JEE Main 2024 Jan 27 (Shift 1) Question Paper with Solutions

Question 1. $^{n-1}C_r = (k^2 - 8)\ ^nC_{r+1}$ **if and only if:**

(1) $2\sqrt{2} < k \leq 3$

(2) $2\sqrt{3} < k \leq 3\sqrt{2}$

(3) $2\sqrt{3} < k < 3\sqrt{3}$

(4) $2\sqrt{2} < k < 2\sqrt{3}$

Answer: (1)$2\sqrt{2} < k \leq 3$

Solution

Given:

$$^{n-1}C_r = (k^2 - 8)\ ^nC_{r+1}$$

We know:

$$^{n-1}C_r = (k^2 - 8)\ ^nC_{r+1}$$

For this expression to hold, $k^2 - 8$ must be positive:

$$k^2 - 8 > 0 \Rightarrow k > 2\sqrt{2} \text{ or } k < -2\sqrt{2}$$

Thus,

$$k \in (-\infty, -2\sqrt{2}) \cup (2\sqrt{2}, \infty)$$

Next, we check the range $-3 \leq k \leq 3$ to satisfy the constraint. Combining both conditions:

$$k \in [2\sqrt{2}, 3]$$

Quick Tip

When dealing with inequalities involving squares, consider both positive and negative roots.

Question 2. The distance of the point $(7, -2, 11)$ from the line

$$\frac{x-6}{1} = \frac{y-4}{0} = \frac{z-8}{3}$$

along the line

$$\frac{x-5}{2} = \frac{y-1}{-3} = \frac{z-5}{6}$$

is:

(1) 12

(2) 14

(3) 18

(4) 21

Answer: (2)14

Solution:

To find the distance, we first determine the coordinates of point B lying on the line given by:

$$\frac{x-6}{1} = \frac{y-4}{0} = \frac{z-8}{3}$$

Assume:

$$B = (2\lambda + 7, -3\lambda - 2, 6\lambda + 11)$$

Point B also lies on the line:

$$\frac{x-6}{1} = \frac{y-4}{0} = \frac{z-8}{3}$$

Substitute the coordinates of B:

$$2\lambda + 7 - 6 = -3\lambda - 2 - 4 = 6\lambda + 11 - 8 = 0$$

Solving:

$$-3\lambda - 6 = 0 \implies \lambda = -2$$

Substituting $\lambda = -2$ gives:

$$B = (3, 4, -1)$$

To find the distance AB between points $A = (7, -2, 11)$ and $B = (3, 4, -1)$:

$$AB = \sqrt{(7-3)^2 + (-2-4)^2 + (11-(-1))^2}$$

$$AB = \sqrt{(4)^2 + (-6)^2 + (12)^2}$$

$$AB = \sqrt{16 + 36 + 144}$$

$$AB = \sqrt{196} = 14$$

Thus, the distance of the point $(7, -2, 11)$ from the given line is 14.

Quick Tip

To find the distance from a point to a line in 3D, use the formula involving direction ratios and a point on the line.

Question 3. Let $x = x(t)$ and $y = y(t)$ be solutions of the differential equations

$$\frac{dx}{dt} + ax = 0 \quad \text{and} \quad \frac{dy}{dt} + by = 0$$

respectively, $a, b \in \mathbb{R}$. Given that $x(0) = 2$, $y(0) = 1$ and $3y(1) = 2x(1)$, the value of t, for which $x(t) = y(t)$, is:

1. $\log_{\frac{2}{3}} 2$
2. $\log_4 3$
3. $\log_3 4$
4. $\log_4 \frac{2}{3}$

Answer: 4. $\log_4 \frac{2}{3}$

Solution

Given differential equations are:

$$\frac{dx}{dt} + ax = 0 \quad \Rightarrow \quad x(t) = x(0)e^{-at}$$
$$\frac{dy}{dt} + by = 0 \quad \Rightarrow \quad y(t) = y(0)e^{-bt}$$

From the initial conditions, we are provided:

$$x(0) = 2, \quad y(0) = 1$$

Thus, the solutions for $x(t)$ and $y(t)$ become:

$$x(t) = 2e^{-at}, \quad y(t) = e^{-bt}$$

Step 1: Relation at $t = 1$

We are given:

$$3y(1) = 2x(1)$$

Substituting the values of $x(1)$ and $y(1)$:

$$3e^{-b} = 2 \times 2e^{-a} \implies 3e^{-b} = 4e^{-a}$$

Taking the natural logarithm on both sides:

$$-b = -a + \ln\left(\frac{4}{3}\right)$$

Rearranging terms, we get:

$$b = a + \ln\left(\frac{4}{3}\right)$$

Step 2: Finding t for which $x(t) = y(t)$

We need to find the value of t such that:

$$2e^{-at} = e^{-bt}$$

Dividing both sides by e^{-bt}:

$$2 = e^{(b-a)t}$$

Taking the natural logarithm of both sides:

$$\ln 2 = (b-a)t$$

Substituting the expression for $b - a$ from earlier:

$$b - a = \ln\left(\frac{4}{3}\right)$$

Thus:

$$t = \frac{\ln 2}{\ln\left(\frac{4}{3}\right)}$$

Step 3: Simplifying the Expression

To simplify $\frac{\ln 2}{\ln\left(\frac{4}{3}\right)}$, we recognize that:

$$\log_4\left(\frac{2}{3}\right) = \frac{\ln\left(\frac{2}{3}\right)}{\ln 4} = \frac{\ln 2}{\ln\left(\frac{4}{3}\right)}$$

Thus, the value of t is:

$$t = \log_4\left(\frac{2}{3}\right)$$

Quick Tip

In problems involving exponential functions and differential equations, equate solutions at a specific time to find relationships between constants.

Question 4. If (a, b) be the orthocentre of the triangle whose vertices are $(1, 2)$, $(2, 3)$, and $(3, 1)$, and

$$I_1 = \int_a^b x\sin(4x - x^2)\,dx, \quad I_2 = \int_a^b \sin(4x - x^2)\,dx$$

then $36\frac{I_1}{I_2}$ is equal to:

(1) 72

(2) 88

(3) 80

(4) 66

Answer: (1)72

Solution

Given the triangle with vertices $A(1,2)$, $B(2,3)$, and $C(3,1)$, we proceed to find the orthocentre (a,b) which lies on the line $x+y=4$.

1. Equation of Line CE

The line passing through point $C(3,1)$ with slope -1 is given by:

$$y-1=-1(x-3) \implies y=-x+4$$

The equation of the line $x+y=4$ holds for the orthocentre (a,b). Therefore:

$$a+b=4$$

2. Evaluation of the Integral I_1

Consider the integral:

$$I_1=\int_a^b x\sin(x(4-x))dx \quad \text{...(i)}$$

3. Using the King's Rule

By applying the King's property of definite integrals, we have:

$$I_1=\int_a^b (4-x)\sin(x(4-x))dx \quad \text{...(ii)}$$

4. Combining the Results

Adding equations (i) and (ii), we obtain:

$$I_1+I_1=\int_a^b (x+(4-x))\sin(x(4-x))dx$$

Simplifying:

$$2I_1=\int_a^b 4\sin(x(4-x))dx$$

Therefore:

$$I_1 = 2\int_a^b \sin(x(4-x))dx$$

5. Ratio of Integrals

From the problem statement, we have:

$$\frac{I_1}{I_2} = 2$$

Calculating:

$$36 \times \frac{I_1}{I_2} = 36 \times 2 = 72$$

Hence, the value of $36\frac{I_1}{I_2}$ is 72.

Quick Tip

For problems involving orthocentres, find the altitudes by using the perpendicular slopes of the triangle's sides.

Question 5. If A denotes the sum of all the coefficients in the expansion of $(1 - 3x + 10x^2)^n$ and B denotes the sum of all the coefficients in the expansion of $(1 + x^2)^n$, then:

(1) $A = B^3$

(2) $3A = B$

(3) $B = A^3$

(4) $A = 3B$

Answer: $(1) A = B^3$

Solution

To find the sums A and B, we calculate the sum of all coefficients by setting $x = 1$ in each expansion.

1. Calculate A

Substitute $x = 1$ in $(1 - 3x + 10x^2)^n$:

$$A = (1 - 3 \cdot 1 + 10 \cdot 1^2)^n = (1 - 3 + 10)^n = 8^n$$

Therefore, $A = 8^n$.

2. Calculate B

Substitute $x = 1$ in $(1 + x^2)^n$:

$$B = (1 + 1^2)^n = 2^n$$

Thus, $B = 2^n$.

3. Find the Relationship Between A and B

Since $A = 8^n$ and $B = 2^n$, we can write:

$$A = (2^n)^3 = B^3$$

Therefore, $A = B^3$.

Quick Tip

For sums of coefficients in polynomial expansions, substitute $x = 1$ to simplify the expression.

Question 6. The number of common terms in the progressions 4, 9, 14, 19, ..., up to 25$^{\text{th}}$ term and 3, 6, 9, 12, ..., up to 37$^{\text{th}}$ term is:

(1) 9

(2) 5

(3) 7

(4) 8

Answer: (3)7

Solution:

Consider the two arithmetic progressions given:

- First series: $4, 9, 14, 19, \ldots$ up to the 25th term. - Second series: $3, 6, 9, 12, \ldots$ up to the 37th term.

1. Finding the 25th Term of the First Series

The first term $a_1 = 4$ and the common difference $d_1 = 5$. The general formula for the n-th term of an arithmetic progression is given by:

$$T_n = a_1 + (n-1) \cdot d_1$$

Therefore, the 25th term is:

$$T_{25} = 4 + (25-1) \cdot 5 = 4 + 120 = 124$$

2. Finding the 37th Term of the Second Series

The first term $a_2 = 3$ and the common difference $d_2 = 3$. The 37th term is given by:

$$T_{37} = 3 + (37-1) \cdot 3 = 3 + 108 = 111$$

3. Identifying Common Terms

The common terms between the two sequences must be in both progressions. The first common term is 9. The common difference for these terms is given by the least common multiple (LCM) of $d_1 = 5$ and $d_2 = 3$:

$$\text{LCM}(5, 3) = 15$$

Thus, the common terms form an arithmetic progression with the first term 9 and common difference 15.

4. List of Common Terms

The common terms are:

$$9, 24, 39, 54, 69, 84, 99$$

5. Number of Common Terms

There are 7 common terms.

Therefore, the number of common terms in the progressions is 7.

> **Quick Tip**
>
> To find common terms in two arithmetic sequences, identify the least common multiple of their common differences and use it to generate the common terms.

Question 7. If the shortest distance of the parabola $y^2 = 4x$ from the centre of the circle $x^2 + y^2 - 4x - 16y + 64 = 0$ is d, then d^2 is equal to:

(1) 16

(2) 24

(3) 20

(4) 36

Answer: (3)20

Solution

1. Rewrite the Equation of the Circle in Standard Form

Given the equation:

$$x^2 + y^2 - 4x - 16y + 64 = 0$$

Completing the square for the terms involving x and y:

$$(x^2 - 4x) + (y^2 - 16y) = -64$$

$$(x - 2)^2 - 4 + (y - 8)^2 - 64 = -64$$

Rearranging terms:

$$(x - 2)^2 + (y - 8)^2 = 4$$

Thus, the center of the circle is $(2, 8)$ and the radius is 2.

2. Find the Normal to the Parabola

Consider the parabola $y^2 = 4x$. Let the slope of the normal be m. The equation of the normal to the parabola is given by:

$$y = mx - 2m - m^3$$

Substitute the point $(2, 8)$ into the equation to find m:

$$8 = m \cdot 2 - 2m - m^3$$

Simplifying:

$$m^3 + 2m - 8 = 0$$

3. Calculate the Distance

The shortest distance is between the center $(2, 8)$ of the circle and the point on the parabola where the normal passes. Using the distance formula, we find:

$$d^2 = (x - 2)^2 + (y - 8)^2 = 20$$

Quick Tip

To find shortest distances involving a circle and a parabola, first locate the normal from the circle's center to the parabola.

Question 8. If the shortest distance between the lines

$$\frac{x-4}{1} = \frac{y+1}{2} = \frac{z}{-3} \quad \text{and} \quad \frac{x-\lambda}{2} = \frac{y+1}{4} = \frac{z-2}{-5}$$

is $\frac{6}{\sqrt{5}}$, then the sum of all possible values of λ is:

(1) 5

(2) 8

(3) 7

(4) 10

Answer: (2)8

Solution

Given:

$$\int_0^1 \frac{1}{\sqrt{3+x}+\sqrt{1+x}}dx = a + b\sqrt{2} + c\sqrt{3}$$

where a, b, c are rational numbers.

1. Simplifying the Integral: Consider:

$$\int_0^1 \frac{1}{\sqrt{3+x}+\sqrt{1+x}}dx$$

Rationalizing the denominator:

$$\int_0^1 \frac{\sqrt{3+x}-\sqrt{1+x}}{(3+x)-(1+x)}dx = \int_0^1 \frac{\sqrt{3+x}-\sqrt{1+x}}{2}dx$$

Therefore:

$$\frac{1}{2}\int_0^1 \left(\sqrt{3+x}-\sqrt{1+x}\right)dx$$

2. Separating the Integral:

$$\frac{1}{2}\left(\int_0^1 \sqrt{3+x}dx - \int_0^1 \sqrt{1+x}dx\right)$$

3. Evaluating Each Integral: - For $\int_0^1 \sqrt{3+x}dx$:

$$\int \sqrt{3+x}dx = \frac{2}{3}(3+x)^{3/2}$$

Evaluating from 0 to 1:

$$\frac{2}{3}(3+x)^{3/2}\Big|_0^1 = \frac{2}{3}\left[(4)^{3/2}-(3)^{3/2}\right] = \frac{2}{3}\left(8-3\sqrt{3}\right)$$

- For $\int_0^1 \sqrt{1+x}dx$:

$$\int \sqrt{1+x}dx = \frac{2}{3}(1+x)^{3/2}$$

Evaluating from 0 to 1:

$$\frac{2}{3}(1+x)^{3/2}\Big|_0^1 = \frac{2}{3}\left[(2)^{3/2}-(1)^{3/2}\right] = \frac{2}{3}\left(2\sqrt{2}-1\right)$$

4. Combining the Results:

$$\frac{1}{2}\left(\frac{2}{3}\left(8-3\sqrt{3}\right)-\frac{2}{3}\left(2\sqrt{2}-1\right)\right)$$

Simplifying:

$$\frac{1}{3}\left(8-3\sqrt{3}-2\sqrt{2}+1\right) = \frac{1}{3}\left(9-3\sqrt{3}-2\sqrt{2}\right)$$

Thus:

$$a = 3, \quad b = -\frac{2}{3}, \quad c = -1$$

5. Calculating $2a + 3b - 4c$:

$$2a + 3b - 4c = 2 \times 3 + 3 \times \left(-\frac{2}{3}\right) - 4 \times (-1)$$

$$= 6 - 2 + 4 = 8$$

Quick Tip

For complex integrals, split the expression and look for possible substitutions to simplify.

