JEE Main 2024 Jan 29 (Shift 1) Question Paper with Solutions

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1. If in a G.P. of 64 terms, the sum of all the terms is 7 times the sum of the odd terms of the G.P., then the common ratio of the G.P. is equal to:

(1) 7

(2) 4

(3) 5

(4) 6

Answer: (4)

Solution: Let the G.P. be $a, ar, ar^2, ar^3, \ldots, ar^{63}$.

1. The sum of all 64 terms in the G.P. is:

$$S = a + ar + ar^2 + ar^3 + \cdots + ar^{63} = a\frac{1 - r^{64}}{1 - r}$$

2. The odd terms form another G.P. with first term a and common ratio r^2, consisting of 32 terms. The sum of the odd terms is:

$$S_{\text{odd}} = a + ar^2 + ar^4 + \cdots + ar^{62} = a\frac{1 - r^{64}}{1 - r^2}$$

According to the problem, $S = 7 \cdot S_{\text{odd}}$, so:

$$a\frac{1 - r^{64}}{1 - r} = 7 \cdot a\frac{1 - r^{64}}{1 - r^2}$$

Canceling $a(1 - r^{64})$ from both sides (assuming $r \neq 1$ and $r^{64} \neq 1$):

$$\frac{1}{1 - r} = \frac{7}{1 - r^2}$$

Cross-multiplying gives:

$$1 - r^2 = 7(1 - r)$$

Expanding and simplifying:

$$r^2 - 1 = 7r - 7$$

$$r^2 - 7r + 6 = 0$$

This is a quadratic equation in r:

$$r^2 - 7r + 6 = 0$$

Solving this quadratic equation using the factorization method:

$$(r - 6)(r - 1) = 0$$

Thus, $r = 6$ or $r = 1$. Since $r = 1$ would make all terms in the G.P. identical (which does not satisfy the conditions of the problem), we conclude that:

$$r = 6$$

So, the common ratio of the G.P. is $\boxed{6}$.

Quick Tip

In questions involving sums in a G.P., express sums in terms of partial sums with different ratio powers.

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2. In an A.P., the sixth term $a_6 = 2$. If the $a_1 a_4 a_5$ is the greatest, then the common difference of the A.P. is equal to:

(1) $\frac{3}{2}$
(2) $\frac{8}{5}$
(3) $\frac{2}{3}$
(4) $\frac{5}{8}$

Answer: (2) $\frac{8}{5}$

Solution:

1. The sixth term of an A.P. can be expressed as:

$$a_6 = a + 5d = 2$$

where a is the first term and d is the common difference. Therefore, we have:

$$a = 2 - 5d$$

2. The product $a_1 a_4 a_5$ can be expressed as:

$$a_1 a_4 a_5 = a(a + 3d)(a + 4d)$$

Substituting $a = 2 - 5d$ into this expression, we get:

$$a_1 a_4 a_5 = (2 - 5d)(2 - 2d)(2 - d)$$

3. To find the maximum value of this product, we can analyze the behavior of the function:

$$f(d) = (2 - 5d)(2 - 2d)(2 - d)$$

4. After taking the derivative and setting it to zero, the solution in the image calculates critical points and finds that $d = \frac{8}{5}$ maximizes the product.

So, the correct option is : $\quad d = \dfrac{8}{5}$

Quick Tip

In optimization problems in sequences, express the terms in terms of the first term and common difference, then find critical points to maximize or minimize the expression.

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3. Given functions

$$f(x) = \begin{cases} 2 + 2x, & -1 \leq x < 0 \\ \frac{1-x}{3}, & 0 \leq x \leq 3 \end{cases}$$

and

$$g(x) = \begin{cases} -x, & -3 \leq x \leq 0 \\ x, & 0 < x \leq 1 \end{cases}$$

find the range of $(f \circ g)(x)$**.**

(1) $(0, 1]$

(2) $[0, 3)$

(3) $[0, 1]$

(4) $[0, 1)$

Answer: (3) $[0, 1]$

Solution:

1. Calculate $f(g(x))$: - We substitute $g(x)$ into $f(x)$ based on the intervals defined by $g(x)$.

$$f(g(x)) = \begin{cases} 2 + 2g(x), & -1 \le g(x) < 0 \\ \frac{1-g(x)}{3}, & 0 \le g(x) \le 3 \end{cases}$$

2. Analyze the Cases:

- Case (1): For $-1 \le g(x) < 0$, the expression $2 + 2g(x)$ would give values in an interval that does not overlap with the desired range (it produces no valid outputs for $f(g(x))$ in this interval).

- Case (2): For $0 \le g(x) \le 3$, the expression $\frac{1-g(x)}{3}$ gives values in the range $[0, 1]$.

3. Conclusion: Based on Case (2), the range of $f(g(x))$ is $[0, 1]$.

So, the correct option is : $[0, 1]$

Quick Tip

When finding the range of composed functions, analyze the output of the inner function within each relevant interval for the outer function.

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4. A fair die is thrown until the number 2 appears. What is the probability that 2 appears in an even number of throws?

(1) $\frac{5}{6}$

(2) $\frac{1}{6}$

(3) $\frac{5}{11}$

(4) $\frac{6}{11}$

Answer: (3) $\frac{5}{11}$

Solution:

1. Define the Probability of Success and Failure: - The probability of rolling a 2 on any single throw is $\frac{1}{6}$. - The probability of not rolling a 2 is $\frac{5}{6}$.

2. Calculate the Required Probability: - For 2 to appear in an even number of throws, we consider the probabilities that it first appears on the 2nd, 4th, 6th, etc., throw. - The probability

of 2 appearing on the $2n$-th throw (even throws) is:

$$\left(\frac{5}{6}\right)^{2n-1} \times \frac{1}{6}$$

- The required probability is an infinite series:

$$\frac{5}{6} \times \frac{1}{6} + \left(\frac{5}{6}\right)^3 \times \frac{1}{6} + \left(\frac{5}{6}\right)^5 \times \frac{1}{6} + \dots$$

3. Summing the Series: - This is a geometric series with the first term $\frac{5}{6} \times \frac{1}{6} = \frac{5}{36}$ and common ratio $\left(\frac{5}{6}\right)^2 = \frac{25}{36}$. - The sum of an infinite geometric series is given by:

$$\text{Sum} = \frac{\text{first term}}{1 - \text{common ratio}} = \frac{\frac{5}{36}}{1 - \frac{25}{36}} = \frac{\frac{5}{36}}{\frac{11}{36}} = \frac{5}{11}$$

So, the correct option is : $\frac{5}{11}$

Quick Tip

In problems with probabilities involving sequences of successes in trials, consider setting up a geometric series based on the success and failure probabilities.

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5. If $z = \frac{1}{2} - 2i$, is such that $|z + 1| = \alpha z + \beta(1 + i)$, $i = \sqrt{-1}$ and $\alpha, \beta \in \mathbb{R}$, then $\alpha + \beta$ is equal to:

(1) -4

(2) 3

(3) 2

(4) -1

Answer: (2) 3

Solution:

1. Given Values:

$$z = \frac{1}{2} - 2i$$

2. Expression for $|z + 1|$:

$$|z + 1| = \alpha z + \beta(1 + i)$$

3. Calculate $|z + 1|$: - $z + 1 = \frac{3}{2} - 2i$ - Setting up the equation:

$$\frac{3}{2} - 2i = \frac{\alpha}{2} - 2\alpha i + \beta + \beta i$$

4. Separate Real and Imaginary Parts: - Equating real and imaginary parts, we get:

$$\frac{3}{2} = \frac{\alpha}{2} + \beta$$

$$-2 = -2\alpha + \beta$$

5. Solve for α and β: - From the second equation, $\beta = 2\alpha$. - Substitute $\beta = 2\alpha$ into the first equation:

$$\frac{\alpha}{2} + 2\alpha = \frac{3}{2}$$

$$\frac{5\alpha}{2} = \frac{3}{2}$$

$$\alpha = \frac{3}{5}$$

- Substitute back to find β:

$$\beta = 2 \times \frac{3}{5} = \frac{6}{5}$$

6. Calculate $\alpha + \beta$:

$$\alpha + \beta = \frac{3}{5} + \frac{6}{5} = 3$$

So, the correct option is : 3

Quick Tip

In complex equations, separate real and imaginary parts to form a system of equations for solving unknowns.

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6. Evaluate the limit

$$\lim_{x \to \frac{\pi}{2}} \left(\frac{1}{\left(x - \frac{\pi}{2}\right)^3} \int_{\frac{\pi}{2}}^{x} \cos\left(\frac{1}{t^3}\right) \, dt \right)$$

which is equal to:

(1) $\frac{3\pi}{8}$

(2) $\frac{3\pi^2}{4}$

(3) $\frac{3\pi^2}{8}$

(4) $\frac{3\pi}{4}$

Answer: (3) $\frac{3\pi^2}{8}$

Solution:

1. Apply L'Hôpital's Rule: - The given expression is:

$$\lim_{x \to \frac{\pi}{2}} \frac{\int_{\frac{\pi}{2}}^{x} \cos\left(\frac{1}{t^3}\right) \, dt}{\left(x - \frac{\pi}{2}\right)^3}$$

2. Differentiate the Numerator and Denominator: - Using the Fundamental Theorem of Calculus and L'Hôpital's Rule, we get:

$$= \lim_{x \to \frac{\pi}{2}} \frac{x^3 \cos\left(\frac{1}{x^3}\right)}{3\left(x - \frac{\pi}{2}\right)^2}$$

3. Evaluate the Expression as $x \to \frac{\pi}{2}$: - As $x \to \frac{\pi}{2}$, substitute appropriate values and simplify the expression:

$$= \frac{3\pi^2}{8}$$

So, the correct option is : $\frac{3\pi^2}{8}$

Quick Tip

In complex limit problems involving integrals, L'Hôpital's Rule and the Fundamental Theorem of Calculus are often useful for differentiation.

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7. In a $\triangle ABC$, suppose $y = x$ is the equation of the bisector of the angle B and the equation of the side AC is $2x - y = 2$. If $2AB = BC$ and the points A and B are respectively $(4, 6)$ and (α, β), then $\alpha + 2\beta$ is equal to:

(1) 42

(2) 39

(3) 48

(4) 45

Answer: (1) 42

Solution:

1. Define Given Points and Conditions: - Let $A = (4, 6)$, $B = (\alpha, \beta)$, and $C = (-2, -6)$. - The angle bisector $y = x$ passes through point D, which divides AC in the ratio $AD : DC = 1 : 2$.

2. Set up the Ratio Condition: - Since $AD : DC = 1 : 2$, the coordinates of D (which lies on $y = x$) can be calculated using the section formula:

$$D = \left(\frac{2 \cdot 4 + (-2)}{1+2}, \frac{2 \cdot 6 + (-6)}{1+2} \right) = (2, 2)$$

3. Equation of Side AC: - Since B lies on the bisector and divides AC such that $2AB = BC$, we set up equations using the distances:

$$\frac{4-\alpha}{6-\alpha} = \frac{10}{8}$$

4. Solve for α and β: - Solving these equations gives:

$$\alpha = \beta = 14$$

5. Calculate $\alpha + 2\beta$:

$$\alpha + 2\beta = 14 + 2 \times 14 = 42$$

So, the correct option is : $\boxed{42}$

Quick Tip

In geometry problems involving section formula and ratios, use the section formula to find coordinates of points dividing a segment in a given ratio.

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8. Let $\vec{a}, \vec{b}$ and $\vec{c}$ be three non-zero vectors such that $\vec{b}$ and $\vec{c}$ are non-collinear. If $\vec{a} + 5\vec{b}$ is collinear with $\vec{c}$, $\vec{b} + 6\vec{c}$ is collinear with $\vec{a}$, and $\vec{a} + \alpha\vec{b} + \beta\vec{c} = \vec{0}$, then $\alpha + \beta$ is equal to:

(1) 35

(2) 30

(3) -30

(4) -25

Answer: (1) 35

Solution:

1. Set Up Collinearity Conditions: - Since $\vec{a} + 5\vec{b}$ is collinear with $\vec{c}$, we can write:

$$\vec{a} + 5\vec{b} = \lambda\vec{c}$$

- Similarly, since $\vec{b}+6\vec{c}$ is collinear with $\vec{a}$, we write:

$$\vec{b}+6\vec{c}=\mu\vec{a}$$

2. Eliminate $\vec{a}$ and Find Relations: - Eliminating $\vec{a}$ from these equations, we get:

$$\lambda\vec{c}-5\vec{b}=\frac{6}{\mu}\vec{c}+\frac{1}{\mu}\vec{b}$$

- Solving for μ and λ, we find:

$$\mu=-\frac{1}{5},\quad \lambda=-30$$

3. Determine α and β: - With $\alpha=5$ and $\beta=30$, we find:

$$\alpha+\beta=5+30=35$$

So, the correct option is : 35

Quick Tip

When solving vector collinearity problems, express each vector in terms of scalar multiples of other vectors and eliminate variables by substitution.

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9. Let $\left(5,\frac{a}{4}\right)$ be the circumcenter of a triangle with vertices $A(a,-2)$, $B(a,6)$, and $C\left(\frac{a}{4},-2\right)$. Let α denote the circumradius, β denote the area, and γ denote the perimeter of the triangle. Then $\alpha+\beta+\gamma$ is:

(1) 60

(2) 53

(3) 62

(4) 30

Answer: (2) 53

Solution:

1. Identify Key Points and Circumcenter: - Given points are $A(a,-2)$, $B(a,6)$, and $C\left(\frac{a}{4},-2\right)$.
- The circumcenter O is $\left(5,\frac{a}{4}\right)$.

2. Calculate AO and BO (Using Distance Formula): - $AO=BO$:

$$(a-5)^2+\left(\frac{a}{4}+2\right)^2=(a-5)^2+\left(\frac{a}{4}-6\right)^2$$

- Solving this gives $a = 8$.

3. Determine Side Lengths of the Triangle: - With $a = 8$: - $AB = 8$, $AC = 6$, $BC = 10$.

4. Calculate Circumradius (α), Area (β), and Perimeter (γ): - Circumradius $\alpha = 5$ - Area $\beta = 24$ - Perimeter $\gamma = 24$

5. Compute $\alpha + \beta + \gamma$:

$$\alpha + \beta + \gamma = 5 + 24 + 24 = 53$$

So, the correct option is : 53

Quick Tip

In geometry problems involving circumcenters and triangle properties, use coordinate geometry and distance formulas to verify relationships and calculate side lengths.

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10. For $x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$, if

$$y(x) = \int \frac{\csc x + \sin x}{\csc x \sec x + \tan x \sin^2 x}\, dx$$

and

$$\lim_{x \to -\frac{\pi}{2}} y(x) = 0$$

then $y\left(\frac{\pi}{4}\right)$ is equal to:

(1) $\tan^{-1}\left(\frac{1}{\sqrt{2}}\right)$
(2) $\frac{1}{2}\tan^{-1}\left(\frac{1}{\sqrt{2}}\right)$
(3) $\frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{1}{\sqrt{2}}\right)$
(4) $\frac{1}{\sqrt{2}}\tan^{-1}\left(-\frac{1}{2}\right)$

Answer: (4) $\frac{1}{\sqrt{2}}\tan^{-1}\left(-\frac{1}{2}\right)$

Solution:

1. Simplify the Integrand: - Rewrite the integrand as:

$$y(x) = \int \frac{(1 + \sin^2 x)\cos x}{1 + \sin^4 x}\, dx$$

- Let $\sin x = t$, so $\cos x\, dx = dt$.

