3}\sqrt{x}\ \text{ms}^{-1}$$

will be the minimum velocity required by a rocket to pull out of the gravitational force of P, where V_e is escape velocity on Earth. The value of x is:

(1) 2

(2) 3

(3) 18

(4) 1

Correct Answer: (1)

Solution:

Escape velocity for a planet is given by:

$$V_{\text{escape}} = \sqrt{\frac{2GM}{R}}$$

For planet P,

$$V_{\text{escape, P}} = \sqrt{\frac{2G\left(\frac{M}{9}\right)}{\frac{R}{2}}}$$

$$= V_e\frac{\sqrt{2}}{3}, \quad x = 2$$

Quick Tip

Escape velocity is directly proportional to the square root of the planet's mass-to-radius ratio.

---

37. 'n' polarizing sheets are arranged such that each makes an angle 45° with the preceding sheet. An unpolarized light of intensity I is incident into this arrangement. The output intensity is found to be

$$\frac{I}{64}.$$

The value of n will be:

(1) 3

(2) 6

(3) 5

(4) 4

Correct Answer: (2)

Solution:

After passing through the first sheet:

$$I_1 = \frac{I}{2}$$

After passing through the second sheet:

$$I_2 = I_1 \cos^2(45^\circ) = \frac{I}{4}$$

After passing through the nth sheet:

$$I_n = \frac{I}{2^n} = \frac{I}{64}$$

Solving for n,

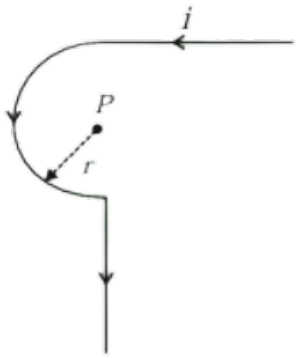
$$n = 6$$

Quick Tip

Malus' Law states that transmitted light intensity through a polarizer is given by $I = I_0 \cos^2\theta$.

---

38. Find the magnetic field at the point P in the figure. The curved portion is a semicircle connected to two long straight wires.



(1) $\frac{\mu_0 I}{2r}\left(1+\frac{2}{\pi}\right)$

(2) $\frac{\mu_0 I}{2r}\left(1+\frac{1}{\pi}\right)$

(3) $\frac{\mu_0 I}{2r}\left(\frac{1}{2}+\frac{1}{2\pi}\right)$

(4) $\frac{\mu_0 I}{2r}\left(1+\frac{1}{\pi}\right)$

Correct Answer: (3)

Solution:

Applying Biot-Savart's Law:

$$B_p = \left(\frac{\mu_0 I}{4r} + \frac{\mu_0 I}{4\pi r}\right) = \frac{\mu_0 I}{2r}\left(\frac{1}{2} + \frac{1}{2\pi}\right)$$

Quick Tip

Use Biot-Savart's Law for calculating the field due to current-carrying loops.

---

39. Which of the following frequencies does not belong to FM broadcast?

(1) 106 MHz

(2) 64 MHz

(3) 99 MHz

(4) 89 MHz

Correct Answer: (2)

Solution:

FM broadcast range is 88 MHz to 108 MHz. Since 64 MHz falls outside this range, it is the correct answer.

Quick Tip

FM radio frequencies lie between **88 MHz and 108 MHz**.

---

40. A steel wire with mass per unit length $7.0 \times 10^{-3}\ kgm^{-1}$ is under tension of 70 N. The speed of transverse waves in the wire will be:

(1) 200π m/s

(2) 100 m/s

(3) 10 m/s

(4) 50 m/s

Correct Answer: (2)

Solution:

The velocity of transverse waves in a stretched string is given by:

$$v = \sqrt{\frac{T}{\mu}}$$

Substituting the values:

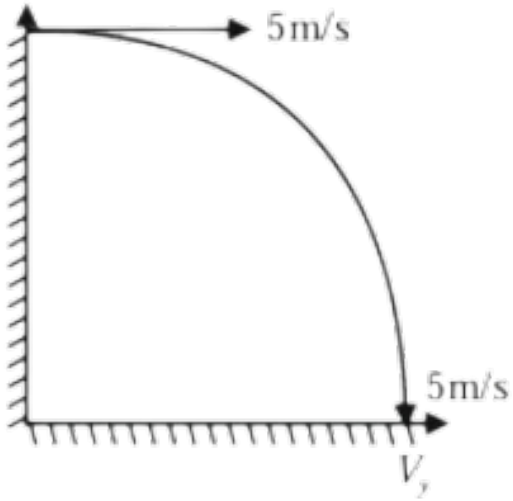
$$v = \sqrt{\frac{70}{7.0 \times 10^{-3}}} = 100 \text{ m/s}$$

Quick Tip

The velocity of waves in a string depends on tension T and linear mass density μ.

---

41. A child stands on the edge of the cliff 10 m above the ground and throws a stone horizontally with an initial speed of 5 m/s. Neglecting air resistance, the speed with which the stone hits the ground will be ___ m/s (given, $g = 10$ m/s^2).



(1) 20

(2) 15

(3) 30

(4) 25

Correct Answer: (2)

Solution:

Using kinematic equation for vertical motion:

$$v_y = \sqrt{2gh} = \sqrt{200}$$

Horizontal velocity remains unchanged:

$$v_x = 5 \text{ m/s}$$

Net velocity:

$$v_{\text{net}} = \sqrt{v_x^2 + v_y^2} = \sqrt{25 + 200} = 15 \text{ m/s}$$

Quick Tip

In projectile motion, horizontal velocity remains constant, and vertical velocity follows free fall equations.

42. A proton moving with one-tenth of velocity of light has a certain de-Broglie wavelength λ. An alpha particle having certain kinetic energy has the same de-Broglie wavelength λ. The ratio of kinetic energy of proton and that of alpha particle is:

(1) 2 : 1

(2) 4 : 1

(3) 1 : 2

(4) 1 : 4

Correct Answer: (2)

Solution:

De-Broglie wavelength is given by:

$$\lambda = \frac{h}{p}$$

Equating for both particles:

$$\frac{KE_p}{KE_\alpha} = \frac{m_\alpha}{m_p} = 4 : 1$$

Quick Tip

De-Broglie wavelength is inversely proportional to momentum.

---

43. A sample of gas at temperature T is adiabatically expanded to double its volume. The work done by the gas in the process is (given, $\gamma = \frac{3}{2}$):

(1) $W = TR[\sqrt{2} - 2]$

(2) $W = \frac{T}{R}[\sqrt{2} - 2]$

(3) $W = RT[2 - \sqrt{2}]$

(4) $W = RT[2 - \sqrt{2}]$

Correct Answer: (4)

Solution:

Using the formula for adiabatic work:

$$W = \frac{R(T_1 - T_2)}{\gamma - 1}$$

Solving for given conditions:

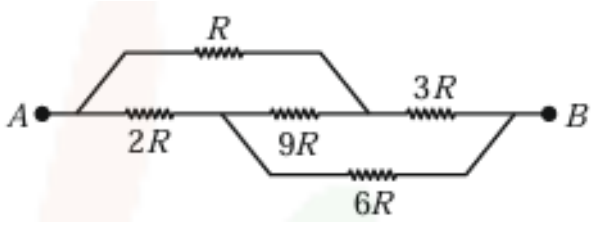
$$W = RT[2 - \sqrt{2}]$$

Quick Tip

For adiabatic expansion, use the Poisson's relation: $PV^{\gamma} = \text{constant}$.

---

44. The equivalent resistance between A and B of the network shown in figure:



(1) $\frac{11R}{3}$

(2) $14R$

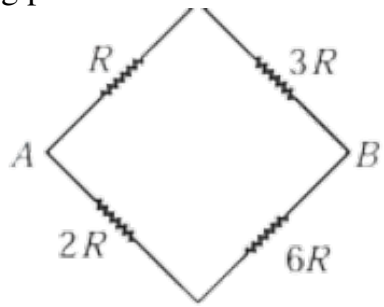
(3) $21R$

(4) $\frac{8R}{3}$

Correct Answer: (4)

Solution:

Using parallel and series resistance formulas:



$$\frac{1}{R_{\text{eq}}} = \frac{1}{4R} + \frac{1}{8R}$$

$$R_{\text{eq}} = \frac{8R}{3}$$

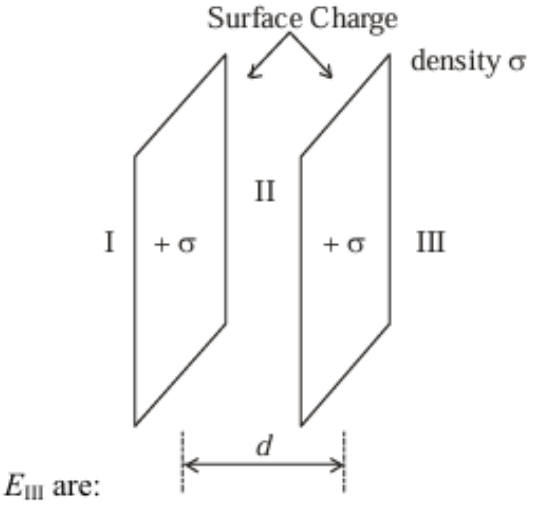
Quick Tip

For resistances in series, add directly. For resistances in parallel, use:

$$\frac{1}{R_{\text{eq}}} = \frac{1}{R_1} + \frac{1}{R_2}$$

---

45. Let σ be the uniform surface charge density of two infinite thin plane sheets shown in the figure. Then the electric fields in three different regions E_I, E_{II} and E_{III} are:



(1) $\vec{E}_I = \frac{2\sigma}{\varepsilon_0}\hat{n}, \quad \vec{E}_{II} = 0, \quad \vec{E}_{III} = \frac{2\sigma}{\varepsilon_0}\hat{n}$

(2) $\vec{E}_I = 0, \quad \vec{E}_{II} = \frac{\sigma}{\varepsilon_0}\hat{n}, \quad \vec{E}_{III} = 0$

(3) $\vec{E}_I = \frac{\sigma}{2\varepsilon_0}\hat{n}, \quad \vec{E}_{II} = 0, \quad \vec{E}_{III} = \frac{\sigma}{2\varepsilon_0}\hat{n}$

(4) $\vec{E}_I = -\frac{\sigma}{\varepsilon_0}\hat{n}, \quad \vec{E}_{II} = 0, \quad \vec{E}_{III} = \frac{\sigma}{\varepsilon_0}\hat{n}$

Correct Answer: (4)

Solution:

Assuming the right-hand side (RHS) to be along $\hat{n}$:

$$\vec{E}_I = \frac{\sigma}{2\varepsilon_0}(-\hat{n}) + \frac{\sigma}{2\varepsilon_0}(-\hat{n}) = -\frac{\sigma}{\varepsilon_0}\hat{n}$$

$$\vec{E}_{II} = 0$$

$$\vec{E}_{III} = \frac{\sigma}{2\varepsilon_0}(\hat{n}) + \frac{\sigma}{2\varepsilon_0}(\hat{n}) = \frac{\sigma}{\varepsilon_0}\hat{n}$$

Quick Tip

For an infinite sheet of charge, the electric field is given by $E = \frac{\sigma}{2\varepsilon_0}$. The net field in different regions depends on the superposition of fields from both sheets.

---

46. A mercury drop of radius 10^{-3} m is broken into 125 equal size droplets. Surface tension of mercury is 0.45 Nm^{-1}. The gain in surface energy is:

(1) 2.26×10^{-5} **J**

(2) 28×10^{-5} **J**

(3) 17.5×10^{-5} **J**

(4) 5×10^{-5} **J**

Correct Answer: (1)

Solution:

Initial surface energy:

$$0.45 \times 4\pi(10^{-3})^2$$

$$\frac{4}{3}\pi(10^{-3})^3 = 125 \times \frac{4\pi}{3}R_{\text{new}}^3$$

$$10^{-3} = 5R_{\text{new}}$$

$$R_{\text{new}} = \frac{10^{-3}}{5}\ \text{m}$$

Final surface energy:

$$0.45 \times 125 \times 4\pi \left(\frac{10^{-3}}{5}\right)^2$$

Increase in energy:

$$0.45 \times 4\pi \times (10^{-3})^2 \left[\frac{125}{25} - 1\right]$$

$$= 4 \times 0.45 \times 4\pi \times 10^{-6}$$

$$= 2.26 \times 10^{-5}\ \text{J}$$

Quick Tip

The total surface area increases when a drop breaks into multiple smaller droplets, leading to an increase in surface energy.

47. The mass of proton, neutron and helium nucleus are respectively 1.0073 u, 1.0087 u **and** 4.0015 u. **The binding energy of helium nucleus is:**

(1) 14.2 MeV

(2) 28.4 MeV

(3) 56.8 MeV

(4) 7.1 MeV

Correct Answer: (2)

Solution:

Binding Energy of Helium:

$$B.E = (2m_p + 2m_n - m_{\text{He}})c^2$$

$$= 28.4 \text{ MeV}$$

Quick Tip

Binding energy is the energy required to separate a nucleus into its constituent protons and neutrons.

---

48. The equation of state for some gases is given by:

$$\left(P + \frac{a}{V^2}\right)(V - b) = RT$$

The physical quantity, which has the same dimensional formula as $\frac{b^2}{a}$**, will be:**

(1) Bulk modulus

(2) Modulus of rigidity

(3) Compressibility

(4) Energy density

Correct Answer: (3)

Solution:

$$[b] = [V]$$

$$\left[\frac{a}{b^2}\right] = [P] \quad \Rightarrow \quad \left[\frac{b^2}{a}\right] = \frac{1}{[P]} = \frac{1}{[B]} = [K]$$

Quick Tip

Compressibility is the reciprocal of bulk modulus, representing how much a material deforms under pressure.

---

49. The average kinetic energy of a molecule of the gas is:

(1) Proportional to absolute temperature

(2) Proportional to volume

(3) Proportional to pressure

(4) Dependent on the nature of the gas

Correct Answer: (1)

Solution:

Translational kinetic energy per molecule:

$$KE = \frac{3}{2}kT$$

It is independent of nature, pressure, and volume.

Quick Tip

The kinetic energy of gas molecules depends only on temperature, not on pressure or volume.