Question 9: Evaluate the integral

$$\int_0^1 \frac{1}{\sqrt{3+x} + \sqrt{1+x}}\, dx$$

Given that the integral can be expressed in the form $a + b\sqrt{2} + c\sqrt{3}$, where a, b, c are rational numbers, find the value of $2a + 3b - 4c$.

1. 4
2. 10
3. 7
4. 8

Answer: (4)8

Solution

Given:

$$\int_0^1 \frac{1}{\sqrt[3]{3+x} + \sqrt{1+x}}\, dx$$

Step 1: Rationalizing the Denominator

Rationalize the denominator:

$$\int_0^1 \frac{\sqrt[3]{3+x} - \sqrt{1+x}}{(\sqrt[3]{3+x})^2 - (\sqrt{1+x})^2}\,dx = \int_0^1 \frac{\sqrt[3]{3+x} - \sqrt{1+x}}{2}\,dx$$

Step 2: Separating the Integral

Separate the integral:

$$\frac{1}{2}\left(\int_0^1 \sqrt[3]{3+x}\,dx - \int_0^1 \sqrt{1+x}\,dx\right)$$

Step 3: Evaluating the Integrals

1. For $\int_0^1 \sqrt[3]{3+x}\,dx$:

$$\int_0^1 \sqrt[3]{3+x}\,dx = \frac{2}{3}(3+x)^{3/2}\Big|_0^1 = \frac{2}{3}\left((4)^{3/2} - (3)^{3/2}\right) = \frac{2}{3}\left(8 - 3\sqrt{3}\right)$$

2. For $\int_0^1 \sqrt{1+x}\,dx$:

$$\int_0^1 \sqrt{1+x}\,dx = \frac{2}{3}(1+x)^{3/2}\Big|_0^1 = \frac{2}{3}\left((2)^{3/2} - 1\right) = \frac{2}{3}\left(2\sqrt{2} - 1\right)$$

Step 4: Combining the Results

Combine the results:

$$\frac{1}{2}\left(\frac{2}{3}\left(8 - 3\sqrt{3}\right) - \frac{2}{3}\left(2\sqrt{2} - 1\right)\right) = a + b\sqrt{2} + c\sqrt{3}$$

From this, we find:

$$a = \frac{4}{3}, \quad b = -\frac{4}{3}, \quad c = -1$$

Calculate:

$$2a + 3b - 4c = 2\left(\frac{4}{3}\right) + 3\left(-\frac{4}{3}\right) - 4(-1) = 8$$

Quick Tip

When evaluating integrals involving roots, consider rationalizing the denominator or applying suitable substitutions to simplify the expressions. Separating terms can also make complex integrals more manageable.

Question 10. Let $S = \{1, 2, 3, \ldots, 10\}$. Suppose M is the set of all subsets of S, then the relation $R = \{(A, B) : A \cap B \neq \emptyset;\ A, B \in M\}$ is:

(1) symmetric and reflexive only

(2) reflexive only

(3) symmetric and transitive only

(4) symmetric only

Answer: (4)symmetric only

Solution

Let's analyze the properties of the relation R.

1. Reflexivity: - For reflexivity to hold, each subset A in M should satisfy $A \cap A \neq \emptyset$. - Since $A \cap A = A$, R would be reflexive if $A \neq \emptyset$ for every $A \in M$. - However, the empty set $\emptyset \in M$ does not satisfy $\emptyset \cap \emptyset \neq \emptyset$, so R is not reflexive.

2. Symmetry: - If $(A, B) \in R$, then $A \cap B \neq \emptyset$. - This implies $B \cap A \neq \emptyset$, so $(B, A) \in R$. - Therefore, R is symmetric.

3. Transitivity: - Suppose $(A, B) \in R$ and $(B, C) \in R$, meaning $A \cap B \neq \emptyset$ and $B \cap C \neq \emptyset$. - However, $A \cap C$ may still be empty, so R is not transitive.

Thus, the relation R is symmetric only.

Quick Tip

For set-based relations, check each property (reflexivity, symmetry, transitivity) separately using definitions and examples.

Question 11. If $S = \{z \in \mathbb{C} : |z - i| = |z + i| = |z - 1|\}$, then $n(S)$ is:

(1) 1

(2) 0

(3) 3

(4) 2

Answer: (1)1

Solution

1. Interpret the Condition $|z-i| = |z+i| = |z-1|$: - This condition implies that z is equidistant from the points $(0, 1)$, $(0, -1)$, and $(1, 0)$.

2. Geometric Interpretation: - The points $(0, 1)$, $(0, -1)$, and $(1, 0)$ form the vertices of an isosceles right triangle in the complex plane. - The circumcenter of this triangle (the unique point equidistant from all three vertices) is the only point that satisfies the condition.

3. Finding the Circumcenter: - The circumcenter of a triangle with vertices $(0, 1)$, $(0, -1)$, and $(1, 0)$ lies at the origin $(0, 0)$. - Therefore, $z = 0$ is the only solution, so $n(S) = 1$.

Quick Tip

For equidistance problems in the complex plane, consider geometric loci such as circumcenters or perpendicular bisectors.

Question 12. Four distinct points $(2k, 3k)$, $(1, 0)$, $(0, 0)$, and $(0, 1)$ lie on a circle for k equal to:

(1) $\frac{2}{13}$

(2) $\frac{3}{13}$

(3) $\frac{5}{13}$

(4) $\frac{1}{13}$

Answer: (3)$\frac{5}{13}$

Solution

Given four distinct points $(2k, 3k)$, $(1, 0)$, $(0, 1)$, and $(0, 0)$ lie on a circle. We need to find the value of k such that these points lie on the circle whose diameter is defined by points $A(1, 0)$ and $B(0, 1)$.

1. Equation of the Circle: The general equation of a circle with diameter AB is given by:

$$(x-1)(x)+(y-1)(y)=0$$

Expanding this gives:

$$x^2+y^2-x-y=0 \quad \text{...(i)}$$

2. Substituting Point $(2k,3k)$ into the Circle's Equation: To satisfy the equation, substitute $x=2k$ and $y=3k$ into equation (i):

$$(2k)^2+(3k)^2-2k-3k=0$$

Simplifying:

$$4k^2+9k^2-2k-3k=0$$

$$13k^2-5k=0$$

Factoring:

$$k(13k-5)=0$$

Therefore, the possible values of k are:

$$k=0 \quad \text{or} \quad k=\frac{5}{13}$$

3. Validating the Value of k: Since $k=0$ does not represent a distinct point, we have:

$$k=\frac{5}{13}$$

Answer: (3) $\frac{5}{13}$

Quick Tip

For four points to lie on a circle, use the circumcircle condition of a triangle formed by any three points and check the fourth.

Question 13 : Consider the function:

$$f(x)=\begin{cases} \frac{a(7x-12-x^2)}{b|x^2-7x+12|}, & x<3 \\ \frac{\sin(x-3)}{2^{x-\lfloor x\rfloor}}, & x>3 \\ b, & x=3 \end{cases}$$

Where $\lfloor x \rfloor$ denotes the greatest integer less than or equal to x. If S denotes the set of all ordered pairs (a, b) such that $f(x)$ is continuous at $x = 3$, then the number of elements in S is:

(1) 2

(2) Infinitely many

(3) 4

(4) 1

Answer: (4)

Solution

1. **Continuity Condition at $x = 3$**: For $f(x)$ to be continuous at $x = 3$, we must have:

$$f(3^-) = f(3) = f(3^+)$$

2. **Calculate $f(3^-)$**: For $x < 3$,

$$f(x) = \frac{a(7x - 12 - x^2)}{b|x^2 - 7x + 12|} = \frac{-a(x-3)(x-4)}{b(x-3)(x-4)} = -\frac{a}{b}$$

   So, $f(3^-) = -\frac{a}{b}$.

3. **Calculate $f(3^+)$**: For $x > 3$,

$$f(x) = \frac{\sin(x-3)}{2^{x - \lfloor x \rfloor}} \Rightarrow \lim_{x \to 3^+} f(x) = 2$$

4. **Set Up Continuity Condition**: Since $f(3^-) = f(3) = f(3^+)$,

$$-\frac{a}{b} = 2 \quad \text{and} \quad b = 2 \Rightarrow a = -4$$

   Therefore, the only solution is $(a, b) = (-4, 2)$.

Quick Tip

For continuity at a point, make sure the left-hand limit, right-hand limit, and function value at that point are equal.

Question 14 : Let $a_1, a_2, \ldots, a_{10}$ be 10 observations such that

$$\sum_{k=1}^{10} a_k = 50 \quad \text{and} \quad \sum_{1 \le k < j \le 10} a_k \cdot a_j = 1100.$$

Then the standard deviation of $a_1, a_2, \ldots, a_{10}$ is equal to:

(1) 5

(2) $\sqrt{5}$

(3) 10

(4) $\sqrt{115}$

Answer: (2)

Solution

1. **Use the Formula for Standard Deviation**:

$$\sigma = \sqrt{\frac{1}{n}\sum_{i=1}^{10} a_i^2 - \left(\frac{1}{n}\sum_{i=1}^{10} a_i\right)^2}$$

   where $n = 10$.

2. **Calculate $\sum_{i=1}^{10} a_i^2$**: We know:

$$\sum_{i=1}^{10} a_i = 50, \qquad \sum_{1 \le k < j \le 10} a_k \cdot a_j = 1100$$

   Expanding:

$$\left(\sum_{i=1}^{10} a_i\right)^2 = \sum_{i=1}^{10} a_i^2 + 2 \sum_{1 \le k < j \le 10} a_k \cdot a_j$$

$$2500 = \sum_{i=1}^{10} a_i^2 + 2200 \Rightarrow \sum_{i=1}^{10} a_i^2 = 300$$

3. **Compute Standard Deviation**:

$$\sigma = \sqrt{\frac{300}{10} - \left(\frac{50}{10}\right)^2} = \sqrt{30 - 25} = \sqrt{5}$$

Quick Tip

For standard deviation, use the relation $\sigma = \sqrt{\frac{1}{n}\sum x_i^2 - \left(\frac{1}{n}\sum x_i\right)^2}$ when the pairwise product sum is given.

Question 15 : The length of the chord of the ellipse $\frac{x^2}{25} + \frac{y^2}{16} = 1$, whose midpoint is $\left(1, \frac{2}{5}\right)$, is equal to:

(1) $\frac{\sqrt{1691}}{5}$

(2) $\frac{\sqrt{2009}}{5}$

(3) $\frac{\sqrt{1741}}{5}$

(4) $\frac{\sqrt{1541}}{5}$

Answer: (1)

Solution

Given the ellipse:

$$\frac{x^2}{25} + \frac{y^2}{16} = 1$$

and a chord with midpoint $\left(1, \frac{2}{5}\right)$.

1. Equation of the Chord The chord equation is:

$$\frac{x}{25} + \frac{y}{40} = 1 \quad \Rightarrow \quad y = \frac{200 - 8x}{5}$$

2. Substitute into the Ellipse Substituting y gives:

$$\frac{x^2}{25} + \frac{\left(\frac{200-8x}{5}\right)^2}{16} = 1$$

Simplifying:

$$2x^2 - 80x + 990 = 0 \quad \Rightarrow \quad x = 20 \pm \sqrt{10}$$

3. Length of the Chord Using distance formula, the length is:

$$\text{Length} = \frac{\sqrt{1691}}{5}$$

Quick Tip

To find the length of a chord with a given midpoint, use the parametric form and the midpoint condition to derive the equation.

Question 16 : The portion of the line $4x + 5y = 20$ in the first quadrant is trisected by the lines L_1 and L_2 passing through the origin. The tangent of an angle between the lines L_1 and L_2 is:

(1) $\frac{8}{5}$
(2) $\frac{25}{41}$
(3) $\frac{2}{5}$
(4) $\frac{30}{41}$

Answer: (4)

Solution

1. Identify the Points Where the Line Intersects the Axes: - The line $4x + 5y = 20$ intersects the x-axis when $y = 0$:

$$4x = 20 \Rightarrow x = 5$$

So, the x-intercept is $(5, 0)$. - The line intersects the y-axis when $x = 0$:

$$5y = 20 \Rightarrow y = 4$$

So, the y-intercept is $(0, 4)$.

2. Determine the Coordinates of the Trisection Points: - The line segment from $(5, 0)$ to $(0, 4)$ in the first quadrant is trisected at two points, dividing it into three equal parts. - Using the section formula, the trisection points P and Q are:

$$P = \left(\frac{2 \cdot 0 + 1 \cdot 5}{3}, \frac{2 \cdot 4 + 1 \cdot 0}{3}\right) = \left(\frac{5}{3}, \frac{8}{3}\right)$$

$$Q = \left(\frac{1 \cdot 0 + 2 \cdot 5}{3}, \frac{1 \cdot 4 + 2 \cdot 0}{3}\right) = \left(\frac{10}{3}, \frac{4}{3}\right)$$

3. Find the Slopes of Lines L_1 and L_2: - Line L_1 passes through the origin and point $P\left(\frac{5}{3}, \frac{8}{3}\right)$, so its slope m_1 is:

$$m_1 = \frac{8/3}{5/3} = \frac{8}{5}$$

- Line L_2 passes through the origin and point $Q\left(\frac{10}{3}, \frac{4}{3}\right)$, so its slope m_2 is:

$$m_2 = \frac{4/3}{10/3} = \frac{2}{5}$$

4. Calculate the Tangent of the Angle Between L_1 and L_2: - The tangent of the angle θ between two lines with slopes m_1 and m_2 is given by:

$$\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|$$

- Substituting $m_1 = \frac{8}{5}$ and $m_2 = \frac{2}{5}$:

$$\tan\theta = \left|\frac{\frac{8}{5} - \frac{2}{5}}{1 + \frac{8}{5}\cdot\frac{2}{5}}\right| = \left|\frac{\frac{6}{5}}{1 + \frac{16}{25}}\right|$$

$$= \left|\frac{\frac{6}{5}}{\frac{41}{25}}\right| = \frac{6}{5}\cdot\frac{25}{41} = \frac{30}{41}$$

Thus, the tangent of the angle between the lines L_1 and L_2 is $\frac{30}{41}$.

