JEE Main 2023 Feb 1 Shift-2 Question Paper

Time Allowed :3 Hours	**Maximum Marks :**300	**Total Questions :**90

General Instructions

Read the following instructions very carefully and strictly follow them:

1. The Duration of test is 3 Hours.

2. This paper consists of 90 Questions.

3. There are three parts in the paper consisting of Physics, Chemistry and Mathematics having 30 questions in each part of equal weightage..

4. Each part (subject) has two sections.
   (i) Section-A: This section contains 20 multiple choice questions which have only one correct answer. Each carries 4 marks for correct answer and –1 mark for wrong answer..

5. (ii) Section-B: This section contains 10 questions. In Section-B, attempt any five questions out of 10. The answer to each of the questions is a numerical value. Each carries 4 marks for correct answer and –1 mark for wrong answer. For Section-B, the answer should be rounded off to the nearest integer.

Physics

Section A

Question 1: A Carnot engine operating between two reservoirs has efficiency $\frac{1}{3}$. When the temperature of the cold reservoir is raised by x, its efficiency decreases to $\frac{1}{6}$. The value of x, if the temperature of the hot reservoir is 99°C, will be:

  (1) 16.5 K

(2) 33 K

(3) 66 K

(4) 62 K

Question 2: Given below are two statements : One is labelled as Assertion A and the other is labelled as Reason R.

  Assertion A: Two metallic spheres are charged to the same potential. One of them is hollow and another is solid, and both have the same radii. Solid sphere will have lower charge than the hollow one.

  Reason R: Capacitance of metallic spheres depend on the radii of spheres.

  In the light of the above statements, choose the correct answer from the options given below.

  (1) A is false but R is true

(2) Both A and R are true and R is the correct explanation of A

(3) A is true but R is false

(4) Both A and R are true but R is not the correct explanation of A

Question 3: As shown in the figure, a long straight conductor with a semicircular arc of radius $\frac{\pi}{10}$ m is carrying current I = 3A. The magnitude of the magnetic field at the center O of the arc is: (The permeability of the vacuum = $4\pi \times 10^{-7}$ NA^{-2})

  (1) 6μT

(2) 1μT

(3) 4μT
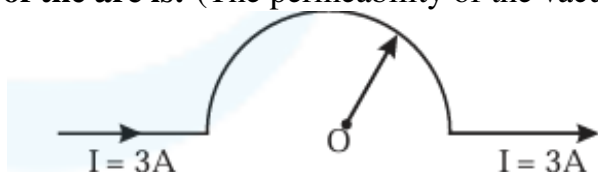
(4) 3μT

Question 4: A coil is placed in a magnetic field such that the plane of the coil is perpendicular to the direction of the magnetic field. The magnetic flux through a coil can be changed:

A. By changing the magnitude of the magnetic field within the coil.

B. By changing the area of the coil within the magnetic field.

C. By changing the angle between the direction of magnetic field and the plane of the coil.

D. By reversing the magnetic field direction abruptly without changing its magnitude.

Choose the most appropriate answer from the options given below:

(1) A and B only

(2) A, B and C only

(3) A, B and D only

(4) A and C only

Question 5: In an amplitude modulation, a modulating signal having amplitude of X V is superimposed with a carrier signal of amplitude Y V in the first case. Then, in the second case, the same modulating signal is superimposed with a different carrier signal of amplitude 2Y V. The ratio of modulation index in the two cases respectively will be:

(1) 1 : 2

(2) 1 : 1

(3) 2 : 1

(4) 4 : 1

Question 6: For a body projected at an angle with the horizontal from the ground, choose the correct statement.

(1) Gravitational potential energy is maximum at the highest point.

(2) The horizontal component of velocity is zero at the highest point.

(3) The vertical component of momentum is maximum at the highest point.

(4) The kinetic energy (K.E.) is zero at the highest point of projectile motion.

Question 7: Two objects A and B are placed at 15 cm and 25 cm from the pole in front of a concave mirror having radius of curvature 40 cm. The distance between images formed by the mirror is:

    (1) 40 cm

(2) 60 cm

(3) 160 cm

(4) 100 cm

Question 8: The Young's modulus of a steel wire of length 6 m and cross-sectional area 3 mm^2, is 2×10^{11} N/m^2. The wire is suspended from its support on a given planet. A block of mass 4 kg is attached to the free end of the wire. The acceleration due to gravity on the planet is $\frac{1}{4}$ of its value on the earth. The elongation of wire is (Take g on the earth = 10 m/s^2):

    (1) 1 cm

(2) 1 mm

(3) 0.1 mm

(4) 0.1 cm

Question 9: Equivalent resistance between the adjacent corners of a regular n-sided polygon of uniform wire of resistance R would be:

    (1) $\frac{(n-1)R}{n^2}$

(2) $\frac{(n-1)R}{2n-1}$

(3) $\frac{n^2 R}{n-1}$

(4) $\frac{(n-1)R}{n}$

Question 10: As shown in the figure, a block of mass 10 kg lying on a horizontal surface is pulled by a force F acting at an angle 30° with horizontal. For $\mu_s = 0.25$, the block will just start to move for the value of F: [Given g = 10 ms^{-2}]
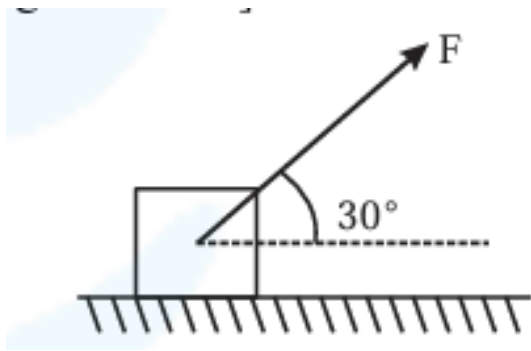
(1) 33.3 N

(2) 25.2 N

(3) 20 N

(4) 35.7 N

Question 11: Given below are two statements : One is labelled as Assertion A and the other is labelled as Reason R.

