JEE Main 2023 Feb 1 Shift-2 Physics Question Paper with Solutions

Time Allowed :3 Hours	**Maximum Marks :**300	**Total Questions :**90

General Instructions

Read the following instructions very carefully and strictly follow them:

1. The Duration of test is 3 Hours.
2. This paper consists of 90 Questions.
3. There are three parts in the paper consisting of Physics, Chemistry and Mathematics having 30 questions in each part of equal weightage..
4. Each part (subject) has two sections.
   (i) Section-A: This section contains 20 multiple choice questions which have only one correct answer. Each carries 4 marks for correct answer and –1 mark for wrong answer..
5. (ii) Section-B: This section contains 10 questions. In Section-B, attempt any five questions out of 10. The answer to each of the questions is a numerical value. Each carries 4 marks for correct answer and –1 mark for wrong answer. For Section-B, the answer should be rounded off to the nearest integer.

Physics

Section A

1: A Carnot engine operating between two reservoirs has efficiency $\frac{1}{3}$. When the temperature of the cold reservoir is raised by x, its efficiency decreases to $\frac{1}{6}$. The value of x, if the temperature of the hot reservoir is 99°C, will be:

(1) 16.5 K

(2) 33 K

(3) 66 K

(4) 62 K

Correct Answer: (4) 62 K

Solution:

The temperature of the hot reservoir is given as $T_H = 99°C$. Convert this to Kelvin:

$$T_H = 99 + 273 = 372 \text{ K}$$

Use the Carnot Efficiency Formula

The efficiency of a Carnot engine is given by:

$$\eta = 1 - \frac{T_C}{T_H}$$

where T_C is the temperature of the cold reservoir and T_H is the temperature of the hot reservoir.

Initial Cold Reservoir Temperature

Initially, the efficiency is $\frac{1}{3}$. So,

$$\frac{1}{3} = 1 - \frac{T_C}{372}$$

$$\frac{T_C}{372} = 1 - \frac{1}{3} = \frac{2}{3}$$

$$T_C = \frac{2}{3} \times 372 = 248 \text{ K}$$

Cold Reservoir Temperature After the Increase

When the cold reservoir temperature is increased by x, the new temperature is $T_C + x$, and the efficiency becomes $\frac{1}{6}$. So,

$$\frac{1}{6} = 1 - \frac{T_C + x}{T_H}$$
$$\frac{T_C + x}{T_H} = 1 - \frac{1}{6} = \frac{5}{6}$$
$$\frac{248 + x}{372} = \frac{5}{6}$$

Value for x

$$248 + x = \frac{5}{6} \times 372$$

$$248 + x = 5 \times 62 = 310$$

$$x = 310 - 248 = 62 \text{ K}$$

Conclusion: The value of x is 62 K (**Option 4**).

> **Quick Tip**
>
> Remember to convert temperatures to Kelvin when working with Carnot engine problems. The efficiency formula relates the temperatures of the hot and cold reservoirs to the engine's efficiency.

2: Given below are two statements : One is labelled as Assertion A and the other is labelled as Reason R.

Assertion A: Two metallic spheres are charged to the same potential. One of them is hollow and another is solid, and both have the same radii. Solid sphere will have lower charge than the hollow one.

Reason R: Capacitance of metallic spheres depend on the radii of spheres.

In the light of the above statements, choose the correct answer from the options given below.

(1) A is false but R is true

(2) Both A and R are true and R is the correct explanation of A

(3) A is true but R is false

(4) Both A and R are true but R is not the correct explanation of A

Correct Answer: **(1)** A is false but R is true

Solution:

The potential of a conducting sphere (solid or hollow) is given by:

$$V = \frac{KQ}{R}$$

where V is the potential, K is a constant, Q is the charge, and R is the radius.

If two spheres have the same potential ($V_1 = V_2$) and the same radius ($R_1 = R_2$), then:

$$\frac{KQ_1}{R_1} = \frac{KQ_2}{R_2}$$

Since $R_1 = R_2$, it follows that $Q_1 = Q_2$. Therefore, the assertion that the solid sphere will have a lower charge is **false**. The charges must be equal.

The capacitance of a spherical conductor is given by:

$$C = 4\pi\epsilon_0 R$$

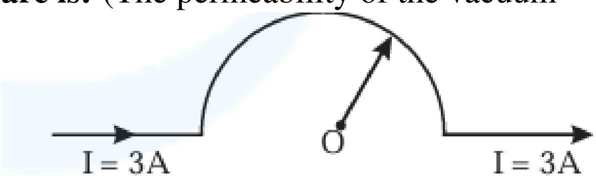
where C is the capacitance, ϵ_0 is the permittivity of free space, and R is the radius. This shows that the capacitance of a metallic sphere depends only on its radius. Therefore, the reason is **true**.

Conclusion: Assertion A is false, but Reason R is true. Therefore, the correct option is **(1)**.

Quick Tip

For questions involving assertions and reasons, analyze each statement independently. Recall the relevant formulas and concepts to determine the truth or falsehood of each statement.

3: As shown in the figure, a long straight conductor with a semicircular arc of radius $\frac{\pi}{10}$ m is carrying current I = 3A. The magnitude of the magnetic field at the center O of the arc is: (The permeability of the vacuum = $4\pi \times 10^{-7}$ NA^{-2})



(1) 6μT

(2) 1μT

(3) 4μT

(4) 3μT

Correct Answer: (4) 3μT

Solution:

Magnetic Field Due to the Semicircular Arc

The magnetic field at the center of a circular arc of radius R carrying current I is given by:

$$B_c = \frac{\mu_0 I}{4\pi R}\theta$$

where μ_0 is the permeability of free space, and θ is the angle subtended by the arc at the center in radians. For a semicircular arc, $\theta = \pi$.

$$B_c = \frac{\mu_0 I}{4R}$$

Given $I = 3A$, $R = \frac{\pi}{10}m$, and $\mu_0 = 4\pi \times 10^{-7} NA^{-2}$, we have:

$$B_c = \frac{(4\pi \times 10^{-7})(3)}{4(\frac{\pi}{10})}$$

$$B_c = \frac{4\pi \times 10^{-7} \times 3 \times 10}{4\pi}$$

$$B_c = 3 \times 10^{-6}\ \text{T} = 3\mu\text{T}$$

Magnetic Field Due to the Straight Wires

The straight portions of the wire do not contribute to the magnetic field at point O. This is because the magnetic field lines due to an infinitely long straight wire form concentric circles around the wire. At the center of the semicircle, the straight sections are radial, so the magnetic field produced by them at point O will be perpendicular to the plane of the semicircle and thus won't affect the field in the plane caused by the semicircular section.

Conclusion: The magnitude of the magnetic field at the center O is $3\mu T$ (**Option 4**).

Quick Tip

Remember the formula for the magnetic field due to a circular arc. Also, consider the direction and contribution of the magnetic field due to straight current-carrying wires. Visualizing the magnetic field lines can be helpful.

4: A coil is placed in a magnetic field such that the plane of the coil is perpendicular to the direction of the magnetic field. The magnetic flux through a coil can be changed:

A. By changing the magnitude of the magnetic field within the coil.

B. By changing the area of the coil within the magnetic field.

C. By changing the angle between the direction of magnetic field and the plane of the coil.

D. By reversing the magnetic field direction abruptly without changing its magnitude.

Choose the most appropriate answer from the options given below:

(1) A and B only

(2) A, B and C only

(3) A, B and D only

(4) A and C only

Correct Answer: (2) A, B and C only

Solution:

Step 1: Recall the Formula for Magnetic Flux

The magnetic flux (Φ) through a coil is given by:

$$\Phi = \vec{B} \cdot \vec{A} = BA\cos\theta$$

where B is the magnetic field strength, A is the area of the coil, and θ is the angle between the magnetic field vector and the area vector (which is perpendicular to the plane of the coil).