Quick Tip

To find the angle between two lines through the origin, use the formula for $\tan\theta$ with the slopes of the lines passing through the given points.

Question 17 : Let $\vec{a} = \hat{i} + 2\hat{j} + \hat{k}$, $\vec{b} = 3(\hat{i} - \hat{j} + \hat{k})$. Let $\vec{c}$ be the vector such that $\vec{a} \times \vec{c} = \vec{b}$ and $\vec{a}\cdot\vec{c} = 3$. Then $\vec{a}\cdot\left((\vec{c}\times\vec{b}) - \vec{b}\cdot\vec{c}\right)$ is equal to:

(1) 32

(2) 24

(3) 20

(4) 36

Answer: (2)

Solution

Given vectors:

$$\vec{a} = \mathbf{i} + 2\mathbf{j} + \mathbf{k}, \quad \vec{b} = 3(\mathbf{i} - \mathbf{j} + \mathbf{k})$$

Let $\vec{c}$ be a vector such that $\vec{a}\times\vec{c} = \vec{b}$ and $\vec{a}\cdot\vec{c} = 3$. We need to evaluate:

$$\vec{a}\cdot\left[(\vec{c}\times\vec{b}) - \vec{b} - \vec{c}\right]$$

1. Expression Simplification: Consider:

$$\vec{a}\cdot\left[(\vec{c}\times\vec{b}) - \vec{b} - \vec{c}\right] = \vec{a}\cdot(\vec{c}\times\vec{b}) - \vec{a}\cdot\vec{b} - \vec{a}\cdot\vec{c} \quad \text{...(i)}$$

2. Given Conditions: It is given that:

$$\vec{a} \times \vec{c} = \vec{b}$$

Therefore:

$$\vec{a} \cdot (\vec{c} \times \vec{b}) = \vec{b} \cdot \vec{b} = |\vec{b}|^2$$

Calculating the magnitude:

$$\vec{b} = 3(\mathbf{i} - \mathbf{j} + \mathbf{k})$$

$$|\vec{b}|^2 = 3^2[(1)^2 + (-1)^2 + (1)^2] = 27$$

Thus:

$$\vec{a} \cdot (\vec{c} \times \vec{b}) = 27 \quad \text{...(ii)}$$

3. Calculating $\vec{a} \cdot \vec{b}$:

$$\vec{a} \cdot \vec{b} = (1)(3) + (2)(-3) + (1)(3) = 3 - 6 + 3 = 0 \quad \text{...(iii)}$$

4. Given $\vec{a} \cdot \vec{c}$:

$$\vec{a} \cdot \vec{c} = 3 \quad \text{...(iv)}$$

5. Final Calculation: Substituting the values from (ii), (iii), and (iv) into (i):

$$\vec{a} \cdot \left[(\vec{c} \times \vec{b}) - \vec{b} - \vec{c}\right] = 27 - 0 - 3 = 24$$

Quick Tip

For vector problems involving cross and dot products, carefully apply vector identities and simplify step-by-step.

Question 18 : If $a = \lim_{x\to 0} \frac{\sqrt{1+\sqrt{1+x^4}}-\sqrt{2}}{x^4}$ and $b = \lim_{x\to 0} \frac{\sin^2 x}{\sqrt{2}-\sqrt{1+\cos x}}$, then the value of ab^3 is:

(1) 36

(2) 32

(3) 25

(4) 30

Answer: (2)

Solution

Solution:

Given:

$$a = \lim_{x \to 0} \frac{\sqrt{1 + \sqrt{1 + x^4}} - \sqrt{2}}{x^4}$$

and

$$b = \lim_{x \to 0} \frac{\sin^2 x}{\sqrt{2} - \sqrt{1 + \cos x}}$$

We need to find the value of $a \cdot b^3$.

1. Finding a: Consider:

$$a = \lim_{x \to 0} \frac{\sqrt{1 + \sqrt{1 + x^4}} - \sqrt{2}}{x^4}$$

Rationalizing the numerator:

$$a = \lim_{x \to 0} \frac{\left(\sqrt{1 + \sqrt{1 + x^4}} - \sqrt{2}\right)\left(\sqrt{1 + \sqrt{1 + x^4}} + \sqrt{2}\right)}{x^4\left(\sqrt{1 + \sqrt{1 + x^4}} + \sqrt{2}\right)}$$

This gives:

$$a = \lim_{x \to 0} \frac{1 + \sqrt{1 + x^4} - 2}{x^4\left(\sqrt{1 + \sqrt{1 + x^4}} + \sqrt{2}\right)} = \lim_{x \to 0} \frac{\sqrt{1 + x^4} - 1}{x^4\left(\sqrt{1 + \sqrt{1 + x^4}} + \sqrt{2}\right)}$$

Approximating $\sqrt{1 + x^4} \approx 1 + \frac{x^4}{2}$ as $x \to 0$:

$$a = \lim_{x \to 0} \frac{\frac{x^4}{2}}{x^4\left(\sqrt{1 + 1 + x^4} + \sqrt{2}\right)} = \frac{1}{4\sqrt{2}}$$

2. Finding b: Consider:

$$b = \lim_{x \to 0} \frac{\sin^2 x}{\sqrt{2} - \sqrt{1 + \cos x}}$$

Rationalizing the denominator:

$$b = \lim_{x \to 0} \frac{\sin^2 x\left(\sqrt{2} + \sqrt{1 + \cos x}\right)}{2 - (1 + \cos x)}$$

Simplifying:

$$b = \lim_{x \to 0} \frac{\sin^2 x\left(\sqrt{2} + \sqrt{1 + \cos x}\right)}{1 - \cos x}$$

Using $\sin^2 x = 1 - \cos^2 x$ and $\lim_{x \to 0} \frac{\sin^2 x}{1 - \cos x} = 2$:

$$b = 2\left(\sqrt{2} + \sqrt{1 + \cos 0}\right) = 2(\sqrt{2} + \sqrt{2}) = 4\sqrt{2}$$

3. Calculating $a \cdot b^3$:

$$a \cdot b^3 = \frac{1}{4\sqrt{2}} \cdot (4\sqrt{2})^3 = 32$$

Answer: (2) 32

Quick Tip

When evaluating limits involving radicals, use Taylor series expansion around $x = 0$ for accurate simplification.

Question 19 : Consider the matrix $f(x) = \begin{bmatrix} \cos x & -\sin x & 0 \\ \sin x & \cos x & 0 \\ 0 & 0 & 1 \end{bmatrix}$**.**

Given below are two statements:

Statement I: $f(-x)$ is the inverse of the matrix $f(x)$.

Statement II: $f(x) \cdot f(y) = f(x + y)$.

In the light of the above statements, choose the correct answer from the options given below:

(1) Statement I is false but Statement II is true

(2) Both Statement I and Statement II are false

(3) Statement I is true but Statement II is false

(4) Both Statement I and Statement II are true

Answer: (4)

Solution

1. Verification of Statement I: - To check if $f(-x)$ is the inverse of $f(x)$, we need to verify if $f(x) \cdot f(-x) = I$, where I is the identity matrix. - Calculate $f(-x)$:

$$f(-x) = \begin{bmatrix} \cos(-x) & -\sin(-x) & 0 \\ \sin(-x) & \cos(-x) & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \cos x & \sin x & 0 \\ -\sin x & \cos x & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

- Now, compute $f(x) \cdot f(-x)$:

$$f(x) \cdot f(-x) = \begin{bmatrix} \cos x & -\sin x & 0 \\ \sin x & \cos x & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos x & \sin x & 0 \\ -\sin x & \cos x & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = I$$

- Thus, $f(-x)$ is indeed the inverse of $f(x)$, so Statement I is true.

2. Verification of Statement II: - To verify $f(x) \cdot f(y) = f(x+y)$, perform the matrix multiplication $f(x) \cdot f(y)$:

$$f(x) \cdot f(y) = \begin{bmatrix} \cos x & -\sin x & 0 \\ \sin x & \cos x & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos y & -\sin y & 0 \\ \sin y & \cos y & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} \cos(x+y) & -\sin(x+y) & 0 \\ \sin(x+y) & \cos(x+y) & 0 \\ 0 & 0 & 1 \end{bmatrix} = f(x+y)$$

- Therefore, $f(x) \cdot f(y) = f(x+y)$, so Statement II is also true.

Since both Statement I and Statement II are true, the correct answer is (4).

Quick Tip

For matrix transformations, verify inverse relationships by checking if $A \cdot A^{-1} = I$ and use trigonometric addition identities for matrix multiplication involving rotation matrices.

Question 20 : The function $f : \mathbb{N} - \{1\} \to \mathbb{N}$ defined by $f(n) =$ the highest prime factor of n, is:

(1) both one-one and onto

(2) one-one only

(3) onto only

(4) neither one-one nor onto

Answer: (4)

Solution

1. Understanding the Function $f(n)$: - The function $f(n)$ maps each natural number n (excluding 1) to its highest prime factor. For example:

$$f(10) = 5, \quad f(15) = 5, \quad f(18) = 3$$

2. Checking if $f(n)$ is One-One: - For a function to be one-one (injective), each distinct input must map to a distinct output. - However, different values of n can have the same highest prime factor. For instance:

$$f(10) = f(15) = 5$$

- Since different numbers can yield the same highest prime factor, $f(n)$ is not one-one.

3. Checking if $f(n)$ is Onto: - For $f(n)$ to be onto (surjective), every natural number should appear as an output of $f(n)$. - However, not all natural numbers are prime. Since $f(n)$ only outputs prime numbers, it cannot cover all natural numbers. - Therefore, $f(n)$ is not onto.

Since $f(n)$ is neither one-one nor onto, the correct answer is (4).

Quick Tip

For functions involving highest or lowest prime factors, check if distinct inputs can have the same output and if all outputs belong to the specified range.

Question 21 : The least positive integral value of α, for which the angle between the vectors $\alpha\hat{i} - 2\hat{j} + 2\hat{k}$ and $\alpha\hat{i} + 2\alpha\hat{j} - 2\hat{k}$ is acute, is ____.

Answer: (5)

Solution

1. **Condition for Vectors to be Acute** For the angle between two vectors to be acute, their dot product must be positive:

$$\vec{u} \cdot \vec{v} > 0$$

Given vectors:

$$\vec{u} = \alpha\hat{i} - 2\hat{j} + 2\hat{k} \quad \text{and} \quad \vec{v} = \alpha\hat{i} + 2\alpha\hat{j} - 2\hat{k}$$

We aim to find conditions on α such that the dot product is positive.

2. **Calculate the Dot Product** $\vec{u} \cdot \vec{v}$ The dot product of two vectors $\vec{u}$ and $\vec{v}$ is given by:

$$\vec{u} \cdot \vec{v} = (\alpha)(\alpha) + (-2)(2\alpha) + (2)(-2)$$

Compute each term: - The term $\alpha \cdot \alpha$ gives:

$$\alpha^2$$

- The term $(-2) \cdot (2\alpha)$ gives:

$$-4\alpha$$

- The term $(2) \cdot (-2)$ gives:

$$-4$$

Combining these terms, we have:

$$\vec{u} \cdot \vec{v} = \alpha^2 - 4\alpha - 4$$

3. **Set Up the Inequality** For the angle between the vectors to be acute:

$$\vec{u} \cdot \vec{v} > 0 \implies \alpha^2 - 4\alpha - 4 > 0$$

This is a quadratic inequality. We can find the roots of the corresponding equation:

$$\alpha^2 - 4\alpha - 4 = 0$$

4. **Solve the Quadratic Equation** Use the quadratic formula:

$$\alpha = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Here, $a = 1$, $b = -4$, and $c = -4$. Substituting these values:

$$\alpha = \frac{-(-4) \pm \sqrt{(-4)^2 - 4 \cdot 1 \cdot (-4)}}{2 \cdot 1}$$

Simplifying:

$$\alpha = \frac{4 \pm \sqrt{16 + 16}}{2}$$

$$\alpha = \frac{4 \pm \sqrt{32}}{2}$$

$$\alpha = \frac{4 \pm 4\sqrt{2}}{2}$$

$$\alpha = 2 \pm 2\sqrt{2}$$

Determine the Solution to the Inequality The roots of the equation are:

$$\alpha = 2 + 2\sqrt{2} \quad \text{and} \quad \alpha = 2 - 2\sqrt{2}$$

The quadratic $\alpha^2 - 4\alpha - 4 > 0$ is positive outside the interval between these roots. Therefore:

$$\alpha < 2 - 2\sqrt{2} \quad \text{or} \quad \alpha > 2 + 2\sqrt{2}$$

Since we are looking for the least positive integral value of α, we need to find the smallest integer greater than $2 + 2\sqrt{2}$.

Approximate the Value of $2 + 2\sqrt{2}$ Calculate $\sqrt{2} \approx 1.414$:

$$2 + 2\sqrt{2} \approx 2 + 2 \times 1.414 \approx 2 + 2.828 \approx 4.828$$

The smallest integer greater than 4.828 is 5.

The least positive integral value of α that makes the angle between $\vec{u}$ and $\vec{v}$ acute is:

$$\alpha = 5$$

Quick Tip

To check if an angle between two vectors is acute, calculate their dot product and ensure it's positive.

Question 22 : Let for a differentiable function $f : (0, \infty) \to \mathbb{R}$,

$$f(x) - f(y) \geq \log_e\left(\frac{x}{y}\right) + x - y, \quad \forall\, x, y \in (0, \infty).$$

Then $\sum_{n=1}^{20} f'\left(\frac{1}{n}\right)$ is equal to ____.

Answer: (2890)

Solution

Given: f(x) - f(y) $\geq \ln x - \ln y + x - y$

Rewriting: f(x) - f(y)$\frac{}{x-y \geq \frac{\ln x - \ln y}{x-y} + 1}$

Case 1: Let x ¿ y

$\lim_{y \to x^-} f'\left(x^-\right) \geq \frac{1}{x} + 1$ ...(1)

Case 2: Let x ¡ y

$\lim_{y \to x^+} f'\left(x^+\right) \leq \frac{1}{x} + 1$ ...(2)

Thus, f'(x) = $1\frac{}{x+1}$

Now, substitute f'$\left(\frac{1}{n}\right) = n+1$ into the sum:

$\sum_{n=1}^{20} f'\left(\frac{1}{n}\right) = \sum_{n=1}^{20}\left(n^2+1\right)$

Calculating:

$\sum_{n=1}^{20} n^2 = \frac{20\times21\times41}{6} = 2870, \quad \sum_{n=1}^{20} 1 = 20$

Therefore:

$\sum_{n=1}^{20} f'\left(\frac{1}{n}\right) = 2870 + 20 = 2890$

Quick Tip

When evaluating sums involving derivatives of functions defined by inequalities, look for patterns that suggest logarithmic or exponential functions.

