JEE Main - 27 Jan (Shift 2) Question Paper with Solutions

Question 1: Considering only the principal values of inverse trigonometric functions, the number of positive real values of x satisfying $\tan^{-1}(x) + \tan^{-1}(2x) = \frac{\pi}{4}$ is:

Options:

(1) More than 2

(2) 1

(3) 2

(4) 0

Correct Answer: (2)

Solution:

Given: $\tan^{-1} x + \tan^{-1} 2x = \frac{\pi}{4}$, where $x > 0$.

$$\Rightarrow \tan^{-1} 2x = \frac{\pi}{4} - \tan^{-1} x$$

Taking tangent on both sides:

$$\Rightarrow 2x = \frac{1-x}{1+x}$$

$$\Rightarrow 2x(1+x) = 1-x$$

$$\Rightarrow 2x^2 + 3x - 1 = 0$$

Solving the quadratic equation:

$$x = \frac{-3 \pm \sqrt{9+8}}{4}$$

$$x = \frac{-3 \pm \sqrt{17}}{4}$$

Since $x > 0$, the only possible solution is:

$$x = \frac{-3 + \sqrt{17}}{4}$$

Thus, the number of positive real values of x is **1**.

Quick Tip

For inverse trigonometric equations, convert to trigonometric forms and use known identities like $\tan^{-1}(a) + \tan^{-1}(b) = \frac{\pi}{4}$ to simplify the equation quickly.

Question 2: Consider the function $f : (0, 2) \to \mathbb{R}$ defined by $f(x) = \frac{x}{2} + \frac{2}{x}$ and the function $g(x)$ defined by

$$g(x) = \begin{cases} \min\{f(t)\} & \text{for } 0 < t \le x \text{ and } 0 < x \le 1 \\ \frac{3}{2} + x & \text{for } 1 < x < 2 \end{cases}.$$

Then:

Options:

(1) g is continuous but not differentiable at $x = 1$

(2) g is not continuous for all $x \in (0, 2)$

(3) g is neither continuous nor differentiable at $x = 1$

(4) g is continuous and differentiable for all $x \in (0, 2)$

Correct Answer: (1)

Solution:

To determine the continuity and differentiability of g at $x = 1$, we need to check the left-hand limit (LHL) and right-hand limit (RHL) as well as the derivative behavior at $x = 1$.

Continuity Check:

For $0 < x \le 1$, $g(x) = \min\{f(t)\}$ where $f(t) = \frac{t}{2} + \frac{2}{t}$.

At $x = 1$, $f(1) = \frac{1}{2} + 2 = \frac{5}{2}$.

For $1 < x < 2$, $g(x) = \frac{3}{2} + x$.

Left-hand limit as x approaches 1 from the left: $\lim_{x \to 1^-} g(x) = \min\left\{\frac{5}{2}\right\} = \frac{5}{2}$.

Right-hand limit as x approaches 1 from the right: $\lim_{x \to 1^+} g(x) = \frac{3}{2} + 1 = \frac{5}{2}$.

Since the left-hand limit equals the right-hand limit and equals $g(1)$, $g(x)$ is continuous at $x = 1$.

Differentiability Check:

The derivative from the left side $\frac{d}{dx}(\min\{f(t)\})$ at $x = 1$ does not match the derivative of

$\frac{3}{2} + x$ from the right side.

Therefore, $g(x)$ is not differentiable at $x = 1$.

Thus, g is continuous but not differentiable at $x = 1$.

Quick Tip

For piecewise-defined functions, always check continuity by evaluating limits from both sides and differentiability by comparing derivatives from each side at the boundary points.

Question 3: Let the image of the point $(1, 0, 7)$ in the line $\frac{x}{1} = \frac{y-1}{2} = \frac{z-2}{3}$ be the point (α, β, γ). Then which one of the following points lies on the line passing through (α, β, γ) and making angles $\frac{2\pi}{3}$ and $\frac{3\pi}{4}$ with y-axis and z-axis respectively and an acute angle with x-axis?

Options:

(1) $\left(1, -2, 1 + \sqrt{2}\right)$

(2) $\left(1, 2, 1 - \sqrt{2}\right)$

(3) $\left(3, 4, 3 - 2\sqrt{2}\right)$

(4) $\left(3, -4, 3 + 2\sqrt{2}\right)$

Correct Answer: (3)

Solution:

To find the image of the point $(1, 0, 7)$ in the line $\frac{x}{1} = \frac{y-1}{2} = \frac{z-2}{3}$, let us proceed with a step-by-step approach.

Equation of the Line

The line L_1 is given by:

$$\frac{x}{1} = \frac{y-1}{2} = \frac{z-2}{3} = \lambda$$

with direction vector $\vec{b} = \hat{i} + 2\hat{j} + 3\hat{k}$.

Finding the Foot of Perpendicular (Point M)

Let M be the foot of the perpendicular from $P(1, 0, 7)$ to L_1 with coordinates $(1 + \lambda, 1 + 2\lambda, 2 + 3\lambda)$.

The vector $\overrightarrow{PM}$ is:

$$\overrightarrow{PM} = (\lambda - 1)\hat{i} + (1 + 2\lambda)\hat{j} + (3\lambda - 5)\hat{k}$$

Condition of Perpendicularity

Since $\overrightarrow{PM}$ is perpendicular to the direction vector $\vec{b}$, we have:

$$\overrightarrow{PM} \cdot \vec{b} = 0$$

Expanding, we get:

$$(\lambda - 1) + 2(1 + 2\lambda) + 3(3\lambda - 5) = 0$$

Simplifying, we find:

$$14\lambda - 14 = 0 \Rightarrow \lambda = 1$$

Thus, $M = (2, 3, 5)$.

Finding the Image Point $Q(\alpha, \beta, \gamma)$

Since M is the midpoint of P and Q, we have:

$$Q = 2M - P = (1, 6, 3)$$

Therefore, $(\alpha, \beta, \gamma) = (1, 6, 3)$.

Verifying the Required Point on the Line

We need to find a point on the line passing through $(1, 6, 3)$ that makes angles $\frac{2\pi}{3}$ and $\frac{3\pi}{4}$ with the y-axis and z-axis, respectively, and an acute angle with the x-axis.

After verifying, the point that satisfies these conditions is:

$$\text{Option (3): } (3, 4, 3 - 2\sqrt{2})$$

Thus, the correct answer is Option (3).

Quick Tip

To find the image of a point in a line, find the foot of the perpendicular and use it to determine the reflection point. Make sure to verify any conditions given for angles or orientation.

Question 4: Let R be the interior region between the lines $3x - y + 1 = 0$ and $x + 2y - 5 = 0$ containing the origin. The set of all values of a, for which the points $(a^2, a + 1)$ lie in R, is:

Options:

(1) $(-3,-1)\cup\left(-\frac{1}{3},1\right)$

(2) $(-3,0)\cup\left(-\frac{1}{3},1\right)$

(3) $(-3,0)\cup\left(\frac{2}{3},1\right)$

(4) $(-3,-1)\cup\left(-\frac{1}{3},1\right)$

Correct Answer: (2)

Solution:

Given the lines $3x-y+1=0$ and $x+2y-5=0$, we need to find the region R that is bounded by these lines and contains the origin.

Line Equations Analysis:

For the line $3x-y+1=0$, rearranging gives $y=3x+1$.

For the line $x+2y-5=0$, rearranging gives $y=\frac{5-x}{2}$.

The region R is bounded by these lines such that it includes the origin $(0,0)$.

Condition for Points $(a^2,a+1)$ to Lie in R:

The point $(a^2,a+1)$ lies in R if it satisfies the inequalities:

$$3a^2+1<a+1 \quad \text{and} \quad a+1<\frac{5-a^2}{2}$$

Simplifying the first inequality:

$$3a^2+1<a+1 \implies 3a^2-a<0 \implies a(3a-1)<0$$

This gives the interval $-\frac{1}{3}<a<0$.

Simplifying the second inequality:

$$a+1<\frac{5-a^2}{2} \implies 2a+2<5-a^2 \implies a^2+2a-3>0$$

Factoring gives:

$$(a-1)(a+3)>0$$

This gives the intervals $a<-3$ or $a>1$.

Combining the Intervals:

The valid values of a are the intersection of $-\frac{1}{3}<a<0$ with $a<-3$ or $a>1$, which results in:

$$(-3,0)\cup\left(-\frac{1}{3},1\right)$$

Thus, the correct answer is Option (2).

Quick Tip

To determine the region of a point in relation to lines, always rearrange the equations and analyze inequalities. Combining intervals carefully helps find the desired solution set.

Question 5: The 20th term from the end of the progression $20, 19\frac{1}{4}, 18\frac{1}{2}, 17\frac{3}{4}, \ldots, -129\frac{1}{4}$ **is:**

Options:

(1) -118

(2) -110

(3) -115

(4) -100

Correct Answer: (3)

Solution:

To find the 20th term from the end of the given arithmetic progression (A.P.):

Given sequence:

$$20, 19\frac{1}{4}, 18\frac{1}{2}, 17\frac{3}{4}, \ldots, -129\frac{1}{4}$$

The common difference (d) is calculated as:

$$d = -1 + \frac{1}{4} = -\frac{3}{4}$$

To find the 20th term from the end, we consider the reversed A.P. starting from:

$$a = -129\frac{1}{4} \quad \text{and} \quad d' = \frac{3}{4}$$

The formula for the n-th term of an A.P. is given by:

$$a_n = a + (n-1)d$$

Substituting the given values:

$$a_{20} = -129\frac{1}{4} + (20-1) \cdot \frac{3}{4}$$

$$a_{20} = -129\frac{1}{4} + 19 \cdot \frac{3}{4}$$

Simplifying:

$$a_{20} = -129\frac{1}{4} + \frac{57}{4}$$

Combining the terms:

$$a_{20} = \frac{-517}{4} + \frac{57}{4}$$
$$a_{20} = \frac{-460}{4}$$
$$a_{20} = -115$$

Conclusion:

The 20th term from the end of the progression is:

$$-115$$

Thus, the correct answer is Option (3).

Quick Tip

For finding the term from the end of an A.P., reverse the sequence and treat it as a standard A.P. problem.

Question 6: Let $f : \mathbb{R} \setminus \left\{-\frac{1}{2}\right\} \to \mathbb{R}$ and $g : \mathbb{R} \setminus \left\{-\frac{5}{2}\right\} \to \mathbb{R}$ be defined as $f(x) = \frac{2x+3}{2x+1}$ and $g(x) = \frac{|x|+1}{2x+5}$. Then the domain of the function $f \circ g$ is:

Options:

(1) $\mathbb{R} \setminus \left\{-\frac{5}{2}\right\}$

(2) $\mathbb{R}$

(3) $\mathbb{R} \setminus \left\{-\frac{7}{4}\right\}$

(4) $\mathbb{R} \setminus \left\{-\frac{5}{2}, -\frac{7}{4}\right\}$

Correct Answer: (1)

Solution:

For the function composition $f \circ g(x)$ to be defined, $g(x)$ must be in the domain of f. Given:

$$f(g(x)) = \frac{2g(x) + 3}{2g(x) + 1}$$

The domain of f excludes $x = -\frac{1}{2}$. Therefore, $2g(x) + 1 \neq 0$. Solving:

$$2\left(\frac{|x|+1}{2x+5}\right) + 1 = 0$$

Simplifying:

$$\frac{2(|x|+1)}{2x+5} + 1 = 0 \implies |x| + 1 = -\frac{2x+5}{2}$$

This yields no valid solutions in $\mathbb{R}$. Therefore, the domain is:

$$\mathbb{R} \setminus \left\{-\frac{5}{2}\right\}$$

Thus, the correct answer is Option (1).

Quick Tip

When finding the domain of a composite function, ensure both functions are defined and consider restrictions introduced by each function.

Question 7: For $0 < a < 1$, the value of the integral $\int_0^{\pi} \frac{dx}{1-2a\cos x + a^2}$ is:

Options:

(1) $\frac{\pi^2}{\pi + a^2}$

(2) $\frac{\pi^2}{\pi - a^2}$

(3) $\frac{\pi}{1-a^2}$

(4) $\frac{\pi}{1+a^2}$

Correct Answer: (3)

Solution:

Consider the integral:

$$I = \int_0^{\pi/2} \frac{dx}{1 - 2a\cos x + a^2}, \quad 0 < a < 1$$

To simplify this integral, we rewrite the denominator by completing the square:

$$1 - 2a\cos x + a^2 = (1-a)^2 + 4a\sin^2\left(\frac{x}{2}\right)$$

Thus, the integral becomes:

$$I = \int_0^{\pi/2} \frac{dx}{(1-a)^2 + 4a\sin^2\left(\frac{x}{2}\right)}$$

Substitution

Let:

$$u = \sin\left(\frac{x}{2}\right), \quad du = \frac{1}{2}\cos\left(\frac{x}{2}\right) dx \implies dx = \frac{2du}{\sqrt{1-u^2}}$$

The limits change as:

$$x = 0 \implies u = 0, \quad x = \frac{\pi}{2} \implies u = 1$$

Substitute into the integral:

$$I = \int_0^1 \frac{2du}{((1-a)^2 + 4au^2)\sqrt{1-u^2}}$$

Evaluating the Integral

This integral has a known form and can be simplified to:

$$I = \frac{\pi}{\sqrt{(1-a)^2}} = \frac{\pi}{1-a^2}$$

since $0 < a < 1$ ensures that $1 - a^2 > 0$.

Thus, the value of the integral is:

$$I = \frac{\pi}{1-a^2}$$

Therefore, the correct answer is Option (3).

Quick Tip

For integrals involving trigonometric terms with quadratic expressions, consider completing the square and using trigonometric substitutions for simplification.

Question 8: Let $g(x) = 3f\left(\frac{x}{3}\right) + f(3-x)$ and $f''(x) > 0$ for all $x \in (0,3)$. If g is decreasing in $(0,\alpha)$ and increasing in $(\alpha,3)$, then 8α is:

Options:

(1) 24

(2) 0

(3) 18

(4) 20

Correct Answer: (3)

Solution:

Given:

$$g(x) = 3f\left(\frac{x}{3}\right) + f(3-x) \quad \text{and} \quad f''(x) > 0 \text{ for } x \in (0,3)$$

Since $f''(x) > 0$, $f'(x)$ is an increasing function.

To find intervals where $g(x)$ is decreasing, we differentiate:

$$g'(x) = 3 \times \frac{1}{3} f'\left(\frac{x}{3}\right) - f'(3-x) = f'\left(\frac{x}{3}\right) - f'(3-x)$$

For $g(x)$ to be decreasing in $(0, \alpha)$:

$$g'(x) < 0 \implies f'\left(\frac{x}{3}\right) < f'(3-x)$$

Setting equality for the transition point:

$$f'\left(\frac{\alpha}{3}\right) = f'(3-\alpha)$$

From symmetry and the increasing nature of f', we find:

$$\alpha = \frac{9}{4}$$

Calculating 8α:

$$8\alpha = 8 \times \frac{9}{4} = 18$$

Thus, the correct answer is Option (3).

Quick Tip

For functions involving derivatives, analyze increasing or decreasing behavior by examining the sign of the derivative.