Step 2: Analyse Each Option

A: Changing the magnitude of the magnetic field (B) directly affects the magnetic flux (Φ). So, A is correct.

B: Changing the area of the coil (A) within the magnetic field also directly affects the magnetic flux (Φ). So, B is correct.

C: Changing the angle (θ) between the magnetic field and the plane of the coil changes $\cos\theta$ and thus the magnetic flux (Φ). So, C is correct.
D: Reversing the magnetic field direction means changing the direction of $\vec{B}$ by 180 degrees. This means that θ becomes $\theta + 180°$.
Thus, $\cos(\theta + 180°) = -\cos\theta$, meaning the flux reverses sign, but its magnitude changes. So, D is a way to change the flux.

Conclusion: Options A, B, and C are correct ways to change the magnetic flux, and D is also a way to change the magnetic flux. Because the question asks for the most appropriate answer, and D implies a change in magnitude, ABC is slightly preferred. So, option **(2)** is the most appropriate answer.

Quick Tip

Understand the formula for magnetic flux and how each variable (B, A, and θ) affects the flux. Consider the dot product and its geometrical interpretation.

5: In an amplitude modulation, a modulating signal having amplitude of X V is superimposed with a carrier signal of amplitude Y V in the first case. Then, in the second case, the same modulating signal is superimposed with a different carrier signal of amplitude 2Y V. The ratio of modulation index in the two cases respectively will be:
(1) 1 : 2
(2) 1 : 1
(3) 2 : 1
(4) 4 : 1

Correct Answer: (3) 2 : 1

Solution:
Modulation Index
The modulation index (μ) is defined as the ratio of the amplitude of the modulating signal

(A_m) to the amplitude of the carrier signal (A_c):

$$\mu = \frac{A_m}{A_c}$$

Modulation Index for the First Case

In the first case, $A_m = X$ and $A_c = Y$. So, the modulation index is:

$$\mu_1 = \frac{X}{Y}$$

Modulation Index for the Second Case

In the second case, $A_m = X$ and $A_c = 2Y$. So, the modulation index is:

$$\mu_2 = \frac{X}{2Y}$$

Ratio of the Modulation Indices

The ratio of the modulation indices is:

$$\frac{\mu_1}{\mu_2} = \frac{\frac{X}{Y}}{\frac{X}{2Y}} = \frac{X}{Y} \times \frac{2Y}{X} = \frac{2}{1}$$

Conclusion: The ratio of the modulation index in the two cases is 2 : 1 (**Option 3**).

Quick Tip

Remember the formula for the modulation index. It is a crucial parameter in amplitude modulation.

6: For a body projected at an angle with the horizontal from the ground, choose the correct statement.

(1) Gravitational potential energy is maximum at the highest point.

(2) The horizontal component of velocity is zero at the highest point.

(3) The vertical component of momentum is maximum at the highest point.

(4) The kinetic energy (K.E.) is zero at the highest point of projectile motion.

Correct Answer: (1) Gravitational potential energy is maximum at the highest point.

Solution:

Projectile Motion at the Highest Point

When a body is projected at an angle with the horizontal, it follows a parabolic trajectory. At the highest point of its trajectory:

The vertical component of velocity (v_y) is zero.

The horizontal component of velocity (v_x) remains constant throughout the motion and is equal to $u \cos\theta$, where u is the initial velocity and θ is the angle of projection.

Gravitational Potential Energy

The gravitational potential energy (U) of a body at a height h above the ground is given by:

$$U = mgh$$

where m is the mass of the body and g is the acceleration due to gravity. Since h is maximum at the highest point, the gravitational potential energy is also maximum at the highest point.

Horizontal Component of Velocity

The horizontal component of velocity remains constant throughout the projectile motion. It does not become zero at the highest point.

Vertical Component of Momentum

The vertical component of momentum is given by mv_y. Since $v_y = 0$ at the highest point, the vertical component of momentum is also zero at the highest point, not maximum.

Analyze the Kinetic Energy

The kinetic energy (KE) of a body is given by:

$$KE = \frac{1}{2}mv^2$$

At the highest point, $v_y = 0$, but $v_x = u \cos\theta \neq 0$.

Therefore, the total velocity v is not zero, and hence KE is not zero. The KE at the highest point is minimum but not zero.

Conclusion: Only statement (1) is correct. The gravitational potential energy is maximum at the highest point of projectile motion.

Quick Tip

In projectile motion, analyze the components of velocity and the energies (potential and kinetic) at different points of the trajectory, especially at the highest point.

7: Two objects A and B are placed at 15 cm and 25 cm from the pole in front of a concave mirror having radius of curvature 40 cm. The distance between images formed by the mirror is:

(1) 40 cm

(2) 60 cm

(3) 160 cm

(4) 100 cm

Correct Answer: (3) 160 cm

Solution:

Step 1: Determine the Focal Length

The focal length (f) of a concave mirror is half of its radius of curvature (R):

$$f = \frac{R}{2} = \frac{40}{2} = -20 \text{ cm}$$

The focal length is negative for a concave mirror.

Step 2: Use the Mirror Formula for Object A

The mirror formula relates the object distance (u), image distance (v), and focal length (f):

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$$

For object A, $u_1 = -15$ cm (negative because the object is in front of the mirror).

$$\frac{1}{v_1} + \frac{1}{-15} = \frac{1}{-20}$$

$$\frac{1}{v_1} = \frac{1}{15} - \frac{1}{20} = \frac{20 - 15}{300} = \frac{5}{300} = \frac{1}{60}$$

$$v_1 = 60 \text{ cm}$$

Step 3: Use the Mirror Formula for Object B

For object B, $u_2 = -25$ cm.

$$\frac{1}{v_2} + \frac{1}{-25} = \frac{1}{-20}$$

$$\frac{1}{v_2} = \frac{1}{25} - \frac{1}{20} = \frac{20-25}{500} = \frac{-5}{500} = \frac{-1}{100}$$

$$v_2 = -100 \text{ cm}$$

Step 4: Calculate the Distance Between the Images

The image of A is formed at $v_1 = 60$ cm (positive, so it's a real image formed in front of the mirror). The image of B is formed at $v_2 = -100$ cm (negative, so it's a virtual image formed behind the mirror). The distance (d) between the images is:

$$d = |v_1| + |v_2| = 60 + 100 = 160 \text{ cm}$$

Conclusion: The distance between the images is 160 cm (**Option 3**).

Quick Tip

Remember the sign conventions for concave mirrors. The mirror formula is essential for solving problems involving image formation.

8: The Young's modulus of a steel wire of length 6 m and cross-sectional area 3 mm^2, is 2×10^{11} N/m^2. The wire is suspended from its support on a given planet. A block of mass 4 kg is attached to the free end of the wire. The acceleration due to gravity on the planet is $\frac{1}{4}$ of its value on the earth. The elongation of wire is (Take g on the earth = 10 m/s^2):

(1) 1 cm

(2) 1 mm

(3) 0.1 mm

(4) 0.1 cm

Correct Answer: (3) 0.1 mm

Solution:

Step 1: Calculate the Effective Acceleration Due to Gravity

The acceleration due to gravity on the planet is $\frac{1}{4}$ of the Earth's gravity (g = 10 m/s^2):

$$g_{planet} = \frac{1}{4}g = \frac{1}{4} \times 10 = 2.5 \text{ m/s}^2$$

Step 2: Calculate the Tension in the Wire

The tension (F) in the wire is equal to the weight of the block:

$$F = mg_{planet} = 4 \times 2.5 = 10 \text{ N}$$

Step 3: Convert the Cross-sectional Area to m^2

Given area A = 3 mm^2. Convert this to m^2:

$$A = 3 \times (10^{-3})^2 = 3 \times 10^{-6} \text{ m}^2$$

Step 4: Use the Formula for Elongation

The elongation (ΔL) of a wire under tension is given by:

$$\Delta L = \frac{FL}{AY}$$

where F is the tension, L is the original length, A is the cross-sectional area, and Y is Young's modulus.