---

50. A block of mass 5 kg is placed at rest on a rough surface. If a force of 30N is applied parallel to the surface and the block slides through a distance of 50 m in 10s, then the coefficient of kinetic friction is (given, $g = 10\ \text{ms}^{-2}$):

(1) 0.60

(2) 0.75

(3) 0.50

(4) 0.25

Correct Answer: (3)

Solution:

Using kinematic equation:

$$S = ut + \frac{1}{2}at^2$$

$$50 = 0 + \frac{1}{2}a \times 100$$

$$a = 1 \text{ m/s}^2$$

Force equation:

$$F - \mu mg = ma$$

$$30 - \mu \times 50 = 5 \times 1$$

$$50\mu = 25$$

$$\mu = \frac{1}{2}$$

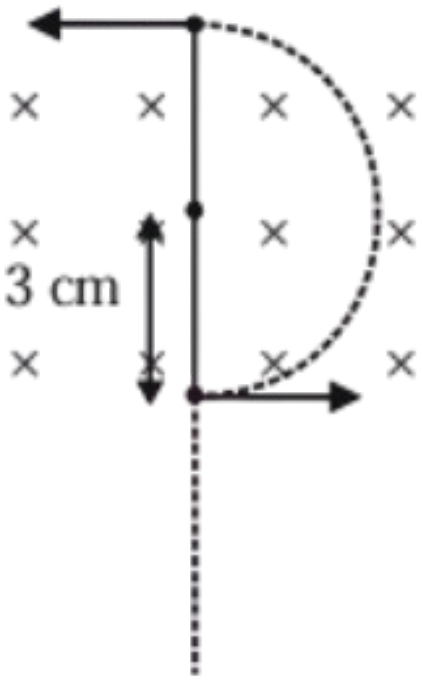
Quick Tip

Kinetic friction depends on the coefficient μ_k and normal force. It acts opposite to motion.

---

51. A charge particle of 2 C accelerated by a potential difference of 100V enters a region of uniform magnetic field of magnitude 4 mT at right angle to the direction of field. The charge particle completes semicircle of radius 3 cm inside magnetic field. The mass of the charge particle is ___ × 10^{-18} kg.

Solution:



$$r = \frac{mv}{qB} = \sqrt{\frac{2km}{qB}}, \quad m = \frac{r^2q^2B^2}{2k}$$

$$m = \frac{1}{100} \times \frac{3}{100} \times 2 \times 2 \times 4 \times 10^{-3} \times 4 \times 10^{-3} \times 10^{-12}$$

$$= 144 \times 10^{-18} \text{ kg}$$

Quick Tip

For a charged particle moving perpendicular to a uniform magnetic field, the motion follows a circular path, and the mass can be determined using the centripetal force relation.

---

52. In an experiment to find emf of a cell using potentiometer, the length of null point for a cell of emf 1.5 V is found to be 60 cm. If this cell is replaced by another cell of emf E, the length-of null point increases by 40 cm. The value of E is $\frac{x}{10}$ V. The value of x is _____.

Solution:

$$\frac{E_1}{E_2} = \frac{l_1}{l_2}$$

$$\frac{1.5}{E_2} = \frac{60}{60+40} = \frac{6}{10} = \frac{3}{5}$$

$$E_2 = \frac{5}{2} = \frac{x}{10}$$

$$x = 25$$

Quick Tip

In a potentiometer experiment, the emf of a cell is directly proportional to the length of the null point.

---

53. A small particle moves to position $5\hat{i} - 2\hat{j} + \hat{k}$ from its initial position $2\hat{i} + 3\hat{j} - 4\hat{k}$ under the action of force $5\hat{i} + 2\hat{j} + 7\hat{k}$ N. The value of work done will be ____ J.

Solution:

$$W = \mathbf{F} \cdot (\mathbf{r_f} - \mathbf{r_i})$$

$$= (5\hat{i} + 2\hat{j} + 7\hat{k}) \cdot [(5\hat{i} - 2\hat{j} + \hat{k}) - (2\hat{i} + 3\hat{j} - 4\hat{k})]$$

$$W = 40 \text{ J}$$

Quick Tip

Work done by a force in moving an object is calculated as the dot product of force and displacement.

---

54. A light of energy 12.75 eV is incident on a hydrogen atom in its ground state. The atom absorbs the radiation and reaches to one of its excited states. The angular momentum of the atom in the excited state is $\frac{x}{\pi} \times 10^{-17}$ eVs. The value of x is

Solution:

$$\text{Energy in ground state} = -13.6 \text{ eV}$$

$$-\frac{13.6}{n^2} = -13.6 + 12.75$$

$$-\frac{13.6}{n^2} = -0.85$$

$$n = \sqrt{16}$$

$$n = 4$$

$$\text{Angular momentum} = \frac{nh}{2\pi} = \frac{4h}{2\pi} = \frac{2h}{\pi}$$

$$\text{Angular momentum} = \frac{2}{\pi} \times 4.14 \times 10^{-15}$$

$$= \frac{828 \times 10^{-17}}{\pi} \text{ eVs}$$

Quick Tip

For a hydrogen atom, angular momentum in the nth excited state is given by $\frac{nh}{2\pi}$.

---

55. A certain pressure P is applied to 1 litre of water and 2 litre of a liquid separately. Water gets compressed to 0.01% whereas the liquid gets compressed to 0.03%. The ratio of Bulk modulus of water to that of the liquid is $\frac{3}{x}$. The value of x is _____.

(1) 1

(2) 2

(3) 3

(4) 4

Correct Answer: (1) 1

Solution:

$$B_{\text{water}} = \frac{-\Delta P}{\left(\frac{\Delta V}{V}\right)}$$

$$B_{\text{water}} = \frac{-\Delta P}{0.01} \times 100$$

$$B_{\text{liquid}} = \frac{-\Delta P}{0.03} \times 100$$

$$\frac{B_{\text{water}}}{B_{\text{liquid}}} = 3$$

$$x = 1$$

Quick Tip

Bulk modulus is a measure of a substance's resistance to uniform compression. Higher bulk modulus means less compressibility.

---

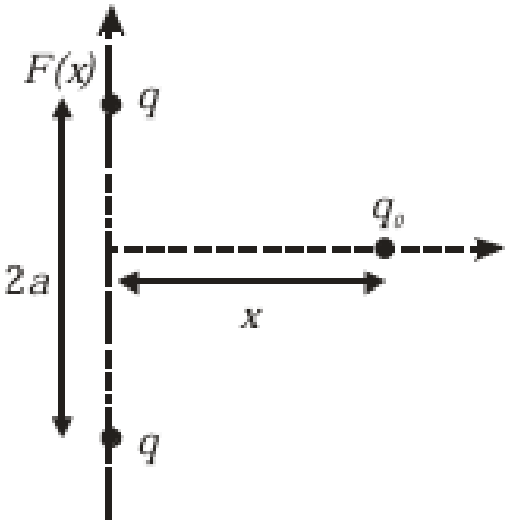
56. Two equal positive point charges are separated by a distance $2a$. The distance of a point from the centre of the line joining two charges on the equatorial line (perpendicular bisector) at which force experienced by a test charge q_0 becomes maximum is $\frac{a}{\sqrt{x}}$. The value of x is _____.

(1) 1

(2) 2

(3) 3

(4) 4



Correct Answer: (2) 2

Solution:

$$F = \frac{2Kqq_0x}{(x^2 + a^2)^{3/2}}$$

For F to be maximum,

$$\frac{dF}{dx} = 0$$

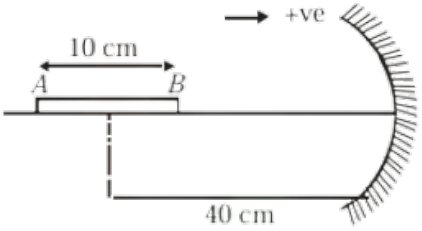
$$x = \frac{a}{\sqrt{2}}$$

Quick Tip

The force experienced by a test charge is maximum when the derivative of the force equation with respect to distance is zero.

---

57. A thin cylindrical rod of length 10 cm is placed horizontally on the principle axis of a concave mirror of focal length 20 cm. The rod is placed in a such a way that mid point of the rod is at 40 cm from the pole of mirror. The length of the image formed by the mirror will be $\frac{x}{3}$ cm. The value of x is _____.



(1) 30

(2) 32

(3) 34

(4) 36

Correct Answer: (2) 32

Solution:

$$U_A = -45 \text{ cm}, \quad f = -20 \text{ cm}$$

$$V_A = \frac{-45 \times (-20)}{-45 - (-20)} = \frac{900}{25} = -36 \text{ cm}$$

And

$$U_B = -35 \text{ cm}$$

$$V_B = \frac{-35 \times (-20)}{-35 - (-20)} = \frac{700}{-15}$$

$$V_A - V_B = \text{length of image}$$

$$= \left(-36 + \frac{140}{3}\right) \text{ cm}$$

$$= \frac{-108 + 140}{3} \text{ cm}$$

$$= \frac{32}{3} \text{ cm}$$

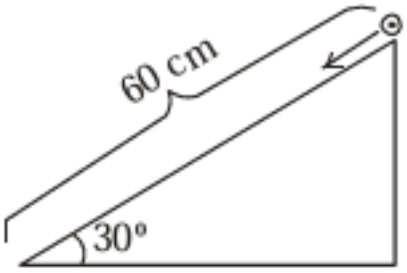
$$\therefore x = 32$$

Quick Tip

For image formation using concave mirrors, apply the mirror formula $\frac{1}{f} = \frac{1}{v} - \frac{1}{u}$.

---

58. A solid cylinder is released from rest from the top of an inclined plane of inclination 30° and length 60 cm. If the cylinder rolls without slipping, its speed upon reaching the bottom of the inclined plane is _____ ms^{-1}. (Given $g = 10$ ms^{-2})



(1) 1

(2) 2

(3) 3

(4) 4

Correct Answer: (2) 2

Solution:

$$v = \sqrt{\frac{2gh}{1 + \frac{k^2}{R^2}}}$$

Where $h = 60 \sin 30^\circ = 30$ cm

$$k^2 = \frac{R^2}{2}$$

$$v = 2 \text{ ms}^{-1}$$

Quick Tip

For rolling motion, use the energy conservation principle: $PE = KE_{\text{linear}} + KE_{\text{rotational}}$.

---

59. The amplitude of a particle executing SHM is 3 cm. The displacement at which its kinetic energy will be 25% more than the potential energy is _____ cm.

(1) 3 cm

(2) 2 cm

(3) 4 cm

(4) 5 cm

Correct Answer: (2) 2 cm

Solution:

$$KE = PE + \frac{PE}{4}$$

$$KE = \frac{5}{4}PE$$

$$\frac{1}{2}m\omega^2(A^2 - x^2) = \frac{5}{4} \times \frac{1}{2}m\omega^2x^2$$

$$[v = \omega\sqrt{A^2 - x^2}]$$

$$A^2 - x^2 = \frac{5}{4}x^2$$

$$\frac{9x^2}{4} = A^2$$

$$\frac{x}{A} = \frac{2}{3}$$

$$\therefore x = \frac{2}{3} \times 3cm$$

$$x = 2cm$$

Quick Tip

Kinetic energy in SHM is given by $KE = \frac{1}{2}m\omega^2(A^2 - x^2)$.

60. A series LCR circuit is connected to an ac source of 220V, 50Hz. The circuit contains a resistance $R = 100\Omega$ and an inductor of inductive reactance $X_L = 79.6\Omega$. The capacitance of the capacitor needed to maximize the average rate at which energy is supplied will be _____ μF.

(1) 30 μF

(2) 40 μF

(3) 50 μF

(4) 60 μF

Correct Answer: (2) 40 μF

Solution:

To maximize the average rate at which energy is supplied i.e., power will be maximum. So in LCR circuit, power will be maximum at the condition of resonance and in resonance condition

$$X_L = X_C$$

$$79.6 = \frac{1}{\omega C}$$

$$C = \frac{1}{2\pi \times 50 \times 79.6}$$

$$C = 40\mu F$$

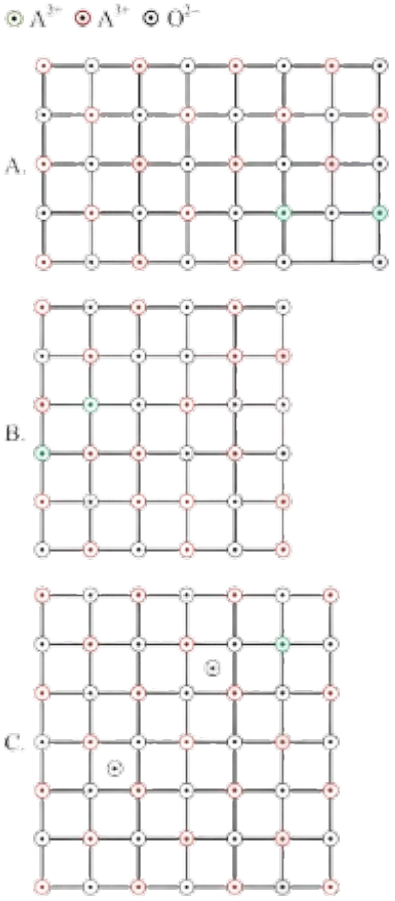
Quick Tip

At resonance in an LCR circuit, the inductive reactance and capacitive reactance cancel each other, leading to maximum power transfer.

CHEMISTRY

Section-A

61. Which of the following represents the lattice structure of $A_{0.95}O$ containing A^{2+}, A^{3+} and O^{2-} ions?



(1) B and C only

(2) B only

(3) A and B only

(4) A only

Correct Answer: (4) A only

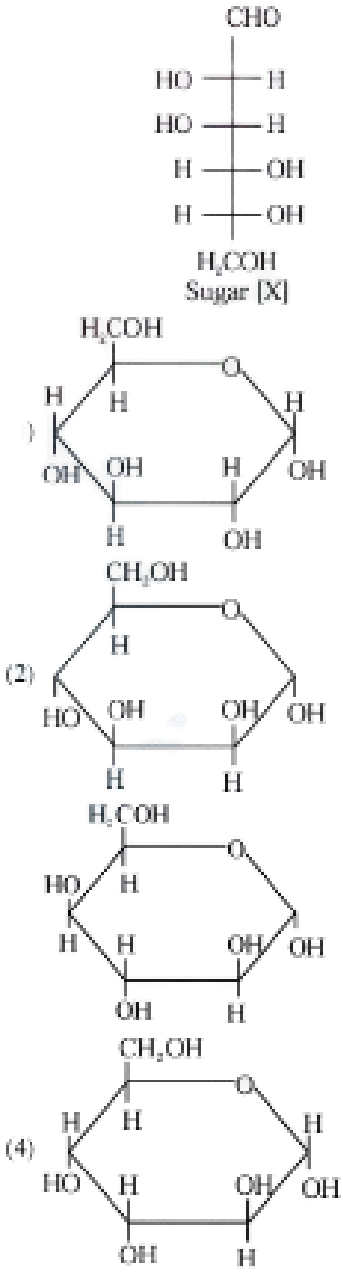
Solution:

The correct representation of the lattice structure of $A_{0.95}O$ containing A^{2+}, A^{3+}, and O^{2-} ions is shown in option A. Thus, the correct answer is option (4).

