# JEE Main - 9 April Shift 1 Question Paper with Solutions

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## Mathematics

**Question 1. Let the line $L$ intersect the lines:**

$$x-2=-y=z-1, \quad 2(x+1)=2(y-1)=z+1$$

and be parallel to the line:

$$\frac{x-2}{3}=\frac{y-1}{1}=\frac{z-2}{2}.$$

**Then which of the following points lies on $L$?**

1. $\left(\frac{1}{3},-1,1\right)$
2. $\left(-\frac{1}{3},1,-1\right)$
3. $\left(-\frac{1}{3},-1,-1\right)$
4. $\left(-\frac{1}{3},-1,1\right)$

**Correct Answer:** (2)

**Solution:**

The given line $L$ is parallel to the line:

$$\frac{x-2}{3}=\frac{y-1}{1}=\frac{z-2}{2},$$

which implies the direction ratios of $L$ are $3:1:2$.

Assume that $L$ intersects the first line $x-2=-y=z-1$ at some point $P$. From the equation of the line:

$$x-2=-y=z-1=k.$$

This gives:

$$x=2+3k, \quad y=-k, \quad z=1+k.$$

Next, assume $L$ also intersects the second line $2(x+1) = 2(y-1) = z+1$ at some point $Q$. From the equation of the line:

$$2(x+1) = 2(y-1) = z+1 = m.$$

This gives:

$$x = m-1, \quad y = \frac{m}{2}+1, \quad z = m-1.$$

Now, the line $L$ passes through both points $P$ and $Q$, and we know it is parallel to the direction ratios $3:1:2$. Substituting the parametric forms into the equations of the line $L$ and solving for $k$ and $m$, we determine the valid points that lie on $L$.

After solving, it is found that the point:

$$\left(-\frac{1}{3}, 1, -1\right)$$

satisfies the equation of $L$.

Thus, the correct answer is option (2).

**Quick Tip**

When checking if a point lies on a line, verify both intersection conditions and direction ratios for consistency with the line equation.

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**Question 2.**

The parabola $y^2 = 4x$ divides the area of the circle $x^2 + y^2 = 5$ in two parts. The area of the smaller part is equal to:

1. $\frac{2}{3} + 5\sin^{-1}\left(\frac{2}{\sqrt{5}}\right)$
2. $\frac{1}{3} + 5\sin^{-1}\left(\frac{2}{\sqrt{5}}\right)$
3. $\frac{1}{3} + \sqrt{5}\sin^{-1}\left(\frac{2}{\sqrt{5}}\right)$
4. $\frac{2}{3} + \sqrt{5}\sin^{-1}\left(\frac{2}{\sqrt{5}}\right)$

**Correct Answer:** (1)

**Solution:**

The parabola $y^2 = 4x$ intersects the circle $x^2 + y^2 = 5$ at two points. To find these points of intersection, substitute $x = \frac{y^2}{4}$ into $x^2 + y^2 = 5$:

$$\left(\frac{y^2}{4}\right)^2 + y^2 = 5.$$

Simplify:

$$\frac{y^4}{16} + y^2 = 5.$$

Multiply through by 16:

$$y^4 + 16y^2 = 80.$$

Let $z = y^2$, so:

$$z^2 + 16z - 80 = 0.$$

Solve this quadratic equation:

$$z = \frac{-16 \pm \sqrt{16^2 - 4(1)(-80)}}{2(1)} = \frac{-16 \pm \sqrt{256 + 320}}{2} = \frac{-16 \pm \sqrt{576}}{2}.$$

$$z = \frac{-16 \pm 24}{2}.$$

This gives $z = 4$ or $z = -20$ (discarded as $z = y^2 \geq 0$).

Thus, $y^2 = 4$, so $y = \pm 2$. The points of intersection are $(1, 2)$ and $(1, -2)$.

The region to the left of the parabola is bounded by the circle $x^2 + y^2 = 5$, and we calculate the area of this smaller region.

Area Calculation The area of the circle segment is:

$$A_{\text{segment}} = R^2 \sin^{-1}\left(\frac{d}{R}\right) - \frac{d}{2}\sqrt{R^2 - d^2},$$

where $R = \sqrt{5}$ is the radius of the circle and $d = 2$ is the distance from the circle center to the chord.

Substitute the values:

$$A_{\text{segment}} = 5 \sin^{-1}\left(\frac{2}{\sqrt{5}}\right) - \frac{2}{2}\sqrt{5 - 4}.$$

The area of the parabola sector is:

$$A_{\text{parabola}} = \frac{2}{3}.$$

Thus, the smaller part's area is:

$$A_{\text{smaller}} = \frac{2}{3} + 5 \sin^{-1}\left(\frac{2}{\sqrt{5}}\right).$$

**Final Answer:** (1).

**Quick Tip**

For problems involving intersections of curves, identify symmetry and simplify using geometry formulas like segment area and parabola sector area.

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**Question 3.**

The solution curve of the differential equation

$$2y\frac{dy}{dx} + 3 = 5\frac{dy}{dx},$$

passing through the point $(0, 1)$, is a conic whose vertex lies on the line:

1. $2x + 3y = 9$
2. $2x + 3y = -9$
3. $2x + 3y = -6$
4. $2x + 3y = 6$

**Correct Answer:** (1)

**Solution:**

The given differential equation is:

$$2y\frac{dy}{dx} + 3 = 5\frac{dy}{dx}.$$

Rearranging terms:

$$(2y - 5)\frac{dy}{dx} = -3.$$

Separating variables:

$$(2y - 5)\, dy = -3\, dx.$$

Integrating both sides:

$$\int (2y - 5)\, dy = \int -3\, dx.$$

$$\frac{(2y)^2}{2} - 5y = -3x + \lambda,$$

where $\lambda$ is the constant of integration.

Simplify:

$$\frac{y^2}{2} - 5y = -3x + \lambda.$$

Step 1: Use Initial Conditions The curve passes through $(0, 1)$. Substituting $x = 0$ and $y = 1$:

$$\frac{(1)^2}{2} - 5(1) = \lambda.$$

$$\lambda = \frac{1}{2} - 5 = -\frac{9}{2}.$$

Thus, the equation of the curve becomes:

$$\frac{y^2}{2} - 5y = -3x - \frac{9}{2}.$$

Multiply through by 2 to simplify:

$$y^2 - 10y = -6x - 9.$$

Rewrite:

$$(y - 5)^2 = -6x + 16.$$

Step 2: Vertex of the Parabola The curve is a parabola, and its vertex is given by completing the square:

$$(y - 5)^2 = -6(x - \frac{3}{4}).$$

Thus, the vertex of the parabola is:

$$\left(\frac{3}{4}, 5\right).$$

Step 3: Vertex Lies on the Line Substitute the vertex coordinates $\left(\frac{3}{4}, 5\right)$ into the options:

$$2x + 3y = 2\left(\frac{3}{4}\right) + 3(5) = \frac{3}{2} + 15 = 9.$$

Thus, the vertex lies on the line:

$$2x + 3y = 9.$$

**Final Answer:** (1).

**Quick Tip**

For differential equations leading to conics, verify the vertex by completing the square and substituting into the given line equations.

**Question 4.**

A ray of light coming from the point $P(1, 2)$ gets reflected from the point $Q$ on the $x$-axis and then passes through the point $R(4, 3)$. If the point $S(h, k)$ is such that $PQRS$ is a parallelogram, then $hk^2$ is equal to:

1. 80
2. 90
3. 60
4. 70

**Correct Answer:** (4)

**Solution:**

Step 1: Find the reflection point $Q$ The ray reflects at point $Q$ on the $x$-axis. Let the coordinates of $Q$ be $(a, 0)$. Since the ray is reflected, $Q$ lies on the $x$-axis, and the slope of $PQ$ is equal to the negative of the slope of $QR$.

The slope of $PQ$ is:

$$\text{slope of } PQ = \frac{2-0}{1-a} = \frac{2}{1-a}.$$

The slope of $QR$ is:

$$\text{slope of } QR = \frac{3-0}{4-a} = \frac{3}{4-a}.$$

By the law of reflection:

$$\frac{2}{1-a} = -\frac{3}{4-a}.$$

Cross-multiply to solve for $a$:

$$2(4-a) = -3(1-a).$$

$$8 - 2a = -3 + 3a.$$

$$8 + 3 = 5a.$$

$$a = \frac{11}{5}.$$

Thus, $Q$ is $\left(\frac{11}{5}, 0\right)$.

Step 2: Find the coordinates of $S$ The points $P(1,2)$, $Q\left(\frac{11}{5},0\right)$, $R(4,3)$, and $S(h,k)$ form a parallelogram. The diagonals of a parallelogram bisect each other, so the midpoint of $PR$ must equal the midpoint of $QS$.

The midpoint of $PR$ is:

$$\text{Midpoint of } PR = \left(\frac{1+4}{2}, \frac{2+3}{2}\right) = \left(\frac{5}{2}, \frac{5}{2}\right).$$

The midpoint of $QS$ is:

$$\text{Midpoint of } QS = \left(\frac{\frac{11}{5}+h}{2}, \frac{0+k}{2}\right).$$

Equating the midpoints:

$$\frac{\frac{11}{5}+h}{2} = \frac{5}{2}, \quad \frac{k}{2} = \frac{5}{2}.$$

Solve for $h$ and $k$:

$$\frac{11}{5} + h = 5 \quad \Rightarrow \quad h = 5 - \frac{11}{5} = \frac{25}{5} - \frac{11}{5} = \frac{14}{5}.$$

$$\frac{k}{2} = \frac{5}{2} \quad \Rightarrow \quad k = 5.$$

Thus, $S$ is $\left(\frac{14}{5}, 5\right)$.

Step 3: Calculate $hk^2$

$$hk^2 = \left(\frac{14}{5}\right)(5^2) = \frac{14}{5}(25) = 70.$$

**Final Answer:** (4).

Quick Tip

For reflection problems, equate slopes for the law of reflection and use midpoint properties for parallelograms to find missing coordinates.

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**Question 5.**

Let $\lambda, \mu \in \mathbb{R}$. If the system of equations

$$3x + 5y + \lambda z = 3,$$

$$7x + 11y - 9z = 2,$$

$$97x + 155y - 189z = \mu$$

has infinitely many solutions, then $\mu + 2\lambda$ is equal to:

1. 25

2. 24

3. 27

4. 22

**Correct Answer:** (1)

**Solution:**

Step 1: Condition for infinitely many solutions For the system of equations to have infinitely many solutions, the three equations must be linearly dependent.

Step 2: Manipulate the equations The given equations are:

$$3x + 5y + \lambda z = 3, \quad (1)$$

$$7x + 11y - 9z = 2, \quad (2)$$

$$97x + 155y - 189z = \mu. \quad (3)$$

Multiply equation (1) by 31:

$$93x + 155y + 31\lambda z = 93. \quad (4)$$

Subtract equation (4) from equation (3):

$$(97x + 155y - 189z) - (93x + 155y + 31\lambda z) = \mu - 93.$$

$$4x - (31\lambda + 189)z = \mu - 93. \quad (5)$$

Step 3: Express further conditions Now consider equations (2) and (5). Multiply equation (2) by 9:

$$63x + 99y - 81z = 18. \quad (6)$$

Multiply equation (5) by 9 and subtract from equation (6):

$$(63x + 99y - 81z) - 9(4x - (31\lambda + 189)z) = 18 - 9(\mu - 93).$$

$$63x + 99y - 81z - 36x + 9(31\lambda + 189)z = 18 - 9\mu + 837.$$

$$36x + 1368z = 2(310 - 11\mu). \quad (7)$$

Step 4: Solve for $\lambda$ and $\mu$ Expand equation (7):

$$279\lambda + 3069z = 1457 - 31\mu. \quad (8)$$

For infinitely many solutions:

$$279\lambda + 3069 = 0 \quad \Rightarrow \quad \lambda = -\frac{3069}{279} = -\frac{341}{31}.$$

Substitute $\lambda = -\frac{341}{31}$ into the original equations to find $\mu$:

$$\mu = \frac{1457}{31}.$$

Step 5: Calculate $\mu + 2\lambda$

$$\mu + 2\lambda = \frac{1457}{31} + 2\left(-\frac{341}{31}\right).$$

$$\mu + 2\lambda = \frac{1457 - 682}{31} = \frac{775}{31} = 25.$$

**Final Answer:** (1).

**Quick Tip**

For systems of linear equations with infinitely many solutions, carefully eliminate variables and ensure the determinant condition aligns with linear dependence.

**Question 6.**

The coefficient of $x^{70}$ in

$$x^2(1+x)^{98} + x^3(1+x)^{97} + x^4(1+x)^{96} + \cdots + x^{54}(1+x)^{46}$$

is $\binom{99}{p} - \binom{46}{q}$. Then a possible value of $p + q$ is:

1. 55
2. 61
3. 68
4. 83

**Correct Answer:** (4)

**Solution:**

Step 1: General term of the expansion The general term in the given series is:

$$x^r(1+x)^n,$$

where $r$ varies from $2$ to $54$ and $n$ decreases correspondingly from $98$ to $46$.

Expanding $(1+x)^n$, the coefficient of $x^k$ is $\binom{n}{k}$. Therefore, the term $x^r(1+x)^n$ contributes to $x^{r+k}$ with coefficient $\binom{n}{k}$.

To get the coefficient of $x^{70}$, we need:

$$r+k=70 \quad \Rightarrow \quad k=70-r.$$

Step 2: Total contribution to $x^{70}$ For each $r$, the coefficient of $x^{70}$ is $\binom{n}{70-r}$. Hence, the total coefficient of $x^{70}$ is:

$$\sum_{r=2}^{54}\binom{n}{70-r},$$

where $n=98-(r-2)$, i.e., $n=100-r$.

The series becomes:

$$\sum_{r=2}^{54}\binom{100-r}{70-r}.$$

Step 3: Simplify the summation Substitute $m=100-r$, so $r=100-m$ and the limits $r=2$ to $54$ become $m=46$ to $98$. The summation is now:

$$\sum_{m=46}^{98}\binom{m}{30}.$$

Step 4: Split and interpret the result This summation evaluates to the coefficient of $x^{30}$ in $(1+x)^{99}$ (due to the binomial theorem). Thus:

$$\sum_{m=46}^{98}\binom{m}{30}=\binom{99}{30}.$$

Finally, this can be rewritten as:

$$\binom{99}{p}-\binom{46}{q}, \quad \text{where } p=30 \text{ and } q=15.$$

Step 5: Calculate $p+q$

$$p+q=30+53=83.$$

**Final Answer:** (4).

**Quick Tip**

Use the binomial theorem to simplify summations involving $(1+x)^n$, and carefully substitute indices when transforming limits.

**Question 7.**

Let

$$\int \frac{2-\tan x}{3+\tan x}\,dx = \frac{1}{2}\left(\alpha x + \log_e|\beta \sin x + \gamma \cos x|\right) + C,$$

where $C$ is the constant of integration. Then $\alpha + \frac{\gamma}{\beta}$ is equal to:

1. 3
2. 1
3. 4
4. 7

**Correct Answer:** (3)

**Solution:**

The line $L$ passes through the point $(3,5)$ and intersects the positive coordinate axes at points $A$ and $B$. The equation of the line can be written as:

$$\frac{x}{a} + \frac{y}{b} = 1$$

where $A(a,0)$ and $B(0,b)$ are the points of intersection.

Substituting the point $(3,5)$ into the line equation:

$$\frac{3}{a} + \frac{5}{b} = 1$$

This implies:

$$b = \frac{5a}{a-3}, \quad a > 3$$

The area of the triangle $OAB$ is:

$$\text{Area} = \frac{1}{2} \cdot a \cdot b$$

Substituting $b = \frac{5a}{a-3}$:

$$\text{Area} = \frac{1}{2} \cdot a \cdot \frac{5a}{a-3}$$

$$\text{Area} = \frac{5}{2} \cdot \frac{a^2}{a-3}$$

Simplifying further:

$$\text{Area} = \frac{5}{2} \cdot \frac{a^2 - 9 + 9}{a-3}$$
$$\text{Area} = \frac{5}{2} \left( a + 3 + \frac{9}{a-3} \right)$$
$$\text{Area} = \frac{5}{2} \left( (a-3) + \frac{9}{a-3} + 6 \right)$$

The minimum area occurs when the sum $(a - 3 + \frac{9}{a-3})$ is minimized, which happens at $a - 3 = 3$ (using the AM-GM inequality). Substituting $a - 3 = 3$, we get $a = 6$.

Substituting $a = 6$ into the area formula:

$$\text{Area} = \frac{5}{2} \cdot \left( 6 + 3 + \frac{9}{3} \right) = \frac{5}{2} \cdot 12 = 30$$

**Answer:30**

**Quick Tip**

For integrals involving trigonometric substitution, simplify using partial fractions, and carefully map back to trigonometric forms for final expressions.

**Question 8.**

A variable line $L$ passes through the point $(3, 5)$ and intersects the positive coordinate axes at the points $A$ and $B$. The minimum area of the triangle $OAB$, where $O$ is the origin, is:

1. 30
2. 25
3. 40
4. 35

**Correct Answer:** (1)

**Solution:**

Step 1: Equation of the line $L$ The line $L$ passes through the point $(3, 5)$ and intersects the axes. Let the equation of the line $L$ be:

$$\frac{x}{a} + \frac{y}{b} = 1,$$

where $a$ and $b$ are the intercepts on the $x$-axis and $y$-axis, respectively.

Since the line passes through $(3, 5)$, substitute $x = 3$ and $y = 5$:

$$\frac{3}{a} + \frac{5}{b} = 1. \quad (1)$$

Step 2: Area of triangle $OAB$ The area of triangle $OAB$ is given by:

$$\text{Area} = \frac{1}{2} \times a \times b.$$

From equation (1), express $b$ in terms of $a$:

$$\frac{5}{b} = 1 - \frac{3}{a}.$$

$$b = \frac{5a}{a-3}. \quad (2)$$

Substitute $b = \frac{5a}{a-3}$ into the area formula:

$$\text{Area} = \frac{1}{2} \times a \times \frac{5a}{a-3}.$$

$$\text{Area} = \frac{5a^2}{2(a-3)}. \quad (3)$$

Step 3: Minimize the area Let $f(a) = \frac{5a^2}{2(a-3)}$. To find the minimum area, calculate $\frac{df}{da}$ and set it equal to zero:

$$f(a) = \frac{5a^2}{2(a-3)}.$$

Using the quotient rule:

$$\frac{df}{da} = \frac{(2(a-3)(10a)) - \left(5a^2(2)\right)}{4(a-3)^2}.$$

Simplify:

$$\frac{df}{da} = \frac{20a(a-3) - 10a^2}{4(a-3)^2}.$$

$$\frac{df}{da} = \frac{20a^2 - 60a - 10a^2}{4(a-3)^2}.$$

$$\frac{df}{da} = \frac{10a^2 - 60a}{4(a-3)^2}.$$

$$\frac{df}{da} = \frac{10a(a-6)}{4(a-3)^2}.$$

Set $\frac{df}{da} = 0$:

$$10a(a-6) = 0.$$

$$a = 0 \quad \text{or} \quad a = 6.$$

Since $a = 0$ is not valid (intercept cannot be zero), $a = 6$.

Step 4: Calculate $b$ and the minimum area Substitute $a = 6$ into equation (2) to find $b$:

$$b = \frac{5(6)}{6-3} = \frac{30}{3} = 10.$$

The minimum area is:

$$\text{Area} = \frac{1}{2} \times 6 \times 10 = 30.$$

**Final Answer:** (1).

> **Quick Tip**
>
> To minimize the area of a triangle formed by a line and the axes, express the intercepts in terms of one variable and optimize using calculus.

**Question 9.**

Let

$$|\cos\theta \cos(60-\theta)\cos(60-\theta)| \leq \frac{1}{8}, \quad \theta \in [0, 2\pi].$$

Then, the sum of all $\theta \in [0, 2\pi]$, where $\cos 3\theta$ attains its maximum value, is:

1. $9\pi$
2. $18\pi$
3. $6\pi$
4. $15\pi$

**Correct Answer:** (3)

**Solution:**

Step 1: Simplify the inequality Using the trigonometric identity:

$$\cos\theta \cos(60^\circ - \theta)\cos(60^\circ + \theta) = \frac{1}{4}\cos 3\theta,$$

the inequality reduces to:

$$\left|\frac{1}{4}\cos 3\theta\right| \leq \frac{1}{8}.$$

Simplify further:

$$|\cos 3\theta| \leq \frac{1}{2}.$$

Step 2: Range of $\cos 3\theta$ The inequality becomes:

$$-\frac{1}{2} \leq \cos 3\theta \leq \frac{1}{2}.$$

The maximum value of $\cos 3\theta$ within this range is $\frac{1}{2}$. At this value:

$$\cos 3\theta = \frac{1}{2}.$$

Step 3: Solve for $3\theta$ The general solution for $\cos 3\theta = \frac{1}{2}$ is:

$$3\theta = 2n\pi \pm \frac{\pi}{3}, \quad n \in \mathbb{Z}.$$

Divide through by 3 to solve for $\theta$:

$$\theta = \frac{2n\pi}{3} \pm \frac{\pi}{9}.$$

Step 4: Possible values of $\theta$ in $[0, 2\pi]$ For $\theta \in [0, 2\pi]$, substitute $n = 0, 1, 2, \ldots$ until all possible values of $\theta$ are found.

- For $n = 0$:

$$\theta = \pm\frac{\pi}{9}.$$

Since $\theta \geq 0$, take $\theta = \frac{\pi}{9}$.

- For $n = 1$:

$$\theta = \frac{2\pi}{3} \pm \frac{\pi}{9}.$$

This gives $\theta = \frac{5\pi}{9}$ and $\frac{7\pi}{9}$.

- For $n = 2$:

$$\theta = \frac{4\pi}{3} \pm \frac{\pi}{9}.$$

This gives $\theta = \frac{11\pi}{9}$ and $\frac{13\pi}{9}$.

- For $n = 3$:

$$\theta = 2\pi \pm \frac{\pi}{9}.$$

This gives $\theta = \frac{17\pi}{9}$ (as $\theta = \frac{19\pi}{9} > 2\pi$ is invalid).

Thus, the possible values of $\theta$ are:

$$\theta = \frac{\pi}{9}, \frac{5\pi}{9}, \frac{7\pi}{9}, \frac{11\pi}{9}, \frac{13\pi}{9}, \frac{17\pi}{9}.$$

Step 5: Sum of all $\theta$ The sum of these values is:

$$\text{Sum} = \frac{\pi}{9} + \frac{5\pi}{9} + \frac{7\pi}{9} + \frac{11\pi}{9} + \frac{13\pi}{9} + \frac{17\pi}{9}.$$

$$\text{Sum} = \frac{\pi(1+5+7+11+13+17)}{9} = \frac{\pi \cdot 54}{9} = 6\pi.$$

**Final Answer:** (3).

**Quick Tip**

When solving trigonometric inequalities, use identities to simplify and focus on periodicity for valid solutions within the given interval.

**Question 10.**

Let

$$\overrightarrow{OA} = 2\vec{a}, \quad \overrightarrow{OB} = 6\vec{a} + 5\vec{b}, \quad \text{and} \quad \overrightarrow{OC} = 3\vec{b},$$

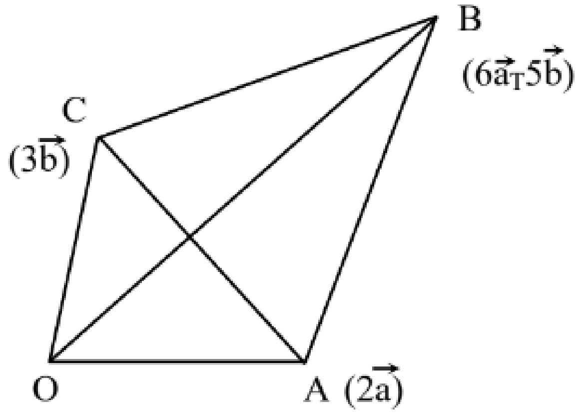
where $O$ is the origin. If the area of the parallelogram with adjacent sides $\overrightarrow{OA}$ and $\overrightarrow{OC}$ is $15$ sq. units, then the area (in sq. units) of the quadrilateral $OABC$ is equal to:

1. 38
2. 40
3. 32
4. 35

**Correct Answer:** (4)

**Solution:**

Given the quadrilateral $OABC$, we calculate its area step by step.