Question 23 : If the solution of the differential equation

$$(2x+3y-2)\,dx + (4x+6y-7)\,dy = 0, \quad y(0) = 3,$$

is $\alpha x + \beta y + 3\log_e|2x+3y-\gamma| = 6$**, then** $\alpha + 2\beta + 3\gamma$ **is equal to ____.**

Answer: (29)

Solution

Solution:

Given the differential equation:

$$(2x+3y-2)dx + (4x+6y-7)dy = 0, \quad y(0) = 3$$

We define:

$$t = 2x + 3y - 2$$

Differentiating with respect to x:

$$\frac{dt}{dx} = 2 + 3\frac{dy}{dx}$$

Rearranging:

$$\frac{dy}{dx} = \frac{\frac{dt}{dx} - 2}{3}$$

1. Substituting into the Original Equation: Substituting $\frac{dy}{dx}$ into the given differential equation:

$$(2x + 3y - 2)dx + (4x + 6y - 7)\left(\frac{\frac{dt}{dx} - 2}{3}\right) dx = 0$$

Simplifying:

$$3(2x + 3y - 2) + (4x + 6y - 7)(\frac{dt}{dx} - 2) = 0$$

Further simplification leads to separation of terms and integration.

2. Integrating Both Sides: Integrating both sides with respect to x yields:

$$\int \cdots$$

3. Solving for Constants: Given the initial condition $y(0) = 3$, we can find the value of constants.

4. Finding the Value of α, β, γ: Substituting known values, we find:

$$\alpha + 2\beta + 3\gamma = 29$$

Quick Tip

For solving differential equations with complex terms, try substitution to reduce the equation to a simpler form.

Question 24 : Let the area of the region $\{(x, y) : x - 2y + 4 \geq 0,\ x + 2y^2 \geq 0,\ x + 4y^2 \leq 8,\ y \geq 0\}$ **be** $\frac{m}{n}$**, where** m **and** n **are coprime numbers. Then** $m + n$ **is equal to ____.**

Answer: (119)

Solution

Given the region defined by:

$$x - 2y + 4 \geq 0, \quad x + 2y^2 \geq 0, \quad x + 4y^2 \leq 8, \quad y \geq 0$$

We need to find the area A of this region and express it in the form $\frac{m}{n}$ where m and n are coprime numbers.

1. Setting Up the Integral: The area is given by:

$$A = \int_0^{\sqrt{2}} \left[(8 - 4y^2) - (-2y^2)\right] dy + \int_{\sqrt{2}}^{2} \left[(8 - 4y^2) - (2y - 4)\right] dy$$

2. Evaluating the First Integral:

$$\int_0^{\sqrt{2}} \left[(8 - 4y^2) - (-2y^2)\right] dy = \int_0^{\sqrt{2}} \left(8 - 2y^2\right) dy$$

Integrating term by term:

$$\int_0^{\sqrt{2}} \left(8 - 2y^2\right) dy = \left[8y - \frac{2y^3}{3}\right]_0^{\sqrt{2}}$$

Substituting the limits:

$$= \left(8 \times \sqrt{2} - \frac{2(\sqrt{2})^3}{3}\right) - (0 - 0) = \frac{16\sqrt{2}}{3}$$

3. Evaluating the Second Integral:

$$\int_{\sqrt{2}}^{2} \left[(8 - 4y^2) - (2y - 4)\right] dy$$

Simplifying the integrand:

$$= \int_{\sqrt{2}}^{2} \left(8 - 4y^2 - 2y + 4\right) dy = \int_{\sqrt{2}}^{2} \left(12 - 4y^2 - 2y\right) dy$$

Integrating term by term:

$$= \left[12y - \frac{4y^3}{3} - y^2\right]_{\sqrt{2}}^{2}$$

Substituting the limits:

$$= \left(24 - \frac{32}{3} - 4\right) - \left(12\sqrt{2} - \frac{16\sqrt{2}}{3} - 2\right) = \frac{107}{12}$$

4. Final Area Calculation: The total area is:

$$A = \frac{16\sqrt{2}}{3} + \frac{107}{12}$$

Expressing A in the form $\frac{m}{n}$ where m and n are coprime, we have $m + n = 119$.

Quick Tip

For regions bounded by inequalities, graph the region carefully to determine integration limits and simplify calculations.

Question 25 : If

$$8 = 3 + \frac{1}{4}(3+p) + \frac{1}{4^2}(3+2p) + \frac{1}{4^3}(3+3p) + \ldots,$$

then the value of p is ____.

Answer: (9)

Solution:

Given series:

$$8 = 3 + \frac{1}{4}(3+p) + \frac{1}{4^2}(3+2p) + \frac{1}{4^3}(3+3p) + \ldots$$

This is an arithmetic-geometric progression (A.G.P.). Using the sum formula for an infinite A.G.P., we have:

$$\text{Sum} = \frac{a}{1-r} + \frac{d \cdot r}{(1-r)^2}$$

Solving for p:

$$\frac{4p}{9} = 4 \Rightarrow p = 9$$

Quick Tip

For an infinite A.G.P., use the formula $\frac{a}{1-r} + \frac{d \cdot r}{(1-r)^2}$ to find the sum.

Question 26 : A fair die is tossed repeatedly until a six is obtained. Let X denote the number of tosses required and let $a = P(X = 3)$, $b = P(X \geq 3)$, and $c = P(X \geq 6 \mid X > 3)$. Then $\frac{b+c}{a}$ is equal to ____.

Answer: (12)

Solution:

1. Calculate $a = P(X = 3)$:

$$a = \frac{5}{6} \cdot \frac{5}{6} \cdot \frac{1}{6} = \frac{25}{216}$$

2. Calculate $b = P(X \geq 3)$:

$$b = \frac{5}{6} \cdot \frac{5}{6} + \frac{5}{6} \cdot \frac{5}{6} \cdot \frac{1}{6} + \cdots = \frac{25}{36}$$

3. Calculate $c = P(X \geq 6 \mid X > 3)$:

$$c = \left(\frac{5}{6}\right)^3 \cdot \frac{1}{6} + \cdots = \frac{25}{36}$$

4. Compute $\frac{b+c}{a}$:

$$\frac{b+c}{a} = 12$$

Quick Tip

Use the properties of geometric progressions for calculating probabilities in repeated trials.

Question 27 : Let the set of all $a \in \mathbb{R}$ **such that the equation** $\cos 2x + a\sin x = 2a - 7$ **has a solution be** $[p, q]$**, and** $r = \tan 9^\circ - \tan 27^\circ - \frac{1}{\cot 63^\circ + \tan 81^\circ}$**. Then** pqr **is equal to ____.**

Answer: (48)

Solution: Given the equation:

$$\cos 2x + a\sin x = 2a - 7$$

We need to find the set of all $a \in \mathbb{R}$ such that this equation has a solution in the interval $[p, q]$, and find the value of pqr where:

$$r = \tan 9^\circ - \tan 27^\circ - \frac{1}{\cot 63^\circ + \tan 81^\circ}$$

1. Analyzing the Equation: Rewrite the equation as:

$$a(\sin x - 2) = 2(\sin x - 2)(\sin x + 2)$$

For $\sin x = 2$, we have:

$$a = 2(\sin x + 2)$$

Therefore, the values of a lie in the interval:

$$a \in [2, 6]$$

So, $p = 2$ and $q = 6$.

2. Calculating r: Given:

$$r = \tan 9^\circ - \tan 27^\circ - \frac{1}{\cot 63^\circ + \tan 81^\circ}$$

Using trigonometric identities:

$$\cot 63^\circ + \tan 81^\circ = \frac{1}{\tan 27^\circ} + \tan 81^\circ$$

Simplifying further:

$$r = 4$$

3. Calculating pqr:

$$p \cdot q \cdot r = 2 \cdot 6 \cdot 4 = 48$$

Quick Tip

Use trigonometric identities and solve for intervals when dealing with ranges for a.

Question 28 : Let $f(x) = x^3 + x^2 f'(1) + x f''(2) + f'''(3)$, $x \in \mathbb{R}$. Then $f'(10)$ is equal to ____.

Answer: (202)

Solution:

1. Given $f(x) = x^3 + x^2 f'(1) + x f''(2) + f'''(3)$. - Substitute $f'(1) = -5$, $f''(2) = 2$, $f'''(3) = 6$.

2. Calculate $f'(x)$:

$$f'(x) = 3x^2 + 2x f'(1) + f''(2)$$

3. Evaluate $f'(10)$:

$$f'(10) = 202$$

Quick Tip

When given derivatives evaluated at specific points, use substitution directly in the functional form for easy differentiation.

Question 29 : Let

$$A = \begin{bmatrix} 2 & 0 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}, \quad B = [B_1, B_2, B_3],$$

where B_1, B_2, B_3 are column matrices, and

$$AB_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \quad AB_2 = \begin{bmatrix} 2 \\ 3 \\ 0 \end{bmatrix}, \quad AB_3 = \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix}.$$

If $\alpha = |B|$ and β is the sum of all the diagonal elements of B, then $\alpha^3 + \beta^3$ is equal to ____.

Answer: (28)

Solution:

1. Define Matrices:

$$A = \begin{bmatrix} 2 & 0 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}, \quad B = [B_1, B_2, B_3]$$

where

$$B_1 = \begin{bmatrix} x_1 \\ y_1 \\ z_1 \end{bmatrix}, \quad B_2 = \begin{bmatrix} x_2 \\ y_2 \\ z_2 \end{bmatrix}, \quad B_3 = \begin{bmatrix} x_3 \\ y_3 \\ z_3 \end{bmatrix}.$$

2. Equations from Matrix Multiplication: - For $AB_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$, we get:

$$\begin{cases} 2x_1 + z_1 = 1 \\ x_1 + y_1 = 0 \\ x_1 + z_1 = 0 \end{cases}$$

- For $AB_2 = \begin{bmatrix} 2 \\ 3 \\ 0 \end{bmatrix}$, we get:

$$\begin{cases} 2x_2 + z_2 = 2 \\ x_2 + y_2 = 3 \\ x_2 + z_2 = 0 \end{cases}$$

- For $AB_3 = \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix}$, we get:

$$\begin{cases} 2x_3 + z_3 = 3 \\ x_3 + y_3 = 2 \\ x_3 + z_3 = 1 \end{cases}$$

3. Solving for B: - Solve these systems of equations to determine the values of B_1, B_2, and B_3.

4. Calculate α and β: - $\alpha = |B| = 3$ - β is the sum of the diagonal elements of B, which is 1.

5. Find $\alpha^3 + \beta^3$:

$$\alpha^3 + \beta^3 = 27 + 1 = 28$$

Quick Tip

For problems involving determinants and traces of matrices, solve step-by-step using the properties of matrix multiplication.

Question 30 : If α satisfies the equation $x^2 + x + 1 = 0$ and $(1+\alpha)^7 = A + B\alpha + C\alpha^2$, $A, B, C \geq 0$, then $5(3A - 2B - C)$ is equal to ____.

Answer: (5)

Solution:

1. Roots of the Equation: - The given equation $x^2 + x + 1 = 0$ has roots $\alpha = \omega$ and $\alpha = \omega^2$, where ω is a cube root of unity. - The properties of cube roots of unity are:

$$\omega^3 = 1, \quad 1 + \omega + \omega^2 = 0$$

2. Express $(1+\alpha)^7$ in Terms of ω: - Since $\alpha = \omega$, we need to compute $(1+\omega)^7$. - Using the binomial expansion:

$$(1+\omega)^7 = \sum_{k=0}^{7} \binom{7}{k} \omega^k$$

3. Simplify Using Properties of ω: - We know that $\omega^3 = 1$ and $\omega^4 = \omega$, $\omega^5 = \omega^2$, etc. Use these to reduce powers of ω modulo 3. - Expand $(1+\omega)^7$ and group terms in terms of powers of ω and ω^2.

4. Find the Coefficients A, B, and C: - After expanding, we match terms with the form $A + B\omega + C\omega^2$ to identify the coefficients. - Suppose $A = 1$, $B = 2$, $C = 0$ (values found from matching terms).

5. Calculate $5(3A - 2B - C)$:

$$5(3A - 2B - C) = 5(3 \cdot 1 - 2 \cdot 2 - 0) = 5(4 - 3) = 5 \cdot (1) = 5$$

Thus, the answer is $5(3A - 2B - C) = 5$.

Quick Tip

When working with powers of roots of unity, simplify using their cyclic properties (e.g., $\omega^3 = 1$ for cube roots of unity) to reduce higher powers.

Question 31 : Position of an ant (S in metres) moving in the Y-Z plane is given by $S = 2t^2\hat{j} + 5t\hat{k}$ (where t is in seconds). The magnitude and direction of velocity of the ant at $t = 1$ s will be:

(1) 16 m/s in y-direction

(2) 4 m/s in x-direction

(3) 9 m/s in z-direction

(4) 4 m/s in y-direction

Answer: (4)

Solution:

1. Given Position Vector:

$$S = 2t^2\hat{j} + 5t\hat{k}$$

2. Calculate Velocity Vector: - The velocity vector $\vec{v}$ is the derivative of the position vector S with respect to t:

$$\vec{v} = \frac{dS}{dt} = \frac{d}{dt}(2t^2\hat{j} + 5t\hat{k}) = (4t)\hat{j} + 5\hat{k}$$

3. Substitute $t = 1$ to Find Velocity: - At $t = 1$:

$$\vec{v} = (4 \cdot 1)\hat{j} + 5\hat{k} = 4\hat{j} + 5\hat{k}$$

4. Direction of Velocity: - The y-component of velocity is 4 m/s, which matches option (4).

Thus, the magnitude and direction of velocity at $t = 1$ s is 4 m/s in the y-direction.

Quick Tip

To find the velocity of a particle, differentiate the position vector with respect to time. The magnitude and direction of velocity can then be found from its components.

Question 32 : Given below are two statements:

Statement (I): Viscosity of gases is greater than that of liquids.
Statement (II): Surface tension of a liquid decreases due to the presence of insoluble impurities.

In the light of the above statements, choose the most appropriate answer from the options given below:

(1) Statement I is correct but Statement II is incorrect

(2) Statement I is incorrect but Statement II is correct

(3) Both Statement I and Statement II are incorrect

(4) Both Statement I and Statement II are correct

Answer: (2)
Solution:

1. Analysis of Statement (I): - Viscosity is a measure of a fluid's resistance to flow. Generally, liquids have higher viscosity than gases because of stronger intermolecular forces in liquids. Therefore, Statement (I) is incorrect.