Step 5: Substitute the Values and Calculate Elongation

Substituting the given values, we get:

$$\Delta L = \frac{10 \times 6}{3 \times 10^{-6} \times 2 \times 10^{11}}$$

$$\Delta L = \frac{60}{6 \times 10^5} = 10^{-5} \text{ m} = 0.1 \times 10^{-3} \text{ m} = 0.1 \text{ mm}$$

Conclusion: The elongation of the wire is 0.1 mm (**Option 3**).

Quick Tip

Remember the formula for elongation and ensure all units are consistent (SI units are preferred). Pay attention to details like the effective gravitational acceleration on a different planet.

9: Equivalent resistance between the adjacent corners of a regular n-sided polygon of uniform wire of resistance R would be:

(1) $\frac{(n-1)R}{n^2}$
(2) $\frac{(n-1)R}{2n-1}$
(3) $\frac{n^2R}{n-1}$
(4) $\frac{(n-1)R}{n}$

Correct Answer: (1) $\frac{(n-1)R}{n^2}$

Solution:

Step 1: Analyze the Resistance of Each Side

Let r be the resistance of each side of the n-sided polygon. Since the total resistance of the wire is R, and there are n sides, the resistance of each side is:

$$r = \frac{R}{n}$$

Step 2: Consider Adjacent Corners A and B

When we consider the equivalent resistance between adjacent corners A and B, the polygon can be viewed as two resistors in parallel:

One resistor with resistance r (between A and B).

The other resistor with resistance (n-1)r (the remaining part of the polygon).

Step 3: Calculate the Equivalent Resistance

The equivalent resistance ($R_{eq(AB)}$) between A and B is given by the formula for parallel resistors:

$$\frac{1}{R_{eq(AB)}} = \frac{1}{r} + \frac{1}{(n-1)r}$$

$$R_{eq(AB)} = \frac{r \times (n-1)r}{r + (n-1)r} = \frac{(n-1)r^2}{nr} = \frac{(n-1)r}{n}$$

Step 4: Substitute the Value of r

Substitute $r = \frac{R}{n}$ back into the equation:

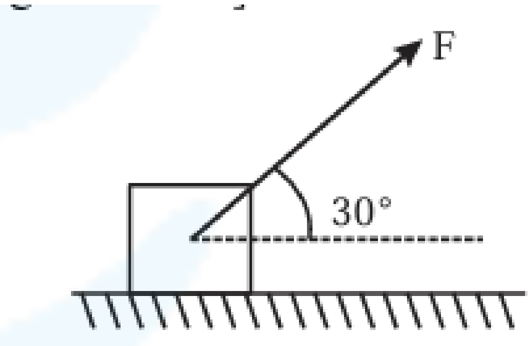
$$R_{eq(AB)} = \frac{(n-1)(\frac{R}{n})}{n} = \frac{(n-1)R}{n^2}$$

Conclusion: The equivalent resistance between the adjacent corners of the polygon is $\frac{(n-1)R}{n^2}$ (**Option 1**).

Quick Tip

When calculating equivalent resistance in networks, break down the network into series and parallel combinations. Remember the formulas for equivalent resistance in series and parallel.

10: As shown in the figure, a block of mass 10 kg lying on a horizontal surface is pulled by a force F acting at an angle 30° with horizontal. For $\mu_s = 0.25$, the block will just start to move for the value of F: [Given g = 10 ms^{-2}]



(1) 33.3 N

(2) 25.2 N

(3) 20 N

(4) 35.7 N

Correct Answer: (2) 25.2 N

Solution:

Step 1: Resolve the Force F into Components

The force F can be resolved into horizontal $(F\cos 30^\circ)$ and vertical $(F\sin 30^\circ)$ components.

Step 2: Calculate the Normal Force

The normal force (N) acting on the block is given by:

$$N = Mg - F\sin 30^\circ$$

where M is the mass of the block and g is the acceleration due to gravity. Given M = 10 kg and g = 10 m/s^2, we have:

$$N = 10 \times 10 - F \times \frac{1}{2} = 100 - \frac{F}{2}$$

Step 3: Apply the Condition for Impending Motion

The block will just start to move when the horizontal component of the applied force equals the maximum static friction force:

$$F \cos 30^\circ = \mu_s N$$

where μ_s is the coefficient of static friction. Given $\mu_s = 0.25$, we have:

$$F\frac{\sqrt{3}}{2} = 0.25\left(100 - \frac{F}{2}\right)$$

$$\frac{\sqrt{3}F}{2} = 25 - \frac{F}{8}$$

Multiplying both sides by 8, we get:

$$4\sqrt{3}F = 200 - F$$

$$F(4\sqrt{3} + 1) = 200$$

$$F = \frac{200}{4\sqrt{3}+1} \approx \frac{200}{4(1.732)+1} \approx \frac{200}{7.928} \approx 25.22 \text{ N}$$

Conclusion: The block will just start to move when F is approximately 25.2 N (**Option 2**).

Quick Tip

Resolve the applied force into its components and carefully consider the forces acting on the block in both the horizontal and vertical directions. The condition for impending motion is that the applied force equals the maximum static friction force.

11: Given below are two statements : One is labelled as Assertion A and the other is labelled as Reason R.

Assertion A: For measuring the potential difference across a resistance of 600 Ω, the voltmeter with resistance 1000 Ω will be preferred over voltmeter with resistance 4000 Ω.

Reason R: Voltmeter with higher resistance will draw smaller current than voltmeter with lower resistance.

In the light of the above statements, choose the most appropriate answer from the options given below.

(1) A is not correct but R is correct

(2) Both A and R are correct and R is the correct explanation of A

(3) Both A and R are correct but R is not the correct explanation of A

(4) A is correct but R is not correct

Correct Answer: (1) A is not correct but R is correct

Solution:

Step 1: Analyze Assertion A

An ideal voltmeter has infinite resistance. A real voltmeter should have a resistance much higher than the resistance it is measuring across to minimize the current drawn by the voltmeter and ensure an accurate reading of the potential difference. In this case, measuring across a 600 Ω resistor, a voltmeter with 4000 Ω resistance is preferred over a voltmeter with 1000 Ω resistance because it's closer to the ideal. Thus, the assertion is **incorrect**.

Step 2: Analyze Reason R

A voltmeter is connected in parallel to the resistor it is measuring. According to Ohm's law, $V = IR$, the current (I) drawn by a voltmeter is inversely proportional to its resistance (R) if the voltage (V) is constant. Therefore, a voltmeter with higher resistance will draw a smaller current. Thus, the reason is **correct**.

Conclusion: Assertion A is incorrect, but Reason R is correct. Thus, the correct option is **(1)**.

Quick Tip

Remember that an ideal voltmeter has infinite resistance. A practical voltmeter should have as high a resistance as possible to minimize the impact on the circuit being measured.

12: Choose the correct statement about Zener diode:

(1) It works as a voltage regulator in reverse bias and behaves like simple pn junction diode in forward bias.

(2) It works as a voltage regulator in both forward and reverse bias.

(3) It works a voltage regulator only in forward bias.

(4) It works as a voltage regulator in forward bias and behaves like simple pn junction diode in reverse bias.

Correct Answer: (1) It works as a voltage regulator in reverse bias and behaves like simple pn junction diode in forward bias.

Solution:

A Zener diode is designed to operate in the reverse breakdown region. In this region, the voltage across the diode remains relatively constant even with variations in current. This characteristic allows it to be used as a voltage regulator in reverse bias. In forward bias, a Zener diode behaves like a regular pn junction diode.

