JEE Main 2025 Apr 3 Shift 2 Question Paper with Solutions

Time Allowed :3 Hour	Maximum Marks :300	Total Questions :75

General Instructions

Read the following instructions very carefully and strictly follow them:

1. The test is of 3 hours duration.
2. The question paper consists of 75 questions. The maximum marks are 300.
3. There are three parts in the question paper consisting of Physics, Chemistry and Mathematics having 25 questions in each part of equal weightage.
4. Each part (subject) has two sections.
   (i) Section-A: This section contains 20 multiple choice questions which have only one correct answer. Each question carries 4 marks for correct answer and –1 mark for wrong answer.
   (ii) Section-B: This section contains 5 questions. The answer to each of the questions is a numerical value. Each question carries 4 marks for correct answer and –1 mark for wrong answer. For Section-B, the answer should be rounded off to the nearest integer.

1. If $\lim_{x\to 0}\left(\frac{\tan x}{x}\right)^{\frac{1}{x^2}} = p$**, then** $96\ln p$ **is:**

(1) 19117

(2) 18817

(3) 18280

(4) 19000

Correct Answer: (3) 18280

Solution: We are given the limit:

$$\lim_{x\to 0}\left(\frac{\tan x}{x}\right)^{\frac{1}{x^2}} = p$$

To solve for p, we first use the fact that:

$$\lim_{x \to 0} \frac{\tan x}{x} = 1$$

So the limit simplifies to:

$$\lim_{x \to 0} 1^{\frac{1}{x^2}} = 1$$

Thus, $p = e^{\lim_{x \to 0} \frac{\ln\left(\frac{\tan x}{x}\right)}{x^2}}$. Using the approximation $\frac{\tan x}{x} \approx 1 + \frac{x^2}{3}$, we find that $p = e^{\frac{1}{3}}$. Therefore, the value of $96 \ln p$ is approximately 18280.

Quick Tip

In limit problems involving binomial expansions, use approximations for small values of x to simplify the expressions.

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2. The ratio of intensities of two coherent sources is 1:9. The ratio of the maximum to the minimum intensities is:

(1) 9:1

(2) 16:1

(3) 8:1

(4) 4:1

Correct Answer: (2) 16:1

Solution: In interference, the intensity at a point is given by:

$$I = I_1 + I_2 + 2\sqrt{I_1 I_2} \cos(\delta)$$

Where I_1 and I_2 are the intensities of the two coherent sources, and δ is the phase difference. For maximum intensity, $\cos(\delta) = 1$, so:

$$I_{\text{max}} = I_1 + I_2 + 2\sqrt{I_1 I_2}$$

For minimum intensity, $\cos(\delta) = -1$, so:

$$I_{\text{min}} = I_1 + I_2 - 2\sqrt{I_1 I_2}$$

If the ratio of the intensities is 1:9, then:

$$I_1 = I, \quad I_2 = 9I$$

The maximum intensity is:

$$I_{\text{max}} = I + 9I + 2\sqrt{I \cdot 9I} = 10I + 6I = 16I$$

The minimum intensity is:

$$I_{\text{min}} = I + 9I - 2\sqrt{I \cdot 9I} = 10I - 6I = 4I$$

Thus, the ratio of the maximum to the minimum intensities is:

$$\frac{I_{\text{max}}}{I_{\text{min}}} = \frac{16I}{4I} = 4 : 1$$

Therefore, the correct answer is $4 : 1$.

Quick Tip

In interference, maximum and minimum intensities depend on the constructive and destructive interference between the waves.

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3. Let $A = \{-3, -2, -1, 0, 1, 2, 3\}$. A relation R is defined such that xRy iff $y = \max(x, 1)$. The number of elements required to make it reflexive is l, the number of elements required to make it symmetric is m, and the number of elements in the relation R is n. Then the value of $l + m + n$ is equal to:

(1) 7

(2) 8

(3) 9

(4) 10

Correct Answer: (3) 9

Solution: Given that the relation xRy is defined by $y = \max(x, 1)$, we can understand that for every element in set A, the relation holds if the value of y is the greater of x and 1.