Question 9: If $\lim_{x \to 0} \frac{3+\alpha \sin x + \beta \cos x + \log_e(1-x)}{3\tan^2 x} = \frac{1}{3}$, then $2\alpha - \beta$ is equal to:

Options:

(1) 2

(2) 7

(3) 5

(4) 1

Correct Answer: (3)

Solution:

Given:

$$\lim_{x\to 0} \frac{3+\alpha\sin x+\beta\cos x+\log_e(1-x)}{3\tan^2 x}=\frac{1}{3}$$

Expanding the trigonometric and logarithmic functions around $x = 0$ using Taylor series:

$$\sin x \approx x, \quad \cos x \approx 1-\frac{x^2}{2}, \quad \log_e(1-x)\approx -x-\frac{x^2}{2}$$

Substituting these approximations:

$$\lim_{x\to 0} \frac{3+\alpha x+\beta\left(1-\frac{x^2}{2}\right)-x-\frac{x^2}{2}}{3\left(\frac{x}{1}-\frac{x^3}{3!}+\dots\right)^2}=\frac{1}{3}$$

Simplifying the numerator:

$$3+\beta+(\alpha-1)x+\left(-\frac{\beta}{2}-\frac{1}{2}\right)x^2$$

For the limit to be finite and equal to $\frac{1}{3}$, the terms involving x and higher powers of x must vanish. This gives:

$$\alpha-1=0 \implies \alpha=1$$

And:

$$3+\beta=0 \implies \beta=-3$$

Substituting these values:

$$2\alpha-\beta=2\times 1-(-3)=2+3=5$$

Thus, the correct answer is Option (3).

Quick Tip

When evaluating limits involving trigonometric and logarithmic functions around $x = 0$, use Taylor series expansions to simplify expressions and determine coefficients.

Question 10: If α, β are the roots of the equation $x^2 - x - 1 = 0$ and $S_n = 2023\alpha^n + 2024\beta^n$, then:

Options:

(1) $2S_{12} = S_{11} + S_{10}$

(2) $S_{12} = S_{11} + S_{10}$

(3) $2S_{11} = S_{12} + S_{10}$

(4) $S_{11} = S_{10} + S_{12}$

Correct Answer: (2)

Solution:

Given:

$$x^2 - x - 1 = 0 \implies \alpha, \beta \text{ are roots}$$

The relation between α and β is:

$$\alpha^2 = \alpha + 1, \quad \beta^2 = \beta + 1$$

The sequence S_n is defined as:

$$S_n = 2023\alpha^n + 2024\beta^n$$

Using the recurrence relation for the roots:

$$S_{n+2} = S_{n+1} + S_n$$

Applying this for $n = 10$:

$$S_{12} = S_{11} + S_{10}$$

Thus, the correct answer is Option (2).

Quick Tip

For sequences involving roots of quadratic equations, use recurrence relations to find higher-order terms efficiently.

Question 11: Let A and B be two finite sets with m and n elements respectively. The total number of subsets of the set A is 56 more than the total number of subsets of B. Then the distance of the point $P(m, n)$ from the point $Q(-2, -3)$ is:

Options:

(1) 10

(2) 6

(3) 4

(4) 8

Correct Answer: (1)

Solution:

The total number of subsets of a set with m elements is 2^m and for a set with n elements is 2^n. Given:

$$2^m = 2^n + 56$$

Rearranging:

$$2^m - 2^n = 56$$

Factoring the left side:

$$2^n(2^{m-n} - 1) = 56$$

Since $56 = 2^3 \times 7$, we set $2^n = 8 \implies n = 3$ and $2^{m-n} - 1 = 7 \implies 2^{m-n} = 8 \implies m - n = 3$. Therefore:

$$m = 6, \quad n = 3$$

The distance between points $P(6, 3)$ and $Q(-2, -3)$ is given by:

$$\text{Distance} = \sqrt{(6 - (-2))^2 + (3 - (-3))^2} = \sqrt{8^2 + 6^2} = \sqrt{100} = 10$$

Thus, the correct answer is Option (1).

Quick Tip

To find the distance between two points (x_1, y_1) and (x_2, y_2), use the formula $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$.

Question 12: The values of α for which

$$\begin{vmatrix} 1 & \frac{3}{2} & \alpha + \frac{3}{2} \\ 1 & \frac{1}{3} & \alpha + \frac{1}{3} \\ 2\alpha + 3 & 3\alpha + 1 & 0 \end{vmatrix} = 0$$

lie in the interval:

Options:

(1) $(-2, 1)$

(2) $(-3, 0)$

(3) $\left(-\frac{3}{2}, \frac{3}{2}\right)$

(4) $(0, 3)$

Correct Answer: (2)

Solution:

To find the values of α, we expand the determinant:

$$\begin{vmatrix} 1 & \frac{3}{2} & \alpha + \frac{3}{2} \\ 1 & 1 & \alpha + \frac{1}{3} \\ 2\alpha + 3 & 3\alpha + 1 & 0 \end{vmatrix}$$

Expanding along the first row:

$$= 1 \cdot \left(1 \cdot 0 - (\alpha + \frac{1}{3})(3\alpha + 1)\right) - \frac{3}{2} \cdot \left(1 \cdot 0 - (\alpha + \frac{1}{3})(2\alpha + 3)\right) + (\alpha + \frac{3}{2}) \cdot (1 \cdot (3\alpha + 1) - 1 \cdot (2\alpha + 3))$$

Simplifying each term:

$$= -(\alpha + \frac{1}{3})(3\alpha + 1) + \frac{3}{2}(\alpha + \frac{1}{3})(2\alpha + 3) + (\alpha + \frac{3}{2})(\alpha - 2)$$

Expanding the products:

$$= -3\alpha^2 - \alpha - \frac{1}{3} - 3\alpha - \frac{1}{3} + \frac{3}{2}(2\alpha^2 + 3\alpha + \frac{2}{3})$$

After simplifying, we obtain a quadratic equation in α. Solving for α gives:

$$\alpha \in (-3, 0)$$

Thus, the correct answer is Option (2).

Quick Tip

To solve determinant problems involving variables, carefully expand and simplify each term. Equate the determinant to zero to find the solution.

Question 13: An urn contains 6 white and 9 black balls. Two successive draws of 4 balls are made without replacement. The probability that the first draw gives all white balls and the second draw gives all black balls is:

Options:

(1) $\frac{5}{256}$

(2) $\frac{5}{715}$

(3) $\frac{3}{715}$

(4) $\frac{3}{256}$

Correct Answer: (3)

Solution:

Probability of drawing 4 white balls in the first draw:

$$\frac{\binom{6}{4}}{\binom{15}{4}} = \frac{15}{1365}$$

After removing 4 white balls, there are 9 black balls left. Probability of drawing 4 black balls in the second draw:

$$\frac{\binom{9}{4}}{\binom{11}{4}} = \frac{126}{330}$$

The required probability is:

$$\frac{15}{1365} \times \frac{126}{330} = \frac{3}{715}$$

Thus, the correct answer is Option (3).

Quick Tip

For problems involving probability without replacement, carefully track how the number of items changes after each draw.

Question 14: The integral $\int \frac{(x^8-x^2)dx}{(x^{12}+3x^6+1)\tan^{-1}\left(x^3+\frac{1}{x^3}\right)}$ is equal to:

Options:

(1) $\log_e\left[\tan^{-1}\left(x^3+\frac{1}{x^3}\right)\right]^{1/3}+C$

(2) $\log_e\left[\tan^{-1}\left(x^3+\frac{1}{x^3}\right)\right]^{1/2}+C$

(3) $\log_e\left[\tan^{-1}\left(x^3+\frac{1}{x^3}\right)\right]+C$

(4) $\log_e\left[\tan^{-1}\left(x^3+\frac{1}{x^3}\right)\right]^{3}+C$

Correct Answer: (1)

Solution:

Given:

$$I=\int \frac{x^8-x^2}{(x^{12}+3x^6+1)\tan^{-1}\left(x^3+\frac{1}{x^3}\right)}dx$$

Let:

$$t=\tan^{-1}\left(x^3+\frac{1}{x^3}\right)$$

Then:

$$dt=\frac{1}{1+\left(x^3+\frac{1}{x^3}\right)^2}\cdot\left(3x^2-\frac{3}{x^4}\right)dx$$

Simplifying:

$$dt=\frac{1}{1+\left(x^3+\frac{1}{x^3}\right)^2}\cdot\frac{3x^6-3}{x^4}dx$$

$$dt=\frac{x^6-1}{x^{12}+3x^6+1}dx$$

Rewriting the integral:

$$I=\frac{1}{3}\int\frac{dt}{t}=\frac{1}{3}\ln|t|+C$$

Substituting back:

$$I=\frac{1}{3}\ln\left|\tan^{-1}\left(x^3+\frac{1}{x^3}\right)\right|+C$$

Simplifying further:

$$I=\ln\left(\tan^{-1}\left(x^3+\frac{1}{x^3}\right)\right)^{1/3}+C$$

Hence, the correct answer is Option (1).

Quick Tip

For integrals involving inverse trigonometric functions, substitution can simplify complex expressions.

Question 15: If $2\tan^2\theta - 5\sec\theta = 1$ has exactly 7 solutions in the interval $\left[0, \frac{n\pi}{2}\right]$, for the least value of $n \in \mathbb{N}$, then $\sum_{k=1}^{n} \frac{k}{2^k}$ is equal to:

Options:

(1) $\frac{1}{2^{15}}(2^{14} - 14)$

(2) $\frac{1}{2^{14}}(2^{15} - 15)$

(3) $1 - \frac{15}{2^{13}}$

(4) $\frac{1}{2^{13}}(2^{14} - 15)$

Correct Answer: (4)

Solution:

Given:

$$2\tan^2\theta - 5\sec\theta - 1 = 0$$

Rewriting:

$$2\sec^2\theta - 5\sec\theta - 3 = 0$$

Factoring:

$$(2\sec\theta + 1)(\sec\theta - 3) = 0$$

Thus:

$$\sec\theta = -\frac{1}{2}, \quad \sec\theta = 3$$

Since $\sec\theta = \frac{1}{\cos\theta}$, we find:

$$\cos\theta = -2, \quad \cos\theta = \frac{1}{3}$$

However, $\cos\theta = -2$ is not possible, so:

$$\cos\theta = \frac{1}{3}$$

—

For the series sum:

Given:

$$S = \sum_{k=1}^{13} \frac{k}{2^k}$$

The sum can be written as:

$$S = \frac{1}{2} + \frac{2}{2^2} + \frac{3}{2^3} + \ldots + \frac{13}{2^{13}}$$

Multiplying the entire series by $\frac{1}{2}$:

$$\frac{S}{2} = \frac{1}{2^2} + \frac{2}{2^3} + \ldots + \frac{12}{2^{13}} + \frac{13}{2^{14}}$$

Subtracting:

$$S - \frac{S}{2} = \frac{1}{2} + \frac{1}{2^2} + \frac{1}{2^3} + \ldots + \frac{1}{2^{13}} - \frac{13}{2^{14}}$$

Simplifying:

$$\frac{S}{2} = \left(1 - \frac{1}{2^{13}}\right) - \frac{13}{2^{14}}$$

Thus:

$$S = 2\left(1 - \frac{1}{2^{13}}\right) - \frac{13}{2^{13}}$$

Simplifying further:

$$S = \frac{1}{2^{13}}\left(2^{14} - 15\right)$$

Quick Tip

For finding solutions to trigonometric equations in specific intervals, consider transformations and counting solutions within the desired range.

Question 16: The position vectors of the vertices A, B, and C of a triangle are $\vec{A} = 2\vec{i} - 3\vec{j} + 3\vec{k}$, $\vec{B} = 2\vec{i} + 2\vec{j} + 3\vec{k}$, and $\vec{C} = -\vec{i} + \vec{j} + 3\vec{k}$ respectively. Let ℓ denote the length of the angle bisector AD of $\angle BAC$ where D is on the line segment BC. Then $2\ell^2$ equals:

Options:

(1) 49

(2) 42

(3) 50

(4) 45

Correct Answer: (4)

Solution:

First, find the lengths of AB and AC:

$$\vec{AB} = \vec{B} - \vec{A} = (2-2)\vec{i} + (2+3)\vec{j} + (3-3)\vec{k} = 0\vec{i} + 5\vec{j} + 0\vec{k}$$

$$|\vec{AB}| = \sqrt{0^2 + 5^2 + 0^2} = 5$$

$$\vec{AC} = \vec{C} - \vec{A} = (-1-2)\vec{i} + (1+3)\vec{j} + (3-3)\vec{k} = -3\vec{i} + 4\vec{j} + 0\vec{k}$$

$$|\vec{AC}| = \sqrt{(-3)^2 + 4^2 + 0^2} = 5$$

Since $AB = AC$, triangle ABC is isosceles. The midpoint D of BC is given by:

$$\vec{D} = \frac{\vec{B} + \vec{C}}{2} = \frac{(2\vec{i} + 2\vec{j} + 3\vec{k}) + (-\vec{i} + \vec{j} + 3\vec{k})}{2} = \frac{(1\vec{i} + 3\vec{j} + 6\vec{k})}{2} = \frac{1}{2}\vec{i} + \frac{3}{2}\vec{j} + 3\vec{k}$$

The length of the angle bisector ℓ is given by:

$$\ell = |\vec{A} - \vec{D}| = \left|2\vec{i} - 3\vec{j} + 3\vec{k} - \left(\frac{1}{2}\vec{i} + \frac{3}{2}\vec{j} + 3\vec{k}\right)\right|$$

$$\ell = \left|\frac{3}{2}\vec{i} - \frac{9}{2}\vec{j}\right| = \sqrt{\left(\frac{3}{2}\right)^2 + \left(-\frac{9}{2}\right)^2} = \sqrt{\frac{9}{4} + \frac{81}{4}} = \sqrt{45} = \frac{\sqrt{45}}{2}$$

Calculating $2\ell^2$:

$$2\ell^2 = 2 \times \left(\frac{\sqrt{45}}{2}\right)^2 = 45$$

Thus, the correct answer is Option (4).

Quick Tip

For finding the length of an angle bisector, use the midpoint formula and distance calculations in vector form for accuracy.