Quick Tip

When identifying lattice structures, ensure that the correct stoichiometry of ions and their respective charges are accurately represented.

62. The correct representation in six-membered pyranose form for the following sugar $[X]$ **is:**



Correct Answer: (2).

Solution:

The correct representation of the six-membered pyranose form for the given sugar is option (2).

Quick Tip

In pyranose forms, the hydroxyl groups are typically positioned on either side of the ring, with the aldehyde or ketone group at the anomeric carbon.

---

63. Highest oxidation state of Mn is exhibited in Mn_2O_7. The correct statements about Mn_2O_7 are:

(A) Mn is tetrahedrally surrounded by oxygen atoms

(B) Mn is octahedrally surrounded by oxygen atoms

(C) Contains Mn-O-Mn bridge

(D) Contains Mn-Mn bond

Choose the correct answer from the options given below:

(1) A and C only

(2) A and D only

(3) B and D only

(4) B and C only

Correct Answer: (1) A and C only.

Solution:

In Mn_2O_7, Mn is in the highest oxidation state of +7. The two Mn atoms are surrounded by oxygen atoms in a way that forms a tetrahedral arrangement. The compound also features a Mn-O-Mn bridge, which is characteristic of Mn_2O_7. Therefore, statement A and statement C are correct.

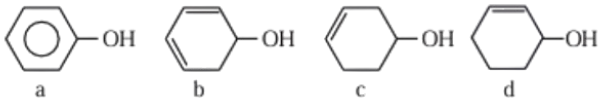
- Statement B is incorrect because Mn is tetrahedrally coordinated, not octahedrally. - Statement D is incorrect because there is no direct Mn-Mn bond in Mn_2O_7; instead, the Mn atoms are linked by oxygen atoms.

Thus, the correct answer is (1) A and C only.

Quick Tip

Mn_2O_7 is a peroxide compound where the Mn atoms are in their highest oxidation state (+7), and the compound features a tetrahedral coordination around Mn.

64. Decreasing order of dehydration of the following alcohols is:



(1) $a > d > b > c$

(2) $b > d > c > a$

(3) $b > a > d > c$

(4) $d > b > c > a$

Correct Answer: (2).

Solution:

The correct decreasing order of dehydration of the alcohols is given by option (2), which corresponds to $b > d > c > a$.

Quick Tip

Dehydration reactions are influenced by the stability of the carbocation intermediates formed during the process. Tertiary alcohols are more easily dehydrated than secondary and primary alcohols.

---

65. Given below are two statements: One is labelled as Assertion A and the other is labelled as Reason R.

Assertion A: Amongst He, Ne, Ar, and Kr; 1 g of activated charcoal adsorbs more of Kr.

Reason R: The critical volume V_c (cm^3 mol^{-1}) and critical pressure P_c (atm) is highest for Krypton but the compressibility factor at critical point Z_c is lowest for Krypton. In the light of the above statements, choose the correct answer from the options given below:

(1) A is true but R is false

(2) A is false but R is true

(3) Both A and R are true but R is NOT the correct explanation of A

(4) Both A and R are true and R is the correct explanation of A

Correct Answer: (1).

Solution:

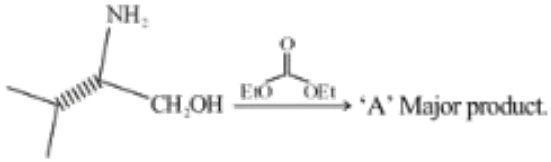
Assertion A is true, as activated charcoal adsorbs more Krypton. Reason R is also true, but it does not explain Assertion A correctly. Thus, the correct answer is (1).

Quick Tip

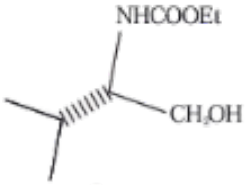
When analyzing the adsorption properties of gases, the adsorbate size and intermolecular forces play a significant role in determining the amount adsorbed.

---

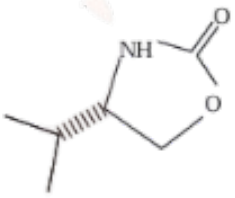
66. In the following reaction, 'A' is:



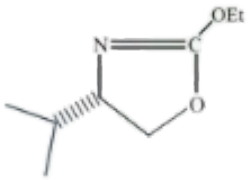
(1)

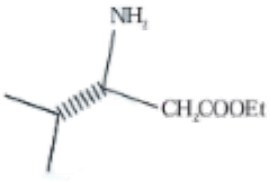


(2)



(3)





Correct Answer: (2).

Solution:

In the given reaction, the starting material is an amino alcohol, and the reaction involves the formation of an amide (NCOEt) as the major product. The reagents involved are likely to

induce dehydration, leading to the formation of an amide group (NCOEt) through the reaction of the amino group with the ester group.

- The reaction typically undergoes dehydration, eliminating water, and forming a stable amide linkage. - Hence, the correct answer is NCOEt.

Quick Tip

When solving reaction mechanism questions, consider the stability of intermediates and possible side reactions that can lead to the major product.

---

67. Match List I with List II.

List-I	List-II
(A) Tranquilizers	(I) Anti blood clotting
(B) Aspirin	(II) Salvarsan
(C) Antibiotic	(III) Antidepressant drugs
(D) Antiseptic	(IV) Soframicine

(1) (A) - IV, (B) - II, (C) - I, (D) - III

(2) (A) - II, (B) - III, (C) - I, (D) - IV

(3) (A) - III, (B) - I, (C) - II, (D) - IV

(4) (A) - II, (B) - IV, (C) - I, (D) - III

Correct Answer: (3).

Solution:

The correct matching of List I with List II is option (3).

Quick Tip

To solve matching questions, match the functional groups with their corresponding activities or classes to ensure accurate pairings.

---

68. Given below are two statements:

Statement I: Chlorine can easily combine with oxygen to form oxides; and the product has a tendency to explode.

Statement II: Chemical reactivity of an element can be determined by its reaction with oxygen and halogens.

In the light of the above statements, choose the correct answer from the options given below:

(1) Both the statements I and II are true

(2) Statement I is true but Statement II is false

(3) Statement I is false but Statement II is true

(4) Both the Statements I and II are false

Correct Answer: (1) Both the statements I and II are true

Solution:

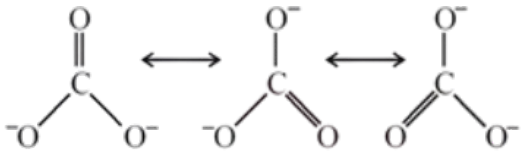
Both statements are correct. Chlorine does react with oxygen to form oxides, which have explosive tendencies. Additionally, the reactivity of elements is often determined by their reactions with oxygen and halogens.

Quick Tip

Reactivity of halogens can be understood by their electronegativity and their tendency to form halides with metals and oxides with oxygen.

---

69. Resonance in carbonate ion (CO_3^{2-}) is:



Which of the following is true?

(1) All these structures are in dynamic equilibrium with each other.

(2) Each structure exists for equal amount of time.

(3) CO_3^{2-} has a single structure i.e., resonance hybrid of the above three structures.

(4) It is possible to identify each structure individually by some physical or chemical method.

Correct Answer: (4) All these structures are in dynamic equilibrium with each other.