2. Substitute and Integrate: - Substituting $\sin x = t$, we get:

$$y(x) = \int \frac{1+t^2}{t^4+1}\,dt = \frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{t-1}{\sqrt{2}}\right) + C$$

3. Determine the Constant C: - At $x = -\frac{\pi}{2}$, $t = -1$. - Since $\lim_{x\to-\frac{\pi}{2}} y(x) = 0$, we find $C = 0$.

4. Calculate $y\left(\frac{\pi}{4}\right)$: - For $x = \frac{\pi}{4}$, $t = \frac{1}{\sqrt{2}}$. - Thus:

$$y\left(\frac{\pi}{4}\right) = \frac{1}{\sqrt{2}}\tan^{-1}\left(-\frac{1}{2}\right)$$

So, the correct option is : $\frac{1}{\sqrt{2}}\tan^{-1}\left(-\frac{1}{2}\right)$

Quick Tip

For integration problems involving trigonometric substitutions, simplify the integrand with trigonometric identities and apply relevant limits.

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11. If α, with $-\frac{\pi}{2} < \alpha < \frac{\pi}{2}$, is the solution of $4\cos\theta + 5\sin\theta = 1$, then the value of $\tan\alpha$ is:

(1) $\frac{10-\sqrt{10}}{6}$

(2) $\frac{10-\sqrt{10}}{12}$

(3) $\frac{\sqrt{10}-10}{12}$

(4) $\frac{\sqrt{10}-10}{6}$

Answer: (3) $\frac{\sqrt{10}-10}{12}$

Solution:

1. Rewrite the Equation in Terms of $\tan\theta$: - Given $4 + 5\tan\theta = \sec\theta$.

2. Square Both Sides: - Squaring both sides to eliminate $\sec\theta$, we get:

$$24\tan^2\theta + 40\tan\theta + 15 = 0$$

3. Solve for $\tan\theta$: - Solving this quadratic equation, we find:

$$\tan\theta = \frac{-10 \pm \sqrt{10}}{12}$$

4. Choose the Correct Value Based on Range: - Since $-\frac{\pi}{2} < \alpha < \frac{\pi}{2}$, we reject $\tan\alpha = -\left(\frac{10+\sqrt{10}}{12}\right)$ and select:

$$\tan\alpha = \frac{\sqrt{10}-10}{12}$$

So, the correct option is : $\dfrac{\sqrt{10}-10}{12}$

Quick Tip

For trigonometric equations, squaring can simplify expressions but may introduce extraneous solutions. Always check solutions within the given range.

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12. A function $y = f(x)$ satisfies

$$f(x)\sin 2x + \sin x - (1+\cos^2 x)f'(x) = 0$$

with the condition $f(0) = 0$. Then $f\left(\frac{\pi}{2}\right)$ is equal to:

(1) 1

(2) 0

(3) -1

(4) 2

Answer: (1) 1

Solution:

1. Rewrite the Differential Equation:

$$\frac{dy}{dx} - \left(\frac{\sin 2x}{1+\cos^2 x}\right)y = \sin x$$

2. Find the Integrating Factor (I.F.): - The integrating factor is:

$$\text{I.F.} = 1+\cos^2 x$$

3. Solve the Differential Equation: - Multiply through by the integrating factor:

$$y\cdot(1+\cos^2 x) = \int (\sin x)\,dx$$

- Integrate:

$$y\cdot(1+\cos^2 x) = -\cos x + C$$

4. Apply Initial Condition $f(0) = 0$: - At $x = 0$:

$$-\cos 0 + C = 0 \Rightarrow C = 1$$

5. Evaluate $y\left(\frac{\pi}{2}\right)$:

$$y\left(\frac{\pi}{2}\right) = 1$$

So, the correct option is : $\quad 1$

Quick Tip

When solving differential equations, always find the integrating factor first and apply initial conditions to determine the constant.

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13. Let O be the origin and the position vectors of A and B be $2\hat{i}+2\hat{j}+\hat{k}$ and $2\hat{i}+4\hat{j}+4\hat{k}$ respectively. If the internal bisector of $\angle AOB$ meets the line AB at C, then the length of OC is:

(1) $\frac{2}{3}\sqrt{31}$
(2) $\frac{2}{3}\sqrt{34}$
(3) $\frac{3}{4}\sqrt{34}$
(4) $\frac{3}{2}\sqrt{31}$

Answer: (2) $\frac{2}{3}\sqrt{34}$

Solution:

1. Find the Coordinates of Points A and B: - $A = (2, 2, 1)$ and $B = (2, 4, 4)$.

2. Use the Internal Division Formula: - The internal bisector of $\angle AOB$ divides AB in the ratio $OA : OB = 1 : 2$. - Using the section formula, the coordinates of C are:

$$C = \frac{1 \cdot B + 2 \cdot A}{1+2} = \frac{1 \cdot (2,4,4) + 2 \cdot (2,2,1)}{3} = \left(2, \frac{8}{3}, \frac{6}{3}\right) = \left(2, \frac{8}{3}, 2\right)$$

3. Calculate the Length of OC: - The vector OC has coordinates $(2, \frac{8}{3}, 2)$. - Using the distance formula:

$$|OC| = \sqrt{2^2 + \left(\frac{8}{3}\right)^2 + 2^2} = \sqrt{4 + \frac{64}{9} + 4} = \frac{\sqrt{136}}{3} = \frac{2\sqrt{34}}{3}$$

$$\textbf{So, the correct option is :} \quad \frac{2}{3}\sqrt{34}$$

Quick Tip

In vector geometry, use the section formula to find the coordinates of a point that divides a line segment in a given ratio.

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14. Consider the function $f : \left[\frac{1}{2}, 1\right] \to \mathbb{R}$ defined by

$$f(x) = 4\sqrt{2}x^3 - 3\sqrt{2}x - 1$$

Consider the statements:

(I) The curve $y = f(x)$ intersects the x-axis exactly at one point.

(II) The curve $y = f(x)$ intersects the x-axis at $x = \cos\frac{\pi}{12}$.

Then

(1) Only (II) is correct

(2) Both (I) and (II) are incorrect

(3) Only (I) is correct

(4) Both (I) and (II) are correct

Answer: (4) Both (I) and (II) are correct

Solution:

1. Check the Derivative $f'(x)$ for Monotonicity:

$$f'(x) = 12\sqrt{2}x^2 - 3\sqrt{2} \geq 0 \quad \text{for} \quad \left[\frac{1}{2}, 1\right]$$

2. Evaluate $f(x)$ at the Endpoints:

$$f\left(\frac{1}{2}\right) < 0$$

$$f(1) > 0$$

- Since $f(x)$ changes sign from negative to positive, there must be exactly one root in $\left[\frac{1}{2}, 1\right]$, confirming that statement (I) is correct.

3. Check if $x = \cos\frac{\pi}{12}$ is a Root: - Rewrite $f(x)$ in terms of $\cos\alpha$:

$$f(x) = \sqrt{2}(4x^3 - 3x) - 1 = 0$$

- Let $\cos\alpha = x$, then $\cos 3\alpha = x$ gives $\alpha = \frac{\pi}{12}$, so:

$$x = \cos\frac{\pi}{12}$$

- This confirms statement (II) is also correct.

So, the correct option is : Both (I) and (II) are correct

Quick Tip

When analyzing functions over a closed interval, checking for monotonicity and evaluating endpoints can help confirm the number of roots.

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15. Let

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \alpha & \beta \\ 0 & \beta & \alpha \end{bmatrix}$$

and $|2A|^3 = 2^{21}$ where $\alpha, \beta \in \mathbb{Z}$. Then a value of α is:

(1) 3

(2) 5

(3) 17

(4) 9

Answer: (2) 5

Solution:

1. Calculate the Determinant of A:

$$|A| = \alpha^2 - \beta^2$$

2. Use the Condition $|2A|^3 = 2^{21}$: - We know that:

$$|2A| = 2^3|A| = 2^{21} \Rightarrow |A| = 2^4 = 16$$

3. Set Up the Equation:

$$\alpha^2 - \beta^2 = 16$$

- Factor as $(\alpha + \beta)(\alpha - \beta) = 16$.

4. Solve for Possible Values of α: - Possible integer solutions for (α, β) that satisfy the equation give $\alpha = 4$ or $\alpha = 5$.

- Since $\alpha = 5$ satisfies the condition, we choose $\alpha = 5$.

So, the correct option is : 5

Quick Tip

When working with matrix determinants and powers, express conditions in terms of determinant values to simplify calculations.

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16. Let PQR be a triangle with $R(-1, 4, 2)$. Suppose $M(2, 1, 2)$ is the midpoint of PQ. The distance of the centroid of $\triangle PQR$ from the point of intersection of the line

$$\frac{x-2}{0} = \frac{y}{2} = \frac{z+3}{-1} \quad \text{and} \quad \frac{x-1}{1} = \frac{y+3}{-3} = \frac{z+1}{1}$$

is:

(1) 69

(2) 9

(3) $\sqrt{69}$

(4) $\sqrt{99}$

Answer: (3) $\sqrt{69}$

Solution:

1. Find the Centroid G of $\triangle PQR$: - Since $M(2, 1, 2)$ is the midpoint of PQ and $R(-1, 4, 2)$ is the third vertex, the centroid G divides MR in the ratio $1 : 2$. - Using the section formula to find G:

$$G = \left(\frac{1 \cdot (-1) + 2 \cdot 2}{1+2}, \frac{1 \cdot 4 + 2 \cdot 1}{1+2}, \frac{1 \cdot 2 + 2 \cdot 2}{1+2} \right) = (1, 2, 2)$$

2. Find the Point of Intersection A of the Given Lines: - Solving the parametric equations of the lines, we find the point of intersection A to be:

$$A = (2, -6, 0)$$

3. Calculate the Distance AG: - Using the distance formula between points $G(1, 2, 2)$ and

$A(2, -6, 0)$:

$$AG = \sqrt{(2-1)^2 + (-6-2)^2 + (0-2)^2} = \sqrt{1+64+4} = \sqrt{69}$$

So, the correct option is : $\sqrt{69}$

Quick Tip

For geometry problems involving centroids and intersections, use the section formula for division points and apply the distance formula to find distances between points.

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17. Let R be a relation on $\mathbb{Z} \times \mathbb{Z}$ defined by

$(a, b)R(c, d)$ if and only if $ad - bc$ is divisible by 5.

Then R is:

(1) Reflexive and symmetric but not transitive

(2) Reflexive but neither symmetric nor transitive

(3) Reflexive, symmetric and transitive

(4) Reflexive and transitive but not symmetric

Answer: (1) Reflexive and symmetric but not transitive

Solution:

1. Check for Reflexivity: - For any (a, b), we have $(a, b)R(a, b)$ since $ad - bc = ab - ab = 0$, which is divisible by 5. - Therefore, R is reflexive.

2. Check for Symmetry: - Suppose $(a, b)R(c, d)$, meaning $ad - bc$ is divisible by 5. - Then $bc - ad = -(ad - bc)$ is also divisible by 5, implying $(c, d)R(a, b)$. - Therefore, R is symmetric.

3. Check for Transitivity: - Consider $(3, 1)R(10, 5)$ and $(10, 5)R(1, 1)$. - However, $(3, 1)$ is not related to $(1, 1)$, so transitivity does not hold. - Therefore, R is not transitive.

So, the correct option is : Reflexive and symmetric but not transitive

Quick Tip

To verify properties of a relation, check reflexivity, symmetry, and transitivity separately by examining the definitions and using counterexamples where necessary.

18. If the value of the integral

$$\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \left(\frac{x^2 \cos x}{1+\pi^x} + \frac{1+\sin^2 x}{1+e^{\sin^x 2023}} \right) dx = \frac{\pi}{4}(\pi + a) - 2,$$

then the value of a is:

(1) 3

(2) $-\frac{3}{2}$

(3) 2

(4) $\frac{3}{2}$

Answer: (1) 3

Solution:

1. Set Up the Integral I:

$$I = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \left(\frac{x^2 \cos x}{1+\pi^x} + \frac{1+\sin^2 x}{1+e^{\sin^x 2023}} \right) dx$$

2. Use Symmetry to Simplify: - Notice that the integrand has symmetry properties, allowing us to split the integral and add:

$$I = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \left(\frac{x^2 \cos x}{1+\pi^x} + \frac{1+\sin^2 x}{1+e^{\sin^{2023}(-x)}} \right) dx$$

3. Combine Integrals: - Adding the two integrals results in:

$$2I = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \left(x^2 \cos x + 1 + \sin^2 x \right) dx$$

4. Evaluate the Integral: - Solving this integral gives:

$$I = \frac{\pi^2}{4} + \frac{3\pi}{4} - 2$$

5. Determine a: - From the given equation, equate terms to find $a = 3$.

So, the correct option is : $a = 3$

Quick Tip

For integrals with symmetric limits, use properties of even and odd functions to simplify the integrand, and consider adding complementary integrals.

19. Suppose

$$f(x)=\frac{\left(2^{x}+2^{-x}\right)\tan x\sqrt{\tan^{-1}\left(x^{2}-x+1\right)}}{\left(7x^{2}+3x+1\right)^{3}},$$

then the value of $f'(0)$ is equal to:

(1) π

(2) 0

(3) $\sqrt{\pi}$

(4) $\frac{\pi}{2}$

Answer: (3) $\sqrt{\pi}$

Solution:

1. Calculate $f'(0)$ Using the Definition of Derivative:

$$f'(0)=\lim_{h\to 0}\frac{f(h)-f(0)}{h}$$

2. Evaluate $f(h)-f(0)$: - Substitute $x=h$ and $x=0$ into $f(x)$:

$$f'(0)=\lim_{h\to 0}\frac{\left(2^{h}+2^{-h}\right)\tan h\sqrt{\tan^{-1}\left(h^{2}-h+1\right)}-0}{\left(7h^{2}+3h+1\right)^{3}\cdot h}$$

3. Simplify the Expression: - Using the limit properties, we get:

$$f'(0)=\sqrt{\pi}$$

So, the correct option is : $\sqrt{\pi}$

Quick Tip

When calculating derivatives at a point using limits, simplify the expression by substituting small values and evaluating the behavior as the limit approaches zero.

20. Let A be a square matrix such that $AA^{\top}=I$. Then

$$\frac{1}{2}A\left[\left(A+A^{\top}\right)^{2}+\left(A-A^{\top}\right)^{2}\right]$$

is equal to:

(1) $A^2 + I$

(2) $A^3 + I$

(3) $A^2 + A^\top$

(4) $A^3 + A^\top$

Answer: (4) $A^3 + A^\top$

Solution:

1. Use the Condition $AA^\top = I = A^\top A$: - Given that $AA^\top = I$, we can substitute this property in the expression.