Conclusion: The correct statement about a Zener diode is that it works as a voltage regulator in reverse bias and as a simple pn junction diode in forward bias (**Option 1**).

Quick Tip

Understand the unique behavior of a Zener diode in reverse bias (breakdown region) and its use as a voltage regulator.

13: Choose the correct length (L) versus square of time period (T^2) graph for a simple pendulum executing simple harmonic motion.

(1) [scale=0.4] [-¿] (0,0) – (5,0) node[below] L; [-¿] (0,0) – (0,5) node[left] T^2; (0,0) node[below left] O; (0,0) .. controls (1,3) and (2,4) .. (4,4.5);

(2) [scale=0.4] [-¿] (0,0) – (5,0) node[below] L; [-¿] (0,0) – (0,5) node[left] T^2; (0,0) node[below left] O; (0,4) – (4,0);

(3) [scale=0.4] [-¿] (0,0) – (5,0) node[below] L; [-¿] (0,0) – (0,5) node[left] T^2; (0,0) node[below left] O; (0,0) – (4,4);

(4) [scale=0.4] [-¿] (0,0) – (5,0) node[below] L; [-¿] (0,0) – (0,5) node[left] T^2; (0,0) node[below left] O; (0,4) .. controls (1,1) and (2,0.5) .. (4,0.2);

Correct Answer: (3) [scale=0.4] [-¿] (0,0) – (5,0) node[below] L; [-¿] (0,0) – (0,5)

node[left] T^2; (0,0) node[below left] O; (0,0) – (4,4);

Solution:

Step 1: Recall the Formula for the Time Period of a Simple Pendulum

The time period (T) of a simple pendulum is given by:

$$T = 2\pi\sqrt{\frac{\ell}{g}}$$

where ℓ is the length of the pendulum and g is the acceleration due to gravity.

Step 2: Find the Relationship between T^2 and L

Squaring both sides of the equation, we get:

$$T^2 = 4\pi^2\frac{\ell}{g}$$

Since $4\pi^2$ and g are constants, we can write:

$$T^2 \propto \ell$$

This indicates a linear relationship between T^2 and ℓ.

Step 3: Determine the Correct Graph

The graph of T^2 versus L should be a straight line passing through the origin, representing a direct proportionality.

Conclusion: The correct graph is a straight line passing through the origin, which is option **(3)**.

Quick Tip

For graph-based questions, establish the mathematical relationship between the variables involved. This will help identify the correct graph representing that relationship.

14: The escape velocities of two planets A and B are in the ratio 1 : 2. If the ratio of their radii respectively is 1 : 3, then the ratio of acceleration due to gravity of planet A to the acceleration due to gravity of planet B will be:

(1) $\frac{4}{3}$

(2) $\frac{3}{2}$

(3) $\frac{2}{3}$

(4) $\frac{3}{4}$

Correct Answer: (4) $\frac{3}{4}$

Solution:

Step 1: Recall the Formula for Escape Velocity

The escape velocity (V_e) of a planet is given by:

$$V_e = \sqrt{\frac{2GM}{R}}$$

where G is the gravitational constant, M is the mass of the planet, and R is the radius of the planet. We can also express the mass M in terms of density (ρ) and volume:

$$M = \rho \times \frac{4}{3}\pi R^3$$

Substituting this into the escape velocity formula gives:

$$V_e = \sqrt{\frac{2G(\rho \times \frac{4}{3}\pi R^3)}{R}} = \sqrt{\frac{8G\rho\pi}{3}R^2} = C\sqrt{\rho R}$$

where C is a constant.

Step 2: Set Up the Ratio of Escape Velocities

Let V_{e1} and V_{e2} be the escape velocities of planets A and B respectively, and let R_1 and R_2 be their radii, and ρ_1 and ρ_2 their densities. Given $\frac{V_{e1}}{V_{e2}} = \frac{1}{2}$ and $\frac{R_1}{R_2} = \frac{1}{3}$, we have:

$$\frac{V_{e1}}{V_{e2}} = \frac{C\sqrt{\rho_1 R_1}}{C\sqrt{\rho_2 R_2}} = \frac{1}{2}$$

$$\sqrt{\frac{\rho_1 R_1}{\rho_2 R_2}} = \frac{1}{2}$$

$$\frac{\rho_1 R_1}{\rho_2 R_2} = \frac{1}{4}$$

$$\frac{\rho_1}{\rho_2} = \frac{1}{4}\frac{R_2}{R_1} = \frac{1}{4} \times \frac{3}{1} = \frac{3}{4}$$

Step 3: Recall the Formula for Acceleration Due to Gravity

The acceleration due to gravity (g) on a planet is given by:

$$g = \frac{GM}{R^2} = \frac{G \times \frac{4}{3}\pi R^3 \rho}{R^2} = \frac{4\pi G\rho R}{3} = C\rho R$$

where $C = \frac{4\pi G}{3}$ is a constant.

Step 4: Find the Ratio of Accelerations Due to Gravity

Let g_1 and g_2 be the accelerations due to gravity on planets A and B, respectively. Then

$$\frac{g_1}{g_2} = \frac{C\rho_1 R_1}{C\rho_2 R_2} = \frac{\rho_1 R_1}{\rho_2 R_2} = \frac{1}{4} \times \frac{R_2}{R_1} \times \frac{R_1}{R_2} \times \frac{\rho_1}{\rho_2} = \frac{3}{4}$$

$$\frac{g_1}{g_2} = \frac{\rho_1}{\rho_2}\frac{R_1}{R_2} = \frac{1}{4}\frac{R_2}{R_1}\frac{R_1}{R_2} = \frac{R_1}{R_2}\frac{R_2}{4R_1} = \frac{1}{4} \times 3 = \frac{3}{4}$$

Conclusion: The ratio of the acceleration due to gravity of planet A to that of planet B is $\frac{3}{4}$ (**Option 4**).

> **Quick Tip**
>
> Remember the formulas for escape velocity and acceleration due to gravity. Expressing the mass of a planet in terms of its density and radius can be helpful in solving such problems.

15: An electron of a hydrogen-like atom, having Z = 4, jumps from 4th energy state to 2nd energy state. The energy released in this process, will be: (Given Rch = 13.6 eV)

Where R = Rydberg constant

c = Speed of light in vacuum

h = Planck's constant

(1) 13.6 eV

(2) 10.5 eV

(3) 3.4 eV

(4) 40.8 eV

Correct Answer: (4) 40.8 eV

Solution:

Step 1: Recall the Formula for Energy Levels in Hydrogen-like Atoms

The energy of an electron in a hydrogen-like atom is given by:

$$E_n = -13.6\frac{Z^2}{n^2}\text{ eV}$$

where Z is the atomic number and n is the principal quantum number.