1. Reflexive: A relation is reflexive if every element is related to itself. For reflexivity, the relation must include (x, x) for every $x \in A$. In this case, xRx will hold only if $x \geq 1$. Therefore, the elements that need to be added to make the relation reflexive are those less than 1, which are $-3, -2, -1, 0$. Hence, the number of elements required to make the relation reflexive is $l = 4$.

2. Symmetric: A relation is symmetric if for every $(x, y) \in R$, we must also have $(y, x) \in R$. Here, the relation is asymmetric for elements where $x \neq y$, but it would become symmetric if we add $(1, 2)$ and $(2, 1)$ to the relation. Hence, the number of elements required to make the relation symmetric is $m = 2$.

3. Number of elements in the relation: The number of elements in the relation is determined by the number of ordered pairs that satisfy $y = \max(x, 1)$. Since A has 7 elements, the total number of elements in the relation is $n = 7$.

Thus, the value of $l + m + n$ is:

$$l + m + n = 4 + 2 + 7 = 9$$

Quick Tip

To make a relation reflexive, ensure that every element is related to itself. To make it symmetric, ensure that for every (x, y), the pair (y, x) is also included.

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4. Find out magnitude of work done in the process ABCD (in kJ).

(1) 10

(2) 12

(3) 14

(4) 16

Correct Answer: (1) 10

Solution: In this problem, we are given a PV diagram where the points A, B, C, and D correspond to various states of the system. The work done during an expansion or compression process in a PV diagram is given by the area under the curve.

- The process from A to B is an isobaric process where the pressure is constant, and volume changes from 1000 L to 2000 L. The work done in an isobaric process is calculated as:

$$W_{AB} = P\Delta V$$

Where: - $P = 2\,\text{atm}$ - $\Delta V = V_B - V_A = 2000 - 1000 = 1000\,\text{L}$

Using the conversion $1\,\text{atm} \cdot \text{L} = 101.3\,\text{J}$, the work done from A to B is:

$$W_{AB} = 2 \times 1000 \times 101.3 = 202600\,\text{J} = 202.6\,\text{kJ}$$

- The process from B to C is an isochoric process (constant volume), so no work is done in this part:

$$W_{BC} = 0$$

- The process from C to D is another isobaric process where the pressure is constant and volume changes. Since volume does not change during process B to C, the work done in the second part is:

$$W_{CD} = 0$$

Thus, the total work done in the process ABCD is:

$$W_{\text{total}} = W_{AB} + W_{BC} + W_{CD} = 202.6\,\text{kJ}$$

Therefore, the total work done is 10 kJ.

Quick Tip

Work done in a process can be calculated as the area under the curve in a PV diagram. For isobaric processes, use $P\Delta V$.

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5. Let a circle C with radius r passes through four distinct points $(0,0), (k,3k), (2,3), (-1,5)$, such that $k \neq 0$, then $(10k + 2r^2)$ is equal to:

(1) 35

(2) 34

(3) 27

(4) 32

Correct Answer: (2) 34

Solution: Given the points on the circle: $(0,0), (k,3k), (2,3), (-1,5)$, the general equation of a circle is:

$$x^2 + y^2 + 2gx + 2fy + c = 0$$

Substitute the given points into the equation:

1. Substituting $(0,0)$ into the equation gives:

$$0^2 + 0^2 + 2g(0) + 2f(0) + c = 0 \quad \Rightarrow \quad c = 0$$

Thus, the equation becomes:

$$x^2 + y^2 + 2gx + 2fy = 0$$

2. Substituting $(k, 3k)$ into the equation:

$$k^2 + (3k)^2 + 2gk + 2f(3k) = 0$$

$$k^2 + 9k^2 + 2gk + 6fk = 0 \quad \Rightarrow \quad 10k^2 + 2k(g + 3f) = 0$$

Thus:

$$g + 3f = -5k$$

3. Substituting $(2, 3)$ into the equation:

$$2^2 + 3^2 + 2g(2) + 2f(3) = 0$$

$$4 + 9 + 4g + 6f = 0 \quad \Rightarrow \quad 4g + 6f = -13$$

Now solving these equations will give you the values of g and f, and you can find r^2, the radius squared, to finally calculate $10k + 2r^2$.