2. Expand the Expression:

$$\frac{1}{2}A\left[\left(A + A^\top\right)^2 + \left(A - A^\top\right)^2\right]$$

Expanding $\left(A + A^\top\right)^2$ and $\left(A - A^\top\right)^2$, we get:

$$= \frac{1}{2}A\left[A^2 + (A^\top)^2 + 2AA^\top + A^2 + (A^\top)^2 - 2AA^\top\right]$$

$$= A\left[A^2 + (A^\top)^2\right]$$

3. Simplify the Result:

$$= A^3 + A^\top$$

So, the correct option is : $\quad A^3 + A^\top$

Quick Tip

In matrix expressions involving transposes and identities, use matrix properties like $AA^\top = I$ to simplify complex expressions.

---

21. Equation of two diameters of a circle are $2x - 3y = 5$ and $3x - 4y = 7$. The line joining the points $\left(-\frac{22}{7}, -4\right)$ and $\left(\frac{1}{7}, 3\right)$ intersects the circle at only one point $P(\alpha, \beta)$. Then $17\beta - \alpha$ is equal to:

(1) 3

(2) 2

(3) 4

(4) -1

Answer: (2) 2

Solution:

1. Find the Centre of the Circle: - The centre C of the circle is the intersection of the diameters $2x - 3y = 5$ and $3x - 4y = 7$. - Solving these equations, we get $C(1, -1)$.

2. Equation of Line AB: - The points $A\left(-\frac{22}{7}, -4\right)$ and $B\left(\frac{1}{7}, 3\right)$ lie on the line AB. - The equation of AB is:

$$7x - 3y + 10 = 0 \quad \text{(i)}$$

3. Equation of Line CP: - Since P lies on the circle, CP is perpendicular to AB with the equation:

$$3x + 7y + 4 = 0 \quad \text{(ii)}$$

4. Solve for α and β: - Solving equations (i) and (ii), we find:

$$\alpha = -\frac{41}{29}, \quad \beta = \frac{1}{29}$$

5. Calculate $17\beta - \alpha$:

$$17\beta - \alpha = 17 \cdot \frac{1}{29} + \frac{41}{29} = 2$$

So, the correct option is : 2

Quick Tip

For problems involving lines and circles, find intersection points using simultaneous equations and use symmetry properties where applicable.

---

22. All the letters of the word "GTWENTY" are written in all possible ways with or without meaning, and these words are arranged as in a dictionary. The serial number of the word "GTWENTY" is:

Answer: 553

Solution:

1. Calculate the Number of Words Starting with Each Letter: - Words starting with E: $6! = 720/2 = 360$ - Words starting with G and second letter E: $5! = 120/2 = 60$ - Words

starting with G and second letter N: $5! = 120/2 = 60$ - Words starting with GTE: $4! = 24$ - Words starting with GTN: $4! = 24$ - Words starting with GTT: $4! = 24$

2. Add the Serial Position of "GTWENTY": - "GTWENTY" itself contributes $+1$.

3. Total Serial Number of "GTWENTY":

$$360 + 60 + 60 + 24 + 24 + 24 + 1 = 553$$

So, the correct option is : 553

Quick Tip

To find the dictionary position of a word, calculate the possible arrangements starting with each preceding letter.

---

24. Let $f(x) = 2^x - x^2$, $x \in \mathbb{R}$. If m and n are respectively the number of points at which the curves $y = f(x)$ and $y = f'(x)$ intersect the x-axis, then the value of $m + n$ is:

Answer: 5

Solution:

1. Find Points where $y = f(x)$ Intersects the x-axis: - For $f(x) = 2^x - x^2 = 0$, we determine the values of x where this equation holds. - This equation intersects the x-axis at three points, so $m = 3$.

2. Find Points where $y = f'(x)$ Intersects the x-axis: - The derivative $f'(x) = 2^x \ln 2 - 2x$. - Setting $f'(x) = 0$, we solve $2^x \ln 2 = 2x$ to find the points where this equation intersects the x-axis. - This yields two intersection points, so $n = 2$.

3. Calculate $m + n$:

$$m + n = 3 + 2 = 5$$

So, the correct option is : 5

Quick Tip

For functions involving exponentials and polynomials, analyze intersections by setting the function and its derivative equal to zero and solving separately.

25. If the points of intersection of two distinct conics $x^2 + y^2 = 4b$ and $\frac{x^2}{16} + \frac{y^2}{b^2} = 1$ lie on the curve $y^2 = 3x^2$, then $3\sqrt{3}$ times the area of the rectangle formed by the intersection points is __.

Answer: 432

Solution:

1. Substitute $y^2 = 3x^2$ in Both Conics: - From $y^2 = 3x^2$, substitute into $x^2 + y^2 = 4b$ and $\frac{x^2}{16} + \frac{y^2}{b^2} = 1$ to find values of b.

2. Solve for b: - By substituting, we get:

$$x^2 = b \quad \text{and} \quad \frac{b}{16} + \frac{3}{b} = 1$$

- Solving this equation gives $b = 4$ or $b = 12$. - Since $b = 4$ makes the curves coincide, we reject it, so $b = 12$.

3. Find Points of Intersection: - With $b = 12$, the points of intersection are $\left(\pm\sqrt{12}, \pm 6\right)$.

4. Calculate the Area of the Rectangle: - The area of the rectangle formed by these points is:

$$\text{Area} = 2 \cdot \sqrt{12} \times 2 \cdot 6 = 4 \cdot \sqrt{12} \cdot 6 = 432$$

So, the correct option is : 432

Quick Tip

For intersection points of conics, substitute auxiliary conditions and solve for possible values, verifying the conditions for distinct intersection.

---

26. If the solution curve $y = y(x)$ of the differential equation $(1 + y^2)(1 + \log_e x)\,dx + x\,dy = 0$, $x > 0$, passes through the point $(1, 1)$ and

$$y(e) = \frac{\alpha - \tan\left(\frac{3}{2}\right)}{\beta + \tan\left(\frac{3}{2}\right)},$$

then $\alpha + 2\beta$ is:

Answer: 3

Solution:

1. Separate Variables in the Differential Equation:

$$\int \left(\frac{1}{x} + \frac{\ln x}{x} \right) dx + \int \frac{dy}{1+y^2} = 0$$

2. Integrate Both Sides: - Integrating, we get:

$$\ln x + \frac{(\ln x)^2}{2} + \tan^{-1} y = C$$

3. Apply Initial Condition $(x, y) = (1, 1)$: - Substitute $x = 1$ and $y = 1$ to find C:

$$\ln 1 + \frac{(\ln 1)^2}{2} + \tan^{-1}(1) = C \Rightarrow C = \frac{\pi}{4}$$

4. Rewrite the Solution with $C = \frac{\pi}{4}$:

$$\ln x + \frac{(\ln x)^2}{2} + \tan^{-1} y = \frac{\pi}{4}$$

5. Evaluate $y(e)$: - Substitute $x = e$ into the equation:

$$\ln e + \frac{(\ln e)^2}{2} + \tan^{-1} y = \frac{\pi}{4}$$

$$1 + \frac{1}{2} + \tan^{-1} y = \frac{\pi}{4}$$

- Solving for y, we get:

$$y = \tan\left(\frac{\pi}{4} - \frac{3}{2} \right) = \frac{1 - \tan\frac{3}{2}}{1 + \tan\frac{3}{2}}$$

6. Identify α and β: - Comparing with the given expression for $y(e)$, we find $\alpha = 1$ and $\beta = 1$.

7. Calculate $\alpha + 2\beta$:

$$\alpha + 2\beta = 1 + 2 \cdot 1 = 3$$

So, the correct option is : 3

Quick Tip

When solving differential equations with initial conditions, use the separation of variables and integration constants to simplify and evaluate the solution at specific points.

---

27. If the mean and variance of the data $65, 68, 58, 44, 48, 45, 60, \alpha, \beta, 60$ **where** $\alpha > \beta$ **are 56 and 66.2 respectively, then** $\alpha^2 + \beta^2$ **is equal to:**

Answer: 6344

Solution:

1. Calculate the Mean $\bar{x}$: - Given that the mean $\bar{x} = 56$.

2. Calculate the Variance σ^2: - Given that the variance $\sigma^2 = 66.2$.

3. Set up the Equations for α and β: - Using the formula for variance with mean and variance values:

$$\frac{\alpha^2 + \beta^2 + 25678}{10} - (56)^2 = 66.2$$

4. Solve for $\alpha^2 + \beta^2$: - Rearranging, we find:

$$\alpha^2 + \beta^2 = 6344$$

So, the correct option is : 6344

Quick Tip

For statistics problems, use the mean and variance formulas directly, especially when dealing with unknown values in the dataset.

---

28. The area (in sq. units) of the part of the circle $x^2 + y^2 = 169$ which is below the line $5x - y = 13$ is

$$\frac{\pi\alpha}{2\beta} \cdot \frac{65}{2} + \frac{\alpha}{\beta} \sin^{-1}\left(\frac{12}{13}\right)$$

where α, β are coprime numbers. Then $\alpha + \beta$ is equal to:

Answer: 171

Solution:

1. Identify the Circle and Line Equation: - The circle is given by $x^2 + y^2 = 169$, which has a radius of $\sqrt{169} = 13$. - The line equation $5x - y = 13$ intersects the circle, creating a segment.

2. Determine Points of Intersection: - The line intersects the circle at points $(5, 12)$ and $(0, -13)$, as shown in the solution diagram.

3. Calculate the Area Below the Line: - The area of the segment below the line is calculated by integrating from $y = -13$ to $y = 12$:

$$\text{Area} = \int_{-13}^{12} \sqrt{169 - y^2}\, dy - \frac{1}{2} \times 25 \times 5$$

4. Simplify the Result: - After integrating, we get:

$$\text{Area} = \frac{\pi}{2} \cdot \frac{169}{2} - \frac{65}{2} + \frac{169}{2} \sin^{-1} \frac{12}{13}$$

5. Determine α and β: - Comparing terms, we find $\alpha = 169$ and $\beta = 2$.

6. Calculate $\alpha + \beta$:

$$\alpha + \beta = 169 + 2 = 171$$

So, the correct option is : 171

Quick Tip

When calculating the area of a segment in a circle cut by a line, integrate the function for the semicircle and subtract any additional areas if needed.

---

29. If

$$\binom{11}{1}\binom{11}{2} + \binom{11}{2}\binom{11}{3} + \ldots + \binom{11}{9}\binom{11}{10} = \frac{n}{m} \quad \text{with } \gcd(n, m) = 1,$$

then $n + m$ is equal to:

Answer: 2041

Solution:

1. Rewrite the Sum in Terms of a Series: - The sum can be expressed as:

$$\sum_{r=1}^{9} \binom{11}{r}\binom{11}{r+1}$$

2. Simplify the Series Using Combinatorial Identities: - This can be simplified by recognizing a pattern and using properties of binomial coefficients:

$$= \frac{1}{12} \sum_{r=1}^{9} \binom{12}{r+1}$$

- Further simplifying, we get:

$$= \frac{1}{12} \left[2^{12} - 26\right] = \frac{2035}{6}$$

3. Determine n and m: - From the simplified result, $n = 2035$ and $m = 6$, with $\gcd(n, m) = 1$.

4. Calculate $n+m$:

$$n+m = 2035 + 6 = 2041$$

So, the correct option is : 2041

Quick Tip

For problems involving binomial coefficients in series, look for combinatorial identities and symmetries to simplify the expressions.

---

30. A line with direction ratios $2,1,2$ meets the lines $x = y+2 = z$ and $x+2 = 2y = 2z$ respectively at the points P and Q. If the length of the perpendicular from the point $(1,2,12)$ to the line PQ is l, then l^2 is:

Answer: 65

Solution:

1. Find Points P and Q: - Let $P(t, t-2, t)$ and $Q(2s-2, s, s)$ be the points where the line with direction ratios $2,1,2$ meets the given lines.

2. Set Up Equations for Direction Ratios of PQ: - The direction ratios (D.R.) of PQ are $2,1,2$. - Equating components:

$$\frac{2s-2-t}{2} = \frac{s-t+2}{1} = \frac{s-t}{2}$$

- Solving these equations, we find $t = 6$ and $s = 2$.

3. Determine Coordinates of P and Q: - Substitute $t = 6$: $P(6,4,6)$. - Substitute $s = 2$: $Q(2,2,2)$.

4. Equation of Line PQ: - The line PQ can be written as:

$$\frac{x-2}{2} = \frac{y-2}{1} = \frac{z-2}{2} = \lambda$$

5. Find the Foot of Perpendicular F from $A(1,2,12)$ to PQ: - Let $F(2\lambda+2, \lambda+2, 2\lambda+2)$ be the foot of the perpendicular. - Since $\overrightarrow{AF} \cdot \overrightarrow{PQ} = 0$, solving gives $\lambda = 2$.

6. Calculate AF: - The coordinates of F are $(6,4,6)$. - Distance $AF = \sqrt{(6-1)^2 + (4-2)^2 + (6-12)^2}$ $\sqrt{65}$.

7. Find l^2:

$$l^2 = 65$$

So, the correct option is : 65

Quick Tip

For finding perpendicular distances from a point to a line in 3D, use the foot of the perpendicular and solve using the dot product condition.

---

31. In the given circuit, the breakdown voltage of the Zener diode is 3.0 V. What is the value of I_Z?

(1) 3.3 mA

(2) 5.5 mA

(3) 10 mA

(4) 7 mA

Answer: (2) 5.5 mA

Solution:

1. Identify the Zener Breakdown Voltage: - Given $V_Z = 3$ V.

2. Determine the Potentials in the Circuit: - Let the potential at point $B = 0$ V. - The potential at $E(V_E) = 10$ V. - Therefore, $V_C = V_A = 3$ V across the Zener diode.

3. Calculate the Total Current I: - Using Ohm's law, the total current I through the 1 kΩ resistor:

$$I = \frac{10-3}{1000} = \frac{7}{1000} = 7 \text{ mA}$$

4. Find I_1, the Current through the 2 kΩ Resistor: - Voltage across the 2 kΩ resistor is 3 V:

$$I_1 = \frac{3}{2000} = 1.5 \text{ mA}$$

5. Calculate I_Z, the Zener Current: - By Kirchhoff's Current Law (KCL), $I = I_Z + I_1$:

$$I_Z = I - I_1 = 7 \text{ mA} - 1.5 \text{ mA} = 5.5 \text{ mA}$$

So, the correct option is : 5.5 mA

Quick Tip

For circuits with Zener diodes, use the breakdown voltage to find the voltage drop and apply Ohm's law to find currents through each branch.