Assertion A: For measuring the potential difference across a resistance of 600 Ω, the voltmeter with resistance 1000 Ω will be preferred over voltmeter with resistance 4000 Ω.

Reason R: Voltmeter with higher resistance will draw smaller current than voltmeter with lower resistance.

In the light of the above statements, choose the most appropriate answer from the options given below.

(1) A is not correct but R is correct

(2) Both A and R are correct and R is the correct explanation of A

(3) Both A and R are correct but R is not the correct explanation of A

(4) A is correct but R is not correct

Question 12: Choose the correct statement about Zener diode:

(1) It works as a voltage regulator in reverse bias and behaves like simple pn junction diode in forward bias.

(2) It works as a voltage regulator in both forward and reverse bias.

(3) It works a voltage regulator only in forward bias.

(4) It works as a voltage regulator in forward bias and behaves like simple pn junction diode in reverse bias.

Question 13: Choose the correct length (L) versus square of time period (T^2) graph for a simple pendulum executing simple harmonic motion.

(1) [scale=0.4] [-¿] (0,0) − (5,0) node[below] L; [-¿] (0,0) − (0,5) node[left] T^2; (0,0) node[below left] O; (0,0) .. controls (1,3) and (2,4) .. (4,4.5);

(2) [scale=0.4] [-¿] (0,0) − (5,0) node[below] L; [-¿] (0,0) − (0,5) node[left] T^2; (0,0) node[below left] O; (0,4) − (4,0);

(3) [scale=0.4] [-¿] (0,0) − (5,0) node[below] L; [-¿] (0,0) − (0,5) node[left] T^2; (0,0) node[below left] O; (0,0) − (4,4);

(4) [scale=0.4] [-¿] (0,0) − (5,0) node[below] L; [-¿] (0,0) − (0,5) node[left] T^2; (0,0) node[below left] O; (0,4) .. controls (1,1) and (2,0.5) .. (4,0.2);

Question 14: The escape velocities of two planets A and B are in the ratio 1 : 2. If the ratio of their radii respectively is 1 : 3, then the ratio of acceleration due to gravity of planet A to the acceleration due to gravity of planet B will be:

(1) $\frac{4}{3}$

(2) $\frac{3}{2}$

(3) $\frac{2}{3}$

(4) $\frac{3}{4}$

Question 15: An electron of a hydrogen-like atom, having Z = 4, jumps from 4th energy state to 2nd energy state. The energy released in this process, will be: (Given Rch = 13.6 eV) Where R = Rydberg constant

c = Speed of light in vacuum

h = Planck's constant
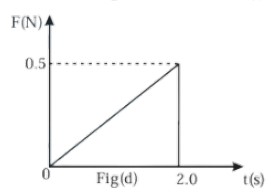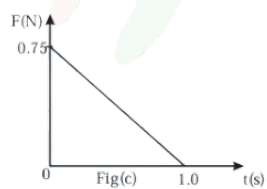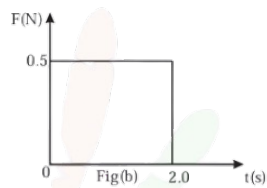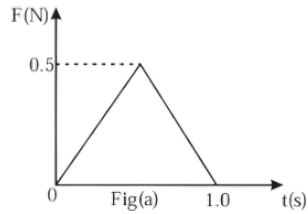
(1) 13.6 eV

(2) 10.5 eV

(3) 3.4 eV

(4) 40.8 eV

Question 16: Figures (a), (b), (c) and (d) show variation of force with time. The impulse is highest in figure:

(1) Fig (c)

(2) Fig (b)

(3) Fig (a)

(4) Fig (d)

Question 17: If the velocity of light c, universal gravitational constant G and Planck's constant h are chosen as fundamental quantities. The dimensions of mass in the new system is:

(1) $[h^{\frac{1}{2}} c^{-\frac{1}{2}} G^{1}]$

(2) $[h^{1} c^{-1} G^{-1}]$

(3) $[h^{-\frac{1}{2}} c^{\frac{1}{2}} G^{\frac{1}{2}}]$

(4) $[h^{\frac{1}{2}} c^{\frac{1}{2}} G^{-\frac{1}{2}}]$

Question 18: For three low density gases A, B, C pressure versus temperature graphs are plotted while keeping them at constant volume, as shown in the figure.

[scale=0.6] [-¿] (0,0) – (5,0) node[anchor=north west] Temperature (°C); [-¿] (0,0) – (0,3) node[anchor=south east] P(atm); (0,0) – (4,1) node[right] Gas C; (0,0) – (4,1.5) node[right] Gas B; (0,0) – (4,2) node[right] Gas A; at (2,-0.3) 0°C; at (-0.3,-0.3) K;

The temperature corresponding to the point 'K' is:

(1) -273°C

(2) -100°C

(3) -373°C

(4) -40°C

Question 19: The ratio of average electric energy density and total average energy density of electromagnetic wave is:

(1) 2

(2) 1

(3) 3

(4) $\frac{1}{2}$

Question 20: The threshold frequency of metal is f_0. When the light of frequency $2f_0$ is incident on the metal plate, the maximum velocity of photoelectron is v_1. When the frequency of incident radiation is increased to $5f_0$, the maximum velocity of photoelectrons emitted is v_2. The ratio of v_1 to v_2 is:

(1) $\frac{v_1}{v_2} = \frac{1}{2}$

(2) $\frac{v_1}{v_2} = \frac{1}{8}$

(3) $\frac{v_1}{v_2} = \frac{1}{16}$

(4) $\frac{v_1}{v_2} = \frac{1}{4}$

Section B

Question 21: For a train engine moving with a speed of 20 ms^{-1}, the driver must apply brakes at a distance of 500 m before the station for the train to come to rest at the station.