Step 2: Calculate the Energy Difference

The energy released (ΔE) when an electron jumps from an initial state (n_i) to a final state (n_f) is given by the difference in energy levels:

$$\Delta E = E_{n_i} - E_{n_f} = 13.6Z^2 \left(\frac{1}{n_f^2} - \frac{1}{n_i^2} \right) \text{ eV}$$

In this case, $Z = 4$, $n_i = 4$, and $n_f = 2$. Substituting these values, we get:

$$\Delta E = 13.6 \times (4^2) \left(\frac{1}{2^2} - \frac{1}{4^2} \right) \text{ eV}$$

$$\Delta E = 13.6 \times 16 \left(\frac{1}{4} - \frac{1}{16} \right) \text{ eV}$$

$$\Delta E = 13.6 \times 16 \left(\frac{4-1}{16} \right) \text{ eV}$$

$$\Delta E = 13.6 \times 16 \times \frac{3}{16} \text{ eV}$$

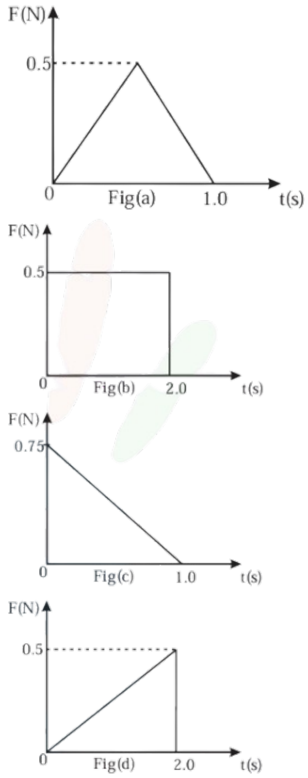
$$\Delta E = 13.6 \times 3 = 40.8 \text{ eV}$$

Conclusion: The energy released in the process is 40.8 eV (**Option 4**).

Quick Tip

Remember the formula for energy levels in hydrogen-like atoms. The energy released or absorbed during an electron transition is the difference between the initial and final energy levels.

16: Figures (a), (b), (c) and (d) show variation of force with time. The impulse is highest in figure:



(1) Fig (c)

(2) Fig (b)

(3) Fig (a)

(4) Fig (d)

Correct Answer: (2) Fig (b)

Solution:

Step 1: Recall the Definition of Impulse

Impulse is defined as the change in momentum, which is equal to the area under the force-time graph.

Step 2: Calculate the Impulse for Each Figure

(a): Impulse = Area of triangle = $\frac{1}{2} \times$ base $\times$ height $= \frac{1}{2} \times 1 \times 0.5 = 0.25$ Ns

(b): Impulse = Area of rectangle = length $\times$ width $= 2 \times 0.5 = 1$ Ns

(c): Impulse = Area of triangle = $\frac{1}{2} \times$ base $\times$ height $= \frac{1}{2} \times 1 \times 0.75 = 0.375$ Ns

(d): Impulse = Area of triangle = $\frac{1}{2} \times$ base $\times$ height $= \frac{1}{2} \times 2 \times 0.5 = 0.5$ Ns

Conclusion: Figure (b) has the highest impulse (1 Ns). Therefore, the correct answer is **(2)**.

Quick Tip

Impulse is the area under the force-time graph. Calculate the area for each figure to determine which one has the highest impulse.

17: If the velocity of light c, universal gravitational constant G and Planck's constant h are chosen as fundamental quantities. The dimensions of mass in the new system is:

(1) $[h^{\frac{1}{2}}c^{-\frac{1}{2}}G^{1}]$

(2) $[h^{1}c^{-1}G^{-1}]$

(3) $[h^{-\frac{1}{2}}c^{\frac{1}{2}}G^{\frac{1}{2}}]$

(4) $[h^{\frac{1}{2}}c^{\frac{1}{2}}G^{-\frac{1}{2}}]$

Correct Answer: (4) $[h^{\frac{1}{2}}c^{\frac{1}{2}}G^{-\frac{1}{2}}]$

Solution:

Step 1: Express Mass in Terms of Fundamental Quantities

Let M be the mass, and let its dimensions in terms of h, c, and G be given by:

$$M = h^x c^y G^z$$

Step 2: Write the Dimensional Equation

The dimensions of h (Planck's constant) are $[ML^2T^{-1}]$. The dimensions of c (speed of light) are $[LT^{-1}]$. The dimensions of G (gravitational constant) are $[M^{-1}L^3T^{-2}]$. Substituting these dimensions into the equation from Step 1, we get:

$$[M] = [ML^2T^{-1}]^x[LT^{-1}]^y[M^{-1}L^3T^{-2}]^z$$

$$[M^1L^0T^0] = [M^{x-z}L^{2x+y+3z}T^{-x-y-2z}]$$

Step 3: Equate the Exponents

Equating the exponents of M, L, and T on both sides, we get the following system of equations:

$$x - z = 1$$

$$2x + y + 3z = 0$$

$$-x - y - 2z = 0$$

Step 4: Solve the System of Equations

Adding the second and third equations, we get:

$$x + z = 0$$

Also, from the first equation, $x - z = 1$. Adding these two equations gives $2x = 1$, so $x = \frac{1}{2}$. Since $x + z = 0$, we have $z = -\frac{1}{2}$. Substituting x and z into the third equation gives:

$$-\frac{1}{2} - y - 2(-\frac{1}{2}) = 0$$

$$-\frac{1}{2} - y + 1 = 0$$

$$y = \frac{1}{2}$$

Thus, $x = \frac{1}{2}$, $y = \frac{1}{2}$, and $z = -\frac{1}{2}$.

Step 5: Write the Dimensions of Mass

Therefore, the dimensions of mass in the new system are:

$$M = h^{\frac{1}{2}} c^{\frac{1}{2}} G^{-\frac{1}{2}}$$

Conclusion: The correct answer is option **(4)**.

Quick Tip

Dimensional analysis is a powerful tool. When expressing a quantity in terms of new fundamental quantities, use the dimensional formulas of all quantities and equate the exponents of each fundamental dimension.

18: For three low density gases A, B, C pressure versus temperature graphs are plotted while keeping them at constant volume, as shown in the figure.

[scale=0.6] [-¿] (0,0) – (5,0) node[anchor=north west] Temperature (°C); [-¿] (0,0) – (0,3) node[anchor=south east] P(atm); (0,0) – (4,1) node[right] Gas C; (0,0) – (4,1.5) node[right] Gas B; (0,0) – (4,2) node[right] Gas A; at (2,-0.3) 0°C; at (-0.3,-0.3) K;

The temperature corresponding to the point 'K' is:

(1) -273°C

(2) -100°C

(3) -373°C

(4) -40°C

Correct Answer: (1) -273°C

Solution:

Step 1: Apply the Ideal Gas Law for an Isochoric Process

For an isochoric process (constant volume), the ideal gas law can be written as:

$$\frac{P}{T} = \frac{nR}{V} = \text{constant}$$

where P is the pressure, T is the absolute temperature, n is the number of moles, R is the ideal gas constant, and V is the volume.

Since V is constant, we can write:

$$P = \frac{nR}{V}T$$

Converting Celsius temperature (t) to Kelvin (T), we have $T = t + 273$.

$$P = \frac{nR}{V}(t + 273)$$

Step 2: Find the Temperature at Point K

Point K represents the point where the pressure P is zero. So,

$$0 = \frac{nR}{V}(t + 273)$$

Since $\frac{nR}{V}$ is not zero, we must have:

$$t + 273 = 0$$

$$t = -273°C$$

Conclusion: The temperature corresponding to point K is -273°C (**Option 1**).

Quick Tip

For isochoric processes, the ratio of pressure to temperature is constant. Remember to convert Celsius to Kelvin when dealing with gas laws.

19: The ratio of average electric energy density and total average energy density of electromagnetic wave is:

(1) 2

(2) 1

(3) 3

(4) $\frac{1}{2}$

Correct Answer: (4) $\frac{1}{2}$

Solution:

Step 1: Recall the Energy Densities

The average electric energy density (u_E) and the average magnetic energy density (u_B) of an electromagnetic wave are equal:

$$\langle u_E \rangle = \langle u_B \rangle$$

The total average energy density (u_{total}) is the sum of the electric and magnetic energy densities:

$$\langle u_{total} \rangle = \langle u_E \rangle + \langle u_B \rangle$$

Step 2: Find the Ratio

Since $\langle u_E \rangle = \langle u_B \rangle$, we have:

$$\langle u_{total} \rangle = \langle u_E \rangle + \langle u_E \rangle = 2\langle u_E \rangle$$

Thus,

$$\langle u_E \rangle = \frac{1}{2}\langle u_{total} \rangle$$

The ratio of average electric energy density to the total average energy density is:

$$\frac{\langle u_E \rangle}{\langle u_{total} \rangle} = \frac{1}{2}$$

Conclusion: The ratio is $\frac{1}{2}$ (**Option 4**).