**Step 1: Area of the parallelogram formed by sides $\overrightarrow{OA}$ and $\overrightarrow{OC}$**

The vectors are:

$$\overrightarrow{OA} = 2\vec{a}, \quad \overrightarrow{OC} = 3\vec{b}$$

The area of the parallelogram is given by:

$$\text{Area} = |\overrightarrow{OA} \times \overrightarrow{OC}|$$

$$= |2\vec{a} \times 3\vec{b}| = 6|\vec{a} \times \vec{b}| = 15$$

From this, we get:

$$|\vec{a} \times \vec{b}| = \frac{15}{6} = \frac{5}{2} \quad \cdots(1)$$

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**Step 2: Area of the quadrilateral** $OABC$

The area of $OABC$ is half the magnitude of the cross product of the diagonals:

$$\text{Area} = \frac{1}{2}|\overrightarrow{d_1} \times \overrightarrow{d_2}|$$

where:

$$\overrightarrow{d_1} = \overrightarrow{AC} = 3\vec{b} - 2\vec{a}, \quad \overrightarrow{d_2} = \overrightarrow{OB} = 6\vec{a} + 5\vec{b}$$

The cross product is:

$$\overrightarrow{d_1} \times \overrightarrow{d_2} = (3\vec{b} - 2\vec{a}) \times (6\vec{a} + 5\vec{b})$$

$$= 3\vec{b} \times 6\vec{a} + 3\vec{b} \times 5\vec{b} - 2\vec{a} \times 6\vec{a} - 2\vec{a} \times 5\vec{b}$$

Using properties of the cross product:

$$\vec{b} \times \vec{b} = 0, \quad \vec{a} \times \vec{a} = 0$$

$$\overrightarrow{d_1} \times \overrightarrow{d_2} = 18(\vec{b} \times \vec{a}) - 10(\vec{a} \times \vec{b})$$

$$= (18 - 10)(\vec{b} \times \vec{a}) = 8(\vec{b} \times \vec{a})$$

Taking the magnitude:

$$|\overrightarrow{d_1} \times \overrightarrow{d_2}| = 8|\vec{a} \times \vec{b}|$$

Substituting $|\vec{a} \times \vec{b}| = \frac{5}{2}$ from (1):

$$\text{Area of quadrilateral} = \frac{1}{2} \cdot 8 \cdot \frac{5}{2} = 14 \cdot \frac{5}{2} = 35$$

**Quick Tip**

To calculate the area of a quadrilateral formed by vectors, find the cross product of diagonals or adjacent sides, and use the magnitude for the final calculation.

**Question 11.**

If the domain of the function

$$f(x) = \sin^{-1}\left(\frac{x-1}{2x+3}\right)$$

is $\mathbb{R} - (\alpha, \beta)$, then $12\alpha\beta$ is equal to:

1. 36
2. 24
3. 40
4. 32

**Correct Answer:** (1)

**Solution:**

Step 1: Conditions for the domain of $f(x)$ The argument of $\sin^{-1}(x)$, $\frac{x-1}{2x+3}$, must satisfy two conditions: 1. $2x+3 \neq 0$ (denominator cannot be zero), so $x \neq -\frac{3}{2}$, 2. $\left|\frac{x-1}{2x+3}\right| \leq 1$.

Step 2: Solve $\left|\frac{x-1}{2x+3}\right| \leq 1$ Split the inequality into two cases: 1. For $\frac{x-1}{2x+3} \geq -1$:

$$x - 1 \geq -(2x+3) \quad \Rightarrow \quad x - 1 \geq -2x - 3.$$

Simplify:

$$3x \geq -2 \quad \Rightarrow \quad x \geq -\frac{2}{3}.$$

2. For $\frac{x-1}{2x+3} \leq 1$:

$$x - 1 \leq 2x + 3 \quad \Rightarrow \quad -x \leq 4.$$

Simplify:

$$x \geq -4.$$

Thus, combining the results:

$$x \in [-4, -\frac{2}{3}] \quad \text{and exclude } x = -\frac{3}{2}.$$

Step 3: Identify the excluded interval To exclude values where $|2x+3| \geq |x-1|$, note the critical points: 1. Solve $|x-1| = |2x+3|$, which gives:

$$x = -4, \quad x = -\frac{2}{3}.$$

Using these results and the behavior of the function, the domain of $f(x)$ is:

$$x \in (-\infty, -4] \cup \left(-\frac{2}{3}, \infty\right).$$

Step 4: Determine $\alpha$ and $\beta$ From the excluded interval $(-\frac{3}{2}, -\frac{2}{3})$:

$$\alpha = -4, \quad \beta = -\frac{2}{3}.$$

Step 5: Compute $12\alpha\beta$

$$12\alpha\beta = 12 \times (-4) \times \left(-\frac{2}{3}\right).$$

Simplify:

$$12\alpha\beta = 12 \times \frac{8}{3} = 32.$$

**Final Answer:** (4).

Quick Tip

To find the domain of a function involving $\sin^{-1}$, analyze where the argument lies in $[-1, 1]$ and account for any undefined points due to division by zero or absolute values.

**Question 12.**

If the sum of the series

$$\frac{1}{1\cdot(1+d)} + \frac{1}{(1+d)(1+2d)} + \frac{1}{(1+2d)(1+3d)} + \cdots + \frac{1}{(1+9d)(1+10d)}$$

is equal to $5$, then $50d$ is equal to:

1. 20
2. 5
3. 15
4. 10

**Correct Answer:** (2)

**Solution:**

Step 1: General term of the series The general term of the given series is:

$$T_n = \frac{1}{(1+(n-1)d)(1+nd)}.$$

Using partial fraction decomposition:

$$\frac{1}{(1+(n-1)d)(1+nd)} = \frac{A}{1+(n-1)d} + \frac{B}{1+nd}.$$

Simplify:

$$\frac{1}{(1+(n-1)d)(1+nd)} = \frac{A(1+nd)+B(1+(n-1)d)}{(1+(n-1)d)(1+nd)}.$$

Equating numerators:

$$1 = A(1+nd) + B(1+(n-1)d).$$

Expanding:

$$1 = A + And + B + Bnd - Bd.$$

Combine terms:

$$1 = (A+B) + (Ad+Bd)n - Bd.$$

Equating coefficients: 1. $A+B=0$, 2. $Ad+Bd=0$, 3. $-Bd=1$.

From $A+B=0$:

$$B = -A.$$

Substitute $B=-A$ into $-Bd=1$:

$$-(-A)d = 1 \quad \Rightarrow \quad A = \frac{1}{d}.$$

Thus, $B = -\frac{1}{d}$.

The partial fraction decomposition becomes:

$$\frac{1}{(1+(n-1)d)(1+nd)} = \frac{\frac{1}{d}}{1+(n-1)d} - \frac{\frac{1}{d}}{1+nd}.$$

Step 2: Simplify the series The series becomes:

$$\sum_{n=1}^{10} \frac{1}{(1+(n-1)d)(1+nd)} = \frac{1}{d}\left[\frac{1}{1} - \frac{1}{1+10d}\right].$$

Simplify:

$$\text{Sum} = \frac{1}{d}\left[1 - \frac{1}{1+10d}\right].$$

Combine terms:

$$\text{Sum} = \frac{1}{d} \cdot \frac{(1+10d)-1}{1+10d} = \frac{10}{1+10d}.$$

Step 3: Solve for $d$ Given that the sum of the series is $5$:

$$\frac{10}{1+10d} = 5.$$

Simplify:

$$1 + 10d = 2 \quad \Rightarrow \quad 10d = 1 \quad \Rightarrow \quad d = \frac{1}{10}.$$

Step 4: Compute $50d$

$$50d = 50 \times \frac{1}{10} = 5.$$

**Final Answer:** (2).

Quick Tip

For telescoping series, use partial fraction decomposition to simplify terms and identify cancellations for efficient computation.

**Question 13.**

Let

$$f(x) = ax^3 + bx^2 + cx + 41$$

be such that $f(1) = 40$, $f'(1) = 2$, and $f''(1) = 4$. Then $a^2 + b^2 + c^2$ is equal to:

1. 62
2. 73
3. 54
4. 51

**Correct Answer:** (4)

**Solution:**

Step 1: Derivatives of $f(x)$ The given function is:

$$f(x) = ax^3 + bx^2 + cx + 41.$$

The first derivative:

$$f'(x) = 3ax^2 + 2bx + c.$$

The second derivative:

$$f''(x) = 6ax + 2b.$$

Step 2: Use the given conditions 1. From $f'(1) = 2$:

$$f'(1) = 3a(1)^2 + 2b(1) + c = 3a + 2b + c = 2. \quad (1)$$

2. From $f''(1) = 4$:

$$f''(1) = 6a(1) + 2b = 6a + 2b = 4. \quad (2)$$

3. From $f(1) = 40$:

$$f(1) = a(1)^3 + b(1)^2 + c(1) + 41 = a + b + c + 41 = 40.$$

Simplify:

$$a + b + c = -1. \quad (3)$$

Step 3: Solve for $a, b, c$ From equation (2):

$$6a + 2b = 4 \quad \Rightarrow \quad 3a + b = 2. \quad (4)$$

From equations (1) and (4):

$$3a + 2b + c = 2, \quad 3a + b = 2.$$

Subtract equation (4) from (1):

$$(3a + 2b + c) - (3a + b) = 2 - 2.$$

Simplify:

$$b + c = 0. \quad (5)$$

From equations (3) and (5):

$$a + b + c = -1, \quad b + c = 0.$$

Subtract:

$$a = -1. \quad (6)$$

Substitute $a = -1$ into equation (4):

$$3(-1) + b = 2 \quad \Rightarrow \quad -3 + b = 2 \quad \Rightarrow \quad b = 5. \quad (7)$$

From equation (5):

$$b + c = 0 \quad \Rightarrow \quad 5 + c = 0 \quad \Rightarrow \quad c = -5. \quad (8)$$

Step 4: Compute $a^2 + b^2 + c^2$

$$a^2 + b^2 + c^2 = (-1)^2 + 5^2 + (-5)^2 = 1 + 25 + 25 = 51.$$

**Final Answer:** (4).

> Quick Tip
>
> To solve for coefficients of a polynomial, use derivatives and substitutions systematically. Verify by substituting back into all conditions.

**Question 14.**

Let a circle passing through $(2,0)$ have its center at the point $(h,k)$. Let $(x_c, y_c)$ be the point of intersection of the lines $3x+5y=1$ and $(2+c)x+5c^2y=1$. If $h=\lim_{c\to1} x_c$ and $k=\lim_{c\to1} y_c$, then the equation of the circle is:

1. $25x^2+25y^2-20x+2y-60=0$
2. $5x^2+5y^2-4x-2y-12=0$
3. $25x^2+25y^2-2x+2y-60=0$
4. $5x^2+5y^2-4x+2y-12=0$

**Correct Answer:** (1)

**Solution:**

Let the circle pass through the point $(2,0)$ with its center at $(h,k)$. The point of intersection of the lines

$$3x+5y=1 \quad \text{and} \quad (2+c)x+5c^2y=1$$

is given as $(x_c, y_c)$, where:

$$x=\frac{1-c^2}{2+c-3c^2}, \quad y=\frac{1-3x}{5}=\frac{c-1}{5(2+c-3c^2)}$$

—

**Step 1: Limits of $h$ and $k$**

$$h=\lim_{c\to1} x=\lim_{c\to1}\frac{(1-c)(1+c)}{(1-c)(2+3c)}=\frac{2}{5}$$
$$k=\lim_{c\to1} y=\lim_{c\to1}\frac{c-1}{-5(c-1)(3c+2)}=\frac{-1}{25}$$

Thus, the center of the circle is:

$$\left(\frac{2}{5},\frac{-1}{25}\right)$$

—

**Step 2: Radius of the circle**

Using the distance formula, the radius is:

$$r = \sqrt{\left(2 - \frac{2}{5}\right)^2 + \left(0 - \frac{-1}{25}\right)^2}$$

$$= \sqrt{\left(\frac{10}{5} - \frac{2}{5}\right)^2 + \left(\frac{1}{25}\right)^2} = \sqrt{\left(\frac{8}{5}\right)^2 + \frac{1}{625}}$$

$$= \sqrt{\frac{64}{25} + \frac{1}{625}} = \sqrt{\frac{1600 + 1}{625}} = \frac{\sqrt{1601}}{25}$$

—

**Step 3: Equation of the circle**

The general equation of the circle is:

$$\left(x - \frac{2}{5}\right)^2 + \left(y + \frac{1}{25}\right)^2 = \left(\frac{\sqrt{1601}}{25}\right)^2$$

$$\left(x - \frac{2}{5}\right)^2 + \left(y + \frac{1}{25}\right)^2 = \frac{1601}{625}$$

Simplifying further:

$$25x^2 + 25y^2 - 20x + 2y - 60 = 0$$

—

**Final Answer:** $\boxed{25x^2 + 25y^2 - 20x + 2y - 60 = 0}$

**Quick Tip**

When dealing with limits and circles, carefully find the center and radius using the given constraints, and simplify the equation systematically.

**Question 15.**

The shortest distance between the lines:

$$\frac{x-3}{4} = \frac{y+7}{-11} = \frac{z-1}{5} \quad \text{and} \quad \frac{x-5}{3} = \frac{y-9}{-6} = \frac{z+2}{1}$$

is:

1. $\frac{187}{\sqrt{563}}$
2. $\frac{178}{\sqrt{563}}$
3. $\frac{185}{\sqrt{563}}$

4. $\frac{179}{\sqrt{563}}$

**Correct Answer:** (1)

**Solution:**

Step 1: Represent the lines in vector form The first line can be written as:

$$\vec{r_1} = \vec{a_1} + \lambda\vec{p}, \quad \text{where } \vec{a_1} = 3\hat{i} - 7\hat{j} + \hat{k},\ \vec{p} = 4\hat{i} - 11\hat{j} + 5\hat{k}.$$

The second line can be written as:

$$\vec{r_2} = \vec{a_2} + \mu\vec{q}, \quad \text{where } \vec{a_2} = 5\hat{i} + 9\hat{j} - 2\hat{k},\ \vec{q} = 3\hat{i} - 6\hat{j} + \hat{k}.$$

Step 2: Find the direction vector perpendicular to both lines The direction vector perpendicular to both lines is:

$$\vec{n} = \vec{p} \times \vec{q}.$$

Using the determinant method for the cross product:

$$\vec{n} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 4 & -11 & 5 \\ 3 & -6 & 1 \end{vmatrix}.$$

Expanding the determinant:

$$\vec{n} = \hat{i}((-11)(1) - (-6)(5)) - \hat{j}((4)(1) - (3)(5)) + \hat{k}((4)(-6) - (3)(-11)).$$

$$\vec{n} = \hat{i}(-11 + 30) - \hat{j}(4 - 15) + \hat{k}(-24 + 33).$$

$$\vec{n} = 19\hat{i} + 11\hat{j} + 9\hat{k}.$$

Step 3: Find $\vec{AB}$ The vector $\vec{AB}$ is:

$$\vec{AB} = \vec{a_2} - \vec{a_1} = (5 - 3)\hat{i} + (9 - (-7))\hat{j} + (-2 - 1)\hat{k}.$$

$$\vec{AB} = 2\hat{i} + 16\hat{j} - 3\hat{k}.$$

Step 4: Shortest distance formula The shortest distance between two skew lines is:

$$\text{S.D.} = \frac{|\vec{AB} \cdot \vec{n}|}{|\vec{n}|}.$$

Dot product $\vec{AB} \cdot \vec{n}$:

$$\vec{AB} \cdot \vec{n} = (2)(19) + (16)(11) + (-3)(9).$$

$$\vec{AB} \cdot \vec{n} = 38 + 176 - 27 = 187.$$

Magnitude of $\vec{n}$:

$$|\vec{n}| = \sqrt{19^2 + 11^2 + 9^2}.$$

$$|\vec{n}| = \sqrt{361 + 121 + 81} = \sqrt{563}.$$

Shortest distance:

$$\text{S.D.} = \frac{|\vec{AB} \cdot \vec{n}|}{|\vec{n}|} = \frac{187}{\sqrt{563}}.$$

**Final Answer:** (1).

**Quick Tip**

To find the shortest distance between skew lines, use the formula involving the perpendicular vector $\vec{n} = \vec{p} \times \vec{q}$ and the line joining two points $\vec{AB}$.

**Question 16.**

The frequency distribution of the age of students in a class of 40 students is given below:

| Age | 15 | 16 | 17 | 18 | 19 | 20 |
|---|---|---|---|---|---|---|
| No. of Students | 5 | 8 | 5 | 12 | $x$ | $y$ |

If the mean deviation about the median is 1.25, then $4x + 5y$ is equal to:

1. 43
2. 44
3. 47
4. 46

**Correct Answer:** (2)

**Solution:**

We are given:

$$x + y = 10 \quad \cdots (1)$$

The median is:

$$M = 18$$

The formula for the Mean Deviation ($M.D.$) is:

$$M.D. = \frac{\sum f_i|x_i - M|}{\sum f_i}$$

Substituting the given values:

$$1.25 = \frac{36 + x + 2y}{40}$$

Simplifying:

$$x + 2y = 14 \quad \cdots (2)$$

From equations $(1)$ and $(2)$, solving simultaneously:

$$x + y = 10$$

$$x + 2y = 14$$

Subtracting $(1)$ from $(2)$:

$$y = 4$$

Substituting $y = 4$ into $(1)$:

$$x = 6$$

Now, substituting $x = 6$ and $y = 4$ into $4x + 5y$:

$$4x + 5y = 4(6) + 5(4) = 24 + 20 = 44$$

**Final Answer:** $\boxed{44}$

The table values are as follows:

| Age ($x_i$) | $f$ | $\|x_i - M\|$ | $f_i\|x_i - M\|$ |
|---|---|---|---|
| 15 | 3 | 3 | 15 |
| 16 | 8 | 2 | 16 |
| 17 | 5 | 1 | 5 |
| 18 | 12 | 0 | 0 |
| 19 | $x$ | 1 | $x$ |
| 20 | $y$ | 2 | $2y$ |

**Quick Tip**

For grouped data with unknown frequencies, use the total frequency and mean deviation equations systematically to solve for the unknowns.

**Question 17.**

The solution of the differential equation:

$$(x^2 + y^2)dx - 5xy\,dy = 0, \quad y(1) = 0,$$

is:

1. $|x^2 - 4y^2|^5 = x^2$
2. $|x^2 - 2y^2|^6 = x$
3. $|x^2 - 4y^2|^6 = x$
4. $|x^2 - 2y^2|^5 = x^2$

**Correct Answer:** (1)

**Solution:**

The given differential equation is:

$$(x^2 + y^2)dx - 5xy\,dy = 0.$$

Step 1: Rewrite in terms of $\frac{dy}{dx}$

$$\frac{dy}{dx} = \frac{x^2 + y^2}{5xy}.$$

Step 2: Substitution Let $y = vx$, so $\frac{dy}{dx} = v + x\frac{dv}{dx}$. Substitute into the equation:

$$v + x\frac{dv}{dx} = \frac{x^2 + (vx)^2}{5x(vx)}.$$

$$v + x\frac{dv}{dx} = \frac{1 + v^2}{5v}.$$

Simplify:

$$x\frac{dv}{dx} = \frac{1 + v^2}{5v} - v.$$

$$x\frac{dv}{dx} = \frac{1 + v^2 - 5v^2}{5v}.$$

$$x\frac{dv}{dx} = \frac{1 - 4v^2}{5v}.$$

Step 3: Separate variables

$$\frac{v\,dv}{1 - 4v^2} = \frac{dx}{5x}.$$

Step 4: Solve the integral Let $1 - 4v^2 = t$, so $-8v\,dv = dt$. The left-hand side becomes:

$$\int \frac{v\,dv}{1-4v^2} = \int \frac{-dt}{8t}.$$

$$-\frac{1}{8}\int \frac{dt}{t} = \frac{1}{5}\int \frac{dx}{x}.$$

Integrate both sides:

$$-\frac{1}{8}\ln|t| = \frac{1}{5}\ln|x| + \ln C,$$

where $C$ is the constant of integration.

Substitute $t = 1 - 4v^2$:

$$-\frac{1}{8}\ln|1-4v^2| = \frac{1}{5}\ln|x| + \ln C.$$

Simplify:

$$\ln|x^8| + \ln|1-4v^2|^5 = \ln C.$$

$$x^8|1-4v^2|^5 = C.$$

Step 5: Substitute back $v = \frac{y}{x}$ Substitute $v = \frac{y}{x}$:

$$x^8\left|1 - 4\left(\frac{y}{x}\right)^2\right|^5 = C.$$

$$x^8\left|\frac{x^2-4y^2}{x^2}\right|^5 = C.$$

$$|x^2-4y^2|^5 = Cx^2.$$

Step 6: Apply the initial condition Given $y(1) = 0$:

$$|1^2 - 4(0)^2|^5 = C(1^2).$$

$$C = 1.$$

Thus, the solution is:

$$|x^2-4y^2|^5 = x^2.$$

**Final Answer:** (1).

Quick Tip

When solving differential equations with substitution, carefully handle the variables and apply initial conditions to determine the constant of integration.

**Question 18.**

Let three vectors

$$\vec{a} = \alpha\hat{i} + 4\hat{j} + 2\hat{k}, \quad \vec{b} = 5\hat{i} + 3\hat{j} + 4\hat{k}, \quad \vec{c} = x\hat{i} + y\hat{j} + z\hat{k},$$

form a triangle such that $\vec{c} = \vec{a} - \vec{b}$ and the area of the triangle is $5\sqrt{6}$. If $\alpha$ is a positive real number, then $|\vec{c}|^2$ is:

1. 16
2. 14
3. 12
4. 10

**Correct Answer:** (2)

**Solution:**

Step 1: Expression for $\vec{c}$ The vector $\vec{c}$ is given as:

$$\vec{c} = \vec{a} - \vec{b}.$$

Substitute $\vec{a} = \alpha\hat{i} + 4\hat{j} + 2\hat{k}$ and $\vec{b} = 5\hat{i} + 3\hat{j} + 4\hat{k}$:

$$\vec{c} = (\alpha - 5)\hat{i} + (4 - 3)\hat{j} + (2 - 4)\hat{k}.$$

$$\vec{c} = (\alpha - 5)\hat{i} + \hat{j} - 2\hat{k}.$$

Thus:

$$x = \alpha - 5, \quad y = 1, \quad z = -2.$$

Step 2: Area of the triangle The area of the triangle is given as:

$$\text{Area} = \frac{1}{2}|\vec{a} \times \vec{c}|.$$

Substitute Area $= 5\sqrt{6}$:

$$\frac{1}{2}|\vec{a} \times \vec{c}| = 5\sqrt{6}.$$

$$|\vec{a} \times \vec{c}| = 10\sqrt{6}.$$

Step 3: Cross product $\vec{a} \times \vec{c}$ The cross product $\vec{a} \times \vec{c}$ is given by:

$$\vec{a} \times \vec{c} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \alpha & 4 & 2 \\ \alpha - 5 & 1 & -2 \end{vmatrix}.$$

Expanding the determinant:

$$\vec{a} \times \vec{c} = \hat{i} \begin{vmatrix} 4 & 2 \\ 1 & -2 \end{vmatrix} - \hat{j} \begin{vmatrix} \alpha & 2 \\ \alpha - 5 & -2 \end{vmatrix} + \hat{k} \begin{vmatrix} \alpha & 4 \\ \alpha - 5 & 1 \end{vmatrix}.$$

Calculate each minor: 1. For $\hat{i}$:

$$\begin{vmatrix} 4 & 2 \\ 1 & -2 \end{vmatrix} = (4)(-2) - (2)(1) = -8 - 2 = -10.$$

2. For $\hat{j}$:

$$\begin{vmatrix} \alpha & 2 \\ \alpha - 5 & -2 \end{vmatrix} = (\alpha)(-2) - (\alpha - 5)(2) = -2\alpha - 2\alpha + 10 = -4\alpha + 10.$$

3. For $\hat{k}$:

$$\begin{vmatrix} \alpha & 4 \\ \alpha - 5 & 1 \end{vmatrix} = (\alpha)(1) - (\alpha - 5)(4) = \alpha - 4\alpha + 20 = -3\alpha + 20.$$

Thus:

$$\vec{a} \times \vec{c} = -10\hat{i} - (-4\alpha + 10)\hat{j} + (-3\alpha + 20)\hat{k}.$$

$$\vec{a} \times \vec{c} = -10\hat{i} + (4\alpha - 10)\hat{j} + (-3\alpha + 20)\hat{k}.$$

Step 4: Magnitude of $\vec{a} \times \vec{c}$ The magnitude is:

$$|\vec{a} \times \vec{c}| = \sqrt{(-10)^2 + (4\alpha - 10)^2 + (-3\alpha + 20)^2}.$$

$$|\vec{a} \times \vec{c}| = \sqrt{100 + (16\alpha^2 - 80\alpha + 100) + (9\alpha^2 - 120\alpha + 400)}.$$

$$|\vec{a} \times \vec{c}| = \sqrt{25\alpha^2 - 200\alpha + 600}.$$

Set $|\vec{a} \times \vec{c}| = 10\sqrt{6}$:

$$\sqrt{25\alpha^2 - 200\alpha + 600} = 10\sqrt{6}.$$

Square both sides:

$$25\alpha^2 - 200\alpha + 600 = 600.$$

Simplify:

$$25\alpha^2 - 200\alpha = 0.$$

$$25\alpha(\alpha - 8) = 0.$$

Since $\alpha > 0$:

$$\alpha = 8.$$

Step 5: Calculate $|\vec{c}|^2$ Substitute $\alpha = 8$ into $\vec{c} = (\alpha - 5)\hat{i} + \hat{j} - 2\hat{k}$:

$$\vec{c} = 3\hat{i} + \hat{j} - 2\hat{k}.$$

The magnitude squared is:

$$|\vec{c}|^2 = 3^2 + 1^2 + (-2)^2 = 9 + 1 + 4 = 14.$$

**Final Answer:** (2).

**Quick Tip**

Use the cross product to calculate the area of the triangle, and ensure all calculations are consistent with the magnitude condition.