Solution:

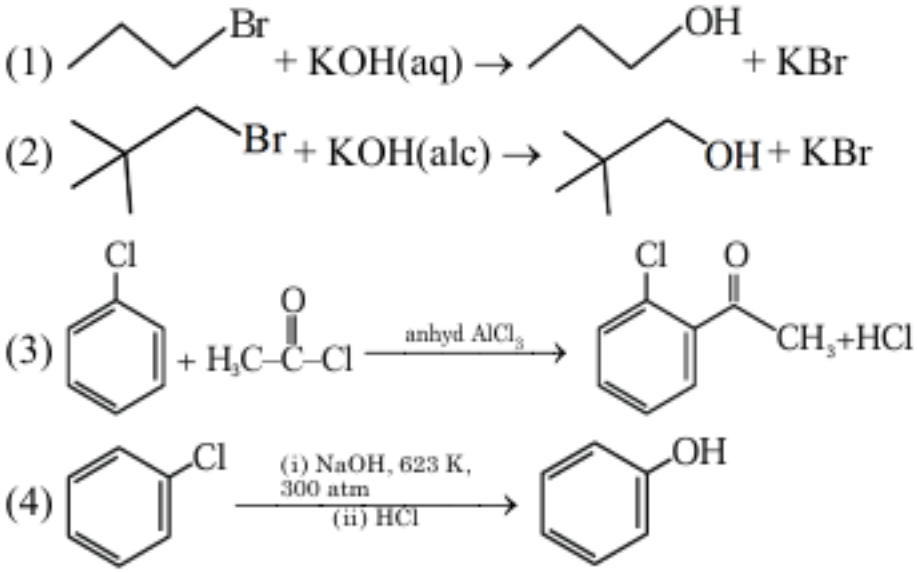
The carbonate ion (CO_3^{2-}) exhibits resonance, where the three resonance structures are in dynamic equilibrium with each other. Each structure doesn't exist for a specific amount of time.

Quick Tip

Resonance structures are different Lewis structures of the same molecule that are in dynamic equilibrium and contribute equally to the actual structure.

---

70. Identify the incorrect option from the following:



Correct Answer: (2)

Solution:

Option (2) is incorrect because alcohols do not normally undergo such reactions to form KBr. Other options depict correct reactions.

Quick Tip

In reactions involving halogens and alcohols, conditions such as solvent and temperature must be carefully considered to determine the correct product.

---

71. A solution of $FeCl_3$, when treated with $K_4[Fe(CN)_6]$, gives a prussian blue precipitate due to the formation of:

(1) $K_3[Fe(CN)_6]$

(2) $Fe[Fe(CN)_6]$

(3) $Fe_3[Fe(CN)_6]_2$

(4) $Fe_4[Fe(CN)_6]_3$

Correct Answer: (4)

Solution:

The reaction between $FeCl_3$ and $K_4[Fe(CN)_6]$ leads to the formation of $Fe_4[Fe(CN)_6]_3$, which precipitates as prussian blue.

Quick Tip

The formation of prussian blue involves a coordination reaction where Fe^{3+} reacts with $[Fe(CN)_6]^{4-}$ to form a complex salt.

---

72. Which of the following are examples of double salts?

(A) $FeSO_4.(NH_4)_2SO_4.6H_2O$

(B) $CuSO_4.4NH_3.H_2O$

(C) $K_2SO_4.Al_2(SO_4)_3.24H_2O$

(D) $Fe(CN)_2.4KCN$

Choose the correct answer:

(1) A and C only

(2) A and B only

(3) A, B and D only

(4) B and D only

Correct Answer: (1) A and C only.

Solution:

Double salts are crystalline salts that consist of two different salts crystallized together in a fixed stoichiometric ratio. - $FeSO_4.(NH_4)_2SO_4.6H_2O$ (Option A) is a double salt consisting of $FeSO_4$ and $(NH_4)_2SO_4$. - $CuSO_4.4NH_3.H_2O$ (Option B) is not a double salt; it is a complex compound. - $K_2SO_4.Al_2(SO_4)_3.24H_2O$ (Option C) is a double salt. - $Fe(CN)_2.4KCN$ (Option D) is not a double salt; it is a coordination complex.
Thus, the correct answer is (1), which includes options A and C.

Quick Tip

Double salts are often formed when two salts crystallize together and retain their individual identities in the crystalline form. Ensure that both salts are of the same ionic nature for it to qualify as a double salt.

---

73. Which of the following complex will show largest splitting of d-orbitals?

(1) $[Fe(C_2O_4)_3]^{3-}$

(2) $[Fe_6(CN)_6]$

(3) $[Fe(CN)_6]^{3-}$

(4) $[Fe(NH_3)_6]^{3+}$

Correct Answer: (3).

Solution:

$[Fe(CN)_6]^{3-}$ will show the largest splitting of d-orbitals because cyanide (CN^-) is a strong field ligand, which leads to a large splitting of the d-orbitals.

Quick Tip

The splitting of d-orbitals in transition metal complexes depends on the ligand field strength, with strong field ligands like CN^- causing greater splitting.

---

74. How can photochemical smog be controlled?

(1) By using tall chimneys

(2) By complete combustion of fuel

(3) By using catalytic converters in the automobiles/industry

(4) By using catalyst

Correct Answer: (3) By using catalytic converters in the automobiles/industry

Solution:

Photochemical smog can be controlled by using catalytic converters in automobiles and industries. Catalytic converters help reduce emissions of nitrogen oxides and hydrocarbons, which are the primary contributors to photochemical smog.

Quick Tip

Catalytic converters are effective in reducing harmful vehicle emissions, thus reducing the formation of photochemical smog in urban areas.

---

75. Match List I with List II.

List I	List II
(A) Slaked lime	(I) NaOH
(B) Dead burnt plaster	(II) $Ca(OH)_2$
(C) Caustic soda	(III) $Na_2CO_3.10H_2O$
(D) Washing soda	(IV) $CaSO_4$

Choose the correct answer from the options given below:

(1) (A) - I, (B) - IV, (C) - II, (D) - III

(2) (A) - III, (B) - IV, (C) - I, (D) - II

(3) (A) - II, (B) - III, (C) - I, (D) - IV

(4) (A) - III, (B) - II, (C) - IV, (D) - I

Correct Answer: (3) (A) - II, (B) - III, (C) - I, (D) - IV.

Solution:

Slaked lime is $Ca(OH)_2$ (Option II), Dead burnt plaster is $CaSO_4$ (Option IV), Caustic soda is $NaOH$ (Option I), and Washing soda is $Na_2CO_3.10H_2O$ (Option III).

Quick Tip

To solve matching questions, first identify the chemical formulas and match them with the correct description of the compounds or elements.

---

76. Choose the correct statement(s): A. Beryllium oxide is purely acidic in nature.

B. Beryllium carbonate is kept in the atmosphere of CO_2.

C. Beryllium sulphate is readily soluble in water.

D. Beryllium shows anomalous behavior.

Choose the correct answer from the options given below:

(1) A, B and C only

(2) B, C and D only

(3) A and B only

(4) A only

Correct Answer: (2) B, C and D only.

Solution:

Beryllium oxide is amphoteric, not purely acidic, so statement A is incorrect. Beryllium carbonate is unstable and decomposes in CO_2 atmosphere, so statement B is correct. Beryllium sulphate is soluble in water, making statement C correct. Beryllium shows anomalous behavior because it differs from other alkaline earth metals in its properties, so statement D is also correct.