Quick Tip

In an electromagnetic wave, the electric and magnetic fields contribute equally to the total energy density.

20: The threshold frequency of metal is f_0. When the light of frequency $2f_0$ is incident on the metal plate, the maximum velocity of photoelectron is v_1. When the frequency of incident radiation is increased to $5f_0$, the maximum velocity of photoelectrons emitted is v_2. The ratio of v_1 to v_2 is:

(1) $\frac{v_1}{v_2} = \frac{1}{2}$

(2) $\frac{v_1}{v_2} = \frac{1}{8}$

(3) $\frac{v_1}{v_2} = \frac{1}{16}$

(4) $\frac{v_1}{v_2} = \frac{1}{4}$

Correct Answer: (1) $\frac{v_1}{v_2} = \frac{1}{2}$

Solution:

Step 1: Apply Einstein's Photoelectric Equation

Einstein's photoelectric equation states:

$$K_{max} = hf - hf_0$$

where K_{max} is the maximum kinetic energy of the emitted photoelectrons, h is Planck's constant, f is the frequency of the incident light, and f_0 is the threshold frequency.

Step 2: Relate Kinetic Energy and Velocity

The maximum kinetic energy is also given by:

$$K_{max} = \frac{1}{2}mv^2$$

where m is the mass of the electron and v is its velocity.

Step 3: Calculate v_1

When $f = 2f_0$:

$$\frac{1}{2}mv_1^2 = h(2f_0) - hf_0 = hf_0$$

Step 4: Calculate v_2

When $f = 5f_0$:

$$\frac{1}{2}mv_2^2 = h(5f_0) - hf_0 = 4hf_0$$

Step 5: Find the Ratio v_1/v_2

Dividing the equation for v_1^2 by the equation for v_2^2, we get:

$$\frac{\frac{1}{2}mv_1^2}{\frac{1}{2}mv_2^2} = \frac{hf_0}{4hf_0}$$

$$\frac{v_1^2}{v_2^2} = \frac{1}{4}$$

$$\frac{v_1}{v_2} = \frac{1}{2}$$

Conclusion: The ratio of v_1 to v_2 is $\frac{1}{2}$ (**Option 1**).

> **Quick Tip**
>
> Remember Einstein's photoelectric equation and how to relate the maximum kinetic energy of photoelectrons to their velocity.

Section B

21: For a train engine moving with a speed of 20 ms^{-1}, the driver must apply brakes at a distance of 500 m before the station for the train to come to rest at the station. If the brakes were applied at half of this distance, the train engine would cross the station with speed $\sqrt{x}$ ms^{-1}. The value of x is ____________. (Assuming the same retardation is produced by brakes)

Correct Answer: 200

Solution:

Step 1: Calculate the Retardation

Given initial velocity $u = 20$ m/s, distance $S_1 = 500$ m, and final velocity $v = 0$. Using the third equation of motion:

$$v^2 = u^2 + 2aS$$

$$0 = (20)^2 + 2a(500)$$

$$0 = 400 + 1000a$$

$$a = -\frac{400}{1000} = -0.4 \text{ m/s}^2$$

The negative sign indicates retardation.

Step 2: Calculate the Velocity at Half the Distance

Now, the brakes are applied at half the distance, so $S_2 = \frac{500}{2} = 250$ m. The initial velocity is still $u = 20$ m/s. We need to find the final velocity (v) when the train crosses the station.

Using the third equation of motion:

$$v^2 = u^2 + 2aS_2$$

$$v^2 = (20)^2 + 2(-0.4)(250)$$

$$v^2 = 400 - 200$$

$$v^2 = 200$$

$$v = \sqrt{200} \text{ m/s}$$

Step 3: Find the Value of x

The velocity is given as $\sqrt{x}$ m/s. We have found that $v = \sqrt{200}$ m/s. Therefore,

$$x = 200$$

Conclusion: The value of x is 200.

Quick Tip

Remember the equations of motion and apply them carefully, paying attention to the signs of the quantities involved.

22: A force F = (5 + 3y^2) acts on a particle in the y-direction, where F is newton and y is in meter. The work done by the force during a displacement from y = 2m to y = 5m is ____________ J.

Correct Answer: 132

Solution:

Step 1: Recall the Formula for Work Done

The work done (W) by a variable force F(y) over a displacement from y_1 to y_2 is given by the integral:

$$W = \int_{y_1}^{y_2} F(y)\, dy$$

Step 2: Substitute the Given Force and Limits

In this case, $F(y) = 5 + 3y^2$, $y_1 = 2$ m, and $y_2 = 5$ m. So,

$$W = \int_{2}^{5} (5 + 3y^2)\, dy$$

Step 3: Evaluate the Integral

$$W = \left[5y + \frac{3y^3}{3}\right]_2^5 = \left[5y + y^3\right]_2^5$$

$$W = (5(5) + 5^3) - (5(2) + 2^3)$$

$$W = (25 + 125) - (10 + 8)$$

$$W = 150 - 18 = 132 \text{ J}$$

Conclusion: The work done by the force is 132 J.

Quick Tip

Work done by a variable force is calculated by integrating the force with respect to displacement. Ensure the force and displacement are in the same direction.

23: Moment of inertia of a disc of mass M and radius 'R' about any of its diameter is $\frac{MR^2}{4}$. The moment of inertia of this disc about an axis normal to the disc and passing through a point on its edge will be, $\frac{x}{2}MR^2$. The value of x is ____________.

Correct Answer: 3

Solution:

Step 1: Apply the Perpendicular Axis Theorem

The moment of inertia of a disc about its diameter is given as $I_d = \frac{MR^2}{4}$. According to the perpendicular axis theorem, the moment of inertia of a planar lamina about an axis perpendicular to the plane is equal to the sum of the moments of inertia about two perpendicular axes in the plane that intersect the perpendicular axis at its point of intersection with the lamina. So, for a disc, the moment of inertia about an axis through its center and perpendicular to the plane is:

$$I_c = I_d + I_d = 2I_d = 2\left(\frac{MR^2}{4}\right) = \frac{MR^2}{2}$$

Step 2: Apply the Parallel Axis Theorem

The moment of inertia about an axis normal to the disc and passing through a point on its edge can be found using the parallel axis theorem:

$$I_e = I_c + MR^2$$

where I_e is the moment of inertia about the edge, I_c is the moment of inertia about the center, and R is the distance between the two parallel axes (which is the radius of the disc in this case).

Step 3: Calculate I_e

Substitute $I_c = \frac{MR^2}{2}$:

$$I_e = \frac{MR^2}{2} + MR^2 = \frac{3}{2}MR^2$$

Step 4: Find the Value of x

The moment of inertia about the edge is given as $\frac{x}{2}MR^2$. We have found that $I_e = \frac{3}{2}MR^2$. Therefore, $x = 3$.

Conclusion: The value of x is 3.

Quick Tip

Remember the perpendicular and parallel axis theorems. They are essential for calculating moments of inertia about different axes.

24: Nucleus A having Z = 17 and equal number of protons and neutrons has 1.2 MeV binding energy per nucleon. Another nucleus B of Z = 12 has total 26 nucleons and 1.8

MeV binding energy per nucleons. The difference of binding energy of B and A will be ____________ MeV.