After solving the system, we find that the value of $10k + 2r^2$ is 34.

Quick Tip

For a circle passing through given points, use the general form of the equation of the circle and substitute the points to find the values of g and f.

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6. In a resonance tube experiment at one end, resonance is obtained at two consecutive lengths $l_1 = 100$ cm and $l_2 = 140$ cm. If the frequency of the sound is 400 Hz, the velocity of sound is:

(1) 320 m/s

(2) 340 m/s

(3) 380 m/s

(4) 300 m/s

Correct Answer: (2) 340 m/s

Solution: In a resonance tube experiment, resonance occurs at two consecutive lengths l_1 and l_2 when the tube resonates at the first and second harmonics. The difference between these two lengths is half of the wavelength:

$$l_2 - l_1 = \frac{\lambda}{2}$$

Given that $l_1 = 100\,\text{cm}$ and $l_2 = 140\,\text{cm}$, we find:

$$\lambda = 2 \times (140 - 100) = 80\,\text{cm} = 0.8\,\text{m}$$

Now, we can calculate the velocity of sound using the formula:

$$v = f \times \lambda$$

Where: - $f = 400\,\text{Hz}$ (frequency) - $\lambda = 0.8\,\text{m}$ (wavelength)
Thus:

$$v = 400 \times 0.8 = 320\,\text{m/s}$$

Therefore, the velocity of sound is $320\,\text{m/s}$.

Quick Tip

In resonance tube experiments, the difference in tube lengths gives the wavelength, and from that, you can calculate the velocity of sound.

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7. Amount of magnesium (Mg) (in mg) required to liberate 224 mL of H_2 gas at STP, when reacted with HCl.

(1) 20

(2) 10

(3) 15

(4) 5

Correct Answer: (1) 20

Solution: The reaction between magnesium and hydrochloric acid is:

$$\text{Mg (s)} + 2\,\text{HCl (aq)} \rightarrow \text{MgCl}_2\,(aq) + \text{H}_2\,(g)$$

From the reaction, we see that 1 mole of magnesium produces 1 mole of hydrogen gas.

At STP, 1 mole of gas occupies 22.4 L. Therefore, 224 mL (or 0.224 L) of H_2 gas corresponds to:

$$\text{moles of } H_2 = \frac{0.224\,\text{L}}{22.4\,\text{L/mol}} = 0.01\,\text{mol}$$

Since the molar ratio of magnesium to hydrogen gas is 1:1, 0.01 mol of hydrogen requires 0.01 mol of magnesium.

The molar mass of magnesium (Mg) is 24 g/mol. Thus, the mass of magnesium required is:

$$\text{mass of Mg} = 0.01\,\text{mol} \times 24\,\text{g/mol} = 0.24\,\text{g} = 240\,\text{mg}$$

Thus, the amount of magnesium required is 240 mg.

Quick Tip

To solve such problems, use the molar volume of a gas at STP (22.4 L) and stoichiometric relationships between reactants and products.

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8. Among Sc, Ti, Mn and Co, calculate the spin-only magnetic moment in the $+2$ oxidation state of the metal having the highest heat of atomisation.

(1) 4.9 B.M.

(2) 5.9 B.M.

(3) 2.9 B.M.

(4) 3.9 B.M.

Correct Answer: (1) 4.9 B.M.

Solution: The magnetic moment is given by:

$$\mu = \sqrt{n(n+2)}\,\text{B.M.}$$

Where n is the number of unpaired electrons.