Quick Tip

When identifying the properties of elements and compounds, it's important to compare them with trends in the periodic table, such as the amphoteric nature of beryllium oxide.

---

77. Given below are two statements: one is labelled as Assertion A and the other is labelled as Reason R.

Assertion A: In an Ellingham diagram, the oxidation of carbon to carbon monoxide shows a

negative slope with respect to temperature.

Reason R: CO tends to get decomposed at higher temperature.

In the light of the above statements, choose the correct answer from the options given below:

(1) Both A and R are correct and R is the correct explanation of A

(2) Both A and R are correct but R is NOT the correct explanation of A

(3) A is false but R is true

(4) A is correct but R is incorrect

Correct Answer: (2) Both A and R are correct but R is NOT the correct explanation of A.

Solution:

In an Ellingham diagram, the oxidation of carbon to carbon monoxide has a negative slope, indicating that carbon is more likely to be oxidized at higher temperatures. However, while CO tends to decompose at high temperatures, it does not directly explain the negative slope of the oxidation reaction.

Quick Tip

Ellingham diagrams are a useful tool for understanding the temperature dependence of the stability of oxides and other compounds.

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78. But-2-yne is reacted separately with one mole of Hydrogen as shown below:

$$\mathrm{B} \xrightarrow{\text{Na, liquid } \mathrm{NH_3}} \mathrm{CH_3C \equiv CH} + \mathrm{H_2} \xrightarrow{\mathrm{Pd/C}} \mathrm{A}$$

A. A is more soluble than B.

B. The boiling point & melting point of A are higher and lower than B respectively.

C. A is more polar than B because dipole moment of A is zero.

D. Br_2 adds easily to B than A.

Identify the incorrect statements from the options given below:

(1) B and C only

(2) B, C and D only

(3) A, C and D only

(4) A and B only

Correct Answer: (3) A, C and D only.

Solution:

In the reaction, But-2-yne undergoes hydrogenation in the presence of sodium in liquid ammonia, followed by catalytic hydrogenation using Pd/C to form the product A. The following statements are provided:

A. A is more soluble than B: This statement is true, as the addition of hydrogen to the triple bond results in a saturated compound that is typically more soluble in organic solvents.

B. The boiling point & melting point of A are higher and lower than B respectively: This statement is incorrect because the hydrogenated product (A) will have higher melting and boiling points compared to the original alkyne (B).

C. A is more polar than B because dipole moment of A is zero: This statement is incorrect as well. While A is saturated, it is not more polar than B; in fact, A has a lower dipole moment than B.

D. Br_2 adds easily to B than A: This statement is true because alkenes (as in B) are more reactive towards electrophilic addition reactions with Br_2 compared to alkanes (as in A).

Thus, the incorrect statements are B, C, and D only, making the correct answer (3).

Quick Tip

In organic reactions, hydrogenation of alkynes to alkenes or alkenes to alkanes results in an increase in the number of hydrogen atoms, which often changes physical properties like solubility and boiling point.

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79. Given below are two statements: one is labelled as Assertion A and the other is labelled as Reason R.

Assertion A: Hydrogen is an environment friendly fuel.

Reason R: The atomic number of hydrogen is 1 and it is a very light element.

In the light of the above statements, choose the correct answer from the options given below:

(1) A is true but R is false

(2) Both A and R are true but R is NOT the correct explanation of A

(3) A is false but R is true

(4) Both A and R are correct and R is the correct explanation of A

Correct Answer: (2) Both A and R are true but R is NOT the correct explanation of A.

Solution:

Hydrogen is indeed an environmentally friendly fuel as it produces only water upon combustion. However, the lightness of hydrogen does not directly explain why it is environmentally friendly.

Quick Tip

Hydrogen is considered a clean fuel because its combustion produces no harmful emissions, making it a promising energy source for the future.

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80. Match List I with List II.

List I	List II
(A) Molisch's Test	(I) Peptide
(B) Biuret Test	(II) Carbohydrate
(C) Carbamine Test	(III) Primary amine
(D) Schiff's Test	(IV) Aldehyde

Choose the correct answer from the options given below:

(1) (A) - I, (B) - II, (C) - III, (D) - IV

(2) (A) - III, (B) - IV, (C) - I, (D) - II

(3) (A) - II, (B) - I, (C) - III, (D) - IV

(4) (A) - III, (B) - II, (C) - IV, (D) - I

Correct Answer: (3) (A) - II, (B) - I, (C) - III, (D) - IV.

Solution:

Molisch's test is used for detecting carbohydrates (Option II), Biuret test detects peptides (Option I), Carbamine test is for primary amines (Option III), and Schiff's test is for

aldehydes (Option IV).

Quick Tip

Matching tests with their corresponding functional groups or compounds is crucial in organic chemistry for identifying functional groups present in unknown compounds.

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81. The density of 3 M solution of NaCl is 1.0 g mL^{-1}. Molality of the solution is ___ $\times 10^{-2}$ m. (Nearest integer)

Given: Molar mass of Na and Cl is 23 and 35.5 g mol^{-1} respectively.

Correct Answer: (364)

Solution:

Molality m is defined as the number of moles of solute per kilogram of solvent. To calculate molality, we first need to calculate the molar mass of NaCl and use the density formula to find the mass of the solvent.

$$m = \frac{\text{moles of solute}}{\text{mass of solvent in kg}} = \frac{\text{moles of NaCl}}{\text{mass of water}}$$

We will calculate using the given density and molarity, which will give us the required molality.

Quick Tip

Use the formula $m = \frac{\text{mol of solute}}{\text{kg of solvent}}$ for calculating molality from molarity and density.

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82. Electrons in a cathode ray tube have been emitted with a velocity of 1000 ms^{-1}. The number of following statements which is/are true about the emitted radiation is —.

Given: $h = 6 \times 10^{-34}$ Js, $m_e = 9 \times 10^{-31}$ kg.

(1) The deBroglie wavelength of the electron emitted is 666.67 nm.

(2) The characteristic of electrons emitted depend upon the material of the electrodes of the cathode ray tube.

(3) The cathode rays start from cathode and move towards anode.

(4) The nature of the emitted electrons depends on the nature of the gas present in cathode ray tube.

Correct Answer: (2)

Solution:

The deBroglie wavelength λ of an electron is given by:

$$\lambda = \frac{h}{m_e v}$$

Where v is the velocity of the electron. Using the given values for h, m_e, and v, we calculate the wavelength.

The emitted electrons' characteristics depend on the material of the electrodes. This statement is true because different materials cause different emissions.

Quick Tip

The deBroglie wavelength is useful for understanding the wave-like properties of particles like electrons. It is calculated by dividing Planck's constant by the product of mass and velocity.

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83. Sum of oxidation states of bromine in bromic acid and perbromic acid is ——.

Correct Answer: (12)

Solution:

In bromic acid, the oxidation state of bromine is +5, and in perbromic acid, the oxidation state of bromine is +7. Summing these oxidation states:

$$5 + 7 = 12$$

Thus, the sum of oxidation states is 12.

Quick Tip

To determine oxidation states in acids, use the known oxidation states of oxygen (usually -2) and hydrogen (usually +1), and balance the equation accordingly.