---

32. The electric current through a wire varies with time as $I = I_0 + \beta t$, where $I_0 = 20\,\mathbf{A}$ and $\beta = 3\,$A/s. The amount of electric charge that crosses through a section of the wire in 20 s is:

(1) $80\,$C

(2) $1000\,$C

(3) $800\,$C

(4) $1600\,$C

Answer: (2) $1000\,$**C**

Solution:

1. Express Current as a Function of Time:

$$I = I_0 + \beta t = 20 + 3t$$

2. Calculate Charge q Over Time: - The current $I = \frac{dq}{dt}$, so we can write:

$$dq = (20 + 3t)\,dt$$

3. Integrate to Find Total Charge q from $t = 0$ to $t = 20$:

$$q = \int_0^{20} (20 + 3t)\,dt$$

- Split the integral:

$$q = \int_0^{20} 20\,dt + \int_0^{20} 3t\,dt$$

4. Evaluate Each Integral:

$$q = [20t]_0^{20} + \left[\frac{3t^2}{2}\right]_0^{20}$$

$$= (20 \times 20) + \frac{3 \times 20^2}{2}$$

$$= 400 + \frac{3 \times 400}{2} = 400 + 600 = 1000\,\mathrm{C}$$

So, the correct option is : $1000\,\mathrm{C}$

Quick Tip

For time-varying currents, integrate $I(t)$ over the time interval to find the total charge passed.

---

33. Given below are two statements:

Statement I: If a capillary tube is immersed first in cold water and then in hot water, the height of capillary rise will be smaller in hot water.

Statement II: If a capillary tube is immersed first in cold water and then in hot water, the height of capillary rise will be smaller in cold water.

In the light of the above statements, choose the most appropriate option from the options given below:

(1) Both Statement I and Statement II are true

(2) Both Statement I and Statement II are false

(3) Statement I is true but Statement II is false

(4) Statement I is false but Statement II is true

Answer: (3) Statement I is true but Statement II is false

Solution:

1. Understanding Capillary Rise: - The height of capillary rise h is given by:

$$h = \frac{2T\cos\theta}{\rho g r}$$

- Where T is the surface tension, θ is the contact angle, ρ is the density of the liquid, g is the acceleration due to gravity, and r is the radius of the capillary tube.

2. Effect of Temperature on Surface Tension: - Surface tension T decreases as the temperature increases. - Since h is directly proportional to T, a decrease in T will lead to a decrease in the height of capillary rise.

3. Analyze Statements: - Statement I is correct because in hot water (higher temperature), the surface tension is lower, resulting in a smaller height of capillary rise. - Statement II is

incorrect because the height of capillary rise is actually larger in cold water due to higher surface tension.

So, the correct option is : (3) Statement I is true but Statement II is false

Quick Tip

The height of capillary rise decreases with an increase in temperature due to the reduction in surface tension.

---

34. A convex mirror of radius of curvature 30 cm forms an image that is half the size of the object. The object distance is:

(1) $-15\,\text{cm}$
(2) $45\,\text{cm}$
(3) $-45\,\text{cm}$
(4) $15\,\text{cm}$

Answer: (1) -15 **cm**

Solution:

1. Given Parameters: - Radius of curvature $R = 30\,\text{cm}$. - For a mirror, focal length $f = \frac{R}{2} = +15\,\text{cm}$ (positive for a convex mirror).

2. Use the Magnification Formula for Mirrors: - Given that the image is half the size of the object, the magnification $m = \frac{1}{2}$. - For a convex mirror, a virtual image is formed for a real object, so m is positive:

$$m = +\frac{1}{2}$$

3. Apply the Mirror Formula: - The magnification formula is $m = \frac{f}{f-u}$. - Substitute $m = \frac{1}{2}$ and $f = 15\,\text{cm}$:

$$\frac{1}{2} = \frac{15}{15-u}$$

4. Solve for u:

$$15 - u = 30 \Rightarrow u = -15\,\text{cm}$$

So, the correct option is : $-15\,\text{cm}$

Quick Tip

For convex mirrors, a positive magnification implies a virtual image. Use the mirror formula and given magnification to solve for object distance.

---

35. Two charges of $5Q$ and $-2Q$ are situated at the points $(3a, 0)$ and $(-5a, 0)$ respectively. The electric flux through a sphere of radius $4a$ having its center at the origin is:

(1) $\frac{2Q}{\varepsilon_0}$
(2) $\frac{5Q}{\varepsilon_0}$
(3) $\frac{7Q}{\varepsilon_0}$
(4) $\frac{3Q}{\varepsilon_0}$

Answer: (2) $\frac{5Q}{\varepsilon_0}$

Solution:

1. Analyze the Position of Charges Relative to the Sphere: - A sphere of radius $4a$ centered at the origin includes the charge $5Q$ located at $(3a, 0)$ since $3a < 4a$. - The charge $-2Q$ at $(-5a, 0)$ lies outside the sphere since $5a > 4a$.

2. Apply Gauss's Law: - According to Gauss's law, the electric flux Φ through a closed surface depends only on the net charge enclosed by the surface:

$$\Phi = \frac{q_{\text{enc}}}{\varepsilon_0}$$

- Since only the $5Q$ charge is inside the sphere, the enclosed charge $q_{\text{enc}} = 5Q$.

3. Calculate the Electric Flux:

$$\Phi = \frac{5Q}{\varepsilon_0}$$

So, the correct option is : $\frac{5Q}{\varepsilon_0}$

Quick Tip

When calculating flux through a Gaussian surface, consider only the charges enclosed within the surface. External charges do not contribute to the net flux.

36. A body starts moving from rest with constant acceleration and covers displacement S_1 in the first $(p-1)$ seconds and S_2 in the first p seconds. The displacement $S_1 + S_2$ will be made in time:

(1) $(2p + 1)$ s

(2) $\sqrt{2p^2 - 2p + 1}$ s

(3) $(2p - 1)$ s

(4) $(2p^2 - 2p + 1)$ s

Answer: (2) $\sqrt{2p^2 - 2p + 1}$ **s**

Solution:

1. Calculate S_1 in the First $(p - 1)$ Seconds: - Since the body starts from rest, using the formula $S = \frac{1}{2}at^2$,

$$S_1 = \frac{1}{2}a(p-1)^2$$

2. Calculate S_2 in the First p Seconds: - Using the same formula for S_2,

$$S_2 = \frac{1}{2}ap^2$$

3. Total Displacement $S_1 + S_2$: - If $S_1 + S_2$ represents the displacement in time t, then:

$$S_1 + S_2 = \frac{1}{2}at^2$$

- Substitute S_1 and S_2 values:

$$\frac{1}{2}a(p-1)^2 + \frac{1}{2}ap^2 = \frac{1}{2}at^2$$

- Simplify by canceling $\frac{1}{2}a$:

$$(p-1)^2 + p^2 = t^2$$

4. Solve for t:

$$t = \sqrt{2p^2 - 2p + 1}$$

So, the correct option is : $\sqrt{2p^2 - 2p + 1}$ s

Quick Tip

For uniformly accelerated motion, use the displacement formula $S = \frac{1}{2}at^2$ and set up equations based on time intervals to find total displacement.

37. The potential energy function (in J) of a particle in a region of space is given as $U = (2x^2 + 3y^3 + 2z)$. Here x, y, and z are in meters. The magnitude of the x-component of force (in N) acting on the particle at point $P(1, 2, 3)$ m is:

(1) 2

(2) 6

(3) 4

(4) 8

Answer: (3) 4

Solution:

1. Given Potential Energy Function:

$$U = 2x^2 + 3y^3 + 2z$$

2. Calculate the x-Component of Force: - The force in the x-direction is given by $F_x = -\frac{\partial U}{\partial x}$. - Differentiating U with respect to x:

$$F_x = -\frac{\partial}{\partial x}(2x^2) = -4x$$

3. Evaluate F_x at $x = 1$: - Substitute $x = 1$:

$$F_x = -4 \times 1 = -4$$

- The magnitude of F_x is $4\,\text{N}$.

$$\textbf{So, the correct option is :} \quad 4\,\text{N}$$

Quick Tip

The force in a specific direction is the negative gradient of the potential energy with respect to that coordinate. Calculate the derivative and evaluate at the given point for the component force.

38. The resistance $R = \frac{V}{I}$ where $V = (200 \pm 5)$ V and $I = (20 \pm 0.2)$ A. The percentage error in the measurement of R is:

(1) 3.5%

(2) 7%

(3) 3%

(4) 5.5%

Answer: (1) 3.5%

Solution:

1. Express R in Terms of V and I:

$$R = \frac{V}{I}$$

2. Calculate the Percentage Error Using Error Analysis: - The relative error in R is given by:

$$\frac{\Delta R}{R} = \frac{\Delta V}{V} + \frac{\Delta I}{I}$$

- Substitute the values:

$$\frac{\Delta R}{R} = \frac{5}{200} + \frac{0.2}{20}$$

3. Simplify the Expression:

$$\frac{\Delta R}{R} = \frac{5}{200} + \frac{0.2}{20} = \frac{5}{200} + \frac{2}{200} = \frac{7}{200}$$

4. Calculate the Percentage Error:

$$\text{Percentage Error} = \frac{\Delta R}{R} \times 100 = \frac{7}{200} \times 100 = 3.5\%$$

So, the correct option is : 3.5%

Quick Tip

For functions involving division, the relative error in the result is the sum of the relative errors of the quantities involved.

---

39. A block of mass 100 kg slides over a distance of 10 m on a horizontal surface. If the coefficient of friction between the surfaces is 0.4, then the work done against friction (in J) is:

(1) $4200\,$J

(2) $3900\,\text{J}$

(3) $4000\,\text{J}$

(4) $4500\,\text{J}$

Answer: (3) $4000\,\textbf{J}$

Solution:

1. Given Data: - Mass $m = 100\,\text{kg}$ - Distance $s = 10\,\text{m}$ - Coefficient of friction $\mu = 0.4$
2. Calculate the Frictional Force f: - The frictional force is given by $f = \mu mg$,

$$f = 0.4 \times 100 \times 10 = 400\,\text{N}$$

3. Calculate the Work Done Against Friction W: - Work done $W = f \times s$,

$$W = 400 \times 10 = 4000\,\text{J}$$

So, the correct option is : $4000\,\text{J}$

Quick Tip

The work done against friction is the product of the frictional force and the distance moved. Use $W = f \times s$ where $f = \mu mg$.

---

40. Match List I with List II

List I	List II
A. $\oint \vec{B} \cdot d\vec{l} = \mu_0 i_c + \mu_0 \varepsilon_0 \frac{d\Phi_E}{dt}$	I. Gauss' law for electricity
B. $\oint \vec{E} \cdot d\vec{l} = -\frac{d\Phi_B}{dt}$	II. Gauss' law for magnetism
C. $\oint \vec{E} \cdot d\vec{A} = \frac{Q}{\varepsilon_0}$	III. Faraday law
D. $\oint \vec{B} \cdot d\vec{A} = 0$	IV. Ampere–Maxwell law

Choose the correct answer from the options given below:

(1) A-IV, B-I, C-III, D-II

(2) A-II, B-III, C-I, D-IV

(3) A-IV, B-III, C-I, D-II

(4) A-I, B-II, C-III, D-IV

Answer: (3) A-IV, B-III, C-I, D-II

Solution:

1. Ampere–Maxwell Law: - $\oint \vec{B} \cdot d\vec{l} = \mu_0 i_c + \mu_0 \varepsilon_0 \frac{d\Phi_E}{dt}$ - This matches with A-IV.

2. Faraday's Law of Electromagnetic Induction: - $\oint \vec{E} \cdot d\vec{l} = -\frac{d\Phi_B}{dt}$ - This matches with B-III.

3. Gauss' Law for Electricity: - $\oint \vec{E} \cdot d\vec{A} = \frac{Q}{\varepsilon_0}$ - This matches with C-I.

4. Gauss' Law for Magnetism: - $\oint \vec{B} \cdot d\vec{A} = 0$ - This matches with D-II.

So, the correct option is : (3) A-IV, B-III, C-I, D-II

Quick Tip

When matching laws, remember that Ampere–Maxwell law relates magnetic fields to electric fields and currents, Faraday's law deals with the induced electric field, and Gauss's laws are for electric and magnetic flux.

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41. If the radius of curvature of the path of two particles of the same mass are in the ratio 3:4, then in order to have constant centripetal force, their velocities will be in the ratio of:

(1) $\sqrt{3} : 2$

(2) $1 : \sqrt{3}$

(3) $\sqrt{3} : 1$

(4) $2 : \sqrt{3}$

Answer: (1) $\sqrt{3} : 2$

Solution:

1. Given Data: - Masses $m_1 = m_2$ - Radius ratio $\frac{r_1}{r_2} = \frac{3}{4}$

2. Use the Centripetal Force Formula: - Centripetal force $F = \frac{mv^2}{r}$. - Since the centripetal force is constant, $F_1 = F_2$:

$$\frac{m_1 v_1^2}{r_1} = \frac{m_2 v_2^2}{r_2}$$

3. Simplify the Equation: - With $m_1 = m_2$, we get:

$$\frac{v_1^2}{r_1} = \frac{v_2^2}{r_2}$$

$$\Rightarrow \frac{v_1}{v_2} = \sqrt{\frac{r_1}{r_2}} = \sqrt{\frac{3}{4}} = \frac{\sqrt{3}}{2}$$

So, the correct option is : $\sqrt{3} : 2$

Quick Tip

For particles with constant centripetal force, the ratio of their velocities is the square root of the inverse ratio of their radii.

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42. A galvanometer having coil resistance 10 Ω shows a full scale deflection for a current of 3 mA. For it to measure a current of 8 A, the value of the shunt should be:

(1) $3 \times 10^{-3}\,\Omega$

(2) $4.85 \times 10^{-3}\,\Omega$

(3) $3.75 \times 10^{-3}\,\Omega$

(4) $2.75 \times 10^{-3}\,\Omega$

Answer: (3) $3.75 \times 10^{-3}\,\Omega$

Solution:

1. Given Data: - Galvanometer resistance $G = 10\,\Omega$ - Full-scale deflection current $I_g = 3\,\text{mA} = 3 \times 10^{-3}\,\text{A}$ - Desired current to be measured $I = 8\,\text{A}$

2. Calculate the Shunt Resistance S: - In order to convert the galvanometer into an ammeter, the shunt resistance S is given by:

$$S = \frac{I_g\,G}{I - I_g}$$

3. Substitute the Values:

$$S = \frac{(3 \times 10^{-3}) \times 10}{8 - 0.003}$$

$$S = \frac{0.03}{7.997} \approx 3.75 \times 10^{-3}\,\Omega$$

So, the correct option is : $3.75 \times 10^{-3}\,\Omega$

Quick Tip

To convert a galvanometer into an ammeter, use the formula for shunt resistance $S = \frac{I_g G}{I - I_g}$ where I is the desired current and I_g is the full-scale deflection current.