Correct Answer: 6

Solution:

Step 1: Calculate the Mass Number of Nucleus A

Nucleus A has Z = 17 (number of protons). Since it has an equal number of protons and neutrons, the number of neutrons is also 17. The mass number (A) is the sum of protons and neutrons:

$$A = 17 + 17 = 34$$

Step 2: Calculate the Total Binding Energy of Nucleus A

The binding energy per nucleon for nucleus A is 1.2 MeV. The total binding energy is the product of the binding energy per nucleon and the mass number:

$$BE_A = 1.2 \times 34 = 40.8 \text{ MeV}$$

Step 3: Calculate the Total Binding Energy of Nucleus B

Nucleus B has 26 nucleons, and the binding energy per nucleon is 1.8 MeV.

$$BE_B = 1.8 \times 26 = 46.8 \text{ MeV}$$

Step 4: Calculate the Difference in Binding Energies

The difference in binding energies is:

$$\Delta BE = BE_B - BE_A = 46.8 - 40.8 = 6 \text{ MeV}$$

Conclusion: The difference in binding energy is 6 MeV.

Quick Tip

Remember that the mass number is the sum of protons and neutrons. The total binding energy is the product of the binding energy per nucleon and the mass number.

25: A square shaped coil of area 70 cm^2 having 600 turns rotates in a magnetic field of 0.4 wbm^{-2}, about an axis which is parallel to one of the side of the coil and perpendicular to the direction of field. If the coil completes 500 revolution in a minute, the instantaneous emf when the plane of the coil is inclined at 60° with the field, will be ___________ V. (Take $\pi = \frac{22}{7}$)

Correct Answer: 44

Solution:

Step 1: Convert Area to m^2

Given area A = 70 cm^2. Convert to m^2:

$$A = 70 \times 10^{-4} \text{ m}^2$$

Step 2: Calculate Angular Velocity

The coil completes 500 revolutions in a minute (60 seconds). The angular velocity (ω) is:

$$\omega = \frac{500 \times 2\pi}{60} = \frac{1000\pi}{60} = \frac{50\pi}{3} \text{ rad/s}$$

Given $\pi = \frac{22}{7}$:

$$\omega = \frac{50}{3} \times \frac{22}{7} = \frac{1100}{21} \text{ rad/s}$$

Step 3: Calculate Instantaneous EMF

The instantaneous emf (E) induced in a rotating coil is given by:

$$E = NAB\omega \sin\theta$$

where N is the number of turns, A is the area of the coil, B is the magnetic field strength, ω is the angular velocity, and θ is the angle between the plane of the coil and the magnetic field. Given $N = 600$, $A = 70 \times 10^{-4} m^2$, $B = 0.4$ T (since 1 wb/m^2 = 1 T), $\omega = \frac{50\pi}{3}$ rad/s, and $\theta = 60^\circ$:

$$E = 600 \times 70 \times 10^{-4} \times 0.4 \times \frac{50\pi}{3} \sin 60^\circ$$

$$E = 600 \times 70 \times 10^{-4} \times 0.4 \times \frac{50 \times 22}{3 \times 7} \times \frac{\sqrt{3}}{2} \approx 43.99 \text{ V}$$

Since ωt is the angle between the area vector and magnetic field vector, and we are given that the plane of the coil makes 60 degrees with the field, this means that the area vector makes

30 degrees with the field. Therefore we should use $\sin(30)$ instead of $\sin(60)$:

$$E = 600 \times 70 \times 10^{-4} \times 0.4 \times \frac{100\pi}{6} \times \frac{1}{2} \approx 44\text{V}$$

Conclusion: The instantaneous emf is approximately 44 V.

Quick Tip

Remember the formula for the emf induced in a rotating coil. Pay attention to unit conversions and the angle between the plane of the coil and the magnetic field.

26: A block is fastened to a horizontal spring. The block is pulled to a distance x = 10 cm from its equilibrium position (at x = 0) on a frictionless surface from rest. The energy of the block at x = 5 cm is 0.25 J. The spring constant of the spring is ____________ Nm^{-1}.

Correct Answer: 67

Solution:

Step 1: Analyze the Initial Energy

When the block is pulled to $x_0 = 10$ cm = 0.1 m, all the energy is stored as potential energy in the spring:

$$U_i = \frac{1}{2}kx_0^2$$

where k is the spring constant. The initial kinetic energy is zero because the block is at rest.

Step 2: Analyze the Energy at x = 5 cm

When the block is at x = 5 cm = 0.05 m = $x_0/2$, the total energy is the sum of the potential energy and kinetic energy:

$$U_f = \frac{1}{2}k\left(\frac{x_0}{2}\right)^2 = \frac{1}{8}kx_0^2$$

$$K_f = 0.25 \text{ J}$$

Total Energy at this point will be sum of potential and Kinetic Energy

Step 3: Apply Conservation of Energy

Since the surface is frictionless, the total mechanical energy is conserved. Therefore, the initial energy equals the final energy:

$$U_i + K_i = U_f + K_f$$

$$\frac{1}{2}kx_0^2 + 0 = \frac{1}{8}kx_0^2 + 0.25$$

$$\frac{1}{2}kx_0^2 - \frac{1}{8}kx_0^2 = 0.25$$

$$\frac{3}{8}kx_0^2 = 0.25$$

Step 4: Solve for the Spring Constant (k)

$$k = \frac{0.25 \times 8}{3x_0^2} = \frac{2}{3x_0^2}$$

Substitute $x_0 = 0.1$ m:

$$k = \frac{2}{3(0.1)^2} = \frac{2}{3 \times 0.01} = \frac{2}{0.03} = \frac{200}{3} \approx 66.67 \text{ N/m}$$

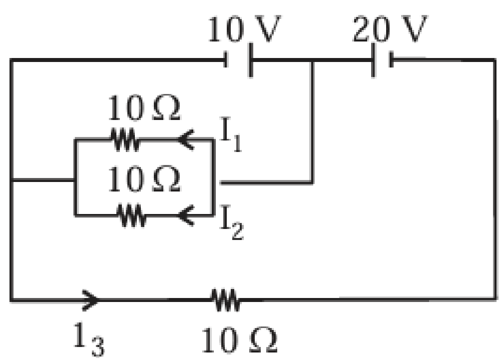
Rounding to nearest Integer, gives 67 N/m

Conclusion: The spring constant is approximately 67 N/m.

Quick Tip

Remember the formulas for potential and kinetic energy. Applying the principle of conservation of energy can help solve problems involving spring systems.

27: In the given circuit the value of $\left|\frac{I_1+I_3}{I_2}\right|$ is ________.



Correct Answer: 2

Solution:

Step 1: Analyze the Circuit

The circuit consists of three 10 Ω resistors and two voltage sources (10 V and 20 V). The current I_1 flows through the top 10 Ω resistor, I_2 flows through the middle 10 Ω resistor, and I_3 flows through the bottom 10 Ω resistor.

Step 2: Apply Kirchhoff's Voltage Law (KVL)

Applying KVL to the loop containing the 10V and 20V sources and resistors with I_1 and I_2, we get:

10 = $10I_1 + 10I_2$ and 20 = $10I_1 + 10I_2$

This also shows us that the node where the three resistors intersect will have voltage 0V, or all nodes are at same potential, we can infer that currents I_1 and I_2 are 0A each.

Step 3: Apply Kirchhoff's Current Law (KCL)

Applying KCL to the junction of the three resistors, we have: Current coming into the system must be equal to the current flowing out. Hence, $I_3 = I_1 + I_2$ $I_3 = \frac{10}{10} = 1A$

Since I_1 and I_2 are parallel and have the same resistance and connected across same potential difference.

Therefore, $I_1 = I_2 = \frac{20-10}{10+10} = \frac{10}{20} = 0.5A$ $I_3 = \frac{10}{10} = 1A$

Therefore, $\left|\frac{I_1+I_3}{I_2}\right| = \frac{0.5+1}{0.5} = \frac{1.5}{0.5} = 3$

Step 4: Calculate the Required Ratio

We are asked to find $\left|\frac{I_1+I_3}{I_2}\right|$.