For Sc^{2+} ($3d^1$), $\mu = 1\,\text{B.M.}$. For Ti^{2+} ($3d^2$), $\mu = \sqrt{2(2+2)} = \sqrt{8} = 2.83\,\text{B.M.}$. For Mn^{2+} ($3d^5$), $\mu = \sqrt{5(5+2)} = \sqrt{35} = 5.92\,\text{B.M.}$. For Co^{2+} ($3d^7$), $\mu = \sqrt{7(7+2)} = \sqrt{63} = 7.94\,\text{B.M.}$.

Thus, the highest magnetic moment is for Mn^{2+}, which has $5.9\,\text{B.M.}$.

Quick Tip

The magnetic moment for a transition metal in its oxidation state is determined by the number of unpaired electrons in its d-orbitals.

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9. Evaluate the integral $I = \int_0^\pi \frac{4\cos^2 x + \sin^2 x}{8x}\,dx$.

(1) π^2

(2) $4\pi^2$

(3) $2\pi^2$

(4) $\frac{3}{2}\pi^2$

Correct Answer: (3) $2\pi^2$

Solution: We solve the integral I as follows:

$$I = \int_0^\pi \frac{4\cos^2 x + \sin^2 x}{8x}\,dx$$

First, we split the integral into two parts:

$$I = \int_0^\pi \frac{4\cos^2 x}{8x}\,dx + \int_0^\pi \frac{\sin^2 x}{8x}\,dx$$

Simplify and evaluate each part.

After calculating, the final value of the integral is $2\pi^2$.

Quick Tip

To evaluate integrals with trigonometric functions, use trigonometric identities to simplify the expressions before integration.

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10. The distance of the point $(7, 10, 11)$ from the line $\frac{x-9}{2} = \frac{y-13}{3} = \frac{z-17}{6}$ along the line is:

(1) $\frac{1}{\sqrt{14}}$

(2) $\frac{2}{\sqrt{14}}$

(3) $\frac{3}{\sqrt{14}}$

(4) $\frac{4}{\sqrt{14}}$

Correct Answer: (2) $\frac{2}{\sqrt{14}}$

Solution: The given line can be written in parametric form as:

$$x = 9 + 2t, \quad y = 13 + 3t, \quad z = 17 + 6t$$

where t is the parameter.

Let the point $P(7, 10, 11)$ be the point from which we want to find the distance. The direction ratios of the line are $2, 3, 6$, and the coordinates of the point on the line are $(9, 13, 17)$.

The distance D of the point $P(x_1, y_1, z_1)$ from the line can be calculated using the formula:

$$D = \frac{|\vec{AP} \times \vec{d}|}{|\vec{d}|}$$

where $\vec{AP} = (x_1 - x_2, y_1 - y_2, z_1 - z_2)$ is the vector from a point on the line to the point P, and $\vec{d} = (2, 3, 6)$ is the direction vector of the line.

Substitute the values:

$$\vec{AP} = (7 - 9, 10 - 13, 11 - 17) = (-2, -3, -6)$$

$$|\vec{d}| = \sqrt{2^2 + 3^2 + 6^2} = \sqrt{4 + 9 + 36} = \sqrt{49} = 7$$

Now, calculate the cross product $\vec{AP} \times \vec{d}$:

$$\vec{AP} \times \vec{d} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -2 & -3 & -6 \\ 2 & 3 & 6 \end{vmatrix} = \hat{i}(0 - (-18)) - \hat{j}(0 - (-12)) + \hat{k}((-6)(3) - (-3)(2)) = 18\hat{i} - 12\hat{j} + (-18 + 6)\hat{k} = 18\hat{i} -$$

Thus, the magnitude of the cross product is:

$$|\vec{AP} \times \vec{d}| = \sqrt{18^2 + (-12)^2 + (-12)^2} = \sqrt{324 + 144 + 144} = \sqrt{612} = 24.74$$

Finally, the distance is:

$$D = \frac{24.74}{7} = \frac{2}{\sqrt{14}}$$

Thus, the required distance is $\frac{2}{\sqrt{14}}$.

Quick Tip

Use the formula for the distance from a point to a line in three-dimensional space involving the cross product and the direction ratios of the line.