---

43. The de-Broglie wavelength of an electron is the same as that of a photon. If the velocity of the electron is 25% of the velocity of light, then the ratio of the K.E. of the electron to the K.E. of the photon will be:

(1) $\frac{1}{1}$

(2) $\frac{1}{8}$

(3) 8 : 1

(4) $\frac{1}{4}$

Answer: (2) $\frac{1}{8}$

Solution:

1. For the Photon: - The energy of a photon E_p is given by:

$$E_p = \frac{hc}{\lambda_p}$$

- Rearranging for wavelength λ_p, we get:

$$\lambda_p = \frac{hc}{E_p}$$

2. For the Electron: - The de-Broglie wavelength of an electron is given by:

$$\lambda_e = \frac{h}{m_e v_e}$$

- The kinetic energy K_e of the electron is related to its velocity by:

$$K_e = \frac{1}{2} m_e v_e^2$$

- Rearranging, the velocity v_e can be expressed as:

$$v_e = \sqrt{\frac{2K_e}{m_e}}$$

3. Equating Wavelengths: - Since the de-Broglie wavelength of the electron is the same as that of the photon, we equate λ_p and λ_e:

$$\frac{hc}{E_p} = \frac{h}{m_e v_e}$$

- Simplifying, we get:

$$E_p = m_e v_e c$$

4. Express v_e in Terms of c: - We are given that $v_e = 0.25c$. - Substitute $v_e = 0.25c$ into the expression for E_p:

$$E_p = m_e(0.25c)c = 0.25m_e c^2$$

5. Calculate the Ratio of Kinetic Energies: - The kinetic energy of the electron is:

$$K_e = \frac{1}{2}m_e v_e^2 = \frac{1}{2}m_e(0.25c)^2 = \frac{1}{2}m_e \cdot 0.0625c^2 = 0.03125m_e c^2$$

- Now, take the ratio $\frac{K_e}{E_p}$:

$$\frac{K_e}{E_p} = \frac{0.03125m_e c^2}{0.25m_e c^2} = \frac{0.03125}{0.25} = \frac{1}{8}$$

So, the correct option is : $\frac{1}{8}$

Quick Tip

When comparing kinetic energies using de-Broglie wavelengths, relate the wavelength expressions for particles and photons, and solve for the ratio based on given velocities.

---

44. The deflection in a moving coil galvanometer falls from 25 divisions to 5 divisions when a shunt of $24\,\Omega$ is applied. The resistance of the galvanometer coil will be:

(1) $12\,\Omega$

(2) $96\,\Omega$

(3) $48\,\Omega$

(4) $100\,\Omega$

Answer: (2) $96\,\Omega$

Solution:

1. Define Variables: - Let x represent the current per division. - Initially, the full-scale deflection current I_g for 25 divisions is:

$$I_g = 25x$$

2. After Applying the Shunt: - With the shunt applied, the deflection drops to 5 divisions, so the new current through the galvanometer becomes:

$$I_g = 5x$$

- The remaining current bypasses through the shunt, giving the total current I:

$$I = 25x$$

- The total current I is divided between the galvanometer and the shunt. So, the current through the shunt is:

$$I - I_g = 25x - 5x = 20x$$

3. Using the Shunt Resistance: - The shunt resistance $S = 24\,\Omega$. - Since the potential difference across the galvanometer and the shunt must be equal, we have:

$$I_g \cdot G = (I - I_g) \cdot S$$

- Substituting $I_g = 5x$, $I - I_g = 20x$, and $S = 24\,\Omega$:

$$5x \cdot G = 20x \cdot 24$$

4. Solving for G: - Cancel x from both sides:

$$5G = 20 \times 24$$

- Simplify:

$$G = \frac{20 \times 24}{5} = 4 \times 24 = 96\,\Omega$$

So, the correct option is : $\boxed{96\,\Omega}$

Quick Tip

When a shunt is applied to a galvanometer, use the condition of equal voltage across the galvanometer and shunt to calculate unknown resistances.

---

45. A biconvex lens of refractive index 1.5 has a focal length of 20 cm in air. Its focal length when immersed in a liquid of refractive index 1.6 will be:

(1) $-16\,\text{cm}$

(2) $-160\,\text{cm}$

(3) $+160\,\text{cm}$

(4) $+16\,\text{cm}$

Answer: (2) -160 **cm**

Solution:

1. Given Data: - Refractive index of the lens $\mu_l = 1.5$ - Refractive index of the medium (liquid) $\mu_m = 1.6$ - Focal length in air $f_a = 20\,\text{cm}$

2. Use the Lens Formula in Different Mediums: - The relationship between the focal length in air f_a and the focal length in the medium f_m is given by:

$$\frac{f_m}{f_a} = \frac{\mu_l - 1}{\mu_l - \mu_m}$$

3. Substitute the Values:

$$\frac{f_m}{20} = \frac{(1.5 - 1)}{(1.5 - 1.6)}$$

$$\frac{f_m}{20} = \frac{0.5}{-0.1}$$

$$f_m = 20 \times -5 = -160\,\text{cm}$$

So, the correct option is : $-160\,\text{cm}$

Quick Tip

When a lens is immersed in a medium with a higher refractive index than the lens material, the focal length becomes negative, indicating a virtual focal point.

---

46. A thermodynamic system is taken from an original state A to an intermediate state B by a linear process as shown in the figure. Its volume is then reduced to the original value from B to C by an isobaric process. The total work done by the gas from A to B and B to C would be:

(1) $33800\,\text{J}$

(2) $2200\,\text{J}$

(3) $600\,\text{J}$

(4) $1200\,\text{J}$

Answer: (BONUS)

Solution:

1. Calculate Work Done from A to B: - Since the process from A to B is linear on the pressure-volume diagram, the work done W_{AB} can be calculated as the area under the line AB. - The average pressure from A to B is $\frac{8000+4000}{2} = 6000\,\text{dyne/cm}^2$. - The volume change from A to B is $4\,\text{m}^3$.

$$W_{AB} = \text{Average Pressure} \times \text{Change in Volume}$$

$$W_{AB} = 6000 \times 4\,\text{dyne/cm}^2 \times \text{m}^3$$

2. Convert Units: - Convert dyne/cm^2 to N/m^2 by using $1\,\text{dyne/cm}^2 = 10^{-5}\,\text{N/m}^2$.

$$W_{AB} = 6000 \times 10^{-5} \times 4\,\text{J} = 800\,\text{J}$$

3. Calculate Work Done from B to C: - From B to C, the process is isobaric (constant pressure), so work done $W_{BC} = \text{Pressure} \times \text{Change in Volume}$. - The pressure at B and C is $4000\,\text{dyne/cm}^2$. - Volume change from B to C is $-4\,\text{m}^3$ (since the volume is reducing).

$$W_{BC} = 4000 \times (-4) \times 10^{-5}\,\text{J} = -800\,\text{J}$$

4. Total Work Done:

$$W_{\text{total}} = W_{AB} + W_{BC} = 800 - 800 = 0\,\text{J}$$

So, the correct option is : BONUS (since the total work done is zero)

Quick Tip

When calculating work done on a PV diagram, find the area under each segment. For processes involving unit conversion, ensure all units are consistent before summing.

---

47. At what distance above and below the surface of the earth a body will have the same weight (take radius of earth as R)?

(1) $\sqrt{5}R - R$

(2) $\frac{\sqrt{5}R-R}{2}$

(3) $\frac{R}{2}$

(4) $\frac{R}{2}(\sqrt{5}-1)$

Answer: (4)

Solution:

1. Define Gravitational Acceleration Above and Below: - Above the earth's surface at height h, gravitational acceleration g_p is:

$$g_p = \frac{gR^2}{(R+h)^2}$$

- Below the earth's surface at depth h, gravitational acceleration g_q is:

$$g_q = g\left(1 - \frac{h}{R}\right)$$

2. Set $g_p = g_q$:

$$\frac{gR^2}{(R+h)^2} = g\left(1 - \frac{h}{R}\right)$$

3. Simplify the Equation:

$$\frac{1}{(1+\frac{h}{R})^2} = 1 - \frac{h}{R}$$

$$\left(1 - \frac{h}{R}\right)\left(1 + \frac{h}{R}\right) = 1$$

4. Let $\frac{h}{R} = x$:

$$(1-x)(1+x) = 1$$

$$1 - x^2 = 1$$

$$x = \frac{\sqrt{5}-1}{2}$$

5. Calculate h:

$$h = \frac{R}{2}(\sqrt{5}-1)$$

So, the correct option is : $h = \frac{R}{2}(\sqrt{5}-1)$

Quick Tip

For problems involving equal weights at different heights and depths, use gravitational formulas for points above and below the earth's surface and equate them.

48. A capacitor of capacitance 100 μF is charged to a potential of 12 V and connected to a 6.4 mH inductor to produce oscillations. The maximum current in the circuit would be:

(1) 3.2 A

(2) 1.5 A

(3) 2.0 A

(4) 1.2 A

Answer: (2) 1.5 A

Solution:

1. Using Energy Conservation in LC Circuit: - The energy stored in the capacitor initially is equal to the maximum energy stored in the inductor:

$$\frac{1}{2}CV^2 = \frac{1}{2}LI_{\text{max}}^2$$

- Solving for I_{max}:

$$I_{\text{max}} = V\sqrt{\frac{C}{L}}$$

2. Substitute Given Values: - $C = 100 \times 10^{-6}\,\text{F}$ - $L = 6.4 \times 10^{-3}\,\text{H}$ - $V = 12\,\text{V}$

$$I_{\text{max}} = 12\sqrt{\frac{100 \times 10^{-6}}{6.4 \times 10^{-3}}}$$

3. Calculate I_{max}:

$$I_{\text{max}} = 12 \times \sqrt{\frac{100 \times 10^{-6}}{6.4 \times 10^{-3}}} = 12 \times \frac{1}{8} = \frac{12 \times 1}{8} = \frac{12}{8} = 1.5\,\text{A}$$

So, the correct option is : $1.5\,\text{A}$

Quick Tip

In an LC circuit, the maximum current can be calculated using $I_{\text{max}} = V\sqrt{\frac{C}{L}}$ by equating the initial energy in the capacitor to the maximum energy in the inductor.

---

49. The explosive in a Hydrogen bomb is a mixture of $^1H^2$, $^1H^3$, and $^3Li^6$ in some condensed form. The chain reaction is given by

$$^3Li^6 +^0 n^1 \rightarrow^2 He^4 +^1 H^3$$

$$^1H^2 +^1 H^3 \rightarrow^2 He^4 +^0 n^1$$

During the explosion, the energy released is approximately

(1) 28.12 MeV

(2) 12.64 MeV

(3) 16.48 MeV

(4) 22.22 MeV

Answer: (4) 22.22 MeV

Solution:

1. Total Reaction: - Combining the two reactions, we get:

$$3\,^3Li^6 +^1 H^2 \rightarrow 2\,^2He^4$$

2. Calculate Energy Released (Q-value): - Using the mass-energy equivalence $Q = \Delta m \cdot c^2$, where Δm is the mass defect. - Given: - $M(^3Li^6) = 6.01690\,\text{amu}$ - $M(^1H^2) = 2.01471\,\text{amu}$ - $M(^2He^4) = 4.00388\,\text{amu}$ - $1\,\text{amu} = 931.5\,\text{MeV}$

3. Calculate Q:

$$Q = \left[M(^3Li^6) + M(^1H^2) - 2 \times M(^2He^4)\right] \times 931.5\,\text{MeV}$$

$$Q = [6.01690 + 2.01471 - 2 \times 4.00388] \times 931.5\,\text{MeV}$$

$$Q = [8.03161 - 8.00776] \times 931.5\,\text{MeV}$$

$$Q = 0.02385 \times 931.5\,\text{MeV}$$

$$Q = 22.22\,\text{MeV}$$

So, the correct option is : $\boxed{22.22\,\text{MeV}}$

Quick Tip

To calculate energy released in nuclear reactions, use the mass defect and convert to energy using $Q = \Delta m \times 931.5\,\text{MeV}$ when mass is given in atomic mass units (amu).

50. Two vessels A and B are of the same size and are at the same temperature. A contains 1 g of hydrogen and B contains 1 g of oxygen. P_A and P_B are the pressures of the gases in A and B respectively, then $\frac{P_A}{P_B}$ is:

(1) 16

(2) 8

(3) 4

(4) 32

Answer: (1) 16

Solution:

1. Use the Ideal Gas Equation:

$$\frac{P_A V_A}{P_B V_B} = \frac{n_A R T_A}{n_B R T_B}$$

- Given $V_A = V_B$ and $T_A = T_B$, the equation simplifies to:

$$\frac{P_A}{P_B} = \frac{n_A}{n_B}$$

2. Calculate Moles of Each Gas: - For hydrogen in vessel A:

$$n_A = \frac{\text{mass of hydrogen}}{\text{molar mass of } H_2} = \frac{1\,\text{g}}{2\,\text{g/mol}} = \frac{1}{2}\,\text{mol}$$

- For oxygen in vessel B:

$$n_B = \frac{\text{mass of oxygen}}{\text{molar mass of } O_2} = \frac{1\,\text{g}}{32\,\text{g/mol}} = \frac{1}{32}\,\text{mol}$$

3. Calculate the Ratio of Pressures:

$$\frac{P_A}{P_B} = \frac{n_A}{n_B} = \frac{\frac{1}{2}}{\frac{1}{32}} = \frac{1}{2} \times 32 = 16$$

So, the correct option is : 16

Quick Tip

When comparing pressures of gases in identical vessels at the same temperature, the ratio of pressures depends on the ratio of moles of each gas.

51. When a hydrogen atom going from $n = 2$ to $n = 1$ emits a photon, its recoil speed is $\frac{x}{5}$ m/s. Where $x =$ _____. (Use: mass of hydrogen atom $= 1.6 \times 10^{-27}$ kg)

Answer: (17)

Solution:

1. Calculate Energy Difference (ΔE): - The energy levels for $n = 1$ and $n = 2$ in a hydrogen atom are given by:

$$E_{n=1} = -13.6\,\text{eV}, \quad E_{n=2} = -3.4\,\text{eV}$$

- The energy difference ΔE when the atom transitions from $n = 2$ to $n = 1$ is:

$$\Delta E = E_{n=1} - E_{n=2} = -13.6\,\text{eV} + 3.4\,\text{eV} = 10.2\,\text{eV}$$

2. Convert Energy to Joules: - $1\,\text{eV} = 1.6 \times 10^{-19}\,\text{J}$, so:

$$\Delta E = 10.2 \times 1.6 \times 10^{-19}\,\text{J} = 1.632 \times 10^{-18}\,\text{J}$$

3. Calculate Recoil Speed (v): - Using $v = \frac{\Delta E}{mc}$, where $m = 1.6 \times 10^{-27}\,\text{kg}$ and $c = 3 \times 10^{8}\,\text{m/s}$:

$$v = \frac{1.632 \times 10^{-18}}{1.6 \times 10^{-27} \times 3 \times 10^{8}}$$

- Simplify:

$$v = 3.4\,\text{m/s} = \frac{17}{5}\,\text{m/s}$$

4. Determine x: - Since the recoil speed is $\frac{x}{5}$, we have $x = 17$.

So, the correct option is : $\boxed{x = 17}$

Quick Tip

In problems involving atomic transitions, calculate the energy difference in electron volts, convert to joules, and apply the momentum-energy relationship to find recoil velocities.

---

52. A ball rolls off the top of a stairway with horizontal velocity u. The steps are 0.1 m high and 0.1 m wide. The minimum velocity u with which the ball just hits the step 5 of the stairway will be $\sqrt{x}$ m/s where $x =$ _____ [use $g = 10\,\text{m/s}^2$]

Answer: (2) $x = 2$

Solution:

1. Determine Horizontal Range to Just Hit Step 5: - The ball needs to cross 4 steps horizontally to just hit the 5th step. - Since each step is 0.1 m wide, the horizontal range R required to reach the 5th step is:

$$R = 0.4\,\text{m}$$

- Using the horizontal motion equation $R = u \cdot t$, we get:

$$t = \frac{R}{u} = \frac{0.4}{u}$$

2. Vertical Motion Analysis: - For vertical displacement, the ball needs to fall a height of $h = 4 \times 0.1 = 0.4\,\text{m}$. - Using the vertical motion equation $h = \frac{1}{2}gt^2$:

$$0.4 = \frac{1}{2} \cdot 10 \cdot \left(\frac{0.4}{u}\right)^2$$

- Simplify to find u:

$$0.4 = 5 \cdot \frac{0.16}{u^2}$$

$$u^2 = 2$$

$$u = \sqrt{2}\,\text{m/s}$$

3. Determine x: - Given that $u = \sqrt{x}$, we find $x = 2$.

So, the correct option is : $x = 2$

Quick Tip

For problems involving projectile motion across steps, calculate the required horizontal range and use vertical displacement to find the initial velocity.

---

53. A square loop of side 10 cm and resistance 0.7 Ω is placed vertically in the east-west plane. A uniform magnetic field of 0.20 T is set up across the plane in the northeast direction. The magnetic field is decreased to zero in 1 s at a steady rate. Then, the magnitude of induced emf is $\sqrt{x} \times 10^{-3}$ V. The value of x is ______.

Answer: (2) $x = 2$

Solution:

1. Calculate Area Vector of the Square Loop: - Side of square $= 10\,\text{cm} = 0.1\,\text{m}$ - Area $A = (0.1)^2 = 0.01\,\text{m}^2$ - Since the loop is placed in the east-west plane, the area vector $\vec{A}$ is along the $\hat{j}$ direction:

$$\vec{A} = 0.01\,\hat{j}\,\text{m}^2$$

2. Calculate the Magnetic Field Vector $\vec{B}$: - The magnetic field $B = 0.20\,\text{T}$ is directed at a $45°$ angle in the northeast direction, so:

$$\vec{B} = \frac{0.20}{\sqrt{2}}\,\hat{i} + \frac{0.20}{\sqrt{2}}\,\hat{j}$$

- Simplify:

$$\vec{B} = 0.1414\,\hat{i} + 0.1414\,\hat{j}\,\text{T}$$

3. Calculate the Magnetic Flux Φ:

$$\Phi = \vec{B}\cdot\vec{A} = \left(0.1414\,\hat{i} + 0.1414\,\hat{j}\right)\cdot\left(0\,\hat{i} + 0.01\,\hat{j}\right)$$

$$\Phi = 0.1414 \times 0.01 = 0.001414\,\text{Wb}$$

4. Calculate Induced EMF (ε): - The magnetic field is reduced to zero in $\Delta t = 1\,\text{s}$, so:

$$\varepsilon = -\frac{\Delta\Phi}{\Delta t} = -\frac{0.001414 - 0}{1} = 0.001414\,\text{V} = \sqrt{2}\times 10^{-3}\,\text{V}$$

5. Determine x: - Since $\varepsilon = \sqrt{x}\times 10^{-3}\,\text{V}$, we have $x = 2$.

So, the correct option is : $x = 2$

Quick Tip

When calculating induced emf in cases where the magnetic field is inclined, resolve the field into components and use only the component perpendicular to the loop's area for flux calculations.

---

54. A cylinder is rolling down on an inclined plane of inclination $60°$. Its acceleration during rolling down will be $\frac{x}{\sqrt{3}}$ m/s^2, where $x =$ _____. (Use $g = 10\,$m/s^2)

Answer: (10)

Solution:

1. Formula for Acceleration in Rolling Motion: - For a cylinder rolling down an incline, the acceleration a is given by:

$$a = \frac{g \sin\theta}{1 + \frac{I_{\text{cm}}}{MR^2}}$$

- For a solid cylinder, $I_{\text{cm}} = \frac{1}{2}MR^2$.

2. Substitute Values:

$$a = \frac{g \sin\theta}{1 + \frac{1}{2}}$$

- Given $g = 10\,\text{m/s}^2$ and $\theta = 60^\circ$ (so $\sin 60^\circ = \frac{\sqrt{3}}{2}$):

$$a = \frac{10 \times \frac{\sqrt{3}}{2}}{1 + \frac{1}{2}}$$

3. Calculate a:

$$a = \frac{10 \times \frac{\sqrt{3}}{2}}{\frac{3}{2}} = \frac{10\sqrt{3}}{3} = \frac{x}{\sqrt{3}}$$

- Therefore, $x = 10$.

So, the correct option is : $x = 10$

Quick Tip

In rolling motion problems involving inclines, use the moment of inertia in the acceleration formula and remember to calculate $\sin\theta$ for the angle of inclination.

---

55. The magnetic potential due to a magnetic dipole at a point on its axis situated at a distance of 20 cm from its center is 1.5×10^{-5} **Tm. The magnetic moment of the dipole is .**

Am^2

Given: $\frac{\mu_0}{4\pi} = 10^{-7}\,\text{Tm/A}$

Answer: (6)

Solution:

1. Formula for Magnetic Potential on the Axis of a Dipole:

$$V = \frac{\mu_0}{4\pi}\frac{M}{r^2}$$

- Where: - $V = 1.5 \times 10^{-5}\,\text{Tm}$ - $\frac{\mu_0}{4\pi} = 10^{-7}\,\text{Tm/A}$ - $r = 20\,\text{cm} = 0.2\,\text{m}$

2. Rearrange to Solve for M:

$$M = \frac{V \cdot r^2}{\frac{\mu_0}{4\pi}}$$

3. Substitute Values:

$$M = \frac{1.5 \times 10^{-5} \times (0.2)^2}{10^{-7}}$$

$$M = \frac{1.5 \times 10^{-5} \times 4 \times 10^{-2}}{10^{-7}}$$

4. Simplify Calculation:

$$M = \frac{1.5 \times 4 \times 10^{-7}}{10^{-7}}$$

$$M = 6\,\text{Am}^2$$

So, the correct option is : $\quad M = 6\,\text{Am}^2$

Quick Tip

For magnetic potential calculations along the axis of a dipole, use $V = \frac{\mu_0}{4\pi}\frac{M}{r^2}$, where M is the dipole moment and r is the distance from the center.

---

56. In a double slit experiment shown in figure, when light of wavelength 400 nm is used, a dark fringe is observed at P. If $D = 0.2$ m, the minimum distance between the slits S_1 and S_2 is ______ mm.

Answer: (0.20 mm)

Solution:

1. Path Difference Condition for Minima: - For a dark fringe (minima) at P, the path difference should be:

$$2\sqrt{D^2 + d^2} - 2D = \frac{\lambda}{2}$$

- Rearrange:

$$\sqrt{D^2 + d^2} - D = \frac{\lambda}{4}$$

- Therefore:

$$\sqrt{D^2 + d^2} = D + \frac{\lambda}{4}$$

2. Square Both Sides:

$$D^2 + d^2 = D^2 + D\lambda + \frac{\lambda^2}{16}$$

- Simplify to solve for d:

$$d^2 = D\lambda + \frac{\lambda^2}{16}$$

3. Substitute Given Values: - $\lambda = 400\,\text{nm} = 400 \times 10^{-9}\,\text{m}$ - $D = 0.2\,\text{m}$

$$d^2 = 0.2 \times 400 \times 10^{-9} + \frac{(400 \times 10^{-9})^2}{16}$$

- Calculate:

$$d^2 \approx 4 \times 10^{-8}\,\text{m}^2$$

$$d \approx 2 \times 10^{-4}\,\text{m} = 0.20\,\text{mm}$$

So, the correct option is : $\quad d = 0.20\,\text{mm}$

Quick Tip

For double slit experiments with dark fringes, use path difference conditions specific to minima and ensure unit conversions for accurate results.

---

57. A $16\,\Omega$ wire is bent to form a square loop. A $9\,\text{V}$ battery with internal resistance $1\,\Omega$ is connected across one of its sides. If a $4\,\mu\text{F}$ capacitor is connected across one of its diagonals, the energy stored by the capacitor will be $\frac{x}{2}\,\mu\text{J}$, where $x =$ _____.

Answer: (81)

Solution:

1. Calculate Equivalent Resistance: - The square loop consists of four $4\,\Omega$ resistors, each forming the sides of the square. - The equivalent resistance R_{eq} between points A and B (opposite sides of the square) is:

$$R_{\text{eq}} = \frac{12 \times 4}{12 + 4} = 3\,\Omega$$

- Including the internal resistance of the battery, the total resistance is $R = 3 + 1 = 4\,\Omega$.

2. Calculate the Current I:

$$I = \frac{V}{R} = \frac{9}{4} = 2.25\,\text{A}$$

3. Determine Current Through Each Side: - Due to symmetry, the current through each $4\,\Omega$ resistor in parallel with the capacitor is I_1:

$$I_1 = \frac{9}{16} = 0.5625\,\text{A}$$

4. Calculate Voltage Across the Capacitor:

$$V_{AB} = I_1 \times 8 = 4.5\,\text{V}$$

5. Calculate Energy Stored in the Capacitor: - Energy stored in a capacitor is given by:

$$U = \frac{1}{2}CV_{AB}^2$$

- Substitute values:

$$U = \frac{1}{2} \times 4 \times (4.5)^2 = \frac{81}{2}\,\mu\text{J}$$

6. Determine x: - Since $U = \frac{x}{2}\,\mu$J, we find $x = 81$.

So, the correct option is : $\boxed{x = 81}$

Quick Tip

In circuit problems involving symmetry, calculate the equivalent resistance carefully and use it to determine the voltage across components.

---

58. When the displacement of a simple harmonic oscillator is one third of its amplitude, the ratio of total energy to the kinetic energy is $\frac{x}{8}$, where $x =$ _____.

Answer: (9)

Solution:

1. Define Total Energy: - The total energy E of a simple harmonic oscillator is given by:

$$E = \frac{1}{2}KA^2$$

- Where K is the spring constant and A is the amplitude.

2. Calculate Potential Energy U at Displacement $\frac{A}{3}$: - When the displacement is $\frac{A}{3}$:

$$U = \frac{1}{2}K\left(\frac{A}{3}\right)^2 = \frac{KA^2}{18} = \frac{E}{9}$$

3. Calculate Kinetic Energy KE: - Kinetic energy is the difference between total energy and potential energy:

$$\text{KE} = E - U = E - \frac{E}{9} = \frac{8E}{9}$$

4. Calculate the Ratio of Total Energy to Kinetic Energy:

$$\frac{\text{Total Energy}}{\text{KE}} = \frac{E}{\frac{8E}{9}} = \frac{9}{8}$$

5. Determine x: - Since the ratio is $\frac{x}{8}$, we have $x = 9$.

So, the correct option is : $x = 9$

Quick Tip

In simple harmonic motion, use the relationship between total, potential, and kinetic energies at various displacements to solve for energy ratios.

---

59. An electron is moving under the influence of the electric field of a uniformly charged infinite plane sheet S having surface charge density $+\sigma$. The electron at $t = 0$ is at a distance of 1 m from S and has a speed of 1 m/s. The maximum value of σ if the electron strikes S at $t = 1$ s is $\alpha \left[\frac{m\epsilon_0}{e}\right]$ C/m^2. The value of α is

Answer: (8)

Solution:

1. Given Information and Variables: - Initial velocity $u = 1\,\text{m/s}$ - Acceleration $a = -\frac{\sigma e}{2\epsilon_0 m}$ - Time $t = 1\,\text{s}$ - Displacement $S = -1\,\text{m}$

2. Use Kinematic Equation: - The kinematic equation for displacement S is:

$$S = ut + \frac{1}{2}at^2$$

- Substitute values:

$$-1 = 1 \times 1 + \frac{1}{2}\left(-\frac{\sigma e}{2\epsilon_0 m}\right) \times (1)^2$$

3. Solve for σ:

$$-1 = 1 - \frac{\sigma e}{4\epsilon_0 m}$$

$$\frac{\sigma e}{4\epsilon_0 m} = 2$$

$$\sigma = 8\frac{\epsilon_0 m}{e}$$

4. Determine α: - Given $\sigma = \alpha\left[\frac{m\epsilon_0}{e}\right]$, we find $\alpha = 8$.

So, the correct option is : $\alpha = 8$

Quick Tip

In problems involving charged particles under uniform electric fields, use kinematic equations and express acceleration in terms of electric field parameters.

---

60. In a test experiment on a model aeroplane in a wind tunnel, the flow speeds on the upper and lower surfaces of the wings are $70\,\mathbf{ms}^{-1}$ and $65\,\mathbf{ms}^{-1}$ respectively. If the wing area is $2\,\mathbf{m}^2$, the lift of the wing is ________ N.

Given: density of air $= 1.2\,\mathrm{kg\,m^{-3}}$

Answer: (810 N)

Solution:

1. Use Bernoulli's Equation for Lift Force:

$$F = \frac{1}{2}\rho(v_1^2 - v_2^2)A$$

- Where: - $\rho = 1.2\,\mathrm{kg/m^3}$ - $v_1 = 70\,\mathrm{m/s}$ - $v_2 = 65\,\mathrm{m/s}$ - $A = 2\,\mathrm{m^2}$

2. Calculate the Lift Force F:

$$F = \frac{1}{2} \times 1.2 \times (70^2 - 65^2) \times 2$$

3. Simplify the Expression:

$$F = \frac{1}{2} \times 1.2 \times (4900 - 4225) \times 2$$

$$F = \frac{1}{2} \times 1.2 \times 675 \times 2$$

$$F = 810\,\mathrm{N}$$

So, the correct option is : $F = 810\,\mathrm{N}$

Quick Tip

For lift calculations involving different airspeeds over wing surfaces, use Bernoulli's equation and ensure the area and air density are correctly applied.

---

61. Given below are two statements: one is labelled as Assertion A and the other is labelled as Reason R:

Assertion A: The first ionisation enthalpy decreases across a period.

Reason R: The increasing nuclear charge outweighs the shielding across the period.

In the light of the above statements, choose the most appropriate from the options given below:

(1) Both A and R are true and R is the correct explanation of A

(2) A is true but R is false

(3) A is false but R is true

(4) Both A and R are true but R is NOT the correct explanation of A

Answer: (3)

Solution:

- The statement in Assertion A is incorrect because the first ionisation enthalpy actually *increases* across a period, not decreases. This is due to the increasing effective nuclear charge as we move across a period, which holds the electrons more tightly, making it harder to remove them. - The statement in Reason R is true; the increase in nuclear charge indeed outweighs the shielding effect across a period, leading to a higher ionisation enthalpy.

Thus, **A is false but R is true**.

So, the correct option is : (3)

Quick Tip

Ionisation enthalpy generally increases across a period due to increased nuclear charge with minimal additional shielding.

---

62. Match List I with List II

LIST-I (Substances)	LIST-II (Element Present)
A. Ziegler catalyst	I. Rhodium
B. Blood Pigment	II. Cobalt
C. Wilkinson catalyst	III. Iron
D. Vitamin B_{12}	IV. Titanium

Choose the correct answer from the options given below:

(1) A-II, B-IV, C-I, D-III

(2) A-II, B-III, C-IV, D-I

(3) A-III, B-II, C-IV, D-I

(4) A-IV, B-III, C-I, D-II

Answer: (4)

Solution:

- Ziegler catalyst $\rightarrow$ Titanium - Blood pigment (hemoglobin) $\rightarrow$ Iron - Wilkinson catalyst $\rightarrow$ Rhodium - Vitamin B_{12} $\rightarrow$ Cobalt

Thus, the correct matching is:

$$\text{A-IV, B-III, C-I, D-II}$$

So, the correct option is : (4)

Quick Tip

For matching questions, remember the key elements associated with specific catalysts and biological molecules like vitamins and pigments.