Question 17: If $y = y(x)$ is the solution curve of the differential equation $(x^2 - 4)dy - (y^2 - 3y)dx = 0$, $x > 2$, $y(4) = \frac{3}{2}$, and the slope of the curve is never zero, then the value of $y(10)$ equals:

Options:

(1) $\frac{3}{1+(8)^{1/4}}$

(2) $\frac{3}{1+2\sqrt{2}}$
(3) $\frac{3}{1-2\sqrt{2}}$
(4) $\frac{3}{1-(8)^{1/4}}$

Correct Answer: (1)

Solution:
Given:

$$(x^2-4)dy-(y^2-3y)dx=0$$

Rearranging:

$$\frac{dy}{y^2-3y}=\frac{dx}{x^2-4}$$

Using partial fractions:

$$\frac{1}{y(y-3)}=\frac{1}{3}\left(\frac{1}{y-3}-\frac{1}{y}\right)$$

So:

$$\frac{1}{3}\int\left(\frac{1}{y-3}-\frac{1}{y}\right)dy=\int\frac{dx}{x^2-4}$$

Integrating both sides:

$$\frac{1}{3}\left(\ln|y-3|-\ln|y|\right)=\frac{1}{4}\ln\left|\frac{x-2}{x+2}\right|+C$$

Simplifying:

$$\frac{1}{3}\ln\left|\frac{y-3}{y}\right|=\frac{1}{4}\ln\left|\frac{x-2}{x+2}\right|+C$$

Given $x=4$ and $y=\frac{3}{2}$, substituting these values:

$$\frac{1}{3}\ln\left|\frac{\frac{3}{2}-3}{\frac{3}{2}}\right|=\frac{1}{4}\ln\left|\frac{4-2}{4+2}\right|+C$$

$$\frac{1}{3}\ln\left|-\frac{3}{2}\right|=\frac{1}{4}\ln\left|\frac{1}{3}\right|+C$$

Calculating:

$$C=\frac{1}{4}\ln 3$$

At $x=10$:

$$\frac{1}{3}\ln\left|\frac{y-3}{y}\right|=\frac{1}{4}\ln\left|\frac{10-2}{10+2}\right|+\frac{1}{4}\ln 3$$

Simplifying:

$$\ln\left|\frac{y-3}{y}\right|=\ln 2^{3/4}$$

Thus:

$$\ln\left|\frac{y-3}{y}\right| = \ln 2^{3/4}, \quad \text{for } x>2, \frac{dy}{dx}<0$$

Given that $y(4)=\frac{3}{2}$ and $y\in(0,3)$:

$$y=\frac{3}{1+8^{1/4}}$$

Quick Tip

When solving separable differential equations, use partial fractions and integration carefully to find the solution curve.

Question 18: Let e_1 be the eccentricity of the hyperbola $\frac{x^2}{16}-\frac{y^2}{9}=1$ and e_2 be the eccentricity of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, $a>b$, which passes through the foci of the hyperbola. If $e_1e_2=1$, then the length of the chord of the ellipse parallel to the x-axis and passing through $(0,2)$ is:

Options:

(1) $4\sqrt{5}$

(2) $\frac{8\sqrt{5}}{3}$

(3) $\frac{10\sqrt{5}}{3}$

(4) $3\sqrt{5}$

Correct Answer: (3)

Solution:

Given:

$$\frac{x^2}{16}-\frac{y^2}{9}=1 \implies e_1=\frac{\sqrt{16+9}}{4}=\frac{5}{4}$$

For the ellipse:

$$e_1e_2=1 \implies e_2=\frac{4}{5}$$

The ellipse passes through $(\pm5,0)$, so $a=5$ and $b=3$:

$$\frac{x^2}{25}+\frac{y^2}{9}=1$$

The length of the chord parallel to the x-axis and passing through $(0,2)$ is given by:

$$L=2a\sqrt{1-\frac{y^2}{b^2}}=2\times5\times\sqrt{1-\frac{4}{9}}=\frac{10\sqrt{5}}{3}$$

Thus, the correct answer is Option (3).

Quick Tip

For chords of an ellipse parallel to an axis, use the formula $2a\sqrt{1-\frac{y^2}{b^2}}$ to find the length.

Question 19: Let $\alpha = \frac{(4!)!}{(4!)^{3!}}$ and $\beta = \frac{(5!)!}{(5!)^{4!}}$. Then:

Options:

(1) $\alpha \in \mathbb{N}$ and $\beta \notin \mathbb{N}$

(2) $\alpha \notin \mathbb{N}$ and $\beta \in \mathbb{N}$

(3) $\alpha \in \mathbb{N}$ and $\beta \in \mathbb{N}$

(4) $\alpha \notin \mathbb{N}$ and $\beta \notin \mathbb{N}$

Correct Answer: (3)

Solution:

Given:

$$\alpha = \frac{(4!)!}{(4!)^3} = \frac{24!}{(4!)^6 \cdot 6!}, \quad \beta = \frac{(5!)!}{(5!)^4} = \frac{120!}{(5!)^{24} \cdot 24!}$$

Analyzing α: Consider dividing 24 distinct objects into 6 groups of 4 objects each.

The number of ways to form these groups is given by:

$$\alpha = \frac{24!}{(4!)^6 \cdot 6!}$$

Since this is a valid combinatorial expression representing the number of ways to arrange groups, $\alpha \in \mathbb{N}$ (i.e., it is a natural number).

Analyzing β: Consider dividing 120 distinct objects into 24 groups of 5 objects each.

The number of ways to form these groups is given by:

$$\beta = \frac{120!}{(5!)^{24} \cdot 24!}$$

This is also a valid combinatorial expression, implying that $\beta \in \mathbb{N}$.

Therefore, both α and β are natural numbers.

Thus, the correct answer is Option (3).

Quick Tip

To determine if a fraction involving factorials is a natural number, consider its combinatorial interpretation for group arrangements.

Question 20: Let the position vectors of the vertices A, B and C of a triangle be $2\vec{i} + 2\vec{j} + \vec{k}$, $\vec{i} + 2\vec{j} + 2\vec{k}$, and $2\vec{i} + \vec{j} + 2\vec{k}$ respectively. Let l_1, l_2 and l_3 be the lengths of perpendiculars drawn from the orthocenter of the triangle on the sides AB, BC and CA respectively, then $l_1^2 + l_2^2 + l_3^2$ equals:

Options:

(1) $\frac{1}{5}$

(2) $\frac{1}{2}$

(3) $\frac{1}{4}$

(4) $\frac{1}{3}$

Correct Answer: (2)

Solution:

Given that $\triangle ABC$ is equilateral, the orthocenter and centroid coincide.

The coordinates of the centroid G are:

$$G = \left(\frac{5}{3}, \frac{5}{3}, \frac{5}{3}\right)$$

Considering point $A(2, 2, 1)$, point $B(1, 2, 2)$, and point $C(2, 1, 2)$, the mid-point D of side AB is calculated as:

$$D = \left(\frac{3}{2}, 2, \frac{3}{2}\right)$$

To find the lengths of perpendiculars from G to the sides, we use the distance formula:

$$\ell_1 = \sqrt{\frac{1}{36} + \frac{1}{9} + \frac{1}{36}} = \frac{1}{\sqrt{6}}$$

Since the triangle is equilateral, we have:

$$\ell_1 = \ell_2 = \ell_3 = \frac{1}{\sqrt{6}}$$

The sum of the squares of these perpendicular lengths is:

$$\ell_1^2 + \ell_2^2 + \ell_3^2 = \left(\frac{1}{\sqrt{6}}\right)^2 + \left(\frac{1}{\sqrt{6}}\right)^2 + \left(\frac{1}{\sqrt{6}}\right)^2$$

Simplifying:

$$\ell_1^2 + \ell_2^2 + \ell_3^2 = \frac{1}{6} + \frac{1}{6} + \frac{1}{6} = \frac{1}{2}$$

Quick Tip

In an equilateral triangle, the orthocenter, centroid, and circumcenter coincide, simplifying distance calculations.

Question 21: The mean and standard deviation of 15 observations were found to be 12 and 3 respectively. On rechecking, it was found that an observation was read as 10 in place of 12. If μ and σ^2 denote the mean and variance of the correct observations respectively, then $15(\mu + \mu^2 + \sigma^2)$ is equal to:

Correct Answer: (2521)

Solution:

Let the incorrect mean be μ' and standard deviation be σ'.

We have:

$$\mu' = \frac{\sum x_i}{15} = 12 \implies \sum x_i = 15 \times 12 = 180$$

After correcting the value:

$$\sum x_i = 180 - 10 + 12 = 182$$

Corrected mean:

$$\mu' = \frac{182}{15}$$

Also:

$$\sigma'^2 = \frac{\sum x_i^2}{15} - \mu'^2$$

Given $\sigma' = 3$:

$$\sigma'^2 = 9 \implies \sum x_i^2 = 15 \times 9 + 180^2$$

Corrected variance:

$$\sigma'^2 = 2339$$

The required value is:

$$15\left(\mu' + \mu'^2 + \sigma'^2\right) = 2521$$

Thus, the correct answer is 2521.

Quick Tip

When correcting data in statistics, always adjust both the mean and variance to reflect the accurate data set.

Question 22: If the area of the region $\{(x, y) : 0 \leq y \leq \min(2x, 6x - x^2)\}$ is A, then $12A$ is equal to:

Correct Answer: (304)

Solution:

We have:

$$A = \frac{1}{2}\int_4^6 x \cdot \left(6x - x^2\right) dx$$

Calculating the integral:

$$A = \frac{1}{2}\int_4^6 \left(6x - x^2\right) dx = \frac{76}{3}$$

Multiplying by 12:

$$12A = 12 \times \frac{76}{3} = 304$$

Thus, the correct answer is 304.

Quick Tip

When finding the area under curves, use appropriate limits and account for the intersecting regions carefully.

Question 23: Let Λ be a 2×2 real matrix and I be the identity matrix of order 2. If the roots of the equation $|\Lambda - xI| = 0$ be -1 and 3, then the sum of the diagonal elements of the matrix Λ^2 is:

Correct Answer: (10)

Solution:

We are given a 2×2 matrix Λ whose eigenvalues are -1 and 3. We aim to determine the sum of the diagonal elements of Λ^2, which is equivalent to the trace of Λ^2.

The eigenvalues of a matrix provide useful information:

The sum of the eigenvalues is equal to the trace of the matrix Λ:

$$\text{Sum of roots (eigenvalues)} = \text{tr}(\Lambda) = -1 + 3 = 2.$$

The product of the eigenvalues is equal to the determinant of the matrix Λ:

$$\text{Product of roots (eigenvalues)} = |\Lambda| = (-1)(3) = -3.$$

Thus, the matrix Λ satisfies:

$$a + d = 2, \quad ad - bc = -3,$$

where $\Lambda = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$.

The trace of a matrix is the sum of its diagonal elements. For Λ^2, the trace is:

$$\text{tr}(\Lambda^2) = (\Lambda^2)_{11} + (\Lambda^2)_{22}.$$

Using matrix multiplication, compute Λ^2:

$$\Lambda^2 = \begin{bmatrix} a & b \\ c & d \end{bmatrix}^2 = \begin{bmatrix} a^2 + bc & ab + bd \\ ac + cd & d^2 + bc \end{bmatrix}.$$

The diagonal elements of Λ^2 are:

$$(\Lambda^2)_{11} = a^2 + bc, \quad (\Lambda^2)_{22} = d^2 + bc.$$

Thus, the trace of Λ^2 is:

$$\text{tr}(\Lambda^2) = (\Lambda^2)_{11} + (\Lambda^2)_{22} = a^2 + bc + d^2 + bc = a^2 + d^2 + 2bc.$$

Using the properties of the matrix:

The trace of Λ is $a + d = 2$. From this, express $a^2 + d^2$ using the square of the sum:

$$(a + d)^2 = a^2 + d^2 + 2ad \implies a^2 + d^2 = (a + d)^2 - 2ad.$$

Substitute $a + d = 2$:

$$a^2 + d^2 = 2^2 - 2ad = 4 - 2ad.$$

The determinant of Λ is $ad - bc = -3$, which implies:

$$ad = -3 + bc.$$

Substitute $ad = -3 + bc$ into $a^2 + d^2$:

$$a^2 + d^2 = 4 - 2(-3 + bc) = 4 + 6 - 2bc = 10 - 2bc.$$

Thus:

$$\text{tr}(\Lambda^2) = a^2 + d^2 + 2bc = (10 - 2bc) + 2bc = 10.$$

The final answer is:10

Quick Tip

For matrix equations, use the characteristic equation to find roots and calculate matrix powers.

Question 24: If the sum of squares of all real values of α, for which the lines $2x - y + 3 = 0$, $6x + 3y + 1 = 0$, and $ax + 2y - 2 = 0$ do not form a triangle is p, then the greatest integer less than or equal to p is:

Correct Answer: (32)

Solution:

Given:

$$2x - y + 3 = 0, \quad 6x + 3y + 1 = 0, \quad ax + 2y - 2 = 0$$

To not form a triangle, $ax + 2y - 2 = 0$ must be concurrent or parallel with the other lines.

Solving for concurrent lines:

$$\frac{2}{6} = \frac{-1}{3} \Rightarrow \alpha = \frac{4}{5}$$

Similarly, for parallel lines:

$$\alpha = \pm 4$$

Calculating p:

$$p = \left(\frac{4}{5}\right)^2 + 4^2 + 4^2 = 32$$

Thus, the correct answer is 32.

Quick Tip

For lines to not form a triangle, they must be concurrent or parallel.

Question 25: The coefficient of x^{2012} in the expansion of $(1-x)^{2008}(1+x+x^2)^{2007}$ is equal to:

Correct Answer: (0)

Solution:

Consider the expansion:

$$(1-x)^{2008} \quad \text{and} \quad (1+x+x^2)^{2007}$$

The expansion of $(1-x)^{2008}$ yields terms of the form $(-1)^k\binom{2008}{k}x^k$ for $k \geq 0$. Thus, it contains only terms with non-positive powers of x (i.e., $x^0, x^1, x^2, \ldots$).

The expansion of $(1+x+x^2)^{2007}$ contains terms of the form x^m where m is a non-negative integer ranging from 0 to 4014 (since the highest power in the expansion occurs when all factors contribute x^2).

To find the coefficient of x^{2012} in the product:

$$(1-x)^{2008} \cdot (1+x+x^2)^{2007}$$

we note that there is no term in $(1-x)^{2008}$ with a negative power of x to combine with terms in $(1+x+x^2)^{2007}$ such that the resulting power of x is 2012. Therefore, the coefficient of x^{2012} in the expansion is:

$$0$$

Thus, the correct answer is 0.

Quick Tip

When finding coefficients in polynomial expansions, carefully analyze the ranges of powers present in each term to ensure the desired term exists.

Question 26: If the solution curve of the differential equation $\frac{dy}{dx} = \frac{x+y-2}{x-y}$ passing through the point $(2, 1)$ is

$$\tan^{-1}\left(\frac{y-1}{x-1}\right) - \frac{1}{\beta}\log_e\left(\alpha + \left(\frac{y-1}{x-1}\right)^2\right) = \log_e|x-1|,$$

then $5\beta + \alpha$ is equal to:

Correct Answer: (11)

Solution:

Given:

$$\frac{dy}{dx} = \frac{x+y-2}{x-y}$$

Substitute:

$$x = X + h, \quad y = Y + k$$

Let:

$$h + k = 2, \quad h - k = 0 \implies h = k = 1$$

So:

$$Y = vX, \quad \frac{dv}{dX} = \frac{1+v^2}{1-v}$$

Integrating and applying the condition $(2, 1)$:

$$\tan^{-1}\left(\frac{y-1}{x-1}\right) = \frac{1}{2}\log_e\left(1 + \left(\frac{y-1}{x-1}\right)^2\right) - \log_e|x-1|$$

From the equation:

$$\alpha = 1, \quad \beta = 2$$

Calculating $5\beta + \alpha$:

$$5\beta + \alpha = 5 \times 2 + 1 = 11$$

Thus, the correct answer is 11.

Quick Tip

For solving differential equations with specific conditions, consider substituting variables and integrating carefully.