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84. At what pH, given half cell MnO_4^- (0.1 M) — Mn^{2+} (0.001 M) will have electrode potential of 1.282 V? ____ (Nearest Integer)

Given $E^\circ_{MnO_4^-/Mn^{2+}} = 1.54\,\text{V}$, $\frac{2.303RT}{F} = 0.059V$

Correct Answer: (3)

Solution:

We can use the Nernst equation to calculate the pH:

$$E = E^\circ - \frac{0.059}{n} \log \left(\frac{[Mn^{2+}]}{[MnO_4^-]} \right)$$

Substituting the given values into the equation, we can solve for pH.

Quick Tip

Use the Nernst equation to calculate electrode potentials under non-standard conditions. Remember, the value of $\frac{2.303RT}{F}$ is given as 0.059 for calculating at 25°C.

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85. Number of isomeric compounds with molecular formula C_9H_10O which (i) do not dissolve in NaOH (ii) do not dissolve in HCl. (iii) do not give orange precipitate with 2, 4 - DNP (iv) on hydrogenation give identical compound with molecular formula C_9H_12O is ____.

Correct Answer: (0)

Solution:

The number of isomeric compounds with the molecular formula $C_9H_{10}O$ that meet all the conditions listed in the question is 0, as no such compound satisfies all the conditions simultaneously.

Quick Tip

For isomeric compounds, consider all the possible structural isomers and check each condition systematically to ensure all criteria are met.

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86. (i) X(g) $\rightleftharpoons$ Y(g) + Z(g), $K_{p1} = 3$

(ii) A(g) $\rightleftharpoons$ 2B(g), $K_{p2} = 1$

If the degree of dissociation and initial concentration of both the reactants X(g) and A(g) are equal, then the ratio of the total pressure at equilibrium $\left(\frac{P_1}{P_2}\right)$ is equal to x : 1. The value of x is ____(Nearest integer)

Correct Answer: (12)

Solution:

To calculate the ratio of the total pressures at equilibrium, use the equilibrium expressions for both reactions and apply the given equilibrium constants K_{p1} and K_{p2}. The degree of dissociation and initial concentrations will help to simplify the calculation.

Quick Tip

Use the equilibrium constant expressions and degree of dissociation to solve for the total pressure at equilibrium.

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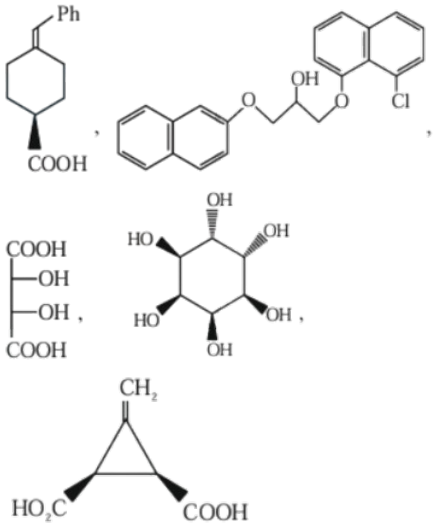
87. The total number of chiral compound/s from the following is _____ .



Correct Answer: (2)

Solution:

The total number of chiral compounds can be determined by analyzing the given structures. Chiral compounds contain a carbon atom bonded to four different groups. In the given compounds, there are 2 chiral centers.

Quick Tip

To determine chiral centers, check for carbon atoms that are bonded to four different groups. These are typically the points of chirality in organic molecules.

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88. A and B are two substances undergoing radioactive decay in a container. The half-life of A is 15 min and that of B is 5 min. If the initial concentration of B is 4 times that of A and they both start decaying at the same time, how much time will it take for the concentration of both of them to be same? _____ min.

Correct Answer: (15)

Solution:

For radioactive decay, the concentration of a substance at time t is given by:

$$C(t) = C_0 \times \left(\frac{1}{2}\right)^{\frac{t}{T_{1/2}}}$$

Where C_0 is the initial concentration and $T_{1/2}$ is the half-life of the substance. Given that the initial concentration of B is 4 times that of A, we can write:

$$C_A(t) = C_0 \times \left(\frac{1}{2}\right)^{\frac{t}{15}}$$

$$C_B(t) = 4C_0 \times \left(\frac{1}{2}\right)^{\frac{t}{5}}$$

We want to find the time t when $C_A(t) = C_B(t)$. Setting the equations equal gives:

$$C_0 \times \left(\frac{1}{2}\right)^{\frac{t}{15}} = 4C_0 \times \left(\frac{1}{2}\right)^{\frac{t}{5}}$$

Simplifying and solving for t gives $t = 15$ minutes.

Quick Tip

When dealing with exponential decay problems, use the equation for half-life and equate the concentrations to solve for time.

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89. At 25°C, the enthalpy of the following processes are given:

$$\mathrm{H_2}(g) + \mathrm{O_2}(g) \rightarrow 2\mathrm{OH}(g)\ \Delta H^\circ = 78\,\mathrm{kJ\ mol^{-1}}$$

$$\mathrm{H_2}(g) + \frac{1}{2}\mathrm{O_2}(g) \rightarrow \mathrm{H_2O}(g)\ \Delta H^\circ = -242\,\mathrm{kJ\ mol^{-1}}$$

$$\mathrm{H_2}(g) \rightarrow 2\mathrm{H}(g)\ \Delta H^\circ = 436\,\mathrm{kJ\ mol^{-1}}$$

$$\frac{1}{2}\mathrm{O_2}(g) \rightarrow \mathrm{O}(g)\ \Delta H^\circ = 249\,\mathrm{kJ\ mol^{-1}}$$

What would be the value of X for the following reaction?

$$\mathrm{H_2O}(g) \rightarrow \mathrm{H}(g) + \mathrm{OH}(g)\ \Delta H^\circ = X\,\mathrm{kJ\ mol^{-1}}$$

Correct Answer: (12)

Solution:

Using Hess's law, we can calculate the enthalpy change for the reaction $\mathrm{H_2O}(g) \rightarrow \mathrm{H}(g) + \mathrm{OH}(g)$. Add the reactions provided in the question in such a way that they result in the target reaction.

$$\Delta H^\circ = (436 + 249) - (78 + (-242))$$

Calculating gives:

$$\Delta H^\circ = 436 + 249 - 78 + 242 = 849\,\text{kJ mol}^{-1}$$

Quick Tip

Use Hess's Law to add or subtract reactions and their enthalpy changes to find the enthalpy change of a target reaction.

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90. 25 mL of an aqueous solution of KCl was found to require 20 mL of 1 M $AgNO_3$ solution when titrated using K_2CrO_4 as an indicator. What is the depression in freezing point of KCl solution of the given concentration? _____ (Nearest integer).
(Given: $K_f = 2.0\,\text{K kg mol}^{-1}$) Assume 1) 100% ionization and 2) density of the aqueous solution as 1 g mL^{-1}

Correct Answer: (3)

Solution:

First, calculate the molality of KCl from the titration data. The number of moles of $AgNO_3$ used is:

$$\text{moles of AgNO}_3 = 1\,\text{M} \times 20\,\text{mL} = 0.02\,\text{mol}$$

Since the reaction between Ag^+ and Cl^- is 1:1, the number of moles of KCl is also 0.02 mol. Using the given volume of KCl solution, we can calculate the molality.

$$\text{molality} = \frac{\text{moles of KCl}}{\text{kg of solvent}}$$

Finally, the depression in freezing point is given by:

$$\Delta T_f = K_f \times m$$

Substituting the values gives the depression in freezing point.

Quick Tip

For freezing point depression, use the formula $\Delta T_f = K_f \times m$, where m is the molality of the solution and K_f is the cryoscopic constant.