---

63. In chromyl chloride test for confirmation of Cl^- ion, a yellow solution is obtained. Acidification of the solution and addition of amyl alcohol and 10% H_2O_2 turns organic layer blue indicating formation of chromium pentoxide. The oxidation state of chromium in that is

(1) $+6$

(2) $+5$

(3) $+10$

(4) $+3$

Answer: (1)

Solution:

- During the chromyl chloride test, the following reaction occurs:

$$Cl^- + K_2Cr_2O_7 + H_2SO_4 \to CrO_2Cl_2 + Cl^-$$

- In basic medium, this results in a yellow solution due to CrO_4^{2-}.

- Upon acidification, addition of amyl alcohol, and 10% H_2O_2, a blue organic layer forms due to the formation of CrO_5 (chromium pentoxide).

- The oxidation state of chromium in CrO_5 is $+6$.

So, the correct option is : $+6$

Quick Tip

In oxidation state determination, note that chromium in CrO_5 exhibits an oxidation state of $+6$ despite unusual bonding.

---

64. The difference in energy between the actual structure and the lowest energy resonance structure for the given compound is

(1) electromeric energy

(2) resonance energy

(3) ionization energy

(4) hyperconjugation energy

Answer: (2)

Solution:

- The difference in energy between the actual structure of a molecule and its most stable resonance structure is known as **resonance energy**. - Resonance energy indicates the extra stability gained by a compound due to resonance, where electrons are delocalized across atoms.

So, the correct option is : Resonance Energy (2)

Quick Tip

Resonance energy provides a measure of the stability due to delocalization of electrons in conjugated systems.

---

65. Given below are two statements:

Statement I: The electronegativity of group 14 elements from Si to Pb gradually decreases.

Statement II: Group 14 contains non-metallic, metallic, as well as metalloid elements.

In the light of the above statements, choose the most appropriate from the options given below:

(1) Statement I is false but Statement II is true

(2) Statement I is true but Statement II is false

(3) Both Statement I and Statement II are true

(4) Both Statement I and Statement II are false

Answer: (1)

Solution:

- The electronegativity values for Group 14 elements from Si to Pb are almost the same, with values approximately as follows:

Element	Electronegativity (EN)
C	2.5
Si	1.8
Ge	1.8
Sn	1.8
Pb	1.9

- Therefore, Statement I is false as there is no gradual decrease in electronegativity from Si to Pb. - Statement II is true because Group 14 does indeed contain non-metallic (C), metalloid (Si, Ge), and metallic (Sn, Pb) elements.

So, the correct option is : (1) Statement I is false but Statement II is true

Quick Tip

In periodic trends, electronegativity does not always decrease uniformly down a group, especially in heavier elements.

---

66. The correct set of four quantum numbers for the valence electron of rubidium atom (Z = 37) is:

(1) $5, 0, 0, +\frac{1}{2}$

(2) $5, 0, 1, +\frac{1}{2}$

(3) $5, 1, 0, +\frac{1}{2}$

(4) $5, 1, 1, +\frac{1}{2}$

Answer: (1)

Solution:

- Rubidium (Rb) has the electron configuration: $[\text{Kr}]5s^1$. - For the valence electron in the $5s$ orbital: - Principal quantum number, $n = 5$. - Azimuthal quantum number, $l = 0$ (since it is an s-orbital). - Magnetic quantum number, $m = 0$ (as m can range from $-l$ to $+l$, and $l = 0$ allows only $m = 0$). - Spin quantum number, $s = +\frac{1}{2}$ (or $-\frac{1}{2}$ as it can have either spin).

Thus, the correct set of quantum numbers is $(5, 0, 0, +\frac{1}{2})$.

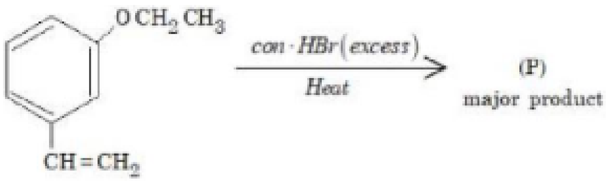
So, the correct option is : (1) $5, 0, 0, +\dfrac{1}{2}$

Quick Tip

The quantum numbers n, l, m, and s are used to uniquely define the state of an electron in an atom, particularly the outermost or valence electron for element identification.

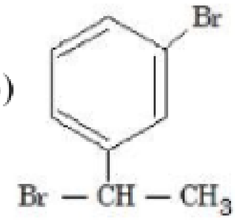
---

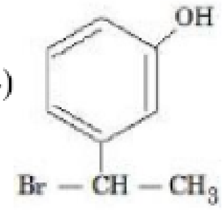
67. The major product (P) in the following reaction is:



(1) 

(2) 

(3) 

(4) 

Answer: (4)

Solution:

In this reaction, an alkene side chain attached to the benzene ring reacts with excess concentrated HBr. The mechanism involves the following steps: 1. The double bond in the alkene reacts with HBr to form a carbocation intermediate. 2. Due to excess HBr, the benzyl carbocation rearranges to a more stable form. 3. The addition of Br takes place, resulting in the formation of the major product, which is the bromo-substituted alkane on the benzene ring.

Thus, the major product is:

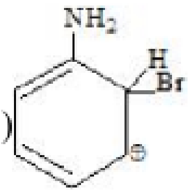
So, the correct option is : (4) $*6(=-=(-CH-CH_3)(Br)-)$

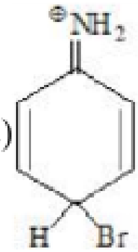
Quick Tip

In electrophilic addition reactions, excess HBr can lead to complete saturation of double bonds in side chains, forming the most stable alkyl halide as the major product.

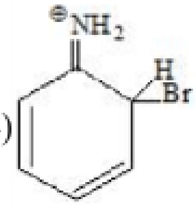
---

68. The arenium ion which is not involved in the bromination of Aniline is:

(1) 

(2) 

(3) 

(4) 

Answer: (3)

Solution:

Since the $-NH_2$ group is ortho/para directing, the arenium ion will not be formed by attack at the meta position. This is due to the electron-donating nature of the $-NH_2$ group, which stabilizes the carbocation at the ortho and para positions relative to itself.

Thus, the arenium ion corresponding to the meta substitution, as shown in option (3), does not participate in the bromination of aniline.

So, the correct option is : (3)

Quick Tip

For aromatic substitution, electron-donating groups like $-NH_2$ direct electrophiles to the ortho and para positions, avoiding the meta position.

---

69. Appearance of blood red colour on treatment of the sodium fusion extract of an organic compound with $FeSO_4$ in presence of concentrated H_2SO_4 indicates the presence of element(s)

(1) Br

(2) N

(3) N and S

(4) S

Answer: (3)

Solution:

$$Fe^{2+} \xrightarrow{H^+,\, \text{Conc. } H_2SO_4} Fe^{3+}$$

$$Fe^{3+} + SCN^- \rightarrow Fe(SCN)_3 \text{ (blood red colour)}$$

The appearance of a blood red color upon this reaction indicates the presence of both nitrogen and sulfur in the compound, as they combine to form thiocyanate ions (SCN^-), which react with Fe^{3+} to produce $Fe(SCN)_3$ with the characteristic blood red color.

So, the correct option is : (3)

Quick Tip

The blood red color in the sodium fusion test is a classic indicator of both nitrogen and sulfur, confirming the formation of thiocyanate ions.

---

70. Given below are two statements: one is labelled as Assertion A and the other is labelled as Reason R:

Assertion A: Aryl halides cannot be prepared by replacement of hydroxyl group of phenol by halogen atom.

Reason R: Phenols react with halogen acids violently.

In the light of the above statements, choose the most appropriate from the options given below:

(1) Both A and R are true but R is NOT the correct explanation of A

(2) A is false but R is true

(3) A is true but R is false

(4) Both A and R are true and R is the correct explanation of A

Answer: (3)

Solution:

Assertion (A): The given statement is correct because in phenol, the hydroxyl group cannot be replaced by a halogen atom through a simple halogen acid reaction. This is due to the resonance stabilization of the hydroxyl group in phenol, which prevents such a substitution.

$$\text{Phenol } (C_6H_5OH) + HX \rightarrow \text{No Reaction}$$

Reason (R): The statement that phenols react violently with halogen acids is incorrect in this context, as this does not relate to the inability to form aryl halides by replacing the hydroxyl group.

Thus, **Assertion (A) is correct but Reason (R) is false**.

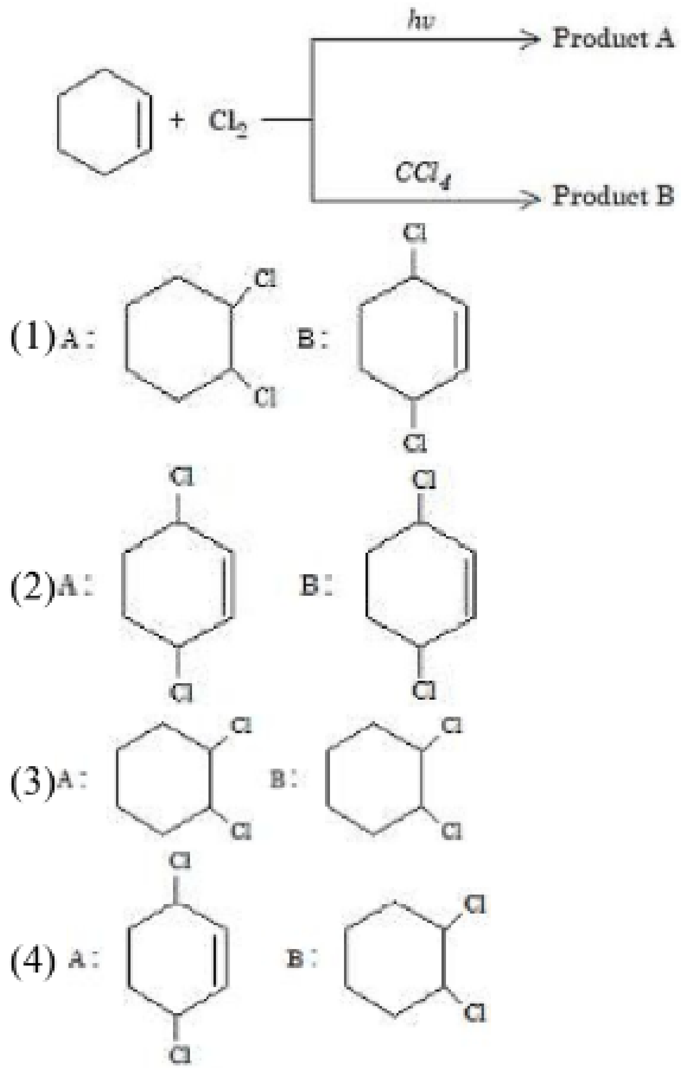
So, the correct option is : (3)

Quick Tip

In organic chemistry, the reactivity of functional groups like the hydroxyl group in phenol can be affected by resonance, preventing typical substitution reactions.

---

71. Identify product A and product B:



Answer: (4)

Solution:

In this reaction:

- Product A is formed through a free radical mechanism. - Product B is formed through an electrophilic addition reaction on the alkene.

The correct answer is option (4), where Product A is formed by the free radical mechanism and Product B is formed by electrophilic addition in the presence of CCl_4.

So, the correct option is : (4)

Quick Tip

Free radical substitution in the presence of light (hv) leads to the addition of Cl atoms on the benzene ring, while the electrophilic addition occurs in the presence of CCl_4.

72. Identify the incorrect pair from the following:

(1) Fluorspar - BF_3

(2) Cryolite - Na_3AlF_6

(3) Fluoroapatite - $3Ca_3(PO_4)_2 \cdot CaF_2$

(4) Carnallite - $KCl \cdot MgCl_2 \cdot 6H_2O$

Answer: (1)

Solution:

Fluorspar is actually $\mathbf{CaF}_2$, not BF_3. BF_3 does not represent Fluorspar, which makes option (1) incorrect.

So, the correct option is : (1)

Quick Tip

In chemistry, it is essential to remember the correct mineral formulas for various compounds, as similar names may refer to entirely different substances.

73. The interaction between π bond and lone pair of electrons present on an adjacent atom is responsible for

(1) Hyperconjugation

(2) Inductive effect

(3) Electromeric effect

(4) Resonance effect

Answer: (4)

Solution:

This interaction is a type of conjugation that leads to **resonance**, where the π electrons and lone pair electrons interact over adjacent atoms, stabilizing the molecule through delocalization of electrons.

So, the correct option is : (4)

Quick Tip

Resonance occurs when there is conjugation between π bonds and lone pairs on adjacent atoms, allowing electron delocalization across multiple atoms, which stabilizes the molecule.

---

74. $KMnO_4$ decomposes on heating at 513 K to form O_2 along with

(1) MnO_2 & K_2O_2

(2) K_2MnO_4 & Mn

(3) Mn & KO_2

(4) K_2MnO_4 & MnO_2

Answer: (4)

Solution:

The decomposition reaction of potassium permanganate ($KMnO_4$) at 513 K is as follows:

$$KMnO_4 \xrightarrow{\Delta} K_2MnO_4 + MnO_2 + O_2$$

So, the correct option is : (4)

Quick Tip

When heating $KMnO_4$, it decomposes to form K_2MnO_4, MnO_2, and releases oxygen gas (O_2), showcasing a common decomposition reaction for permanganates at high temperatures.

---

75. In which one of the following metal carbonyls, CO forms a bridge between metal atoms?

(1) $[Co_2(CO)_8]$

(2) $[Mn_2(CO)_{10}]$

(3) $[Os_3(CO)_{12}]$

(4) $[Ru_3(CO)_{12}]$

Answer: (1)

Solution:

In $[Co_2(CO)_8]$, the CO ligands form bridging bonds between the cobalt (Co) atoms. This bridging CO coordination is a characteristic feature of $[Co_2(CO)_8]$, where two cobalt atoms are connected via CO ligands that act as bridges.

The structures of each option are as follows:

- (1) $[Co_2(CO)_8]$ - Contains bridging CO between Co atoms.
- (2) $[Mn_2(CO)_{10}]$ - No bridging CO.
- (3) $[Os_3(CO)_{12}]$ - No bridging CO.
- (4) $[Ru_3(CO)_{12}]$ - No bridging CO.

Thus, **$[Co_2(CO)_8]$** is the correct answer where CO forms a bridge between metal atoms.

So, the correct option is : (1)

Quick Tip

In metal carbonyls, bridging CO ligands are commonly found in complexes with two or more metal atoms, where they act to stabilize the metal-metal bonds.