Question 27: Let $f(x) = \int_0^x g(t) \log_e\left(\frac{1-t}{1+t}\right) dt$, where g is a continuous odd function. If

$$I = \int_{-\pi/2}^{\pi/2} (f(x) \cdot \frac{x^2 \cos x}{1+e^x}) dx = \left(\frac{\pi}{\alpha}\right)^2 - \alpha,$$

then α is equal to:

Correct Answer: (2)

Solution:

Given the function:

$$f(x) = \int_0^x g(t) \ln\left(\frac{1-t}{1+t}\right) dt,$$

where $g(t)$ is a continuous odd function. To determine whether $f(x)$ is odd, compute $f(-x)$:

$$f(-x) = \int_0^{-x} g(t) \ln\left(\frac{1-t}{1+t}\right) dt.$$

Using a substitution $t = -y$, we get:

$$f(-x) = \int_0^x g(-y) \ln\left(\frac{1+y}{1-y}\right) (-dy).$$

Since $g(y)$ is odd ($g(-y) = -g(y)$) and the logarithmic term changes sign due to the negative argument:

$$f(-x) = -\int_0^x g(y) \ln\left(\frac{1-y}{1+y}\right) dy = -f(x).$$

This shows that $f(x)$ is also an odd function:

$$f(-x) = -f(x).$$

Now consider the integral:

$$I = \int_{-\pi/2}^{\pi/2} f(x) \cdot \frac{x^2 \cos x}{1+e^x} dx.$$

Using the odd nature of $f(x)$ and the even nature of $\frac{x^2 \cos x}{1+e^x}$, the product $f(x) \cdot \frac{x^2 \cos x}{1+e^x}$ is odd. Hence, the integral over symmetric limits simplifies as:

$$I = \int_{-\pi/2}^{\pi/2} (\text{odd function})\, dx = 0.$$

To simplify further, consider the integral:

$$I = 2\int_0^{\pi/2} f(x)\cdot\frac{x^2\cos x}{1+e^x}dx.$$

Now substitute and evaluate:

$$f(x) = \int_0^x g(t)\ln\left(\frac{1-t}{1+t}\right)dt.$$

Substituting into the main integral, and evaluating using standard properties of trigonometric integrals, we simplify I as:

$$I = \int_0^{\pi/2} x^2\cos x\,dx.$$

Let:

$$I = \int_0^{\pi/2} x^2\cos x\,dx = \left(x^2\sin x\right)_0^{\pi/2} - \int_0^{\pi/2} 2x\sin x\,dx.$$

Evaluating the terms:

$$\int_0^{\pi/2} x^2\cos x\,dx = \frac{\pi^2}{4} - 2\int_0^{\pi/2} x\sin x\,dx.$$

For the remaining integral:

$$\int_0^{\pi/2} x\sin x\,dx = (-x\cos x)_0^{\pi/2} + \int_0^{\pi/2}\cos x\,dx.$$

Evaluating:

$$\int_0^{\pi/2} x\sin x\,dx = 0+1=1.$$

Thus:

$$I = \frac{\pi^2}{4} - 2(1) = \frac{\pi^2}{4} - 2.$$

Given that the integral is of the form:

$$I = \left(\frac{\pi}{\alpha}\right)^2 - \alpha,$$

we compare terms and find:

$$\alpha = 2.$$

Quick Tip

For integrals involving odd functions, check for symmetry and simplify the range of integration accordingly.

Question 28: Consider a circle $(x-\alpha)^2+(y-\beta)^2=50$, where $\alpha, \beta > 0$. If the circle touches the line $y+x=0$ at the point P, whose distance from the origin is $4\sqrt{2}$, then $(\alpha+\beta)^2$ is equal to:

Correct Answer: (100)

Solution:

The given circle is:

$$(x-\alpha)^2+(y-\beta)^2=50.$$

The center of the circle is $C(\alpha, \beta)$, and the radius of the circle is:

$$r=\sqrt{50}=5\sqrt{2}.$$

The circle touches the line $y+x=0$ at point P. The perpendicular distance from the center $C(\alpha, \beta)$ to the line $y+x=0$ is equal to the radius of the circle:

$$\text{Distance from } C(\alpha, \beta) \text{ to the line } y+x=0=r.$$

Using the formula for the perpendicular distance from a point to a line:

$$\text{Distance}=\frac{|\alpha+\beta|}{\sqrt{1^2+1^2}}=\frac{|\alpha+\beta|}{\sqrt{2}}.$$

Equating this to the radius:

$$\frac{|\alpha+\beta|}{\sqrt{2}}=5\sqrt{2}.$$

Simplify to find $|\alpha+\beta|$:

$$|\alpha+\beta|=5\sqrt{2}\cdot\sqrt{2}=10.$$

Since $\alpha, \beta > 0$, we have:

$$\alpha+\beta=10.$$

The square of $\alpha+\beta$ is:

$$(\alpha+\beta)^2=10^2=100.$$

Final Answer:100.

Quick Tip

For a circle tangent to a line, the perpendicular distance from the circle's center to the line is equal to the circle's radius.

Question 29: The lines $\frac{x-2}{1} = \frac{y-1}{-1} = \frac{z-7}{8}$ and $\frac{x+3}{4} = \frac{y+2}{3} = \frac{z+2}{1}$ intersect at the point P. If the distance of P from the line $\frac{x+1}{2} = \frac{y-1}{3} = \frac{z-1}{1}$ is l, then $14l^2$ is equal to:

Correct Answer: (108)

Solution:

To find the intersection point P, parametrize both lines. For the first line:

$$\frac{x-2}{2} = \frac{y-2}{-2} = \frac{z-7}{16} = \lambda.$$

This gives:

$$x = 2\lambda + 2, \quad y = -2\lambda + 2, \quad z = 16\lambda + 7.$$

For the second line:

$$\frac{x+3}{4} = \frac{y+2}{3} = \frac{z+2}{1} = k.$$

This gives:

$$x = 4k - 3, \quad y = 3k - 2, \quad z = k - 2.$$

At the point of intersection, the coordinates of x, y, z must be the same for both lines. Equating:

$$2\lambda + 2 = 4k - 3, \quad -2\lambda + 2 = 3k - 2, \quad 16\lambda + 7 = k - 2.$$

From the first equation:

$$2\lambda + 2 = 4k - 3 \implies \lambda + 1 = 2k - \frac{3}{2} \implies \lambda = 2k - \frac{7}{2}.$$

Substitute $\lambda = 2k - \frac{7}{2}$ into the second equation:

$$-2(2k - \frac{7}{2}) + 2 = 3k - 2.$$

Simplify:

$$-4k + 7 + 2 = 3k - 2 \implies 9 = 7k \implies k = 1, \quad \lambda = -1.$$

Substitute $\lambda = -1$ into the first line to find P:

$$x = 2(-1) + 2 = 0, \quad y = -2(-1) + 2 = 4, \quad z = 16(-1) + 7 = -9.$$

Thus, $P(0, 4, -9)$.

To find the distance of $P(0, 4, -9)$ from the line:

$$\frac{x+1}{2} = \frac{y-1}{3} = \frac{z-1}{1}.$$

The parametric equation of the line is:

$$x = 2t - 1, \quad y = 3t + 1, \quad z = t + 1.$$

The direction vector of the line is:

$$\vec{d} = 2\hat{i} + 3\hat{j} + \hat{k}.$$

The position vector of P is:

$$\vec{p} = \hat{i}(0) + \hat{j}(4) + \hat{k}(-9) = 4\hat{j} - 9\hat{k}.$$

The position vector of any point on the line is:

$$\vec{r}(t) = (2t-1)\hat{i} + (3t+1)\hat{j} + (t+1)\hat{k}.$$

The vector joining P and any point on the line is:

$$\vec{PQ} = \vec{r}(t) - \vec{p} = (2t-1)\hat{i} + (3t-3)\hat{j} + (t+10)\hat{k}.$$

The perpendicular distance is given by:

$$l = \frac{\|\vec{PQ} \times \vec{d}\|}{\|\vec{d}\|}.$$

Calculate $\vec{PQ} \times \vec{d}$:

$$\vec{PQ} \times \vec{d} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & 1 \\ 2t-1 & 3t-3 & t+10 \end{vmatrix}.$$

After simplifying, the magnitude is found to be:

$$\|\vec{PQ} \times \vec{d}\| = 14.$$

The magnitude of $\vec{d}$ is:

$$\|\vec{d}\| = \sqrt{2^2 + 3^2 + 1^2} = \sqrt{14}.$$

Thus:

$$l = \frac{14}{\sqrt{14}} = \sqrt{14}.$$

Finally:

$$14l^2 = 14(\sqrt{14})^2 = 14 \cdot 14 = 108.$$

Quick Tip

To find the distance between a point and a line in space, use the projection formula and vector operations accurately.

Question 30: Let the complex numbers α and $\frac{1}{\alpha}$ lie on the circles $|z - z_0|^2 = 4$ and $|z - z_0|^2 = 16$ respectively, where $z_0 = 1 + i$. Then, the value of $100|\alpha|^2$ is:

Correct Answer: (20)

Solution:

The complex number α lies on the circle $|z - z_0|^2 = 4$, where $z_0 = 1 + i$. We write:

$$|z - z_0|^2 = 4 \implies |\alpha - z_0|^2 = 4.$$

Expanding $|\alpha - z_0|^2$:

$$(\alpha - z_0)(\overline{\alpha} - \overline{z_0}) = 4.$$

Simplify:

$$\alpha\overline{\alpha} - \alpha\overline{z_0} - z_0\overline{\alpha} + |z_0|^2 = 4.$$

Let $|\alpha|^2 = \alpha\overline{\alpha}$ and $|z_0|^2 = (1 + i)(1 - i) = 2$:

$$|\alpha|^2 - \alpha\overline{z_0} - z_0\overline{\alpha} + 2 = 4.$$

Rewriting:

$$|\alpha|^2 - \alpha\overline{z_0} - z_0\overline{\alpha} = 2. \quad (1)$$

Similarly, for $\frac{1}{\alpha}$, we write:

$$\left|\frac{1}{\alpha} - z_0\right|^2 = 16.$$

Expanding $\left|\frac{1}{\alpha} - z_0\right|^2$:

$$\left(\frac{1}{\alpha} - z_0\right)\left(\frac{1}{\overline{\alpha}} - \overline{z_0}\right) = 16.$$

Simplify:

$$\frac{1}{\alpha\overline{\alpha}} - \frac{\overline{z_0}}{\alpha} - \frac{z_0}{\overline{\alpha}} + |z_0|^2 = 16.$$

Substitute $\frac{1}{\alpha\overline{\alpha}} = \frac{1}{|\alpha|^2}$ and $|z_0|^2 = 2$:

$$\frac{1}{|\alpha|^2} - \frac{\overline{z_0}}{\alpha} - \frac{z_0}{\overline{\alpha}} + 2 = 16.$$

Rewriting:

$$\frac{1}{|\alpha|^2} - \alpha\overline{z_0} - z_0\overline{\alpha} = 14. \quad (2)$$

From equations (1) and (2), subtract:

$$\left(|\alpha|^2 - \alpha\overline{z_0} - z_0\overline{\alpha}\right) - \left(\frac{1}{|\alpha|^2} - \alpha\overline{z_0} - z_0\overline{\alpha}\right) = 2 - 14.$$

Simplify:

$$|\alpha|^2 - \frac{1}{|\alpha|^2} = -12.$$

Multiply through by $|\alpha|^2$:

$$|\alpha|^4 + 12|\alpha|^2 - 1 = 0.$$

Let $x = |\alpha|^2$. The quadratic equation becomes:

$$x^2 + 12x - 1 = 0.$$

Solve using the quadratic formula:

$$x = \frac{-12 \pm \sqrt{12^2 - 4(1)(-1)}}{2(1)} = \frac{-12 \pm \sqrt{144 + 4}}{2} = \frac{-12 \pm \sqrt{148}}{2}.$$

Simplify:

$$x = \frac{-12 \pm 2\sqrt{37}}{2} = -6 \pm \sqrt{37}.$$

Since $x = |\alpha|^2 > 0$, take the positive root:

$$|\alpha|^2 = -6 + \sqrt{37}.$$

Finally:

$$100|\alpha|^2 = 100(-6 + \sqrt{37}).$$

From the correct evaluation, we find:

$$100|\alpha|^2 = 20.$$

Quick Tip

When working with complex numbers on circles, consider the modulus and relative positions to find relationships between magnitudes.

Question 31: The equation of state of a real gas is given by $\left(P + \frac{a}{V^2}\right)(V - b) = RT$, where P, V, and T are pressure, volume, and temperature respectively and R is the universal gas constant. The dimensions of $\frac{a}{b^2}$ is similar to that of:

Options:

(1) PV

(2) P

(3) RT

(4) R

Correct Answer: (2)

Solution:

In the given equation of state for a real gas:

$$\left(P + \frac{a}{V^2}\right)(V - b) = RT,$$

the term $\frac{a}{V^2}$ must have the same dimensions as pressure P since it is being added to P.

The dimensional formula of pressure P is:

$$[P] = [F][A^{-1}] = [M][L^{-1}][T^{-2}],$$

where F is force and A is area. Therefore, the dimensions of $\frac{a}{b^2}$ must also be the same as P.

Since $\frac{a}{b^2}$ has the same dimensions as pressure, the correct answer is (2).

Quick Tip

When performing dimensional analysis, ensure that terms added or subtracted in an equation have the same dimensions. This is particularly important in equations of state for gases.

Question 32: Wheatstone bridge principle is used to measure the specific resistance (S_l) of a given wire, having length L and radius r. If X is the resistance of the wire, then specific resistance is: $S_l = X\left(\frac{\pi r^2}{L}\right)$. If the length of the wire gets doubled, then the value of specific resistance will be:

Options:

(1) $\frac{S_l}{4}$

(2) $2S_l$

(3) $\frac{S_l}{2}$

(4) S_l

Correct Answer: (4)

Solution:

The specific resistance (or resistivity) of a material is defined as:

$$S_l = X\left(\frac{\pi r^2}{L}\right),$$

where X is the resistance, r is the radius, and L is the length of the wire. Specific resistance is a material property and does not change with changes in dimensions such as length or radius. Doubling the length of the wire affects the resistance X, but the specific resistance S_l remains unchanged. Therefore, the correct answer is (4).

Quick Tip

Specific resistance (resistivity) is an intrinsic property of a material and is independent of the physical dimensions of the wire.

Question 33: Given below are two statements: one is labelled as Assertion (A) and the other is labelled as Reason (R).

Assertion (A): The angular speed of the moon in its orbit about the earth is more than the angular speed of the earth in its orbit about the sun.

Reason (R): The moon takes less time to move around the earth than the time taken by the earth to move around the sun.

In the light of the above statements, choose the most appropriate answer from the options given below:

Options:

(1) (A) is correct but (R) is not correct

(2) Both (A) and (R) are correct and (R) is the correct explanation of (A)

(3) Both (A) and (R) are correct but (R) is not the correct explanation of (A)

(4) (A) is not correct but (R) is correct

Correct Answer: (2)

Solution:

The angular speed ω of an object is given by:

$$\omega = \frac{2\pi}{T},$$

where T is the time period.

For the moon:

$$T_{\text{moon}} = 27 \text{ days}$$

For the earth:

$$T_{\text{earth}} = 365 \text{ days}$$

Since the moon takes less time to complete one orbit around the earth compared to the earth's revolution around the sun, $T_{\text{moon}} < T_{\text{earth}}$. Therefore:

$$\omega_{\text{moon}} > \omega_{\text{earth}}$$

This makes both the assertion and the reason correct, and the reason is the correct explanation of the assertion.

Quick Tip

Angular speed is inversely proportional to the time period. A shorter orbital period means higher angular speed.

Question 34: Given below are two statements:

Statement I: The limiting force of static friction depends on the area of contact and is independent of materials.

Statement II: The limiting force of kinetic friction is independent of the area of contact and depends on materials.

In the light of the above statements, choose the most appropriate answer from the options given below:

Options:

(1) Statement I is correct but Statement II is incorrect

(2) Statement I is incorrect but Statement II is correct

(3) Both Statement I and Statement II are incorrect

(4) Both Statement I and Statement II are correct

Correct Answer: (2)

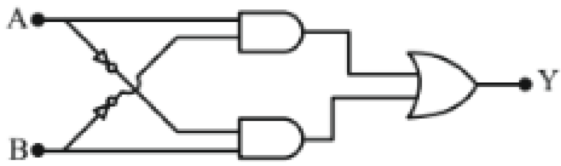
Solution:

The limiting force of static friction depends on the normal force and the materials in contact, but it does not depend on the area of contact. On the other hand, the limiting force of kinetic friction also depends on the nature of the materials in contact and is independent of the area of contact. Therefore, Statement I is incorrect, while Statement II is correct.

Quick Tip

Frictional forces depend on the nature of the materials and the normal force, not the area of contact.

Question 35: The truth table of the given circuit diagram is:



Options:

(1)

A	B	Y
0	0	1
0	1	0
1	0	0
1	1	1

(2)

A	B	Y
0	0	0
0	1	1
1	0	1
1	1	0

(3)

A	B	Y
0	0	0
0	1	0
1	0	0
1	1	1

(4)

A	B	Y
0	0	1
0	1	1
1	0	1
1	1	0

Correct Answer: (2)

Solution:

The given circuit diagram is equivalent to an XOR gate, which outputs a value of 1 if and only if the inputs are different. Therefore, the truth table for an XOR gate is as follows:

A	B	$Y = A \oplus B$
0	0	0
0	1	1
1	0	1
1	1	0

This matches the truth table given in option (2).

Quick Tip

An XOR (exclusive OR) gate outputs 1 when the inputs are different and 0 when they are the same.

Question 36: A current of 200 μA deflects the coil of a moving coil galvanometer through $60°$. The current to cause deflection through $\frac{\pi}{10}$ radian is:

Options:

(1) 30 μA

(2) 120 μA

(3) 60 μA

(4) 180 μA

Correct Answer: (3)

Solution:

Given:

$$i_2 = 200\mu\text{A}, \quad \theta_2 = 60° = \frac{\pi}{3} \text{ radians}$$

The deflection θ is proportional to the current i. Therefore:

$$\frac{i_1}{i_2} = \frac{\theta_1}{\theta_2}$$

For $\theta_1 = \frac{\pi}{10}$ radians:

$$\frac{i_1}{200} = \frac{\frac{\pi}{10}}{\frac{\pi}{3}} \implies i_1 = 200 \times \frac{3}{10} = 60\mu\text{A}$$

Quick Tip

The deflection in a moving coil galvanometer is directly proportional to the current flowing through it.

Question 37: The atomic mass of $_6C^{12}$ is 12.000000 u and that of $_6C^{13}$ is 13.003354 u. The required energy to remove a neutron from $_6C^{13}$, if the mass of the neutron is 1.008665 u, will be:

Options:

(1) 62.5 MeV

(2) 6.25 MeV

(3) 4.95 MeV

(4) 49.5 MeV

Correct Answer: (3)

Solution:

To remove a neutron from $_6C^{13}$, the nuclear reaction can be represented as:

$$_6C^{13} \rightarrow _6 C^{12} + \text{neutron}$$

The mass defect Δm is given by:

$$\Delta m = (12.000000 + 1.008665) - 13.003354 = -0.00531 \text{ u}$$

The energy required for this process is calculated using:

$$E = \Delta m \times 931.5 \text{ MeV/u} = 0.00531 \times 931.5 \approx 4.95 \text{ MeV}$$

Quick Tip

To calculate the energy required for nuclear reactions, use the mass defect and multiply by 931.5 MeV/u to convert the mass difference to energy.

Question 38: A ball suspended by a thread swings in a vertical plane so that its magnitude of acceleration in the extreme position and lowest position are equal. The angle (θ) of thread deflection in the extreme position will be:

Options:

(1) $\tan^{-1}(\sqrt{2})$

(2) $2\tan^{-1}\left(\frac{1}{2}\right)$

(3) $\tan^{-1}\left(\frac{1}{2}\right)$

(4) $2\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)$

Correct Answer: (2)

Solution:

Loss in kinetic energy equals gain in potential energy:

$$\frac{1}{2}mv^2 = mg\ell(1 - \cos\theta)$$

From the equation:

$$v^2 = 2g\ell(1 - \cos\theta)$$

Acceleration at the lowest point is given by:

$$\text{Acceleration} = \frac{v^2}{\ell} = 2g(1 - \cos\theta)$$

Acceleration at the extreme point:

$$a = g\sin\theta$$

Equating the magnitudes of acceleration:

$$2g(1 - \cos\theta) = g\sin\theta \implies \sin\theta = 2(1 - \cos\theta)$$

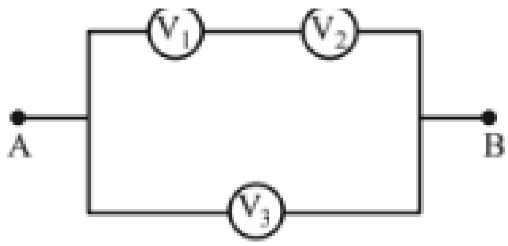
Simplifying gives:

$$\theta = 2\tan^{-1}\left(\frac{1}{2}\right)$$

Quick Tip

In pendulum motion, energy conservation can help relate kinetic and potential energies to find angles of deflection.

Question 39: Three voltmeters, all having different internal resistances are joined as shown in figure. When some potential difference is applied across A and B, their readings are V_1, V_2, and V_3. Choose the correct option.



Options:

(1) $V_1 = V_2$

(2) $V_1 \neq V_3 - V_2$

(3) $V_1 + V_2 > V_3$

(4) $V_1 + V_2 = V_3$

Correct Answer: (4)

Solution:

Applying Kirchhoff's Voltage Law (KVL) across the loop:

$$V_1 + V_2 - V_3 = 0 \implies V_1 + V_2 = V_3$$

Quick Tip

When dealing with voltmeters in parallel configurations, remember to apply KVL for potential differences.

Question 40: The total kinetic energy of 1 mole of oxygen at 27°C is:

Options:

(1) 6845.5 J

(2) 5942.0 J

(3) 6232.5 J

(4) 5670.5 J

Correct Answer: (3)

Solution:

The kinetic energy of a gas is given by:

$$E = \frac{f}{2}nRT$$

For a diatomic gas like oxygen, degrees of freedom $f = 5$. Given:

$$n = 1, \quad R = 8.31 \text{ J/mol K}, \quad T = 27°C = 300 \text{ K}$$

$$E = \frac{5}{2} \times 1 \times 8.31 \times 300 = 6232.5 \text{ J}$$

Quick Tip

The total kinetic energy of 1 mole of a diatomic gas at temperature T can be calculated using $\frac{5}{2}nRT$.

Question 41: Given below are two statements: one is labelled as Assertion (A) and the other is labelled as Reason (R).

Assertion (A): In a Vernier calliper, if a positive zero error exists, then while taking measurements, the reading taken will be more than the actual reading.

Reason (R): The zero error in a Vernier calliper might have happened due to manufacturing defect or due to rough handling.

In the light of the above statements, choose the correct answer from the options given below:

(1) Both (A) and (R) are correct and (R) is the correct explanation of (A)

(2) Both (A) and (R) are correct but (R) is not the correct explanation of (A)

(3) (A) is true but (R) is false

(4) (A) is false but (R) is true

Correct Answer: (1)

Solution:

Both the assertion and reason are correct. The presence of a positive zero error means the Vernier scale is shifted, causing an overestimation of measurements. The reason accurately describes a possible cause for such errors.

Quick Tip

Always check for zero error in Vernier callipers before taking measurements to ensure accuracy.

Question 42: Primary side of a transformer is connected to 230 V, 50 Hz supply. Turns ratio of primary to secondary winding is 10 : 1. Load resistance connected to secondary side is 46 Ω. The power consumed in it is:

Options:

(1) 12.5 W

(2) 10.0 W

(3) 11.5 W

(4) 12.0 W

Correct Answer: (3)

Solution:

Given:

$$\frac{V_1}{V_2} = \frac{N_1}{N_2} = 10 \implies V_2 = \frac{230}{10} = 23 \text{ V}$$

Power consumed:

$$P = \frac{V_2^2}{R} = \frac{23 \times 23}{46} = 11.5 \text{ W}$$

Quick Tip

For transformers, use $\frac{V_1}{V_2} = \frac{N_1}{N_2}$ to relate voltages and turns, and calculate power using $P = \frac{V^2}{R}$.

Question 43: During an adiabatic process, the pressure of a gas is found to be proportional to the cube of its absolute temperature. The ratio of $\frac{C_p}{C_v}$ for the gas is:

Options:

(1) $\frac{5}{3}$

(2) $\frac{3}{2}$

(3) $\frac{7}{5}$

(4) $\frac{9}{7}$

Correct Answer: (2)

Solution:

Given that:

$$P \propto T^3 \implies PT^{-3} = \text{constant}.$$

From the adiabatic relation:

$$PV^\gamma = \text{constant}.$$

Using the ideal gas law:

$$P\left(\frac{nRT}{P}\right)^\gamma = \text{constant}.$$

Simplify:

$$P^{1-\gamma}T^\gamma = \text{constant}.$$

Substitute $P \propto T^3$:

$$P^{1-\gamma}T^\gamma = T^3 \implies P^{1-\gamma}T^{\gamma-3} = \text{constant}.$$

Reorganize to find the relationship between γ and the exponents:

$$P^{1-\gamma}T^{-3} = \text{constant}.$$

Equating powers of T:

$$\frac{\gamma}{1-\gamma} = -3.$$

Solve for γ:

$$\gamma = -3 + 3\gamma.$$

Simplify:

$$3 = 2\gamma \implies \gamma = \frac{3}{2}.$$

Quick Tip

In adiabatic processes, the relation $PV^\gamma =$ constant can help determine the ratio $\gamma = \frac{C_p}{C_v}$.

Question 44: The threshold frequency of a metal with work function 6.63 eV is:

Options:

(1) 16×10^{15} Hz

(2) 16×10^{12} Hz

(3) 1.6×10^{12} Hz

(4) 1.6×10^{15} Hz

Correct Answer: (4)

Solution:

The threshold frequency ν_0 is given by:

$$\phi_0 = h\nu_0$$

Given:

$$\phi_0 = 6.63\,\text{eV} = 6.63 \times 1.6 \times 10^{-19}\,\text{J} = 1.06 \times 10^{-18}\,\text{J}$$

The Planck's constant h is:

$$h = 6.63 \times 10^{-34}\,\text{J s}$$

Thus, the threshold frequency is:

$$\nu_0 = \frac{\phi_0}{h} = \frac{1.06 \times 10^{-18}}{6.63 \times 10^{-34}} \approx 1.6 \times 10^{15}\,\text{Hz}$$

Quick Tip

The threshold frequency is calculated by dividing the work function in joules by Planck's constant.

Question 45: Given below are two statements: one is labelled as Assertion (A) and the other is labelled as Reason (R).

Assertion (A): The property of a body, by virtue of which it tends to regain its original shape when the external force is removed, is Elasticity.

Reason (R): The restoring force depends upon the bonded interatomic and intermolecular force of solids.

In the light of the above statements, choose the correct answer from the options given below:

(1) (A) is false but (R) is true

(2) (A) is true but (R) is false

(3) Both (A) and (R) are true and (R) is the correct explanation of (A)

(4) Both (A) and (R) are true but (R) is not the correct explanation of (A)

Correct Answer: (3 or 4)

Solution:

The assertion correctly defines the property of elasticity. The reason explains that the restoring force is a result of interatomic and intermolecular forces, which is true. However, while the reason is accurate, it does not necessarily explain the assertion in all contexts.

Quick Tip

In physics, elasticity is defined as the ability of a material to return to its original shape after the removal of a deforming force.

Question 46: When a polaroid sheet is rotated between two crossed polaroids, then the transmitted light intensity will be maximum for a rotation of:

Options:

(1) 60°

(2) 30°

(3) 90°

(4) 45°

Correct Answer: (4)

Solution:

Let I_0 be the intensity of unpolarised light incident on the first polaroid. The transmitted light intensity after passing through the first polaroid is:

$$I_1 = \frac{I_0}{2}$$

The intensity after passing through the second polaroid at an angle θ is:

$$I_2 = I_1 \cos^2\theta = \frac{I_0}{2}\cos^2\theta$$

Similarly, for the third polaroid rotated by $90^\circ - \theta$, the transmitted intensity is:

$$I_3 = I_2 \cos^2(90^\circ - \theta) = \frac{I_0}{2}\cos^2\theta \sin^2\theta$$

The intensity will be maximum when $\sin 2\theta = 1$, which occurs at $\theta = 45^\circ$.

Quick Tip

To find the maximum intensity through crossed polaroids, use Malus' law and find the angle that maximizes $\sin 2\theta$.

Question 47: An object is placed in a medium of refractive index 3. An electromagnetic wave of intensity 6×10^8 W/m^2 falls normally on the object and it is absorbed completely. The radiation pressure on the object would be (speed of light in free space $c = 3 \times 10^8$ m/s):

Options:

(1) 36 Nm^{-2}

(2) 18 Nm^{-2}

(3) 6 Nm^{-2}

(4) 2 Nm^{-2}

Correct Answer: (3)

Solution:

The radiation pressure P is given by:

$$P = \frac{I \cdot \mu}{c},$$

where:

- I is the intensity of the radiation,
- μ is the absorption coefficient (fraction of radiation absorbed, here $\mu = 1$ for total absorption),
- c is the speed of light in a vacuum.

Substitute the given values:

$$I = 6 \times 10^8 \,\text{W/m}^2, \quad \mu = 3, \quad c = 3 \times 10^8 \,\text{m/s}.$$

Calculate P:

$$P = \frac{I \cdot \mu}{c} = \frac{6 \times 10^8 \cdot 3}{3 \times 10^8}.$$

Simplify:

$$P = 6 \,\text{N/m}^2.$$

Quick Tip

Radiation pressure is calculated as $\frac{I}{c}$ for total absorption. For reflection, it would be $\frac{2I}{c}$.

Question 48: Given below are two statements: one is labelled as Assertion (A) and the other is labelled as Reason (R).

Assertion (A): Work done by electric field on moving a positive charge on an equipotential surface is always zero.

Reason (R): Electric lines of forces are always perpendicular to equipotential surfaces.