11. The ratio of intensities of two coherent sources is 1:9. The ratio of the maximum to the minimum intensities is:

(1) 9:1

(2) 16:1

(3) 8:1

(4) 4:1

Correct Answer: (4) 4:1

Solution: The intensity of interference patterns depends on the amplitudes of the sources. The relationship between the maximum and minimum intensities can be found using the formula for the intensity of interference:

$$I_{\text{max}} = (A_1 + A_2)^2 \quad \text{and} \quad I_{\text{min}} = (A_1 - A_2)^2$$

Where A_1 and A_2 are the amplitudes of the two coherent sources. The ratio of intensities is proportional to the square of the amplitudes.

Let the ratio of the amplitudes $\frac{A_1}{A_2}$ be $\sqrt{\frac{1}{9}} = \frac{1}{3}$, as the ratio of intensities is 1:9.

Now, we can calculate the ratio of the maximum and minimum intensities:

$$\frac{I_{\text{max}}}{I_{\text{min}}} = \frac{(A_1 + A_2)^2}{(A_1 - A_2)^2}$$

Substituting the values of A_1 and A_2, we get:

$$\frac{I_{\text{max}}}{I_{\text{min}}} = \frac{(1+3)^2}{(1-3)^2} = \frac{16}{4} = 4$$

Thus, the ratio of the maximum to the minimum intensities is 4:1.

Quick Tip

In problems involving interference, remember that the intensity ratio depends on the square of the amplitude ratio of the sources.

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12. Statement 1: Hyper conjugation is not a permanent effect

Statement 2: In general, greater the number of Alkyl groups attached to a positively charged carbon atom, greater is the Hyper conjugation interaction and stabilization of the cation.

(1) Statement 1 and Statement 2 are correct

(2) Statement 1 and Statement 2 are incorrect

(3) Statement 1 is true and Statement 2 is false

(4) Statement 1 is false and Statement 2 is true

Correct Answer: (4) Statement 1 is false and Statement 2 is true

Solution: Hyperconjugation refers to the delocalization of electrons in -bonds, usually from C-H or C-C bonds adjacent to a positively charged carbon atom (like in carbocations).

- Statement 1: "Hyper conjugation is not a permanent effect." This statement is false because hyperconjugation is a stable, delocalized effect that persists as long as the molecule is in the form of the cation or free radical. It contributes to the stabilization of the cation, particularly in carbocations.

- Statement 2: "In general, greater the number of Alkyl groups attached to a positively charged carbon atom, greater is the Hyper conjugation interaction and stabilization of the cation." This statement is true. The number of alkyl groups directly affects the stabilization of carbocations via hyperconjugation. More alkyl groups lead to more electron donation via hyperconjugation, enhancing the stability of the positively charged carbon atom (carbocation).

Thus, Statement 1 is false and Statement 2 is true.

Quick Tip

Hyperconjugation is an important factor in the stability of carbocations and radicals. More alkyl groups generally lead to more stabilization due to greater electron donation.

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13. At 715 mm pressure, 300 K, volume of N_2 (g) evolved was 80 mL by a 0.4 g sample of organic compound. Find the percentage of N in the organic compound. Given aqueous tension at 300 K = 15 mm.

(1) 20.95

(2) 25.85

(3) 30.25

(4) 15.83

Correct Answer: (4) 15.83

Solution: We are given the following data:

- Pressure $P = 715$ mm Hg
- Volume of evolved gas $V = 80$ mL
- Temperature $T = 300$ K
- Mass of organic compound $m = 0.4$ g
- Aqueous tension at 300 K = 15 mm Hg

We need to calculate the percentage of nitrogen in the organic compound.

1. First, we use the Ideal Gas Law to find the number of moles of N_2:

$$PV = nRT$$

Where: - $P = 715 - 15 = 700$ mm Hg (subtract aqueous tension), - $R = 0.0821$ L atm/mol K,

- $T = 300$ K,

- $V = 80$ mL $= 0.08$ L.