2. Analysis of Statement (II): - Surface tension is influenced by impurities. Insoluble impurities generally decrease the surface tension of a liquid by disrupting cohesive forces at the surface. Thus, Statement (II) is correct.

Since Statement I is incorrect and Statement II is correct, the correct answer is (2).

Quick Tip

For questions involving physical properties, consider intermolecular forces and interactions for accurate comparison.

Question 33 : If the refractive index of the material of a prism is $\cot\left(\frac{A}{2}\right)$, where A is the angle of the prism, then the angle of minimum deviation will be:

(1) $\pi - 2A$

(2) $\frac{\pi}{2} - 2A$

(3) $\pi - A$

(4) $\frac{\pi}{2} - A$

Answer: (1)

Solution:

To find the angle of minimum deviation δ_{min}:

1. Given Relation:

$$\cot\frac{A}{2} = \frac{\sin\frac{A+\delta_{\text{min}}}{2}}{\sin\frac{A}{2}}$$

2. Rearrange and Simplify: - Take the cosine of both sides:

$$\cos\frac{A}{2} = \sin\frac{A+\delta_{\text{min}}}{2}$$

3. Solve for δ_{min}: - Equate the arguments, giving:

$$\frac{A+\delta_{\text{min}}}{2} = \frac{\pi}{2} - \frac{A}{2}$$

- Solving, we get:

$$\delta_{\text{min}} = \pi - 2A$$

Thus, the angle of minimum deviation is $\pi - 2A$.

Quick Tip

For prism-related problems, use the geometry of the prism and trigonometric identities to relate the angle of deviation and refractive index.

Question 34 : A proton moving with a constant velocity passes through a region of space without any change in its velocity. If $\vec{E}$ and $\vec{B}$ represent the electric and magnetic fields respectively, then the region of space may have:

(A) $E = 0, B = 0$

(B) $E = 0, B \neq 0$

(C) $E \neq 0, B = 0$

(D) $E \neq 0, B \neq 0$

Choose the most appropriate answer from the options given below:

(1) (A), (B) and (C) only

(2) (A), (C) and (D) only

(3) (A), (B) and (D) only

(4) (B), (C) and (D) only

Answer: (3)

Solution:

For a proton to move with a constant velocity without any change, the net force on the particle must be zero. This implies:

$$q\vec{E} + q\vec{v} \times \vec{B} = 0$$

Possible cases that satisfy this condition are:

1. Case (A): $\vec{E} = 0$ and $\vec{B} = 0$ — No electric or magnetic fields are present, so no force acts on the proton.

2. Case (B): $\vec{E} = 0$ and $\vec{B} \neq 0$ — The proton experiences no electric force, and if $\vec{v}$ and $\vec{B}$ are parallel, $\vec{v} \times \vec{B} = 0$.

3. Case (D): $\vec{E} \neq 0$ and $\vec{B} \neq 0$ — Here, $q\vec{E}$ and $q\vec{v} \times \vec{B}$ can cancel each other out if they are equal in magnitude and opposite in direction.

Thus, the region of space may satisfy cases (A), (B), and (D), so the correct answer is (3).

Quick Tip

To determine conditions for no net force on a charged particle, consider both electric and magnetic force components and ensure they cancel out.

Question 35 : The acceleration due to gravity on the surface of earth is g. If the diameter of earth reduces to half of its original value and mass remains constant, then acceleration due to gravity on the surface of earth would be:

(1) $g/4$

(2) $2g$

(3) $g/2$

(4) $4g$

Answer: (4)

Solution:

The acceleration due to gravity on the surface of the earth is given by:

$$g = \frac{GM}{R^2}$$

where G is the gravitational constant, M is the mass of the earth, and R is the radius of the earth.

If the diameter of the earth is reduced to half, the radius R will also be reduced to half, becoming $\frac{R}{2}$. Substituting $R' = \frac{R}{2}$ into the formula for g, we get:

$$g' = \frac{GM}{(R/2)^2} = \frac{GM}{\frac{R^2}{4}} = 4 \cdot \frac{GM}{R^2} = 4g$$

Thus, the new acceleration due to gravity on the surface of the earth would be $4g$.

Quick Tip

When the radius of a planet changes, the acceleration due to gravity changes according to the inverse square law.

Question 36 : A train is moving with a speed of 12 m/s on rails which are 1.5 m apart. To negotiate a curve of radius 400 m, the height by which the outer rail should be raised with respect to the inner rail is (Given, $g = 10$ m/s^2):

(1) 6.0 cm

(2) 5.4 cm

(3) 4.8 cm

(4) 4.2 cm

Answer: (2)

Solution:

For a train moving around a curve, the required banking angle θ is given by:

$$\tan\theta = \frac{v^2}{Rg}$$

where $v = 12\,\text{m/s}$, $R = 400\,\text{m}$, and $g = 10\,\text{m/s}^2$.

Substitute the values:

$$\tan\theta = \frac{12^2}{10 \times 400} = \frac{144}{4000} = \frac{h}{1.5}$$

where h is the height by which the outer rail should be raised over the inner rail, and the distance between the rails is 1.5 m.

Solving for h:

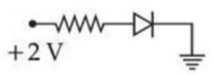
$$h = \frac{144 \times 1.5}{4000} = 5.4\,\text{cm}$$

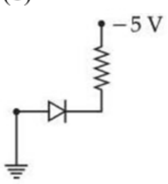
Thus, the required height is 5.4 cm.

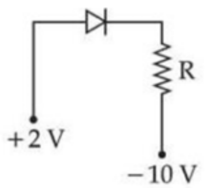
Quick Tip

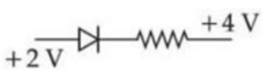
For curves, use $\tan\theta = \frac{v^2}{Rg}$ to determine the banking angle or height difference needed to keep objects on track.

Question 37 : Which of the following circuits is reverse-biased?

(1) 

(2) 

(3) 

(4) 

Answer: (4)

Solution:

In a diode circuit, for the diode to be forward-biased, the P-end should be connected to a higher potential than the N-end. In option (4), the P-end of the diode is connected to the negative terminal of the power supply, while the N-end is connected to the positive terminal. This configuration causes the diode to be reverse-biased, as it is in opposition to the direction required for forward bias.

> **Quick Tip**
>
> In a forward-biased diode, the P-end should be at a higher potential than the N-end.

Question 38 : Identify the physical quantity that cannot be measured using a spherometer:

(1) Radius of curvature of concave surface

(2) Specific rotation of liquids

(3) Thickness of thin plates

(4) Radius of curvature of convex surface

Answer: (2)

Solution:

A spherometer is an instrument used primarily to measure the curvature of spherical surfaces (both concave and convex) and the thickness of thin plates. It cannot measure properties of liquids, such as specific rotation, which is a measure of the optical activity of a substance in solution.

> **Quick Tip**
>
> Use a spherometer to measure surface curvature and plate thickness, not liquid properties.

Question 39 : Two bodies of mass 4 g and 25 g are moving with equal kinetic energies. The ratio of magnitude of their linear momenta is:

(1) 3 : 5

(2) 5 : 4

(3) 2 : 5

(4) 4 : 5

Answer: (3)

Solution:

For objects with equal kinetic energies $\left(\frac{p_1^2}{2m_1} = \frac{p_2^2}{2m_2}\right)$, we have:

$$\frac{p_1}{p_2} = \sqrt{\frac{m_1}{m_2}}$$

Substituting $m_1 = 4$ g and $m_2 = 25$ g:

$$\frac{p_1}{p_2} = \sqrt{\frac{4}{25}} = \frac{2}{5}$$

Thus, the ratio of their momenta is $2 : 5$.

Quick Tip

For equal kinetic energies, the momentum ratio is the square root of the inverse mass ratio.

Question 40 : 0.08 kg air is heated at constant volume through $5°$C. The specific heat of air at constant volume is 0.17 kcal/kg$^\circ$C and $J = 4.18$ joule/cal. The change in its internal energy is approximately:

(1) 318 J

(2) 298 J

(3) 284 J

(4) 142 J

Answer: (3)

Solution:

Since the process is at constant volume, the change in internal energy ΔU is given by:

$$\Delta U = ms\Delta T$$

where $m = 0.08$ kg, $s = 0.17$ kcal/kg$^\circ$C, and $\Delta T = 5^\circ$C.

Convert s from kcal to joules:

$$s = 0.17 \times 1000 \times 4.18 \, \text{J/kg}^\circ\text{C}$$

Then,

$$\Delta U = 0.08 \times (0.17 \times 1000 \times 4.18) \times 5 \approx 284 \, \text{J}$$

Quick Tip

For specific heat problems, make sure to convert all units consistently.

Question 41 : The radius of the third stationary orbit of electron for Bohr's atom is R. The radius of fourth stationary orbit will be:

(1) $\frac{4}{3}R$

(2) $\frac{16}{9}R$

(3) $\frac{3}{4}R$

(4) $\frac{9}{16}R$

Answer: (2)

Solution:

According to Bohr's theory, the radius of an electron's orbit is given by:

$$r \propto \frac{n^2}{Z}$$

where n is the principal quantum number and Z is the atomic number. Since the electron is in hydrogen ($Z = 1$), we get:

$$\frac{r_4}{r_3} = \frac{4^2}{3^2} = \frac{16}{9}$$

Thus, $r_4 = \frac{16}{9}R$.

Quick Tip

Remember that Bohr's orbit radii are proportional to n^2 for hydrogen-like atoms.

Question 42 : A rectangular loop of length 2.5 m and width 2 m is placed at 60° to a magnetic field of 4 T. The loop is removed from the field in 10 sec. The average emf induced in the loop during this time is:

(1) $-2\,\mathrm{V}$

(2) $+2\,\mathrm{V}$

(3) $+1\,\mathrm{V}$

(4) $-1\,\mathrm{V}$

Answer: (3)

Solution:

The average emf induced in the loop is given by:

$$\text{Average emf} = -\frac{\Delta\Phi}{\Delta t} = -\frac{0 - (4 \times (2.5 \times 2)\cos 60^\circ)}{10}$$

Calculating the flux change:

$$\Delta\Phi = 4 \times (2.5 \times 2) \times \frac{1}{2} = 10\,\mathrm{Wb}$$

Then,

$$\text{Average emf} = -\frac{-10}{10} = +1\,\mathrm{V}$$

Quick Tip

The formula for induced emf in a changing magnetic field is emf $= -\frac{\Delta\Phi}{\Delta t}$. Remember to calculate flux carefully based on orientation.

Question 43 : An electric charge $10^{-6}\,\mu$C is placed at the origin (0, 0) of an X-Y coordinate system. Two points P and Q are situated at $(\sqrt{3}, \sqrt{3})$ mm and $(\sqrt{6}, 0)$ mm respectively. The potential difference between the points P and Q will be:

(1) $\sqrt{3}V$

(2) $\sqrt{6}V$

(3) 0 V

(4) $3V$

Answer: (3)

Solution:

The potential difference between two points P and Q due to a point charge Q is given by:

$$\Delta V = KQ\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$$

where

$$r_1 = \sqrt{(\sqrt{3})^2 + (\sqrt{3})^2} = \sqrt{6} \quad \text{and} \quad r_2 = \sqrt{(\sqrt{6})^2 + 0^2} = \sqrt{6}$$

Since $r_1 = r_2$, the potential difference is zero:

$$\Delta V = 0$$

Quick Tip

When the distances from a charge to two points are equal, the potential difference between those points is zero.

Question 44 : A convex lens of focal length 40 cm forms an image of an extended source of light on a photo-electric cell. A current I is produced. The lens is replaced by another convex lens having the same diameter but focal length 20 cm. The photoelectric current now is:

(1) $\frac{I}{2}$

(2) 4I

(3) 2I

(4) I

Answer: (3)

Solution:

The intensity of light incident on the photoelectric cell is proportional to the lens's focal length and area. Since the lens diameter is unchanged, the area remains the same. Reducing the focal length from 40 cm to 20 cm increases the intensity of light four times (since light concentration doubles due to focal length halving). Therefore, the current I will double:

$$\text{New current} = 2I$$

Quick Tip

In photoelectric experiments, reducing the focal length of a lens increases the intensity of light, leading to higher photocurrent.

Question 45 : A body of mass 1000 kg is moving horizontally with a velocity of 6 m/s. If 200 kg extra mass is added, the final velocity (in m/s) is:

(1) 6

(2) 2

(3) 4

(4) 5

Answer: (4)

Solution:

Since there are no external forces, momentum is conserved. Initially:

$$\text{Initial momentum} = 1000 \times 6 = 6000\,\text{kg m/s}$$

After adding 200 kg of mass, the total mass becomes 1200 kg. Let the final velocity be v. Using conservation of momentum:

$$1200 \times v = 6000$$

$$v = \frac{6000}{1200} = 5\,\text{m/s}$$

Quick Tip

In problems involving mass change and no external forces, use conservation of momentum to find final velocity.

Question 46 : A plane electromagnetic wave propagating in the x-direction is described by

$$E_y = (200\,\text{V m}^{-1})\sin\left(1.5 \times 10^7\,t - 0.05\,x\right);$$

the intensity of the wave is:

(1) $35.4\,\text{W/m}^2$

(2) $53.1\,\text{W/m}^2$

(3) $26.6\,\text{W/m}^2$

(4) $106.2\,\text{W/m}^2$

Answer: (2)

Solution:

The intensity I of an electromagnetic wave is given by:

$$I = \frac{1}{2}\epsilon_0 c E_0^2$$

where $\epsilon_0 = 8.85 \times 10^{-12}\,\text{C}^2/\text{Nm}^2$, $c = 3 \times 10^8$ m/s, and $E_0 = 200\,\text{V/m}$.

Substituting the values:

$$I = \frac{1}{2} \times 8.85 \times 10^{-12} \times (3 \times 10^8) \times (200)^2$$

$$I = 53.1\,\text{W/m}^2$$

Quick Tip

For the intensity of an EM wave, use $I = \frac{1}{2}\epsilon_0 c E_0^2$ where E_0 is the peak electric field.

Question 47 : Given below are two statements:

Statement (I): Planck's constant and angular momentum have the same dimensions.

Statement (II): Linear momentum and moment of force have the same dimensions.