---

76. Type of amino acids obtained by hydrolysis of proteins is:

(1) β

(2) α

(3) δ

(4) γ

Answer: (2)

Solution:

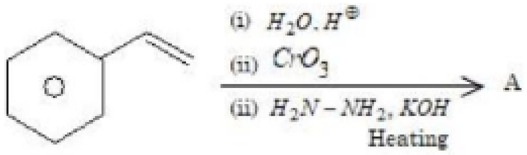
Proteins are natural polymers composed of α-amino acids, which are connected by peptide linkages. Upon acidic hydrolysis, proteins break down into α-amino acids due to the cleavage of these peptide bonds.

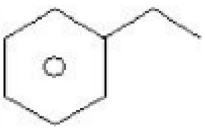
So, the correct option is : (2)

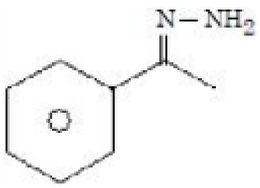
> Quick Tip
>
> α-amino acids are the building blocks of proteins and are released during the hydrolysis of protein chains.

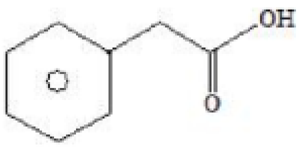
---

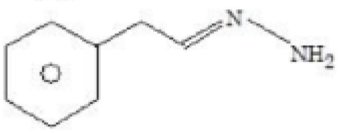
77. The final product A formed in the following multistep reaction sequence is:



(1) 

(2) 

(3) 

(4) 

Answer: (1)

Solution:

The reaction sequence involves the following steps: 1. The initial compound undergoes hydration with H_2O, H^+, forming a primary alcohol. 2. Oxidation with CrO_3 converts the alcohol to a ketone. 3. Finally, treatment with NH_2NH_2, KOH and heating (Wolff-Kishner reduction) reduces the ketone to an alkane.

Thus, the final product **A** is a simple alkyl chain attached to the benzene ring: Answer: (1).

So, the correct option is : (1)

Quick Tip

The Wolff-Kishner reduction is commonly used to reduce ketones or aldehydes to alkanes, completely removing the oxygen functionality.

---

78. Which of the following is not correct?

(1) ΔG is negative for a spontaneous reaction

(2) ΔG is positive for a spontaneous reaction

(3) ΔG is zero for a reversible reaction

(4) ΔG is positive for a non-spontaneous reaction

Answer: (2)

Solution:

For a process to be spontaneous, the Gibbs free energy change (ΔG) must be negative at constant temperature and pressure, indicating that the reaction proceeds forward without external energy input. Conversely, for a non-spontaneous process, ΔG is positive, meaning the reaction does not proceed unless energy is supplied. For a reaction in equilibrium (reversible), $\Delta G = 0$.

Thus, statement (2) is incorrect as it suggests ΔG is positive for a spontaneous reaction, which contradicts the condition for spontaneity.

$$(\Delta G)_{P,T} = (+) \text{ for non-spontaneous process}$$

So, the correct option is : (2)

Quick Tip

In thermodynamics, a negative Gibbs free energy change ($\Delta G < 0$) indicates a spontaneous reaction, while a positive ΔG indicates non-spontaneity.

---

79. Chlorine undergoes disproportionation in alkaline medium as shown below:

$$a\,\mathrm{Cl_2(g)} + b\,\mathrm{OH^-(aq)} \to c\,\mathrm{ClO^-(aq)} + d\,\mathrm{Cl^-(aq)} + e\,\mathrm{H_2O(l)}$$

The values of a, b, c, and d in a balanced redox reaction are respectively:

(1) 1, 2, 1, and 1

(2) 2, 2, 1, and 3

(3) 3, 4, 4, and 2

(4) 2, 4, 1, and 3

Answer: (1)

Solution:

The disproportionation reaction of chlorine in alkaline medium can be balanced as follows. Chlorine (Cl_2) is both reduced and oxidized:

$$\mathrm{Cl_2 + 2OH^- \to Cl^- + ClO^- + H_2O}$$

Thus, the values of $a, b, c,$ and d are 1, 2, 1, and 1 respectively.

So, the correct option is : (1)

Quick Tip

In a disproportionation reaction, a single substance is simultaneously oxidized and reduced, as seen with chlorine in alkaline medium.

---

80. In alkaline medium, MnO_4^- oxidises I^- to:

(1) IO_4^-

(2) IO^-

(3) I_2

(4) IO_3^-

Answer: (4)

Solution:

The reaction of MnO_4^- in an alkaline medium with I^- produces IO_3^- as the product.

$$2MnO_4^- + H_2O + I^- \rightarrow 2MnO_2 + 2OH^- + IO_3^-$$

Thus, the oxidation of I^- in the presence of MnO_4^- under alkaline conditions leads to the formation of IO_3^-.

So, the correct option is : (4)

Quick Tip

In alkaline medium, permanganate ions MnO_4^- often act as strong oxidizing agents and can oxidize halides like I^- to their higher oxidation states.

---

81. Number of compounds with one lone pair of electrons on central atom amongst following is _

O_3, H_2O, SF_4, ClF_3, NH_3, BrF_5, XeF_4

(1) 2

(2) 3

(3) 5

(4) 4

Answer: (4)

Solution:

To determine the number of compounds with one lone pair on the central atom, let's examine each compound:

- O_3: Contains one lone pair on the central oxygen atom.
- H_2O: Contains two lone pairs on the central oxygen atom.
- SF_4: Contains one lone pair on the sulfur atom.
- ClF_3: Contains two lone pairs on the chlorine atom.
- NH_3: Contains one lone pair on the nitrogen atom.
- BrF_5: Contains one lone pair on the bromine atom.

- XeF_4: Contains two lone pairs on the xenon atom.

From the above, the compounds with exactly one lone pair on the central atom are: O_3, SF_4, NH_3, and BrF_5.

Total: 4 Compounds

So, the correct option is : (4)

Quick Tip

Identifying lone pairs on central atoms can help determine the shape and polarity of molecules based on VSEPR theory.

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82. The mass of zinc produced by the electrolysis of zinc sulphate solution with a steady current of 0.015 A for 15 minutes is $\_\_ \times 10^{-4}$ **g**.

(Atomic mass of zinc = 65.4 amu)

(1) 45.75

(2) 46

(3) 45

(4) 47

Answer: (45.75) or (46)

Solution:

The reaction for the deposition of zinc is as follows:

$$\mathbf{Zn}^{2+} + 2\mathbf{e}^- \rightarrow \mathbf{Zn}$$

Using the formula for electrolysis:

$$W = \frac{Z \times i \times t}{F}$$

where

- $Z = \frac{65.4}{2}$ **(Equivalent weight of zinc),**
- $i = 0.015$ **A (current),**

- $t = 15 \times 60$ **seconds,**

- $F = 96500$ **C/mol (Faraday constant).**

Calculating the mass of zinc:

$$W = \frac{65.4}{2 \times 96500} \times 0.015 \times 15 \times 60$$

$$W = 45.75 \times 10^{-4}\,\mathbf{g}$$

Since the answer can be approximated, we also consider 46×10^{-4} **g.**

So, the correct option is : (45.75) or (46)

Quick Tip

In electrolysis calculations, always verify the time in seconds and use the correct equivalent weight for the element being deposited.

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83. For a reaction taking place in three steps at the same temperature, the overall rate constant $K = \frac{K_1 K_2}{K_3}$**. If** E_{a1}, E_{a2}, **and** E_{a3} **are 40, 50, and 60 kJ/mol respectively, the overall** E_a **is ____ kJ/mol.**

(1) 20

(2) 40

(3) 30

(4) 50

Answer: (30)

Solution:

For the overall rate constant:

$$K = \frac{K_1 \cdot K_2}{K_3} = \frac{A_1 \cdot A_2}{A_3} \cdot e^{\frac{(E_{a1}+E_{a2}-E_{a3})}{RT}}$$

Therefore,

$$K = \frac{A \cdot e^{-E_a/RT}}{A_3} = \frac{A_1 A_2}{A_3} \cdot e^{\frac{(E_{a1}+E_{a2}-E_{a3})}{RT}}$$

Given:

$$E_a = E_{a1} + E_{a2} - E_{a3} = 40 + 50 - 60 = 30\,\textbf{kJ/mol}$$

So, the correct option is : (30)

Quick Tip

When calculating the overall activation energy in multi-step reactions, sum the activation energies of steps in the numerator and subtract those in the denominator according to the rate constant expression.

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84. For the reaction $N_2O_4(g) \rightleftharpoons 2NO_2(g)$, $K_p = 0.492$ atm at 300K. K_c for the reaction at the same temperature is ___ $\times 10^{-2}$. (Given: $R = 0.082$ L atm mol^{-1}K^{-1})

(1) 1

(2) 2

(3) 3

(4) 4

Answer: (2)

Solution:

To convert K_p to K_c:

$$K_p = K_c \cdot (RT)^{\Delta n_g}$$

For the reaction $N_2O_4(g) \rightleftharpoons 2NO_2(g)$,

$$\Delta n_g = 2 - 1 = 1$$

Therefore:

$$K_c = \frac{K_p}{RT} = \frac{0.492}{0.082 \times 300} = 2 \times 10^{-2}$$

So, the correct option is : (2)

Quick Tip

Use the relation $K_p = K_c \cdot (RT)^{\Delta n_g}$ to convert between K_p and K_c, where Δn_g is the difference in moles of gaseous products and reactants.

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85. A solution of H_2SO_4 is 31.4% H_2SO_4 by mass and has a density of 1.25 g/mL. The molarity of the H_2SO_4 solution is ___ M (nearest integer). (Given molar mass of $H_2SO_4 = 98\,\mathbf{g\ mol^{-1}}$)

(1) 3

(2) 5

(3) 6

(4) 4

Answer: (4)

Solution:

To find the molarity (M), use the formula:

$$M = \frac{n_{\text{solute}}}{V} \times 1000$$

Given that the solution is 31.4% H_2SO_4, with a density of 1.25 g/mL:

1. Calculate the mass of H_2SO_4 in 100 g of solution:

$$\textbf{Mass of } H_2SO_4 = 31.4\,\mathbf{g}$$

2. Convert this to moles:

$$\frac{31.4}{98} = 0.32\,\mathbf{mol}$$

3. Find the volume of the solution:

$$\textbf{Volume} = \frac{\textbf{Mass of solution}}{\textbf{Density}} = \frac{100}{1.25} = 80\,\mathbf{mL}$$

4. Calculate molarity:

$$M = \frac{0.32\,\mathbf{mol}}{80\,\mathbf{mL}} \times 1000 = 4.005 \approx 4\,\mathbf{M}$$

So, the correct option is : (4)

Quick Tip

For solutions with a given percentage by mass and density, calculate molarity by converting mass percent to moles and using the density to find the solution volume.

86. The osmotic pressure of a dilute solution is 7×10^5 Pa at 273 K. Osmotic pressure of the same solution at 283 K is ___ $\times 10^4$ Nm^{-2}.

Answer: (72.56) or (73)

Solution:

The osmotic pressure (π) is given by the formula:

$$\pi = CRT$$

Since concentration (C) and R are constants, we can use the ratio:

$$\frac{\pi_1}{\pi_2} = \frac{T_1}{T_2}$$

Rearranging to find π_2:

$$\pi_2 = \pi_1 \frac{T_2}{T_1} = 7 \times 10^5 \times \frac{283}{273}$$

Calculating π_2:

$$\pi_2 = 72.56 \times 10^4 \, \mathbf{Nm}^{-2}$$

Thus, the osmotic pressure at 283 K is approximately:

So, the correct option is : (72.56) or (73)

Quick Tip

To calculate osmotic pressure at a different temperature, use the relation $\frac{\pi_1}{T_1} = \frac{\pi_2}{T_2}$ when the concentration is constant.

---

87. Number of compounds among the following which contain sulfur as a heteroatom is ___.

Compounds: Furan, Thiophene, Pyridine, Pyrrole, Cysteine, Tyrosine

Answer: (2)

Solution:

The compounds containing sulfur as a heteroatom are:

- **Thiophene - Contains sulfur in its five-membered aromatic ring.**
- **Cysteine - Contains sulfur in the form of a thiol group (-SH).**

Thus, the correct count of compounds with sulfur as a heteroatom is:

So, the correct option is : (2)

Quick Tip

Remember that sulfur can appear in heterocyclic rings (like in thiophene) or as functional groups in amino acids (like cysteine).

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88. The number of species from the following which are paramagnetic and with bond order equal to one is ___.

Species: $H_2, He_2^+, O_2^+, N_2^-, O_2^{2-}, F_2, Ne_2^+, B_2$

Answer: (1)

Solution:

We analyze the magnetic behavior and bond order for each species:

Species	Magnetic Behaviour	Bond Order
H_2	**Diamagnetic**	1
He_2^+	**Paramagnetic**	0.5
O_2^+	**Paramagnetic**	2.5
N_2^-	**Paramagnetic**	2
O_2^{2-}	**Diamagnetic**	1
F_2	**Diamagnetic**	1
Ne_2^+	**Paramagnetic**	0.5
B_2	**Paramagnetic**	1

Among these, the only species that is both paramagnetic and has a bond order of 1 is:

$$B_2$$

Thus, the correct count is:

So, the correct option is : (1)

Quick Tip

Paramagnetic species have unpaired electrons, and bond order can be calculated based on molecular orbital theory to determine bond strength.

89. From the compounds given below, the number of compounds which give a positive Fehling's test is ___.

Compounds: Benzaldehyde, Acetaldehyde, Acetone, Acetophenone, Methanal, 4-nitrobenzaldeh cyclohexane carbaldehyde.

Answer: (3)

Solution:

Fehling's test is positive for compounds with an aldehyde group (except for aromatic aldehydes) or alpha-hydroxy ketones.

The compounds that give a positive Fehling's test are:

- **Acetaldehyde** $(\mathbf{CH_3CHO})$
- **Methanal** $(\mathbf{HCHO})$
- **Cyclohexane carbaldehyde**

Other compounds, such as benzaldehyde, acetophenone, and 4-nitrobenzaldehyde, do not give a positive Fehling's test because they are aromatic aldehydes or ketones.

Thus, the correct count is:

$$\textbf{So, the correct option is :} \quad \mathbf{(3)}$$

Quick Tip

Fehling's test is a qualitative test used to detect the presence of aldehydes (except aromatic aldehydes) and certain alpha-hydroxy ketones.

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90. Consider the given reaction. The total number of oxygen atoms present per molecule of the product (P) is ___.

$$\textbf{Reaction:} \quad \mathbf{CH_3 - CH = CH - CH_3} \xrightarrow{(i)\,\mathbf{O_3},\,(ii)\,\mathbf{Zn/H_2O}} 2\,\mathbf{CH_3 - C = O}$$

Answer: (1)

Solution:

Upon ozonolysis, the compound $CH_3 - CH = CH - CH_3$ is cleaved at the double bond, resulting in two molecules of $CH_3 - C = O$.

Each resulting molecule contains one oxygen atom.

Hence, the total number of oxygen atoms present per molecule of the product is:

So, the correct option is : (1)

Quick Tip

Ozonolysis of alkenes cleaves the double bond and introduces an oxygen atom at each end, resulting in carbonyl compounds (aldehydes or ketones).