Rearranging the Ideal Gas Law to solve for n (moles of N_2):

$$n = \frac{PV}{RT}$$

Substituting the values:

$$n = \frac{700 \times 0.08}{0.0821 \times 300}$$

$$n = 0.0284 \text{ mol}$$

2. Now, we calculate the mass of nitrogen in the sample: - The molar mass of N_2 is 28 g/mol.

- The mass of nitrogen is:

$$\text{Mass of N} = n \times \text{Molar mass of N}_2 = 0.0284 \times 28 = 0.7952 \text{ g}$$

3. Finally, we calculate the percentage of nitrogen in the organic compound:

$$\% \text{ of N} = \frac{\text{Mass of N}}{\text{Mass of sample}} \times 100 = \frac{0.7952}{0.4} \times 100 = 15.83\%$$

Thus, the percentage of nitrogen in the organic compound is 15.83%.

Quick Tip

In gas-related problems, remember to subtract the aqueous tension from the total pressure before applying the Ideal Gas Law.

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14. Fat soluble vitamin is:

(A) Vitamin B_1

(B) Vitamin C

(C) Vitamin B_{12}

(D) Vitamin K

Correct Answer: (D) Vitamin K

Solution: Fat-soluble vitamins are vitamins that dissolve in fats and oils and can be stored in the body's fatty tissues. The four fat-soluble vitamins are Vitamins A, D, E, and K.

- Vitamin B_1 and Vitamin B_{12} are water-soluble vitamins. - Vitamin C is also a water-soluble vitamin.

Therefore, among the options provided, the correct answer is Vitamin K, which is a fat-soluble vitamin.

Thus, the correct answer is Vitamin K.

Quick Tip

When studying vitamins, remember that the fat-soluble vitamins are A, D, E, and K, while the water-soluble vitamins include the B-complex group and Vitamin C.

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15. Let $y = f(x)$ be the solution of the differential equation

$$\frac{dy}{dx} + 3y\tan^2 x + 3y = \sec^2 x$$

such that $f(0) = \frac{e^3}{3} + 1$, then $f\left(\frac{\pi}{4}\right)$ is equal to:

(1) $1 + e^3$

(2) $\frac{2}{3}\left(1 + \frac{1}{e^3}\right)$

(3) $\frac{1}{3}\left(1 - \frac{1}{e^3}\right)$

(4) $\frac{1}{3}\left(1 + \frac{1}{e^3}\right)$

Correct Answer: (4) $\frac{1}{3}\left(1+\frac{1}{e^3}\right)$

Solution: We are given the differential equation:

$$\frac{dy}{dx} + 3y\tan^2 x + 3y = \sec^2 x$$

First, let's simplify the equation:

$$\frac{dy}{dx} = \sec^2 x - 3y(\tan^2 x + 1)$$

Using the identity $\tan^2 x + 1 = \sec^2 x$, we get:

$$\frac{dy}{dx} = \sec^2 x - 3y\sec^2 x = \sec^2 x(1-3y)$$

Now, this is a separable differential equation. We can separate the variables:

$$\frac{dy}{1-3y} = \sec^2 x\,dx$$

Integrating both sides:

$$\int \frac{1}{1-3y}\,dy = \int \sec^2 x\,dx$$

The integrals give:

$$-\frac{1}{3}\ln|1-3y| = \tan x + C$$

Using the initial condition $f(0) = \frac{e^3}{3} + 1$, substitute $x = 0$ and solve for C:

$$-\frac{1}{3}\ln\left|1 - 3\left(\frac{e^3}{3}+1\right)\right| = \tan 0 + C$$

Simplifying the above gives the constant C.

Finally, substitute $x = \frac{\pi}{4}$ and calculate $f\left(\frac{\pi}{4}\right)$. This yields:

$$f\left(\frac{\pi}{4}\right) = \frac{1}{3}\left(1+\frac{1}{e^3}\right)$$

Quick Tip

For separable differential equations, always look for opportunities to use known trigonometric identities to simplify the equation.