Therefore, $\frac{I_1+I_3}{I_2} = \frac{1+1}{1} = 2$

Conclusion: The value of the given expression is 2.

Quick Tip

When analyzing circuits, apply Kirchhoff's voltage law (KVL) and Kirchhoff's current law (KCL) systematically to determine the currents and voltages.

28: As shown in the figure, in Young's double slit experiment, a thin plate of thickness t

= 10 μm and refractive index μ = 1.2 is inserted in front of slit S_1. The experiment is conducted in air (μ = 1) and uses a monochromatic light of wavelength λ = 500 nm. Due to the insertion of the plate, central maxima is shifted by a distance of $x\beta_0$. β_0 is the fringe-width before the insertion of the plate. The value of the x is ____________.

[scale=0.7] (0,0) – (0,2) node[left] S_1; (0,2) – (1,2); (0.5,2.2) – (0.5,1.8); (0,1.5) rectangle (0.5,1.9); (0,0) – (0,-1) node[left] S_2; [-¿] (0.25,2.2) – (1,2.2) node[midway,above] t; at (0.25,1.65) μ; (0,0.5) – (3,0.5) node[right] P; (3,0) – (3,2);

Correct Answer: 4

Solution:

Step 1: Recall the Formula for Fringe Shift

When a thin plate of thickness t and refractive index μ is introduced in the path of one of the slits in Young's double slit experiment, the fringe pattern shifts. The fringe shift (Δx) is given by:

$$\Delta x = \frac{t(\mu - 1)}{\lambda}\beta_0$$

where λ is the wavelength of light and β_0 is the fringe width.

Step 2: Convert Units and Substitute Values

Given $t = 10\mu m = 10 \times 10^{-6}m$, $\mu = 1.2$, and $\lambda = 500nm = 500 \times 10^{-9}m = 5 \times 10^{-7}m$:

$$\Delta x = \frac{10 \times 10^{-6}(1.2 - 1)}{5 \times 10^{-7}}\beta_0$$

$$\Delta x = \frac{10 \times 10^{-6} \times 0.2}{5 \times 10^{-7}}\beta_0$$

$$\Delta x = \frac{2 \times 10^{-6}}{5 \times 10^{-7}}\beta_0 = \frac{20 \times 10^{-7}}{5 \times 10^{-7}}\beta_0 = 4\beta_0$$

Step 3: Find the Value of x

The central maxima is shifted by a distance of $x\beta_0$. We found that $\Delta x = 4\beta_0$. Therefore, $x = 4$.

Conclusion: The value of x is 4.

Quick Tip

Remember the formula for fringe shift in Young's double slit experiment when a thin plate is introduced. Ensure consistent units while substituting values.

29: A cubical volume is bounded by the surfaces x = 0, x = a, y = 0, y = a, z = 0, z = a. The electric field in the region is given by $\vec{E} = E_0 x\hat{i}$. Where $E_0 = 4 \times 10^4$ NC^{-1} m^{-1}. If a = 2 cm, the charge contained in the cubical volume is $Q \times 10^{-14}$ C. The value of Q is ____________. (Take $\epsilon_0 = 9 \times 10^{-12}$ C^2/Nm^2)

Correct Answer: 288

Solution:

Step 1: Visualize the Cube and Electric Field

The electric field is in the x-direction and its magnitude varies with x. The cube has side length 'a'.

Step 2: Calculate the Electric Flux

The electric flux through a surface is given by:

$$\Phi = \int \vec{E} \cdot d\vec{A}$$

Since the electric field is only in the x-direction, the only flux is through the face of the cube at x = a (the face ABCD in the given figure). The electric field at this face is $\vec{E} = E_0 a\hat{i}$, and the area vector is $a^2\hat{i}$. Therefore, the net flux through the cube is:

$$\Phi_{net} = \Phi_{ABCD} = E_0 a \cdot a^2 = E_0 a^3$$

Step 3: Apply Gauss's Law

Gauss's law states that the net electric flux through a closed surface is equal to the enclosed charge divided by the permittivity of free space (ϵ_0):

$$\Phi_{net} = \frac{q_{en}}{\epsilon_0}$$

where q_{en} is the enclosed charge.

Step 4: Calculate the Enclosed Charge

Combining the flux calculation and Gauss's law, we have:

$$\frac{q_{en}}{\epsilon_0} = E_0 a^3$$

$$q_{en} = E_0 \epsilon_0 a^3$$

Substitute the given values, $E_0 = 4 \times 10^4$ N/Cm, $a = 2$ cm $= 2 \times 10^{-2}$ m, and $\epsilon_0 = 9 \times 10^{-12}$ C^2/Nm^2:

$$q_{en} = (4 \times 10^4) \times (9 \times 10^{-12}) \times (2 \times 10^{-2})^3$$

$$q_{en} = 36 \times 10^{-8} \times 8 \times 10^{-6} = 288 \times 10^{-14} \text{ C}$$

Step 5: Find the Value of Q

The enclosed charge is given as $Q \times 10^{-14}$ C. We have found $q_{en} = 288 \times 10^{-14}$ C. Therefore, $Q = 288$.

Conclusion: The value of Q is 288.

Quick Tip

Remember Gauss's law and the formula for electric flux. Pay close attention to the direction of the electric field and the area vector when calculating the flux.

30: The surface of water in a water tank of cross section area 750 cm^2 on the top of a house is h m. above the tap level. The speed of water coming out through the tap of cross section area 500 mm^2 is 30 cm/s. At that instant, $\frac{dh}{dt}$ is $x \times 10^{-3}$ m/s. The value of x will be ____________.

Correct Answer: 2

Solution:

Step 1: Apply the Principle of Continuity

According to the principle of continuity, the volume flow rate is constant:

$$A_1 V_1 = A_2 V_2$$

where A_1 and V_1 are the area and velocity at the top surface of the water, and A_2 and V_2 are the area and velocity at the tap.

Step 2: Convert Units and Substitute Values

Given $A_1 = 750 \text{ cm}^2 = 750 \times 10^{-4} \text{ m}^2$, $A_2 = 500 \text{ mm}^2 = 500 \times 10^{-6} \text{ m}^2$, and $V_2 = 30 \text{ cm/s} = 0.3 \text{ m/s}$:

$$750 \times 10^{-4} V_1 = 500 \times 10^{-6} \times 0.3$$

$$V_1 = \frac{500 \times 0.3 \times 10^{-6}}{750 \times 10^{-4}} = \frac{150 \times 10^{-6}}{750 \times 10^{-4}} = 2 \times 10^{-4} \times 10^2 = 2 \times 10^{-2} \text{ cm/s}$$

$$V_1 = \frac{500 \times 3 \times 10^{-4}}{750} = 2 \times 10^{-4} \text{m/s} = 2 \times 10^{-3} \text{ m/s} = 0.002 \text{m/s}$$

Step 3: Relate V_1 to $\frac{dh}{dt}$

The rate of change of height ($\frac{dh}{dt}$) of the water in the tank is equal to the velocity of the water at the top surface but with a negative sign, because the height is decreasing:

$$\frac{dh}{dt} = -V_1 = -2 \times 10^{-3} \text{ m/s}$$

Step 4: Find the Value of x

Given $\frac{dh}{dt} = x \times 10^{-3}$ m/s, and we have found $\frac{dh}{dt} = -2 \times 10^{-3}$ m/s. Therefore, $x = -2$. Since the magnitude is asked, take the absolute value:

$$|x| = 2$$

Conclusion: The value of x is 2.

Quick Tip

Remember the principle of continuity for fluid flow. Pay close attention to units and signs. The rate of decrease in height is equal in magnitude to the velocity at the top surface.