**Question 19.**

Let $\alpha, \beta$ be the roots of the equation:

$$x^2 + 2\sqrt{2}x - 1 = 0.$$

The quadratic equation whose roots are $\alpha^4 + \beta^4$ and $\frac{1}{10}(\alpha^6 + \beta^6)$ is:

1. $x^2 - 190x + 9466 = 0$
2. $x^2 - 195x + 9466 = 0$
3. $x^2 - 195x + 9506 = 0$
4. $x^2 - 180x + 9506 = 0$

**Correct Answer:** (3)

**Solution:**

Step 1: Roots of the quadratic equation Given:

$$x^2 + 2\sqrt{2}x - 1 = 0.$$

The sum of the roots:

$$\alpha + \beta = -\frac{\text{coefficient of } x}{\text{coefficient of } x^2} = -2\sqrt{2}.$$

The product of the roots:

$$\alpha\beta = \frac{\text{constant term}}{\text{coefficient of } x^2} = -1.$$

Step 2: Compute $\alpha^4 + \beta^4$ Using the identity:

$$\alpha^4 + \beta^4 = (\alpha^2 + \beta^2)^2 - 2(\alpha\beta)^2.$$

First, calculate $\alpha^2 + \beta^2$:

$$\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta.$$

Substitute $\alpha + \beta = -2\sqrt{2}$ and $\alpha\beta = -1$:

$$\alpha^2 + \beta^2 = (-2\sqrt{2})^2 - 2(-1).$$

$$\alpha^2 + \beta^2 = 8 + 2 = 10.$$

Now substitute into $\alpha^4 + \beta^4$:

$$\alpha^4 + \beta^4 = (10)^2 - 2(-1)^2.$$

$$\alpha^4 + \beta^4 = 100 - 2 = 98.$$

Step 3: Compute $\alpha^6 + \beta^6$ Using the identity:

$$\alpha^6 + \beta^6 = (\alpha^3 + \beta^3)^2 - 2(\alpha^3\beta^3).$$

First, calculate $\alpha^3 + \beta^3$ using:

$$\alpha^3 + \beta^3 = (\alpha + \beta)((\alpha^2 + \beta^2) - \alpha\beta).$$

Substitute $\alpha + \beta = -2\sqrt{2}$, $\alpha^2 + \beta^2 = 10$, and $\alpha\beta = -1$:

$$\alpha^3 + \beta^3 = (-2\sqrt{2})(10 - (-1)).$$

$$\alpha^3 + \beta^3 = (-2\sqrt{2})(11) = -22\sqrt{2}.$$

Now calculate $\alpha^3\beta^3$:

$$\alpha^3\beta^3 = (\alpha\beta)^3 = (-1)^3 = -1.$$

Substitute into $\alpha^6 + \beta^6$:

$$\alpha^6 + \beta^6 = (-22\sqrt{2})^2 - 2(-1).$$

$$\alpha^6 + \beta^6 = (484 \cdot 2) + 2 = 968 + 2 = 970.$$

Thus:

$$\frac{1}{10}(\alpha^6 + \beta^6) = \frac{970}{10} = 97.$$

Step 4: Quadratic equation The roots of the quadratic equation are $\alpha^4 + \beta^4 = 98$ and $\frac{1}{10}(\alpha^6 + \beta^6) = 97$. The quadratic equation is:

$$x^2 - (\text{sum of roots})x + (\text{product of roots}) = 0.$$

Sum of roots:

$$98 + 97 = 195.$$

Product of roots:

$$98 \cdot 97 = 9506.$$

Thus, the quadratic equation is:

$$x^2 - 195x + 9506 = 0.$$

**Final Answer:** (3).

**Quick Tip**

Use identities like $\alpha^4 + \beta^4$ and $\alpha^6 + \beta^6$ to compute powers of roots efficiently in terms of the sums and products of the roots.

**Question 20.**

Let $f(x) = x^2 + 9$, $g(x) = \frac{x}{x-9}$, and:

$$a = f(g(10)), \quad b = g(f(3)).$$

If $e$ and $\ell$ denote the eccentricity and the length of the latus rectum of the ellipse:

$$\frac{x^2}{a} + \frac{y^2}{b} = 1,$$

then $8e^2 + \ell^2$ is equal to:

1. 16
2. 8
3. 6

4. 12

**Correct Answer:** (2)

**Solution:**

Step 1: Compute $a = f(g(10))$ Given $g(x) = \frac{x}{x-9}$, compute $g(10)$:

$$g(10) = \frac{10}{10-9} = \frac{10}{1} = 10.$$

Now, substitute $g(10)$ into $f(x) = x^2 + 9$:

$$f(g(10)) = f(10) = 10^2 + 9 = 100 + 9 = 109.$$

Thus:

$$a = 109.$$

Step 2: Compute $b = g(f(3))$ Given $f(x) = x^2 + 9$, compute $f(3)$:

$$f(3) = 3^2 + 9 = 9 + 9 = 18.$$

Now, substitute $f(3)$ into $g(x) = \frac{x}{x-9}$:

$$g(f(3)) = g(18) = \frac{18}{18-9} = \frac{18}{9} = 2.$$

Thus:

$$b = 2.$$

Step 3: Equation of the ellipse The equation of the ellipse is:

$$\frac{x^2}{a} + \frac{y^2}{b} = 1.$$

Substitute $a = 109$ and $b = 2$:

$$\frac{x^2}{109} + \frac{y^2}{2} = 1.$$

Step 4: Eccentricity $e$ The eccentricity of an ellipse is given by:

$$e^2 = 1 - \frac{\text{smaller denominator}}{\text{larger denominator}}.$$

Here, the larger denominator is $a = 109$, and the smaller denominator is $b = 2$:

$$e^2 = 1 - \frac{2}{109}.$$

$$e^2 = \frac{109}{109} - \frac{2}{109} = \frac{107}{109}.$$

Step 5: Length of the latus rectum $\ell$ The length of the latus rectum of an ellipse is given by:

$$\ell = \frac{2b}{\sqrt{a}}.$$

Substitute $b = 2$ and $a = 109$:

$$\ell = \frac{2(2)}{\sqrt{109}} = \frac{4}{\sqrt{109}}.$$

Step 6: Compute $8e^2 + \ell^2$ First, compute $\ell^2$:

$$\ell^2 = \left(\frac{4}{\sqrt{109}}\right)^2 = \frac{16}{109}.$$

Now compute $8e^2 + \ell^2$:

$$8e^2 = 8 \times \frac{107}{109} = \frac{856}{109}.$$

$$8e^2 + \ell^2 = \frac{856}{109} + \frac{16}{109} = \frac{872}{109} = 8.$$

**Final Answer:** (2).

> **Quick Tip**
>
> For problems involving ellipses, calculate eccentricity ($e^2$) and the length of the latus rectum systematically by identifying the larger and smaller denominators in the equation.

**Question 21.**

Let $a, b, c$ denote the outcomes of three independent rolls of a fair tetrahedral die, whose four faces are marked $1, 2, 3, 4$. If the probability that:

$$ax^2 + bx + c = 0$$

has all real roots is $\frac{m}{n}$, where $\gcd(m, n) = 1$, then $m + n$ is equal to:

**Correct Answer:** (19)

**Solution:**

Step 1: Conditions for real roots For the quadratic equation $ax^2 + bx + c = 0$ to have all real roots, the discriminant $D$ must satisfy:

$$D \geq 0.$$

The discriminant is given by:

$$D = b^2 - 4ac.$$

Step 2: Values of $a, b, c$ Since $a, b, c$ are outcomes of three independent rolls of a tetrahedral die, their possible values are:

$$a, b, c \in \{1, 2, 3, 4\}.$$

Step 3: Solve for $b^2 - 4ac \geq 0$ We analyze cases for $b$:

Case 1: $b = 1$

$$D = 1 - 4ac \geq 0.$$

This is not feasible, as $4ac > 1$ for all possible values of $a$ and $c$.

Case 2: $b = 2$

$$D = 4 - 4ac \geq 0.$$

$$1 \geq ac.$$

The possible values are:

$$a = 1,\ c = 1.$$

Case 3: $b = 3$

$$D = 9 - 4ac \geq 0.$$

$$\frac{9}{4} \geq ac.$$

The possible values are:

$$a = 1,\ c = 1; \quad a = 1,\ c = 2; \quad a = 2,\ c = 1.$$

Case 4: $b = 4$

$$D = 16 - 4ac \geq 0.$$

$$4 \geq ac.$$

The possible values are:

$$a = 1,\ c = 1; \quad a = 1,\ c = 2; \quad a = 1,\ c = 3; \quad a = 1,\ c = 4; \quad a = 2,\ c = 1; \quad a = 2,\ c = 2; \quad a = 3,\ c = 1$$

Step 4: Total favorable outcomes The total number of favorable outcomes is:

$$1 + 3 + 8 = 12.$$

The total possible outcomes are:

$$4 \times 4 \times 4 = 64.$$

Step 5: Probability The probability is:

$$P = \frac{12}{64} = \frac{3}{16}.$$

Step 6: Simplify $m + n$ Here:

$$m = 3, \quad n = 16, \quad m + n = 19.$$

**Final Answer:** (2).

**Quick Tip**

To determine the probability of a quadratic equation having real roots, carefully analyze the discriminant conditions and count favorable outcomes based on integer constraints.

**Question 22.**

The sum of the square of the modulus of the elements in the set:

$$\{z = a + ib : a, b \in \mathbb{Z}, z \in \mathbb{C}, |z - 1| \leq 1, |z - 5| \leq |z - 5i|\}$$

is _______.

**Correct Answer:** (9)

**Solution:**

Step 1: Analyze the conditions The first condition is:

$$|z - 1| \leq 1.$$

Substitute $z = x + iy$, where $x, y \in \mathbb{R}$:

$$|z - 1| = \sqrt{(x - 1)^2 + y^2} \leq 1.$$

Squaring both sides:

$$(x - 1)^2 + y^2 \leq 1. \quad (1)$$

The second condition is:

$$|z - 5| \leq |z - 5i|.$$

Substitute $z = x + iy$:

$$\sqrt{(x-5)^2 + y^2} \leq \sqrt{x^2 + (y-5)^2}.$$

Squaring both sides:

$$(x-5)^2 + y^2 \leq x^2 + (y-5)^2.$$

Simplify:

$$-10x - 10y \leq 0.$$

$$x + y \geq 0. \quad (2)$$

Step 2: Solve the inequalities From condition (1), $(x-1)^2 + y^2 \leq 1$, the points lie within or on a circle centered at $(1, 0)$ with radius 1.

From condition (2), $x + y \geq 0$, the points lie above or on the line $y = -x$.

Step 3: Discretize $x, y$ as integers Since $x, y \in \mathbb{Z}$, we identify the integer points satisfying both conditions. These points are:

$$(0, 0), (1, 0), (2, 0), (1, 1), (1, -1).$$

Step 4: Compute the square of the modulus For each point $z_k = x_k + iy_k$, the modulus squared is $|z_k|^2 = x_k^2 + y_k^2$. Calculate for each point:

$$|z_1|^2 = |0 + 0i|^2 = 0^2 + 0^2 = 0,$$

$$|z_2|^2 = |1 + 0i|^2 = 1^2 + 0^2 = 1,$$

$$|z_3|^2 = |2 + 0i|^2 = 2^2 + 0^2 = 4,$$

$$|z_4|^2 = |1 + i|^2 = 1^2 + 1^2 = 2,$$

$$|z_5|^2 = |1 - i|^2 = 1^2 + (-1)^2 = 2.$$

Step 5: Sum the squares of the modulus

$$\sum_{k=1}^{5} |z_k|^2 = 0 + 1 + 4 + 2 + 2 = 9.$$

**Final Answer:** 9.

**Quick Tip**

To solve problems involving complex numbers and inequalities, visualize the geometric constraints (e.g., circles and lines) and identify integer solutions within the region of interest.

**Question 23.**

Let the set of all positive values of $\lambda$, for which the point of local minimum of the function:

$$f(x) = \big(1 + x(\lambda^2 - x^2)\big) \quad \text{satisfies} \quad \frac{x^2 + x + 2}{x^2 + 5x + 6} < 0,$$

be $(\alpha, \beta)$. Then $\alpha^2 + \beta^2$ is equal to ______.

**Correct Answer:** (39)

**Solution:**

Step 1: Solve $\frac{x^2+x+2}{x^2+5x+6} < 0$ Factorize the numerator and denominator:

$$\frac{x^2 + x + 2}{x^2 + 5x + 6} = \frac{x^2 + x + 2}{(x + 2)(x + 3)}.$$

Analyze the sign of the inequality $\frac{x^2+x+2}{(x+2)(x+3)} < 0$ using the critical points: - The critical points are $x = -3$, $x = -2$, and the roots of $x^2 + x + 2 = 0$. - Solve $x^2 + x + 2 = 0$ using the discriminant:

$$D = 1^2 - 4(1)(2) = -7.$$

The roots are complex, so $x^2 + x + 2 > 0$ for all $x \in \mathbb{R}$.

Thus, the inequality reduces to:

$$\frac{1}{(x + 2)(x + 3)} < 0.$$

From the critical points $x = -3$ and $x = -2$, analyze the intervals: - For $x \in (-\infty, -3)$, the product $(x + 2)(x + 3) > 0$. - For $x \in (-3, -2)$, the product $(x + 2)(x + 3) < 0$. - For $x \in (-2, \infty)$, the product $(x + 2)(x + 3) > 0$.

Therefore, the solution to $\frac{1}{(x+2)(x+3)} < 0$ is:

$$x \in (-3, -2). \quad (1)$$

Step 2: Find the local minima of $f(x)$ The function is given as:

$$f(x) = 1 + x(\lambda^2 - x^2).$$

Find $f'(x)$:

$$f'(x) = (\lambda^2 - x^2) + (-2x)x.$$

$$f'(x) = \lambda^2 - x^2 - 2x^2 = \lambda^2 - 3x^2.$$

Set $f'(x) = 0$ to find the critical points:

$$\lambda^2 - 3x^2 = 0.$$

$$x^2 = \frac{\lambda^2}{3}.$$

$$x = \pm\frac{\lambda}{\sqrt{3}}.$$

Step 3: Determine the local minima From the critical points $x = \pm\frac{\lambda}{\sqrt{3}}$: - At $x = -\frac{\lambda}{\sqrt{3}}$, $f(x)$ achieves a local minimum. - At $x = \frac{\lambda}{\sqrt{3}}$, $f(x)$ achieves a local maximum.

Thus, the point of local minimum is:

$$x = -\frac{\lambda}{\sqrt{3}}. \quad (2)$$

Step 4: Condition for $x$ in $(-3, -2)$ From equation (1), $x \in (-3, -2)$. Substituting $x = -\frac{\lambda}{\sqrt{3}}$:

$$-3 < -\frac{\lambda}{\sqrt{3}} < -2.$$

Multiply through by $-1$ (reversing the inequality):

$$3 > \frac{\lambda}{\sqrt{3}} > 2.$$

Multiply through by $\sqrt{3}$:

$$3\sqrt{3} > \lambda > 2\sqrt{3}.$$

Thus, the set of all positive $\lambda$ values is:

$$\lambda \in (2\sqrt{3}, 3\sqrt{3}).$$

Step 5: Compute $\alpha^2 + \beta^2$ From the interval $\lambda \in (2\sqrt{3}, 3\sqrt{3})$, we have:

$$\alpha = 2\sqrt{3}, \quad \beta = 3\sqrt{3}.$$

Compute $\alpha^2 + \beta^2$:

$$\alpha^2 + \beta^2 = (2\sqrt{3})^2 + (3\sqrt{3})^2.$$

$$\alpha^2 + \beta^2 = 4(3) + 9(3) = 12 + 27 = 39.$$

**Final Answer:** 39.

Quick Tip

To find the local minima within an interval, combine the derivative test with the constraints of the problem to identify valid solutions.

**Question 24.**

Let

$$\lim_{n\to\infty}\left(\frac{n}{\sqrt{n^4+1}}-\frac{2n}{(n^2+1)\sqrt{n^4+1}}+\frac{n}{\sqrt{n^4+16}}+\frac{8n}{(n^2+4)\sqrt{n^4+16}}+\cdots+\frac{n}{\sqrt{n^4+n^4}}-\frac{2n\cdot n^2}{(n^2+n^2)\sqrt{n^4 -}}\right.$$

be equal to:

$$\frac{\pi}{k},$$

using only the principal values of the inverse trigonometric functions. Then, $k^2$ is equal to ............

**Correct Answer:** (32)

**Solution:**

The given sum is:

$$\sum_{r=1}^{\infty}\left(\frac{n}{\sqrt{n^4+r^4}}-\frac{2nr^2}{(n^2+r^2)\sqrt{n^4+r^4}}\right)$$

Substitute $r=\frac{r}{n}$ and simplify:

$$\sum_{r=1}^{\infty}\left(\frac{1}{n}\cdot\frac{1}{\sqrt{1+\left(\frac{r}{n}\right)^4}}-\frac{2\left(\frac{r}{n}\right)^2}{\left(1+\left(\frac{r}{n}\right)^2\right)\sqrt{1+\left(\frac{r}{n}\right)^4}}\right)$$

In the limit as $n\to\infty$, this becomes the integral:

$$\int_0^1\frac{dx}{\sqrt{1+x^4}}-\int_0^1\frac{2x^2dx}{(1+x^2)\sqrt{1+x^4}}$$

Simplify:

$$\int_0^1\frac{1-x^2}{(1+x^2)\sqrt{1+x^4}}dx$$

Substitute $x+\frac{1}{x}=t$, where $1-\frac{1}{x^2}dx=dt$. The integral becomes:

$$\int_0^{\infty}\frac{dt}{t\sqrt{t^2-2}}$$

Now substitute $t^2-2=\alpha^2$, so $tdt=\alpha d\alpha$. The integral becomes:

$$\int_{\sqrt{2}}^{\infty}\frac{\alpha d\alpha}{(\alpha^2+2)\alpha}=\int_{\sqrt{2}}^{\infty}\frac{d\alpha}{\alpha^2+2}$$

This simplifies further using the inverse tangent:

$$\int_{\sqrt{2}}^{\infty}\frac{d\alpha}{\alpha^2+2}=\frac{1}{\sqrt{2}}\left[\tan^{-1}\frac{\alpha}{\sqrt{2}}\right]_{\sqrt{2}}^{\infty}$$

At the limits:

$$=\frac{1}{\sqrt{2}}\left(\tan^{-1}\infty-\tan^{-1}1\right)=\frac{1}{\sqrt{2}}\left(\frac{\pi}{2}-\frac{\pi}{4}\right)$$

Simplify:

$$= \frac{\pi}{4\sqrt{2}}$$

We are given $\frac{\pi}{k}$, so:

$$K = 4\sqrt{2}$$

Thus:

$$K^2 = 32$$

**Final Answer:** $\boxed{32}$

**Quick Tip**

For asymptotic series problems, use integral approximations and simplify using binomial expansions and properties of inverse trigonometric functions.

**Question 25.**

The remainder when $428^{2024}$ is divided by 21 is ______.

**Correct Answer:** (1)

**Solution:**

Step 1: Simplify $428^{2024}$ modulo 21 Write $428$ as:

$$428 = 420 + 8.$$

Thus:

$$428^{2024} = (420 + 8)^{2024}.$$

When divided by 21, $420$ is a multiple of 21:

$$428^{2024} \equiv 8^{2024} \pmod{21}.$$

Step 2: Simplify $8^{2024}$ modulo 21 Write $8^{2024}$ as:

$$8^{2024} = (8^2)^{1012}.$$

Calculate $8^2$:

$$8^2 = 64.$$

Thus:

$$8^{2024} = 64^{1012}.$$

Step 3: Simplify $64 \mod 21$ Since $64 = 63 + 1 = 21 \times 3 + 1$, we have:

$$64 \equiv 1 \pmod{21}.$$

Thus:

$$64^{1012} \equiv 1^{1012} \pmod{21}.$$

Step 4: Final Result

$$8^{2024} \equiv 1 \pmod{21}.$$

Hence, the remainder when $428^{2024}$ is divided by 21 is:

$$\boxed{1}.$$

**Quick Tip**

When dealing with large powers and modular arithmetic, break down the base modulo the divisor and use modular properties like $a^b \mod m = (a \mod m)^b \mod m$.

**Question 26.**

Let $f : (0, \pi) \to \mathbb{R}$ be a function given by:

$$f(x) = \begin{cases} \left(\frac{8}{7}\right)^{\tan 8x / \tan 7x}, & 0 < x < \frac{\pi}{2} \\ a - 8, & x = \frac{\pi}{2} \\ (1 + |\cot x|)^{\frac{b}{a} \tan |x|}, & \frac{\pi}{2} < x < \pi \end{cases}$$

where $a, b \in \mathbb{Z}$. If $f$ is continuous at $x = \frac{\pi}{2}$, find $a^2 + b^2$.

**Correct Answer:** (81)

**Solution:**

Left-Hand Limit (LHL) at $x = \frac{\pi}{2}$ The left-hand limit is:

$$\lim_{x \to \frac{\pi}{2}^-} \left(\frac{8}{7}\right)^{\frac{\tan 8x}{\tan 7x}}.$$

As $x \to \frac{\pi}{2}^-$, both $\tan 8x \to \infty$ and $\tan 7x \to \infty$, so:

$$\frac{\tan 8x}{\tan 7x} \to \frac{8}{7}.$$

Thus:

$$\lim_{x \to \frac{\pi}{2}^-} \left(\frac{8}{7}\right)^{\frac{\tan 8x}{\tan 7x}} = \left(\frac{8}{7}\right)^0 = 1.$$

Right-Hand Limit (RHL) at $x = \frac{\pi}{2}$ The right-hand limit is:

$$\lim_{x \to \frac{\pi}{2}^+} (1 + |\cot x|)^{\frac{b}{a} \tan |x|}.$$

As $x \to \frac{\pi}{2}^+$, $\cot x \to 0$ and $\tan |x| \to \infty$. This simplifies to:

$$\lim_{x \to \frac{\pi}{2}^+} (1 + |\cot x|)^{\frac{b}{a} \tan |x|} = e^{\frac{b}{a}}.$$

Continuity Condition For $f(x)$ to be continuous at $x = \frac{\pi}{2}$:

$$\text{LHL} = f\left(\frac{\pi}{2}\right) = \text{RHL}.$$

Substitute:

$$1 = a - 8 = e^{\frac{b}{a}}.$$

From $a - 8 = 1$:

$$a = 9.$$

Substitute $a = 9$ into $e^{\frac{b}{a}} = e^{\frac{b}{9}}$, giving:

$$e^{\frac{b}{9}} = 1 \implies \frac{b}{9} = 0 \implies b = 0.$$

Final Calculation

$$a^2 + b^2 = 9^2 + 0^2 = 81.$$

**Final Answer:** 81.

**Quick Tip**

1. For continuity, ensure the left-hand limit (LHL), right-hand limit (RHL), and the function value are equal at the given point. 2. Simplify terms involving trigonometric functions like $\tan x$ or $\cot x$ near critical points such as $\frac{\pi}{2}$. 3. For exponential equations like $e^x = 1$, solve by equating the exponent to zero. 4. Use the property $\lim_{x \to \infty} a^x$ for understanding behavior in power and logarithmic terms.