If the brakes were applied at half of this distance, the train engine would cross the station with speed $\sqrt{x}$ ms^{-1}. The value of x is _____. (Assuming the same retardation is produced by brakes)

Question 22: A force F = (5 + 3y^2) acts on a particle in the y-direction, where F is newton and y is in meter. The work done by the force during a displacement from y = 2m to y = 5m is _____ J.

Question 23: Moment of inertia of a disc of mass M and radius 'R' about any of its diameter is $\frac{MR^2}{4}$. The moment of inertia of this disc about an axis normal to the disc and passing through a point on its edge will be, $\frac{x}{2}MR^2$. The value of x is _____.

Question 24: Nucleus A having Z = 17 and equal number of protons and neutrons has 1.2 MeV binding energy per nucleon. Another nucleus B of Z = 12 has total 26 nucleons and 1.8 MeV binding energy per nucleons. The difference of binding energy of B and A will be _____ MeV.

Question 25: A square shaped coil of area 70 cm^2 having 600 turns rotates in a magnetic field of 0.4 wbm^{-2}, about an axis which is parallel to one of the side of the coil and perpendicular to the direction of field. If the coil completes 500 revolution in a minute, the instantaneous emf when the plane of the coil is inclined at 60° with the field, will be _____ V. (Take $\pi = \frac{22}{7}$)
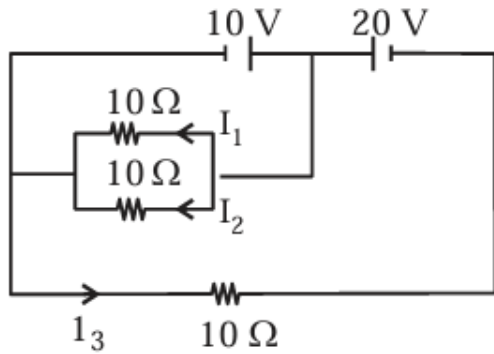

Question 26: A block is fastened to a horizontal spring. The block is pulled to a distance x = 10 cm from its equilibrium position (at x = 0) on a frictionless surface from rest. The energy of the block at x = 5 cm is 0.25 J. The spring constant of the spring is _____ Nm^{-1}.

Question 27: In the given circuit the value of $\left|\frac{I_1+I_3}{I_2}\right|$ is _____.

10 V   20 V

10 Ω
←I_1
10 Ω
←I_2

I_3 10 Ω

Question 28: As shown in the figure, in Young's double slit experiment, a thin plate of thickness t = 10 μm and refractive index μ = 1.2 is inserted in front of slit S_1. The experiment is conducted in air (μ = 1) and uses a monochromatic light of wavelength λ = 500 nm. Due to the insertion of the plate, central maxima is shifted by a distance of $x\beta_0$. β_0 is the fringe-width before the insertion of the plate. The value of the x is _____.

[scale=0.7] (0,0) – (0,2) node[left] S_1; (0,2) – (1,2); (0.5,2.2) – (0.5,1.8); (0,1.5) rectangle (0.5,1.9); (0,0) – (0,-1) node[left] S_2; [-¿] (0.25,2.2) – (1,2.2) node[midway,above] t; at (0.25,1.65) μ; (0,0.5) – (3,0.5) node[right] P; (3,0) – (3,2);


Question 29: A cubical volume is bounded by the surfaces x = 0, x = a, y = 0, y = a, z = 0, z = a. The electric field in the region is given by $\vec{E} = E_0 x\hat{i}$. Where $E_0 = 4 \times 10^4$ NC^{-1} m^{-1}. If a = 2 cm, the charge contained in the cubical volume is $Q \times 10^{-14}$ C. The value of Q is _____. (Take $\epsilon_0 = 9 \times 10^{-12}$ C^2/Nm^2)

Question 30: The surface of water in a water tank of cross section area 750 cm^2 on the top of a house is h m. above the tap level. The speed of water coming out through the tap of cross section area 500 mm^2 is 30 cm/s. At that instant, $\frac{dh}{dt}$ is $x \times 10^{-3}$ m/s. The value of x will be _____.

Chemistry

Section A

Question 31: In a reaction,
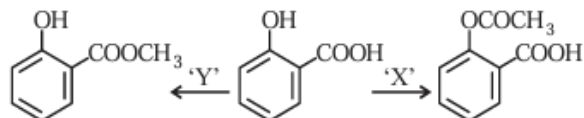
reagents 'X' and 'Y' respectively are :

(1) $(CH_3CO)_2O/H+$ and $CH_3OH/H+, \Delta$

(2) $(CH_3CO)_2O/H+$ and $(CH_3CO)_2O/H+$

(3) $CH_3OH/H+, \Delta$ and $CH_3OH/H+, \Delta$

(4) $CH_3OH/H+ \Delta$ and $(CH_3CO)_2O/H+$

Question 32: The correct order of bond enthalpy (kJ mol^{-1}) is:

(1) Si – Si > C – C > Sn – Sn > Ge – Ge

(2) Si – Si > C – C > Ge – Ge > Sn – Sn

(3) C – C > Si – Si > Sn – Sn > Ge – Ge

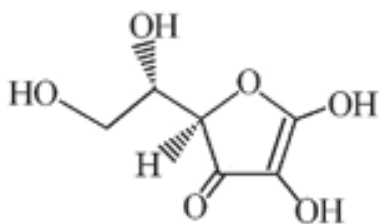(4) C – C > Si – Si > Ge – Ge > Sn – Sn

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Question 33: All structures given below are of vitamin C. Most stable of them is :
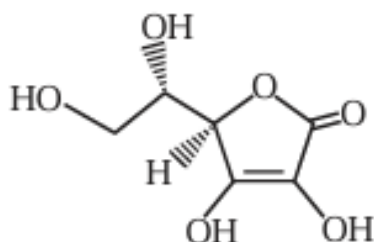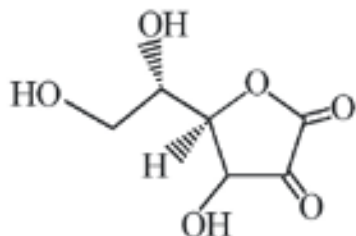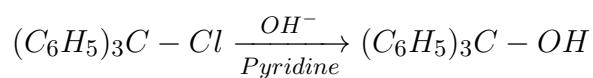

(1)
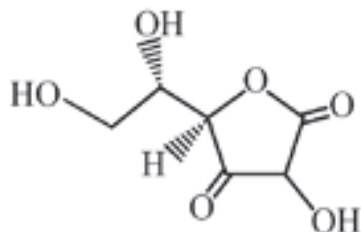
(2)

(3)

(4)

Question 34: The graph which represents the following reaction is:

$$(C_6H_5)_3C - Cl \xrightarrow[Pyridine]{OH^-} (C_6H_5)_3C - OH$$

(1) [scale=0.4] [-¿] (0,0) − (5,0) node[below] $[(C_6H_5)_3C - Cl]$; [-¿] (0,0) − (0,5) node[left] rate; (0,3) − (4,3);

(2) [scale=0.4] [-¿] (0,0) − (5,0) node[below] $[OH^-]$; [-¿] (0,0) − (0,5) node[left] rate; (0,0) − (4,4);

(3) [scale=0.4] [-¿] (0,0) − (5,0) node[below] $[(C_6H_5)_3C - Cl]$; [-¿] (0,0) − (0,5) node[left] rate; (0,0) − (4,4);

(4) [scale=0.4] [-¿] (0,0) − (5,0) node[below] [Pyridine]; [-¿] (0,0) − (0,5) node[left] rate; (0,0) − (4,4);

Question 35:

'X' is : $\xrightarrow[\Delta]{HF}$ X
Major product

(1)

(2)

(3)

(4)
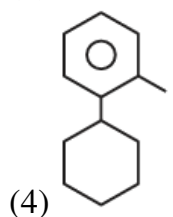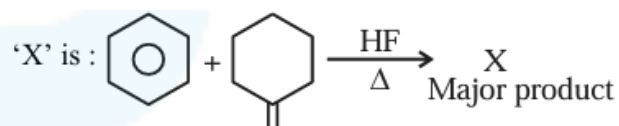
Question 36: The complex cation which has two isomers is:

(1) $[Co(H_2O)_6]^{3+}$

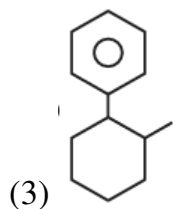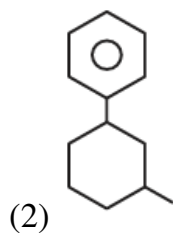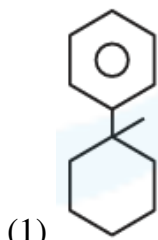(2) $[Co(NH_3)_5Cl]^{2+}$

(3) $[Co(NH_3)_5NO_2]^{2+}$

(4) $[Co(NH_3)_5Cl]^{+}$

Question 37: Given below are two statements :

Statement I : Sulphanilic acid gives esterification test for carboxyl group.

Statement II : Sulphanilic acid gives red colour in Lassaigne's test for extra element detection.

In the light of the above statements, choose the **most appropriate** answer from the options given below :

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(1) Statement I is correct but Statement II is incorrect.

(2) Both Statement I and Statement II are incorrect.

(3) Both Statement I and Statement II are correct.

(4) Statement I is incorrect but Statement II is correct.

Question 38: Given below are two statements : one is labelled as Assertion (A) and the other is labelled as Reason (R).

Assertion (A) : Gypsum is used for making fireproof wall boards.

Reason (R) : Gypsum is unstable at high temperatures.

In the light of the above statements, choose the correct answer from the options given below : (1) Both (A) and (R) are correct but (R) is not the correct explanation of (A).

(2) (A) is correct but (R) is not correct.

(3) (A) is not correct but (R) is correct.

(4) Both (A) and (R) are correct and (R) is the correct explanation of (A).

Question 39: Which element is not present in Nessler's reagent ?

(1) Mercury

(2) Potassium

(3) Iodine

(4) Oxygen

Question 40: Given below are two statements : one is labelled as Assertion (A) and the other is labelled as Reason (R).

Assertion (A) : α-halocarboxylic acid on reaction with dil. NH_3 gives good yield of α-amino carboxylic acid whereas the yield of amines is very low when prepared from alkyl halides.

Reason (R) : Amino acids exist in zwitter ion form in aqueous medium.

In the light of the above statements, choose the **correct** answer from the options given below :

(1) Both (A) and (R) are correct and (R) is the correct explanation of (A).

(2) Both (A) and (R) are correct but (R) is not the correct explanation of (A).

(3) (A) is correct but (R) is not correct.

(4) (A) is not correct but (R) is correct.

Question 41: The industrial activity held least responsible for global warming is :

(1) manufacturing of cement

(2) steel manufacturing

(3) Electricity generation in thermal power plants.

(4) Industrial production of urea

Question 42: The structures of major products A, B and C in the following reaction are sequence.

(1)

(2)

(3)

15

A = (structure: HO, CN, H with ethyl and methyl substituents),

B = (structure: HO, —NH$_2$, H with ethyl and methyl substituents),

C = (structure: HO, CO$_2$H, H with ethyl and methyl substituents)

(4)

Question 43: Given below are two statements : one is labelled as Assertion (A) and the other is labelled as Reason (R).

Assertion (A) : Cu^{2+} in water is more stable than Cu+.

Reason (R) : Enthalpy of hydration for Cu^{2+} is much less than that of Cu+.

In the light of the above statements, choose the **correct** answer from the options given below : (1) Both (A) and (R) are correct and (R) is the correct explanation of (A).
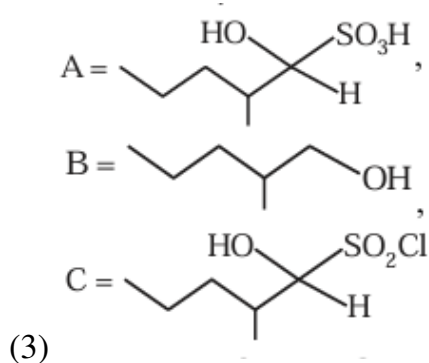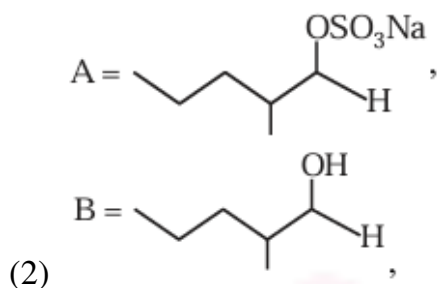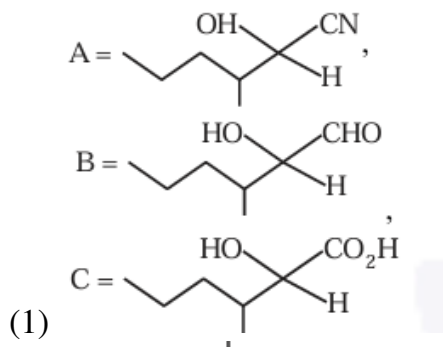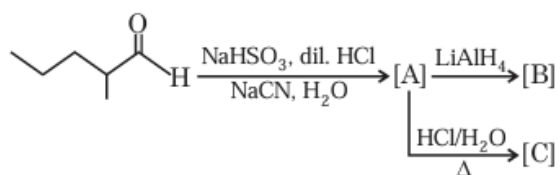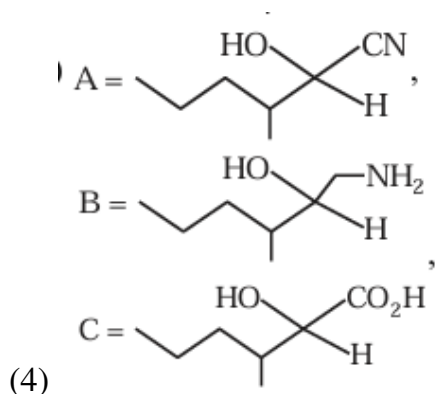
(2) (A) is correct but (R) is not correct.

(3) (1) is not correct but (R) is correct.

(4) Both (A) and (R) are correct but (R) is not the correct explanation of (A).

Question 44: The starting material for convenient preparation of deuterated hydrogen peroxide (D$_2$O$_2$) in laboratory is:

(1) K$_2$S$_2$O$_8$

(2) 2-ethylanthraquinol

(3) BaO$_2$

(4) BaO

Question 45: In figure, a straight line is given for Freundrich Adsorption ($y = 3x + 2.505$). The value of $\frac{1}{n}$ and log K are respectively.

[scale=0.6] [-¿] (0,0) − (4,0) node[below] log P; [-¿] (0,0) − (0,4) node[left] $\log \frac{x}{m}$; (0,1) − (3,3); [¡-¿] (0.2,0) − (0.2,1) node[midway, right] log K; [¡-¿] (3,1.2) − (3,3) node[midway, left] 1; [¡-¿] (1.5,1.5) − (3,1.5) node[midway, below] $\frac{1}{n}$; at (4,-0.3) X; at (0,4.3) Y;

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(1) 0.3 and log 2.505

(2) 0.3 and 0.7033

(3) 3 and 2.505

(4) 3 and 0.7033

Question 46: Given below are two statements : one is labelled as Assertion (A) and the other is labelled as Reason (R).

Assertion (A) : An aqueous solution of KOH when for volumetric analysis, its concentration should be checked before the use.

Reason (R) : On aging, KOH solution absorbs atmospheric CO_2.

In the light of the above statements, choose the correct answer from the options given below.

(1) (A) is not correct but (R) is correct

(2) Both (A) and (R) are correct but (R) is not the correct explanation of (A)

(3) Both (A) and (R) are correct and (R) is the correct explanation of (A)

(4) (A) is correct but (R) is not correct

Question 47: Which one of the following sets of ions represents a collection of isoelectronic species? (Given : Atomic Number : F :9, Cl : 17, Na = 11, Mg = 12, Al = 13, K = 19, Ca = 20, Sc = 21)

(1) (Li+, Na+, Mg^{2+}, Ca^{2+})

(2) (Ba^{2+}, Sr^{2+}, K+, Ca^{2+})

(3) (N^{3-}, O^{2-}, F−, S^{2-})

(4) (K^+, Cl^-, Ca^{2+}, Sc^{3+})

Question 48: The effect of addition of helium gas to the following reaction in equilibrium state, is :

$$PCl_5(g) \rightleftharpoons PCl_3(g) + Cl_2(g)$$

(1) the equilibrium will shift in the forward direction and more of Cl_2 and PCl_3 gases will be produced.

(2) the equilibrium will go backward due to suppression of dissociation of PCl_5.

(3) helium will deactivate PCl_5 and reaction will stop.

(4) addition of helium will not affect the equilibrium.

Question 49: For electron gain enthalpies of the elements denoted as $\Delta_{eg}H$, the incorrect option is :

(1) $\Delta_{eg}H$ (Cl) $< \Delta_{eg}H$ (F)

(2) $\Delta_{eg}H$ (Se) $< \Delta_{eg}H$ (S)

(3) $\Delta_{eg}H$ (I) $< \Delta_{eg}H$ (At)

(4) $\Delta_{eg}H$ (Te) $< \Delta_{eg}H$ (Po)

Question 50: O-O bond length in H_2O_2 is _X_ than the O-O bond length in F_2O_2. The O – H bond length in H_2O_2 is __Y__ than that of the O-F bond in F_2O_2. Choose the correct option for _X_ and _Y_ from the given below.

(1) X – shorter,   Y – shorter

(2) X – shorter,   Y – longer

(3) X – longer,   Y – longer

(4) X – longer,   Y – shorter

Section B

Question 51: 0.3 g of ethane undergoes combustion at $27°C$ in a bomb calorimeter. The temperature of calorimeter system (including the water) is found to rise by $0.5°C$. The heat evolved during combustion of ethane at constant pressure is _____ kJ mol^{-1}. (Nearest integer)

[Given : The heat capacity of the calorimeter system is 20 kJ K^{-1}, R = 8.3 JK^{-1} mol^{-1}. Assume ideal gas behaviour. Atomic mass of C and H are 12 and 1 g mol^{-1} respectively]

Question 52: Among following compounds, the number of those present in copper matte is _____.

    A. $CuCO_3$

B. Cu_2S

C. Cu_2O

D. FeO

Question 53: Among the following, the number of tranquilizer/s is/are _____.

    A. Chlordiazepoxide

B. Veronal

C. Valium

D. Salvarsan

Question 54: $A \rightarrow B$

The above reaction is of zero order. Half life of this reaction is 50 min. The time taken for the concentration of A to reduce to one-fourth of its initial value is ___ min. (Nearest integer)

Question 55: 20% of acetic acid is dissociated when its 5 g is added to 500 mL of water. The depression in freezing point of such water is ___ $\times 10^{-3}\,^{\circ}C$. Atomic mass of C, H and O are 12, 1 and 16 a.m.u. respectively.

[Given : Molal depression constant and density of water are 1.86 K kg mol^{-1} and 1 g cm^{-3} respectively.]

Question 56: The molality of a 10% (v/v) solution of di-bromine solution in CCl_4 (carbon tetrachloride) is 'x'. x = ___ $\times 10^{-2}$ M. (Nearest integer)

    Given:

- Molar mass of Br_2 = 160 g mol^{-1}

- Atomic mass of C = 12 g mol^{-1}

- Atomic mass of Cl = 35.5 g mol^{-1}

- Density of dibromine = 3.2 g cm^{-3}
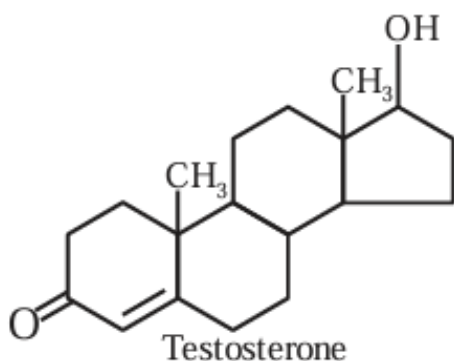
- Density of CCl$_4$ = 1.6 g cm^{-3}

Question 57: **1 $\times 10^{-5}$ M AgNO$_3$ is added to 1 L of saturated solution of AgBr. The conductivity of this solution at 298 K is ____ $\times 10^{-8}$ S m^{-1}.**

Given:

- K$_{sp}$(AgBr) = 4.9 $\times 10^{-13}$ at 298K

- $\lambda^0_{Ag^+}$ = 6 $\times 10^{-3}$ Sm^2 mol^{-1}

- $\lambda^0_{Br^-}$ = 8 $\times 10^{-3}$ Sm^2 mol^{-1}

- $\lambda^0_{NO_3^-}$ = 7 $\times 10^{-3}$ Sm^2 mol^{-1}

Question 58: Testosterone, which is a steroidal hormone, has the following structure.

Testosterone

The total number of asymmetric carbon atom/s in testosterone is _____

Question 59: The spin only magnetic moment of [Mn(H$_2$O)$_6$]$^{2+}$ complexes is _____ B.M. (Nearest integer)

Given: Atomic no. of Mn is 25

Question 60: A metal M crystallizes into two lattices: face centred cubic (fcc) and body centred cubic (bcc) with unit cell edge length of 2.0 and 2.5 $\mathring{A}$ respectively. The ratio of densities of lattices fcc to bcc for the metal M is _____. (Nearest integer)

Mathematics

Section A

Question 61: The sum $\sum_{n=1}^{\infty} \frac{2n^2+3n+4}{(2n)!}$ is equal to:

(1) $\frac{11e}{2} + \frac{7}{2e}$

(2) $\frac{13e}{4} + \frac{5}{4e} - 4$

(3) $\frac{11e}{2} + \frac{7}{2e} - 4$

(4) $\frac{13e}{4} + \frac{5}{4e}$

Question 62: Let $S = \{x \in \mathbb{R} : 0 < x < 1 \text{ and } 2\tan^{-1}\left(\frac{1-x}{1+x}\right) = \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)\}$. If $n(S)$ denotes the number of elements in S, then:

(1) $n(S) = 2$ and only one element in S is less than $\frac{1}{2}$.

(2) $n(S) = 1$ and the element in S is more than $\frac{1}{2}$.

(3) $n(S) = 1$ and the element in S is less than $\frac{1}{2}$.

(4) $n(S) = 0$.

Question 63: Let $\vec{a} = 2\hat{i} - 7\hat{j} + 5\hat{k}$, $\vec{b} = \hat{i} + \hat{k}$, and $\vec{c} = \hat{i} + 2\hat{j} - 3\hat{k}$ be three given vectors. If $\vec{r}$ is a vector such that $\vec{r} \times \vec{a} = \vec{c} \times \vec{a}$ and $\vec{r} \cdot \vec{b} = 0$, then $|\vec{r}|$ is equal to:

(1) $\frac{11}{7}\sqrt{2}$

(2) $\frac{11}{7}$

(3) $\frac{11}{5}\sqrt{2}$

(4) $\frac{\sqrt{914}}{7}$

Question 64: If $A = \frac{1}{2}\begin{bmatrix} 1 & \sqrt{3} \\ -\sqrt{3} & 1 \end{bmatrix}$, then:

(1) $A^{30} - A^{25} = 2I$

(2) $A^{30} + A^{25} + A = I$

(3) $A^{30} + A^{25} - A = I$

(4) $A^{30} = A^{25}$

Question 65: Two dice are thrown independently. Let A be the event that the number

appeared on the 1ˢᵗ die is less than the number appeared on the 2ⁿᵈ die, B be the event that the number appeared on the 1ˢᵗ die is even and that on the 2ⁿᵈ die is odd, and C be the event that the number appeared on the 1ˢᵗ die is odd and that on the 2ⁿᵈ die is even. Then:

(1) The number of favourable cases of the event $(A \cup B) \cap C$ is 6.

(2) A and B are mutually exclusive.

(3) The number of favourable cases of the events $A, B,$ and C are 15, 6, and 6 respectively.

(4) B and C are independent.

Question 66: Which of the following statements is a tautology?

(1) $p \rightarrow (p \wedge (p \rightarrow q))$

(2) $(p \wedge q) \rightarrow \sim (p \rightarrow q)$

(3) $(p \wedge (p \rightarrow q)) \rightarrow \sim q$

(4) $p \vee (p \wedge q)$

Question 67: The number of integral values of k, for which one root of the equation $2x^2 - 8x + k = 0$ lies in the interval $(1, 2)$ and its other root lies in the interval $(2, 3)$, is:

(1) 2

(2) 0

(3) 1

(4) 3

Question 68: Let $f : \mathbb{R} - \{0, 1\} \rightarrow \mathbb{R}$ be a function such that $f(x) + f\left(\frac{1}{1-x}\right) = 1 + x$. Then $f(2)$ is equal to:

(1) $\frac{9}{2}$

(2) $\frac{9}{4}$

(3) $\frac{7}{4}$

(4) $\frac{7}{3}$

Question 69: Let the plane P pass through the intersection of the planes $2x + 3y - z = 2$ and $x + 2y + 3z = 6$, and be perpendicular to the plane $2x + y - z + 1 = 0$. If d is the distance of P from the point $(-7, 1, 1)$, then d^2 is equal to:

(1) $\frac{250}{83}$

(2) $\frac{15}{53}$

(3) $\frac{25}{83}$

(4) $\frac{250}{82}$

Question 70: Let a, b be two real numbers such that $ab < 0$. If the complex number $\frac{1+ai}{b+i}$ is of unit modulus and $a + ib$ lies on the circle $|z - 1| = |2z|$, then a possible value of $\frac{1+\lfloor a \rfloor}{4b}$, where $\lfloor t \rfloor$ is the greatest integer function, is:

(1) $-\frac{1}{2}$

(2) -1

(3) 1

(4) $\frac{1}{2}$

Question 71: The sum of the absolute maximum and minimum values of the function $f(x) = |x^2 - 5x + 6| - 3x + 2$ in the interval $[-1, 3]$ is equal to:

(1) 10

(2) 12

(3) 13

(4) 24

Question 72: Let $P(S)$ denote the power set of $S = \{1, 2, 3, \ldots, 10\}$. Define the relations R_1 and R_2 on $P(S)$ as AR_1B if

$$(A \cap B^c) \cup (B \cap A^c) = \varnothing,$$

and AR_2B if

$$A \cup B^c = B \cup A^c,$$

for all $A, B \in P(S)$. Then:

(1) Both R_1 and R_2 are equivalence relations.

(2) Only R_1 is an equivalence relation.

(3) Only R_2 is an equivalence relation.

(4) Both R_1 and R_2 are not equivalence relations.

Question 73: The area of the region given by $\{(x, y) : xy \leq 8, \ 1 \leq y \leq x^2\}$ **is:**

(1) $8 \ln_e 2 - \frac{13}{3}$

(2) $16 \ln_e 2 - \frac{14}{3}$

(3) $8 \ln_e 2 + \frac{7}{6}$

(4) $16 \ln_e 2 + \frac{7}{3}$

Question 74: Let $\alpha x = \exp(x^\beta y^\gamma)$ **be the solution of the differential equation** $2x^2 y \, dy - (1 - xy^2) \, dx = 0$, $x > 0$, $y(2) = \sqrt{\ln_e 2}$. **Then** $\alpha + \beta - \gamma$ **equals:**

(1) 1

(2) -1

(3) 0

(4) 3

Question 75: The value of the integral

$$\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \frac{x + \frac{\pi}{4}}{2 - \cos 2x} \, dx \text{ is:}$$

(1) $\frac{\pi^2}{6}$

(2) $\frac{\pi^2}{12\sqrt{3}}$

(3) $\frac{\pi^2}{3\sqrt{3}}$

(4) $\frac{\pi^2}{6\sqrt{3}}$

Question 76: Let $9 = x_1 < x_2 < \cdots < x_7$ **be in an A.P. with common difference** d. **If the standard deviation of** $x_1, x_2, \ldots, x_7$ **is** 4 **and the mean is** $\bar{x}$, **then** $\bar{x} + x_6$ **is equal to:**

(1) $18 \left(1 + \frac{1}{\sqrt{3}} \right)$

(2) 34

(3) $2 \left(9 + \frac{8}{\sqrt{7}} \right)$

(4) 25

Question 77: For the system of linear equations $ax + y + z = 1$, $x + ay + z = 1$, $x + y + az = \beta$, **which one of the following statements is NOT correct?**

(1) It has infinitely many solutions if $\alpha = 2$ and $\beta = -1$.

(2) It has no solution if $\alpha = -2$ and $\beta = 1$.

(3) $x + y + z = \frac{3}{4}$ if $\alpha = 2$ and $\beta = 1$.

(4) It has infinitely many solutions if $\alpha = 1$ and $\beta = 1$.

Question 78: Let $\vec{a} = 5\hat{i} - \hat{j} - 3\hat{k}$ and $\vec{b} = \hat{i} + 3\hat{j} + 5\hat{k}$ be two vectors. Then which one of the following statements is TRUE?

(1) Projection of $\vec{a}$ on $\vec{b}$ is $\frac{17}{\sqrt{35}}$ and the direction of the projection vector is the same as $\vec{b}$.

(2) Projection of $\vec{a}$ on $\vec{b}$ is $\frac{-17}{\sqrt{35}}$ and the direction of the projection vector is opposite to $\vec{b}$.

(3) Projection of $\vec{a}$ on $\vec{b}$ is $\frac{17}{\sqrt{35}}$ and the direction of the projection vector is opposite to $\vec{b}$.

(4) Projection of $\vec{a}$ on $\vec{b}$ is $\frac{-17}{\sqrt{35}}$ and the direction of the projection vector is opposite to $\vec{b}$.

Question 79: Let $P(x_0, y_0)$ be the point on the hyperbola $3x^2 - 4y^2 = 36$, which is nearest to the line $3x + 2y = 1$. Then $\sqrt{2}\,(y_0 - x_0)$ is equal to:

(1) -3

(2) 9

(3) -9

(4) 3

Question 80: If $y(x) = x^x$, $x > 0$, then $y''(2) - 2y'(2)$ is equal to:

(1) $8\log_e 2 - 2$

(2) $4\log_e 2 + 2$

(3) $4(\log_e 2)^2 - 2$

(4) $4(\log_e 2)^2 + 2$

Section B

Question 81: The total number of six-digit numbers, formed using the digits $4, 5, 9$ only and divisible by 6, is ____.

Question 82: Number of integral solutions to the equation $x + y + z = 21$, where $x \geq 1, y \geq 3, z \geq 4$, is ____.

Question 83: The line $x = 8$ is the directrix of the ellipse $E : \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ with the corresponding focus $(2, 0)$. If the tangent to E at the point P in the first quadrant passes through the point $(0, 4\sqrt{3})$ and intersects the x-axis at Q, then $(3PQ)^2$ is equal to ____.

Question 84:

If the x-intercept of a focal chord of the parabola $y^2 = 8x + 4y + 4$ is 3, then the length of this chord is equal to ____.

Question 85: If $\int_0^\pi \frac{5^{\cos x}(1 + \cos x \cos 3x + \cos^2 x + \cos^3 x \cos 3x)}{1 + 5^{\cos x}} dx = \frac{k\pi}{16}$, then k is equal to ____.

Question 86: Let the sixth term in the binomial expansion of

$$\left(\sqrt{2^{\log_2(10 - 3^x)}} + 5 \cdot \sqrt{2^{(x-2)\log_2 3}} \right)^m,$$

in the increasing powers of $2^{(x-2)\log_2 3}$, be 21. If the binomial coefficients of the second, third, and fourth terms in the expansion are respectively the first, third, and fifth terms of an A.P., then the sum of the squares of all possible values of x is ____.

Question 87: If the term without x in the expansion of

$$\left(x^{\frac{2}{3}} + \frac{\alpha}{x^3} \right)^{22}$$

is 7315, then $|\alpha|$ is equal to ____.

Question 88: The sum of the common terms of the following three arithmetic progressions:

- 3, 7, 11, 15, . . . , 399,

- 2, 5, 8, 11, . . . , 359,

- 2, 7, 12, 17, . . . , 197,

is equal to ____.

Question 89: Let $\alpha x + \beta y + \gamma z = 1$ be the equation of a plane passing through the point $(3, -2, 5)$ and perpendicular to the line joining the points $(1, 2, 3)$ and $(-2, 3, 5)$. Then the value of $\alpha\beta\gamma$ is equal to ____.

Question 90: The point of intersection C of the plane $8x + y + 2z = 0$ and the line joining the points $A(-3, -6, 1)$ and $B(2, 4, -3)$ divides the line segment AB internally in the ratio $k : 1$. If a, b, c ($|a|, |b|, |c|$ are coprime) are the direction ratios of the perpendicular from the point C on the line $\frac{1-x}{1} = \frac{y+4}{2} = \frac{z+2}{3}$, then $|a + b + c|$ is equal to ____.