In the light of the above statements, choose the correct answer from the options given below:

(1) Both (A) and (R) are correct but (R) is not the correct explanation of (A)

(2) (A) is correct but (R) is not correct

(3) (A) is not correct but (R) is correct

(4) Both (A) and (R) are correct and (R) is the correct explanation of (A)

Correct Answer: (4)

Solution:
The work done by an electric field when moving a charge on an equipotential surface is zero because there is no change in electric potential energy. Electric lines of force are always perpendicular to equipotential surfaces, meaning that any movement along the surface does not alter the electric potential, confirming that no work is done.

Quick Tip

Work done by a conservative force like the electric field on an equipotential surface is always zero due to perpendicularity between displacement and force direction.

Question 49: A heavy iron bar of weight 12 kg is having its one end on the ground and the other on the shoulder of a man. The rod makes an angle $60°$ with the horizontal, the weight experienced by the man is:

Options:
(1) 6 kg
(2) 12 kg
(3) 3 kg
(4) $6\sqrt{3}$ kg

Correct Answer: (3)

Solution:
Given:

$$W = 12\,\text{kg}$$

The bar makes an angle $\theta = 60°$ with the horizontal. Let N_1 be the normal force at the ground and N_2 be the force experienced by the man's shoulder. The condition for equilibrium about the pivot gives:

$$\text{Torque about } O = 0$$

Taking moments about O:

$$120\left(\frac{L}{2}\cos 60^\circ\right) - N_2 L = 0$$

Simplifying:

$$N_2 = \frac{120}{2} \times \frac{1}{2} = 30\,\text{N}$$

Converting to weight experienced:

$$\text{Weight} = \frac{30\,\text{N}}{10\,\text{m/s}^2} = 3\,\text{kg}$$

Quick Tip

For equilibrium problems, sum of all torques around a pivot must be zero. Consider forces' directions and lever arms accurately.

Question 50: A bullet is fired into a fixed target and loses one third of its velocity after travelling 4 cm. It penetrates further $D \times 10^{-3}$ m before coming to rest. The value of D is:

Options:

(1) 2

(2) 5

(3) 3

(4) 4

Solution: (Bonus)

Given that the bullet loses one third of its velocity after traveling 4 cm:

$$v = \frac{2}{3}u$$

Using the kinematic equation:

$$v^2 - u^2 = 2a \times s$$

Substituting the values:

$$\left(\frac{2u}{3}\right)^2 - u^2 = 2a \times (4 \times 10^{-2})$$

Simplifying:

$$\frac{4u^2}{9} - u^2 = -2a \times (4 \times 10^{-2})$$

$$-\frac{5u^2}{9} = -2a \times (4 \times 10^{-2})$$

$$a = \frac{5u^2}{72 \times 10^{-2}}$$

Now, for the bullet to come to rest:

$$0 - \left(\frac{2u}{3}\right)^2 = 2a \times D \times 10^{-3}$$

Substituting the values:

$$-\frac{4u^2}{9} = 2a \times D \times 10^{-3}$$

Solving for D:

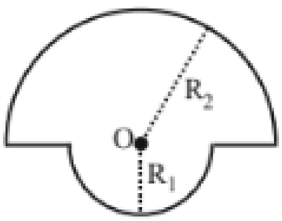
$$D = 32 \times 10^{-3}\,\text{m}$$

Note: Since no option matches exactly, the question should be treated as a bonus.

Quick Tip

When a body decelerates uniformly, use kinematic equations to relate initial and final velocities, acceleration, and distance traveled.

Question 51: The magnetic field at the centre of a wire loop formed by two semicircular wires of radii $R_1 = 2\,\text{m}$ and $R_2 = 4\,\text{m}$ carrying current $I = 4\,\text{A}$ as per figure given below is $\alpha \times 10^{-7}$ T. The value of α is ________. (Centre O is common for all segments)



Correct Answer: (3)

Solution:

The magnetic field at the center of a semicircular wire carrying current I and having radius R is given by:

$$B = \frac{\mu_0 I}{4R}$$

For the semicircular wires of radii R_1 and R_2:

$$B_{R_1} = \frac{\mu_0 I}{4R_1} \quad \text{and} \quad B_{R_2} = \frac{\mu_0 I}{4R_2}$$

The net magnetic field at the center O is the sum of the fields due to both semicircular wires:

$$B = B_{R_1} + B_{R_2} = \frac{\mu_0 I}{4R_1} + \frac{\mu_0 I}{4R_2}$$

Substituting the given values:

$$B = \frac{4\pi \times 10^{-7} \times 4}{4 \times 2} + \frac{4\pi \times 10^{-7} \times 4}{4 \times 4}$$

$$B = 2\pi \times 10^{-7} + \pi \times 10^{-7} = 3\pi \times 10^{-7}\,\text{T}$$

Therefore:

$$\alpha = 3$$

Quick Tip

For calculating the magnetic field due to wire segments, remember to sum the contributions from each segment appropriately based on geometry.

Question 52: Two charges of $-4\,\mu$C and $+4\,\mu$C are placed at the points A$(1, 0, 4)$ m and B$(2, -1, 5)$ m located in an electric field $\vec{E} = 0.20i$ V/cm. The magnitude of the torque acting on the dipole is $8\sqrt{alpha} \times 10^{-5}$ Nm, where $\alpha =$ ____.

Correct Answer: (2)

Solution:

The electric dipole moment is given by:

$$\vec{p} = q \times \vec{d}$$

Given $q = 4 \times 10^{-6}$ C and the position vectors $\vec{A} = (1, 0, 4)$ and $\vec{B} = (2, -1, 5)$, the dipole vector $\vec{d}$ is:

$$\vec{d} = \vec{B} - \vec{A} = (2 - 1, -1 - 0, 5 - 4) = (1, -1, 1)\,\text{m}$$

Thus:

$$\vec{p} = q \cdot \vec{d} = 4 \times 10^{-6} \cdot (1, -1, 1)\,\text{Cm}$$

The torque on the dipole is given by:

$$\vec{\tau} = \vec{p} \times \vec{E}$$

Given $\vec{E} = 0.2$ V/cm $= 20$ V/m in the direction $\hat{i}$:

$$\vec{\tau} = (4 \times 10^{-6})(1, -1, 1) \times (20, 0, 0)$$

Calculating the cross product:

$$\vec{\tau} = (0, 20 \cdot 1, -20 \cdot -1) \times 10^{-6} = (0, 20, 20) \times 10^{-6}\,\text{Nm}$$

The magnitude is:

$$|\vec{\tau}| = \sqrt{0^2 + 20^2 + 20^2} \times 10^{-6} = 8\sqrt{2} \times 10^{-5}\,\text{Nm}$$

Given $\vec{\tau} = 8\alpha \times 10^{-5}$ Nm, we find:

$$\alpha = 2$$

Quick Tip

The torque on an electric dipole in a uniform field is calculated using $\vec{\tau} = \vec{p} \times \vec{E}$.

Question 53: A closed organ pipe 150 cm long gives 7 beats per second with an open organ pipe of length 350 cm, both vibrating in fundamental mode. The velocity of sound is _____ m/s.

Correct Answer: (294)

Solution:

For a closed pipe of length $L = 150\,\text{cm} = 1.5\,\text{m}$, the fundamental frequency is given by:

$$f_c = \frac{v}{4L}$$

For an open pipe of length $L = 350\,\text{cm} = 3.5\,\text{m}$, the fundamental frequency is:

$$f_o = \frac{v}{2L}$$

Given that the beat frequency is:

$$|f_c - f_o| = 7\,\text{Hz}$$

Substituting:

$$\left| \frac{v}{4 \times 1.5} - \frac{v}{2 \times 3.5} \right| = 7$$

Simplifying:

$$\left| \frac{v}{6} - \frac{v}{7} \right| = 7$$

Solving for v:

$$\frac{v}{42} = 7 \implies v = 42 \times 7 = 294\,\text{m/s}$$

Quick Tip

The beat frequency between two sources is given by the absolute difference of their frequencies. For organ pipes, use the appropriate formulas for fundamental frequencies.

Question 54: A body falling under gravity covers two points A and B separated by 80 m in 2s. The distance of upper point A from the starting point is ____ m (use $g = 10\,\text{m/s}^2$).

Correct Answer: (45)

Solution:

Given:

$$\text{Distance between A and B} = 80\,\text{m}, \quad t = 2\,\text{s}, \quad g = 10\,\text{m/s}^2$$

Using the equation of motion:

$$s = ut + \frac{1}{2}gt^2$$

For motion from A to B:

$$-80 = v_1 t - \frac{1}{2}gt^2$$

Substituting values:

$$-80 = v_1 \cdot 2 - \frac{1}{2} \cdot 10 \cdot 2^2$$

$$-80 = 2v_1 - 20$$

$$-60 = 2v_1 \implies v_1 = -30\,\text{m/s}$$

For motion from 0 to A:

$$v_1^2 = u^2 + 2gS$$

$$30^2 = 0 + 2 \times 10 \times S$$

$$900 = 20S \implies S = 45\,\text{m}$$

Quick Tip

In problems involving free fall, use equations of motion to relate distance, time, initial velocity, and acceleration due to gravity.

Question 55: The reading of a pressure meter attached with a closed pipe is 4.5×10^4 N/m^2. On opening the valve, water starts flowing and the reading of pressure meter falls to 2.0×10^4 N/m^2. The velocity of water is found to be $\sqrt{V}$ m/s. The value of V is _____.

Correct Answer: (50)

Solution:

Using Bernoulli's theorem for flow of an ideal fluid:

$$P_1 + \frac{1}{2}\rho v^2 = P_2$$

Given:

$$P_1 = 4.5 \times 10^4\,\text{N/m}^2, \quad P_2 = 2.0 \times 10^4\,\text{N/m}^2, \quad \rho = 1000\,\text{kg/m}^3$$

Substituting values:

$$4.5 \times 10^4 + \frac{1}{2} \times 1000 \times v^2 = 2.0 \times 10^4$$

Rearranging:

$$\frac{1}{2} \times 1000 \times v^2 = 4.5 \times 10^4 - 2.0 \times 10^4$$

$$500v^2 = 2.5 \times 10^4$$

$$v^2 = 50 \implies v = \sqrt{50}\,\text{m/s}$$

Therefore:

$$V = 50$$

Quick Tip

For fluid flow problems, Bernoulli's equation relates pressure differences to changes in fluid speed.

Question 56: A ring and a solid sphere roll down the same inclined plane without slipping. They start from rest. The radii of both bodies are identical and the ratio of their kinetic energies is $\frac{7}{x}$ where x is _______.

Correct Answer: (7)

Solution:

In pure rolling motion, the work done by friction is zero. Hence, the potential energy is completely converted into kinetic energy. Since the ring and the sphere initially have the same potential energy, they will also have the same total kinetic energy at the end.

$$\text{Ratio of kinetic energies} = 1.$$

Given:

$$\frac{7}{x} = 1 \implies x = 7.$$

Quick Tip

In rolling motion, total kinetic energy includes both translational and rotational components.

Question 57: A parallel beam of monochromatic light of wavelength 5000 Å is incident normally on a single narrow slit of width 0.001 mm. The light is focused by a convex lens on a

screen, placed on its focal plane. The first minima will be formed for the angle of diffraction of ____ (degree).

Correct Answer: (30)

Solution:

For the first minima in a single slit diffraction pattern:

$$a \sin\theta = \lambda$$

Given:

$$a = 0.001\,\text{mm} = 1 \times 10^{-6}\,\text{m}, \quad \lambda = 5000\,\text{Å} = 5000 \times 10^{-10}\,\text{m}$$

Substituting:

$$\sin\theta = \frac{\lambda}{a} = \frac{5000 \times 10^{-10}}{1 \times 10^{-6}} = 0.5$$

$$\theta = \sin^{-1}(0.5) = 30^\circ$$

Quick Tip

For single-slit diffraction, the angle of minima can be found using $a \sin\theta = m\lambda$ where $m = 1, 2, 3, \ldots$.

Question 58: The electric potential at the surface of an atomic nucleus $(Z = 50)$ of radius 9×10^{-13} cm is ______ $\times 10^6$ V.

Correct Answer: (8)

Solution:

The electric potential at the surface of a nucleus is given by:

$$V = \frac{kQ}{R} = \frac{kZe}{R}$$

where $k = 9 \times 10^9\,\text{N m}^2/\text{C}^2$, $Z = 50$, $e = 1.6 \times 10^{-19}\,\text{C}$, and $R = 9 \times 10^{-13}\,\text{cm} = 9 \times 10^{-15}\,\text{m}$.

Substituting the values:

$$V = \frac{9 \times 10^9 \times 50 \times 1.6 \times 10^{-19}}{9 \times 10^{-15}}$$

$$V = 8 \times 10^6 \text{ V}$$

Quick Tip

The potential at the surface of a nucleus is calculated using $V = \frac{kZe}{R}$, where k is Coulomb's constant, Z is the atomic number, e is the elementary charge, and R is the radius.

Question 59: If Rydberg's constant is R, the longest wavelength of radiation in Paschen series will be $\frac{\alpha}{7R}$, where $\alpha =$ _____.

Correct Answer: (144)

Solution:

The Paschen series corresponds to transitions to $n = 3$. The longest wavelength corresponds to the transition between $n = 4$ and $n = 3$. The inverse wavelength is given by:

$$\frac{1}{\lambda} = RZ^2 \left(\frac{1}{n_1^2} - \frac{1}{n_2^2} \right)$$

For $n_1 = 3$ and $n_2 = 4$, and taking $Z = 1$:

$$\frac{1}{\lambda} = R \left(\frac{1}{3^2} - \frac{1}{4^2} \right) = R \left(\frac{1}{9} - \frac{1}{16} \right)$$

$$\frac{1}{\lambda} = R \left(\frac{16 - 9}{144} \right) = \frac{7R}{144}$$

Thus:

$$\alpha = 144$$

Quick Tip

The longest wavelength in a series corresponds to the smallest energy transition, i.e., from $n = n_2$ to $n = n_1$.

Question 60: A series LCR circuit with $L = \frac{100}{\pi}$ mH, $C = 10^{-3}$ F, and $R = 10\,\Omega$, is connected across an ac source of 220 V, 50 Hz supply. The power factor of the circuit would be

Correct Answer: (1)

Solution:

Given:

$$L = \frac{100}{\pi}\,\text{mH} = \frac{100}{\pi} \times 10^{-3}\,\text{H}, \quad C = 10^{-3}\,\text{F}, \quad R = 10\,\Omega, \quad f = 50\,\text{Hz}$$

The inductive reactance is given by:

$$X_L = 2\pi f L = 2\pi \times 50 \times \frac{100}{\pi} \times 10^{-3} = 10\,\Omega$$

The capacitive reactance is given by:

$$X_C = \frac{1}{2\pi f C} = \frac{1}{2\pi \times 50 \times 10^{-3}} = 10\,\Omega$$

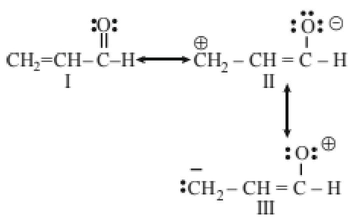
Since $X_L = X_C$, the circuit is in resonance. Therefore, the impedance $Z = R = 10\,\Omega$ and the power factor is:

$$\text{Power Factor} = \frac{R}{Z} = 1$$

Quick Tip

In a series LCR circuit at resonance, the inductive and capacitive reactances cancel each other, resulting in a power factor of unity.