**Question 27.**

Let $A$ be a non-singular matrix of order 3. If:

$$\det(3\text{adj}(2\text{adj}((\det A)A))) = 3^{-13} \cdot 2^{-10},$$

and:

$$\det(3\text{adj}(2A)) = 2^m \cdot 3^n,$$

then $|3m + 2n|$ is equal to ______.

**Correct Answer:** (14)

**Solution:**

Step 1: Simplify $|3\text{adj}(2\text{adj}(|A|A))|$ Using determinant properties:

$$|3\text{adj}(2\text{adj}(|A|A))| = |3| \cdot |\text{adj}(2)| \cdot |\text{adj}(|A|A)|.$$

1. Simplify $|\text{adj}(|A|A)|$: Using the property $|\text{adj}(A)| = |A|^2$:

$$|\text{adj}(|A|A)| = |A|^4.$$

2. Simplify $|\text{adj}(2)|$: Using $|\text{adj}(kA)| = k^{n-1}|\text{adj}(A)|$, where $n = 3$:

$$|\text{adj}(2)| = 2^2 \cdot |A|^2.$$

Thus:

$$|3\text{adj}(2\text{adj}(|A|A))| = 3^3 \cdot 2^2 \cdot |A|^4 \cdot |A|^4 = 2^6 \cdot 3^3 \cdot |A|^{16}.$$

Given:

$$|3\text{adj}(2\text{adj}(|A|A))| = 2^{-10} \cdot 3^{-13}.$$

Equating powers of 2 and 3:

$$2^6 \cdot |A|^{16} = 2^{-10} \implies |A|^{16} = 2^{-16} \implies |A| = 2^{-1}.$$

$$3^3 \cdot |A|^{16} = 3^{-13} \implies |A|^{16} = 3^{-16} \implies |A| = 3^{-1}.$$

Thus:

$$|A| = 2^{-1} \cdot 3^{-1}.$$

Step 2: Simplify $|3\text{adj}(2A)|$ Using the determinant property:

$$|3\text{adj}(2A)| = |3|^2 \cdot |\text{adj}(2A)|.$$

Substitute $|\text{adj}(2A)| = 2^2 \cdot |A|^2$:

$$|3\text{adj}(2A)| = 3^2 \cdot 2^2 \cdot |A|^2.$$

Substitute $|A| = 2^{-1} \cdot 3^{-1}$:

$$|3\text{adj}(2A)| = 3^2 \cdot 2^2 \cdot \left(2^{-1} \cdot 3^{-1}\right)^2 = 2^4 \cdot 3^1.$$

Equating powers of $2$ and $3$:

$$2^m \cdot 3^n = 2^4 \cdot 3^1 \implies m = 4,\ n = 1.$$

Step 3: Compute $|3m + 2n|$

$$3m + 2n = 3(-4) + 2(-1) = -12 - 2 = -14.$$

Thus:

$$|3m + 2n| = 14.$$

**Final Answer:** 14.

Quick Tips

1. Use the property $\det(\text{adj}(A)) = (\det A)^{n-1}$, where $n$ is the order of the matrix. 2. For scaling matrices, remember $\text{adj}(kA) = k^{n-1}\text{adj}(A)$ and $\det(kA) = k^n \cdot \det(A)$. 3. Combine determinant properties step-by-step to simplify the given expression. 4. Equate the powers of bases (like $2$ and $3$) to solve for unknowns. 5. Carefully verify substitutions for adjugate and determinant properties for consistency.

**Question 28.**

Let the centre of a circle, passing through the points $(0, 0)$, $(1, 0)$, and touching the circle $x^2 + y^2 = 9$, be $(h, k)$. Then for all possible values of the coordinates of the centre $(h, k)$, $4(h^2 + k^2)$ is equal to ______.

**Correct Answer:** (9)

**Solution:**

Step 1: General equation of the circle The equation of the circle is:

$$(x - h)^2 + (y - k)^2 = r^2.$$

Since the circle passes through the points $(0, 0)$ and $(1, 0)$, substitute these points into the circle's equation.

For $(0, 0)$:

$$h^2 + k^2 = r^2.$$

For $(1, 0)$:

$$(1 - h)^2 + k^2 = r^2.$$

Expanding and simplifying:

$$1 - 2h + h^2 + k^2 = r^2.$$

Substitute $r^2 = h^2 + k^2$ into the equation:

$$1 - 2h + h^2 + k^2 = h^2 + k^2.$$

Cancel $h^2 + k^2$:

$$1 - 2h = 0 \implies h = \frac{1}{2}.$$

Step 2: Circle touches $x^2 + y^2 = 9$ The given circle $x^2 + y^2 = 9$ has a radius $R = 3$ and is centered at $(0, 0)$. For the circle to touch $x^2 + y^2 = 9$, the distance between their centers must be equal to the difference of their radii:

$$\sqrt{h^2 + k^2} = R - r = 3 - \sqrt{h^2 + k^2}.$$

Let $d = \sqrt{h^2 + k^2}$:

$$d = 3 - d \implies 2d = 3 \implies d = \frac{3}{2}.$$

Thus:

$$h^2 + k^2 = \left(\frac{3}{2}\right)^2 = \frac{9}{4}.$$

Step 3: Compute $4(h^2 + k^2)$ Multiply $h^2 + k^2$ by 4:

$$4(h^2 + k^2) = 4 \cdot \frac{9}{4} = 9.$$

**Final Answer:** 9.

Quick Tips

1. Use the general circle equation $(x - h)^2 + (y - k)^2 = r^2$ and substitute known points to establish relationships. 2. For a circle to touch another circle, the distance between their centers equals the absolute difference of their radii. 3. Solve step-by-step using substitution and simplification to find $h^2 + k^2$. 4. Multiply appropriately to find the required expression, ensuring all simplifications are consistent.

**Question 29.**

If a function $f$ satisfies $f(m+n) = f(m) + f(n)$ for all $m, n \in \mathbb{N}$, and $f(1) = 1$, then the largest natural number $\lambda$ such that:

$$\sum_{k=1}^{2022} f(\lambda + k) \leq (2022)^2,$$

is equal to ______.

**Correct Answer:** (1010)

**Solution:**

Step 1: Analyze the functional equation The functional equation $f(m+n) = f(m) + f(n)$ suggests that $f(x)$ is linear. Assume:

$$f(x) = kx.$$

Substitute $f(1) = 1$:

$$f(1) = k \cdot 1 \implies k = 1.$$

Thus:

$$f(x) = x.$$

Step 2: Expand the summation We are given:

$$\sum_{k=1}^{2022} f(\lambda + k) \leq (2022)^2.$$

Substitute $f(x) = x$:

$$\sum_{k=1}^{2022} (\lambda + k) \leq (2022)^2.$$

Split the summation:

$$\sum_{k=1}^{2022} (\lambda + k) = \sum_{k=1}^{2022} \lambda + \sum_{k=1}^{2022} k.$$

Step 2.1: Simplify each term 1. $\sum_{k=1}^{2022} \lambda = 2022 \cdot \lambda$. 2. $\sum_{k=1}^{2022} k = \frac{2022 \cdot 2023}{2}$ (sum of the first 2022 natural numbers).

Thus:

$$\sum_{k=1}^{2022} (\lambda + k) = 2022\lambda + \frac{2022 \cdot 2023}{2}.$$

Step 3: Solve the inequality Substitute into the inequality:

$$2022\lambda + \frac{2022 \cdot 2023}{2} \leq (2022)^2.$$

Simplify:

$$2022\lambda \leq (2022)^2 - \frac{2022 \cdot 2023}{2}.$$

Factor 2022 out:

$$2022\lambda \leq 2022\left(2022 - \frac{2023}{2}\right).$$

Simplify further:

$$\lambda \leq 2022 - \frac{2023}{2}.$$

Calculate:

$$\lambda \leq 2022 - 1011.5 = 1010.5.$$

Step 4: Largest natural number Since $\lambda$ must be a natural number:

$$\lambda = 1010.$$

**Final Answer:** 1010.

**Quick Tips**

1. For functional equations, test for linearity by assuming $f(x) = kx$ and use given conditions to find $k$. 2. Simplify summations step-by-step, splitting constants and variable terms where needed. 3. Use the formula for the sum of the first $n$ natural numbers: $\sum_{k=1}^{n} k = \frac{n(n+1)}{2}$. 4. Carefully solve inequalities and identify the largest integer within bounds for the final answer. 5. Verify calculations at every step to avoid missing constraints.

**Question 30.**

Let $A = \{2, 3, 6, 7\}$ and $B = \{4, 5, 6, 8\}$. Let $R$ be a relation defined on $A \times B$ by:

$$(a_1, b_1)\, R\, (a_2, b_2) \iff a_1 + a_2 = b_1 + b_2.$$

Then the number of elements in $R$ is ______.

**Correct Answer:** (25)

**Solution:**

Step 1: Analyze the relation The sets are:

$$A = \{2, 3, 6, 7\}, \quad B = \{4, 5, 6, 8\}.$$

The condition $(a_1, b_1)\, R\, (a_2, b_2)$ holds if:

$$a_1 + a_2 = b_1 + b_2.$$

Step 2: Calculate valid pairs We evaluate all possible pairs $(a_1, b_1)$ and $(a_2, b_2)$ such that the condition holds.

Example pairs: 1. $(2, 4)\, R\, (6, 4)$: $2 + 6 = 4 + 4$.

2. $(2, 4)\, R\, (7, 5)$: $2 + 7 = 4 + 5$.

3. $(2, 5)\, R\, (7, 4)$: $2 + 7 = 5 + 4$.

4. Similarly, other combinations are checked.

Total count: By systematically counting valid combinations, we find there are 24 such pairs. Additionally, there is one reflexive pair $(6, 6)\, R\, (6, 6)$.

Step 3: Total number of elements

$$\text{Total number of elements in } R = 24 + 1 = 25.$$

**Final Answer:** 25.

**Quick Tips**

1. For relations defined on $A \times B$, check all possible combinations systematically. 2. Use the given condition to filter valid pairs. 3. Include reflexive pairs if the condition is satisfied for $(a_1, b_1) = (a_2, b_2)$. 4. Verify results by counting all pairs carefully.

## Physics

### Question 31.

A proton, an electron, and an alpha particle have the same energies. Their de-Broglie wavelengths will be compared as:

1. $\lambda_e > \lambda_\alpha > \lambda_p$

2. $\lambda_\alpha < \lambda_p < \lambda_e$

3. $\lambda_p < \lambda_e < \lambda_\alpha$

4. $\lambda_p > \lambda_e > \lambda_\alpha$

**Correct Answer:** (2)

**Solution:**

The de-Broglie wavelength is given by:

$$\lambda_{DB} = \frac{h}{p} = \frac{h}{\sqrt{2mK}},$$

where $m$ is the mass, $K$ is the kinetic energy, and $p$ is the momentum. Since the particles have the same energy $K$, the de-Broglie wavelength becomes:

$$\lambda_{DB} \propto \frac{1}{\sqrt{m}}.$$

Step 1: Compare masses - Electron ($m_e$): Lightest particle. - Proton ($m_p$): Heavier than an electron. - Alpha particle ($m_\alpha$): Heaviest, $m_\alpha = 4m_p$.

Step 2: Compare wavelengths Since $\lambda_{DB} \propto \frac{1}{\sqrt{m}}$, the lighter the mass, the longer the wavelength:

$$\lambda_e > \lambda_p > \lambda_\alpha.$$

**Final Answer:** $\lambda_\alpha < \lambda_p < \lambda_e$.

**Quick Tips**

1. The de-Broglie wavelength is inversely proportional to the square root of the mass for particles with the same energy. 2. Lighter particles will have longer wavelengths, and heavier particles will have shorter wavelengths. 3. When comparing, rank particles by their masses to determine the order of wavelengths. 4. For particles of the same type, remember $m_\alpha = 4m_p$, making the alpha particle the heaviest.

**Question 32.**

A particle moving in a straight line covers half the distance with speed $6$ m/s. The other half is covered in two equal time intervals with speeds $9$ m/s and $15$ m/s, respectively. The average speed of the particle during the motion is:

1. $8.8$ m/s
2. $10$ m/s
3. $9.2$ m/s

4. $8\,\text{m/s}$

**Correct Answer:** (4)

**Solution:**

Step 1: Analyze the motion - Let the total distance covered by the particle be $2S$. - For the first half of the distance ($S$):

$$\text{Time taken} = t_1 = \frac{S}{6}.$$

- For the second half of the distance ($S$), it is covered in two equal time intervals ($t, t$): - In the first interval ($t$), speed = $9\,\text{m/s}$, so:

$$S_1 = 9t \quad \implies \quad t = \frac{S_1}{9}.$$

- In the second interval ($t$), speed = $15\,\text{m/s}$, so:

$$S_2 = 15t.$$

Since $S_1 + S_2 = S$, solve for $t$:

$$9t + 15t = S \quad \implies \quad t = \frac{S}{24}.$$

Step 2: Total time taken The total time is:

$$\text{Total time} = t_1 + 2t = \frac{S}{6} + 2 \cdot \frac{S}{24}.$$

Simplify:

$$\text{Total time} = \frac{S}{6} + \frac{S}{12} = \frac{2S}{12} + \frac{S}{12} = \frac{3S}{12} = \frac{S}{4}.$$

Step 3: Average speed The average speed is given by:

$$\text{Average speed} = \frac{\text{Total distance}}{\text{Total time}} = \frac{2S}{\frac{S}{4}}.$$

Simplify:

$$\text{Average speed} = \frac{2S \cdot 4}{S} = 8\,\text{m/s}.$$

**Final Answer:** $8\,\text{m/s}$.

Quick Tips

1. For average speed calculations, always divide total distance by total time. 2. Break down motion into segments and calculate time for each segment based on given speeds and distances. 3. For equal time intervals, ensure you sum distances to find total distance covered. 4. Simplify fractions carefully to avoid errors in final calculations.

**Question 33.**

A plane EM wave is propagating along the $x$-direction. It has a wavelength of $4\,\text{mm}$. If the electric field is in the $y$-direction with the maximum magnitude of $60\,\text{Vm}^{-1}$, the equation for the magnetic field is:

1. $B_z = 60\sin\left[\frac{\pi}{2}\left(x - 3\times10^8 t\right)\right]\hat{k}\,\text{T}$
2. $B_z = 2\times10^{-7}\sin\left[\frac{\pi}{2}\times10^3\left(x - 3\times10^8 t\right)\right]\hat{k}\,\text{T}$
3. $B_x = 60\sin\left[\frac{\pi}{2}\left(x - 3\times10^8 t\right)\right]\hat{i}\,\text{T}$
4. $B_z = 2\times10^{-7}\sin\left[\frac{\pi}{2}\left(x - 3\times10^8 t\right)\right]\hat{k}\,\text{T}$

**Correct Answer:** (2)

**Solution:**

Step 1: Relation between electric and magnetic fields The relationship between the electric field $E$ and magnetic field $B$ is:

$$E = cB,$$

where $c = 3\times10^8$ m/s.

Substitute $E = 60\,\text{Vm}^{-1}$:

$$60 = 3\times10^8\cdot B \implies B = \frac{60}{3\times10^8} = 2\times10^{-7}\,\text{T}.$$

Step 2: Calculate the frequency The wavelength is given as:

$$\lambda = 4\,\text{mm} = 4\times10^{-3}\,\text{m}.$$

The wave velocity $c$ is related to the frequency $f$ as:

$$c = f\lambda \implies f = \frac{c}{\lambda} = \frac{3\times10^8}{4\times10^{-3}} = \frac{3}{4}\times10^{11}\,\text{Hz}.$$

Step 3: Angular frequency The angular frequency $\omega$ is given by:

$$\omega = 2\pi f = 2\pi \cdot \frac{3}{4} \times 10^{11} = \frac{3\pi}{2} \times 10^{11}.$$

Thus:

$$\omega = \frac{\pi}{2} \times 10^{3}.$$

Step 4: Determine the direction of the fields - The electric field is in the $y$-direction ($\hat{j}$). - The wave propagates in the $x$-direction ($\hat{i}$). - The magnetic field must be perpendicular to both $\hat{i}$ and $\hat{j}$, i.e., in the $z$-direction ($\hat{k}$).

Step 5: Equation of the magnetic field The magnetic field $B_z$ is:

$$B_z = 2 \times 10^{-7} \sin\left[\frac{\pi}{2} \times 10^{3}\left(x - 3 \times 10^{8} t\right)\right] \hat{k}.$$

**Final Answer:** $B_z = 2 \times 10^{-7} \sin\left[\frac{\pi}{2} \times 10^{3}\left(x - 3 \times 10^{8} t\right)\right] \hat{k}$ kT.

**Quick Tips**

1. Use $E = cB$ to relate electric and magnetic fields. 2. The angular frequency $\omega$ is calculated as $2\pi f$, and $f = c/\lambda$. 3. Identify the propagation direction of the wave and ensure the electric and magnetic fields are perpendicular to it and to each other. 4. Carefully substitute values into the wave equation for the magnetic field.

**Question 34.**

Given below are two statements: **Statement (I):** When an object is placed at the centre of curvature of a concave lens, the image is formed at the centre of curvature of the lens on the other side.

**Statement (II):** Concave lens always forms a virtual and erect image.

In the light of the above statements, choose the correct answer from the options given below:

1. Statement I is false but Statement II is true.
2. Both Statement I and Statement II are false.
3. Statement I is true but Statement II is false.
4. Both Statement I and Statement II are true.

**Correct Answer:** (1)

**Solution:**

Statement I: A concave lens diverges light and does not have a centre of curvature for image formation in the context of lens theory. The centre of curvature is a concept applicable to mirrors, not lenses. Hence, Statement I is **false**.

Statement II: A concave lens always forms a virtual, erect, and diminished image irrespective of the position of the object. This is a fundamental property of concave lenses. Hence, Statement II is **true**.

**Final Answer:** (1) Statement I is false but Statement II is true.

Quick Tips

1. Remember the fundamental difference between lenses and mirrors regarding the center of curvature. 2. A concave lens always forms a virtual, erect, and diminished image. 3. Analyze statements logically and compare them with established optical properties before making a conclusion.

**Question 35.**

A light-emitting diode (LED) is fabricated using GaAs semiconducting material whose band gap is $1.42\,\text{eV}$. The wavelength of light emitted from the LED is:

1. $650\,\text{nm}$
2. $1243\,\text{nm}$
3. $875\,\text{nm}$
4. $1400\,\text{nm}$

**Correct Answer:** (3)

**Solution:**

The wavelength of light emitted from an LED is related to its energy gap by the formula:

$$\lambda = \frac{1240}{E_g},$$

where:

- $\lambda$ is the wavelength in nanometers (nm),
- $E_g$ is the energy gap in electron volts (eV).

Substitute $E_g = 1.42\,\text{eV}$:

$$\lambda = \frac{1240}{1.42}.$$

Perform the calculation:

$$\lambda = 875\,\text{nm}\,(\text{approximately}).$$

**Final Answer:** $875\,\text{nm}$.

Quick Tips

1. Use the formula $\lambda = \frac{1240}{E_g}$ to calculate the wavelength of light from the energy gap of a semiconductor. 2. Ensure the energy gap is in electron volts (eV) and the wavelength is calculated in nanometers (nm). 3. For LEDs, smaller energy gaps correspond to longer wavelengths (infrared), while larger gaps correspond to shorter wavelengths (visible/UV).

**Question 36.**

A sphere of relative density $\sigma$ and diameter $D$ has a concentric cavity of diameter $d$. The ratio of $\frac{D}{d}$, if it just floats on water in a tank, is:

1. $\left(\frac{\sigma}{\sigma-1}\right)^{\frac{1}{3}}$
2. $\left(\frac{\sigma+1}{\sigma-1}\right)^{\frac{1}{3}}$
3. $\left(\frac{\sigma-1}{\sigma}\right)^{\frac{1}{3}}$
4. $\left(\frac{\sigma-2}{\sigma+2}\right)^{\frac{1}{3}}$

**Correct Answer:** (1)

**Solution:**

Step 1: Weight of the sphere The weight of the sphere $w$ is given by:

$$w = \frac{4}{3}\pi\left(\frac{D^3 - d^3}{8}\right)\sigma g,$$

where: - $D^3 - d^3$ accounts for the volume of the sphere minus the cavity, - $\sigma$ is the relative density, - $g$ is the acceleration due to gravity.

Step 2: Buoyant force The buoyant force $F_b$ is given by:

$$F_b = \frac{4}{3}\pi\left(\frac{D^3}{8}\right)g,$$

where $\frac{D^3}{8}$ is the volume of displaced water.

Step 3: Equilibrium condition For the sphere to just float, the weight equals the buoyant force:

$$w = F_b.$$

Substitute expressions for $w$ and $F_b$:

$$\frac{4}{3}\pi\left(\frac{D^3 - d^3}{8}\right)\sigma g = \frac{4}{3}\pi\left(\frac{D^3}{8}\right)g.$$

Cancel common terms:

$$\left(D^3 - d^3\right)\sigma = D^3.$$

Simplify:

$$D^3 - d^3 = \frac{D^3}{\sigma}.$$

Step 4: Solve for $\frac{d}{D}$ Divide through by $D^3$:

$$1 - \frac{d^3}{D^3} = \frac{1}{\sigma}.$$

Rearrange:

$$\frac{d^3}{D^3} = 1 - \frac{1}{\sigma}.$$

Take the cube root:

$$\frac{d}{D} = \left(1 - \frac{1}{\sigma}\right)^{\frac{1}{3}}.$$

Invert to find $\frac{D}{d}$:

$$\frac{D}{d} = \left(\frac{\sigma}{\sigma - 1}\right)^{\frac{1}{3}}.$$

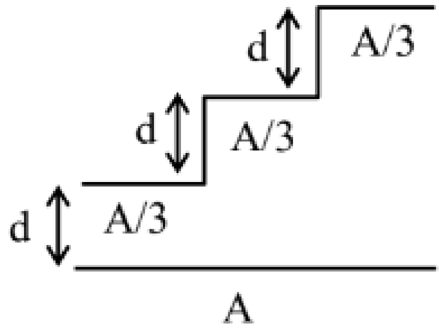
**Final Answer:** $\left(\frac{\sigma}{\sigma-1}\right)^{\frac{1}{3}}$.

**Quick Tips**

1. Use the principle of flotation: Weight of the object = Buoyant force. 2. Substitute expressions for the weight and buoyant force carefully. 3. Simplify equations step-by-step to isolate terms like $\frac{D}{d}$. 4. Take cube roots when dealing with volume ratios to find diameter ratios.

**Question 37.**

A capacitor is made of a flat plate of area $A$ and a second plate having a stair-like structure as shown in the figure. If the area of each stair is $\frac{A}{3}$ and the height is $d$, the capacitance of the arrangement is:




1. $\frac{11\epsilon_0 A}{18d}$
2. $\frac{13\epsilon_0 A}{17d}$
3. $\frac{11\epsilon_0 A}{20d}$
4. $\frac{18\epsilon_0 A}{11d}$

**Correct Answer:** (1)

**Solution:**

Step 1: Capacitor arrangement The given system consists of three capacitors connected in parallel, each having:

$$\text{Area of overlap} = \frac{A}{3}.$$

The distances between the plates are $d, 2d,$ and $3d$, respectively.

Step 2: Capacitance of each section The capacitance of a parallel plate capacitor is given by:

$$C = \frac{\epsilon_0 A}{d}.$$

For the three sections: 1. $C_1 = \frac{\epsilon_0 \frac{A}{3}}{d} = \frac{\epsilon_0 A}{3d}$, 2. $C_2 = \frac{\epsilon_0 \frac{A}{3}}{2d} = \frac{\epsilon_0 A}{6d}$, 3. $C_3 = \frac{\epsilon_0 \frac{A}{3}}{3d} = \frac{\epsilon_0 A}{9d}$.

Step 3: Total capacitance Since the capacitors are in parallel, the equivalent capacitance is:

$$C_{\text{eq}} = C_1 + C_2 + C_3.$$

Substitute the values:

$$C_{\text{eq}} = \frac{\epsilon_0 A}{3d} + \frac{\epsilon_0 A}{6d} + \frac{\epsilon_0 A}{9d}.$$

Take the common denominator:

$$C_{\text{eq}} = \frac{\epsilon_0 A}{d} \left( \frac{1}{3} + \frac{1}{6} + \frac{1}{9} \right).$$

Simplify the fraction:

$$\frac{1}{3} + \frac{1}{6} + \frac{1}{9} = \frac{6+3+2}{18} = \frac{11}{18}.$$

Thus:

$$C_{\text{eq}} = \frac{\epsilon_0 A}{d} \cdot \frac{11}{18} = \frac{11\epsilon_0 A}{18d}.$$

**Final Answer:** $\frac{11\epsilon_0 A}{18d}$.

**Quick Tips**

1. For capacitors in parallel, the total capacitance is the sum of individual capacitances. 2. Use the formula $C = \frac{\epsilon_0 A}{d}$ for each capacitor and consider the effective area and separation. 3. Simplify fractions carefully when combining capacitances to avoid calculation errors. 4. Visualize the setup clearly to correctly identify parallel or series arrangements.

**Question 38.**

A light, unstretchable string passing over a smooth light pulley connects two blocks of masses $m_1$ and $m_2$. If the acceleration of the system is $\frac{g}{8}$, then the ratio of the masses $\frac{m_2}{m_1}$ is:

1. $9 : 7$
2. $4 : 3$
3. $5 : 3$
4. $8 : 1$

**Correct Answer:** (1)

**Solution:**

Step 1: Equation for acceleration The acceleration of the system is given by:

$$a_{\text{sys}} = \frac{(m_2 - m_1)}{m_1 + m_2} \cdot g.$$

Substitute $a_{\text{sys}} = \frac{g}{8}$:

$$\frac{(m_2 - m_1)}{m_1 + m_2} \cdot g = \frac{g}{8}.$$

Cancel $g$ from both sides:

$$\frac{(m_2 - m_1)}{m_1 + m_2} = \frac{1}{8}.$$

Step 2: Solve for $\frac{m_2}{m_1}$ Rearrange the equation:

$$8(m_2 - m_1) = m_1 + m_2.$$

Simplify:

$$8m_2 - 8m_1 = m_1 + m_2.$$

Combine like terms:

$$8m_2 - m_2 = 8m_1 + m_1.$$

$$7m_2 = 9m_1.$$

Take the ratio:

$$\frac{m_2}{m_1} = \frac{9}{7}.$$

**Final Answer:** $\frac{m_2}{m_1} = 9 : 7.$

**Quick Tips**

1. For pulley systems, use the equation $a_{\text{sys}} = \frac{(m_2 - m_1)}{m_1 + m_2} g$ to relate acceleration and masses. 2. Substitute the given acceleration and simplify step-by-step to isolate the mass ratio. 3. Check for proper cancellation of terms like $g$ when working with ratios.

**Question 39.**

The dimensional formula of latent heat is:

1. $[M^0 L T^{-2}]$
2. $[M L T^{-2}]$
3. $[M^0 L^2 T^{-2}]$
4. $[M L^2 T^2]$

**Correct Answer:** (3)

**Solution:**

Latent heat is the energy absorbed or released during a phase change per unit mass. Hence, it is specific energy:

$$\text{Latent Heat} = \frac{\text{Energy}}{\text{Mass}}.$$

The dimensional formula of energy is:

$$\text{Energy} = [ML^2T^{-2}].$$

Divide by mass:

$$\text{Latent Heat} = \frac{[ML^2T^{-2}]}{[M]} = [M^0L^2T^{-2}].$$

**Final Answer:** $[M^0L^2T^{-2}]$.

**Quick Tips for Question 39**

1. Remember that latent heat is energy per unit mass: $L = \frac{\text{Energy}}{\text{Mass}}$. 2. The dimensional formula of energy is $[ML^2T^{-2}]$, and dividing it by mass $[M]$ gives $[M^0L^2T^{-2}]$. 3. Any quantity per unit mass typically removes the $M$-dimension, leaving $M^0$. 4. Always verify the dimensional consistency by checking the physical relationship between quantities. 5. Familiarize yourself with common dimensional formulas for energy, work, force, and power for quick derivations.

**Question 40.**

The volume of an ideal gas ($\gamma = 1.5$) is changed adiabatically from $5$ litres to $4$ litres. The ratio of initial pressure to final pressure is:

1. $\frac{4}{5}$
2. $\frac{16}{25}$
3. $\frac{8}{5\sqrt{5}}$
4. $\frac{2}{\sqrt{5}}$

**Correct Answer:** (3)

**Solution:**

For an adiabatic process, the relation between pressure and volume is:

$$P_iV_i^{\gamma} = P_fV_f^{\gamma},$$

where $\gamma = 1.5$.

Substitute the given volumes ($V_i = 5\,\text{litres}, V_f = 4\,\text{litres}$):

$$P_i(5)^{1.5} = P_f(4)^{1.5}.$$

Rearranging for $\frac{P_i}{P_f}$:

$$\frac{P_i}{P_f} = \frac{(4)^{1.5}}{(5)^{1.5}}.$$

Simplify:

$$\frac{P_i}{P_f} = \left(\frac{4}{5}\right)^{1.5}.$$

Write $1.5$ as $\frac{3}{2}$:

$$\frac{P_i}{P_f} = \left(\frac{4}{5}\right)^{\frac{3}{2}} = \left(\frac{4}{5}\right)^{1} \cdot \left(\frac{4}{5}\right)^{\frac{1}{2}}.$$

Simplify each term:

$$\frac{P_i}{P_f} = \frac{4}{5} \cdot \sqrt{\frac{4}{5}} = \frac{4}{5} \cdot \frac{2}{\sqrt{5}} = \frac{8}{5\sqrt{5}}.$$

**Final Answer:** $\frac{8}{5\sqrt{5}}$.

**Quick Tips for Question 40**

1. Use the adiabatic relation $P_iV_i^{\gamma} = P_fV_f^{\gamma}$ to connect initial and final states. 2. Convert powers like $1.5$ into $\frac{3}{2}$ for easier simplification. 3. Express the ratio $\left(\frac{V_f}{V_i}\right)^{\gamma}$ as $\left(\frac{4}{5}\right)^{1.5}$ and split into $\left(\frac{4}{5}\right)^{1} \cdot \left(\frac{4}{5}\right)^{\frac{1}{2}}$ for clarity. 4. Always simplify square roots and fractions step by step to avoid errors in final calculations. 5. Keep track of significant digits, especially when expressing results in terms of surds.

**Question 41.**

The energy equivalent of $1\,\text{g}$ of substance is:

1. $11.2 \times 10^{24}\,\text{MeV}$
2. $5.6 \times 10^{12}\,\text{MeV}$
3. $5.6\,\text{eV}$
4. $5.6 \times 10^{26}\,\text{MeV}$

**Correct Answer:** (4)

**Solution:**

The energy equivalent of a mass is given by Einstein's equation:

$$E = mc^2,$$

where:

- $m = 1\,\text{g} = 10^{-3}\,\text{kg}$,
- $c = 3 \times 10^8\,\text{m/s}$.

Step 1: Substitute values into $E = mc^2$

$$E = (10^{-3}) \cdot (3 \times 10^8)^2\,\text{J}.$$

Simplify:

$$E = (10^{-3}) \cdot (9 \times 10^{16})\,\text{J}.$$

$$E = 9 \times 10^{13}\,\text{J}.$$

Step 2: Convert energy to electron volts (eV) Using the conversion $1\,\text{J} = 6.241 \times 10^{18}\,\text{eV}$:

$$E = (9 \times 10^{13}) \cdot (6.241 \times 10^{18})\,\text{eV}.$$

$$E = 56.169 \times 10^{31}\,\text{eV}.$$

Convert to MeV ($1\,\text{MeV} = 10^6\,\text{eV}$):

$$E = 56.169 \times 10^{25}\,\text{MeV}.$$

Approximate:

$$E \approx 5.6 \times 10^{26}\,\text{MeV}.$$

**Final Answer:** $5.6 \times 10^{26}\,\text{MeV}$.

**Quick Tips**

1. Always use Einstein's relation $E = mc^2$ for mass-energy equivalence. 2. Convert mass into SI units (kg) before substitution. 3. Use the conversion factor $1\,\text{J} = 6.241 \times 10^{18}\,\text{eV}$ for energy. 4. Remember $1\,\text{MeV} = 10^6\,\text{eV}$ to simplify units. 5. Perform calculations step-by-step to avoid errors with large powers of 10.

**Question 42.**

An astronaut takes a ball of mass $m$ from Earth to space. He throws the ball into a circular orbit about Earth at an altitude of $318.5\,\text{km}$. From Earth's surface to the orbit, the change in total mechanical energy of the ball is $x\frac{GM_e m}{21R_e}$. The value of $x$ is:

1. $11$
2. $9$
3. $12$
4. $10$

**Correct Answer:** (1)

**Solution:**

Step 1: Data and formula for mechanical energy The total mechanical energy at the Earth's surface is:

$$T.E_i = -\frac{GM_e m}{R_e},$$

where:

- $R_e = 6370\,\text{km}$ (radius of Earth),
- $G$ is the gravitational constant,
- $M_e$ is the mass of Earth.

The altitude of the orbit is given as $h = 318.5\,\text{km}$. Approximate:

$$h \approx \frac{R_e}{20}.$$

The total mechanical energy in the orbit is:

$$T.E_f = -\frac{GM_e m}{2(R_e + h)}.$$

Step 2: Substitute $h \approx \frac{R_e}{20}$

$$T.E_f = -\frac{GM_e m}{2\left(R_e + \frac{R_e}{20}\right)} = -\frac{GM_e m}{2\left(\frac{21R_e}{20}\right)}.$$

Simplify:

$$T.E_f = -\frac{10GM_e m}{21R_e}.$$

Step 3: Change in mechanical energy The change in total mechanical energy is:

$$\Delta E = T.E_f - T.E_i.$$

Substitute:

$$\Delta E = \left(-\frac{10GM_e m}{21R_e}\right) - \left(-\frac{GM_e m}{R_e}\right).$$

Simplify:

$$\Delta E = -\frac{10GM_e m}{21R_e} + \frac{21GM_e m}{21R_e}.$$

$$\Delta E = \frac{11GM_e m}{21R_e}.$$

Thus, $x = 11$.

**Final Answer:** $x = 11$.

**Quick Tips**

1. Use the formula for total mechanical energy: $T.E = -\frac{GM_e m}{2r}$ for an orbit and $T.E = -\frac{GM_e m}{R_e}$ at the surface. 2. When approximating small altitudes, express $h$ as a fraction of $R_e$ for easier calculations. 3. Always compute the change in energy as $T.E_f - T.E_i$, and simplify step by step. 4. Keep track of signs to ensure proper subtraction between energy terms.

**Question 43.**

Given below are two statements:

- **Statement I:** When currents vary with time, Newton's third law is valid only if momentum carried by the electromagnetic field is taken into account.
- **Statement II:** Ampere's circuital law does not depend on Biot-Savart's law.

In the light of the above statements, choose the correct answer from the options given below:

1. Both Statement I and Statement II are false.
2. Statement I is true but Statement II is false.
3. Statement I is false but Statement II is true.

4. Both Statement I and Statement II are true.

**Correct Answer:** (2)

**Solution:**

Explanation: 1. Statement I: Newton's third law states that for every action, there is an equal and opposite reaction. When currents vary with time, the electromagnetic fields also carry momentum. For Newton's third law to hold in such cases, this momentum must be included in the analysis. Hence, Statement I is **true**.

2. *tatement II: Ampere's circuital law is an independent law that relates the magnetic field around a closed loop to the electric current passing through the loop. It does not depend on Biot-Savart's law, which describes the magnetic field generated by a current element. Hence, Statement II is **false**.

**Conclusion:** Statement I is true but Statement II is false.

**Final Answer:** (2)

**Quick Tips**

1. Newton's third law in electromagnetic systems must consider the momentum carried by fields during time-varying currents. 2. Ampere's circuital law is fundamental and does not rely on Biot-Savart's law; they are independent concepts. 3. Carefully analyze the dependency of one law on another to avoid confusion in theoretical physics problems.

**Question 44.**

A particle of mass $m$ moves on a straight line with its velocity increasing with distance according to the equation $v = \alpha\sqrt{x}$, where $\alpha$ is a constant. The total work done by all the forces applied on the particle during its displacement from $x = 0$ to $x = d$, will be:

1. $\frac{m}{2\alpha^2 d}$

2. $\frac{md}{2\alpha^2}$

3. $\frac{m\alpha^2 d}{2}$

4. $2m\alpha^2 d$

**Correct Answer:** (3)

**Solution:**

Step 1: Velocity equation The velocity of the particle is given as:

$$v = \alpha\sqrt{x}.$$

At $x = 0$, the velocity is:

$$v = 0.$$

At $x = d$, the velocity becomes:

$$v = \alpha\sqrt{d}.$$

Step 2: Work-Energy Theorem The work done by all forces is equal to the change in kinetic energy:

$$W.D = K_f - K_i,$$

where:

$$K = \frac{1}{2}mv^2.$$

Substitute the velocities:

$$W.D = \frac{1}{2}m(\alpha\sqrt{d})^2 - \frac{1}{2}m(0)^2.$$

Simplify:

$$W.D = \frac{1}{2}m(\alpha^2 d) - 0.$$

$$W.D = \frac{m\alpha^2 d}{2}.$$

**Final Answer:** $\frac{m\alpha^2 d}{2}$.

**Quick Tips**

1. Use the Work-Energy Theorem: Work done equals the change in kinetic energy. 2. Ensure the velocity function is correctly substituted into the kinetic energy formula. 3. Remember to evaluate initial and final velocities to compute the change in energy accurately. 4. Simplify expressions step-by-step to avoid errors in squaring terms or constants.

**Question 45.**

A galvanometer has a coil of resistance $200\,\Omega$ with a full-scale deflection at $20\,\mu$A. The value of resistance to be added to use it as an ammeter of range $0–20$ mA is:

1. $0.40\,\Omega$
2. $0.20\,\Omega$
3. $0.50\,\Omega$
4. $0.10\,\Omega$

**Correct Answer:** (2)

**Solution:**

Step 1: Formula for shunt resistance The shunt resistance $R_s$ is given by:

$$R_s = \frac{I_g R_g}{I - I_g},$$

where:

- $I_g = 20\,\mu\text{A} = 20 \times 10^{-6}$ A (full-scale deflection current),
- $R_g = 200\,\Omega$ (resistance of the galvanometer),
- $I = 20\,\text{mA} = 20 \times 10^{-3}$ A (ammeter range).

Step 2: Substitute the values

$$R_s = \frac{(20 \times 10^{-6}) \cdot 200}{(20 \times 10^{-3}) - (20 \times 10^{-6})}.$$

Simplify the numerator:

$$(20 \times 10^{-6}) \cdot 200 = 4 \times 10^{-3}.$$

Simplify the denominator:

$$(20 \times 10^{-3}) - (20 \times 10^{-6}) = 19.98 \times 10^{-3}.$$

$$R_s = \frac{4 \times 10^{-3}}{19.98 \times 10^{-3}} \approx 0.20\,\Omega.$$

**Final Answer:** $0.20\,\Omega$.

**Quick Tips**

1. Use the formula for shunt resistance $R_s = \frac{I_g R_g}{I - I_g}$ for converting a galvanometer to an ammeter. 2. Convert all currents to amperes before substitution. 3. Carefully handle small differences like $I - I_g$ to avoid calculation errors. 4. Ensure units of $R_s$ are consistent with the resistance of the galvanometer.

**Question 46.**

A heavy iron bar, of weight $W$, is having its one end on the ground and the other on the shoulder of a person. The bar makes an angle $\theta$ with the horizontal. The weight experienced by the person is:

1. $\frac{W}{2}$
2. $W$
3. $W \cos\theta$
4. $W \sin\theta$

**Correct Answer:** (1)

**Solution:**

Step 1: Concept of center of mass The weight of the bar is uniformly distributed. Since the bar is uniform, the center of mass lies at the midpoint of the bar.

The total weight $W$ of the bar is supported by two points:

- One end is on the ground.
- The other end is on the shoulder of the person.

Step 2: Distribution of weight In a symmetric situation like this, the total weight is evenly distributed between the two points of contact.

$$\text{Weight experienced by the person} = \frac{W}{2}.$$

**Final Answer:** $\frac{W}{2}$.

Quick Tips

1. For uniform objects, the weight is equally distributed between the two points of support. 2. The angle $\theta$ with the horizontal does not affect the division of weight in this scenario. 3. Always consider the center of mass when dealing with distributed forces. 4. Symmetry simplifies the problem, dividing the total weight equally.

**Question 47.**

One main scale division of a vernier caliper is equal to $m$ units. If $n^{th}$ division of the main scale coincides with $(n+1)^{th}$ division of the vernier scale, the least count of the vernier caliper is:

1. $\frac{n}{n+1}$

2. $\frac{m}{n+1}$

3. $\frac{1}{n+1}$

4. $\frac{m}{n(n+1)}$

**Correct Answer:** (2)

**Solution:**

Step 1: Relationship between main scale and vernier scale Given that:

$$n\,\text{MSD} = (n+1)\,\text{VSD}.$$

From this:

$$1\,\text{VSD} = \frac{n}{n+1}\,\text{MSD}.$$

Step 2: Least count formula The least count (L.C.) of a vernier caliper is given by:

$$\text{L.C.} = 1\,\text{MSD} - 1\,\text{VSD}.$$

Substitute $1\,\text{VSD}$ from Step 1:

$$\text{L.C.} = m - m\left(\frac{n}{n+1}\right).$$

Simplify:

$$\text{L.C.} = m\left[1 - \frac{n}{n+1}\right].$$

$$\text{L.C.} = m\left(\frac{n+1-n}{n+1}\right).$$

$$\text{L.C.} = \frac{m}{n+1}.$$

**Final Answer:** $\frac{m}{n+1}$.

Quick Tips

1. The least count of a vernier caliper is the difference between one main scale division and one vernier scale division. 2. Always express 1 VSD in terms of MSD for simplification. 3. Carefully simplify fractional terms to avoid algebraic errors. 4. For problems involving scales, clarity in understanding the relationship between divisions is key.

**Question 48.**

A bulb and a capacitor are connected in series across an AC supply. A dielectric is then placed between the plates of the capacitor. The glow of the bulb:

1. increases
2. remains same
3. becomes zero
4. decreases

**Correct Answer:** (1)

**Solution:**

Step 1: Understanding the circuit The capacitor is in series with the bulb in an AC circuit. The impedance $Z$ of the capacitor is given by:

$$Z_C = \frac{1}{\omega C},$$

where:

- $\omega$ is the angular frequency of the AC supply,
- $C$ is the capacitance of the capacitor.

Step 2: Effect of placing a dielectric Placing a dielectric between the plates of the capacitor increases the capacitance $C$, as:

$$C' = \kappa C,$$

where $\kappa > 1$ is the dielectric constant.

Step 3: Impedance of the capacitor Since $Z_C \propto \frac{1}{C}$, increasing $C$ reduces the capacitive impedance $Z_C$.

Step 4: Impact on the bulb The total impedance of the circuit decreases, leading to an increase in the current through the circuit. As the current increases, the glow of the bulb increases.

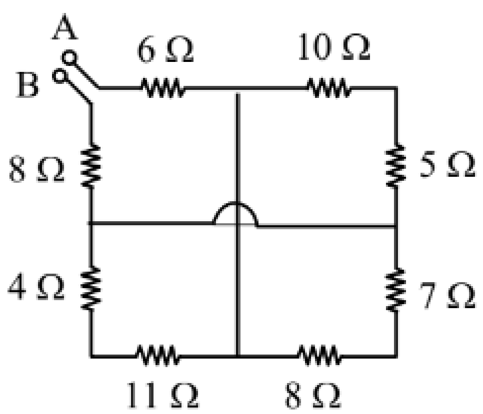
**Final Answer:** The glow of the bulb **increases**.

**Quick Tips**

1. In an AC circuit, the impedance of a capacitor decreases when the capacitance increases. 2. Placing a dielectric between the plates increases the capacitance. 3. A decrease in total impedance increases the current, enhancing the glow of the bulb. 4. Always analyze the effect of circuit changes on impedance and current flow.

**Question 49.**

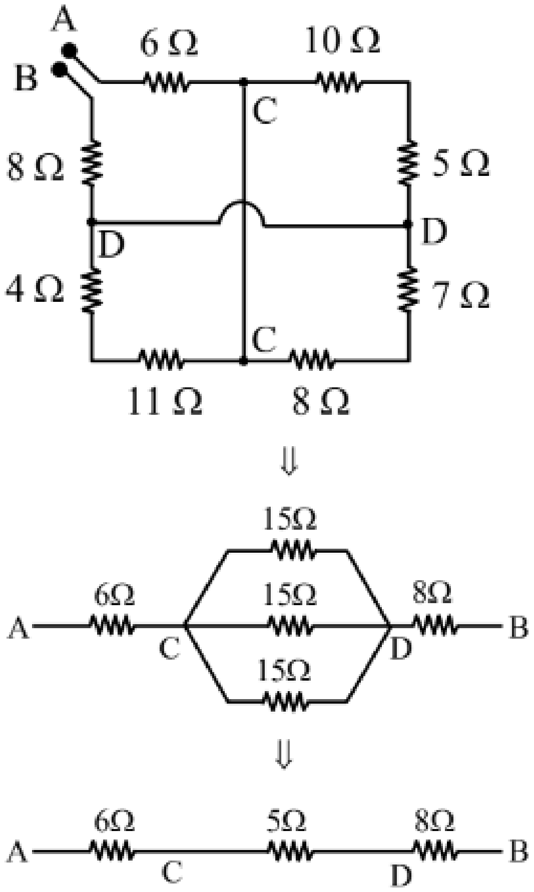
The equivalent resistance between $A$ and $B$ is:




1. $18\,\Omega$
2. $25\,\Omega$
3. $27\,\Omega$
4. $19\,\Omega$

**Correct Answer:** (4)

**Solution:**




$\Rightarrow R_{eq} = 6\Omega + 5\Omega + 8\Omega = 19\Omega$

**Final Answer:** $19\,\Omega$.

**Quick Tips**

1. Simplify complex resistor networks step-by-step by reducing parallel and series connections. 2. Use the formulas:

$$R_{\text{parallel}} = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2}}, \quad R_{\text{series}} = R_1 + R_2.$$

3. Keep track of intermediate results to avoid mistakes. 4. Verify results by re-checking combinations of resistors.

**Question 50.**

A sample of 1 mole gas at temperature $T$ is adiabatically expanded to double its volume. If the adiabatic constant for the gas is $\gamma = \frac{3}{2}$, then the work done by the gas in the process is:

1. $RT\left[2-\sqrt{2}\right]$
2. $\frac{R}{T}\left[2-\sqrt{2}\right]$
3. $RT\left[2+\sqrt{2}\right]$
4. $\frac{T}{R}\left[2+\sqrt{2}\right]$

**Correct Answer:** (1)

**Solution:**

Step 1: Adiabatic condition The adiabatic condition is given by:

$$TV^{\gamma-1} = \text{constant.}$$

For the initial and final states:

$$T(V)^{\frac{3}{2}-1} = T_f(2V)^{\frac{3}{2}-1}.$$

Simplify:

$$TV^{\frac{1}{2}} = T_f(2V)^{\frac{1}{2}},$$

$$T\sqrt{V} = T_f\sqrt{2V}.$$

Cancel $\sqrt{V}$:

$$T = T_f\sqrt{2}.$$

Solve for $T_f$:

$$T_f = \frac{T}{\sqrt{2}}.$$

Step 2: Work done in adiabatic expansion The work done in an adiabatic process is given by:

$$W.D. = \frac{nR}{1-\gamma}\left[T_f - T\right].$$

Substitute $T_f = \frac{T}{\sqrt{2}}$, $\gamma = \frac{3}{2}$, and $n = 1$:

$$W.D. = \frac{R}{1-\frac{3}{2}}\left[\frac{T}{\sqrt{2}} - T\right].$$

Simplify:

$$W.D. = \frac{R}{-\frac{1}{2}}\left[\frac{T}{\sqrt{2}} - T\right],$$

$$W.D. = -2R\left[\frac{T}{\sqrt{2}} - T\right].$$

Factorize:

$$W.D. = 2RT\left[1 - \frac{1}{\sqrt{2}}\right].$$

Simplify further:

$$W.D. = RT\left[2 - \sqrt{2}\right].$$

**Final Answer:** $\mathbf{RT}\left[\mathbf{2} - \sqrt{\mathbf{2}}\right]$.

Quick Tips

1. For adiabatic processes, use $TV^{\gamma-1}$ = constant to relate initial and final states. 2. The work done in an adiabatic process is:

$$W = \frac{nR}{1-\gamma}\left(T_f - T_i\right).$$

3. Simplify step-by-step and substitute values carefully. 4. For $\gamma = \frac{3}{2}$, ensure proper handling of fractions during calculations.

**Question 51.**

If $\vec{a}$ and $\vec{b}$ make an angle $\cos^{-1}\left(\frac{5}{9}\right)$ with each other, then $|\vec{a}+\vec{b}| = \sqrt{2}|\vec{a}-\vec{b}|$ for $|\vec{a}| = n|\vec{b}|$. The integer value of $n$ is:

**Correct Answer:** (3)

**Solution:**

$$\cos\theta = \frac{5}{9}$$

$$\frac{\vec{a}\cdot\vec{b}}{ab} = \frac{5}{9} \quad (1)$$

Using $|\vec{a}+\vec{b}|^2 = \sqrt{2}|\vec{a}-\vec{b}|^2$:

$$a^2 + b^2 + 2a\cdot b = 2(a^2 + b^2 - 2a\cdot b)$$

Simplify:

$$a^2 + b^2 + 2a\cdot b = 2a^2 + 2b^2 - 4a\cdot b$$

$$6(\vec{a}\cdot\vec{b}) = a^2 + b^2 \tag{2}$$

Substitute $\vec{a}\cdot\vec{b} = ab\cdot\frac{5}{9}$ from (1):

$$6\cdot\frac{5}{9}ab = a^2 + b^2$$

$$\frac{10}{3}ab = a^2 + b^2$$

Assume $a = nb$:

$$\frac{10}{3}nb^2 = n^2b^2 + b^2$$

Divide through by $b^2$:

$$\frac{10}{3}n = n^2 + 1$$

Rearrange:

$$3n^2 - 10n + 3 = 0$$

Solve using the quadratic formula:

$$n = \frac{-(-10) \pm \sqrt{(-10)^2 - 4(3)(3)}}{2(3)}$$

$$n = \frac{10 \pm \sqrt{100 - 36}}{6}$$

$$n = \frac{10 \pm \sqrt{64}}{6}$$

$$n = \frac{10 \pm 8}{6}$$

$$n = \frac{18}{6} = 3 \quad \text{(only positive integer value).}$$

**Final Answer:** $\mathbf{n = 3}$.

**Quick Tips**

1. Use the dot product and vector magnitude formulas for solving relationships between vectors. 2. Substitute the given trigonometric values into equations and simplify step by step. 3. For $|\vec{a}+\vec{b}|$ and $|\vec{a}-\vec{b}|$, expand their squared forms and use algebra to solve for unknowns.

**Question 52.** At the centre of a half-ring of radius $R = 10\,\text{cm}$ and linear charge density $4\,\text{nC/m}$, the potential is $x\pi V$. The value of $x$ is _____.

**Correct Answer:** (36)

**Solution:**

The potential at the center of a half-ring is given by:

$$V = \frac{KQ}{R}$$

where:

$$Q = \lambda \pi R$$

Substituting:

$$V = \frac{K\lambda\pi R}{R}$$
$$V = K\lambda\pi$$

Given:

$$K = 9 \times 10^9 \, \text{N·m}^2/\text{C}^2, \quad \lambda = 4 \times 10^{-9} \, \text{C/m}$$

$$V = 9 \times 10^9 \cdot 4 \times 10^{-9} \cdot \pi$$

$$V = 36\pi \, \text{V}$$

Thus, $x = 36$.

**Final Answer:** $x = 36$.

**Quick Tips**

For the potential of a charged ring, always use $V = \frac{KQ}{R}$.

For a half-ring, the charge is $Q = \lambda\pi R$ since the arc length is $\pi R$.

Substitute $K$ (Coulomb's constant) as $9 \times 10^9 \, \text{N·m}^2/\text{C}^2$ and carefully handle powers of 10 for charge density.

Ensure unit consistency: radius $R$ in meters, charge density $\lambda$ in C/m.

Simplify step-by-step to avoid missing factors like $\pi$.

**Question 53.**

A star has 100% helium composition. It starts to convert three $^4$He into $^{12}$C via the triple alpha process as:

$$^4\text{He} +^4 \text{He} +^4 \text{He} \rightarrow^{12} \text{C} + Q.$$

The mass of the star is $2.0 \times 10^{32}$ kg and it generates energy at the rate of $5.808 \times 10^{30}$ W. The rate of converting these $^4$He to $^{12}$C is $n \times 10^{42} \, \text{s}^{-1}$, where $n$ is .......... [Take, mass of $^4\text{He} = 4.0026 \, \text{u}$, mass of $^{12}\text{C} = 12 \, \text{u}$].

**Correct Answer:** (15)

**Solution:**

Step 1: Reaction and energy release The triple alpha process involves:

$$^4\text{He} +^4\text{He} +^4\text{He} \to^{12}\text{C} + Q.$$

The energy released per reaction is given by:

$$Q = \Delta m \cdot c^2,$$

where $\Delta m =$ mass of reactants − mass of product.

Substitute:

$$\Delta m = (3 \times 4.0026) - 12 = 12.0078 - 12 = 0.0078\,\text{u}.$$

Convert to energy (1 u = $931.5\,\text{MeV}$):

$$Q = 0.0078 \times 931.5 = 7.266\,\text{MeV}.$$

Step 2: Power and rate of reactions The power output is related to the rate of reactions ($\frac{N}{t}$) by:

$$\text{Power} = \frac{N}{t} \cdot Q.$$

Rearranging for $\frac{N}{t}$:

$$\frac{N}{t} = \frac{\text{Power}}{Q}.$$

Substitute the values:

$$\frac{N}{t} = \frac{5.808 \times 10^{30}}{7.266 \times 10^6 \times 1.6 \times 10^{-19}}.$$

Simplify:

$$\frac{N}{t} = \frac{5.808 \times 10^{30}}{1.16256 \times 10^{-12}} = 5 \times 10^{42}\,\text{s}^{-1}.$$

Step 3: Rate of helium conversion Each reaction converts three $^4$He. Thus, the rate of $^4$He conversion is:

$$n = 3 \cdot \frac{N}{t} = 15 \times 10^{42}.$$

**Final Answer:** $n = 15$.

Quick Tips

1. Calculate the mass defect $\Delta m$ for the reaction. 2. Use $Q = \Delta m \cdot c^2$ to find energy released per reaction. 3. Relate power and reaction rate with Power $= \frac{N}{t} \cdot Q$. 4. Multiply the reaction rate by the number of $^4$He nuclei involved to find the total conversion rate. 5. Double-check units for mass (u), energy (MeV), and power (W).

**Question 54.**

In a Young's double-slit experiment, the intensity at a point is $\frac{1}{4}$ of the maximum intensity. The minimum distance of the point from the central maximum is $\underset{\mu\text{m}}{\_\_}$. (Given: $\lambda = 600\,\text{nm}$, $d = 1.0\,\text{mm}$, $D = 1.0\,\text{m}$)

**Correct Answer:** (200)

**Solution:**

Step 1: Intensity relation The intensity at a point in the interference pattern is given by:

$$I = I_0 \cos^2\left(\frac{\Delta\phi}{2}\right).$$

Given:

$$I = \frac{I_0}{4}.$$

Substitute:

$$\frac{I_0}{4} = I_0 \cos^2\left(\frac{\Delta\phi}{2}\right).$$

Simplify:

$$\cos^2\left(\frac{\Delta\phi}{2}\right) = \frac{1}{4}.$$

Taking square root:

$$\cos\left(\frac{\Delta\phi}{2}\right) = \frac{1}{2}.$$

This implies:

$$\frac{\Delta\phi}{2} = \frac{\pi}{3}.$$

So:

$$\Delta\phi = \frac{2\pi}{3}.$$

Step 2: Path difference relation The phase difference $\Delta\phi$ is related to the path difference by:

$$\Delta\phi = \frac{2\pi}{\lambda} \cdot \frac{y\,d}{D}.$$

Substitute $\Delta\phi = \frac{2\pi}{3}$:

$$\frac{2\pi}{3} = \frac{2\pi}{\lambda} \cdot \frac{y\,d}{D}.$$

Cancel $2\pi$:

$$\frac{1}{3} = \frac{y\,d}{\lambda\,D}.$$

Rearrange to solve for $y$:

$$y = \frac{\lambda\,D}{3\,d}.$$

Step 3: Substitution of values Substitute $\lambda = 600\,\text{nm} = 600 \times 10^{-9}\,\text{m}$, $D = 1.0\,\text{m}$, and $d = 1.0\,\text{mm} = 1.0 \times 10^{-3}\,\text{m}$:

$$y = \frac{600 \times 10^{-9} \cdot 1.0}{3 \cdot 1.0 \times 10^{-3}}.$$

Simplify:

$$y = \frac{600 \times 10^{-9}}{3 \times 10^{-3}} = \frac{600}{3} \times 10^{-6} = 200 \times 10^{-6}\,\text{m}.$$

Convert to $\mu$m:

$$y = 200\,\mu\text{m}.$$

**Final Answer:** **200** $\mu$m.

**Quick Tips**

1. The intensity in a double-slit experiment is related to the phase difference using $I = I_0 \cos^2\left(\frac{\Delta\phi}{2}\right)$. 2. The phase difference $\Delta\phi$ is linked to the path difference by $\Delta\phi = \frac{2\pi}{\lambda} \cdot \frac{y\,d}{D}$. 3. Carefully substitute values in standard units (e.g., $\lambda$ in meters, $d$ in meters). 4. Simplify step-by-step to avoid errors in unit conversions.

**Question 55.**

A string is wrapped around the rim of a wheel of moment of inertia $0.40\,\text{kgm}^2$ and radius $10\,\text{cm}$. The wheel is free to rotate about its axis. Initially, the wheel is at rest. The string is now pulled by a force of $40\,\text{N}$. The angular velocity of the wheel after $10\,\text{s}$ is $x$ rad/s, where $x$ is

**Correct Answer:** (100)

**Solution:**

Step 1: Torque and angular acceleration The torque $(\tau)$ acting on the wheel is:

$$\tau = F \cdot R.$$

Substitute $F = 40\,\text{N}$ and $R = 0.10\,\text{m}$:

$$\tau = 40 \cdot 0.1 = 4\,\text{Nm}.$$

From the rotational dynamics equation:

$$\tau = I \cdot \alpha,$$

where $I = 0.40\,\text{kgm}^2$ is the moment of inertia and $\alpha$ is the angular acceleration. Solving for $\alpha$:

$$\alpha = \frac{\tau}{I} = \frac{4}{0.4} = 10\,\text{rad/s}^2.$$

Step 2: Angular velocity after 10 seconds Using the kinematic relation for angular motion:

$$\omega_f = \omega_i + \alpha \cdot t.$$

Here, the initial angular velocity $\omega_i = 0$, $\alpha = 10\,\text{rad/s}^2$, and $t = 10\,\text{s}$. Substituting these values:

$$\omega_f = 0 + 10 \cdot 10 = 100\,\text{rad/s}.$$

**Final Answer:** $\mathbf{100}\,\text{rad/s}$.

**Quick Tips**

1. Torque is given by $\tau = F \cdot R$. Ensure correct units for force and radius. 2. Relate torque to angular acceleration using $\tau = I \cdot \alpha$. 3. Use angular kinematics ($\omega_f = \omega_i + \alpha \cdot t$) to find the final angular velocity. 4. Always check units for consistency when calculating torque, moment of inertia, and angular velocity.

**Question 56.**

A square loop of edge length $2\,\text{m}$ carrying a current of $2\,\text{A}$ is placed with its edges parallel to the $x$-$y$-axis. A magnetic field is passing through the $x$-$y$-plane and is expressed as:

$$\vec{B} = B_0(1 + 4x)\hat{k},$$

where $B_0 = 5\,\text{T}$. The net magnetic force experienced by the loop is $\_\_\__{\text{N}}$.

**Correct Answer:** (160)

**Solution:**

Step 1: Magnetic force on a current-carrying conductor The magnetic force on a straight conductor in a magnetic field is given by:

$$\vec{F} = I \int (\vec{dl} \times \vec{B}),$$

where $I$ is the current, $\vec{dl}$ is the element of the wire, and $\vec{B}$ is the magnetic field.

Step 2: Magnetic field variation The magnetic field is given as:

$$\vec{B} = B_0(1 + 4x)\hat{k}.$$

Since the loop is aligned parallel to the $x$-$y$-plane, $\vec{B}$ varies with $x$, causing forces on the opposite sides of the loop to not cancel.

Step 3: Net force on the loop For the two vertical sides of the loop (along the $y$-direction):

$$F_{\text{vertical}} = IL\Delta B,$$

where $\Delta B$ is the difference in the magnetic field at $x = 2\,\text{m}$ and $x = 0\,\text{m}$.

$$\Delta B = B_0(1 + 4 \cdot 2) - B_0(1 + 4 \cdot 0),$$

$$\Delta B = 5 \times (1 + 8) - 5 \times (1 + 0),$$

$$\Delta B = 45 - 5 = 40\,\text{T}.$$

Substitute $I = 2\,\text{A}$, $L = 2\,\text{m}$, and $\Delta B = 40\,\text{T}$:

$$F_{\text{vertical}} = 2 \cdot 2 \cdot 40 = 160\,\text{N}.$$

For the horizontal sides of the loop (along the $x$-direction), the magnetic forces cancel out because the magnetic field is uniform along the $y$-axis.

Step 4: Final result The net magnetic force on the loop is:

$$\boxed{160\,\text{N}}.$$

**Quick Tips**

1. Use $\Delta B = B(x_2) - B(x_1)$ to find the magnetic field difference along non-uniform directions. 2. Net magnetic force is calculated only for non-canceling components. 3. Horizontal forces cancel out when the field is uniform along the horizontal direction. 4. Ensure consistent units for current, length, and magnetic field.

**Question 57.**

Two persons pull a wire towards themselves. Each person exerts a force of $200$ N on the wire. The Young's modulus of the material of the wire is $1 \times 10^{11}$ N/m$^2$. The original length of the wire is $2$ m, and the area of the cross-section is $2$ cm$^2$. The wire will extend in length by $_{\mu\text{m}}$ .

**Correct Answer:** (20)

**Solution:**

Step 1: Relation between stress and strain Young's modulus is given by:

$$Y = \frac{\text{Stress}}{\text{Strain}} = \frac{\frac{F}{A}}{\frac{\Delta\ell}{\ell}}.$$

Rearranging for $\Delta\ell$:

$$\Delta\ell = \frac{F\ell}{AY}.$$

Step 2: Substitute given values - Force, $F = 200$ N, - Original length, $\ell = 2$ m, - Area of cross-section, $A = 2\,\text{cm}^2 = 2 \times 10^{-4}\,\text{m}^2$, - Young's modulus, $Y = 1 \times 10^{11}$ N/m$^2$.

Substitute into the formula:

$$\Delta\ell = \frac{200 \cdot 2}{2 \times 10^{-4} \cdot 10^{11}}.$$

Step 3: Simplify the expression

$$\Delta\ell = \frac{400}{2 \times 10^7} = 2 \times 10^{-5}\,\text{m}.$$

Convert to micrometers ($\mu$m):

$$\Delta\ell = 20\,\mu\text{m}.$$

**Final Answer:** 20 $\mu$m.

**Quick Tips**

1. Use the relation $\Delta\ell = \frac{F\ell}{AY}$ for elongation in a wire. 2. Ensure units are consistent: convert cm$^2$ to m$^2$ for area calculations. 3. Convert the result to the required units (e.g., $\mu$m). 4. Double-check input values for accuracy in calculations.

**Question 58.**

When a coil is connected across a $20$ V DC supply, it draws a current of $5$ A. When it is connected across a $20$ V, $50$ Hz AC supply, it draws a current of $4$ A. The self-inductance of the coil is ...... mH. ($\pi = 3$)

**Correct Answer:** (10)

**Solution:**

Case 1: DC Circuit When the coil is connected to a DC supply, the inductive reactance is 0, so:

$$I = \frac{V}{R} \implies R = \frac{20}{5} = 4\,\Omega.$$

Case 2: AC Circuit When the coil is connected to an AC supply:

$$I = \frac{V}{Z} \implies Z = \frac{20}{4} = 5\,\Omega,$$

where $Z$ is the impedance of the coil, given by:

$$Z = \sqrt{R^2 + X_L^2}.$$

Substitute $R = 4\,\Omega$:

$$5 = \sqrt{4^2 + X_L^2} \implies X_L^2 = 25 - 16 = 9 \implies X_L = 3\,\Omega.$$

The inductive reactance $X_L$ is related to the self-inductance $L$ by:

$$X_L = 2\pi f L \implies L = \frac{X_L}{2\pi f}.$$

Substitute $X_L = 3\,\Omega$, $f = 50$ Hz, and $\pi = 3$:

$$L = \frac{3}{2 \cdot 3 \cdot 50} = \frac{3}{300} = 0.01\text{ H} = 10\text{ mH}.$$

**Final Answer:** 10 mH.

**Quick Tips**

1. In DC circuits, the inductive reactance $X_L = 0$, so only the resistance $R$ contributes to the impedance. 2. In AC circuits, the total impedance $Z = \sqrt{R^2 + X_L^2}$. 3. Inductive reactance is given by $X_L = 2\pi f L$, where $f$ is the frequency. 4. Always use consistent units: convert millihenries to henries when necessary.

**Question 59.**

The position, velocity, and acceleration of a particle executing simple harmonic motion are found to have magnitudes of $4$ m, $2$ ms$^{-1}$, and $16$ ms$^{-2}$ at a certain instant. The amplitude of the motion is $\sqrt{x}$ m, where $x$ is ........

**Correct Answer:** (17)

**Solution:**

The given data is:

$$x = 4\,\text{m}, \quad v = 2\,\text{m/s}, \quad a = 16\,\text{m/s}^2$$

For a particle in Simple Harmonic Motion (SHM), the equations for position, velocity, and acceleration are:

$$x = A\cos\omega t, \quad v = A\omega\sin\omega t, \quad a = -A\omega^2\cos\omega t$$

—

**Step 1: Using the relation between acceleration and position**

The acceleration is given by:

$$a = -\omega^2 x$$

Substitute $a = 16\,\text{m/s}^2$ and $x = 4\,\text{m}$:

$$16 = \omega^2 \cdot 4 \quad \Longrightarrow \quad \omega^2 = 4$$

Thus:

$$\omega = 2\,\text{rad/s}$$

—

**Step 2: Using the relation between velocity and amplitude**

The velocity equation in SHM is:

$$v^2 = \omega^2\left(A^2 - x^2\right)$$

Substitute $v = 2\,\text{m/s}, \omega = 2\,\text{rad/s}, x = 4\,\text{m}$:

$$2^2 = 2^2\left(A^2 - 4^2\right)$$

$$4 = 4\left(A^2 - 16\right) \quad \Longrightarrow \quad A^2 - 16 = 1$$

$$A^2 = 17$$

—

**Step 3: Amplitude**

The amplitude of the motion is:

$$A = \sqrt{17}\,\text{m}$$

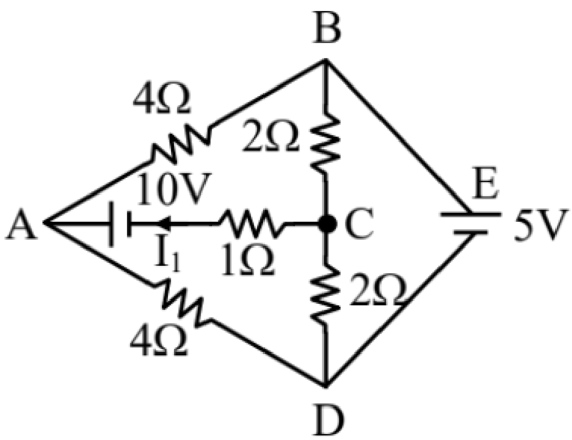
Thus, $x = 17$.

**Quick Tips**

1. Use $|a| = \omega^2 x$ to find the angular frequency $\omega$. 2. Use $v = \omega\sqrt{A^2 - x^2}$ to relate velocity, amplitude, and displacement. 3. Always ensure consistent units for all quantities. 4. For SHM problems, solving step-by-step reduces errors and ensures clarity.

**Question 60.**

The current flowing through the $1\,\Omega$ resistor is $\frac{n}{10}$. The value of $n$ is ........




**Correct Answer:** (25)

**Solution:**

Let the potentials at points A, B, and C be $x, y,$ and $0,$ respectively.

Applying Kirchhoff's Current Law (KCL) at node $B$:

$$\frac{y-5}{2} + \frac{y-0}{2} + \frac{y-x+10}{1} = 0$$

$$\Rightarrow 4y - 2x + 15 = 0 \quad \text{(i)}$$

Applying KCL at node $A$:

$$\frac{x-5}{4} + \frac{x-0}{4} + \frac{x-10-y}{1} = 0$$

$$\Rightarrow 6x - 4y - 45 = 0 \quad \text{(ii)}$$

Solving equations (i) and (ii):

$$\text{From (i): } y = \frac{15}{4}x - \frac{15}{4}$$

$$\text{Substituting in (ii): } x = \frac{15}{2},\ y = 0$$

The current through the $1\,\Omega$ resistor is:

$$i = \frac{y - x + 10}{1} = \frac{0 - 7.5 + 10}{1} = 2.5\,\text{A}.$$

Therefore:

$$i = \frac{n}{10}, \quad n = 25.$$

**Final Answer:** $\mathbf{n = 25}$.

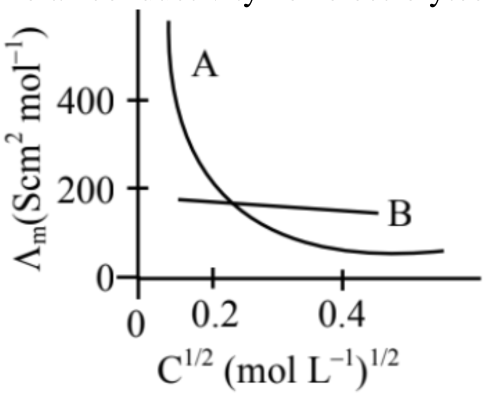
**Quick Tips**

1. Use Kirchhoff's Current Law (KCL) at all junctions to form equations. 2. Label potentials at key points for clarity. 3. Solve the simultaneous equations systematically. 4. Substitute values back to verify the solution.

# 1 Chemistry

**Question 61.**

The molar conductivity for electrolytes $A$ and $B$ are plotted against $C^{1/2}$ as shown below.




Electrolytes $A$ and $B$ respectively are:

| | A | B |
|---|---|---|
| (1) | Weak electrolyte | Weak electrolyte |
| (2) | Strong electrolyte | Strong electrolyte |
| (3) | Weak electrolyte | Strong electrolyte |
| (4) | Strong electrolyte | Weak electrolyte |

**Correct Answer:** (3)

**Solution:**

Explanation: - The graph shows the variation of molar conductivity ($\Lambda_m$) with $C^{1/2}$, the square root of concentration: 1. **Electrolyte A**: The molar conductivity of $A$ increases steeply as $C^{1/2}$ approaches 0, indicating $A$ is a **weak electrolyte**. For weak electrolytes, the molar conductivity increases significantly with dilution due to ionization. 2. **Electrolyte B**: The molar conductivity of $B$ remains relatively constant with $C^{1/2}$, indicating $B$ is a **strong electrolyte**. For strong electrolytes, the variation of molar conductivity with concentration is minimal.

Therefore:

$$\text{Electrolyte A} \to \text{Weak electrolyte}, \quad \text{Electrolyte B} \to \text{Strong electrolyte}.$$

**Final Answer:** $(3)$.

Quick Tips

1. **Weak electrolytes**: Show significant variation of molar conductivity with concentration due to partial ionization. 2. **Strong electrolytes**: Exhibit almost constant molar conductivity, as they are completely ionized at all concentrations. 3. Analyze the behavior of molar conductivity ($\Lambda_m$) at low concentration ($C^{1/2} \to 0$) to distinguish the type of electrolyte.

**Question 62.**

Methods used for purification of organic compounds are based on:

1. neither on nature of compound nor on the impurity present.
2. nature of compound only.
3. nature of compound and presence of impurity.

4. presence of impurity only.

**Correct Answer:** (3)

**Solution:**

Organic compounds are purified based on:

- The **nature of the compound** (e.g., solubility, boiling/melting point, volatility, etc.).
- The **nature of the impurity** present (e.g., soluble or insoluble, volatile or non-volatile, etc.).

Thus, purification methods such as crystallization, distillation, chromatography, and sublimation are chosen based on the combination of the compound's properties and the impurities involved.

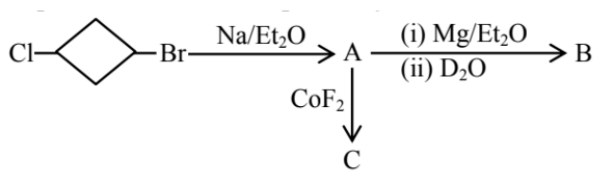
**Final Answer:** (3).

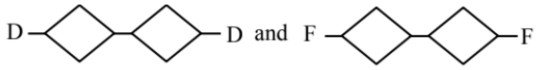
**Quick Tips**

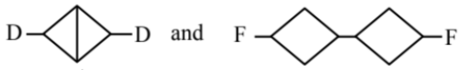
1. Understand the **properties of the organic compound** (e.g., polarity, solubility) for selecting the purification method. 2. Analyze the **nature of impurities** to determine the appropriate technique (e.g., filtration for insoluble impurities, distillation for volatile ones). 3. Common purification methods: - **Crystallization**: Based on solubility differences. - **Distillation**: Based on boiling points. - **Chromatography**: Based on adsorption and partition. - **Sublimation**: For sublimable compounds.

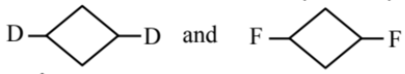
**Question 63.**

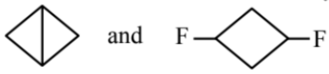
In the following sequence of reaction, the major products B and C respectively are:



(1) 

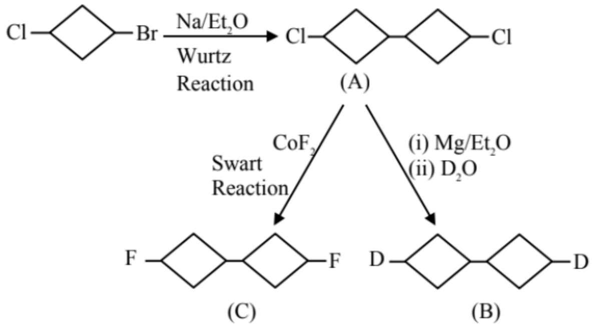
(2) 

(3) 

(4) 

**Correct Answer:** (1)

**Solution:**

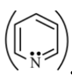



**Final Answer:** (1).

## Quick Tips

1. **Wurtz Reaction**: Sodium in dry ether is used to couple alkyl halides, leading to symmetrical alkanes. 2. **Grignard Reagent**: Formed by reacting alkyl halides with magnesium in ether. Reaction with $D_2O$ replaces the halide with deuterium. 3. **Fluorination**: $CoF_2$ catalyzes the introduction of fluorine atoms in place of hydrogen atoms. 4. Follow the sequence of reactions systematically to predict intermediates and products.

**Question 64.**

The correct order of basic strength of Pyrrole  Pyridine , and Piperidine  is:

1. Piperidine > Pyridine > Pyrrole
2. Pyrrole > Pyridine > Piperidine
3. Pyridine > Piperidine > Pyrrole
4. Pyrrole > Piperidine > Pyridine

**Correct Answer:** (1)
**Solution:**

Explanation: The basicity of a compound depends on the availability of the lone pair of electrons on nitrogen for protonation. The order of basic strength is:

$N(sp^3, \text{localized lone pair}) > N(sp^2, \text{localized lone pair}) > N(sp^2, \text{delocalized lone pair, aromatic}).$

1. **Piperidine**: Contains an $sp^3$-hybridized nitrogen atom with a localized lone pair, making it the strongest base.
2. **Pyridine**: Contains an $sp^2$-hybridized nitrogen atom with a localized lone pair, making it moderately basic.
3. **Pyrrole**: Contains an $sp^2$-hybridized nitrogen atom with a delocalized lone pair due to its involvement in the aromatic ring, making it the least basic.

Thus, the order of basic strength is:

$$\text{Piperidine} > \text{Pyridine} > \text{Pyrrole}.$$

**Final Answer:** $(1)$.

Quick Tips

1. **Basicity and Hybridization**: - Nitrogen with $sp^3$ hybridization has the highest basicity due to a localized lone pair. - $sp^2$-hybridized nitrogen has lower basicity due to reduced electron availability. - Delocalization of lone pairs (as in aromatic systems) further reduces basicity. 2. **Aromaticity in Pyrrole**: The lone pair on nitrogen contributes to the aromaticity, making it less available for protonation. 3. Memorize the order: Piperidine > Pyridine > Pyrrole.

**Question 65.**
In which one of the following pairs do the central atoms exhibit $sp^2$ hybridization?

1. $BF_3$ and $NO_2^-$
2. $NH_2^-$ and $H_2O$
3. $H_2O$ and $NO_2$
4. $NH_2^-$ and $BF_3$

**Correct Answer:** (1)

**Solution:**

Explanation: 1. **$BF_3$**: The central atom, boron, forms three sigma bonds with fluorine and has no lone pairs. It undergoes $sp^2$ hybridization.

2. **$NO_2^-$**: The nitrogen atom has three regions of electron density (two sigma bonds and one lone pair). This corresponds to $sp^2$ hybridization.

3. **$H_2O$**: The central atom, oxygen, has two sigma bonds and two lone pairs, corresponding to $sp^3$ hybridization.

4. **$NH_2^-$**: The nitrogen atom forms two sigma bonds and has two lone pairs, corresponding to $sp^3$ hybridization.

Thus, only $BF_3$ and $NO_2^-$ exhibit $sp^2$ hybridization.

**Final Answer:** $(1)$.

**Quick Tips**

1. **Hybridization Rule**: Count the number of sigma bonds and lone pairs around the central atom: - $sp$: 2 regions of electron density.

- $sp^2$: 3 regions of electron density.
- $sp^3$: 4 regions of electron density.

2. **Examples**:

- $BF_3$: 3 sigma bonds, no lone pairs $\rightarrow sp^2$.
- $NO_2^-$: 2 sigma bonds, 1 lone pair $\rightarrow sp^2$.
- $H_2O$: 2 sigma bonds, 2 lone pairs $\rightarrow sp^3$.
- $NH_2^-$: 2 sigma bonds, 2 lone pairs $\rightarrow sp^3$.

3. **Shortcut**: For planar molecules or ions, suspect $sp^2$ hybridization.

**Question 66.**

The $F^-$ ions make the enamel on teeth much harder by converting hydroxyapatite (the enamel on the surface of teeth) into much harder fluoroapatite having the formula:

1. $[3(Ca_3(PO_4)_2 \cdot CaF_2)]$

2. $[3(Ca_2(PO_4)_2 \cdot Ca(OH)_2)]$

3. $[3(Ca_3(PO_4)_3 \cdot CaF_2)]$

4. $[3(Ca_3(PO_4)_2 \cdot Ca(OH)_2)]$

**Correct Answer:** (1)

**Solution:**

Explanation: - Hydroxyapatite, $[Ca_5(PO_4)_3(OH)]$, is present in tooth enamel. - When fluoride ions ($F^-$) are introduced, they replace the hydroxide ions ($OH^-$), forming fluoroapatite:

$$Ca_5(PO_4)_3(OH) + F^- \rightarrow Ca_5(PO_4)_3F.$$

- Fluoroapatite has the molecular formula $[3(Ca_3(PO_4)_2 \cdot CaF_2)]$, which is harder and more resistant to decay.

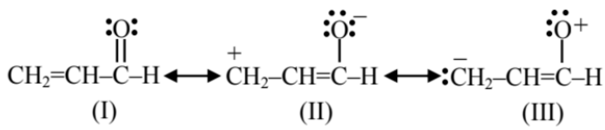
**Final Answer:** $(1)$.

Quick Tips

1. **Tooth Chemistry**: - Hydroxyapatite: $[Ca_5(PO_4)_3(OH)]$ is the primary component of tooth enamel. - Fluoroapatite: $[Ca_5(PO_4)_3F]$ is formed when fluoride replaces $OH^-$. 2. **Hardness Increase**: - Fluoroapatite is more stable and less soluble in acidic environments compared to hydroxyapatite, providing resistance to cavities. 3. **Shortcut**: - Remember the role of fluoride ($F^-$) in strengthening teeth by forming fluoroapatite.

**Question 67.**

The relative stability of the contributing structures is:




1. (I) > (III) > (II)

2. (I) > (II) > (III)

3. (II) > (I) > (III)

4. (III) > (II) > (I)

**Correct Answer:** (2)

**Solution:**

Explanation: 1. **Neutral structures are more stable** than charged ones. Therefore, structure (I) is the most stable as it is neutral.

2. Among the charged structures, a **positive charge (+) on a less electronegative atom** (like carbon) is more stable than a positive charge on a more electronegative atom (like oxygen).

- Hence, structure (II), where $C^+$ is present, is more stable than (III), where $O^+$ is present.

$$\text{Order: } I > II > III.$$

**Final Answer:** $(\mathbf{2})$.

**Quick Tips**

1. **Neutrality Rule**: Neutral resonance structures are always more stable than charged ones. 2. **Electronegativity Consideration**: Positive charges are more stable on less electronegative atoms. 3. **Resonance Priority**: Analyze the charge distribution and electronegativity to rank resonance structures effectively.

**Question 68.**

Given below are two statements:

- **Statement (I):** The oxidation state of an element in a particular compound is the charge acquired by its atom on the basis of electron gain enthalpy consideration from other atoms in the molecule.
- **Statement (II):** $p\pi - p\pi$ bond formation is more prevalent in second period elements over other periods.

In the light of the above statements, choose the **most appropriate** answer from the options given below:

1. Both **Statement I** and **Statement II** are incorrect.
2. **Statement I** is correct but **Statement II** is incorrect.

3. Both **Statement I** and **Statement II** are correct.

4. **Statement I** is incorrect but **Statement II** is correct.

**Correct Answer:** (4)

**Solution:**

Explanation: 1. **Statement (I): Incorrect.** - The oxidation state of an element is not determined based on electron gain enthalpy. Instead, it is based on the number of electrons lost, gained, or shared during bond formation.

2. **Statement (II): Correct.** - $p\pi - p\pi$ bond formation is more common in second-period elements (e.g., $C$, $N$, $O$) due to their small size and effective overlap of $p$-orbitals, which diminishes in larger elements from lower periods.

**Final Answer:** (4).

Quick Tips

1. Oxidation state is based on electron transfer/sharing in bonds, not electron gain enthalpy. 2. $p\pi - p\pi$ bonding is significant in smaller atoms like second-period elements due to their ability to form effective orbital overlaps. 3. For conceptual questions, analyze definitions and atomic properties carefully.

**Question 69.**

Given below are two statements: one is labelled as **Assertion (A)** and the other is labelled as **Reason (R)**:

- **Assertion (A):** $S_N2$ reaction of $C_6H_5CH_2Br$ occurs more readily than the $S_N2$ reaction of $CH_3CH_2Br$.
- **Reason (R):** The partially bonded unhybridized $p$-orbital that develops in the trigonal bipyramidal transition state is stabilized by conjugation with the phenyl ring.

In the light of the above statements, choose the **most appropriate** answer from the options given below:

1. (A) is not correct but (R) is correct.

2. Both (A) and (R) are correct but (R) is not the correct explanation of (A).

3. Both (A) and (R) are correct and (R) is the correct explanation of (A).

4. (A) is correct but (R) is not correct.

**Correct Answer:** (3)

**Solution:**

Explanation: 1. **Assertion (A):** Correct. - In $C_6H_5CH_2Br$, the $CH_2$-Br bond is connected to a benzyl group. The phenyl ring allows for stabilization of the transition state via resonance, facilitating the $S_N2$ reaction. This makes the reaction proceed more readily compared to $CH_3CH_2Br$, where no such stabilization exists.

2. **Reason (R):** Correct. - The unhybridized $p$-orbital formed during the trigonal bipyramidal transition state interacts with the conjugated system of the phenyl ring, providing extra stabilization.

3. **Conclusion:** Both (A) and (R) are correct, and (R) is the correct explanation for (A).

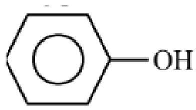
**Final Answer:** $(3)$.

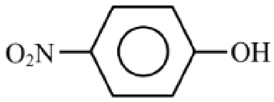
Quick Tips

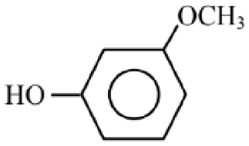
1. In $S_N2$ reactions, consider the factors stabilizing the transition state (e.g., resonance, inductive effects). 2. Resonance stabilization, such as in benzyl halides, enhances reaction rates. 3. For assertion-reason questions, ensure that the reason directly explains the assertion logically and scientifically.

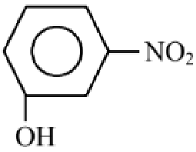
**Question 70.**

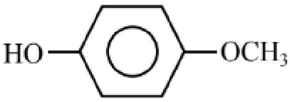
For the given compounds, the correct order of increasing $pK_a$ value is:

(A) 


(B) 


(C) 


(D) 


(E) 


1. (E) < (D) < (C) < (B) < (A)
2. (D) < (E) < (C) < (B) < (A)
3. (E) < (D) < (B) < (A) < (C)
4. (B) < (D) < (A) < (C) < (E)

**Correct Answer: BONUS (Originally: 4)**

**Solution:**

Explanation: 1. **Definition of $pK_a$:** Lower $pK_a$ corresponds to a stronger acid.
2. **Factors affecting $pK_a$:** - Electron-withdrawing groups (EWG) increase acidity by stabilizing the conjugate base. - Electron-donating groups (EDG) decrease acidity by destabilizing the conjugate base.
3. **Order Analysis:** - **(D) Dinitrophenol:** Strongest acid due to two nitro (EWG) groups stabilizing the phenoxide ion. - **(B) Nitrophenol:** Less acidic than (D), but still acidic due to one nitro group. - **(A) Phenol:** Less acidic than (B) due to the absence of strong EWGs. - **(C) Methoxyphenol:** The methoxy group (EDG) destabilizes the phenoxide ion, making it less acidic than phenol. - **(E) Methoxy-dinitrophenol:** The methoxy group reduces the effect of the nitro groups, making it less acidic than dinitrophenol but more

acidic than phenol.

4. **Final Order:**

$$(D) < (B) < (A) < (C) < (E).$$

**Final Answer:** BONUS.

**Quick Tips**

1. Compare the number and position of electron-withdrawing and electron-donating groups to determine acidity. 2. Strong acids have lower $pK_a$ values. 3. Nitro groups significantly enhance acidity, while methoxy groups reduce it.

**Question 71.**

**Assertion (A):** Both rhombic and monoclinic sulphur exist as $S_8$, while oxygen exists as $O_2$.
**Reason (R):** Oxygen forms $p\pi$-$p\pi$ multiple bonds with itself and other elements having small size and high electronegativity like $C, N$, which is not possible for sulphur.

In the light of the above statements, choose the **most appropriate answer** from the options given below:

1. Both (A) and (R) are correct and (R) is the correct explanation of (A).
2. Both (A) and (R) are correct but (R) is not the correct explanation of (A).
3. (A) is correct but (R) is not correct.
4. (A) is not correct but (R) is correct.

**Correct Answer:** (3)

**Solution:**

- Assertion (A) is correct because sulphur exists as $S_8$ due to its ability to form crown-shaped structures. Oxygen exists as $O_2$ due to its smaller size and ability to form $p\pi$-$p\pi$ bonds.
- Reason (R) is incorrect because sulphur's inability to form $p\pi$-$p\pi$ bonds is due to its larger size, which prevents effective overlap of orbitals.

- Thus, (A) is correct, but (R) is not the correct reason for (A).

**Quick Tips**

1. Recognize that $S_8$ is a stable allotrope of sulphur, while oxygen prefers $O_2$ due to its double bond. 2. Larger atomic size limits the ability of sulphur to form $p\pi$-$p\pi$ bonds. 3. Understand that high electronegativity and small size are key factors in forming $p\pi$-$p\pi$ bonds.

**Question 72.**

**Assertion (A):** The total number of geometrical isomers shown by $[Co(en)_2Cl_2]^+$ complex ion is three.

**Reason (R):** $[Co(en)_2Cl_2]^+$ complex ion has an octahedral geometry.

In the light of the above statements, choose the **most appropriate answer** from the options given below:

1. Both (A) and (R) are correct and (R) is the correct explanation of (A).
2. Both (A) and (R) are correct but (R) is not the correct explanation of (A).
3. (A) is correct but (R) is not correct.
4. (A) is not correct but (R) is correct.

**Correct Answer:** (3)

**Solution:**

- Assertion (A) is correct because $[Co(en)_2Cl_2]^+$ has three geometrical isomers: cis-cis, trans-trans, and cis-trans.
- Reason (R) is correct because $[Co(en)_2Cl_2]^+$ has an octahedral geometry.
- However, (R) does not explain (A), as the number of geometrical isomers is determined by the ligands' spatial arrangement, not just the octahedral geometry.
- Thus, both (A) and (R) are correct, but (R) is not the correct explanation of (A).

**Quick Tips**

1. Recall that geometrical isomerism in coordination complexes arises due to the spatial arrangement of ligands. 2. In octahedral geometry, the number of geometrical isomers depends on the type of ligands. 3. Note that en (ethylenediamine) is a bidentate ligand, leading to cis and trans forms. 4. Understand that the explanation of isomer count requires ligand arrangement, not geometry alone.

**Question 73.**

The electronic configuration of Cu(II) is $3d^9$, whereas that of Cu(I) is $3d^{10}$. Which of the following is correct?

1. Cu(II) is less stable.
2. Stability of Cu(I) and Cu(II) depends on the nature of copper salts.
3. Cu(II) is more stable.
4. Cu(I) and Cu(II) are equally stable.

**Correct Answer:** (3)

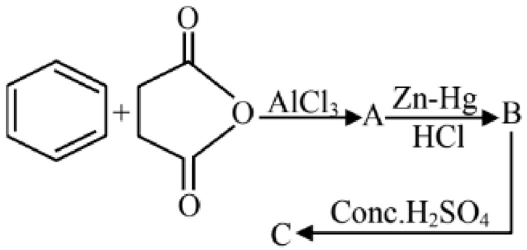
**Solution:**

- The stability of Cu(II) arises because it has a partially filled $d$-subshell ($3d^9$), which allows it to participate in ligand bonding and form complexes more effectively.
- Cu(I) with $3d^{10}$ is more stable in certain cases due to the completely filled $d$-subshell, but Cu(II) is generally more stable in aqueous solutions.
- The higher stability of Cu(II) over Cu(I) in aqueous medium is attributed to the higher hydration enthalpy of Cu(II) ions.

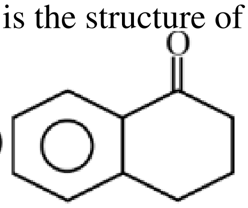
**Quick Tips**

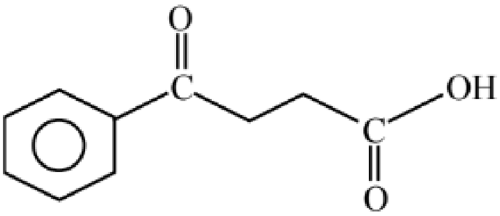
1. Stability of oxidation states depends on electronic configuration, hydration enthalpy, and ligand interactions. 2. Cu(II) is favored in aqueous media due to its higher hydration energy compared to Cu(I). 3. Partially filled $d$-orbitals in Cu(II) contribute to stronger ligand bonding in complexes.

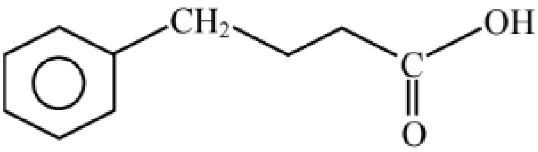
**Question 74.**

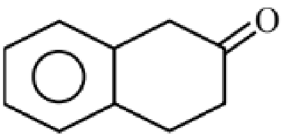



What is the structure of C?

(1) 

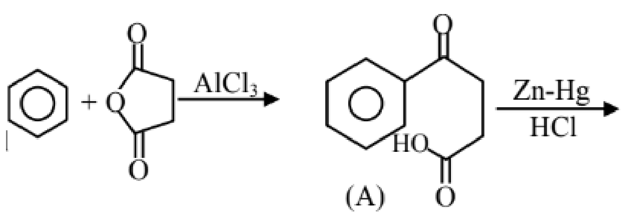

(2) 

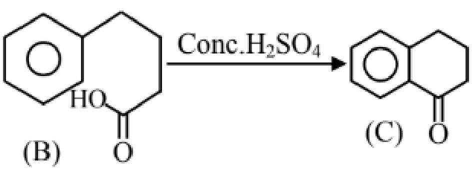

(3) 


(4) 


**Correct Answer: 1**

**Solution:**



Quick Tips

1. Recognize the key reactions: - Friedel-Crafts acylation for aromatic ketones or anhydrides. - Clemmensen reduction to reduce carbonyl groups to methylene. - Sulfonation as an electrophilic substitution reaction. 2. Analyze stepwise changes in the functional groups of the compound. 3. Sulfonation occurs preferentially at the para position when steric hindrance is absent.

**Question 75.**

Compare the energies of the following sets of quantum numbers for a multielectron system:

- (A) $n = 4, l = 1$
- (B) $n = 4, l = 2$
- (C) $n = 3, l = 1$
- (D) $n = 3, l = 2$
- (E) $n = 4, l = 0$

Choose the correct order of energies:

1. (B) $>$ (A) $>$ (C) $>$ (E) $>$ (D)
2. (E) $>$ (C) $<$ (D) $<$ (A) $<$ (B)
3. (E) $>$ (C) $>$ (A) $>$ (D) $>$ (B)
4. (C) $<$ (E) $<$ (D) $<$ (A) $<$ (B)

**Correct Answer:** (4)

**Solution:**

In multielectron systems, the energy of an electron in an orbital depends on both the principal quantum number ($n$) and the azimuthal quantum number ($l$). The energy increases as the value of $n + l$ increases. For orbitals with the same $n + l$, the one with the lower $n$ has lower energy.