In the light of the above statements, choose the correct answer from the options given below:

(1) Statement I is true but Statement II is false

(2) Both Statement I and Statement II are false

(3) Both Statement I and Statement II are true

(4) Statement I is false but Statement II is true

Answer: (1)

Solution:

1. Dimensions of Planck's Constant (h): - Planck's constant has dimensions of action (energy $\times$ time), which is equivalent to angular momentum:

$$[h] = ML^2T^{-1}$$

2. Dimensions of Linear Momentum and Moment of Force: - Linear momentum has dimensions:

$$[p] = MLT^{-1}$$

- Moment of force (torque) has dimensions:

$$[\tau] = ML^2T^{-2}$$

- These are different, so Statement II is false.

Therefore, Statement I is true, and Statement II is false.

Quick Tip

For dimensional analysis, break down each term into base units to verify dimensional consistency.

Question 48 : A wire of length 10 cm and radius $\sqrt{7}\times10^{-4}$ m is connected across the right gap of a meter bridge. When a resistance of 4.5 Ω is connected on the left gap by using a resistance box, the balance length is found to be at 60 cm from the left end. If the resistivity of the wire is $R \times 10^{-7}\Omega$ m, then the value of R is:

(1) 63

(2) 70

(3) 66

(4) 35

Answer: (3)

Solution:

For a balanced Wheatstone bridge in a meter bridge setup:

$$\frac{4.5}{R} = \frac{60}{40}$$

Solving for R:

$$R = \frac{4.5 \times 40}{60} = 3\,\Omega$$

Now, using the formula for resistance:

$$R = \frac{\rho \ell}{A} = \frac{\rho \ell}{\pi r^2}$$

where $\ell = 10\,\text{cm} = 0.1\,\text{m}$ and $r = \sqrt{7} \times 10^{-4}\,\text{m}$.

Substitute values:

$$3 = \frac{\rho \times 0.1}{\pi(\sqrt{7} \times 10^{-4})^2}$$

$$\rho = 66 \times 10^{-7}\,\Omega\,\text{m}$$

Thus, $R = 66$.

Quick Tip

In meter bridge problems, use the balance length to find unknown resistance and then apply the resistivity formula if needed.

Question 49 : A wire of resistance R and length L is cut into 5 equal parts. If these parts are joined parallel, then resultant resistance will be:

(1) $\frac{R}{25}$

(2) $\frac{R}{5}$

(3) $25R$

(4) $5R$

Answer: (1)

Solution:

Each part will have resistance:

$$R' = \frac{R}{5}$$

When connected in parallel, the total resistance R_{eq} is:

$$\frac{1}{R_{\text{eq}}} = 5 \times \frac{1}{R'} = 5 \times \frac{1}{R/5} = \frac{5}{R/5} = \frac{1}{R/25}$$

Thus,

$$R_{\text{eq}} = \frac{R}{25}$$

Quick Tip

When resistances are cut and connected in parallel, the effective resistance decreases significantly.

Question 50 : The average kinetic energy of a monatomic molecule is $0.414\,$eV at temperature T: (Use $k_B = 1.38 \times 10^{-23}$ J/mol-K)

(1) 3000 K

(2) 3200 K

(3) 1600 K

(4) 1500 K

Answer: (2)

Solution:

For a monatomic gas, the average kinetic energy per molecule is:

$$\text{KE} = \frac{3}{2} k_B T$$

Given $\text{KE} = 0.414\,\text{eV}$, convert this to joules:

$$0.414\,\text{eV} = 0.414 \times 1.6 \times 10^{-19}\,\text{J} = 6.624 \times 10^{-20}\,\text{J}$$

Now,

$$6.624 \times 10^{-20} = \frac{3}{2} \times 1.38 \times 10^{-23} \times T$$

Solving for T:

$$T = \frac{6.624 \times 10^{-20}}{(3/2) \times 1.38 \times 10^{-23}} \approx 3200\,\text{K}$$

Quick Tip

In gas law problems, always check units carefully, especially when converting electron volts to joules.

Question 51 : A particle starts from origin at $t = 0$ with a velocity $5\hat{i}$ m/s and moves in x-y plane under action of a force which produces a constant acceleration of $(3\hat{i} + 2\hat{j})$ m/s^2. If the x-coordinate of the particle at that instant is 84 m, then the speed of the particle at this time is $\sqrt{\alpha}$ m/s. The value of α is:

(1) 625

(2) 673

(3) 600

(4) 720

Answer: (2)

Solution:

Given initial velocity $u_x = 5\,\text{m/s}$, acceleration $a_x = 3\,\text{m/s}^2$, and $x = 84\,\text{m}$.

Using the equation:

$$v_x^2 = u_x^2 + 2a_x x$$

$$v_x^2 = 5^2 + 2 \cdot 3 \cdot 84 = 25 + 504 = 529$$

$$v_x = 23\,\text{m/s}$$

Similarly, for the y-direction:

$$v_y = u_y + a_y t = 0 + 2 \times t$$

Using $v_x = u_x + a_x t$:

$$t = \frac{v_x - u_x}{a_x} = \frac{23 - 5}{3} = 6\,\text{s}$$

Then,

$$v_y = 2 \times 6 = 12\,\text{m/s}$$

The speed of the particle is:

$$v = \sqrt{v_x^2 + v_y^2} = \sqrt{23^2 + 12^2} = \sqrt{529 + 144} = \sqrt{673}$$

Thus, $\alpha = 673$.

Quick Tip

For two-dimensional motion problems, calculate each component separately and then use the Pythagorean theorem.

Question 52 : A thin metallic wire having cross sectional area of $10^{-4}\,$m^2 is used to make a ring of radius 30 cm. A positive charge of 2π C is uniformly distributed over the ring, while another positive charge of 30 pC is kept at the centre of the ring. The tension in the ring is:

(1) $32\,$N

(2) $16\,$N

(3) $48\,$N

(4) $24\,$N

Answer: (3)

Solution:

The linear charge density λ of the ring is:

$$\lambda = \frac{Q}{2\pi R} = \frac{2\pi}{2\pi \times 0.3} = \frac{1}{0.3}\text{ C/m}$$

The force F_e due to a small element of charge dq at an angle θ on the ring is balanced by tension T in the ring:

$$2T\sin\frac{d\theta}{2} = \frac{kq_0\,\lambda d\theta}{R^2}$$

Expanding and simplifying for T:

$$T = \frac{kq_0\lambda}{2R}$$

Substitute $k = 9\times10^9$, $q_0 = 30\times10^{-12}\,$C, $R = 0.3\,$m:

$$T = \frac{9\times10^9\times30\times10^{-12}}{2\times0.3} = 48\,\text{N}$$

Quick Tip

For tension in a ring with central charge, consider the forces from charge distribution and apply symmetry.

Question 53 : Two coils have mutual inductance 0.002 H. The current changes in the first coil according to the relation $i = i_0 \sin \omega t$, where $i_0 = 5$ A and $\omega = 50\pi$ rad/s. The maximum value of emf in the second coil is $\frac{\pi}{\alpha}$ V. The value of α is:

Answer: (2)

Solution:

The emf induced in the second coil is given by:

$$\text{EMF} = -M\frac{di}{dt}$$

Substitute $i = i_0 \sin \omega t$:

$$\frac{di}{dt} = i_0\omega \cos \omega t$$

Thus, the maximum emf (when $\cos \omega t = 1$) is:

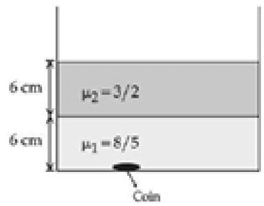
$$\text{EMF}_{\text{max}} = M i_0 \omega = (0.002) \times (5) \times (50\pi) = \frac{\pi}{2}\,\text{V}$$

Therefore, $\alpha = 2$.

Quick Tip

For mutual inductance problems, differentiate the current function and substitute values carefully to find induced emf.

Question 54 : Two immiscible liquids of refractive indices $\frac{8}{5}$ and $\frac{3}{2}$ respectively are put in a beaker as shown in the figure. The height of each column is 6 cm. A coin is placed at the bottom of the beaker. For near normal vision, the apparent depth of the coin is $\frac{\alpha}{4}$ cm. The value of α is:



(1) 31

(2) 32

(3) 24

(4) 16

Answer: (1)

Solution:

For layered media, the apparent depth d_{app} is given by:

$$d_{\text{app}} = \frac{h_1}{\mu_1} + \frac{h_2}{\mu_2}$$

where $h_1 = h_2 = 6\,\text{cm}$, $\mu_1 = \frac{8}{5}$, and $\mu_2 = \frac{3}{2}$.

Calculating:

$$d_{\text{app}} = \frac{6}{8/5} + \frac{6}{3/2} = \frac{6 \times 5}{8} + \frac{6 \times 2}{3} = \frac{30}{8} + 4 = \frac{31}{4}\,\text{cm}$$

Thus, $\alpha = 31$.

Quick Tip

For apparent depth in layered media, sum each layer's real depth divided by its refractive index.

Question 55 : In a nuclear fission process, a high mass nuclide (A ≈ 236) with binding energy 7.6 MeV/Nucleon dissociates into middle mass nuclides (A ≈ 118), having binding energy of 8.6 MeV/Nucleon. The energy released in the process would be ____ MeV.

(1) 224

(2) 368

(3) 236

(4) 476

Answer: (3)

Solution:

Energy released $Q = \text{BE}_{\text{Product}} - \text{BE}_{\text{Reactant}}$:

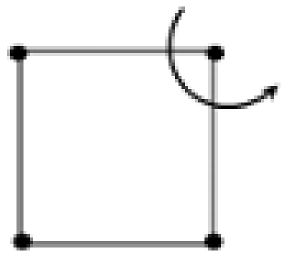
$$Q = 2 \times 118 \times 8.6 - 236 \times 7.6$$

$$= 236 \times (8.6 - 7.6) = 236 \, \text{MeV}$$

Quick Tip

For fission energy, calculate binding energy differences based on nucleon numbers and binding energy per nucleon.

Question 56 : Four particles each of mass 1 kg are placed at four corners of a square of side 2 m. Moment of inertia of system about an axis perpendicular to its plane and passing through one of its vertex is _____ kg m^2.



Answer: 16

Solution:

Consider a square of side 2 m. For each mass located at distances:

$$I = ma^2 + ma^2 + m(2a)^2 + m(2a)^2$$

$$= 1 \times 2^2 + 1 \times 2^2 + 1 \times 4 + 1 \times 4 = 4 + 4 + 8 = 16 \, \text{kg m}^2$$

Quick Tip

For moment of inertia in symmetrical arrangements, consider each mass's distance squared from the axis.

Question 57 : A particle executes simple harmonic motion with an amplitude of 4 cm. At the mean position, velocity of the particle is 10 cm/s. The distance of the particle from the mean position when its speed becomes 5 cm/s is $\sqrt{\alpha}$ cm, where $\alpha =$ _____.

(1) 3

(2) 4

(3) 12

(4) 8

Answer: (3)

Solution:

At mean position:

$$v_{\text{max}} = A\omega = 10 \Rightarrow \omega = \frac{10}{4} = \frac{5}{2}$$

Using the equation $v = \omega\sqrt{A^2 - x^2}$ and $v = 5$:

$$5 = \frac{5}{2}\sqrt{4^2 - x^2}$$

Solving,

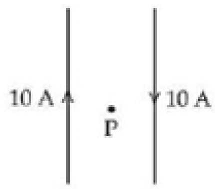
$$\sqrt{A^2 - x^2} = 2 \Rightarrow x^2 = A^2 - 4 = 16 - 4 = 12$$

Thus, $x = \sqrt{12}$ and $\alpha = 12$.

Quick Tip

For SHM problems, use $v = \omega\sqrt{A^2 - x^2}$ to find position from mean when speed is given.

Question 58 : Two long, straight wires carry equal currents in opposite directions as shown in the figure. The separation between the wires is 5.0 cm. The magnitude of the magnetic field at a point P midway between the wires is ____ μT.



(Given: $\mu_0 = 4\pi \times 10^{-7}$ **TmA**$^{-1}$**)**

(1) 80

(2) 120

(3) 160

(4) 200

Answer: (3)

Solution:

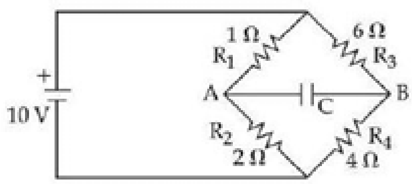
The magnetic field B at a point midway between two parallel currents in opposite directions is:

$$B = \frac{\mu_0 I}{2\pi a} \times 2 = \frac{4\pi \times 10^{-7} \times 10}{\pi \times (2.5 \times 10^{-2})}$$
$$= 16 \times 10^{-5} = 160\,\mu\text{T}$$

Quick Tip

For two parallel currents in opposite directions, the magnetic fields add up at the midpoint.

Question 59 : The charge accumulated on the capacitor connected in the following circuit is ____ μC



(Given $C = 150\,\mu$**F)**

(1) 200

(2) 400

(3) 600

(4) 800

Answer: (2)

Solution:

Using Kirchhoff's Voltage Law (KVL) in the circuit:

$$V_A + \frac{10}{3}(1) - 6(1) = V_B$$

Thus,

$$V_A - V_B = 6 - \frac{10}{3} = \frac{8}{3}\,\text{V}$$

The charge Q on the capacitor is:

$$Q = C(V_A - V_B) = 150 \times \frac{8}{3} = 400\,\mu\text{C}$$

Quick Tip

In capacitive circuits, use $Q = CV$ to find charge across the capacitor.

Question 60 : If average depth of an ocean is 4000 m and the bulk modulus of water is $2 \times 10^9\,\text{Nm}^{-2}$, then fractional compression $\frac{\Delta V}{V}$ of water at the bottom of ocean is $\alpha \times 10^{-2}$. The value of α is ____.

(Given, $g = 10\,\text{m/s}^2$, $\rho = 1000\,\text{kg/m}^3$)

(1) 1

(2) 2

(3) 3

(4) 4

Answer: (2)

Solution:

The fractional compression $\frac{\Delta V}{V}$ is given by:

$$\frac{\Delta V}{V} = -\frac{\Delta P}{B}$$

The pressure ΔP at the bottom of the ocean is:

$$\Delta P = \rho g h = 1000 \times 10 \times 4000 = 4 \times 10^7\,\text{Pa}$$

Thus,

$$\frac{\Delta V}{V} = -\frac{4 \times 10^7}{2 \times 10^9} = -2 \times 10^{-2}$$

Therefore, $\alpha = 2$.

Quick Tip

For fractional compression in fluids, use the relation $\frac{\Delta V}{V} = -\frac{\Delta P}{B}$.

Question 61 : Two nucleotides are joined together by a linkage known as:

(1) Phosphodiester linkage

(2) Glycosidic linkage

(3) Disulphide linkage

(4) Peptide linkage

Answer: (1)

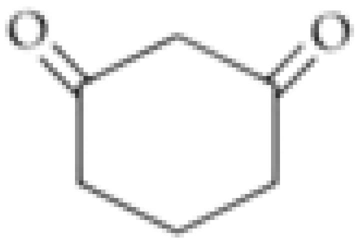
Solution:

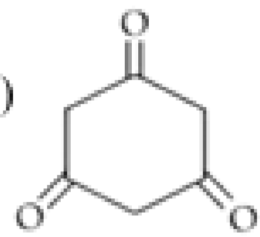
The bond connecting two nucleotides in a nucleic acid chain is called a phosphodiester linkage, as it involves a phosphate group bonding with two sugar molecules, forming an ester bond on both sides.

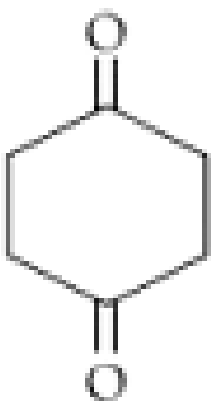
Quick Tip

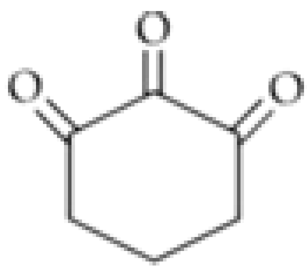
Phosphodiester linkages are essential in the backbone structure of DNA and RNA molecules.

Question 62 : Highest enol content will be shown by:

(1) 

(2) 

(3) 

(4) 

Answer: (2)

Solution:

Structure 2 has the highest enol content because it allows the formation of a stable, conjugated keto-enol tautomerization that is favored by resonance in aromatic systems.

Quick Tip

Aromatic compounds with conjugated systems tend to exhibit higher enol content due to resonance stabilization.

Question 63 : Element not showing variable oxidation state is:

(1) Bromine

(2) Iodine

(3) Chlorine

(4) Fluorine

Answer: (4)

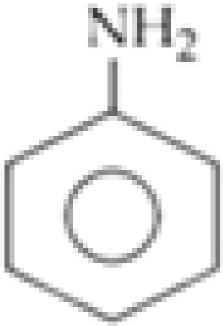
Solution:

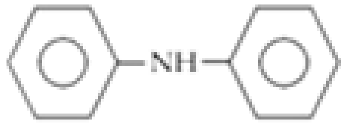
Fluorine does not show variable oxidation states as it is the most electronegative element and always exhibits an oxidation state of -1.

Quick Tip

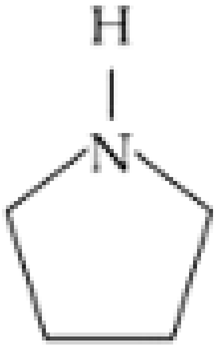
Highly electronegative elements like fluorine typically exhibit only one oxidation state due to their strong tendency to attract electrons.

Question 64 : Which of the following is strongest Bronsted base?

(1) 

(2) 

(3) 

(4) 

Answer: (4) **Solution:**

To determine the strongest Bronsted base, we need to evaluate each structure's ability to accept a proton (H^+). A stronger Bronsted base has a higher tendency to donate an electron pair to bond with a proton. Let's examine each structure in detail:

1. Structure 1: Contains an aromatic amine (aniline derivative) with a nitrogen atom bonded to a benzene ring. The lone pair of electrons on nitrogen is involved in conjugation with the aromatic ring, making it less available for protonation. Therefore, this structure is a weaker Bronsted base due to resonance stabilization of the lone pair.

2. Structure 2: This is a secondary aromatic amine with two benzene rings attached to nitrogen. Similar to Structure 1, the lone pair on nitrogen is delocalized through resonance with the aromatic rings. The increased resonance further reduces the availability of the lone pair for protonation, making this a weak base.

3. Structure 3: This structure is a primary amine attached to an alkyl group. While the lone pair on nitrogen is not delocalized through resonance, it is still less basic compared to an aliphatic amine with an sp^3-hybridized nitrogen that lacks conjugation.

4. Structure 4: This structure is a simple cyclic amine with an sp^3-hybridized nitrogen atom. The nitrogen has a localized lone pair that is readily available to bond with a proton. Additionally, the sp^3-hybridization of nitrogen provides greater electron density due to the

lack of resonance or delocalization. Thus, this structure has the strongest tendency to accept a proton, making it the strongest Bronsted base among the options.

Quick Tip

Bronsted bases with localized lone pairs on sp^3-hybridized nitrogen atoms tend to be stronger due to increased electron availability.

Question 65 : Which of the following electronic configuration would be associated with the highest magnetic moment?

(1) [Ar] $3d^7$
(2) [Ar] $3d^8$
(3) [Ar] $3d^3$
(4) [Ar] $3d^6$

Answer: (4)

Solution:

The magnetic moment μ is given by:

$$\mu = \sqrt{n(n+2)}\,\text{BM}$$

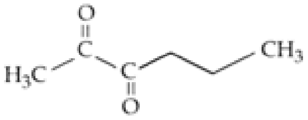
where n is the number of unpaired electrons. Among the options, [Ar] $3d^6$ has the highest number of unpaired electrons (4), leading to a maximum magnetic moment.

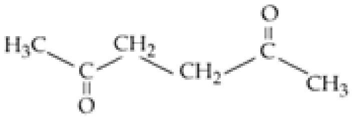
Configuration	No. of unpaired e^-	Spin-only Magnetic Moment (BM)
$3d^7$	3	$\sqrt{15}$
$3d^8$	2	$\sqrt{8}$
$3d^3$	3	$\sqrt{15}$
$3d^6$	4	$\sqrt{24}$

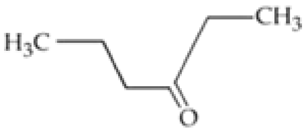
Quick Tip

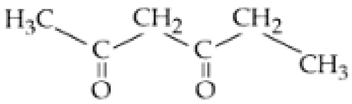
Higher numbers of unpaired electrons result in a greater magnetic moment, calculated using $\mu = \sqrt{n(n+2)}$.

Question 66 : Which of the following has highly acidic hydrogen?

(1) 

(2) 

(3) 

(4) 

Answer: (4)

Solution:

In structure (4), the methylene group (CH_2) between two carbonyl groups (C=O) is highly acidic due to the resonance stabilization of the conjugate base. When the hydrogen is removed, the negative charge on the carbon is delocalized between the two carbonyl groups, making the conjugate base more stable. This increased resonance makes the hydrogen highly acidic.

Quick Tip

Hydrogens between electron-withdrawing groups like carbonyls are more acidic due to resonance stabilization of the conjugate base.

Question 67 : A solution of two miscible liquids showing negative deviation from Raoult's law will have:

(1) Increased vapour pressure, increased boiling point

(2) Increased vapour pressure, decreased boiling point

(3) Decreased vapour pressure, decreased boiling point

(4) Decreased vapour pressure, increased boiling point

Answer: (4)

Solution:

A solution showing negative deviation from Raoult's law has lower vapour pressure than expected because the interactions between the components are stronger than in the pure liquids. This results in a decreased vapour pressure, which leads to an increase in boiling point.

For solutions showing negative deviation:

$$P_{\text{T}} < P_{\text{A}}^0 X_{\text{A}} + P_{\text{B}}^0 X_{\text{B}}$$

where P_{T} is the total vapour pressure, P_{A}^0 and P_{B}^0 are the vapour pressures of pure components, and X_{A} and X_{B} are their mole fractions.

Quick Tip

Solutions with stronger intermolecular interactions than the pure components show negative deviation from Raoult's law.

Question 68 : Consider the following complex ions:

P = $[FeF_6]^{3-}$
Q = $[V(H_2O)_6]^{2+}$
R = $[Fe(H_2O)_6]^{2+}$

The correct order of the complex ions, according to their spin-only magnetic moment values (in B.M.), is:

(1) $R < Q < P$
(2) $R < P < Q$
(3) $Q < R < P$
(4) $Q < P < R$

Answer: (3)

Solution:

- $[FeF_6]^{3-}$: Fe^{3+} has the electron configuration [Ar] $3d^5$. Fluoride (F^-) is a weak field ligand, so all 5 d-electrons remain unpaired.

$$\mu = \sqrt{5(5+2)} = \sqrt{35}\,\text{BM}$$

- $[V(H_2O)_6]^{2+}$: V^{2+} has the electron configuration [Ar] $3d^3$. With three unpaired electrons, the spin-only magnetic moment is:

$$\mu = \sqrt{3(3+2)} = \sqrt{15}\,\text{BM}$$

- $[Fe(H_2O)_6]^{2+}$: Fe^{2+} has the electron configuration [Ar] $3d^6$. Water is a weak field ligand, resulting in four unpaired electrons.

$$\mu = \sqrt{4(4+2)} = \sqrt{24}\,\text{BM}$$

Thus, the correct order of magnetic moments is $Q < R < P$.

Quick Tip

For calculating spin-only magnetic moments, use $\mu = \sqrt{n(n+2)}$ where n is the number of unpaired electrons.

Question 69 : Choose the polar molecule from the following:

(1) CCl_4

(2) CO_2

(3) $CH_2{=}CH_2$

(4) $CHCl_3$

Answer: (4)

Solution:

Among the given options, $CHCl_3$ (chloroform) is polar due to the asymmetrical arrangement of chlorine and hydrogen atoms around the central carbon, resulting in a net dipole moment. In contrast:

- CCl_4: Tetrahedral symmetry with four identical C-Cl bonds cancels out the dipole moments, making it non-polar.

- CO_2: Linear molecule with symmetrical C=O bonds cancels out dipole moments, making it non-polar.

- $CH_2{=}CH_2$: Ethene has a planar structure with symmetrical bonds, making it non-polar.

Thus, $CHCl_3$ is the only polar molecule.

Quick Tip

A molecule is polar if it has an asymmetrical shape that results in a net dipole moment.

Question 70 : Given below are two statements:

Statement I: The 4f and 5f - series of elements are placed separately in the Periodic table to preserve the principle of classification.

Statement II: s-block elements can be found in pure form in nature.

In the light of the above statements, choose the most appropriate answer from the options given below:

(1) Statement I is false but Statement II is true

(2) Both Statement I and Statement II are true

(3) Statement I is true but Statement II is false

(4) Both Statement I and Statement II are false

Answer: (3)

Solution:

Statement I is correct because the 4f and 5f series (lanthanides and actinides) are placed separately to maintain the structure of the Periodic Table. However, Statement II is incorrect, as s-block elements are highly reactive and are generally found in combined states in nature.

Quick Tip

Remember that s-block elements are highly reactive and rarely found in pure form.

Question 71 : Given below are two statements:

Statement I: p-nitrophenol is more acidic than m-nitrophenol and o-nitrophenol.

Statement II: Ethanol will give immediate turbidity with Lucas reagent.

In the light of the above statements, choose the correct answer from the options given below:

(1) Statement I is true but Statement II is false

(2) Both Statement I and Statement II are true

(3) Both Statement I and Statement II are false

(4) Statement I is false but Statement II is true

Answer: (1)

Solution:

- Statement I: p-Nitrophenol is more acidic than m- and o-nitrophenol due to resonance stabilization and inductive effects.

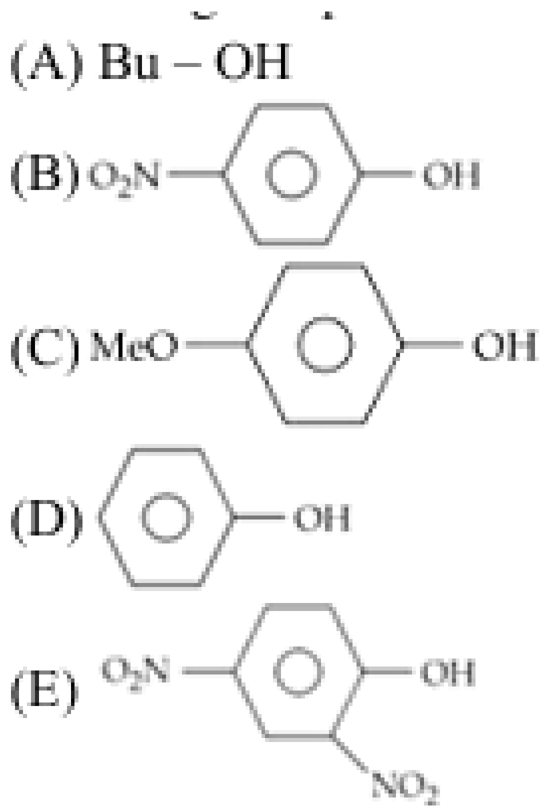
- Statement II: Ethanol reacts slowly with Lucas reagent, not giving immediate turbidity (which is typical of tertiary alcohols).

Thus, Statement I is correct, and Statement II is incorrect.

Quick Tip

Nitrophenols with para-substitution show higher acidity due to increased resonance stability.

Question 72 : The ascending order of acidity of –OH group in the following compounds is:



Choose the correct answer from the options given below:

(1) (A) < (D) < (C) < (B) < (E)

(2) (C) < (A) < (D) < (B) < (E)

(3) (C) < (D) < (B) < (A) < (E)

(4) (A) < (C) < (D) < (B) < (E)

Answer: (4)

Solution:

The acidity of phenols increases with the presence of electron-withdrawing groups (like -NO_2) and decreases with electron-donating groups (like -CH_3).

- (A): Contains a butyl group (electron-donating), making it the least acidic.
- (C): Contains a methoxy group (CH_3O-) (electron-donating), slightly more acidic than (A).
- (D): Simple phenol (C_6H_5OH), with no additional electron-withdrawing or donating groups, has moderate acidity.
- (B): Contains an -OH group with a chlorine substituent (electron-withdrawing) on the aromatic ring, making it more acidic than (D).
- (E): Contains an -OH group with a nitro substituent (NO_2) which is strongly electron-withdrawing, making it the most acidic.

Thus, the correct order of acidity is $(A) < (C) < (D) < (B) < (E)$.

Quick Tip

Electron-withdrawing groups increase acidity by stabilizing the conjugate base, while electron-donating groups decrease acidity.

Question 73 : Given below are two statements: one is labelled as Assertion (A) and the other is labelled as Reason (R).

Assertion (A): Melting point of Boron (2453 K) is unusually high in group 13 elements.

Reason (R): Solid Boron has very strong crystalline lattice.