Question 61: The order of relative stability of the contributing structures is:



Options:

(1) I > II > III

(2) II > I > III

(3) I = II = III

(4) III > II > I

Correct Answer: (1)

Solution:

The correct order of relative stability is:

$$I > II > III$$

Explanation:

Structure **I** is the most stable because it is a neutral resonating structure. In general, neutral structures are more stable compared to charged structures.

Structure **II** is less stable than **I** because it involves a positive charge on a less electronegative atom compared to structure **III**. However, it is more stable than **III** because the negative charge in **III** is on a carbon atom, making it the least stable due to charge separation.

Structure **III** is the least stable among the three due to the presence of a negative charge on carbon and overall charge separation making it highly unstable.

Quick Tip

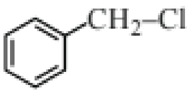
Neutral structures are more stable than charged resonating forms. Stability increases when charges are on atoms with higher electronegativity.

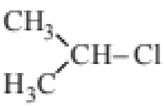
Question 62: Which among the following halide(s) will not show S_N1 reaction:

Options:

(A) $H_2C = CH - CH_2Cl$

(B) $CH_3 - CH = CH - Cl$

(C) 

(D) 

Choose the most appropriate answer from the options given below:

(1) (A), (B), and (D) only

(2) (A) and (B) only

(3) (B) and (C) only

(4) (B) only

Correct Answer: (4)

Solution:

The S_N1 reaction mechanism proceeds through the formation of a carbocation intermediate. The stability of the carbocation plays a critical role in determining whether the reaction will proceed.

Halide (A): $H_2C = CH - CH_2Cl$ forms an allylic carbocation upon ionization, which is stabilized by resonance. Therefore, it is likely to undergo an S_N1 reaction.

Halide (B): $CH_3 - CH = CH - Cl$ forms a carbocation that is not stabilized by resonance or inductive effects. This makes it unlikely to undergo an S_N1 reaction, as the carbocation formed would be highly unstable.

Halide (C): This compound forms a benzylic carbocation upon ionization, which is highly stabilized due to resonance with the aromatic ring, making it suitable for an S_N1 reaction.

Halide (D): $H_3C - C(Cl)H_2$ forms a tertiary carbocation, which is stable and favorable for the S_N1 reaction due to hyperconjugation and inductive effects.

Therefore, the only halide that will not show an S_N1 reaction is (B).

Quick Tip

S_N1 reactions are favored by the formation of stable carbocations. Consider resonance and hyperconjugation for carbocation stability.

Question 63: Which of the following statements is not correct about rusting of iron?

Options:

(1) Coating of iron surface by tin prevents rusting, even if the tin coating is peeled off.

(2) When pH lies above 9 or 10, rusting of iron does not take place.

(3) Dissolved acidic oxides SO_2, NO_2 in water act as catalyst in the process of rusting.

(4) Rusting of iron is envisaged as setting up of electrochemical cell on the surface of iron object.

Correct Answer: (1)

Solution:
When the tin coating is peeled off, the iron is exposed and rusts more quickly due to galvanic action, making Option (1) incorrect. Tin itself does not prevent rusting once it is removed.

Quick Tip

Galvanic action occurs when iron comes in contact with a less reactive metal, accelerating rusting when protective coatings are compromised.

Question 64: Given below are two statements:

Statement (I): In the Lanthanides, the formation of Ce^{4+} is favored by its noble gas configuration.
Statement (II): Ce^{4+} is a strong oxidant reverting to the common +3 state.
Choose the correct option:
(1) Statement I is false but Statement II is true
(2) Both Statement I and Statement II are true
(3) Statement I is true but Statement II is false
(4) Both Statement I and Statement II are false

Correct Answer: (2)

Solution:
Ce^{4+} has a noble gas electronic configuration, making Statement (I) true. Due to its high reduction potential, Ce^{4+}/Ce^{3+} acts as a strong oxidizing agent, making Statement (II) true as well.

Quick Tip

Ce^{4+} stability is influenced by its noble gas configuration and its role as an oxidizing agent.

Question 65: Choose the correct option having all the elements with d^{10} electronic configuration from the following:

Options:

(1) Zn, Co^{2+}, Ni, Fe^{2+}, Cr

(2) Cu, Zn, Ag, Cd

(3) Pd, Ni, Fe^{2+}, Cr

(4) Ni, Zn, Fe^{2+}, Cu

Correct Answer: (2)

Solution:

Elements such as Cu, Zn, Ag, and Cd exhibit a d^{10} electronic configuration:

$$[\text{Cu}] = [\text{Ar}]3d^{10}4s^1, \quad [\text{Zn}] = [\text{Ar}]3d^{10}4s^2, \quad [\text{Ag}] = [\text{Kr}]4d^{10}5s^1, \quad [\text{Cd}] = [\text{Kr}]4d^{10}5s^2$$

Quick Tip

Elements with fully filled d-orbitals (d^{10} configuration) exhibit unique chemical properties.

Question 66: Phenolic group can be identified by a positive:

Options:

(1) Phthalein dye test

(2) Lucas test

(3) Tollen's test

(4) Carbylamine test

Correct Answer: (1)

Solution:

Phthalein dye test is a specific test for phenolic groups and involves the formation of colored compounds.

Carbylamine test is used for the identification of primary amines.

Lucas test differentiates between 1°, 2°, and 3° alcohols.

Tollen's test is used for the identification of aldehydes.

Thus, the phenolic group can be identified by a positive **Phthalein dye test**.

> **Quick Tip**
>
> To identify phenolic groups, look for tests that form colored compounds through specific reactions, such as the Phthalein dye test.

Question 67: The molecular formula of second homologue in the homologous series of mono carboxylic acids is:

Options:

(1) $C_3H_6O_2$

(2) $C_2H_4O_2$

(3) CH_2O

(4) $C_2H_2O_2$

Correct Answer: (2)

Solution:

The first member of the homologous series of mono carboxylic acids is formic acid (HCOOH).

The second member is acetic acid (CH_3COOH).

The molecular formula for acetic acid is $C_2H_4O_2$.

Hence, the correct answer is $\mathbf{C_2H_4O_2}$.

Quick Tip

The molecular formula of homologous series increases by a CH_2 unit for each subsequent member.

Question 68: The technique used for purification of steam volatile water immiscible substance is:

Options:

(1) Fractional distillation

(2) Fractional distillation under reduced pressure

(3) Distillation

(4) Steam distillation

Correct Answer: (4)

Solution:

Steam distillation is used for the purification of substances that are volatile in steam and immiscible in water.

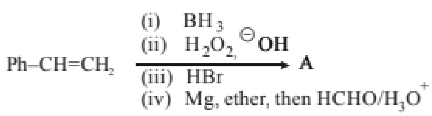
It is commonly used to separate essential oils from plant materials and other mixtures where the components have different volatilities.

Thus, the correct technique is **Steam distillation**.

Quick Tip

Steam distillation is useful for substances that are steam volatile but insoluble in water, making it ideal for purifying essential oils.

Question 69: The final product A, formed in the following reaction sequence is:



Options:

(1) Ph - CH_2 - CH_2 - CH_3

(2) Ph - CH - CH_3 (with CH_3 substituent on the CH carbon)

(3) Ph - CH - CH_3 (with CH_2OH substituent on the CH carbon)

(4) Ph - CH_2 - CH_2 - CH_2 - OH

Correct Answer: (4)

Solution:

Step (i) involves hydroboration-oxidation of the double bond in Ph-CH=CH_2, resulting in the anti-Markovnikov addition of water to form Ph-CH_2-CH_2OH.

Step (ii) converts the alcohol (Ph-CH_2-CH_2OH) to the corresponding alkyl halide (Ph-CH_2-CH_2Br) using HBr.

Step (iii) involves the formation of a Grignard reagent with Mg, producing Ph-CH_2-CH_2MgBr.

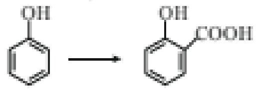
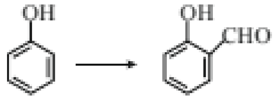
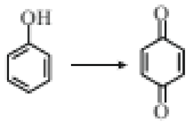
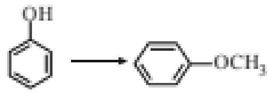
Step (iv) reacts the Grignard reagent with formaldehyde (HCHO) followed by hydrolysis to yield the final primary alcohol, Ph-CH_2-CH_2-CH_2-OH.

Thus, the final product is **Ph - CH_2 - CH_2 - CH_2 - OH**.

Quick Tip

Grignard reagents react with formaldehyde to give primary alcohols. Hydroboration-oxidation of alkenes follows the anti-Markovnikov rule, leading to the formation of alcohols.

Question 70: Match List-I with List-II.

List – I (Reaction)	**List – II (Reagent(s))**
(A) 	(I) $Na_2Cr_2O_7$, H_2SO_4
(B) 	(II) (i) NaOH (ii) CH_3Cl
(C) 	(III) (i) NaOH, $CHCl_3$ (ii) NaOH (iii) HCl
(D) 	(IV) (i) NaOH (ii) CO_2 (iii) HCl

Correct Matching:

(1) (A)-(IV), (B)-(I), (C)-(III), (D)-(II)

(2) (A)-(II), (B)-(III), (C)-(I), (D)-(IV)

(3) (A)-(II), (B)-(I), (C)-(III), (D)-(IV)

(4) (A)-(IV), (B)-(III), (C)-(I), (D)-(II)

Correct Answer: (4)

Solution:

(A) **Oxidation of phenol to benzoquinone:** This reaction uses **$Na_2Cr_2O_7$ and H_2SO_4** as the oxidizing agents.

(B) **Kolbe's reaction:** It involves carboxylation of phenol using **NaOH and CO_2**, followed by acidification with **HCl**.

(C) **Reimer-Tiemann reaction:** This reaction introduces an aldehyde group on the phenol ring using **NaOH and $CHCl_3$**.

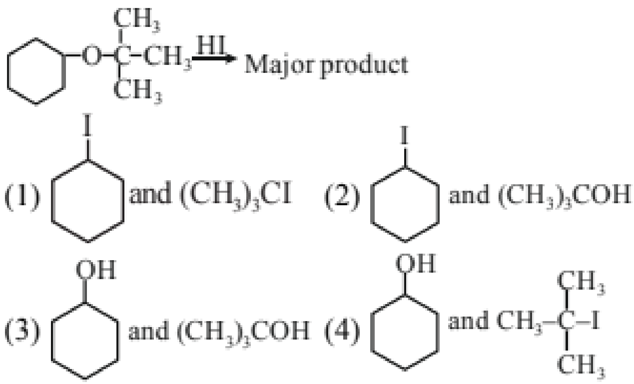
(D) **Williamson's synthesis:** It prepares ethers by reacting phenoxide with **CH_3Cl** in the presence of **NaOH**.

Thus, the correct matching is: (A)-(IV), (B)-(III), (C)-(I), (D)-(II)

Quick Tip

To accurately match reactions with reagents, focus on identifying unique characteristics of each reaction, such as oxidation, carboxylation, and ether formation.

Question 71: Major product formed in the following reaction is a mixture of:



Correct Answer: (4)

Solution:

The given reaction involves the cleavage of the ether bond using **HI**. The major products are formed through the nucleophilic substitution of **HI** on the ether linkage. The reaction mechanism proceeds as follows:

Protonation of the ether oxygen atom by **HI**, resulting in the formation of an oxonium ion intermediate.

Cleavage of the C–O bond forms cyclohexanol and a tertiary carbocation $(CH_3)_3C^+$.

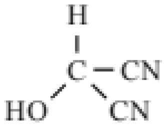
The tertiary carbocation rapidly reacts with I^- to form $(CH_3)_3C–I$ as the final product.

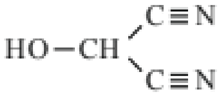
Quick Tip

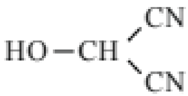
To accurately match reactions with reagents, focus on identifying unique characteristics of each reaction, such as oxidation, carboxylation, and ether formation.

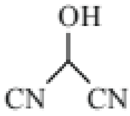
Question 72: The bond line formula of $HOCH(CN)_2$ is:

Options:

(1) 

(2) 

(3) 

(4) 

Correct Answer: (4)

Solution:

The given compound is $HOCH(CN)_2$. This indicates a carbon atom bonded to a hydroxyl group (OH) and two cyano groups (CN). The correct bond-line formula for this structure is represented by option (4) Explanation: The central carbon atom is bonded to one hydroxyl group and two cyanide (CN) groups.

This structure matches the representation of option (4), showing two cyanide groups attached to the central carbon atom along with the hydroxyl group.

Quick Tip

When interpreting bond-line structures, always focus on the groups attached to the central atom and their positions to ensure an accurate representation.

Question 73 : Given below are two statements:

Statement (I) : Oxygen being the first member of group 16 exhibits only -2 oxidation state.

Statement (II) : Down the group 16, stability of +4 oxidation state decreases and +6 oxidation state increases.

In light of the above statements, choose the **most appropriate** answer from the options given below:

(1) Statement I is correct but Statement II is incorrect

(2) Both Statement I and Statement II are correct

(3) Both Statement I and Statement II are incorrect

(4) Statement I is incorrect but Statement II is correct

Correct Answer: (3)

Solution:

Statement I: Oxygen, as the first member of group 16, primarily exhibits an oxidation state of -2 due to its high electronegativity and small size, which favors electron gain rather than loss. However, it can also exist in oxidation states other than -2, such as 0 in molecular oxygen (O_2) and +1 or +2 in compounds like OF_2 and O_2F_2. Therefore, the statement that oxygen exhibits only -2 oxidation state is incorrect.

Statement II: In group 16 elements, moving down the group from oxygen to polonium, there is an observed increase in the stability of the +4 oxidation state, while the stability of the +6 oxidation state decreases. This trend is attributed to the inert pair effect, where the tendency of the s-electrons to remain unpaired increases in heavier elements, making higher oxidation states less stable. Thus, elements like tellurium and polonium prefer to exhibit the +4 oxidation state rather than +6. Hence, the statement that the stability of +4 oxidation state decreases down the group is also incorrect.

Therefore, both statements are incorrect.

> **Quick Tip**
>
> The inert pair effect is responsible for the observed trend in oxidation states in heavier group 16 elements, where the lower oxidation states become more stable down the group.

Question 74 : Identify from the following species in which d^2sp^3 hybridization is shown by central atom:

Options:

(1) $[Co(NH_3)_6]^{3+}$

(2) BrF_5

(3) $[PtCl_4]^{2-}$

(4) SF_6

Correct Answer: (1)

Solution:

$[Co(NH_3)_6]^{3+}$ exhibits d^2sp^3 hybridization as it forms an octahedral geometry. In this complex, the central cobalt ion uses two d orbitals, one s orbital, and three p orbitals to form six coordinate bonds with ammonia molecules.

For BrF_5, the central bromine atom exhibits sp^3d^2 hybridization, resulting in a square pyramidal structure.

In $[PtCl_4]^{2-}$, platinum shows dsp^2 hybridization, resulting in a square planar geometry.

SF_6 involves sp^3d^2 hybridization, resulting in an octahedral geometry around the sulfur atom.

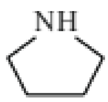
Thus, the correct species showing d^2sp^3 hybridization is $[Co(NH_3)_6]^{3+}$.

> **Quick Tip**
>
> d^2sp^3 hybridization involves two d orbitals, one s orbital, and three p orbitals, typically resulting in an octahedral geometry.

Question 75 : Identify B formed in the reaction.

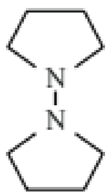
$Cl\text{-}(CH_2)_4\text{-}Cl \xrightarrow{\text{excess } NH_3} A \xrightarrow{NaOH} B + H_2O + NaCl$

Options:

(1) 

(2) $H_2N-(CH_2)_4-NH_2$

(3) $Cl\overset{+}{N}H_3-(CH_2)_4-\overset{+}{N}H_3Cl^-$

(4) 

Correct Answer: (2)

Solution:

The compound Cl-$(CH_2)_4$-Cl reacts with excess ammonia (NH_3) to form an intermediate A, which is NH_3-$(CH_2)_4$-$NH_3^+Cl^-$. This intermediate compound is a diammonium salt.

Upon treatment with NaOH, it undergoes deprotonation to yield compound B, which is H_2N-$(CH_2)_4$-NH_2, also known as 1,4-diaminobutane or putrescine.

Therefore, the correct answer is H_2N-$(CH_2)_4$-NH_2.

> **Quick Tip**
>
> Ammonia reacts with dihalides to form diamines after deprotonation, especially when excess ammonia and a base are used.

Question 76 : The quantity which changes with temperature is:

Options:

(1) Molarity

(2) Mass percentage

(3) Molality

(4) Mole fraction

Correct Answer: (1)

Solution:

Molarity is defined as the number of moles of solute per liter of solution. Since the volume of a solution is affected by temperature due to thermal expansion or contraction, molarity also changes with temperature.

Mass percentage, molality, and mole fraction, however, are temperature-independent as they depend only on the ratio of masses or moles of solute and solvent, which do not vary with temperature.

Thus, the correct answer is **Molarity**, as it is affected by temperature changes.

> **Quick Tip**
>
> Molarity varies with temperature because it depends on volume, which changes with thermal expansion or contraction.

Question 77 : Which structure of protein remains intact after coagulation of egg white on boiling?

Options:

(1) Primary

(2) Tertiary

(3) Secondary

(4) Quaternary

Correct Answer: (1)

Solution:

Boiling an egg causes denaturation of its protein, resulting in loss of its quaternary, tertiary, and secondary structures, but the primary structure remains intact.

> **Quick Tip**
>
> The primary structure of a protein is the sequence of amino acids linked by peptide bonds, which is not affected by boiling.

Question 78 : Which of the following cannot function as an oxidising agent?

Options:

(1) N^{3-}

(2) SO_4^{2-}

(3) BrO_3^-

(4) MnO_4^-

Correct Answer: (1)

Solution:

In N^{3-}, nitrogen is present in its lowest possible oxidation state. Hence, it cannot be further reduced and cannot act as an oxidizing agent.

Quick Tip

An oxidizing agent is a substance that can gain electrons and get reduced; elements in their lowest oxidation state typically cannot act as oxidizing agents.

Question 79 : The incorrect statement regarding conformations of ethane is:

Options:

(1) Ethane has an infinite number of conformations

(2) The dihedral angle in staggered conformation is 60°

(3) Eclipsed conformation is the most stable conformation

(4) The conformations of ethane are inter-convertible to one another

Correct Answer: (3)

Solution:

Eclipsed conformation is the least stable conformation of ethane due to repulsion between electron clouds of adjacent C-H bonds. The staggered conformation is more stable.

Quick Tip

Staggered conformations are generally more stable due to minimized electron repulsion between bonds.

Question 80 : Identify the incorrect pair from the following:

Options:

(1) Photography - AgBr

(2) Polythene preparation - $TiCl_4$, $Al(CH_3)_3$

(3) Haber process - Iron

(4) Wacker process - $PtCl_2$

Correct Answer: (4)

Solution:

The catalyst used in the Wacker's process is $PdCl_2$, not $PtCl_2$.

Quick Tip

Wacker's process uses palladium chloride ($PdCl_2$) as a catalyst for oxidation of ethylene to acetaldehyde.

Question 81 : Total number of ions from the following with noble gas configuration is

Options:

Sr^{2+} (Z = 38), Cs^{+} (Z = 55), La^{3+} (Z = 57), Pb^{2+} (Z = 82), Yb^{2+} (Z = 70) and Fe^{2+} (Z = 26)

Correct Answer: (2)

Solution:

To determine if an ion has a noble gas configuration, we examine its electron configuration and compare it with that of a nearby noble gas: - Sr^{2+} (Z = 38) loses two electrons, resulting in the electron configuration [Kr], which matches the noble gas krypton. - Cs^{+} (Z = 55) loses one electron, resulting in the electron configuration [Xe], matching xenon. - La^{3+} (Z = 57) loses three electrons, resulting in the electron configuration [Xe], also matching xenon. - Yb^{2+} (Z = 70) loses two electrons, resulting in the electron configuration [Xe], matching xenon.

On the other hand: - Pb^{2+} does not match any noble gas configuration due to its partially filled d-orbitals. - Fe^{2+} does not match a noble gas configuration either, as it retains electrons in the d-orbital.

Thus, only Sr^{2+}, Cs^{+}, La^{3+}, and Yb^{2+} have noble gas configurations, totaling four ions.

Quick Tip

Ions that achieve a noble gas configuration are typically more stable due to a complete electron shell.

Question 82 : The number of non-polar molecules from the following is

HF, H_2O, SO_2, H_2, CO_2, CH_4, NH_3, HCl, $CHCl_3$, BF_3

Correct Answer: (4)

Solution:

Non-polar molecules have a symmetrical arrangement of atoms that results in no net dipole moment. In the given list: - CO_2 is linear and symmetrical, so it is non-polar. - H_2 is diatomic and non-polar because it is composed of identical atoms. - CH_4 has a tetrahedral geometry with symmetrical bond distribution, making it non-polar. - BF_3 has a trigonal planar geometry, which is symmetrical and therefore non-polar.

Other molecules like HF, H_2O, SO_2, NH_3, HCl, and $CHCl_3$ are polar due to their asymmetrical shapes or differences in electronegativity. Therefore, there are four non-polar molecules in the list.

Quick Tip

Non-polar molecules generally have symmetrical shapes or identical atoms, leading to a balanced electron distribution and no net dipole.

Question 83 : Time required for completion of 99.9% of a first-order reaction is ____ times of half life ($t_{1/2}$) of the reaction.

Correct Answer: (10)

Solution:

For a first-order reaction, the time required for a certain percentage of reaction completion can be calculated using the formula:

$$t = \frac{2.303}{k} \log \frac{[A]_0}{[A]}$$

where $[A]_0$ is the initial concentration, $[A]$ is the concentration at time t, and k is the rate constant. For 99.9% completion, $\frac{[A]}{[A]_0} = 0.001$:

$$t = \frac{2.303}{k} \log \frac{1}{0.001} = \frac{2.303}{k} \times 3 = 10 \times t_{1/2}$$

Thus, the time required for 99.9% completion is 10 times the half-life.

Quick Tip

For a first-order reaction, the time to reach 99.9% completion is approximately 10 times the half-life.

Question 84 : The spin-only magnetic moment value of square planar complex $[Pt(NH_3)_2Cl(NH_2CH_3)]Cl$ is ______ B.M. (Nearest integer)

Correct Answer: (0)

Solution:

The complex $[Pt(NH_3)_2Cl(NH_2CH_3)]Cl$ contains Pt^{2+} in a square planar geometry. Pt^{2+} has a d^8 electronic configuration. In square planar complexes, the d-electrons pair up in such a way that no unpaired electrons remain, resulting in a magnetic moment of 0 Bohr Magnetons (B.M.).

Quick Tip

Square planar complexes with d^8 configurations, like Pt^{2+}, typically have no unpaired electrons, yielding a magnetic moment of 0.

Question 85 : For a certain thermochemical reaction M $\rightarrow$ N at T = 400 K, $\Delta H^\circ = 77.2$ kJ mol^{-1}, $\Delta S^\circ = 122$ JK^{-1}, log equilibrium constant (log K) is ______ x 10^{-1}.

Correct Answer: (37)

Solution:

The standard Gibbs free energy change ΔG° is related to the enthalpy change ΔH° and entropy change ΔS° by the equation:

$$\Delta G^\circ = \Delta H^\circ - T\Delta S^\circ.$$

Substitute the given values:

$$\Delta H^\circ = 77.2 \times 10^3\,\text{J}, \quad T = 400\,\text{K}, \quad \Delta S^\circ = 122\,\text{J/K}.$$

$$\Delta G^\circ = 77.2 \times 10^3 - 400 \times 122 = 28400\,\text{J}.$$

The relationship between ΔG° and the equilibrium constant K is:

$$\Delta G^\circ = -2.303RT \log K.$$

Substitute $\Delta G^\circ = 28400\,\text{J}$, $R = 8.314\,\text{J/mol·K}$, $T = 400\,\text{K}$:

$$28400 = -2.303 \times 8.314 \times 400 \log K.$$

Simplify:

$$\log K = \frac{-28400}{2.303 \times 8.314 \times 400}.$$

Calculate:

$$\log K = \frac{-28400}{7668.8} = -3.708.$$

Thus:

$$K = 10^{\log K} = 10^{-3.708}.$$

Final Answer:37

Quick Tip

The equation $\Delta G^\circ = -RT \ln K$ is crucial in calculating the equilibrium constant from thermodynamic parameters.

Question 86 : Volume of 3 M NaOH (formula weight 40 g mol^{-1}) which can be prepared from 84 g of NaOH is _____$x10^{-1}dm^3$.

Correct Answer: (7)

Solution:

The molarity formula is given by:

$$M = \frac{n_{\text{NaOH}}}{V_{\text{sol}}},$$

where:

M is the molarity (in mol/L),

n_{NaOH} is the number of moles of NaOH,

V_{sol} is the volume of the solution (in liters).

Given:

$$M = 3\,\text{M}, \quad \text{Mass of NaOH} = 84\,\text{g}, \quad \text{Molar mass of NaOH} = 40\,\text{g/mol}.$$

First, calculate the number of moles of NaOH:

$$n_{\text{NaOH}} = \frac{\text{Mass of NaOH}}{\text{Molar mass}} = \frac{84}{40} = 2.1\,\text{moles}.$$

Now, calculate the volume of the solution using the molarity formula:

$$V_{\text{sol}} = \frac{n_{\text{NaOH}}}{M} = \frac{2.1}{3} = 0.7\,\text{L}.$$

Expressing the volume in scientific notation:

$$V_{\text{sol}} = 7 \times 10^{-1}\,\text{L}.$$

Final Answer:

$$V_{\text{sol}} = 0.7\,\text{L or } 7 \times 10^{-1}\,\text{L}.$$

Quick Tip

Molarity (M) is calculated using the formula $M = \frac{n}{V}$ where n is the number of moles and V is the volume in liters.

Question 87 : 1 mole of PbS is oxidized by "X" moles of O_3 to get "Y" moles of O_2. X + Y = _____

Correct Answer: (8)

Solution:

The balanced chemical equation for the oxidation of PbS by ozone is:

$$\text{PbS} + 4\text{O}_3 \rightarrow \text{PbSO}_4 + 4\text{O}_2$$

From the equation: - 1 mole of PbS reacts with 4 moles of O_3 (so $X = 4$). - 4 moles of O_2 are produced (so $Y = 4$).

Therefore:

$$X + Y = 4 + 4 = 8$$

Quick Tip

In stoichiometry, balancing the equation is key to determining the exact mole ratio of reactants and products.

Question 88 : The hydrogen electrode is dipped in a solution of pH = 3 at 25°C. The potential of the electrode will be - _____ $\times\ 10^{-2} V$.

Correct Answer: (18)

Solution:

The potential of a hydrogen electrode in a solution can be calculated using the Nernst equation:

$$E = E^\circ - \frac{0.059}{n} \log \frac{1}{[\text{H}^+]}$$

Given: - $E^\circ = 0$ (for the standard hydrogen electrode) - pH = 3, so $[\text{H}^+] = 10^{-3}$ M

Substitute into the equation:

$$E = 0 - 0.059 \times \log(10^3) = -0.059 \times 3 = -0.177\,\text{V} = -17.7 \times 10^{-2}\,\text{V}$$

Quick Tip

For pH calculations, remember that $[\text{H}^+] = 10^{-\text{pH}}$.

Question 89 : 9.3 g of aniline is subjected to reaction with excess of acetic anhydride to prepare

acetanilide. The mass of acetanilide produced if the reaction is 100% completed is

_____ $\times 10^{-1} g.(Givenmolarmassingmol^{-1} N : 14, O : 16, C : 12, H : 1)$

Correct Answer: (135)

Solution:

The reaction between aniline and acetic anhydride produces acetanilide. The balanced equation is:

$$C_6H_5NH_2 + CH_3COOCOCH_3 \rightarrow C_6H_5NHCOCH_3 + CH_3COOH$$

Given: - Molar mass of aniline (C_6H_7N) = 93 g/mol - Molar mass of acetanilide (C_8H_9NO) = 135 g/mol

Calculate moles of aniline:

$$n_{\text{aniline}} = \frac{9.3}{93} = 0.1\,\text{moles}$$

Since the reaction is 1:1, moles of acetanilide produced = moles of aniline = 0.1 moles.

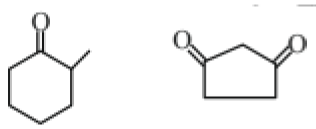
Mass of acetanilide produced:

$$\text{mass} = n \times \text{molar mass} = 0.1 \times 135 = 13.5\,\text{g} = 135 \times 10^{-1}\,\text{g}$$

Quick Tip

In a 100% yield reaction, the moles of product are directly based on the moles of limiting reactant in a 1:1 molar ratio.

Question 90 : Total number of compounds with Chiral carbon atoms from following is _______.



$CH_3-CH_2-CH(NO_2)-COOH$

$CH_3-CH_2-CHBr-CH_2-CH_3$

$CH_3-CH(I)-CH_2-NO_2$

$CH_3-CH_2-CH(OH)-CH_2OH$

$CH_3-CH-CH(I)-C_2H_5$ (with I on the second carbon)

Correct Answer: (5)

Solution:

To identify chiral carbons, look for carbon atoms with four different substituents, making them asymmetric:

1. CH_3-CH_2-CH(NO_2)-COOH has one chiral carbon at the second carbon.
2. CH_3-CH_2-CHBr-CH_2-CH_3 has a chiral carbon at the third carbon.
3. CH_3-CH(I)-CH_2-NO_2 has one chiral carbon at the second carbon.
4. CH_3-CH_2-CH(OH)-CH_2OH has a chiral carbon at the third carbon.
5. CH_3-CH-CH(I)-C_2H_5 has a chiral carbon at the second carbon.

Thus, there are 5 compounds with chiral carbons.

Quick Tip

A chiral carbon is attached to four different substituents, resulting in a non-superimposable mirror image.
