JEE Main 2025 April 4 Shift 1 Mathematics Question Paper

Time Allowed :3 Hours	Maximum Marks :300	Total Questions :75

General Instructions

Read the following instructions very carefully and strictly follow them:

1. Multiple choice questions (MCQs)
2. Questions with numerical values as answers.
3. There are three sections: **Mathematics, Physics, Chemistry**.
4. **Mathematics:** 25 (20+5) 10 Questions with answers as a numerical value. Out of 10 questions, 5 questions are compulsory.
5. **Physics:** 25 (20+5) 10 Questions with answers as a numerical value. Out of 10 questions, 5 questions are compulsory..
6. **Chemistry:** 25 (20+5) 10 Questions with answers as a numerical value. Out of 10 questions, 5 questions are compulsory.
7. Total: 75 Questions (25 questions each).
8. 300 Marks (100 marks for each section).
9. **MCQs:** Four marks will be awarded for each correct answer and there will be a negative marking of one mark on each wrong answer.
10. **Questions with numerical value answers:** Candidates will be given four marks for each correct answer and there will be a negative marking of 1 mark for each wrong answer.

Mathematics

Section - A

1. Let $f, g : (1, \infty) \to R$ be defined as $f(x) = \frac{2x+3}{5x+2}$ and $g(x) = \frac{2-3x}{1-x}$. If the range of the function $fog : [2, 4] \to R$ is $[\alpha, \beta]$, then $\frac{1}{\beta-\alpha}$ is equal to

(1) 68
(2) 29
(3) 2
(4) 56

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2. Consider the sets $A = \{(x, y) \in R \times R : x^2 + y^2 = 25\}$,
$B = \{(x, y) \in R \times R : x^2 + 9y^2 = 144\}$, $C = \{(x, y) \in Z \times Z : x^2 + y^2 \leq 4\}$, and $D = A \cap B$.
The total number of one-one functions from the set D to the set C is:

(1) 15120
(2) 19320
(3) 17160
(4) 18290

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3. Let $A = \{1, 6, 11, 16, \ldots\}$ and $B = \{9, 16, 23, 30, \ldots\}$ be the sets consisting of the first 2025 terms of two arithmetic progressions. Then $n(A \cup B)$ is

(1) 3814
(2) 4027
(3) 3761
(4) 4003

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4. For an integer $n \geq 2$, if the arithmetic mean of all coefficients in the binomial expansion of $(x + y)^{2n-3}$ is 16, then the distance of the point $P(2n - 1, n^2 - 4n)$ from the line $x + y = 8$ is:

(1) $\sqrt{2}$
(2) $2\sqrt{2}$
(3) $5\sqrt{2}$
(4) $3\sqrt{2}$

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5. The probability of forming a 12 persons committee from 4 engineers, 2 doctors, and 10 professors containing at least 3 engineers and at least 1 doctor is:

(1) $\frac{129}{182}$
(2) $\frac{103}{182}$
(3) $\frac{17}{26}$
(4) $\frac{19}{26}$

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6. Let the shortest distance between the lines $\frac{x-3}{3} = \frac{y-\alpha}{-1} = \frac{z-3}{1}$ and $\frac{x+3}{-3} = \frac{y+7}{2} = \frac{z-\beta}{4}$ be $3\sqrt{30}$. Then the positive value of $5\alpha + \beta$ is

(1) 42
(2) 46
(3) 48
(4) 40

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7. If $\lim_{x \to 1} \frac{(x-1)(6+\lambda\cos(x-1))+\mu\sin(1-x)}{(x-1)^3} = -1$, where $\lambda, \mu \in R$, then $\lambda + \mu$ is equal to

(1) 18

(2) 20
(3) 19
(4) 17

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8. Let $f : [0, \infty) \to R$ be a differentiable function such that $f(x) = 1 - 2x + \int_0^x e^{x-t} f(t)\, dt$ for all $x \in [0, \infty)$. Then the area of the region bounded by $y = f(x)$ and the coordinate axes is
(1) $\sqrt{5}$
(2) $\frac{1}{2}$
(3) $\sqrt{2}$
(4) 2

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9. Let A and B be two distinct points on the line $L : \frac{x-6}{3} = \frac{y-7}{2} = \frac{z-7}{-2}$. Both A and B are at a distance $2\sqrt{17}$ from the foot of perpendicular drawn from the point $(1, 2, 3)$ on the line L. If O is the origin, then $\overrightarrow{OA} \cdot \overrightarrow{OB}$ is equal to:
(1) 49
(2) 47
(3) 21
(4) 62

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10. Let $f : R \to R$ be a continuous function satisfying $f(0) = 1$ and $f(2x) - f(x) = x$ for all $x \in R$. If $\lim_{n\to\infty} \left\{ f(x) - f\left(\frac{x}{2^n}\right) \right\} = G(x)$, then $\sum_{r=1}^{10} G(r^2)$ is equal to
(1) 540
(2) 385
(3) 420
(4) 215

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11. 1 + 3 + 5^2 + 7 + 9^2 + ... upto 40 terms is equal to
(1) 43890
(2) 41880
(3) 33980
(4) 40870

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12. In the expansion of $\left(\sqrt{5} + \frac{1}{\sqrt{5}}\right)^n$, $n \in N$, if the ratio of 15^{th} term from the beginning to the 15^{th} term from the end is $\frac{1}{6}$, then the value of ${}^nC_3$ is:
(1) 4060
(2) 1040
(3) 2300

(4) 4960

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13. Considering the principal values of the inverse trigonometric functions, $\sin^{-1}\left(\frac{\sqrt{3}}{2}x+\frac{1}{2}\sqrt{1-x^2}\right)$, $-\frac{1}{2}<x<\frac{1}{\sqrt{2}}$, is equal to

(1) $\frac{\pi}{4}+\sin^{-1}x$
(2) $\frac{\pi}{6}+\sin^{-1}x$
(3) $\frac{-5\pi}{6}-\sin^{-1}x$
(4) $\frac{5\pi}{6}-\sin^{-1}x$

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14. Consider two vectors $\vec{u}=3\hat{i}-\hat{j}$ and $\vec{v}=2\hat{i}+\hat{j}-\lambda\hat{k}$, $\lambda>0$. The angle between them is given by $\cos^{-1}\left(\frac{\sqrt{5}}{2\sqrt{7}}\right)$. Let $\vec{v}=\vec{v}_1+\vec{v}_2$, where $\vec{v}_1$ is parallel to $\vec{u}$ and $\vec{v}_2$ is perpendicular to $\vec{u}$. Then the value $|\vec{v}_1|^2+|\vec{v}_2|^2$ is equal to

(1) $\frac{23}{2}$
(2) 14
(3) $\frac{25}{2}$
(4) 10

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15. Let the three sides of a triangle are on the lines $4x-7y+10=0$, $x+y=5$, and $7x+4y=15$. Then the distance of its orthocenter from the orthocenter of the triangle formed by the lines $x=0$, $y=0$, and $x+y=1$ is

(1) 5
(2) $\sqrt{5}$
(3) $\sqrt{20}$
(4) 20

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16. The value of $\int_{-1}^{1}\frac{(1+\sqrt{|x|-x})e^x+(\sqrt{|x|-x})e^{-x}}{e^x+e^{-x}}\,dx$ is equal to

(1) $3-\frac{2\sqrt{2}}{3}$
(2) $2+\frac{2\sqrt{2}}{3}$
(3) $1-\frac{2\sqrt{2}}{3}$
(4) $1+\frac{2\sqrt{2}}{3}$

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17. The length of the latus-rectum of the ellipse, whose foci are $(2,5)$ and $(2,-3)$ and eccentricity is $\frac{4}{5}$, is

(1) $\frac{6}{5}$
(2) $\frac{50}{3}$
(3) $\frac{10}{3}$
(4) $\frac{18}{5}$

18. **Consider the equation $x^2 + 4x - n = 0$, where $n \in [20, 100]$ is a natural number. Then the number of all distinct values of n, for which the given equation has integral roots, is equal to**
(1) 7
(2) 8
(3) 6
(4) 9

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19. **A box contains 10 pens of which 3 are defective. A sample of 2 pens is drawn at random and let X denote the number of defective pens. Then the variance of X is**
(1) $\frac{11}{15}$
(2) $\frac{28}{75}$
(3) $\frac{2}{15}$
(4) $\frac{3}{5}$

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20. **If $10\sin^4\theta + 15\cos^4\theta = 6$, then the value of $\frac{27\csc^6\theta + 8\sec^6\theta}{16\sec^8\theta}$ is:**
(1) $\frac{2}{5}$
(2) $\frac{3}{4}$
(3) $\frac{3}{5}$
(4) $\frac{1}{5}$

SECTION-B

21. **If the area of the region $\{(x, y) : |x - 5| \leq y \leq 4\sqrt{x}\}$ is A, then $3A$ is equal to**
(1) 368
(2) 360
(3) 370
(4) 380

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22. **Let $A = \begin{bmatrix} \cos\theta & 0 & -\sin\theta \\ 0 & 1 & 0 \\ \sin\theta & 0 & \cos\theta \end{bmatrix}$. If for some $\theta \in (0, \pi)$, $A^2 = A^T$, then the sum of the diagonal elements of the matrix $(A + I)^3 + (A - I)^3 - 6A$ is equal to**
(1) 6
(2) 12
(3) 10
(4) 8

23. Let $A = \{z \in C : |z - 2 - i| = 3\}$, $B = \{z \in C : \mathbf{Re}(z - iz) = 2\}$, and $S = A \cap B$. Then $\sum_{z \in S} |z|^2$ is equal to

(1) 22
(2) 20
(3) 24
(4) 18

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24. Let C be the circle $x^2 + (y-1)^2 = 2$, E_1 and E_2 be two ellipses whose centres lie at the origin and major axes lie on the x-axis and y-axis respectively. Let the straight line $x + y = 3$ touch the curves C, E_1, and E_2 at $P(x_1, y_1)$, $Q(x_2, y_2)$, and $R(x_3, y_3)$ respectively. Given that P is the mid-point of the line segment QR and $PQ = \frac{2\sqrt{2}}{3}$, the value of $9(x_1y_1 + x_2y_2 + x_3y_3)$ is equal to

(1) 46
(2) 48
(3) 44
(4) 50

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25. Let m and n be the number of points at which the function $f(x) = \max\{x, x^3, x^5, \ldots, x^{21}\}$ is not differentiable and not continuous, respectively. Then $m + n$ is equal to

(1) 3
(2) 4
(3) 5
(4) 6