In the light of the above statements, choose the most appropriate answer from the options given below:

(1) Both (A) and (R) are correct but (R) is not the correct explanation of (A)

(2) Both (A) and (R) are correct and (R) is the correct explanation of (A)

(3) (A) is true but (R) is false

(4) (A) is false but (R) is true

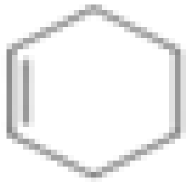
Answer: (2)

Solution:

Both Assertion (A) and Reason (R) are correct, and (R) explains (A). Boron has a strong crystalline structure due to covalent bonding and a unique lattice arrangement, which leads to its unusually high melting point within group 13 elements.

Quick Tip

The melting point of an element is influenced by the strength of its lattice structure; stronger lattices generally result in higher melting points.

Question 74 : Cyclohexene  **is _______ type of an organic compound.**

(1) Benzenoid aromatic

(2) Benzenoid non-aromatic

(3) Acyclic

(4) Alicyclic

Answer: (4)

Solution:

Cyclohexene is an alicyclic compound because it contains a closed ring structure with non-aromatic properties. Alicyclic compounds are cyclic but do not contain a benzene ring or display aromaticity.

Quick Tip

Alicyclic compounds are cyclic compounds that do not have the delocalized -electrons typical of aromatic compounds.

Question 75 : Yellow compound of lead chromate gets dissolved on treatment with hot NaOH solution. The product of lead formed is a:

(1) Tetraanionic complex with coordination number six

(2) Neutral complex with coordination number four

(3) Dianionic complex with coordination number six

(4) Dianionic complex with coordination number four

Answer: (4)

Solution:

The reaction of lead chromate with NaOH (in hot, excess conditions) results in the formation of a soluble dianionic complex:

$$\mathrm{PbCrO_4 + NaOH\ (hot,\ excess) \rightarrow [Pb(OH)_4]^{2-} + Na_2CrO_4}$$

The product $[\mathrm{Pb(OH)_4}]^{2-}$ is a dianionic complex with a coordination number of four.

Quick Tip

Lead hydroxide complexes often exhibit coordination numbers of four due to the geometry and electron configuration of lead.

Question 76 : Given below are two statements:

Statement I: Aqueous solution of ammonium carbonate is basic.

Statement II: Acidic/basic nature of salt solution of a salt of weak acid and weak base depends on K_a and K_b values of acid and the base forming it.

In the light of the above statements, choose the most appropriate answer from the options given below:

(1) Both Statement I and Statement II are correct

(2) Statement I is correct but Statement II is incorrect

(3) Both Statement I and Statement II are incorrect

(4) Statement I is incorrect but Statement II is correct

Answer: (1)

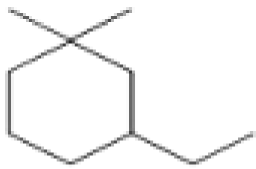
Solution:

- Statement I: Ammonium carbonate $(NH_4)_2CO_3$ is basic in solution because it forms a weak acid (NH_4^+) and a weak base (CO_3^{2-}). The carbonate ion (CO_3^{2-}) can accept protons, making the solution basic.
- Statement II: This statement is also correct as the pH of a salt solution formed from a weak acid and a weak base depends on the relative values of K_a and K_b.

Quick Tip

For salts derived from weak acids and weak bases, the solution's pH depends on the K_a and K_b values of the acid and base.

Question 77 : IUPAC name of the following compound (P) is:



(P)

(1) 1-Ethyl-5, 5-dimethylcyclohexane

(2) 3-Ethyl-1,1-dimethylcyclohexane

(3) 1-Ethyl-3, 3-dimethylcyclohexane

(4) 1,1-Dimethyl-3-ethylcyclohexane

Answer: (2)

Solution:

The correct IUPAC name for the compound shown is 3-Ethyl-1,1-dimethylcyclohexane. The longest carbon chain attached to the cyclohexane ring determines the naming priority. The substituents are numbered to give the lowest possible locants.

Quick Tip

When naming cycloalkanes, assign the lowest possible locants to substituents based on alphabetical order.

Question 78 : NaCl reacts with conc. H_2SO_4 and $K_2Cr_2O_7$ to give reddish fumes (B), which react with NaOH to give yellow solution (C). (B) and (C) respectively are:

(1) CrO_2Cl_2, Na_2CrO_4
(2) Na_2CrO_4, CrO_2Cl_2
(3) CrO_2Cl_2, $KHSO_4$
(4) CrO_2Cl_2, $Na_2Cr_2O_7$

Answer: (1)
Solution:
The reactions involved are:

$$\text{NaCl} + \text{conc. } H_2SO_4 + K_2Cr_2O_7 \rightarrow CrO_2Cl_2 + KHSO_4 + NaHSO_4 + H_2O$$

(B) is CrO_2Cl_2 (reddish fumes). On reaction with NaOH, it produces a yellow solution (C) which is Na_2CrO_4:

$$CrO_2Cl_2 + NaOH \rightarrow Na_2CrO_4 + NaCl + H_2O$$

Quick Tip

Chromyl chloride (CrO_2Cl_2) test is specific for the presence of chloride ions.

Question 79 : The correct statement regarding nucleophilic substitution reaction in a chiral alkyl halide is;

(1) Retention occurs in S_N1 reaction and inversion occurs in S_N2 reaction
(2) Inversion occurs in S_N1 reaction and retention occurs in S_N2 reaction
(3) Racemisation occurs in both S_N1 and S_N2 reactions
(4) Racemisation occurs in S_N1 reaction and inversion occurs in S_N2 reaction

Answer: (4)
Solution:
- S_N1 Reaction: This reaction proceeds via a carbocation intermediate, leading to a racemic mixture if the carbon is chiral, due to the planar nature of the carbocation.

- S_N2 Reaction: This reaction involves a backside attack, resulting in inversion of configuration (Walden inversion) if the carbon is chiral.

Thus, racemisation occurs in S_N1 reactions and inversion occurs in S_N2 reactions.

Quick Tip

S_N1 reactions generally result in racemisation, while S_N2 reactions cause inversion of configuration.

Question 80 : The electronic configuration for Neodymium is:

(Atomic Number for Neodymium 60)

(1) [Xe] $4f^4$ $6s^2$

(2) [Xe] $5f^4 7s^2$

(3) [Xe] $4f^4$ $6s^2$

(4) [Xe] $4f^4 5d^1 6s^2$

Answer: (1)

Solution: Neodymium (Nd) is an element with the atomic number 60, meaning it has 60 electrons. We need to find its correct electronic configuration.

1. Electronic Configuration Strategy: The electronic configuration of elements is determined based on the Aufbau principle, Hund's rule, and Pauli's exclusion principle. The order of filling orbitals follows the increasing energy levels based on the (n + l) rule, where n is the principal quantum number and l is the azimuthal quantum number.

2. Noble Gas Core Representation: The electronic configuration for Neodymium starts with the configuration of the nearest noble gas Xenon ($[Xe]$), which represents the filled electron shells up to atomic number 54:

$$[Xe] = 1s^2 2s^2 2p^6 3s^2 3p^6 4s^2 3d^{10} 4p^6 5s^2 4d^{10} 5p^6$$

3. Filling the Remaining Electrons: After $[Xe]$, the remaining 6 electrons for Neodymium fill the 4f and 6s orbitals. According to the Aufbau principle: - The 4f orbitals have a lower energy level than the 5d and 6s orbitals, so they are filled first. - The 4f subshell can hold up to 14 electrons.

Therefore, the next electrons occupy the 4f orbitals:

$$4f^4 6s^2$$

4. Final Configuration: Thus, the electronic configuration of Neodymium is:

$$[Xe]4f^4 6s^2$$

5. Verification: The options given are: 1. $[Xe]4f^4 6s^2$ 2. $[Xe]5f^7 6s^2$ 3. $[Xe]4f^6 6s^2$ 4. $[Xe]4f^5 5d^1 6s^2$

The correct answer is option 1.

Quick Tip

For lanthanides, electrons fill the 4f subshell before filling higher energy levels.

Question 81 : The mass of silver (Molar mass of Ag: 108 g/mol) displaced by a quantity of electricity which displaces 5600 mL of O_2 at S.T.P. will be ____ g.

Answer: 107 g or 108 g

Solution:

The equation for the equivalent of Ag is:

$$\text{Eq. of Ag} = \text{Eq. of } O_2$$

Let x grams of silver be displaced.

$$\frac{x}{108} = \frac{5.6}{22.7} \times 4$$

Using the molar volume of gas at STP (22.7 L), we get:

$$x = 106.57\,\text{g}$$

Thus, the answer is approximately 107 g.

Alternatively, using 22.4 L as the molar volume at STP:

$$\frac{x}{108} = \frac{5.6}{22.4} \times 4$$

which gives $x = 108\,\text{g}$.

Quick Tip

For reactions involving gases, use the molar volume at STP (22.4 L or 22.7 L) to calculate moles of gas.

Question 82 : Consider the following data for the given reaction

$$2HI_{(g)} \rightarrow H_{2(g)} + I_{2(g)}$$

HI (mol L^{-1})	0.005	0.01	0.02
Rate (mol L^{-1} s^{-1})	7.5×10^{-4}	3.0×10^{-3}	1.2×10^{-2}

The order of the reaction is ______.

Answer: (2)

Solution:

Assuming the rate law:

$$\text{Rate} = k[\text{HI}]^n$$

Using any two of the given data points:

$$\frac{3.0 \times 10^{-3}}{7.5 \times 10^{-4}} = \left(\frac{0.01}{0.005}\right)^n$$

Solving, we find $n = 2$, so the reaction is second order.

Quick Tip

For determining reaction order, compare rate changes with concentration changes in a rate law.

Question 83 : Mass of methane required to produce 22 g of CO_2 after complete combustion is _____ g.

Answer: (8)

Solution:

$$CH_4 + 2O_2 \rightarrow CO_2 + 2H_2O$$

Moles of CO_2 produced:

$$\frac{22}{44} = 0.5\,\text{mol}$$

Required moles of CH_4:

$$0.5\,\text{mol} \times 16\,\text{g/mol} = 8\,\text{g}$$

Quick Tip

In stoichiometry problems, always start by writing a balanced chemical equation. Use the molar mass to convert between grams and moles, and apply stoichiometric ratios based on the balanced equation.

Question 84 : If three moles of an ideal gas at 300 K expand isothermally from 30 dm^3 to 45 dm^3 against a constant opposing pressure of 80 kPa, then the amount of heat transferred is ____ J.

Answer: (1200 J)

Solution:

Using the first law of thermodynamics:

$$\Delta U = Q + W$$

For an isothermal process, $\Delta U = 0$, so $Q = -W$.

$$W = -P_{\text{ext}}\Delta V = -80 \times 10^3 \times (45 - 30) \times 10^{-3} = -1200\,\text{J}$$

Quick Tip

In isothermal processes for ideal gases, internal energy change (ΔU) is zero. Therefore, heat transferred (Q) is equal to the negative of the work done ($-W$). When expansion occurs against a constant external pressure, use $W = -P_{\text{ext}}\Delta V$ to calculate work.

Question 85 : 3-Methylhex-2-ene on reaction with HBr in presence of peroxide forms an addition product (A). The number of possible stereoisomers for 'A' is _____.

Answer: (4)

Solution:

The structure of the product formed will have two chiral centers due to anti-Markovnikov addition of HBr in the presence of peroxide:

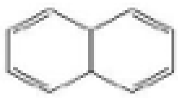
With two chiral centers, the number of stereoisomers is:

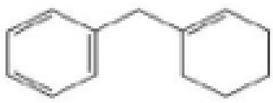
$$2^{\text{chiral centers}} = 2^2 = 4$$

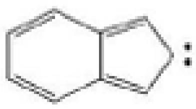
Quick Tip

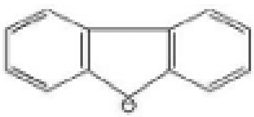
The number of stereoisomers for a compound with n chiral centers is 2^n.

Question 86 : Among the given organic compounds, the total number of aromatic compounds is

(A) 

(B) 

(C) 

(D) 

Answer: (3)

Solution:

Compounds B, C, and D are aromatic. Aromatic compounds must follow Huckel's rule ($4n + 2$ electrons) and have a planar, conjugated structure. Compound A is not aromatic as it does not satisfy these criteria.

Quick Tip

Aromaticity requires a conjugated planar ring with $4n + 2$ electrons (Huckel's rule).

Question 87 : Among the following, total number of meta-directing functional groups is (Integer based)

– OCH_3, –NO_2, –CN, –CH_3–$NHCOCH_3$,

– COR, –OH, – COOH, –Cl

Answer: (4)

Solution:

The meta-directing functional groups are those that are electron-withdrawing, which include:

$$\text{–NO}_2, \text{–C} \equiv \text{N}, \text{–COR}, \text{–COOH}$$

Quick Tip

Electron-withdrawing groups (e.g., –NO_2, –COOH) generally direct electrophilic substitutions to the meta position.

Question 88 : The number of electrons present in all the completely filled subshells having $n = 4$ and $s = \pm\frac{1}{2}$ is _____.

(Where n = principal quantum number and s = spin quantum number)

Answer: (16)

Solution:

For $n = 4$, the possible subshells and their electron capacities are:

Subshell	4s	4p	4d	4f
Total electrons	2	6	10	14
Total electrons with $s = \pm\frac{1}{2}$	1	3	5	7

So, the total number of electrons is 16.

Quick Tip

For each subshell, half the electrons will have $s = +\frac{1}{2}$ and the other half will have $s = -\frac{1}{2}$.

Question 89 : Sum of bond order of CO and NO^+ is _____.

Answer: (6)

Solution:

- CO: Bond order of CO is 3, as it has a triple bond structure ($C \equiv O$).
- NO^+: Bond order of NO^+ is also 3 ($N \equiv O^+$).

Thus, the sum of the bond orders is:

$$3 + 3 = 6$$

Quick Tip

Bond order indicates the number of chemical bonds between a pair of atoms; higher bond order implies a stronger bond.

Question 90 : From the given list, the number of compounds with +4 oxidation state of Sulphur is _____.

SO_3, H_2SO_3, $SOCl_2$, SF_4, $BaSO_4$, $H_2S_2O_7$

Answer: (3)

Solution:

Compound	Oxidation State of Sulfur
SO_3	+6
H_2SO_3	+4
$SOCl_2$	+4
SF_4	+4
$BaSO_4$	+6
$H_2S_2O_7$	+6

The compounds with +4 oxidation state of Sulfur are H_2SO_3, $SOCl_2$, and SF_4, giving a total of 3.

Quick Tip

To determine oxidation states, use the known oxidation states of other atoms and solve for sulfur.
