JEE Main 2023 Feb 1 Shift 1 Mathematics Question Paper

Time Allowed : 3 Hours	**Maximum Marks :** 300	**Total Questions :** 90

General Instructions

Read the following instructions very carefully and strictly follow them:

1. The test is of 3 hours duration.
2. The question paper consists of 90 questions, out of which 75 are to attempted. The maximum marks are 300.
3. There are three parts in the question paper consisting of Physics, Chemistry and Mathematics having 30 questions in each part of equal weightage.
4. Each part (subject) has two sections.
   (i) Section-A: This section contains 20 multiple choice questions which have only one correct answer. Each question carries 4 marks for correct answer and –1 mark for wrong answer.
   (ii) Section-B: This section contains 10 questions. In Section-B, attempt any five questions out of 10. The answer to each of the questions is a numerical value. Each question carries 4 marks for correct answer and –1 mark for wrong answer. For Section-B, the answer should be rounded off to the nearest integer

MATHEMATICS

Section-A

1. If

$$\lim_{n\to\infty}\left(\frac{1}{1+n}+\frac{1}{2+n}+\frac{1}{3+n}+\cdots+\frac{1}{2n}\right)$$

is equal to:

(1) 0

(2) $\log_e 2$

(3) $\log_e\left(\frac{3}{2}\right)$

(4) $\log_e\left(\frac{2}{3}\right)$

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2. The negation of the expression

$$q \vee (\neg q \wedge p)$$

is equivalent to:

(1) $(\neg p) \wedge (\neg q)$

(2) $p \wedge (\neg q)$

(3) $(\neg p) \vee (\neg q)$

(4) $(\neg p) \vee q$

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3. In a binomial distribution $B(n,p)$, the sum and product of the mean and variance are 5 and 6, respectively. Then $6(n+p-q)$ is equal to:

(1) 51

(2) 52

(3) 53

(4) 50

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4. The sum to 10 terms of the series

$$\frac{1}{1+1^2+1^4}+\frac{2}{1+2^2+2^4}+\frac{3}{1+3^2+3^4}+\dots$$

is:

(1) $\frac{59}{111}$

(2) $\frac{55}{111}$

(3) $\frac{56}{111}$

(4) $\frac{58}{111}$

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5. The value of

$$\frac{1}{1!50!} + \frac{1}{3!48!} + \frac{1}{5!46!} + \cdots + \frac{1}{49!2!} + \frac{1}{51!1!}$$

is:

(1) $\frac{2^{50}}{50!}$

(2) $\frac{2^{50}}{51!}$

(3) $\frac{2^{51}}{51!}$

(4) $\frac{2^{51}}{50!}$

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6. If the orthocentre of a triangle, whose vertices are $(1, 2)$, $(2, 3)$, and $(3, 1)$ **is** (α, β), **then the quadratic equation whose roots are** $\alpha + 4\beta$ **and** $4\alpha + \beta$ **is:**

(1) $x^2 - 19x + 90 = 0$

(2) $x^2 - 18x + 80 = 0$

(3) $x^2 - 22x + 120 = 0$

(4) $x^2 - 20x + 99 = 0$

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1. For a triangle ABC, the value of

$$\cos 2A + \cos 2B + \cos 2C$$

is least. If its inradius is 3 and incentre is M, then which of the following is NOT correct?

(1) Perimeter of $\triangle ABC$ is $18\sqrt{3}$

(2) $\sin 2A + \sin 2B + \sin 2C = \sin A + \sin B + \sin C$

(3) $\overline{MA} \cdot \overline{MB} = -18$

(4) Area of $\triangle ABC$ is $\frac{27\sqrt{3}}{2}$

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8. The combined equation of the two lines

$$ax + by + c = 0 \quad \text{and} \quad a'x + b'y + c' = 0$$

can be written as

$$(ax + by + c)(a'x + b'y + c') = 0$$

The equation of the angle bisectors of the lines represented by the equation

$$2x^2 + xy - 3y^2 = 0$$

is:

(1) $3x^2 + 5xy + 2y^2 = 0$

(2) $x^2 - y^2 + 10xy = 0$

(3) $3x^2 + xy - 2y^2 = 0$

(4) $x^2 - y^2 - 10xy = 0$

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9. The shortest distance between the lines

$$\frac{x-5}{1} = \frac{y-2}{2} = \frac{z-4}{-3}, \quad \frac{x+3}{1} = \frac{y+5}{4} = \frac{z-1}{-5}$$

is:

(1) $7\sqrt{3}$

(2) $5\sqrt{3}$

(3) $6\sqrt{3}$

(4) $4\sqrt{3}$

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10. Let S denote the set of all real values of λ such that the system of equations

$$\lambda x + y + z = 1$$

$$x + \lambda y + z = 1$$

$$x + y + \lambda z = 1$$

is inconsistent. Then

$$\sum_{\lambda \in S} (|\lambda|^2 + |\lambda|)$$

is equal to:

(1) 2

(2) 12

(3) 4

(4) 6

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11. Let

$$S = \left\{x : x \in \mathbb{R} \text{ and } (\sqrt{3}+\sqrt{2})^{x-4} + (\sqrt{3}-\sqrt{2})^{x-4} = 10\right\}$$

Then $|S|$ is equal to:

(1) 2

(2) 4

(3) 6

(4) 0

---

12. Let S be the set of all solutions of the equation

$$\cos^{-1}(2x) - 2\cos^{-1}(\sqrt{1-x^2}) = \pi, \quad x \in \left[-\frac{1}{2}, \frac{1}{2}\right].$$

Then

$$\sum_{x \in S} 2\sin^{-1}(x^2 - 1)$$

is equal to:

(1) 0

(2) $\frac{-2\pi}{3}$

(3) $\pi - \sin^{-1}\left(\frac{\sqrt{3}}{4}\right)$

(4) $\pi - 2\sin^{-1}\left(\frac{\sqrt{3}}{4}\right)$

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13. If the center and radius of the circle

$$\left|\frac{z-2}{z-3}\right| = 2$$

are respectively (α, β) and γ, then

$$3(\alpha + \beta + \gamma)$$

is equal to:

(1) 11

(2) 9

(3) 10

(4) 12

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14. If $y = y(x)$ is the solution curve of the differential equation

$$\frac{dy}{dx} + y \tan x = x \sec x, \quad 0 \leq x \leq \frac{\pi}{3}$$

and $y(0) = 1$, then $y\left(\frac{\pi}{6}\right)$ is equal to:

(1) $\frac{\pi}{12} + \frac{\sqrt{3}}{2} \log_e \left(\frac{2}{e\sqrt{3}}\right)$

(2) $\frac{\pi}{12} + \frac{\sqrt{3}}{2} \log_e \left(\frac{2\sqrt{3}}{e}\right)$

(3) $\frac{\pi}{12} - \frac{\sqrt{3}}{2} \log_e \left(\frac{2\sqrt{3}}{e}\right)$

(4) $\frac{\pi}{12} + \frac{\sqrt{3}}{2} \log_e \left(\frac{2}{e\sqrt{3}}\right)$

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15. Let R be a relation on $\mathbb{R}$, given by

$$R = \{(a, b) : 3a - 3b + \sqrt{7} \text{ is an irrational number}\}$$

Then R is:

(1) Reflexive but neither symmetric nor transitive

(2) Reflexive and transitive but not symmetric

(3) Reflexive and symmetric but not transitive

(4) An equivalence relation

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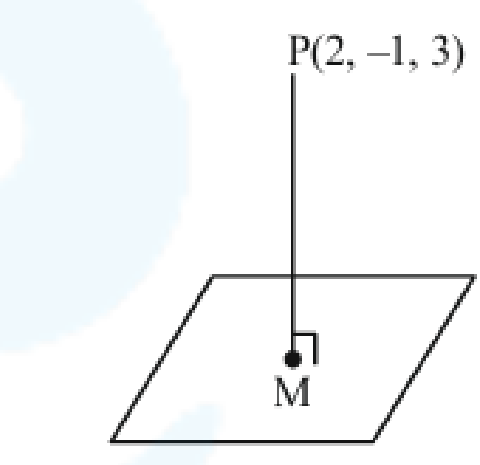
16. Let the image of the point $P(2, -1, 3)$ in the plane

$$x + 2y - z = 0$$

be Q. Then the distance of the plane

$$3x + 2y + z + 29 = 0$$

from the point Q is:



(1) $\frac{22\sqrt{2}}{7}$

(2) $\frac{24\sqrt{2}}{7}$

(3) $2\sqrt{14}$

(4) $3\sqrt{14}$

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17. Let

$$f(x) = \begin{bmatrix} 1+\sin^2 x & \cos^2 x & \sin 2x \\ \sin^2 x & 1+\cos^2 x & \sin 2x \\ \sin^2 x & \cos^2 x & 1+\sin 2x \end{bmatrix}$$

where $x \in \left[\frac{\pi}{6}, \frac{\pi}{3}\right]$. **If** α, β **are respectively the maximum and the minimum values of** f**, then**

(1) $\beta^2 - 2\sqrt{\alpha} = \frac{19}{4}$

(2) $\beta^2 + 2\sqrt{\alpha} = \frac{19}{4}$

(3) $\alpha^2 - \beta^2 = 4\sqrt{3}$

(4) $\alpha^2 + \beta^2 = \frac{9}{2}$

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18. Let

$$f(x) = 2x + \tan^{-1} x, \quad g(x) = \log_e(\sqrt{1+x^2} + x)$$

where $x \in [0,3]$. **Then:**

(1) There exists $x \in [0,3]$ such that $f'(x) < g'(x)$

(2) $\max f(x) > \max g(x)$

(3) There exist $0 < x_1 < x_2 < 3$ such that $f(x) < g(x)$, $\forall x \in (x_1, x_2)$

(4) $\min f'(x) = 1 + \max g'(x)$

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19. The mean and variance of 5 observations are 5 and 8, respectively. If 3 observations are 1, 3, 5, then the sum of cubes of the remaining two observations is:

(1) 1072

(2) 1792

(3) 1216

(4) 1456

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20. The area enclosed by the closed curve C given by the differential equation

$$\frac{dy}{dx} + \frac{x+a}{y-2} = 0, \quad y(1) = 0$$

is 4π. **Let P and Q be the points of intersection of the curve C and the y-axis. If normals at P and Q on the curve C intersect the x-axis at points R and S respectively, then the length of the line segment RS is:**

(1) $2\sqrt{3}$

(2) $\frac{2\sqrt{3}}{3}$

(3) 2

(4) $\frac{4\sqrt{3}}{3}$

Section-B

21. Let $a_1 = 8, a_2, a_3, \ldots a_n$ **be an A.P. If the sum of its first four terms is 50 and the sum of its last four terms is 170, then the product of its middle two terms is:**

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22. A(2, 6, 2), B(-4, 0, λ**), C(2, 3, -1) and D(4, 5, 0), where** $|\lambda| \leq 5$ **are the vertices of a quadrilateral ABCD. If its area is 18 square units, then**

$5 - 6\lambda$ is equal to:

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23. The number of 3-digit numbers that are divisible by either 2 or 3 but not divisible by 7 is:

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24. The remainder when $19^{200} + 23^{200}$ **is divided by 49, is:**

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25. If

$$\int_0^1 (x^{21} + x^4 + x^7)(2x^{14} + 3x^7 + 6)^{1/7} dx = \frac{1}{7}(11)^{m/n}$$

where $l, m, n \in \mathbb{N}$, **and** m, n **are coprime, then**

$l + m + n$ is equal to:

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26. If

$$f(x) = x^2 + g'(1)x + g''(2)$$

and

$$g(x) = f(1)x^2 + xf'(x) + f''(x),$$

then the value of $f(4) - g(4)$ **is equal to:**

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27. Let

$$\vec{v} = \alpha\hat{i} + 2\hat{j} - 3\hat{k}, \quad \vec{w} = 2\alpha\hat{i} + \hat{j} - \hat{k}, \quad \text{and } \hat{u} \text{ be a vector such that } |\hat{u}| = \alpha > 0.$$

If the minimum value of the scalar triple product

$$[\vec{u} \quad \vec{v} \quad \vec{w}]$$

is $-\alpha\sqrt{3401}$, **and**

$$|\hat{u} \cdot \hat{i}|^2 = \frac{m}{n}$$

where m **and** n **are coprime natural numbers, then**

$m + n$ is equal to:

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28. The number of words, with or without meaning, that can be formed using all the letters of the word ASSASSINATION so that the vowels occur together, is:

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30. Let $f : \mathbb{R} \to \mathbb{R}$ be a differentiable function such that

$$f'(x) + f(x) = \int_0^2 f(t)dt.$$

If $f(0) = e^{-2}$, then

$2f(0) - f(2)$ is equal to:
