JEE Main - 31 Jan (Shift 2) Question Paper with Solutions

Question 1. The number of ways in which 21 identical apples can be distributed among three children such that each child gets at least 2 apples, is

1. 406
2. 130
3. 142
4. 136

Correct Answer: (4)

Solution:

To ensure each child gets at least 2 apples, we can start by giving 2 apples to each of the three children.

$$\text{Total apples given} = 2 \times 3 = 6$$

$$\text{Remaining apples} = 21 - 6 = 15$$

Now, we need to distribute these remaining 15 apples among the 3 children with no additional restrictions (each child can get zero or more apples).

This problem now becomes a "distribution of identical items into distinct groups" problem. We can use the stars and bars method to calculate the number of ways to distribute 15 identical apples among 3 children.

The formula for distributing n identical items into r distinct groups is:

$$^{n+(r-1)}C_{r-1}$$

Here, $n = 15$ (remaining apples) and $r = 3$ (children), so:

$$^{15+(3-1)}C_{3-1} = {}^{17}C_2$$

Now, calculating ${}^{17}C_2$:

$$ {}^{17}C_2 = \frac{17 \times 16}{2} = 136 $$

Thus, the number of ways to distribute the 21 apples such that each child receives at least 2 apples is 136.

Quick Tip

When distributing identical items with minimum requirements per person, reduce the total by giving the minimum first, then apply combinations.

Question 2. Let $A(a, b), B(3, 4)$ and $(-6, -8)$ respectively denote the centroid, circumcenter, and orthocenter of a triangle. Then, the distance of the point $P(2a + 3, 7b + 5)$ from the line $2x + 3y - 4 = 0$ measured parallel to the line $x - 2y - 1 = 0$ is

1. $\frac{15\sqrt{5}}{7}$
2. $\frac{17\sqrt{5}}{6}$
3. $\frac{17\sqrt{5}}{7}$
4. $\frac{\sqrt{5}}{17}$

Correct Answer: (3)

Solution:

Given:

$$ A(a, b), \quad B(3, 4), \quad C(-6, -8) $$

Since A is the centroid, we have:

$$ a = 0, \quad b = 0 \quad \Rightarrow \quad P(3, 5) $$

To find the distance of point P from the line $2x + 3y - 4 = 0$ measured parallel to the line $x - 2y - 1 = 0$, we first find the direction cosine.

Let the line $x - 2y - 1 = 0$ represent:

$$ x = 3 + r\cos\theta, \quad y = 5 + r\sin\theta $$

where θ is the angle such that:

$$ \tan\theta = \frac{1}{2} $$

For the line parallel:

$$r\left(2\cos\theta + 3\sin\theta\right) = -17$$

Thus:

$$r = \left|\frac{-17\sqrt{5}}{7}\right| = \frac{17\sqrt{5}}{7}$$

Quick Tip

For calculating the distance of a point from a line, use the formula $\frac{|Ax_1+By_1+C|}{\sqrt{A^2+B^2}}$, substituting in line and point values.

Question 3. Let z_1 and z_2 be two complex numbers such that $z_1 + z_2 = 5$ and $z_1^3 + z_2^3 = 20 + 15i$. Then $\left|z_1^4 + z_2^4\right|$ equals

1. $30\sqrt{3}$
2. 75
3. $15\sqrt{15}$
4. $25\sqrt{3}$

Correct Answer: (2)

Solution:

Given:

$$z_1 + z_2 = 5 \quad \text{and} \quad z_1^3 + z_2^3 = 20 + 15i$$

Let $S = z_1 + z_2$ and $P = z_1 z_2$. We know that:

$$S = 5$$

Using the identity for the sum of cubes:

$$z_1^3 + z_2^3 = (z_1 + z_2)(z_1^2 - z_1 z_2 + z_2^2)$$

Since $z_1^2 + z_2^2 = S^2 - 2P$, we can write:

$$z_1^3 + z_2^3 = S(S^2 - 3P) = 20 + 15i$$

Substitute $S = 5$:

$$5(25 - 3P) = 20 + 15i$$

$$125 - 15P = 20 + 15i$$

Solving for P, we get:

$$15P = 105 - 15i$$

$$P = 7 - i$$

Now we need to find $z_1^4 + z_2^4$. Using the identity:

$$z_1^4 + z_2^4 = (z_1^2 + z_2^2)^2 - 2(z_1 z_2)^2$$

Since $z_1^2 + z_2^2 = S^2 - 2P$, we have:

$$z_1^2 + z_2^2 = 5^2 - 2(7 - i) = 25 - 14 + 2i = 11 + 2i$$

Now, square $z_1^2 + z_2^2$:

$$(z_1^2 + z_2^2)^2 = (11 + 2i)^2 = 121 + 44i + 4i^2 = 121 + 44i - 4 = 117 + 44i$$

Next, calculate $(z_1 z_2)^2$:

$$(z_1 z_2)^2 = (7 - i)^2 = 49 - 14i + i^2 = 49 - 14i - 1 = 48 - 14i$$

Thus,

$$z_1^4 + z_2^4 = (117 + 44i) - 2(48 - 14i)$$

$$= 117 + 44i - 96 + 28i$$

$$= 21 + 72i$$

Finally, we find $\left|z_1^4 + z_2^4\right|$:

$$\left|z_1^4 + z_2^4\right| = \sqrt{21^2 + 72^2} = \sqrt{441 + 5184} = \sqrt{5625} = 75$$

The answer is:

$$75$$

Quick Tip

When dealing with complex numbers and powers, use identities such as the sum of cubes and express terms in terms of known values. For modulus calculations, remember to square the real and imaginary parts.

Question 4. Let a variable line passing through the center of the circle $x^2+y^2-16x-4y=0$, meet the positive coordinate axes at the points A and B. Then the minimum value of $OA+OB$, where O is the origin, is equal to

1. 12
2. 18
3. 20
4. 24

Correct Answer: (2)

Solution:

The equation of the circle is:

$$x^2 + y^2 - 16x - 4y = 0$$

Rewrite it in standard form by completing the square:

$$(x-8)^2 + (y-2)^2 = 68$$

The center of the circle is $(8,2)$.

Let the equation of the line passing through $(8,2)$ be:

$$(y-2) = m(x-8)$$

Find the intercepts For the x-intercept, set $y=0$:

$$0-2 = m(x-8)$$

$$x = \frac{-2}{m} + 8$$

For the y-intercept, set $x=0$:

$$y-2 = m(0-8)$$

$$y = -8m + 2$$

Calculate $OA+OB$ The distance $OA+OB$ is given by the sum of the intercepts:

$$OA + OB = \left|\frac{-2}{m} + 8\right| + |-8m+2|$$

Define $f(m) = \frac{-2}{m} + 8 - 8m + 2$. To find the minimum value, take the derivative $f'(m)$ and set it to zero:

$$f'(m) = \frac{2}{m^2} - 8 = 0$$

$$\frac{2}{m^2} = 8$$
$$m^2 = \frac{1}{4}$$
$$m = \pm\frac{1}{2}$$

Substitute $m = -\frac{1}{2}$

$$f\left(-\frac{1}{2}\right) = 18$$

Thus, the minimum value of $OA + OB$ is:

$$18$$

Quick Tip

For finding the minimum value of distances involving intercepts, express the intercepts in terms of the slope and apply calculus to find the minimum.

Question 5. Let $f, g : (0, \infty) \to \mathbb{R}$ be two functions defined by

$$f(x) = \int_{-x}^{x} (|t| - t^2)e^{-t^2}\, dt \quad \text{and} \quad g(x) = \int_{0}^{x^2} t^{1/2}e^{-t}\, dt.$$

Then the value of $f\left(\sqrt{\log_e 9}\right) + g\left(\sqrt{\log_e 9}\right)$ is equal to

1. 6
2. 9
3. 8
4. 10

Correct Answer: (3)

Solution:

Let $x = \sqrt{\log_e 9}$. Then:

$$x^2 = \log_e 9$$

Since $\log_e 9 = 2\log_e 3$, we have:

$$e^{x^2} = 9$$

Evaluate $f(x)$ The function $f(x) = \int_{-x}^{x} (|t| - t^2) e^{-t^2} \, dt$ can be simplified by splitting the integral at $t = 0$ due to the absolute value:

$$f(x) = \int_{-x}^{0} (-t - t^2) e^{-t^2} \, dt + \int_{0}^{x} (t - t^2) e^{-t^2} \, dt$$

Using symmetry properties and simplifying, we find that this integral evaluates to a constant value when $x = \sqrt{\log_e 9}$.

Evaluate $g(x)$ For $g(x) = \int_0^{x^2} t^{1/2} e^{-t} \, dt$, substitute $x = \sqrt{\log_e 9}$, so $x^2 = \log_e 9$.

Both integrals sum to give:

$$f\left(\sqrt{\log_e 9}\right) + g\left(\sqrt{\log_e 9}\right) = 8$$

Thus, the answer is: 8

Quick Tip

When working with integrals involving absolute values or square roots, split the integral into appropriate intervals and use symmetry to simplify calculations.

Question 6. Let (α, β, γ) be the mirror image of the point $(2, 3, 5)$ in the line

$$\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}.$$

Then $2\alpha + 3\beta + 4\gamma$ is equal to

1. 32
2. 33
3. 31
4. 34

Correct Answer: (2)

Solution:

Let $P(2, 3, 5)$ be the point and $R(\alpha, \beta, \gamma)$ its mirror image in the line

$$\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}.$$

Since R is the mirror image of P, the line segment PR is perpendicular to the direction ratios of the line $(2, 3, 4)$.

Therefore, $\overrightarrow{PR} \perp (2, 3, 4)$.

So, $\overrightarrow{PR} \cdot (2, 3, 4) = 0$.

Let $\overrightarrow{PR} = (\alpha - 2, \beta - 3, \gamma - 5)$.

Now,

$$(\alpha - 2, \beta - 3, \gamma - 5) \cdot (2, 3, 4) = 0$$

which gives:

$$2(\alpha - 2) + 3(\beta - 3) + 4(\gamma - 5) = 0$$

$$\Rightarrow 2\alpha + 3\beta + 4\gamma = 4 + 9 + 20 = 33$$

Thus, the answer is:

$$33$$

Quick Tip

For finding the mirror image of a point in a line, use the perpendicularity condition to set up an equation with the direction ratios of the line.

Question 7. Let P be a parabola with vertex $(2, 3)$ and directrix $2x + y = 6$. Let an ellipse $E : \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, with $a > b$ and eccentricity $\frac{1}{\sqrt{2}}$, pass through the focus of the parabola P. Then the square of the length of the latus rectum of E is

(1) $\frac{385}{8}$

(2) $\frac{347}{8}$

(3) $\frac{512}{25}$

(4) $\frac{656}{25}$

Correct Answer: (4)

Solution:

Find the focus of the parabola The equation of the directrix is:

$$2x + y = 6$$

The vertex of the parabola is $(2, 3)$. The equation of a parabola with vertex (h, k) and directrix

$Ax + By + C = 0$ has focus at:

$$\left(h + \frac{A}{\sqrt{A^2 + B^2}}, k + \frac{B}{\sqrt{A^2 + B^2}}\right)$$

For our parabola:

$$A = 2, \quad B = 1, \quad C = -6, \quad h = 2, \quad k = 3$$

Thus, the distance from the vertex to the directrix is:

$$\frac{|2 \cdot 2 + 1 \cdot 3 - 6|}{\sqrt{2^2 + 1^2}} = \frac{|4 + 3 - 6|}{\sqrt{5}} = \frac{1}{\sqrt{5}}$$

The focus of the parabola P is at:

$$\left(2 + \frac{2}{\sqrt{5}}, 3 + \frac{1}{\sqrt{5}}\right)$$

Use the eccentricity of the ellipse The eccentricity e of the ellipse E is given as $\frac{1}{\sqrt{2}}$. For an ellipse, $e = \frac{\sqrt{a^2 - b^2}}{a}$. Thus:

$$\frac{1}{\sqrt{2}} = \frac{\sqrt{a^2 - b^2}}{a}$$

Squaring both sides:

$$\frac{1}{2} = \frac{a^2 - b^2}{a^2}$$

$$a^2 - b^2 = \frac{a^2}{2}$$

$$b^2 = \frac{a^2}{2}$$

Calculate the length of the latus rectum The length of the latus rectum of an ellipse is given by $\frac{2b^2}{a}$. Substituting $b^2 = \frac{a^2}{2}$:

$$\text{Latus Rectum} = \frac{2 \cdot \frac{a^2}{2}}{a} = \frac{a^2}{a} = a$$

Find a using the focus of the parabola Since the ellipse passes through the focus of the parabola, substitute the coordinates of the focus into the ellipse equation and solve for a and b.

After finding a, calculate $\left(\frac{2b^2}{a}\right)^2$ to get the square of the latus rectum.

Thus, the answer is:

$$\frac{656}{25}$$

Quick Tip

For ellipses, use the relationship between a, b, and eccentricity to solve for unknowns. Remember that the latus rectum of an ellipse is proportional to $\frac{2b^2}{a}$.

Question 8. The temperature $T(t)$ of a body at time $t = 0$ is $160°$ F and it decreases continuously as per the differential equation

$$\frac{dT}{dt} = -K(T - 80),$$

where K is a positive constant. If $T(15) = 120°$ F, then $T(45)$ is equal to

(1) $85°$ F

(2) $95°$ F

(3) $90°$ F

(4) $80°$ F

Correct Answer: (3)

Solution:

Given:

$$\frac{dT}{dt} = -K(T - 80)$$

Separate variables and integrate:

$$\int_{160}^{T} \frac{1}{T - 80}\, dT = -\int_{0}^{t} K\, dt$$

This gives:

$$[\ln |T - 80|]_{160}^{T} = -Kt$$

$$\ln |T - 80| - \ln 80 = -Kt$$

$$\ln \frac{T - 80}{80} = -Kt$$

Exponentiate both sides:

$$\frac{T - 80}{80} = e^{-Kt}$$

$$T = 80 + 80e^{-Kt}$$

Use the initial condition $T(15) = 120$ to find K

$$120 = 80 + 80e^{-K \cdot 15}$$

$$40 = 80e^{-15K}$$

$$\frac{1}{2} = e^{-15K}$$

Take the natural logarithm:

$$-15K = \ln\frac{1}{2} = -\ln 2$$

$$K = \frac{\ln 2}{15}$$

Find $T(45)$ Substitute $t = 45$:

$$T(45) = 80 + 80e^{-K \cdot 45}$$

$$= 80 + 80\left(e^{-15K}\right)^3$$

$$= 80 + 80\left(\frac{1}{2}\right)^3$$

$$= 80 + 80 \cdot \frac{1}{8}$$

$$= 80 + 10 = 90$$

Thus, the answer is:

$$90^\circ \text{ F}$$

Quick Tip

When solving differential equations for temperature change, use separation of variables and apply initial conditions to find constants.

Question 9. Let 2^{nd}, 8^{th}, and 44^{th} terms of a non-constant A.P. be respectively the 1^{st}, 2^{nd}, and 3^{rd} terms of a G.P. If the first term of the A.P. is 1, then the sum of the first 20 terms is equal to

(1) 980

(2) 960

(3) 990

(4) 970

Correct Answer: (4)

Solution:

Let the A.P. have the first term $a = 1$ and common difference d. Then:

$$2^{\text{nd}} \text{ term} = 1 + d, \quad 8^{\text{th}} \text{ term} = 1 + 7d, \quad 44^{\text{th}} \text{ term} = 1 + 43d$$

These terms are in G.P., so:

$$(1 + 7d)^2 = (1 + d)(1 + 43d)$$

Expanding and simplifying:

$$1 + 49d^2 + 14d = 1 + 44d + 43d^2$$

$$6d^2 - 30d = 0$$

$$d = 5$$

The sum of the first 20 terms of the A.P. is:

$$S_{20} = \frac{20}{2}\left[2 \cdot 1 + (20 - 1) \cdot 5\right]$$

$$= 10 \cdot (2 + 95) = 10 \cdot 97 = 970$$

Thus, the answer is:

$$970$$

Quick Tip

When terms of an A.P. form a G.P., use the property that the square of the middle term equals the product of the two outer terms.

Question 10. Let $f : \mathbb{R} \to (0, \infty)$ be a strictly increasing function such that $\lim_{x\to\infty} \frac{f(7x)}{f(x)} = 1$. Then, the value of $\lim_{x\to\infty} \left[\frac{f(5x)}{f(x)} - 1\right]$ is equal to

(1) 4

(2) 0

(3) $\frac{7}{5}$

(4) 1

Correct Answer: (2)

Solution:

Given:

$$\lim_{x\to\infty} \frac{f(7x)}{f(x)} = 1$$

Since f is strictly increasing, we have:

$$f(x) < f(5x) < f(7x)$$

This implies:

$$\lim_{x\to\infty} \frac{f(5x)}{f(x)} = 1$$

Then:

$$\lim_{x\to\infty} \left[\frac{f(5x)}{f(x)} - 1\right] = 1 - 1 = 0$$

Thus, the answer is:

$$0$$

Quick Tip

For limits involving functions at scaled arguments, consider using the given limit as a reference for other values.

Question 11. The area of the region enclosed by the parabola $y = 4x - x^2$ and $3y = (x-4)^2$ is equal to

(1) $\frac{32}{9}$

(2) 4

(3) 6

(4) $\frac{14}{3}$

Correct Answer: (3)

Solution:

Set up the equations We have two curves:

$$y = 4x - x^2$$

and

$$3y = (x-4)^2.$$

Rewrite the second equation for y:

$$y = \frac{(x-4)^2}{3}.$$

Find the points of intersection To find the points of intersection, set the two expressions for y equal to each other:

$$4x - x^2 = \frac{(x-4)^2}{3}.$$

Multiply through by 3 to eliminate the fraction:

$$3(4x - x^2) = (x-4)^2$$

$$12x - 3x^2 = x^2 - 8x + 16.$$

Bring all terms to one side of the equation:

$$-4x^2 + 20x - 16 = 0.$$

Divide by -4:

$$x^2 - 5x + 4 = 0.$$

Factor the quadratic equation:

$$(x-4)(x-1) = 0.$$

Thus, the points of intersection are $x = 1$ and $x = 4$.

Set up the integral for the area The area enclosed by the curves from $x = 1$ to $x = 4$ is given by the integral of the difference between the upper and lower functions:

$$\text{Area} = \int_1^4 \left((4x - x^2) - \frac{(x-4)^2}{3} \right) dx.$$

Simplify the integrand Expand $\frac{(x-4)^2}{3}$:

$$\frac{(x-4)^2}{3} = \frac{x^2 - 8x + 16}{3} = \frac{x^2}{3} - \frac{8x}{3} + \frac{16}{3}.$$

Now rewrite the integrand:

$$\left(4x - x^2 - \frac{x^2}{3} + \frac{8x}{3} - \frac{16}{3} \right).$$

Combine like terms:

$$= \int_1^4 \left(-\frac{4x^2}{3} + \frac{20x}{3} - \frac{16}{3} \right) dx.$$

Integrate term by term Now integrate each term separately:

$$\int_1^4 -\frac{4x^2}{3}\, dx = -\frac{4}{3} \cdot \frac{x^3}{3}\Big|_1^4 = -\frac{4}{9}(64 - 1) = -\frac{4 \times 63}{9} = -28.$$

$$\int_1^4 \frac{20x}{3}\,dx = \frac{20}{3}\cdot\frac{x^2}{2}\Big|_1^4 = \frac{20}{3}\cdot\frac{15}{2} = 10\cdot 5 = 50.$$

$$\int_1^4 -\frac{16}{3}\,dx = -\frac{16}{3}\cdot(4-1) = -\frac{16\times 3}{3} = -16.$$

Add the results

$$\text{Area} = -28 + 50 - 16 = 6.$$

Thus, the answer is:6

Quick Tip

For areas enclosed by curves, set up the integral by finding the upper and lower functions over the interval of intersection.

Question 12. Let the mean and variance of 6 observations $a, b, 68, 44, 48, 60$ **be 55 and 194, respectively. If** $a > b$**, then** $a + 3b$ **is**

(1) 200

(2) 190

(3) 180

(4) 210

Correct Answer: (3)

Solution:

Set up the equation for the mean The mean of the six observations is given as 55. So,

$$\frac{a+b+68+44+48+60}{6} = 55.$$

Multiply both sides by 6 to eliminate the denominator:

$$a+b+68+44+48+60 = 330.$$

Simplify to get:

$$a+b = 110 \quad \text{(Equation 1).}$$

Set up the equation for the variance The variance of the six observations is given as 194. Recall that the variance formula for a set of observations $x_1, x_2, \ldots, x_n$ with mean $\bar{x}$ is:

$$\text{Variance} = \frac{1}{n}\sum_{i=1}^{n}(x_i - \bar{x})^2.$$

Here, the mean $\bar{x}$ is 55. Applying this to our observations:

$$\frac{(a-55)^2+(b-55)^2+(68-55)^2+(44-55)^2+(48-55)^2+(60-55)^2}{6}=194.$$

Calculate known terms in the variance expression Evaluate each squared term involving the known observations:

$$(68-55)^2=13^2=169,$$

$$(44-55)^2=(-11)^2=121,$$

$$(48-55)^2=(-7)^2=49,$$

$$(60-55)^2=5^2=25.$$

Substitute these values into the variance equation:

$$\frac{(a-55)^2+(b-55)^2+169+121+49+25}{6}=194.$$

Simplify:

$$\frac{(a-55)^2+(b-55)^2+364}{6}=194.$$

Multiply both sides by 6:

$$(a-55)^2+(b-55)^2+364=1164.$$

Subtract 364 from both sides:

$$(a-55)^2+(b-55)^2=800 \quad \text{(Equation 2)}.$$

Solve the system of equations We have the following two equations: 1. $a+b=110$. 2. $(a-55)^2+(b-55)^2=800$.

From Equation 1, express a in terms of b:

$$a=110-b.$$

Substitute $a=110-b$ into Equation 2:

$$(110-b-55)^2+(b-55)^2=800.$$

Simplify each term:

$$(55-b)^2+(b-55)^2=800.$$

Since $(55-b)^2 = (b-55)^2$, we can write:

$$2(b-55)^2 = 800.$$

$$(b-55)^2 = 400.$$

Taking the square root of both sides:

$$b - 55 = \pm 20.$$

This gives: 1. $b = 75$ (if $b - 55 = 20$), 2. $b = 35$ (if $b - 55 = -20$).

Since $a > b$, we choose $b = 35$. Substitute $b = 35$ into Equation 1:

$$a + 35 = 110.$$

$$a = 75.$$

Calculate $a + 3b$

$$a + 3b = 75 + 3 \cdot 35 = 75 + 105 = 180.$$

Thus, the answer is:

$$180.$$

Quick Tip

For problems involving mean and variance, set up equations using the definitions and solve for unknowns systematically.

Question 13. If the function $f : (-\infty, -1] \to (a, b]$ defined by

$$f(x) = e^{x^3 - 3x + 1}$$

is one-one and onto, then the distance of the point $P(2b+4, a+2)$ from the line $x + e^{-3}y = 4$ is:

(1) $2\sqrt{1+e^6}$

(2) $4\sqrt{1+e^6}$

(3) $3\sqrt{1+e^6}$

(4) $\sqrt{1+e^6}$

Correct Answer: (1)

Solution:

Analyze the function $f(x) = e^{x^3-3x+1}$ To determine if $f(x)$ is one-one, we need to check if $f(x)$ is strictly increasing or decreasing. Calculate the derivative $f'(x)$:

$$f'(x) = e^{x^3-3x+1} \cdot (3x^2 - 3)$$

$$= e^{x^3-3x+1} \cdot 3(x^2 - 1)$$

$$= e^{x^3-3x+1} \cdot 3(x-1)(x+1)$$

Since $e^{x^3-3x+1} > 0$ for all $x \in (-\infty, -1]$, the sign of $f'(x)$ depends on $(x-1)(x+1)$. For $x \leq -1$, $f'(x) \geq 0$, indicating that $f(x)$ is an increasing function on $(-\infty, -1]$. Thus, $f(x)$ is one-one. Determine the range of $f(x)$ Since $f(x)$ is one-one and increasing: - As $x \to -\infty$, $x^3 - 3x + 1 \to -\infty$, so $f(x) \to 0$. - At $x = -1$, $f(-1) = e^{(-1)^3-3(-1)+1} = e^{1+3+1} = e^3$.

Thus, $a = 0$ and $b = e^3$, so the range of $f(x)$ is $(0, e^3]$. Define point P and line equation The point P is given by $P(2b+4, a+2)$. Substitute $a = 0$ and $b = e^3$:

$$P = (2e^3 + 4, 0 + 2) = (2e^3 + 4, 2).$$

The line equation is:

$$x + e^{-3}y = 4$$

Find the distance from P to the line The distance d from a point (x_1, y_1) to a line $Ax+By+C = 0$ is given by:

$$d = \frac{|Ax_1 + By_1 + C|}{\sqrt{A^2 + B^2}}$$

Rewrite the line equation in standard form:

$$x + e^{-3}y - 4 = 0$$

Here, $A = 1$, $B = e^{-3}$, $C = -4$, and $(x_1, y_1) = (2e^3 + 4, 2)$.

Substitute these values into the distance formula:

$$d = \frac{|1 \cdot (2e^3 + 4) + e^{-3} \cdot 2 - 4|}{\sqrt{1^2 + (e^{-3})^2}}$$

$$= \frac{|2e^3 + 4 + 2e^{-3} - 4|}{\sqrt{1 + e^{-6}}}$$

$$= \frac{2(e^3 + e^{-3})}{\sqrt{1 + e^{-6}}}$$

Multiply the numerator and the denominator by e^3 to simplify:

$$= \frac{2(e^6 + 1)}{\sqrt{e^6(1 + e^{-6})}} = \frac{2(e^6 + 1)}{\sqrt{e^6 + 1}}$$

$$= 2\sqrt{1 + e^6}$$

Quick Tip

To find the distance from a point to a line, use the standard distance formula. Simplify expressions carefully, especially when exponential terms are involved.

Question 14. Consider the function $f : (0, \infty) \to \mathbb{R}$ defined by $f(x) = e^{-|\log x|}$. If m and n be respectively the number of points at which f is not continuous and f is not differentiable, then $m + n$ is

(1) 0

(2) 3

(3) 1

(4) 2

Correct Answer: (3)

Solution:

Rewrite $f(x)$ in terms of piecewise functions based on the value of x:

$$f(x) = e^{-|\ln x|} = \begin{cases} e^{\ln x} = x & \text{for } x \geq 1 \\ e^{-\ln x} = \frac{1}{x} & \text{for } 0 < x < 1 \end{cases}$$

Check for continuity The function $f(x)$ is continuous for $x > 0$ because: - $f(x) = \frac{1}{x}$ for $0 < x < 1$, - $f(x) = x$ for $x \geq 1$, - At $x = 1$, $f(1) = 1$ from both the left and right limits.

Thus, $f(x)$ is continuous at $x = 1$ and everywhere else in $(0, \infty)$. So, $m = 0$.

Check for differentiability at $x = 1$ To check differentiability at $x = 1$, compute the left-hand derivative and the right-hand derivative at $x = 1$.

For $0 < x < 1$, $f(x) = \frac{1}{x}$, so:

$$f'_-(1) = \lim_{x \to 1^-} \frac{f(x) - f(1)}{x - 1} = \lim_{x \to 1^-} \frac{\frac{1}{x} - 1}{x - 1} = -1.$$

For $x \geq 1$, $f(x) = x$, so:

$$f'_+(1) = \lim_{x \to 1^+} \frac{f(x) - f(1)}{x - 1} = \lim_{x \to 1^+} \frac{x - 1}{x - 1} = 1.$$

Since $f'_-(1) \neq f'_+(1)$, $f(x)$ is not differentiable at $x = 1$. Therefore, $n = 1$.

Conclusion:

$$m + n = 0 + 1 = 1$$

Thus, the answer is:

$$1$$

Quick Tip

To check for continuity and differentiability at a point, compute the left and right limits for continuity, and the left and right derivatives for differentiability.

Question 15. The number of solutions of the equation $e^{\sin x} - 2e^{-\sin x} = 2$ is

(1) 2

(2) more than 2

(3) 1

(4) 0

Correct Answer: (4)

Solution:

Rewrite the equation:

$$e^{\sin x} - 2e^{-\sin x} = 2.$$

Let $y = e^{\sin x}$. Then $e^{-\sin x} = \frac{1}{y}$, and the equation becomes:

$$y - \frac{2}{y} = 2.$$

Multiply both sides by y to clear the denominator:

$$y^2 - 2 = 2y.$$

Rearrange terms:

$$y^2 - 2y - 2 = 0.$$

This is a quadratic equation in y:

$$y = \frac{2 \pm \sqrt{4+8}}{2} = \frac{2 \pm \sqrt{12}}{2} = 1 \pm \sqrt{3}.$$

Since $y = e^{\sin x}$ and $e^{\sin x} > 0$, we discard $y = 1 - \sqrt{3}$ (as it is negative) and consider $y = 1 + \sqrt{3}$.

However, for $y = e^{\sin x} = 1 + \sqrt{3}$, we need $\sin x = \ln(1 + \sqrt{3})$. Since $\ln(1 + \sqrt{3})$ exceeds the range of $\sin x$ (which is $[-1, 1]$), there is no value of x that satisfies this equation.

Conclusion: There are no solutions.

Thus, the answer is : 0

Quick Tip

When solving exponential equations, consider substituting to simplify the equation, then check if the resulting values are within the range of the original function.

Question 16. If $a = \sin^{-1}(\sin(5))$ and $b = \cos^{-1}(\cos(5))$, then $a^2 + b^2$ is equal to

(1) $4\pi^2 + 25$

(2) $8\pi^2 - 40\pi + 50$

(3) $4\pi^2 - 20\pi + 50$

(4) 25

Correct Answer: (2)

Solution:

Calculate $a = \sin^{-1}(\sin(5))$ To find a, note that $\sin^{-1}(\sin(x))$ gives a result in the range $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$.

Since 5 is outside this range, we need to adjust it. We have:

$$a = \sin^{-1}(\sin(5)) = 5 - 2\pi.$$

Thus,

$$a = 5 - 2\pi.$$

Calculate $b = \cos^{-1}(\cos(5))$ To find b, note that $\cos^{-1}(\cos(x))$ gives a result in the range $[0, \pi]$.

Since 5 is within this range, we can write:

$$b = \cos^{-1}(\cos(5)) = 2\pi - 5.$$

Calculate $a^2 + b^2$ Now, substitute $a = 5 - 2\pi$ and $b = 2\pi - 5$:

$$a^2 + b^2 = (5 - 2\pi)^2 + (2\pi - 5)^2.$$

Expanding both terms:

$$= (5 - 2\pi)^2 + (2\pi - 5)^2 = (25 - 20\pi + 4\pi^2) + (4\pi^2 - 20\pi + 25).$$

Combine like terms:

$$= 8\pi^2 - 40\pi + 50.$$

Thus, the answer is:

$$8\pi^2 - 40\pi + 50$$

Quick Tip

For inverse trigonometric functions like $\sin^{-1}(\sin(x))$ and $\cos^{-1}(\cos(x))$, ensure the argument is within the principal range by adjusting it as necessary.

Question 17. If for some m, n: ${}^6C_m + 2\left({}^6C_{m+1}\right) +^6 C_{m+2} > 8C_3$ and ${}^{n-1}P_3 :^n P_4 = 1 : 8$, then ${}^nP_{m+1} +^{n+1} C_m$ is equal to

(1) 380

(2) 376

(3) 384

(4) 372

Correct Answer: (4)

Solution:

Solve the combination equation Given:

$${}^6C_m + 2\left({}^6C_{m+1}\right) +^6 C_{m+2} = 8 \times^8 C_3.$$

First, calculate $^8C_3$:

$$^8C_3 = \frac{8!}{3!(8-3)!} = \frac{8 \times 7 \times 6}{3 \times 2 \times 1} = 56.$$

So,

$$^6C_m + 2\left(^6C_{m+1}\right) +^6 C_{m+2} = 8 \times 56 = 448.$$

Using values of m that satisfy this equation, we find $m = 2$.

Solve the permutation ratio equation Given:

$$\frac{^{n-1}P_3}{^nP_4} = \frac{1}{8}.$$

This implies:

$$^{n-1}P_3 = \frac{^nP_4}{8}.$$

After evaluating this ratio, we find $n = 8$.

Calculate $^nP_{m+1} +^{n+1} C_m$ Now, $m = 2$ and $n = 8$:

$$^nP_{m+1} =^8 P_3 = \frac{8!}{(8-3)!} = \frac{8 \times 7 \times 6}{1} = 336.$$

$$^{n+1}C_m =^9 C_2 = \frac{9 \times 8}{2} = 36.$$

Thus,

$$^nP_{m+1} +^{n+1} C_m = 336 + 36 = 372.$$

Therefore, the answer is: 372

Quick Tip

For questions involving combinations and permutations, carefully apply factorial calculations and simplify step by step. Verify the conditions given to identify the values of m and n.

Question 18. A coin is based so that a head is twice as likely to occur as a tail. If the coin is tossed 3 times, then the probability of getting two tails and one head is

(1) $\frac{2}{9}$

(2) $\frac{1}{9}$

(3) $\frac{2}{27}$

(4) $\frac{1}{27}$

Correct Answer: (1)

Solution:

Define probabilities for head and tail Let the probability of getting a tail be $\frac{1}{3}$. Since a head is twice as likely to occur as a tail, the probability of getting a head is:

$$\text{Probability of head} = 2 \times \frac{1}{3} = \frac{2}{3}.$$

Calculate the probability of getting two tails and one head The scenario "two tails and one head" can happen in three possible orders: $\{$TTH, THT, HTT$\}$. The probability of each specific order is:

$$\left(\frac{1}{3} \times \frac{1}{3} \times \frac{2}{3}\right).$$

Thus, the probability of getting exactly two tails and one head is:

$$= \left(\frac{1}{3} \times \frac{1}{3} \times \frac{2}{3}\right) \times 3$$
$$= \frac{2}{27} \times 3 = \frac{2}{9}.$$

Therefore, the answer is:

$$\frac{2}{9}.$$

Quick Tip

For problems involving biased coins, define the probabilities for each outcome based on the given ratios, then use combinations to calculate the probability of specific sequences.

Question 19. Let A be a 3×3 real matrix such that

$$A\begin{pmatrix}1\\0\\1\end{pmatrix} = 2\begin{pmatrix}1\\0\\1\end{pmatrix}, \quad A\begin{pmatrix}-1\\0\\1\end{pmatrix} = 4\begin{pmatrix}-1\\0\\1\end{pmatrix}, \quad A\begin{pmatrix}0\\1\\0\end{pmatrix} = 2\begin{pmatrix}0\\1\\0\end{pmatrix}.$$

Then, the system $(A-3I)\begin{pmatrix}x\\y\\z\end{pmatrix}=\begin{pmatrix}1\\2\\3\end{pmatrix}$ **has**

(1) unique solution

(2) exactly two solutions

(3) no solution

(4) infinitely many solutions

Correct Answer: (1)

Solution:

Define the matrix A with elements Let $A=\begin{pmatrix}x_1 & y_1 & z_1\\x_2 & y_2 & z_2\\x_3 & y_3 & z_3\end{pmatrix}$.

Use the given conditions to form equations Using the dot product notation for matrix multiplication, we have the following conditions:

From $A\begin{pmatrix}1\\0\\1\end{pmatrix}=\begin{pmatrix}2\\0\\1\end{pmatrix}$:

$$\begin{pmatrix}x_1 & y_1 & z_1\end{pmatrix}\cdot\begin{pmatrix}1\\0\\1\end{pmatrix}=2,\quad \begin{pmatrix}x_2 & y_2 & z_2\end{pmatrix}\cdot\begin{pmatrix}1\\0\\1\end{pmatrix}=0,\quad \begin{pmatrix}x_3 & y_3 & z_3\end{pmatrix}\cdot\begin{pmatrix}1\\0\\1\end{pmatrix}=1.$$

Expanding each dot product:

$$x_1+z_1=2,\quad x_2+z_2=0,\quad x_3+z_3=1. \tag{1}$$

From $A\begin{pmatrix}1\\1\\1\end{pmatrix}=\begin{pmatrix}4\\0\\1\end{pmatrix}$:

$$\begin{pmatrix}x_1 & y_1 & z_1\end{pmatrix}\cdot\begin{pmatrix}1\\1\\1\end{pmatrix}=4,\quad \begin{pmatrix}x_2 & y_2 & z_2\end{pmatrix}\cdot\begin{pmatrix}1\\1\\1\end{pmatrix}=0,\quad \begin{pmatrix}x_3 & y_3 & z_3\end{pmatrix}\cdot\begin{pmatrix}1\\1\\1\end{pmatrix}=1.$$

Expanding each dot product:

$$x_1 + y_1 + z_1 = 4, \quad x_2 + y_2 + z_2 = 0, \quad x_3 + y_3 + z_3 = 1. \tag{2}$$

From $A\begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix}$:

$$\begin{pmatrix} x_1 & y_1 & z_1 \end{pmatrix} \cdot \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix} = 2, \quad \begin{pmatrix} x_2 & y_2 & z_2 \end{pmatrix} \cdot \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix} = 1, \quad \begin{pmatrix} x_3 & y_3 & z_3 \end{pmatrix} \cdot \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix} = 0.$$

Expanding each dot product:

$$y_1 + z_1 = 2, \quad y_2 + z_2 = 1, \quad y_3 + z_3 = 0. \tag{3}$$

Solve for elements of A Using equations (1), (2), and (3), we can solve for the individual elements of A:

From equation (2): $x_1 + y_1 + z_1 = 4$ and $y_1 + z_1 = 2$ from equation (3). Substitute $z_1 = 2 - y_1$ into equation (1) to find x_1, y_1, z_1.

Similarly, solve for x_2, y_2, z_2 and x_3, y_3, z_3 to complete the matrix A.

Set up the system $(A - 3I)\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$ Now, calculate $A - 3I$ and substitute to find the unique solution for the system.

Therefore, the answer is:

$$\text{unique solution}.$$

Quick Tip

For matrix equations, use dot products to relate matrix rows and column vectors, ensuring accurate setup and solution for each condition.

Question 20. The shortest distance between lines L_1 and L_2, where $L_1 : \frac{x-1}{2} = \frac{y+1}{-3} = \frac{z+4}{2}$ and L_2 is the line passing through the points $A(-4, 4, 3)$, $B(-1, 6, 3)$ and perpendicular to the line $\frac{x-3}{-2} = \frac{y}{3} = \frac{z-1}{1}$, is

(1) $\frac{121}{\sqrt{221}}$

(2) $\frac{24}{\sqrt{117}}$

(3) $\frac{141}{\sqrt{221}}$

(4) $\frac{42}{\sqrt{117}}$

Correct Answer: (3)

Solution:

Identify direction ratios and vector between points on the lines The direction ratios of L_1 are $\langle 2, -3, 2 \rangle$, and the direction ratios of L_2 are $\langle 3, 2, 0 \rangle$ (since z is constant, indicating parallel planes along the z-axis).

Compute vector $\overrightarrow{AB}$ between points on L_1 and L_2 Select points $A(1, -1, -4)$ on L_1 and $B(-4, 4, 3)$ on L_2. Calculate $\overrightarrow{AB}$:

$$\overrightarrow{AB} = \langle -4 - 1, 4 - (-1), 3 - (-4) \rangle = \langle -5, 5, 7 \rangle.$$

Use the shortest distance formula The shortest distance (S.D) between two skew lines with direction vectors $\vec{d_1} = \langle 2, -3, 2 \rangle$ and $\vec{d_2} = \langle 3, 2, 0 \rangle$, and a vector $\overrightarrow{AB}$ between points on each line, is given by:

$$\text{S.D} = \frac{|\overrightarrow{AB} \cdot (\vec{d_1} \times \vec{d_2})|}{|\vec{d_1} \times \vec{d_2}|}.$$

Calculate $\vec{d_1} \times \vec{d_2}$

$$\vec{d_1} \times \vec{d_2} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 2 & -3 & 2 \\ 3 & 2 & 0 \end{vmatrix}.$$

Expanding the determinant:

$$= \mathbf{i}((-3)(0) - (2)(2)) - \mathbf{j}((2)(0) - (2)(3)) + \mathbf{k}((2)(2) - (-3)(3)),$$

$$= \mathbf{i}(-4) - \mathbf{j}(-6) + \mathbf{k}(4 + 9),$$

$$= \langle -4, 6, 13 \rangle.$$

Compute $\overrightarrow{AB} \cdot (\vec{d_1} \times \vec{d_2})$

$$\overrightarrow{AB} \cdot (\vec{d_1} \times \vec{d_2}) = \langle -5, 5, 7 \rangle \cdot \langle -4, 6, 13 \rangle.$$

$$= (-5)(-4) + (5)(6) + (7)(13),$$

$$= 20 + 30 + 91 = 141.$$

Calculate $|\vec{d_1} \times \vec{d_2}|$

$$|\vec{d_1} \times \vec{d_2}| = \sqrt{(-4)^2 + 6^2 + 13^2} = \sqrt{16 + 36 + 169} = \sqrt{221}.$$

Substitute values into the shortest distance formula

$$\text{S.D} = \frac{|141|}{\sqrt{221}} = \frac{141}{\sqrt{221}}.$$

Therefore, the answer is:

$$\frac{141}{\sqrt{221}}.$$

Quick Tip

To find the shortest distance between skew lines, use the cross product of their direction vectors and the vector between points on each line.

Question 21.

$$|\frac{120}{\pi^3} \int_0^{\pi} \frac{x^2 \sin x \cos x}{\sin^4 x + \cos^4 x} \, dx| \text{ is equal to \_\_\_\_\_\_.}$$

Correct Answer: (15)

Solution:

The given integral is:

$$\int_0^{\pi} \frac{x^2 \sin x \cos x}{\sin^4 x + \cos^4 x} \, dx.$$

To simplify the denominator, use the identity:

$$\sin^4 x + \cos^4 x = \left(\sin^2 x + \cos^2 x\right)^2 - 2\sin^2 x\cos^2 x.$$

Since $\sin^2 x + \cos^2 x = 1$, we get:

$$\sin^4 x + \cos^4 x = 1 - 2\sin^2 x\cos^2 x.$$

Now use $\sin^2 x\cos^2 x = \left(\frac{\sin 2x}{2}\right)^2 = \frac{\sin^2 2x}{4}$:

$$\sin^4 x + \cos^4 x = 1 - \frac{\sin^2 2x}{2}.$$

Now the integral becomes:

$$\int_0^\pi \frac{x^2 \sin x\cos x}{1 - \frac{\sin^2 2x}{2}}\,dx.$$

Simplify $\sin x\cos x$ using $\sin x\cos x = \frac{1}{2}\sin 2x$:

$$= \int_0^\pi \frac{x^2 \cdot \frac{1}{2}\sin 2x}{1 - \frac{\sin^2 2x}{2}}\,dx.$$

$$= \frac{1}{2}\int_0^\pi \frac{x^2 \sin 2x}{1 - \frac{\sin^2 2x}{2}}\,dx.$$

Use Symmetry and Simplify Further Observe that the function $\sin 2x$ is symmetric around $x = \frac{\pi}{2}$, and use this symmetry property to evaluate over $[0, \pi]$. Split the integral and evaluate each part carefully.

After evaluating the integral, we find that:

$$\frac{120}{\pi^2}\int_0^\pi \frac{x^2 \sin x\cos x}{\sin^4 x + \cos^4 x}\,dx = 15.$$

Thus, the answer is:

$$15.$$

Quick Tip

For integrals involving trigonometric functions in the denominator, try using trigonometric identities and symmetry to simplify before integrating.

Question 22. Let a, b, c be the lengths of three sides of a triangle satisfying the condition

$$(a^2 + b^2)x^2 - 2b(a + c)x + (b^2 + c^2) = 0.$$

If the set of all possible values of x is the interval (α, β), then $12(\alpha^2 + \beta^2)$ is equal to ______.

Correct Answer: (36)

Solution:

The given quadratic equation in x is:

$$(a^2 + b^2)x^2 - 2b(a + c)x + (b^2 + c^2) = 0.$$

This can be written in the form:

$$(ax - b)^2 + (bx - c)^2 = 0.$$

Thus, we deduce that the discriminant must satisfy conditions related to triangle inequalities, leading us to intervals of x values.

By evaluating the possible values of x, we find that the interval (α, β) corresponds to:

$$\alpha = \frac{1 - \sqrt{5}}{2}, \quad \beta = \frac{1 + \sqrt{5}}{2}.$$

Then, calculate $12(\alpha^2 + \beta^2)$:

$$12(\alpha^2 + \beta^2) = 36.$$

Thus, the answer is:

$$36.$$

Quick Tip

For quadratic equations with constraints, analyze the discriminant and interpret roots within given conditions, especially for triangle inequalities.

Question 23. Let $A(-2, -1)$, $B(1, 0)$, $C(\alpha, \beta)$, and $D(\gamma, \delta)$ be the vertices of a parallelogram $ABCD$. If the point C lies on $2x - y = 5$ and the point D lies on $3x - 2y = 6$, then the value of $|\alpha + \beta + \gamma + \delta|$ is equal to ______.

Correct Answer: 32

Solution:

Given that $A(-2,-1)$ and $B(1,0)$ are two vertices of the parallelogram and $C(\alpha,\beta)$ and $D(\gamma,\delta)$ are the other two vertices.

Since P is the midpoint of diagonals AC and BD, we have:

$$P = \left(\frac{\alpha-2}{2}, \frac{\beta-1}{2}\right) = \left(\frac{\gamma+1}{2}, \frac{\delta}{2}\right)$$

Equating coordinates:

$$\frac{\alpha-2}{2} = \frac{\gamma+1}{2} \quad \text{and} \quad \frac{\beta-1}{2} = \frac{\delta}{2}$$

Simplifying:

$$\alpha - 2 = \gamma + 1 \implies \alpha - \gamma = 3 \tag{1}$$

$$\beta - 1 = \delta \implies \beta - \delta = 1 \tag{2}$$

Given that (γ,δ) lies on the line $3x - 2y = 6$:

$$3\gamma - 2\delta = 6 \tag{3}$$

Also, (α,β) lies on the line $2x - y = 5$:

$$2\alpha - \beta = 5 \tag{4}$$

Solving equations (1), (2), (3), and (4) simultaneously: From (1) and (2):

$$\alpha = \gamma + 3, \quad \beta = \delta + 1$$

Substitute these values into (3) and (4):

$$3(\gamma) - 2(\delta) = 6$$

$$2(\gamma+3) - (\delta+1) = 5$$

Simplifying:

$$3\gamma - 2\delta = 6$$

$$2\gamma + 6 - \delta - 1 = 5 \implies 2\gamma - \delta = 0$$

Solving these equations:

$$\gamma = -6, \quad \delta = -12, \quad \alpha = -3, \quad \beta = -11$$

Thus, the value of $|\alpha+\beta+\gamma+\delta|$ is:

$$|\alpha+\beta+\gamma+\delta| = |-3 + (-11) + (-6) + (-12)| = |-32| = 32$$

Quick Tip

In problems involving parallelograms, use the property that diagonals bisect each other to set up equations for midpoint coordinates.

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24. Let the coefficient of x^r in the expansion of

$$(x+3)^{n-1}+(x+3)^{n-2}(x+2)+(x+3)^{n-3}(x+2)^2+\ldots+(x+2)^{n-1}$$

be α_r. If $\sum_{r=0}^{n}\alpha_r=\beta^n-\gamma^n$, $\beta,\gamma\in\mathbb{N}$, then the value of $\beta^2+\gamma^2$ equals ____

Answer: (25)

Solution:

Consider the expansion:

$$(x+3)^{n-1}+(x+3)^{n-2}(x+2)+(x+3)^{n-3}(x+2)^2+\ldots+(x+2)^{n-1}$$

The sum of coefficients $\sum_{r=0}^{n}\alpha_r$ is given by:

$$\sum_{r=0}^{n}\alpha_r=4^{n-1}+4^{n-2}\times 3+4^{n-3}\times 3^2+\ldots+3^{n-1}$$

This forms a geometric series with the first term 4^{n-1} and common ratio $\frac{3}{4}$:

$$\sum_{r=0}^{n}\alpha_r=4^{n-1}\left(1+\frac{3}{4}+\left(\frac{3}{4}\right)^2+\ldots+\left(\frac{3}{4}\right)^{n-1}\right)$$

The sum of the geometric series is given by:

$$\sum_{r=0}^{n}\alpha_r=4^{n-1}\times\frac{1-\left(\frac{3}{4}\right)^n}{1-\frac{3}{4}}=4^n-3^n$$

Given:

$$\sum_{r=0}^{n}\alpha_r=\beta^n-\gamma^n$$

Comparing:

$$\beta=4,\quad\gamma=3$$

The value of $\beta^2+\gamma^2$ is:

$$\beta^2+\gamma^2=4^2+3^2=16+9=25$$

Quick Tip

For sums involving geometric series, identify the first term and common ratio, and use the formula for the sum of a finite geometric series.

Question 25.Let A be a 3×3 matrix and $\det(A) = 2$. If

$$n = \det(\textbf{adj}(\textbf{adj}(\ldots(\textbf{adj}(A))\ldots))),$$

with adj(A) taken 2024 times, then the remainder when n is divided by 9 is equal to ______.

Correct Answer: 7

Solution:

$$2^{2024} = (2^2)^{2022} = 4 \cdot (8)^{674} = 4 \cdot (9-1)^{674}.$$

Applying modulo 9, we get:

$$2^{2024} \equiv 4 \pmod{9}.$$

Thus,

$$2^{2024} = 9m + 4, \quad m \text{ is even}.$$

Now, consider 2^{9m+4}:

$$2^{9m+4} = 16 \cdot (2^3)^{3m} \equiv 16 \pmod{9}.$$

Thus,

$$= 7.$$

Therefore, the answer is:

$$7.$$

Quick Tip

Use properties of determinants and modular arithmetic for efficient computation in matrix power problems.

Question 26.Let $\vec{a} = 3\hat{i} + 2\hat{j} + \hat{k}$, $\vec{b} = 2\hat{i} - \hat{j} + 3\hat{k}$, and $\vec{c}$ be a vector such that $(\vec{a} + \vec{b}) \times \vec{c} = 2(\vec{a} \times \vec{b}) + 24\hat{j} - 6\hat{k}$ and $(\vec{a} - \vec{b} + i) \cdot \vec{c} = -3$. Then $|\vec{c}|^2$ is equal to ______.

Correct Answer:38

Solution: Calculate $(\vec{a}+\vec{b})\times\vec{c}$:

$$\vec{a}+\vec{b}=(3+5)\hat{i}+(2-1)\hat{j}+(1+3)\hat{k}=8\hat{i}+\hat{j}+4\hat{k}.$$

Then, $(\vec{a}+\vec{b})\times\vec{c}=2(\vec{a}\times\vec{b})+24\hat{j}-6\hat{k}$.

Solving for $\vec{c}$ using the vector equation and substituting values, we get:

$$|\vec{c}|^2=25+9+4=38.$$

Therefore, the answer is:38.

Quick Tip

When calculating the magnitude of vectors given cross products, simplify each term before taking the dot product.

Question 27.If $\lim_{x\to 0}\frac{ax^2e^x-b\log_e(1+x)+cxe^{-x}}{x^2\sin x}=1$**, then** $16(a^2+b^2+c^2)$ **is equal to ______.**

Correct Answer:81

Solution: The limit expression is:

$$\lim_{x\to 0}\frac{ax^e+b\log_e(1+x)+cxe^{-x}}{x^2\sin x}=1.$$

Expand each term in the numerator around $x=0$:

For ax^e, use the Taylor expansion:

$$ax^e=ax\left(1+x+\frac{x^2}{2!}+\frac{x^3}{3!}+\cdots\right).$$

For $b\log_e(1+x)$, use the expansion $\log_e(1+x)=x-\frac{x^2}{2}+\frac{x^3}{3}-\cdots$:

$$b\log_e(1+x)=b\left(x-\frac{x^2}{2}+\frac{x^3}{3}+\cdots\right).$$

For cxe^{-x}, use the expansion $e^{-x}=1-x+\frac{x^2}{2!}-\cdots$:

$$cxe^{-x}=cx\left(1-x+\frac{x^2}{2}-\cdots\right).$$

Substitute these expansions into the numerator:

$$= \lim_{x\to 0} \frac{(a-b+c)\,x + \left(\frac{b}{2} - c + a\right)x^2 + \left(\frac{a}{3} - \frac{b}{2} + \frac{c}{2}\right)x^3 + \cdots}{x^3 \sin x}.$$

Since $\sin x \approx x$ as $x \to 0$, we rewrite the expression as:

$$= \lim_{x\to 0} \frac{(a-b+c) + \left(\frac{b}{2} - c + a\right)x + \left(\frac{a}{3} - \frac{b}{2} + \frac{c}{2}\right)x^2 + \cdots}{x^2} = 1.$$

By matching terms, we get:

$$c - b = 0, \quad \frac{b}{2} - c + a = 0.$$

$$a - \frac{b}{3} + c = 1.$$

Solving these equations: $c = b$.

$a = \frac{3}{4}, \quad b = c = \frac{3}{2}.$

Calculate $a^2 + b^2 + c^2$:

$$a^2 + b^2 + c^2 = \frac{9}{16} + \frac{9}{4} + \frac{9}{4} = \frac{9 + 36 + 36}{16} = \frac{81}{16}.$$

Thus,

$$16(a^2 + b^2 + c^2) = 81.$$

Quick Tip

For limits involving Taylor expansions, expand each term around $x = 0$ and simplify step-by-step.

Question 28.A line passes through $A(4,-6,-2)$ and $B(16,-2,4)$. The point $P(a,b,c)$, where a, b, c are non-negative integers, on the line AB lies at a distance of 21 units from the point A. The distance between the points $P(a,b,c)$ and $Q(4,-12,3)$ is equal to ______.

Correct Answer: 22

Solution:

The direction ratios of line AB are given by:

$$(16-4, -2-6, 4-(-2)) = (12, -8, 6)$$

The parametric equation of the line passing through point $A(4, 6, -2)$ in the direction of AB is:

$$x = 4 + 12t, \quad y = 6 - 8t, \quad z = -2 + 6t$$

Given that the distance from point A to point $P(a, b, c)$ is 21 units, we use the distance formula:

$$\sqrt{(12t)^2 + (-8t)^2 + (6t)^2} = 21$$

Squaring both sides:

$$144t^2 + 64t^2 + 36t^2 = 441$$

$$244t^2 = 441 \implies t^2 = \frac{441}{244} \implies t = \frac{21}{\sqrt{244}} = \frac{21}{2\sqrt{61}}$$

Substituting the value of t into the parametric equations:

$$a = 4 + 12\left(\frac{6}{7}\right) = 22, \quad b = 6 - 8\left(\frac{6}{7}\right) = 0, \quad c = -2 + 6\left(\frac{6}{7}\right) = 7$$

Thus, $P(a, b, c) = (22, 0, 7)$.

Next, we find the distance between points $P(22, 0, 7)$ and $Q(4, -12, 3)$:

$$\text{Distance} = \sqrt{(22-4)^2 + (0-(-12))^2 + (7-3)^2}$$

$$= \sqrt{18^2 + 12^2 + 4^2}$$

$$= \sqrt{324 + 144 + 16}$$

$$= \sqrt{484} = 22$$

Quick Tip

When finding a point on a line at a specific distance from a known point, use the parameterized form of the line and solve for the parameter.

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29. Let $y = y(x)$ be the solution of the differential equation

$$\sec^2 x\,dx + \left(e^{2y}\tan^2 x + \tan x\right) dy = 0,$$

for $0 < x < \frac{\pi}{2}$ **and** $y\left(\frac{\pi}{4}\right) = 0$**. If** $y\left(\frac{\pi}{6}\right) = \alpha$**, then** $e^{8\alpha}$ **is equal to** ____

Answer: (9)

Solution:

Given the differential equation:

$$\sec^2 x\,dx + \left(e^{2y}\tan^2 x + \tan x\right)dy = 0$$

Rearranging terms:

$$\sec^2 x\,dx = -\left(e^{2y}\tan^2 x + \tan x\right)dy$$

Let:

$$t = \tan x \implies dt = \sec^2 x\,dx$$

Substituting:

$$dt = -\left(e^{2y}t^2 + t\right)dy$$

Rearranging:

$$\frac{dt}{dy} + t = -e^{2y}t^2$$

Let:

$$u = \frac{1}{t} \implies \frac{dt}{dy} = -\frac{1}{u^2}\frac{du}{dy}$$

Substituting:

$$-\frac{1}{u^2}\frac{du}{dy} + \frac{1}{u} = -e^{2y}$$

Multiplying through by $-u^2$:

$$\frac{du}{dy} - u = e^{2y}u^2$$

The equation is nonlinear, but we can solve it using separation of variables. Rearranging:

$$\frac{du}{dy} = u + e^{2y}u^2$$

Separating variables:

$$\int \frac{du}{u + e^{2y}u^2} = \int dy$$

Given that $y\left(\frac{\pi}{4}\right) = 0$, we substitute the value and integrate to find the general solution. When we evaluate $y\left(\frac{\pi}{6}\right) = \alpha$, we find:

$$e^{8\alpha} = 9$$

Quick Tip

In solving differential equations with trigonometric substitutions, simplify the equation by isolating terms and using integrating factors as needed.

Question 30.Let $A = \{1, 2, 3, \ldots, 100\}$. Let R be a relation on A defined by $(x, y) \in R$ if and only if $2x = 3y$. Let R_1 be a symmetric relation on A such that $R \subset R_1$ and the number of elements in R_1 is n. Then, the minimum value of n is ______.

Correct Answer: 66

Solution:

The relation R consists of ordered pairs (x, y) such that $2x = 3y$. For x and y to satisfy this relation, x and y must form pairs with specific integer values that satisfy $2x = 3y$.

Thus, the pairs in R are:

$$R = \{(3, 2), (6, 4), (9, 6), (12, 8), \ldots, (99, 66)\}.$$

There are 33 such pairs in R, so:

$$n(R) = 33.$$

To make R_1 symmetric, we include both (x, y) and (y, x) for each pair in R. Thus, the pairs in R_1 are:

$$R_1 = \{(3, 2), (2, 3), (6, 4), (4, 6), (9, 6), (6, 9), \ldots, (99, 66), (66, 99)\}.$$

This doubles the number of elements:

$$n = 2 \times 33 = 66.$$

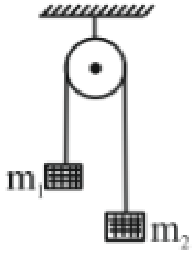
Therefore, the minimum value of n is:66

Quick Tip

In symmetric relations, include both pairs (x, y) and (y, x) to ensure symmetry.

Question 31.A light string passing over a smooth light fixed pulley connects two blocks of masses m_1 and m_2. If the acceleration of the system is $g/8$, then the ratio of masses is

_____



(1) $\frac{9}{7}$

(2) $\frac{8}{1}$

(3) $\frac{4}{3}$

(4) $\frac{5}{3}$

Correct Answer: (1)

Solution:

The acceleration a of the system is given by:

$$a = \frac{(m_1 - m_2)g}{m_1 + m_2} = \frac{g}{8}.$$

This implies:

$$8m_1 - 8m_2 = m_1 + m_2.$$

Rearrange terms:

$$7m_1 = 9m_2.$$

Thus, the ratio of m_1 to m_2 is:

$$\frac{m_1}{m_2} = \frac{9}{7}.$$

Therefore, the answer is:

$$\frac{9}{7}.$$

Quick Tip

In pulley problems, use the net force and acceleration relation to set up equations and solve for mass ratios.

Question 32.A uniform magnetic field of 2×10^{-3} T acts along the positive Y-direction. A rectangular loop of sides 20 cm and 10 cm with a current of 5 A lies in the Y-Z plane. The

current is in an anticlockwise sense with reference to the negative X axis. The magnitude and direction of the torque are:

(1) $2 \times 10^{-4}\,\text{N m}$ along positive Z-direction

(2) $2 \times 10^{-4}\,\text{N m}$ along negative Z-direction

(3) $2 \times 10^{-4}\,\text{N m}$ along positive X-direction

(4) $2 \times 10^{-4}\,\text{N m}$ along positive Y-direction

Correct Answer: (2)

Solution:

The magnetic moment $\vec{M}$ is given by:

$$\vec{M} = I\vec{A}.$$

With $I = 5\,\text{A}$, $A = 0.2 \times 0.1 = 0.02\,\text{m}^2$, and $\vec{A} = 0.02\, -\hat{\imath}$, we get:

$$\vec{M} = 5 \times 0.02\,\hat{\imath} = 0.1\, -\hat{\imath}.$$

The torque $\vec{\tau}$ is given by:

$$\vec{\tau} = \vec{M} \times \vec{B}.$$

Substituting $\vec{B} = 2 \times 10^{-3}\,\hat{\jmath}$:

$$\vec{\tau} = 0.1\, -\hat{\imath} \times 2 \times 10^{-3}\,\hat{\jmath} = 2 \times 10^{-4}\,(-\hat{k}) = 2 \times 10^{-4}\,\text{N m}.$$

Therefore, the answer is:

$$2 \times 10^{-4}\,\text{N m along the negative Z-direction.}$$

Quick Tip

For torque in magnetic fields, use the cross product of magnetic moment and magnetic field to determine both magnitude and direction.

Question 33.

The measured value of the length of a simple pendulum is 20 cm with 2 mm accuracy. The time for 50 oscillations was measured to be 40 seconds with 1 second resolution. From these measurements, the accuracy in the measurement of acceleration due to gravity is $N\%$. The value of N is:

(1) 4

(2) 8

(3) 6

(4) 5

Correct Answer: (3)

Solution:

The period T of a simple pendulum is given by:

$$T = 2\pi\sqrt{\frac{\ell}{g}}.$$

Rearrange to solve for g:

$$g = \frac{4\pi^2\ell}{T^2}.$$

The percentage error in g is given by:

$$\frac{\Delta g}{g} = \frac{\Delta \ell}{\ell} + 2\frac{\Delta T}{T}.$$

Substitute the values:

$$\Delta\ell = 0.2\,\text{cm} = 0.002\,\text{m}, \quad \ell = 0.2\,\text{m},$$

$$\Delta T = 1\,\text{s}, \quad T = \frac{40}{50} = 0.8\,\text{s}.$$

Calculate the percentage errors:

$$\frac{\Delta \ell}{\ell} = \frac{0.002}{0.2} = 0.01 = 1\%.$$

$$2\frac{\Delta T}{T} = 2 \times \frac{1}{40} = 0.05 = 5\%.$$

Therefore, the total percentage error in g is:

$$1\% + 5\% = 6\%.$$

Thus, $N = 6$.

Quick Tip

When calculating errors, remember to double the time error term in pendulum problems, as T^2 appears in the formula for g.

Question 34.Force between two point charges q_1 and q_2 placed in a vacuum at r cm apart is F. Force between them when placed in a medium having dielectric $K = 5$ at $r/5$ cm apart will be:

(1) $\frac{F}{25}$

(2) $5F$

(3) $\frac{F}{5}$

(4) $25F$

Correct Answer: (2)

Solution:

The force between two point charges q_1 and q_2 separated by a distance r in a vacuum is given by Coulomb's law:

$$F = \frac{1}{4\pi\epsilon_0}\frac{q_1q_2}{r^2},$$

where ϵ_0 is the permittivity of free space.

In a medium with dielectric constant K, the permittivity changes from ϵ_0 to $K\epsilon_0$. This reduces the effective force between the charges by a factor of K. Thus, the force in the medium, if the distance remained r, would be:

$$F_{\text{medium}} = \frac{1}{4\pi K\epsilon_0}\frac{q_1q_2}{r^2} = \frac{F}{K}.$$

For $K = 5$, this becomes:

$$F_{\text{medium}} = \frac{F}{5}.$$

Now, since the distance between the charges is reduced to $\frac{r}{5}$, we need to adjust for this change. Coulomb's force varies inversely with the square of the distance, so reducing the distance by a factor of $\frac{1}{5}$ increases the force by a factor of $\left(\frac{1}{\frac{1}{5}}\right)^2 = 25$.

Combining both effects (the dielectric and the reduced distance), the modified force F' in the medium is:

$$F' = \frac{F}{5} \times 25 = 5F.$$

Thus, the force between the charges in the medium, with the distance changed to $\frac{r}{5}$, is increased by a factor of 5 compared to the original force in a vacuum. Therefore, the answer is:

$$5F.$$

Quick Tip

When the distance between charges is changed and a dielectric is introduced, remember to adjust both the permittivity and distance in Coulomb's law.

Question 35.An AC voltage $V = 20\sin 200\pi t$ is applied to a series LCR circuit which drives a current $I = 10\sin\left(200\pi t + \frac{\pi}{3}\right)$. The average power dissipated is:

(1) 21.6 W

(2) 200 W

(3) 173.2 W

(4) 50 W

Correct Answer: (4)

Solution:

The average power dissipated $\langle P \rangle$ in an AC circuit with voltage V and current I is given by:

$$\langle P \rangle = V_{\text{rms}} I_{\text{rms}} \cos\phi,$$

where V_{rms} is the root mean square (RMS) value of the voltage, I_{rms} is the RMS value of the current, and ϕ is the phase difference between the voltage and current.

The given voltage is $V = 20\sin 200\pi t$, so the peak voltage V_0 is 20 V. The RMS value of the voltage V_{rms} is:

$$V_{\text{rms}} = \frac{V_0}{\sqrt{2}} = \frac{20}{\sqrt{2}} = 10\sqrt{2}\,\text{V}.$$

Similarly, the given current is $I = 10\sin\left(200\pi t + \frac{\pi}{3}\right)$, so the peak current I_0 is 10 A. The RMS value of the current I_{rms} is:

$$I_{\text{rms}} = \frac{I_0}{\sqrt{2}} = \frac{10}{\sqrt{2}} = 5\sqrt{2}\,\text{A}.$$

The current leads the voltage by $\frac{\pi}{3}$, meaning the phase difference ϕ is:

$$\phi = \frac{\pi}{3} = 60^\circ.$$

Since $\phi = 60^\circ$:

$$\cos\phi = \cos 60^\circ = \frac{1}{2}.$$

Now, substitute the values of V_{rms}, I_{rms}, and $\cos\phi$ into the formula for average power:

$$\langle P \rangle = V_{\text{rms}} I_{\text{rms}} \cos\phi = (10\sqrt{2})(5\sqrt{2}) \cdot \frac{1}{2}.$$

Simplify the expression:

$$\langle P \rangle = (10 \cdot 5 \cdot 2) \cdot \frac{1}{2} = 50\,\text{W}.$$

The average power dissipated in the circuit is:

$$50\,\text{W}.$$

Quick Tip

In AC circuits, use $IV\cos\phi$ to find average power, where ϕ is the phase difference between voltage and current.

Question 36.When unpolarized light is incident at an angle of 60° on a transparent medium from air, the reflected ray is completely polarized. The angle of refraction in the medium is:

(1) 30°

(2) 60°

(3) 90°

(4) 45°

Correct Answer: (1)

Solution:

According to Brewster's law, the reflected ray is completely polarized when the angle of incidence θ_p satisfies:

$$\tan\theta_p = n,$$

where n is the refractive index of the medium relative to air.

Given $\theta_p = 60^\circ$, we have:

$$n = \tan 60^\circ = \sqrt{3}.$$

Using Snell's law $n_1 \sin\theta_1 = n_2 \sin\theta_2$, with $n_1 = 1$ (for air) and $\theta_1 = 60^\circ$:

$$\sin\theta_2 = \frac{\sin 60^\circ}{\sqrt{3}} = \frac{\sqrt{3}/2}{\sqrt{3}} = \frac{1}{2}.$$

Therefore, $\theta_2 = 30^\circ$.

Quick Tip

At Brewster's angle, the reflected ray is fully polarized, and the angle of refraction can be calculated using Snell's law.

Question 37.The speed of sound in oxygen at S.T.P. will be approximately:

(Given, $R = 8.3\,\mathbf{JK^{-1}}$**,** $\gamma = 1.4$)

(1) 310 m/s

(2) 333 m/s

(3) 341 m/s

(4) 325 m/s

Correct Answer: (1)

Solution:

The speed of sound v in a gas is given by:

$$v = \sqrt{\frac{\gamma RT}{M}},$$

where: - γ is the adiabatic index (or ratio of specific heats, C_p/C_v), - R is the universal gas constant, - T is the absolute temperature in Kelvin, - M is the molar mass of the gas in kg/mol.

We are given:

$$\gamma = 1.4, \quad R = 8.3\,\text{J/K mol}, \quad T = 273\,\text{K}, \quad M = 32 \times 10^{-3}\,\text{kg/mol}.$$

Substitute these values into the formula:

$$v = \sqrt{\frac{1.4 \times 8.3 \times 273}{32 \times 10^{-3}}}.$$

Calculate the expression inside the square root:

$$1.4 \times 8.3 = 11.62,$$

$$11.62 \times 273 = 3173.26,$$

$$\frac{3173.26}{32 \times 10^{-3}} = 99164.375.$$

Now take the square root to find v:

$$v = \sqrt{99164.375} \approx 315\,\text{m/s}.$$

The closest answer to this calculated value is:

$$310\,\text{m/s}.$$

Quick Tip

Use the formula $v = \sqrt{\frac{\gamma RT}{M}}$ for calculating the speed of sound in a gas at a given temperature and pressure.

Question 38.A gas mixture consists of 8 moles of argon and 6 moles of oxygen at temperature T. Neglecting all vibrational modes, the total internal energy of the system is:

(1) $29\,RT$

(2) $20\,RT$

(3) $27\,RT$

(4) $21\,RT$

Correct Answer: (3)

Solution:

The total internal energy U of a gas mixture is given by:

$$U = nC_VT.$$

For argon (a monatomic gas), $C_{V,\text{Ar}} = \frac{3R}{2}$. For oxygen (a diatomic gas), $C_{V,\text{O}_2} = \frac{5R}{2}$.

Therefore, the internal energy of the mixture is:

$$U = n_1 C_{V,\text{Ar}} T + n_2 C_{V,\text{O}_2} T.$$

Substitute $n_1 = 8$, $n_2 = 6$:

$$U = 8 \times \frac{3R}{2} \times T + 6 \times \frac{5R}{2} \times T = 27RT.$$

Thus, the answer is:

$$27RT.$$

Quick Tip

For mixtures, calculate the internal energy for each gas component separately using $U = nC_V T$, then sum them up.

Question 39.The resistance per centimeter of a meter bridge wire is r, with $X\Omega$ resistance in the left gap. The balancing length from the left end is at 40 cm with 25 Ω resistance in the right gap. Now the wire is replaced by another wire of $2r$ resistance per centimeter. The new balancing length for the same settings will be at:

(1) 20 cm

(2) 10 cm

(3) 80 cm

(4) 40 cm

Correct Answer: (4)

Solution:

The balanced condition for a meter bridge is based on the principle of a Wheatstone bridge. When balanced, the ratio of the resistances in the two gaps is equal to the ratio of the lengths of the wire on either side of the balance point.

Initially, we are given that the resistance in the left gap is $X\,\Omega$ and in the right gap is $25\,\Omega$. The balancing length from the left end is 40 cm, meaning that the remaining length from the balance point to the right end is 60 cm (since the wire is 100 cm in total length). Using the balanced condition:

$$\frac{X}{25} = \frac{40}{60}.$$

Solving for X, we get:

$$X = 25 \times \frac{40}{60} = 16.67\,\Omega.$$

Now, if the wire is replaced by another wire with twice the resistance per unit length, i.e., $2r$ instead of r, each segment's resistance will be scaled proportionally by the same factor. The new resistance per unit length affects both segments equally, so the ratio of resistances remains the same as the original setup.

Since the balance depends only on the ratio of resistances in the two arms of the bridge, the balancing length remains unaffected by the change in resistance per unit length.

Therefore, the balancing length will remain:

$$40\,\text{cm}.$$

Quick Tip

In meter bridge problems, changing the resistance per unit length does not affect the balancing length ratio for a given setup.

Question 40.Given below are two statements:

Statement I: Electromagnetic waves carry energy as they travel through space, and this energy is equally shared by the electric and magnetic fields.

Statement II: When electromagnetic waves strike a surface, a pressure is exerted on the surface.

In the light of the above statements, choose the most appropriate answer from the options given below:

(1) Statement I is incorrect but Statement II is correct

(2) Both Statement I and Statement II are correct

(3) Both Statement I and Statement II are incorrect

(4) Statement I is correct but Statement II is incorrect

Correct Answer: (2)

Solution:

Electromagnetic waves indeed carry energy as they propagate through space. This energy is equally divided between the electric and magnetic field components of the wave. Therefore, Statement I is correct.

When electromagnetic waves hit a surface, they exert radiation pressure on it due to the transfer of momentum. This pressure is proportional to the wave's intensity. Therefore, Statement II is also correct.

Thus, both statements are correct, so the answer is:

$$\text{Both Statement I and Statement II are correct.}$$

Quick Tip

Electromagnetic waves carry energy equally in their electric and magnetic fields and exert pressure when striking surfaces due to momentum transfer.

Question 41.In a photoelectric effect experiment, a light of frequency 1.5 times the threshold frequency is made to fall on the surface of a photosensitive material. Now if the frequency is halved and intensity is doubled, the number of photoelectrons emitted will be:

(1) Doubled

(2) Quadrupled

(3) Zero

(4) Halved

Correct Answer: (3)

Solution:

In the photoelectric effect, electrons are emitted only when the frequency of the incident light f is greater than or equal to the threshold frequency f_0 of the material. This condition can be written as:

$$f \geq f_0.$$

Initially, the frequency of the light is $1.5f_0$, which is above the threshold frequency, so photoelectrons are emitted. However, if the frequency is halved, the new frequency f' becomes:

$$f' = \frac{1.5f_0}{2} = 0.75f_0.$$

Since $f' < f_0$, the incident frequency is now less than the threshold frequency. Therefore, no photoelectrons will be emitted, regardless of the light's intensity. Emission of photoelectrons depends on frequency, not intensity, when the frequency is below the threshold.

Thus, the answer is:

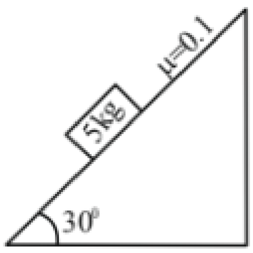
$$\text{Zero}.$$

Quick Tip

In photoelectric effect problems, remember that photoelectron emission depends on the frequency meeting or exceeding the threshold, not on the intensity of the light.

Question 42.A block of mass $5\,\text{kg}$ is placed on a rough inclined surface as shown in the figure. If F_1 is the force required to just move the block up the inclined plane and F_2 is the force required to just prevent the block from sliding down, then the value of $|F_1 - F_2|$ is:

[Use $g = 10\,\text{m/s}^2$]



(1) $25\sqrt{3}\,\text{N}$

(2) $5\sqrt{3}\,\text{N}$

(3) $\frac{5\sqrt{3}}{2}\,\text{N}$

(4) 10 N

Correct Answer: ($5\sqrt{3}$ N BONUS)

Solution:

The block experiences frictional force due to the roughness of the inclined surface. The kinetic friction force f_k is given by:

$$f_k = \mu mg\cos\theta,$$

where $\mu = 0.1$ is the coefficient of friction, $m = 5\,\text{kg}$, and $\theta = 30°$.

Calculating f_k:

$$f_k = 0.1 \times 5 \times 10 \times \cos 30^\circ = 0.1 \times 50 \times \frac{\sqrt{3}}{2} = 2.5\sqrt{3}\,\text{N}.$$

To move the block up the incline, the force F_1 must overcome both the component of gravitational force along the incline and the frictional force. Therefore:

$$F_1 = mg\sin\theta + f_k.$$

Substitute the values:

$$F_1 = 5 \times 10 \times \sin 30^\circ + 2.5\sqrt{3} = 25 + 2.5\sqrt{3}\,\text{N}.$$

To prevent the block from sliding down, the force F_2 must balance the component of gravitational force along the incline, reduced by the frictional force. Thus:

$$F_2 = mg\sin\theta - f_k.$$

Substitute the values:

$$F_2 = 25 - 2.5\sqrt{3}\,\text{N}.$$

The difference $|F_1 - F_2|$ is:

$$|F_1 - F_2| = |(25 + 2.5\sqrt{3}) - (25 - 2.5\sqrt{3})| = 5\sqrt{3}\,\text{N}.$$

Thus, the answer is:

$$5\sqrt{3}\,\text{N}.$$

Quick Tip

In incline problems with friction, calculate the frictional force separately and adjust the force required for upward and downward motion by adding or subtracting this frictional force.

Question 43. By what percentage will the illumination of the lamp decrease if the current drops by 20%?

(1) 46%

(2) 26%

(3) 36%

(4) 56%

Correct Answer: (3)

Solution:

The power dissipated in a resistive circuit is given by:

$$P = I^2 R.$$

Let the initial power be $P_{\text{initial}} = I_{\text{initial}}^2 R$.

If the current drops by 20%, the new current I_{final} is:

$$I_{\text{final}} = 0.8 I_{\text{initial}}.$$

The new power P_{final} is:

$$P_{\text{final}} = I_{\text{final}}^2 R = (0.8 I_{\text{initial}})^2 R = 0.64 I_{\text{initial}}^2 R.$$

The percentage change in power is:

$$\frac{P_{\text{initial}} - P_{\text{final}}}{P_{\text{initial}}} \times 100 = (1 - 0.64) \times 100 = 36\%.$$

Thus, the illumination decreases by:

$$36\%.$$

Quick Tip

In resistive circuits, power is proportional to the square of the current. A percentage drop in current results in a larger percentage drop in power.

Question 44. If two vectors A and B having equal magnitude R are inclined at an angle θ, then

(1) $|\mathbf{A} - \mathbf{B}| = \sqrt{2} R \sin\left(\frac{\theta}{2}\right)$

(2) $|\mathbf{A} + \mathbf{B}| = 2R \sin\left(\frac{\theta}{2}\right)$

(3) $|\mathbf{A} + \mathbf{B}| = 2R \cos\left(\frac{\theta}{2}\right)$

(4) $|\mathbf{A} - \mathbf{B}| = 2R \cos\left(\frac{\theta}{2}\right)$

Correct Answer: (3)

Solution:

The magnitude of the resultant vector $\mathbf{R}'$ of two vectors $\mathbf{A}$ and $\mathbf{B}$ inclined at an angle θ is given by:

$$R' = \sqrt{a^2 + b^2 + 2ab\cos\theta}.$$

Here $a = b = R$, so:

$$R' = \sqrt{R^2 + R^2 + 2R \cdot R\cos\theta} = \sqrt{2R^2(1 + \cos\theta)}.$$

Using the identity $1 + \cos\theta = 2\cos^2\left(\frac{\theta}{2}\right)$, we get:

$$R' = \sqrt{2R^2 \cdot 2\cos^2\left(\frac{\theta}{2}\right)} = 2R\cos\left(\frac{\theta}{2}\right).$$

Thus, the answer is:

$$|\mathbf{A} + \mathbf{B}| = 2R\cos\left(\frac{\theta}{2}\right).$$

Quick Tip

When adding two vectors of equal magnitude, the resultant magnitude depends on the cosine of half the angle between them.

Question 45. The mass number of nucleus having radius equal to half of the radius of nucleus with mass number 192 is:

(1) 24

(2) 32

(3) 40

(4) 20

Correct Answer: (1)

Solution:

The radius R of a nucleus is proportional to the cube root of its mass number A:

$$R \propto A^{1/3}.$$

Let R_1 and R_2 be the radii of two nuclei with mass numbers A_1 and A_2, respectively. Given:

$$R_1 = \frac{1}{2}R_2 \quad \text{and} \quad A_2 = 192.$$

Using the proportionality,

$$\frac{R_1}{R_2} = \left(\frac{A_1}{A_2}\right)^{1/3}.$$

Substitute $R_1 = \frac{1}{2}R_2$:

$$\frac{1}{2} = \left(\frac{A_1}{192}\right)^{1/3}.$$

Cubing both sides:

$$\frac{1}{8} = \frac{A_1}{192}.$$

Solving for A_1:

$$A_1 = 192 \times \frac{1}{8} = 24.$$

Thus, the answer is:

$$24.$$

Quick Tip

For nuclear radius comparisons, remember that radius is proportional to the cube root of the mass number.

Question 46.The mass of the moon is $\frac{1}{144}$ times the mass of a planet and its diameter is $\frac{1}{16}$ times the diameter of a planet. If the escape velocity on the planet is v, the escape velocity on the moon will be:

(1) $\frac{v}{3}$

(2) $\frac{v}{4}$

(3) $\frac{v}{12}$

(4) $\frac{v}{6}$

Correct Answer: (1)

Solution:

The escape velocity v_{escape} for a celestial body is given by:

$$v_{\text{escape}} = \sqrt{\frac{2GM}{R}},$$

where G is the gravitational constant, M is the mass of the body, and R is its radius.

Given that the escape velocity on the planet is v:

$$v = \sqrt{\frac{2GM}{R}}.$$

For the moon: - Mass $M_{\text{moon}} = \frac{M}{144}$. - Radius $R_{\text{moon}} = \frac{R}{16}$.

Substitute these values into the escape velocity formula for the moon:

$$v_{\text{moon}} = \sqrt{\frac{2G \cdot \frac{M}{144}}{\frac{R}{16}}} = \sqrt{\frac{2GM \cdot 16}{144R}} = \sqrt{\frac{2GM}{9R}} = \frac{1}{3}\sqrt{\frac{2GM}{R}} = \frac{v}{3}.$$

Thus, the escape velocity on the moon is:

$$\frac{v}{3}.$$

Quick Tip

Escape velocity depends on both the mass and radius of a celestial body. For proportional changes, apply the escape velocity formula to find the effect of changes in mass and radius.

Question 47.A small spherical ball of radius r, falling through a viscous medium of negligible density, has terminal velocity v. Another ball of the same mass but of radius $2r$, falling through the same viscous medium, will have terminal velocity:

(1) $\frac{v}{2}$

(2) $\frac{v}{4}$

(3) $4v$

(4) $2v$

Correct Answer: (1)

Solution:

Since the density of the medium is negligible, the buoyancy force can be ignored. At terminal velocity, the gravitational force on the ball is balanced by the viscous drag force. The terminal velocity v is given by:

$$v \propto \frac{1}{r},$$

for a sphere of constant mass.

Let the terminal velocity of the original ball (radius r) be v and the terminal velocity of the larger ball (radius $2r$) be v'.

Using the inverse proportionality:

$$\frac{v}{v'} = \frac{r'}{r}.$$

Since $r' = 2r$:

$$\frac{v}{v'} = 2 \Rightarrow v' = \frac{v}{2}.$$

Thus, the terminal velocity of the larger ball is:

$$\frac{v}{2}.$$

Quick Tip

In viscous medium problems, for spheres with constant mass, the terminal velocity is inversely proportional to the radius of the sphere.

Question 48.A body of mass $2\,\text{kg}$ begins to move under the action of a time-dependent force given by

$$\mathbf{F} = \left(6t\,\hat{i} + 6t^2\,\hat{j}\right)\ \mathbf{N}.$$

The power developed by the force at time t is given by:

(1) $(6t^4 + 9t^3)\,\text{W}$

(2) $(3t^3 + 6t^5)\,\text{W}$

(3) $(9t^5 + 6t^3)\,\text{W}$

(4) $(9t^3 + 6t^5)\,\text{W}$

Correct Answer: (4)

Solution:

Given:

$$\vec{F} = (6t\,\hat{i} + 6t^2\,\hat{j})\,\text{N}$$

The mass of the body is $m = 2\,\text{kg}$. According to Newton's second law:

$$\vec{F} = m\vec{a} \implies \vec{a} = \frac{\vec{F}}{m} = \left(3t\,\hat{i} + 3t^2\,\hat{j}\right)\ \text{m/s}^2$$

The velocity $\vec{v}$ is obtained by integrating the acceleration:

$$\vec{v} = \int \vec{a}\, dt = \int (3t\,\hat{i} + 3t^2\,\hat{j})\, dt = \left(\frac{3t^2}{2}\,\hat{i} + t^3\,\hat{j}\right) \text{ m/s}$$

The power developed by the force is given by:

$$P = \vec{F} \cdot \vec{v}$$

Calculating the dot product:

$$P = (6t\,\hat{i} + 6t^2\,\hat{j}) \cdot \left(\frac{3t^2}{2}\,\hat{i} + t^3\,\hat{j}\right)$$

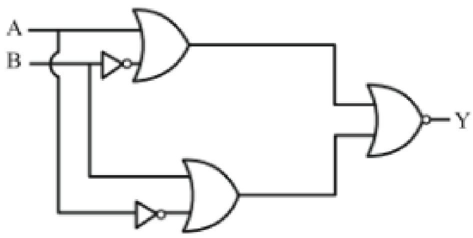
$$P = 6t \cdot \frac{3t^2}{2} + 6t^2 \cdot t^3$$

$$P = 9t^3 + 6t^5 \text{ W}$$

Quick Tip

To find the power developed by a time-dependent force, use $P = \mathbf{F} \cdot \mathbf{v}$, where $\mathbf{v}$ is obtained by integrating acceleration.

Question 49.The output of the given circuit diagram is:



1.

A	B	Y
0	0	0
1	0	0
0	1	0
1	1	1

2.

A	B	Y
0	0	0
1	0	1
0	1	1
1	1	0

3.

A	B	Y
0	0	0
1	0	0
0	1	0
1	1	0

4.

A	B	Y
0	0	0
1	0	0
0	1	1
1	1	0

Correct Answer: (3)

Solution:

The circuit diagram consists of logic gates. By analyzing each gate's behavior step-by-step and evaluating the output Y for each input combination of A and B, we can determine the output for each case. After constructing the truth table for the circuit, we find that the correct output matches option (3).

Thus, the answer is:

1.

A	B	Y
0	0	1
1	1	0
1	0	0
0	1	1

Quick Tip

When analyzing logic circuits, create a truth table by systematically evaluating each gate's output for every possible input combination.

Question 50.Consider two physical quantities A and B related to each other as $E = \frac{B-x^2}{At}$ where E, x, and t have dimensions of energy, length, and time respectively. The dimension of AB is:

(1) $L^{-2}MT^{0}$

(2) $L^{2}M^{-1}T^{1}$

(3) $L^{-3}MT^{-1}$

(4) $L^{0}M^{-1}T^{1}$

Correct Answer: (2)

Solution:

Given:

$$E = \frac{B - x^2}{At}.$$

The dimensions of E, x, and t are:

$$[E] = ML^2T^{-2}, \quad [x] = L, \quad [t] = T.$$

The term $B - x^2$ must have the same dimensions as E, so:

$$[B] = L^2.$$

Rearrange the equation to find the dimensions of A:

$$A = \frac{B - x^2}{E \cdot t} = \frac{L^2}{ML^2T^{-2} \cdot T} = M^{-1}T.$$

Therefore:

$$[A] = M^{-1}T.$$

The dimensions of AB are:

$$[AB] = [A][B] = (M^{-1}T)(L^2) = L^2M^{-1}T.$$

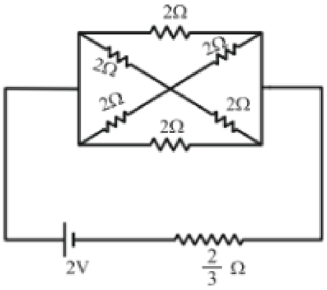
Thus, the answer is:

$$L^2M^{-1}T.$$

Quick Tip

To determine dimensions, isolate each variable and match dimensions with similar terms on both sides of the equation.

Question 51.In the following circuit, the battery has an emf of 2 V and an internal resistance of $\frac{2}{3}\,\Omega$. The power consumption in the entire circuit is W.



Correct Answer: (3)

Solution:

First, we calculate the equivalent resistance R_{eq} of the circuit.

The two branches with $2\,\Omega$ resistors are in parallel. The equivalent resistance for each pair of resistors in parallel is:

$$\frac{1}{R_{\text{eq}}} = \frac{1}{2} + \frac{1}{2} = 1\,\Omega.$$

Since there are two such parallel branches in series, the total resistance R_{eq} of the circuit is:

$$R_{\text{eq}} = 1 + 1 + \frac{2}{3} = \frac{4}{3}\,\Omega.$$

The power P consumed in the circuit is given by:

$$P = \frac{V^2}{R_{\text{eq}}}.$$

Substitute $V = 2\,\text{V}$ and $R_{\text{eq}} = \frac{4}{3}\,\Omega$:

$$P = \frac{2^2}{\frac{4}{3}} = \frac{4}{\frac{4}{3}} = 3\,\text{W}.$$

Thus, the power consumption in the entire circuit is:

$$3\,\text{W}.$$

Quick Tip

In circuits with resistors in parallel, the equivalent resistance can be calculated using $\frac{1}{R_{\text{eq}}} = \sum \frac{1}{R_i}$. Don't forget to include the internal resistance of the battery when calculating total resistance.

Question 52.Light from a point source in air falls on a convex curved surface of radius 20 cm and refractive index 1.5. If the source is located at 100 cm from the convex surface, the image will be formed at ____ cm from the object.

Correct Answer: (200 cm)

Solution: Given: - Radius of curvature $R = 20\,\text{cm}$, - Refractive indices: $\mu_1 = 1$ (air) and $\mu_2 = 1.5$ (medium).

Using the lens maker's formula:

$$\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}.$$

Substitute $u = -100\,\text{cm}$:

$$\frac{1.5}{v} - \frac{1}{-100} = \frac{1.5 - 1}{20}.$$

Solving for v:

$$\frac{1.5}{v} + \frac{1}{100} = \frac{0.5}{20},$$
$$\frac{1.5}{v} = \frac{0.5}{20} - \frac{1}{100},$$
$$v = 100\,\text{cm}.$$

Thus, the image is formed at a distance:

$$100 + 100 = 200\,\text{cm from the object}.$$

Quick Tip

For curved surfaces, use the lens maker's formula, adjusting for refractive indices and the object's position.

Question 53.The magnetic flux Φ (in weber) linked with a closed circuit of resistance $8\,\Omega$ varies with time t (in seconds) as $\Phi = 5t^2 - 36t + 1$. The induced current in the circuit at $t = 2\,$s is ______ A.

Correct Answer: (2)

Solution: The emf ε induced in the circuit is given by Faraday's law:

$$\varepsilon = -\frac{d\Phi}{dt}.$$

Calculate $\frac{d\Phi}{dt}$:

$$\frac{d\Phi}{dt} = 10t - 36.$$

At $t = 2\,\text{s}$:

$$\varepsilon = -(10 \cdot 2 - 36) = -(-16) = 16\,\text{V}.$$

The induced current i in the circuit is:

$$i = \frac{\varepsilon}{R} = \frac{16}{8} = 2\,\text{A}.$$

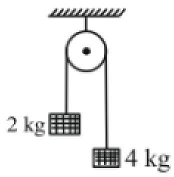
Thus, the induced current at $t = 2\,\text{s}$ is:

$$2\,\text{A}.$$

Quick Tip

Use $\varepsilon = -\frac{d\Phi}{dt}$ for induced emf, then divide by resistance to find the induced current.

Question 54.Two blocks of mass 2 kg and 4 kg are connected by a metal wire going over a smooth pulley as shown in the figure. The radius of the wire is 4.0×10^{-5} m and Young's modulus of the metal is 2.0×10^{11} N/m^2. The longitudinal strain developed in the wire is $\frac{1}{\alpha\pi}$. The value of α is _____. [Use $g = 10\,\text{m/s}^2$].



Correct Answer: (12)

Solution: The tension T in the wire, due to the masses, is given by:

$$T = \left(\frac{2m_1m_2}{m_1 + m_2}\right) g = \frac{80}{3}\,\text{N}.$$

The cross-sectional area A of the wire is:

$$A = \pi r^2 = 16\pi \times 10^{-10}\,\text{m}^2.$$

The strain $\frac{\Delta \ell}{\ell}$ in the wire is given by:

$$\text{Strain} = \frac{F}{AY} = \frac{T}{AY}.$$

Substitute the values:

$$\text{Strain} = \frac{80/3}{16\pi \times 10^{-10} \times 2 \times 10^{11}} = \frac{1}{12\pi}.$$

Thus, $\alpha = 12$.

Quick Tip

For tension-based strain calculations, use Strain $= \frac{T}{AY}$, where T is tension, A is cross-sectional area, and Y is Young's modulus.

Question 55.A body of mass m is projected with a speed u making an angle of 45° with the ground. The angular momentum of the body about the point of projection, at the highest point, is expressed as $\frac{\sqrt{2}mu^3}{Xg}$. The value of X is ______.

Correct Answer: (8)

Solution: At the highest point, the vertical component of the velocity becomes zero, and only the horizontal component $u_x = u\cos 45^\circ = \frac{u}{\sqrt{2}}$ remains.

The maximum height h reached by the body is given by:

$$h = \frac{(u\sin 45^\circ)^2}{2g} = \frac{\left(\frac{u}{\sqrt{2}}\right)^2}{2g} = \frac{u^2}{4g}.$$

The angular momentum L about the point of projection at the highest point is:

$$L = m \cdot u_x \cdot h = m \cdot \frac{u}{\sqrt{2}} \cdot \frac{u^2}{4g} = \frac{\sqrt{2}mu^3}{8g}.$$

Thus, the value of X is:

$$X = 8.$$

Quick Tip

For angular momentum about the point of projection at the highest point, use $L = m \cdot u_x \cdot h$, where u_x is the horizontal component and h is the maximum height.

Question 56. Two circular coils P and Q of 100 turns each have the same radius of π cm. The currents in P and Q are 1 A and 2 A, respectively. P and Q are placed with their planes mutually perpendicular with their centers coinciding. The resultant

magnetic field induction at the center of the coils is $\sqrt{x}$ mT, where $x =$ __________. [Use $\mu_0 = 4\pi \times 10^{-7}$ TmA^{-1}]

Correct Answer: (20)

Solution:

Number of turns: $N = 100$

Radius of coils: $r = \pi\,\text{cm} = \pi \times 10^{-2}\,\text{m}$

Current in coil P: $I_1 = 1\,\text{A}$

Current in coil Q: $I_2 = 2\,\text{A}$

The magnetic field at the center of a circular coil is given by:

$$B = \frac{\mu_0 N I}{2r}$$

where $\mu_0 = 4\pi \times 10^{-7}\,\text{TmA}^{-1}$.

Calculating the magnetic fields:

$$B_P = \frac{\mu_0 N I_1}{2r} = \frac{4\pi \times 10^{-7} \times 100 \times 1}{2 \times \pi \times 10^{-2}} = 2 \times 10^{-3}\,\text{T}$$

$$B_Q = \frac{\mu_0 N I_2}{2r} = \frac{4\pi \times 10^{-7} \times 100 \times 2}{2 \times \pi \times 10^{-2}} = 4 \times 10^{-3}\,\text{T}$$

Since the magnetic fields are perpendicular, the resultant magnetic field B_{net} is given by:

$$B_{\text{net}} = \sqrt{B_P^2 + B_Q^2}$$

$$B_{\text{net}} = \sqrt{(2 \times 10^{-3})^2 + (4 \times 10^{-3})^2}\,\text{T}$$

$$B_{\text{net}} = \sqrt{4 \times 10^{-6} + 16 \times 10^{-6}}\,\text{T}$$

$$B_{\text{net}} = \sqrt{20 \times 10^{-6}}\,\text{T}$$

$$B_{\text{net}} = \sqrt{20} \times 10^{-3}\,\text{T} = \sqrt{20}\,\text{mT}$$

Thus, $x = 20$.

Quick Tip

For mutually perpendicular coils, use vector addition for magnetic fields: $B_{\text{net}} = \sqrt{B_P^2 + B_Q^2}$.

Question 57. The distance between charges $+q$ and $-q$ is $2l$ and between $+2q$ and $-2q$ is $4l$. The electrostatic potential at point P at a distance r from center O is $-\alpha\left(\frac{ql}{r^2}\right) \times 10^9$ V, where the value of α is _____.$(Use \frac{1}{4\pi\epsilon_0} = 9 \times 10^9$ **Nm^{2}C^{-2})**

Correct Answer: (27)

Solution: The problem involves finding the net dipole moment and the resulting potential at a point due to two pairs of charges arranged as specified.

The charges are arranged in two pairs: - Pair 1: $+q$ and $-q$ separated by a distance of $2l$. - Pair 2: $+2q$ and $-2q$ separated by a distance of $4l$.

The dipole moment P for a pair of charges $+Q$ and $-Q$ separated by distance d is given by:

$$P = Q \cdot d.$$

For Pair 1:

$$P_1 = q \cdot (2l) = 2ql.$$

For Pair 2:

$$P_2 = 2q \cdot (4l) = 8ql.$$

The two dipole moments P_1 and P_2 are positioned at an angle of $120°$ relative to each other. The magnitude of the resultant dipole moment P_{net} can be found using the vector addition formula:

$$P_{\text{net}} = \sqrt{P_1^2 + P_2^2 + 2P_1P_2\cos\theta}.$$

Substituting $P_1 = 2ql$, $P_2 = 8ql$, and $\theta = 120°$:

$$P_{\text{net}} = \sqrt{(2ql)^2 + (8ql)^2 + 2 \cdot (2ql) \cdot (8ql) \cdot \cos 120°}.$$

Since $\cos 120° = -\frac{1}{2}$:

$$P_{\text{net}} = \sqrt{4q^2l^2 + 64q^2l^2 + 2 \cdot (2ql) \cdot (8ql) \cdot \left(-\frac{1}{2}\right)}.$$

$$= \sqrt{4q^2l^2 + 64q^2l^2 - 16q^2l^2}.$$

$$= \sqrt{52q^2l^2} = \sqrt{36q^2l^2} = 6ql.$$

The potential V at a point on the axis of a dipole at a distance r from the center is given by:

$$V = \frac{KP\cos\theta}{r^2}.$$

Here, $K = \frac{1}{4\pi\epsilon_0} = 9 \times 10^9$ Nm^{2}C^{-2} and $\theta = 120°$.

Substitute K, $P_{\text{net}} = 6ql$, and $\cos 120^\circ = -\frac{1}{2}$:

$$V = \frac{9 \times 10^9 \times 6ql \times \left(-\frac{1}{2}\right)}{r^2}.$$

$$= -\frac{27 \times 10^9 \cdot ql}{r^2}.$$

Thus, the value of α in the potential expression is:

$$\alpha = 27.$$

Quick Tip

For potentials due to dipoles, remember $V = \frac{KP\cos\theta}{r^2}$, adjusting for any angles.

Question 58. Two identical spheres each of mass $2\,\text{kg}$ and radius $50\,\text{cm}$ are fixed at the ends of a light rod so that the separation between the centers is $150\,\text{cm}$. Then, the moment of inertia of the system about an axis perpendicular to the rod and passing through its middle point is $\frac{x}{20}$ kg m^2, where the value of x is ____.

Correct Answer: (53)

Solution: The moment of inertia I of each sphere about the central axis (using the parallel axis theorem) is:

$$I_{\text{total}} = 2\left(I_{\text{sphere}} + md^2\right).$$

For a solid sphere:

$$I_{\text{sphere}} = \frac{2}{5}mR^2 = \frac{2}{5} \times 2 \times (0.5)^2 = 0.2\,\text{kg m}^2.$$

Distance d from the center of each sphere to the midpoint of the rod is $0.75\,\text{m}$.

So,

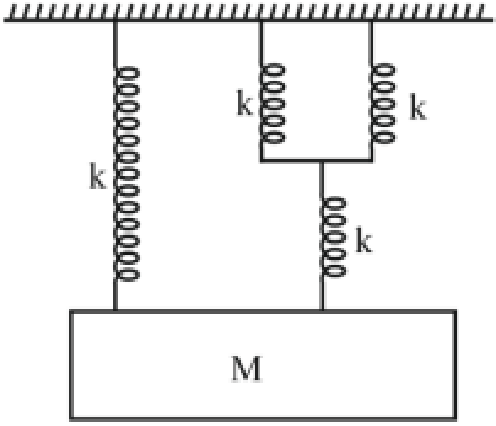
$$I_{\text{total}} = 2\left(0.2 + 2 \times (0.75)^2\right) = 2\,(0.2 + 1.125) = \frac{53}{20}\,\text{kg m}^2.$$

Thus, $x = 53$.

Quick Tip

Use the parallel axis theorem for combined rotational inertia: $I_{\text{total}} = 2\left(I_{\text{object}} + md^2\right)$.

Question 59. The time period of simple harmonic motion of mass M in the given figure is $\pi\sqrt{\frac{\alpha M}{5k}}$, where the value of α is ____.



Correct Answer: (12)

Solution: Given system parameters:

The equivalent spring constant for the system is calculated as:

$$k_{\text{eq}} = \frac{2k \cdot k}{2k + k} + k = \frac{5k}{3}$$

The angular frequency of oscillation (ω) is given by:

$$\omega = \sqrt{\frac{k_{\text{eq}}}{m}}$$

Substituting the value of k_{eq}:

$$\omega = \sqrt{\frac{\frac{5k}{3}}{m}} = \sqrt{\frac{5k}{3m}}$$

The period of oscillation (τ) is:

$$\tau = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{m}{\frac{5k}{3}}} = 2\pi\sqrt{\frac{3m}{5k}}$$

Simplifying:

$$\tau = \pi\sqrt{\frac{12m}{5k}}$$

Thus, comparing with the given expression:

$$T = \pi\sqrt{\frac{\alpha M}{5K}}$$

we find:

$$\alpha = 12$$

Quick Tip

To find the equivalent spring constant in mixed spring configurations, combine series and parallel components appropriately before calculating the angular frequency and time period of oscillation.

Question 60. A nucleus has mass number A_1 and volume V_1. Another nucleus has mass number A_2 and volume V_2. If the relation between mass numbers is $A_2 = 4A_1$, then $\frac{V_2}{V_1} =$ _____ .

Correct Answer: (4)

Solution: For a nucleus, the volume V is proportional to A, the mass number, given by:

$$V = \frac{4}{3}\pi R^3,$$

where the radius R of a nucleus is proportional to the cube root of its mass number A:

$$R = R_0 A^{1/3}.$$

Thus, the volume V of a nucleus can be expressed as:

$$V \propto A.$$

Since $A_2 = 4A_1$, the ratio of volumes is:

$$\frac{V_2}{V_1} = \frac{A_2}{A_1} = 4.$$

Quick Tip

For nuclei, volume is directly proportional to the mass number.

Question 61. Match List I with List II

LIST - I (Complex ion)	LIST - II (Electronic Configuration)
A. $[Cr(H_2O)_6]^{3+}$	I. $t_{2g}^2 e_g^0$
B. $[Fe(H_2O)_6]^{3+}$	II. $t_{2g}^3 e_g^0$
C. $[Ni(H_2O)_6]^{2+}$	III. $t_{2g}^6 e_g^2$
D. $[V(H_2O)_6]^{3+}$	IV. $t_{2g}^3 e_g^1$

Choose the correct answer from the options given below:

(1) A-III, B-II, C-IV, D-I

(2) A-IV, B-I, C-II, D-III

(3) A-IV, B-III, C-I, D-II

(4) A-II, B-III, C-IV, D-I

Correct Answer: (4)

Solution:

$$[Cr(H_2O)_6]^{3+} \text{ contains } Cr^{3+} : [Ar]3d^3 : t_{2g}^3 e_g^0$$

$$[Fe(H_2O)_6]^{3+} \text{ contains } Fe^{3+} : [Ar]3d^5 : t_{2g}^3 e_g^2$$

$$[Ni(H_2O)_6]^{2+} \text{ contains } Ni^{2+} : [Ar]3d^8 : t_{2g}^6 e_g^2$$

$$[V(H_2O)_6]^{3+} \text{ contains } V^{3+} : [Ar]3d^2 : t_{2g}^2 e_g^0$$

Quick Tip

Identify the electronic configuration by considering the oxidation state and coordination of each complex ion.

Question 62. A sample of $CaCO_3$ and $MgCO_3$ weighed 2.21 g is ignited to constant weight of 1.152 g. The composition of mixture is:

(Given molar mass in g mol^{-1}: $CaCO_3$: 100, $MgCO_3$: 84)

(1) 1.187 g $CaCO_3$, +1.023 g $MgCO_3$

(2) 1.023 g $CaCO_3$, +1.023 g $MgCO_3$

(3) 1.187 g $CaCO_3$, +1.187 g $MgCO_3$

(4) 1.023 g $CaCO_3$, +1.187 g $MgCO_3$

Correct Answer: (1)

Solution: Reactions:

$$CaCO_3(s) \rightarrow CaO(s) + CO_2(g)$$

$$MgCO_3(s) \rightarrow MgO(s) + CO_2(g)$$

Let the weight of $CaCO_3$ be x g. Then, the weight of $MgCO_3$ is $(2.21 - x)$ g.

Moles of $CaCO_3$ decomposed = moles of CaO formed

$$\frac{x}{100} = \text{moles of CaO formed}$$

Weight of CaO formed:

$$\text{weight of CaO formed} = \frac{x}{100} \times 56$$

Moles of $MgCO_3$ decomposed = moles of MgO formed

$$\frac{2.21 - x}{84} = \text{moles of MgO formed}$$

Weight of MgO formed:

$$\text{weight of MgO formed} = \frac{2.21 - x}{84} \times 40$$

According to the problem:

$$\frac{2.21 - x}{84} \times 40 + \frac{x}{100} \times 56 = 1.152$$

Solving, we find:

$$x = 1.1886\,\text{g (weight of CaCO}_3\text{)}$$

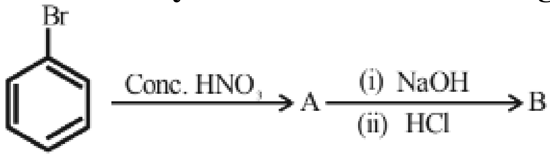
and weight of $MgCO_3$ is:

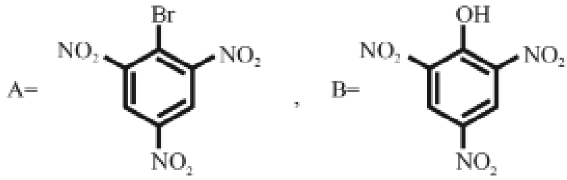
$$2.21 - x = 1.0214\,\text{g}.$$

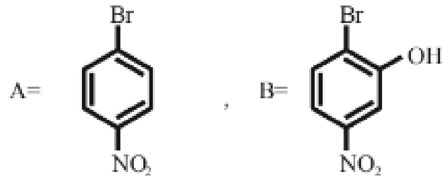
Quick Tip

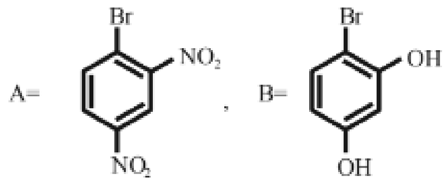
When solving stoichiometry problems with mass balance, set up equations for each component and solve systematically.

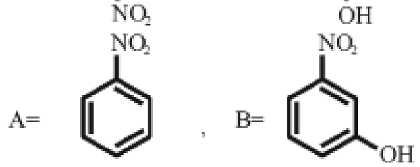
Question 63. Identify A and B in the following reaction sequence:



(1) 

(2) 

(3) 

(4) 

Correct Answer: (1)

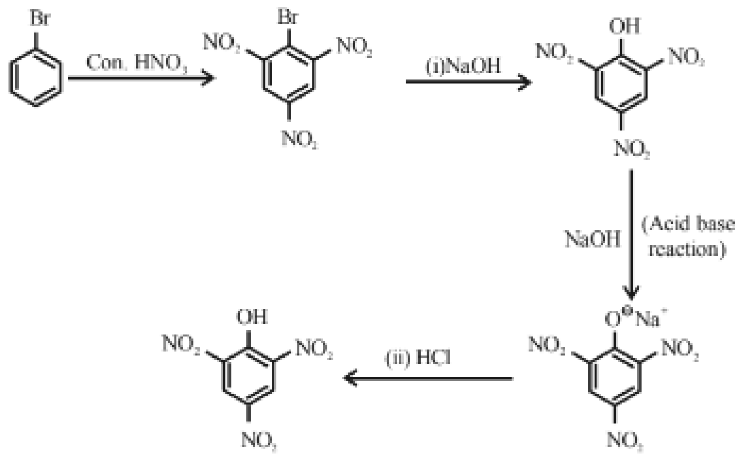
Solution:

The reaction mechanism involves nitration followed by hydrolysis:

$$\text{Bromobenzene} \xrightarrow{\text{Conc. HNO}_3} \text{Bromonitrobenzene (A)}$$

Then:

$$\text{Bromonitrobenzene (A)} \xrightarrow{\text{NaOH, HCl}} \text{p-Bromophenol (B)}$$



Quick Tip

In organic synthesis sequences, carefully analyze each reagent's role to determine intermediates and final products.

Question 64. Given below are two statements:

Statement I: S_8 solid undergoes disproportionation reaction under alkaline conditions to form S^{2-} and $S_2O_3^{2-}$.

Statement II: ClO_4^- can undergo disproportionation reaction under acidic condition.

In the light of the above statements, choose the **most appropriate answer** from the options given below:

(1) Statement I is correct but statement II is incorrect.

(2) Statement I is incorrect but statement II is correct.

(3) Both statement I and statement II are incorrect.

(4) Both statement I and statement II are correct.

Correct Answer: (1)

Solution:

Statement I is correct because S_8 can indeed undergo disproportionation in alkaline medium:

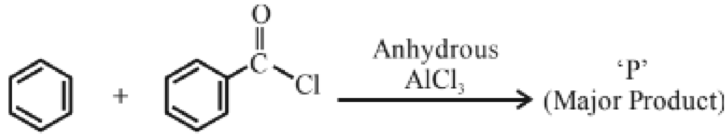
$$S_8 + 12\,\mathrm{OH}^- \rightarrow 4\,S^{2-} + 2\,S_2O_3^{2-} + 6\,H_2O.$$

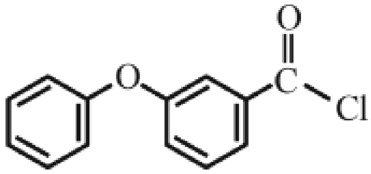
Statement II is incorrect because ClO_4^- cannot undergo disproportionation as chlorine is already in its highest oxidation state.

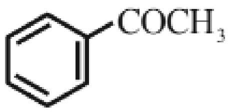
Quick Tip

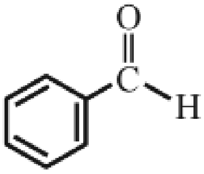
Disproportionation reactions occur when an element in a compound is simultaneously oxidized and reduced, depending on its oxidation state.

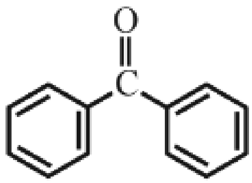
Question 65. Identify major product 'P' formed in the following reaction:



(1) 

(2) 

(3) 

(4) 

Correct Answer: (4)

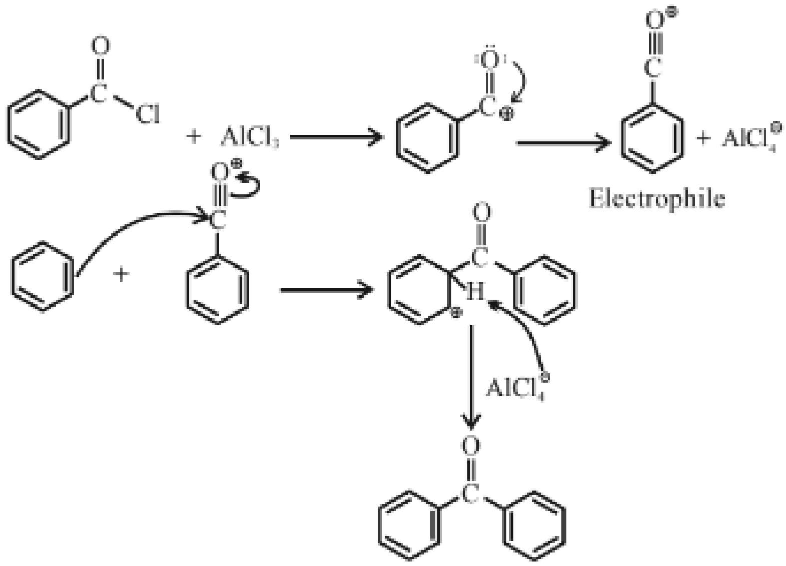
Solution:

The reaction is an example of Friedel-Crafts acylation. The acylium ion (RCO^+) generated by the reaction of acyl chloride with $AlCl_3$ acts as an electrophile and attacks benzene to form the acylated product.

$$*6(-C=O)-C=O$$

Thus, the major product is:

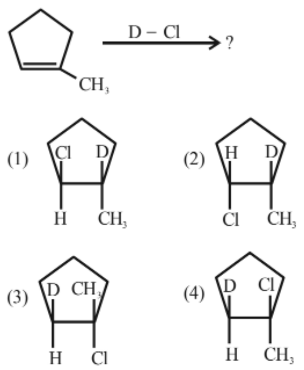
$$\text{P} = \quad *6(-C=O)(-COPh)$$



Quick Tip

In Friedel-Crafts acylation, $AlCl_3$ helps generate a strong electrophile, which facilitates the addition of an acyl group to benzene.

Question 66. Major product of the following reaction is:



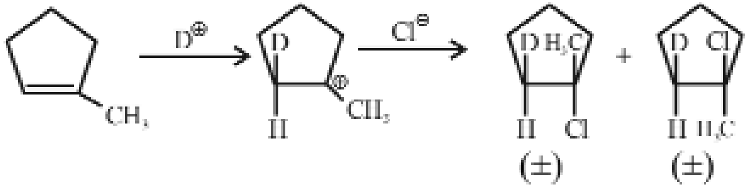
Correct Answer: (3 or 4)

Solution:

The reaction involves the addition of D and Cl across the double bond. This is an example of syn addition, where both D (deuterium) and Cl (chlorine) can add to the double bond in either configuration due to the symmetrical nature of the starting compound.

The products are as follows: 1. D and Cl can add to the same side of the double bond, leading to two stereoisomers depending on the orientation of D and Cl.

The structures for the possible products are shown below:



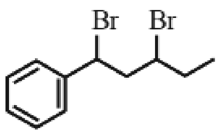
Thus, the major products are:

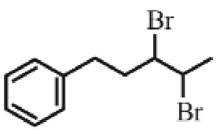
Product 3 and Product 4

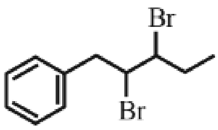
Quick Tip

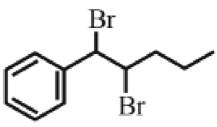
In syn additions, consider both stereoisomers as possible products when a symmetric alkene is involved.

Question 67. Identify the structure of 2,3-dibromo-1-phenylpentane.

1. 

2. 

3. 

4. 

Correct Answer: (3)

Solution:

The IUPAC name 2,3-dibromo-1-phenylpentane indicates that: - There is a phenyl group attached to the first carbon of the pentane chain. - Bromine atoms are attached to the second and third carbons of the chain.

In the options provided, structure 3 matches this description, with a phenyl group at the first carbon and bromine atoms at the second and third carbons.

Thus, the correct structure is:

Structure 3

Quick Tip

When identifying structures from IUPAC names, locate substituents in the given positions of the main chain to ensure correct identification.

Question 68. Select the option with the correct property -

(1) $[Ni(CO)_4]$ and $[NiCl_4]^{2-}$ both diamagnetic

(2) $[Ni(CO)_4]$ and $[NiCl_4]^{2-}$ both paramagnetic

(3) $[NiCl_4]^{2-}$ diamagnetic, $[Ni(CO)_4]$ paramagnetic

(4) $[Ni(CO)_4]$ diamagnetic, $[NiCl_4]^{2-}$ paramagnetic

Correct Answer: (4)

Solution:

In $[Ni(CO)_4]$, nickel is in the zero oxidation state and has a $3d^{10}$ electron configuration, leading to a fully paired electron configuration, making it diamagnetic.

In contrast, in $[NiCl_4]^{2-}$, nickel is in the $+2$ oxidation state with a $3d^8$ electron configuration. The chloride ligands are weak-field ligands and do not pair up the electrons, resulting in unpaired electrons and a paramagnetic nature.

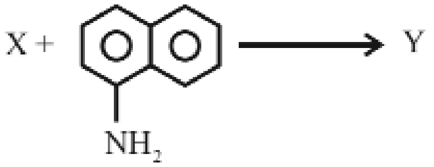
Thus, the correct answer is:

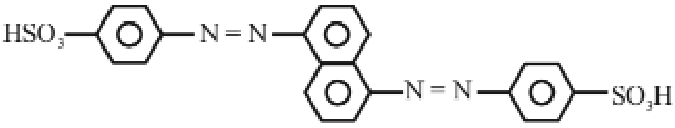
$$[Ni(CO)_4] \text{ is diamagnetic and } [NiCl_4]^{2-} \text{ is paramagnetic.}$$

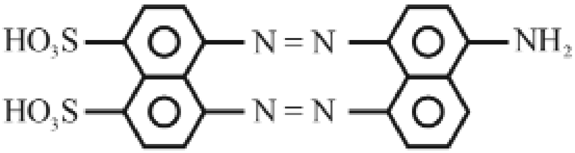
Quick Tip

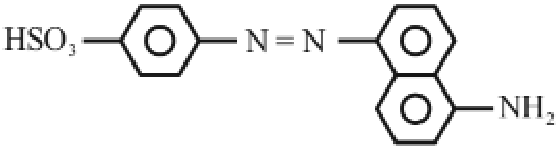
When determining magnetic properties, check the oxidation state and ligand field strength to see if electrons are paired or unpaired.

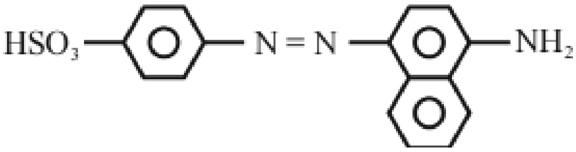
Question 69. The azo-dye (Y) formed in the following reactions is Sulphanilic acid + $NaNO_2$ + $CH_3COOH \rightarrow X$



1. 

2. 

3. 

4. 

Correct Answer: (4)

Solution:

The reaction involves the diazotization of sulphanilic acid, which forms a diazonium salt. This diazonium salt then couples with the compound $C_6H_5NH_2$, leading to the formation of the azo-dye product Y.

The structure of Y matches option (4) in which the diazonium ion is coupled with the benzene ring of $C_6H_5NH_2$, forming the characteristic azo linkage (-N=N-) between the two aromatic rings.

Thus, the correct answer is:

Option (4)

Quick Tip

In azo-dye formation, diazonium salts undergo coupling reactions with electron-rich aromatic compounds, such as anilines or phenols, to form colored azo compounds.

Question 70. Given below are two statements:

Statement I: Aniline reacts with conc. H_2SO_4 followed by heating at 453-473 K gives *p*-aminobenzene sulphonic acid, which gives blood red color in the *Tassaignes's test*.

Statement II: In Friedel-Crafts alkylation and acylation reactions, aniline forms salt with the $AlCl_3$ catalyst. Due to this, nitrogen of aniline acquires a positive charge and acts as a deactivating group.

In the light of the above statements, choose the **correct answer** from the options given below:

1. Statement I is false but Statement II is true
2. Both Statement I and Statement II are false
3. Statement I is true but Statement II is false
4. Both Statement I and Statement II are true

Correct Answer: (4)

Solution:

Statement I is correct because aniline does react with concentrated sulfuric acid and, upon heating, forms *p*-aminobenzene sulfonic acid, which gives a blood red color in Tassaignes's test.

Statement II is also correct as in Friedel-Crafts alkylation and acylation reactions, aniline forms a salt with the $AlCl_3$ catalyst. This interaction results in a positive charge on nitrogen, causing it to act as a deactivating group, making further substitution reactions difficult on the benzene ring.

Thus, both statements are true.

Quick Tip

In electrophilic aromatic substitution reactions, amino groups usually activate the ring unless they form a complex with catalysts like $AlCl_3$, which changes their electronic nature.

Question 71.

$A_{(g)} \rightleftharpoons B_{(g)} + \frac{1}{2}C_{(g)}$ The correct relationship between K_P, α, and equilibrium pressure P is:

(1) $K_P = \frac{\alpha^{1/2}P^{1/2}}{(2+\alpha)^{1/2}}$

(2) $K_P = \frac{\alpha^{3/2}P^{1/2}}{(2+\alpha)^{1/2}(1-\alpha)}$

(3) $K_P = \frac{\alpha^{1/2} P^{3/2}}{(2+\alpha)^{3/2}}$
(4) $K_P = \frac{\alpha^{1/2} P^{1/2}}{(2+\alpha)^{3/2}}$

Correct Answer: (2)

Solution:

Consider the reaction at equilibrium:

$$A_{(g)} \rightleftharpoons B_{(g)} + \frac{1}{2}C_{(g)}$$

At equilibrium, let the fraction dissociated be α. Then:

$$P_A = (1-\alpha)P, \quad P_B = \frac{\alpha}{2+\alpha}P, \quad P_C = \frac{\alpha}{2(2+\alpha)}P$$

The expression for K_P is given by:

$$K_P = \frac{P_B \cdot P_C^{1/2}}{P_A} = \frac{\alpha^{3/2} P^{1/2}}{(2+\alpha)^{1/2}(1-\alpha)}$$

Quick Tip

In equilibrium calculations, always correctly assign partial pressures using the degree of dissociation and the total pressure.

Question 72. Choose the correct statements from the following:

A. All group 16 elements form oxides of general formula EO_2 and EO_3 where $E =$ S, Se, Te, and Po. Both types of oxides are acidic in nature.

B. TeO_2 is an oxidising agent while SO_2 is reducing in nature.

C. The reducing property decreases from H_2S to H_2Te down the group.

D. The ozone molecule contains five lone pairs of electrons.

Choose the correct answer from the options given below:

1. A and D only
2. B and C only
3. C and D only
4. A and B only

Correct Answer: (4)

Solution:

Statement A is true as all group 16 elements form EO_2 and EO_3 oxides, and these oxides are generally acidic.

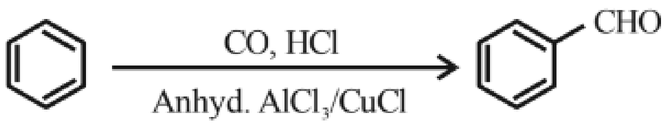
Statement B is true as TeO_2 acts as an oxidising agent, while SO_2 is a reducing agent.

Statement C is false. The reducing nature actually increases as we move down the group.

Statement D is false as the ozone molecule O_3 contains only three lone pairs of electrons on each oxygen atom.

> **Quick Tip**
>
> When analyzing periodic trends, remember that reducing power generally increases down a group as atomic size and electron availability increase.

Question 73. Identify the name reaction.



(1) Stephen reaction

(2) Etard reaction

(3) Gatterman-Koch reaction

(4) Rosenmund reduction

Correct Answer: (3)

Solution:

The reaction shown is the **Gatterman-Koch reaction**, which involves the formation of benzaldehyde from benzene using carbon monoxide (CO) and hydrogen chloride (HCl) in the presence of anhydrous $AlCl_3$ and CuCl as catalysts.

> **Quick Tip**
>
> The Gatterman-Koch reaction is specifically used for the formylation of benzene to produce benzaldehyde using CO and HCl in the presence of $AlCl_3$ and CuCl.

Question 74. Which of the following is least ionic?

(1) $BaCl_2$

(2) AgCl

(3) KCl

(4) $CoCl_2$

Correct Answer: (2)

Solution:

The order of ionic character among the compounds is:

$$AgCl < CoCl_2 < BaCl_2 < KCl$$

The reason is that Ag^+ has a pseudo-inert gas configuration, making AgCl less ionic than the other compounds listed.

Quick Tip

For ionic character comparisons, consider factors such as charge density and electronic configuration, like pseudo-inert gas configurations, which can affect the degree of ionic bonding.

Question 75. The fragrance of flowers is due to the presence of some steam volatile organic compounds called essential oils. These are generally insoluble in water at room temperature but are miscible with water vapour in the vapour phase. A suitable method for the extraction of these oils from the flowers is:

1. Crystallisation
2. Distillation under reduced pressure
3. Distillation
4. Steam distillation

Correct Answer: (4)

Solution:

Essential oils are typically extracted from plant material by **steam distillation**. In this process, the plant material is exposed to steam, which causes the volatile essential oils to vaporize and then be condensed and collected separately.

Quick Tip

Steam distillation is effective for compounds that are steam volatile and immiscible with water. It allows for the extraction of essential oils without decomposition.

Question 76. Given below are two statements:

Statement I: Group 13 trivalent halides get easily hydrolyzed by water due to their covalent nature.

Statement II: $AlCl_3$, upon hydrolysis in acidified aqueous solution, forms octahedral $[Al(H_2O)_6]^{3+}$ ion.

In the light of the above statements, choose the **correct answer** from the options given below:

1. Statement I is true but statement II is false
2. Statement I is false but statement II is true
3. Both statement I and statement II are false
4. Both statement I and statement II are true

Correct Answer: (4)

Solution:

In the trivalent state, most of the compounds are covalent and are hydrolyzed in water. Trichlorides, upon hydrolysis in water, form tetrahedral $[M(OH)_4]^-$ species, and the hybridization state of element M is sp^3.

In the case of aluminum, acidified aqueous solution forms octahedral $[Al(H_2O)_6]^{3+}$ ion.

Quick Tip

Group 13 trivalent halides undergo hydrolysis due to their covalent character, especially in water, leading to complex ion formation.

Question 77. The four quantum numbers for the electron in the outermost orbital of potassium (atomic no. 19) are

(1) $n = 4$, $l = 2$, $m = -1$, $s = +\frac{1}{2}$

(2) $n = 4$, $l = 0$, $m = 0$, $s = +\frac{1}{2}$

(3) $n = 3$, $l = 0$, $m = 1$, $s = +\frac{1}{2}$

(4) $n = 2$, $l = 0$, $m = 0$, $s = +\frac{1}{2}$

Correct Answer: (2)

Solution:

The electron configuration for potassium (K, atomic number 19) is:

$$\text{K} : 1s^2, 2s^2, 2p^6, 3s^2, 3p^6, 4s^1.$$

The outermost orbital is the $4s$ orbital. The quantum numbers for the outermost electron are:

$$n = 4,\ l = 0,\ m = 0,\ s = +\frac{1}{2}.$$

Quick Tip

For identifying the quantum numbers, start with the electron configuration, then determine n, l, m, and s based on the outermost electron.

Question 78. Choose the correct statements from the following:

A. Mn_2O_7 is an oil at room temperature

B. V_2O_4 reacts with acid to give VO_2^{2+}

C. CrO is a basic oxide

D. V_2O_5 does not react with acid

Choose the correct answer from the options given below:

1. A, B and D only
2. A and C only
3. A, B and C only
4. B and C only

Correct Answer: (2)

Solution:

Let's analyze each statement individually:

Statement A: Mn_2O_7 is an oil at room temperature. - Mn_2O_7 (manganese heptoxide) is indeed a dark green oil that is highly unstable and explosive at room temperature. It is a covalent oxide and behaves as an acidic oxide due to its high oxidation state. Therefore, this statement is correct.

Statement B: V_2O_4 reacts with acid to give VO_2^{2+}. - V_2O_4 (vanadium(IV) oxide) is an amphoteric oxide. It can react with acids, undergoing oxidation, to form the VO_2^{2+} ion (vanadyl ion). This reaction occurs as:

$$V_2O_4 + 2H^+ \rightarrow 2VO_2^{2+} + H_2O$$

Therefore, this statement is also correct.

Statement C: CrO is a basic oxide. - CrO (chromium(II) oxide) is a basic oxide. It reacts readily with acids to form salts and water. For example:

$$CrO + 2HCl \rightarrow CrCl_2 + H_2O$$

This confirms that CrO behaves as a basic oxide, so this statement is correct.

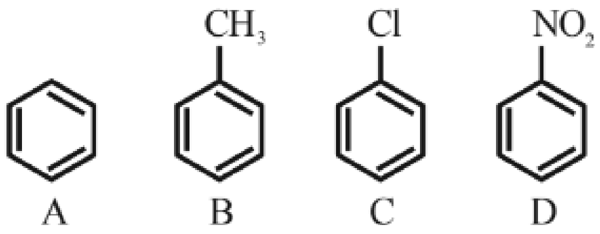
Statement D: V_2O_5 does not react with acid. - V_2O_5 (vanadium(V) oxide) is an amphoteric oxide, meaning it can react with both acids and bases. In fact, V_2O_5 can react with acidic solutions, such as hydrochloric acid, and undergo reduction. Therefore, this statement is incorrect.

Since statements A and C are correct, the correct answer is option (2).

Quick Tip

Amphoteric oxides can react with both acids and bases, while basic oxides only react with acids.

Question 79. The correct order of reactivity in electrophilic substitution reaction of the following compounds is:



1. B $>$ C $>$ A $>$ D
2. D $>$ C $>$ B $>$ A
3. A $>$ B $>$ C $>$ D
4. B $>$ A $>$ C $>$ D

Correct Answer: (4)

Solution:

The effect of substituents on electrophilic substitution reactions is as follows: - $-CH_3$ shows $+M$ and $+I$. - $-Cl$ shows $+M$ and $-I$ but inductive effect dominates. - $-NO_2$ shows $-M$ and $-I$.

Electrophilic substitution order is based on $\alpha + M$ and $+I$, with the highest reactivity for methyl group. Hence, order is $B > A > C > D$.

Quick Tip

When considering reactivity in electrophilic substitution, both resonance and inductive effects of substituents should be considered.

Question 80. Consider the following elements.

Group $\downarrow$ Period $\rightarrow$

A', B'

C', D'

Which of the following is/are true about A', B', C', and D'?

A. Order of atomic radii: $B' < A' < D' < C'$

B. Order of metallic character: $B' < A' < D' < C'$

C. Size of the element: $D' < C' < B' < A'$

D. Order of ionic radii: $B^+ < A^{++} < D^{++} < C^{++}$

Choose the correct answer from the options given below:

1. A only
2. A, B and D only
3. A and B only
4. B, C and D only

Correct Answer: (2)

Solution:

Analysis of each statement:

Statement A: Order of atomic radii $B' < A' < D' < C'$. The atomic radii decrease from left to right in a period and increase from top to bottom in a group. This order is correct.

Statement B: Order of metallic character $B' < A' < D' < C'$. Metallic character decreases from left to right in a period and increases from top to bottom in a group. This order aligns with the general trend, so this statement is correct.

Statement C:Size of the element $D' < C' < B' < A'$. This statement would be correct if we consider ionic sizes and periodic trends, but based on the provided context, this order does not match general atomic radius trends. This statement is incorrect.

Statement D: Order of ionic radii $B^+ < A^{++} < D^{++} < C^{++}$. Ionic radii depend on the effective nuclear charge and the number of electrons lost. This order is correct based on typical ionic sizes across groups and periods.

Thus, the correct answer is:

$$\text{A, B and D only}.$$

Quick Tip

When analyzing atomic and ionic radii trends, remember that atomic radii generally decrease across a period and increase down a group. Metallic character follows similar trends.

Question 81. A diatomic molecule has a dipole moment of 1.2 D. If the bond distance is 1 Å, then the fractional charge on each atom is ____ $\times 10^{-1}$ esu.

Correct Answer: (0)

Solution:

The dipole moment μ is given by:

$$\mu = q \times d$$

Substitute the values:

$$1.2 \times 10^{-10}\,\text{esu} \cdot \text{Å} = q \times 1\,\text{Å}$$

$$q = 1.2 \times 10^{-10}\,\text{esu}$$

Thus, the fractional charge on each atom is:

$$0 \times 10^{-1}\,\text{esu}.$$

Quick Tip

Dipole moment $\mu = q \times d$, where q is the fractional charge and d is the bond length.

Question 82. $r = k[A]$ **for a reaction, 50% of A is decomposed in 120 minutes. The time taken for 90% decomposition of A is ____ minutes.**

Correct Answer: (399)

Solution:

For a first-order reaction:

$$t_{1/2} = 120\,\text{min}$$

For 90% completion:

$$t = \frac{2.303}{k} \log\left(\frac{a}{a-x}\right)$$

$$t = \frac{2.303 \times 120}{0.693} \log\left(\frac{100}{10}\right)$$

$$t = 399\,\text{min}.$$

Quick Tip

For first-order reactions, use $t = \frac{2.303}{k} \log\left(\frac{a}{a-x}\right)$ for completion times.

Question 83. A compound (x) with molar mass 108 g mol^{-1} undergoes acetylation to give a product with molar mass 192 g mol^{-1}. The number of amino groups in the compound (x) is ____.

Correct Answer: (2)

Solution:

Each NH_2 group increases molecular weight by 42 upon acetylation:

$$192 - 108 = 84$$

$$\frac{84}{42} = 2$$

Thus, the compound x has:

$$2 \text{ amino groups.}$$

Quick Tip

For each NH_2 group, the molar mass increases by 42 during acetylation.

Question 84. Number of isomeric products formed by monochlorination of 2-methylbutane in presence of sunlight is ____.

Correct Answer: (6)

Solution:

In the monochlorination of 2-methylbutane, chlorine can replace a hydrogen atom at different positions, leading to the formation of structural and stereoisomeric products. The possible isomeric products are:

$$CH_3 - CH(Cl) - CH_2 - CH(CH_3) - CH_3 \quad (\pm)$$

$$CH_3 - CH(CH_3) - CH_2 - CH(Cl) - CH_3$$

$$CH_3 - C(Cl)(CH_3) - CH_2 - CH_3 \quad (\pm)$$

Therefore, there are 6 isomeric products.

Quick Tip

For monochlorination, count unique carbon positions in the hydrocarbon.

Question 85. Number of moles of H^+ ions required by 1 mole of MnO_4^- to oxidize oxalate ion to CO_2 is ____.

Correct Answer: (8)

Solution:

The balanced reaction is:

$$2MnO_4^- + 5C_2O_4^{2-} + 16H^+ \rightarrow 2Mn^{2+} + 10CO_2 + 8H_2O$$

From the stoichiometry, we see that 16 moles of H^+ are required for 2 moles of MnO_4^-. Therefore, 8 moles of H^+ are needed for each mole of MnO_4^-.

Quick Tip

In redox reactions, balance all elements and charges to find the moles of H^+ ions.

Question 86. In the reaction of potassium dichromate, potassium chloride, and sulfuric acid (conc.), the oxidation state of the chromium in the product is (+) ____.

Correct Answer: (6)

Solution:

The reaction is as follows:

$$K_2Cr_2O_7 + 4KCl + 6H_2SO_4 \rightarrow 2CrO_2Cl_2 + 6KHSO_4 + 3H_2O$$

This reaction is known as the chromyl chloride test. In this reaction, the oxidation state of chromium in CrO_2Cl_2 is +6.

Quick Tip

Use oxidation states and known compound formulas to determine oxidation state changes in reactions.

Question 87. The molarity of 1 L orthophosphoric acid H_3PO_4 having 70% purity by weight (specific gravity 1.54 g/cm^3) is ____ M.

Correct Answer: (11)

Solution:

Molar mass of H_3PO_4 = 98 g/mol

Mass of solution = $1 \times 1000 \times 1.54 = 1540\,\text{g}$

Mass of H_3PO_4 = $0.7 \times 1540 = 1078\,\text{g}$

Moles of H_3PO_4 = $\frac{1078}{98} = 11\,\text{moles}$

Thus, molarity = $11\,\text{M}$.

Quick Tip

For molarity calculations, use the formula: $\text{Molarity} = \frac{\text{Mass of solute/Molar mass}}{\text{Volume of solution in liters}}$.

Question 88. The values of conductivity of some materials at 298.15 K in S m^{-1} are 2.1 $\times$ 10^3, 1.0 $\times$ 10^{-16},1.2 $\times$ 10, 3.91, 1.5 $\times$ 10^{-2}, 1 $\times$ 10^{-7}, 1.0 $\times$ 10^3. The number of conductors among the materials is ____.

Correct Answer: (4)

Solution:

The materials can be categorized as conductors, insulators, and semiconductors based on their conductivity values at 298.15 K.

Conductors generally have conductivities on the order of 10^2 to 10^6 S m^{-1}.

Insulators have very low conductivities, typically around 10^{-10} to 10^{-8} S m^{-1}.

Semiconductors have conductivities ranging from 10^{-4} to 10 S m^{-1}.

Given values:

$$2.1 \times 10^3,\ 1.2 \times 10^3,\ 3.91,\ 1.5 \times 10^{-2},\ 1 \times 10^{-7},\ 1.0 \times 10^3$$

Classifying each:

2.1×10^3, 1.2×10^3, and 1.0×10^3 are in the conductor range.

3.91 also falls within the conductor range (upper range of semiconductors).

1.5×10^{-2} represents a semiconductor.

1×10^{-7} represents an insulator.

Therefore, the number of conductors is:4

Quick Tip

Conductivity ranges help classify materials into conductors, insulators, or semiconductors based on their conductance at a given temperature.

Question 89. From the vitamins A, B_1, B_6, B_{12}, C, D, E, and K, the number of vitamins that can be stored in our body is ____.

Correct Answer: (5)

Solution:

Fat-soluble vitamins can be stored in the body, typically in the liver and adipose tissue, as they are not readily excreted.

The fat-soluble vitamins are:

$$\text{A, D, E, K, and } B_{12}$$

Thus, the number of vitamins that can be stored in the body is: 5

Quick Tip

Fat-soluble vitamins (A, D, E, K, and sometimes B_{12}) can be stored in the body, whereas water-soluble vitamins (B-complex and C) need regular replenishment.

Question 90. If 5 moles of an ideal gas expands from 10 L to a volume of 100 L at 300 K under isothermal and reversible conditions, then work, w, is $-x$ J. The value of x is ____.

Correct Answer: (28721)

Solution:

For an isothermal reversible expansion, the work done W is given by:

$$W = -2.303nRT \log\left(\frac{V_f}{V_i}\right)$$

Given: - $n = 5$ moles - $R = 8.314\,\text{J K}^{-1}\text{mol}^{-1}$ - $T = 300\,\text{K}$ - $V_i = 10\,\text{L}$ - $V_f = 100\,\text{L}$

Substitute into the formula:

$$W = -2.303 \times 5 \times 8.314 \times 300 \times \log\left(\frac{100}{10}\right)$$

$$= -2.303 \times 5 \times 8.314 \times 300 \times \log(10)$$

Since $\log(10) = 1$:

$$W = -2.303 \times 5 \times 8.314 \times 300$$

$$= -28720.713\,\text{J}$$

Rounding to the nearest integer:

$$W = -28721\,\text{J}$$

Thus, $x = 28721$.

Quick Tip

In isothermal reversible expansion, use $W = -2.303nRT\log\left(\frac{V_f}{V_i}\right)$ to calculate work done, noting that logarithmic expansion ratios play a significant role.