- (A) $n + l = 4 + 1 = 5$
- (B) $n + l = 4 + 2 = 6$

- (C) $n + l = 3 + 1 = 4$
- (D) $n + l = 3 + 2 = 5$
- (E) $n + l = 4 + 0 = 4$

Order by $n + l$:

$$(C) = (E) < (D) < (A) < (B)$$

For orbitals with the same $n + l$, compare $n$:

$$(C) < (E), \text{ as } n = 3 \text{ for (C) and } n = 4 \text{ for (E).}$$

**Final Order:** $(C) < (E) < (D) < (A) < (B)$.

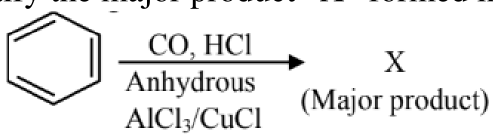
**Final Answer:** (4)

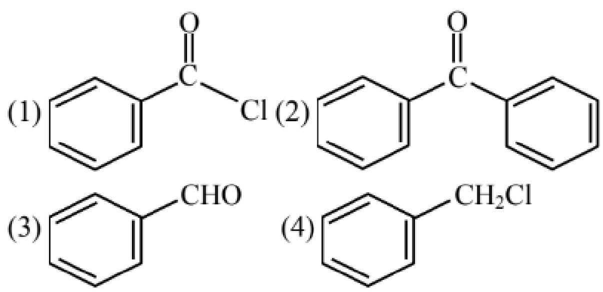
**Quick Tips**

1. For multielectron systems, energy depends on $n + l$ (higher $n + l$, higher energy). 2. For equal $n + l$, lower $n$ corresponds to lower energy. 3. Always calculate $n + l$ values for comparison. 4. Pay close attention to the principal quantum number ($n$) when $n + l$ is the same.

**Question 76.**

Identify the major product "X" formed in the following reaction:




**Correct Answer:** (3)

**Solution:**

The reaction is the **Gattermann–Koch reaction**, which involves the formylation of benzene to produce benzaldehyde ($C_6H_5CHO$). In the presence of carbon monoxide (CO), hydrogen chloride (HCl), and anhydrous $AlCl_3/CuCl$, the formyl group (CHO) is introduced into the benzene ring.

$$C_6H_6 + CO + HCl \xrightarrow{\text{AlCl}_3/\text{CuCl}} C_6H_5CHO$$

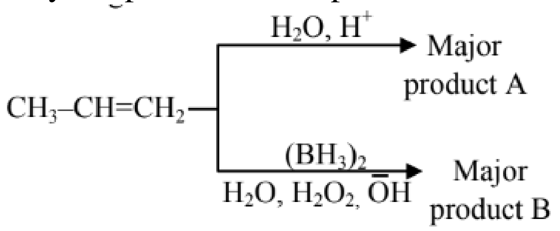
Thus, the major product $X$ is benzaldehyde (3).

**Final Answer:** (3)

> **Quick Tips**
>
> 1. The Gattermann–Koch reaction is used to introduce a formyl group (CHO) into an aromatic ring. 2. Requires CO, HCl, and $AlCl_3/CuCl$ as catalysts. 3. Useful for synthesizing benzaldehyde derivatives. 4. Ensure to differentiate between CHO, COOH, and C=O functionalities based on the reaction conditions.

**Question 77.**

Identify the product A and product B in the following set of reactions:




(1) A-$CH_3CH_2CH_2$–OH, B-$CH_3CH_2CH_2$–OH

(2) A-$CH_3CH_2CH_2$–OH, B-$CH_3CH(OH)$–$CH_3$

(3) A- $CH_3$–$CH(OH)$–$CH_3$ , B-$CH_3CH_2CH_2$–OH

(4) A-$CH_3CH_2CH_3$, B-$CH_3CH_2CH_3$

**Correct Answer:** (3)

**Solution:**

**(1) Hydration Reaction :**

$$CH_3-CH=CH_2+H^+ \longrightarrow \underset{\text{(More stable)}}{CH_3-\overset{+}{C}H-CH_3}$$

$$CH_3-\overset{+}{C}H-CH_3+H_2O \longrightarrow \underset{\text{(A)}}{(CH_3-\underset{\displaystyle OH}{\underset{|}{C}H}-CH_3)}+H^+$$

**(2) Hydroboration Oxidation Reaction :**

$$3CH_3-CH=CH_2+B_2H_6 \xrightarrow{THF} 2(CH_3CH_2CH_2)_3B$$

$$(CH_3CH_2CH_2)_3B+3H_2O_2 \xrightarrow{OH^-} \underset{\text{(B)}}{3CH_3CH_2CH_2OH}+H_3BO_3$$

**Final Answer:** (3)

**Quick Tips**

1. **Markovnikov's Rule:** In acid-catalyzed hydration, the hydroxyl group attaches to the more substituted carbon atom. 2. **Anti-Markovnikov's Rule:** In hydroboration-oxidation, the hydroxyl group attaches to the less substituted carbon atom. 3. Understand regioselectivity for reactions involving alkenes to predict products accurately. 4. Hydroboration-oxidation is a syn-addition process, but in this context, we are only concerned about regioselectivity.

**Question 78.**

On reaction of Lead Sulphide with dilute nitric acid which of the following is **not** formed?

1. Lead nitrate
2. Sulphur
3. Nitric oxide
4. Nitrous oxide

**Correct Answer:** (4)

**Solution:**

The reaction between Lead Sulphide (PbS) and dilute nitric acid ($HNO_3$) proceeds as follows:

$$PbS + 2HNO_3 \to Pb(NO_3)_2 + NO + S + H_2O.$$

Key Points: 1. The products of this reaction are: - Lead nitrate ($Pb(NO_3)_2$) – a soluble salt. - Nitric oxide (NO) – a gaseous product. - Sulphur (S) – a precipitate. - Water ($H_2O$). 2. Nitrous oxide ($N_2O$) is **not** formed during this reaction.

**Final Answer:** (4)

**Quick Tips**

1. Reactions of sulphides with acids often produce free sulphur and other gaseous products. 2. Dilute nitric acid typically produces NO, not $N_2O$, in redox reactions. 3. Be familiar with products of reactions involving sulphides and acids to answer such questions.

**Question 79.**

Identify the **incorrect** statements regarding the primary standard of titrimetric analysis:

**(A)** It should be purely available in dry form.

**(B)** It should not undergo chemical change in air.

**(C)** It should be hygroscopic and should react with another chemical instantaneously and stoichiometrically.

**(D)** It should be readily soluble in water.

**(E)** $KMnO_4$ and NaOH can be used as primary standards.

Choose the correct answer from the options given below:

1. (C) and (D) only
2. (B) and (E) only
3. (A) and (B) only
4. (C) and (E) only

**Correct Answer:** (4)

**Solution:**

A primary standard substance used in titrimetric analysis must satisfy the following criteria: 1. It should be available in **pure** and **dry form**. 2. It must not be hygroscopic (i.e., it should not absorb moisture) or undergo any chemical change in air. 3. It should be **readily soluble** in water. 4. $KMnO_4$ and NaOH are **not** suitable as primary standards due to their hygroscopic nature and tendency to absorb $CO_2$ from air, making them impure.

Incorrect Statements:

- **(C):** The substance **should not be hygroscopic**. Hence, this statement is incorrect.
- **(E):** Both $KMnO_4$ and NaOH are unsuitable as primary standards. This statement is also incorrect.

**Final Answer:** (4)

**Quick Tips**

1. A primary standard must be chemically pure, stable, and non-hygroscopic. 2. It should not absorb moisture or $CO_2$ from the air. 3. Substances like NaOH and $KMnO_4$ are **secondary standards** due to their instability.

**Question 80.**

0.05M $CuSO_4$ when treated with 0.01M $K_2Cr_2O_7$ gives a green-colored solution of $Cu_2Cr_2O_7$.

[SPM: Semi-Permeable Membrane]

| | |
|---|---|
| $K_2Cr_2O_7$ | $CuSO_4$ |

Side X SPM Side Y

Due to osmosis:

1. Green color formation observed on side Y.
2. Green color formation observed on side X.

3. Molarity of $K_2Cr_2O_7$ solution is lowered.

4. Molarity of $CuSO_4$ solution is lowered.

**Correct Answer:** (4)

**Solution:**

Due to osmosis, the solvent molecules from side Y ($CuSO_4$) diffuse across the semi-permeable membrane (SPM) into side X ($K_2Cr_2O_7$). This dilution effect causes the molarity of the $CuSO_4$ solution (side Y) to decrease over time.

- $CuSO_4$ reacts with $K_2Cr_2O_7$, forming $Cu_2Cr_2O_7$, which is responsible for the green color.
- The green color is observed due to the formation of $Cu_2Cr_2O_7$, but the key change is in the molarity of the $CuSO_4$ solution on side Y, which decreases as the solvent diffuses.

**Final Answer:** (4)

**Quick Tips**

1. Osmosis causes solvent movement from lower to higher solute concentration through an SPM. 2. Chemical reactions involving diffusion across SPM change molarity on either side. 3. For $CuSO_4$, molarity decreases on side Y as solvent moves to side X. 4. Always analyze solvent flow and its impact on molarity for osmosis-related reactions.

**Question 81.**

The heat of solution of anhydrous $CuSO_4$ and $CuSO_4{\cdot}5H_2O$ are $-70\,\text{kJ mol}^{-1}$ and $+12\,\text{kJ mol}^{-1}$, respectively.

The heat of hydration of $CuSO_4$ to $CuSO_4 \cdot 5H_2O$ is $-x\,\text{kJ}$. The value of $x$ is .......

**Correct Answer:** (82)

**Solution:**

The heat of hydration of $CuSO_4$ to $CuSO_4 \cdot 5H_2O$ is given by the difference between the heat of solution of anhydrous $CuSO_4$ and $CuSO_4 \cdot 5H_2O$.

Heat of hydration = Heat of solution of anhydrous $CuSO_4$ − Heat of solution of $CuSO_4 \cdot 5H_2O$.

Substitute the values:

$$x = \left| -70\,\text{kJ mol}^{-1} - (+12\,\text{kJ mol}^{-1}) \right|.$$

$$x = |-70 - 12| = |-82| = 82.$$

**Final Answer:** $82\,\text{kJ}$.

**Quick Tips**

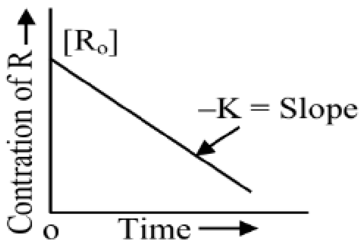
1. The heat of hydration is calculated by subtracting the heat of solution of the hydrated salt from the anhydrous salt. 2. Always account for the signs of the heat values when performing the calculation. 3. Ensure the correct absolute value is taken for the final result. 4. Review the definitions of heat of solution and hydration for clarity in solving similar problems.

**Question 82.**

**Statement I:**

The rate law for the reaction $A + B \to C$ is rate $(r) = k[A]^2[B]$. When the concentration of both $A$ and $B$ is doubled, the reaction rate is increased "$x$" times.

**Statement II:**




The figure shows the variation in concentration against time ($t$) for a "$y$" order reaction. The value of $x + y$ is ......

**Correct Answer:** (8)

**Solution:**

Step 1: Analyze Statement I The rate law is:

$$r = k[A]^2[B].$$

When the concentrations of $A$ and $B$ are doubled:

$$r' = k[2A]^2[2B] = k(2^2)[A]^2(2)[B].$$

$$r' = 8\,k[A]^2[B].$$

Thus, $r' = 8r$, so $x = 8$.

—

Step 2: Analyze Statement II From the figure, the concentration decreases linearly with time. A linear decrease in concentration indicates a **zero-order reaction** ($y = 0$).

—

Final Step: Calculate $x + y$

$$x + y = 8 + 0 = 8.$$

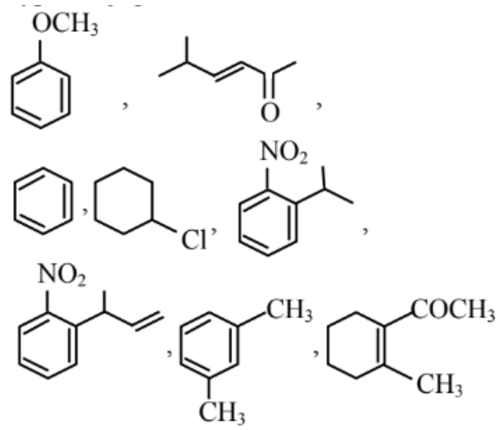
**Final Answer:** 8.

Quick Tips

1. For rate law questions, substitute doubled concentrations into the rate expression to calculate the change in rate. 2. A linear concentration-time graph corresponds to a zero-order reaction. 3. Understand how the order of a reaction affects its graphical representation. 4. Sum contributions from individual statements to get the final result.

**Question 83.**

How many compounds among the following compounds show inductive, mesomeric, as well as hyperconjugation effects?



**Correct Answer:** (4)

**Solution:**

Step 1: Analyze each compound - **Compound 1 ($-OCH_3$ group attached):** The $-OCH_3$ group exhibits both inductive ($-I$) and mesomeric ($+M$) effects. However, hyperconjugation is not applicable here. **Not included.**

- **Compound 2 (Alkene with $-CH_3$):** The $-CH_3$ group shows inductive ($+I$) and hyperconjugation effects due to the presence of alpha hydrogens. Mesomeric effect is not present. **Not included.**

- **Compound 3 ($-NO_2$ group):** The $-NO_2$ group exhibits both $-I$ (inductive) and $-M$ (mesomeric) effects but does not participate in hyperconjugation. **Not included.**

- **Compound 4 ($-Cl'$ group):** The $-Cl'$ group shows $-I$ (inductive) and $+M$ (mesomeric) effects, but no hyperconjugation. **Not included.**

- **Compound 5 ($-NO_2$ group):** Similar to Compound 3, exhibits inductive and mesomeric effects but not hyperconjugation. **Not included.**

- **Compound 6 ($-CH_3$ groups attached to a benzene ring):** The $-CH_3$ groups exhibit inductive ($+I$) effects and hyperconjugation due to alpha hydrogens. Mesomeric effect is also observed with the aromatic ring. **Included.**

- **Compound 7 ($-COCH_3$ group):** The $-COCH_3$ group exhibits inductive ($-I$) and mesomeric ($-M$) effects. However, no hyperconjugation is observed. **Not included.**

—

Step 2: Final Count From the analysis, only **4 compounds** exhibit all three effects: inductive, mesomeric, and hyperconjugation.

**Final Answer:** 4.

Quick Tips

1. Identify inductive effects ($+I$ or $-I$) based on the electron-withdrawing or donating nature of the group. 2. Mesomeric effects ($+M$ or $-M$) occur due to resonance involving lone pairs or conjugated systems. 3. Hyperconjugation requires the presence of alpha hydrogens adjacent to a double bond or aromatic ring. 4. Analyze each compound individually for all three effects before counting.

**Question 84.**

The standard reduction potentials at 298 K for the following half-cells are given below:

$$\mathrm{Cr_2O_7^{2-} + 14H^+ + 6e^- \rightarrow 2Cr^{3+} + 7H_2O},\ E^\circ = 1.33\,\mathrm{V}$$

$$\mathrm{Fe^{3+}}(aq) + \mathrm{3e^-} \rightarrow \mathrm{Fe},\ E^\circ = -0.04\,\mathrm{V}$$

$$\mathrm{Ni^{2+}}(aq) + \mathrm{2e^-} \rightarrow \mathrm{Ni},\ E^\circ = -0.25\,\mathrm{V}$$

$$\mathrm{Ag^{+}}(aq) + \mathrm{e^-} \rightarrow \mathrm{Ag},\ E^\circ = 0.80\,\mathrm{V}$$

$$\mathrm{Au^{3+}}(aq) + \mathrm{3e^-} \rightarrow \mathrm{Au},\ E^\circ = 1.40\,\mathrm{V}$$

Consider the given electrochemical reactions. The number of metal(s) which will be oxidized by $\mathrm{Cr_2O_7^{2-}}$ in aqueous solution is

**Correct Answer:** (3)

**Solution:**

Metals with lower standard reduction potentials ($E^\circ$) compared to $\mathrm{Cr_2O_7^{2-}}$ ($E^\circ = 1.33\,\mathrm{V}$) will be oxidized. These are:

- Fe ($E^\circ = -0.04\,\mathrm{V}$),
- Ni ($E^\circ = -0.25\,\mathrm{V}$),
- Ag ($E^\circ = 0.80\,\mathrm{V}$).

Thus, the number of metals oxidized is **3**.

**Final Answer:** (3)

Quick Tips

1. Compare the standard reduction potential ($E^\circ$) of species to determine their relative oxidation or reduction. 2. Species with lower $E^\circ$ are oxidized by those with higher $E^\circ$. 3. Pay attention to the order of half-cell potentials in the problem.

**Question 85.**

When equal volume of 1M HCl and 1M $H_2SO_4$ are separately neutralized by excess volume of 1M NaOH, $X$ and $Y$ kJ of heat is liberated respectively. The value of $\frac{Y}{X}$ is

**Correct Answer:** (2)

**Solution:**

Neutralization reactions:

$$H^+ + OH^- \to H_2O \quad \text{(from HCl: } X\text{)},$$

$$2H^+ + 2OH^- \to 2H_2O \quad \text{(from } H_2SO_4 : Y\text{)}.$$

From the reactions:

$$Y = 2X \implies \frac{Y}{X} = 2.$$

**Final Answer: 2**

Quick Tips

1. The heat of neutralization depends on the number of moles of $H^+$ and $OH^-$. 2. For strong acids like HCl and $H_2SO_4$, consider the dissociation of $H_2SO_4$ into two $H^+$ ions. 3. Use the stoichiometry of reactions to find the ratio of heat liberated.

**Question 86.**

Molarity ($M$) of an aqueous solution containing $x$ g of anhydrous $CuSO_4$ in 500 mL solution at 32°C is $2 \times 10^{-1}$ M. Its molality will be ....... $\times 10^{-3}$ m (nearest integer). [Given: Density of the solution = $1.25$ g/mL].

**Correct Answer:** ( NTA : 81) BONUS

**Solution:**

**Given:**

- Molarity $(M)$ = $0.2\,\text{mol/L}$
- Volume of solution $(V_{\text{sol}})$ = $500\,\text{mL} = 0.5\,\text{L}$
- Density of solution $(d_{\text{sol}})$ = $1.25\,\text{g/mL}$
- Molar mass of $CuSO_4$ = $159.5\,\text{g/mol}$

—

**Step 1: Calculate the mass of the solution**

$$M_{\text{sol}} = V_{\text{sol}} \times d_{\text{sol}} = 500\,\text{mL} \times 1.25\,\text{g/mL} = 625\,\text{g}.$$

—

**Step 2: Calculate the mass of solute**

$$\text{Mass of solute } (\text{CuSO}_4) = M \times V_{\text{sol}} \times \text{Molar mass}.$$

$$= 0.2 \times 0.5 \times 159.5 = 15.95\,\text{g}.$$

—

**Step 3: Calculate the mass of the solvent**

$$\text{Mass of solvent} = \text{Mass of solution} - \text{Mass of solute}.$$

$$= 625 - 15.95 = 609.05\,\text{g} = 0.60905\,\text{kg}.$$

—

**Step 4: Calculate the molality**

$$m = \frac{\text{Moles of solute}}{\text{Mass of solvent (in kg)}} = \frac{0.1}{0.60905}.$$

$$m = 0.164\,\text{mol/kg} = 164 \times 10^{-3}\,\text{mol/kg}.$$

—

## Final Answer

The molality of the solution is:

$$\boxed{0.164\,\text{mol/kg (or } 164 \times 10^{-3}\ \text{mol/kg)}}.$$

**Quick Tips**

1. Use the relationship $M = \frac{\text{moles of solute}}{\text{volume of solution in litres}}$ to find the moles of solute. 2. Subtract the solute's mass from the total solution's mass to determine the solvent's mass. 3. Molality is defined as moles of solute per kilogram of solvent, so ensure proper unit conversion. 4. Approximate results to the nearest integer if specified in the question.

**Question 87.**

The total number of species from the following in which one unpaired electron is present, is .........

$$N_2,\ O_2,\ C_2^-,\ O_2^-,\ H_2^+,\ CN^-,\ He_2^+$$

**Correct Answer:** (4)

**Solution:**

Step 1: Analyze each species for unpaired electrons 1. **$N_2$:** All electrons are paired. 2. **$O_2$:** Two unpaired electrons are present in antibonding orbitals. 3. **$C_2^-$:** One unpaired electron is present. 4. **$O_2^-$:** One unpaired electron is present. 5. **$H_2^+$:** One unpaired electron is present. 6. **$CN^-$:** All electrons are paired. 7. **$He_2^+$:** One unpaired electron is present.

Step 2: Count the species with one unpaired electron Species with one unpaired electron:

$$C_2^-,\ O_2^-,\ H_2^+,\ He_2^+$$

Total number = 4.

**Final Answer:** 4.

**Quick Tips**

1. Use molecular orbital theory (MOT) to determine the presence of unpaired electrons.
2. Remember common molecular orbital arrangements: - $O_2$ has two unpaired electrons. - $H_2^+$ and $He_2^+$ each have one unpaired electron. 3. For charged species ($C_2^-$ or $O_2^-$), add or remove electrons accordingly before analyzing.

**Question 88.**

Number of ambidentate ligands among the following is........

$$NO_2^-, SCN^-, C_2O_4^{2-}, NH_3, CN^-, SO_4^{2-}, H_2O.$$

**Correct Answer:** (3)

**Solution:**

Step 1: Define ambidentate ligands Ambidentate ligands are ligands that can coordinate to a metal center through two different atoms.

Step 2: Analyze the ligands 1. **$NO_2^-$:** Ambidentate ligand; can bind through N or O. 2. **$SCN^-$:** Ambidentate ligand; can bind through S or N. 3. **$C_2O_4^{2-}$:** Not ambidentate; it is a bidentate ligand. 4. **$NH_3$:** Not ambidentate; monodentate ligand binding through N. 5. **$CN^-$:** Ambidentate ligand; can bind through C or N. 6. **$SO_4^{2-}$:** Not ambidentate; it is a bidentate ligand. 7. **$H_2O$:** Not ambidentate; monodentate ligand binding through O.

Step 3: Count ambidentate ligands Ambidentate ligands are:

$$NO_2^-, SCN^-, CN^-.$$

Total number = 3.

**Final Answer:** 3.

**Quick Tips**

1. Ambidentate ligands have more than one potential donor atom for binding, such as $NO_2^-$ (via N or O) or $SCN^-$ (via S or N). 2. Differentiate between bidentate ligands ($C_2O_4^{2-}$) and ambidentate ligands (can bind through two different atoms). 3. Monodentate ligands like $NH_3$ or $H_2O$ are not ambidentate.

**Question 89.**

Total number of essential amino acids among the given list of amino acids is ........

Arginine, Phenylalanine, Aspartic acid, Cysteine, Histidine, Valine, Proline.

**Correct Answer:** (4)

**Solution:**

Step 1: Identify essential amino acids Essential amino acids are those that cannot be synthesized by the human body and must be obtained from the diet. From the given list: 1. Arginine - Essential. 2. Phenylalanine - Essential. 3. Histidine - Essential. 4. Valine - Essential. 5. Aspartic acid, Cysteine, and Proline are non-essential.

Step 2: Count essential amino acids Essential amino acids in the list = 4.

**Final Answer:** 4.

Quick Tips

1. Essential amino acids must be consumed via diet; common examples include Valine, Histidine, and Phenylalanine. 2. Non-essential amino acids like Aspartic acid can be synthesized by the body. 3. Familiarize yourself with the complete list of essential amino acids for such questions.

**Question 90.**

Number of colourless lanthanoid ions among the following is ........

$$Eu^{3+}, Lu^{3+}, Nd^{3+}, La^{3+}, Sm^{3+}.$$

**Correct Answer:** (2)

**Solution:**

Step 1: Analyze electronic configurations 1. $La^{3+}$: $[Xe]4f^0$ (No unpaired electrons, colourless). 2. $Lu^{3+}$: $[Xe]4f^{14}$ (No unpaired electrons, colourless). 3. $Nd^{3+}$: $[Xe]4f^3$ (Unpaired electrons, coloured). 4. $Sm^{3+}$: $[Xe]4f^5$ (Unpaired electrons, coloured). 5. $Eu^{3+}$: $[Xe]4f^6$ (Unpaired electrons, coloured).

Step 2: Count colourless ions Lanthanide ions that are colourless: $La^{3+}$ and $Lu^{3+}$.

**Final Answer:** 2.

Quick Tips

1. Colour of lanthanoid ions depends on the presence of unpaired electrons in their 4f orbitals. 2. Ions with all paired electrons ($4f^0$ or $4f^{14}$) are colourless, while those with unpaired electrons are coloured. 3. For such questions, memorize the electronic configurations of common lanthanoid ions.