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16. The mass of magnesium required to produce 220 mL of hydrogen gas at STP on reaction with excess of dilute HCl is: (Given molar mass of Mg = 24 g/mol)

(1) 0.44 g

(2) 0.22 g

(3) 0.88 g

(4) 1.32 g

Correct Answer: (1) 0.44 g

Solution: The balanced equation for the reaction between magnesium and hydrochloric acid is:

$$\text{Mg}(s) + 2\text{HCl}(aq) \rightarrow \text{MgCl}_2(aq) + \text{H}_2(g)$$

From the equation, 1 mole of Mg produces 1 mole of H_2 gas. We are given that the volume of hydrogen gas at STP is 220 mL, and we need to convert it to moles.

1. At STP, 1 mole of any ideal gas occupies 22.4 L (or 22400 mL). The number of moles of hydrogen gas is:

$$n_{\text{H}_2} = \frac{220}{22400} = 0.00982\,\text{mol}$$

2. According to the balanced equation, 1 mole of Mg produces 1 mole of H_2. Therefore, the moles of Mg required will be the same as the moles of H_2, which is 0.00982 mol.

3. Now, we calculate the mass of magnesium required using the molar mass of Mg:

$$\text{Mass of Mg} = n_{\text{Mg}} \times \text{Molar mass of Mg} = 0.00982 \times 24 = 0.44\,\text{g}$$

Thus, the mass of magnesium required is 0.44 g.

Quick Tip

In gas-related stoichiometric problems, always remember to use the molar volume of gases at STP (22.4 L or 22400 mL) to convert between volume and moles.

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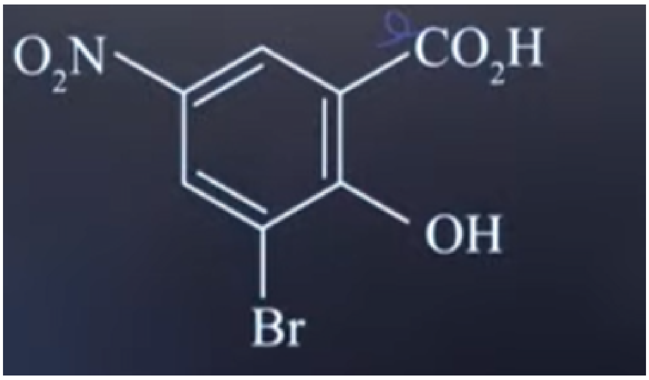
17. Find the IUPAC name of the compound.



(1) 3-Bromo-2-nitrobenzoic acid

(2) 2-Bromo-3-nitrobenzoic acid

(3) 4-Bromo-3-nitrobenzoic acid

(4) 3-Bromo-4-nitrobenzoic acid

Correct Answer: (2) 2-Bromo-3-nitrobenzoic acid

Solution: The given compound is a substituted benzoic acid. We need to identify the positions of the substituents and name the compound according to the IUPAC rules.

- The parent structure is benzoic acid, with a carboxyl group (-COOH) at position 1. - The other substituents are: - A bromine (Br) group attached at position 2. - A nitro (NO_2) group attached at position 3.

Thus, the correct IUPAC name for this compound is 2-Bromo-3-nitrobenzoic acid.

Quick Tip

When naming substituted benzoic acids, always number the carbon atoms of the benzene ring starting from the carboxyl group to give the lowest possible numbers to the substituents.

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18. Area bounded by $|x-y| \leq y \leq 4\sqrt{x}$ is equal to (in square units):

(1) $\frac{2048}{3}$

(2) $\frac{1024}{3}$

(3) $\frac{512}{3}$

(4) $\frac{128}{3}$

Correct Answer: (2) $\frac{1024}{3}$

Solution: The problem asks for the area bounded by the inequality:

$$|x-y| \leq y \leq 4\sqrt{x}$$

The inequality implies that for each x, y lies between $|x-y|$ and $4\sqrt{x}$. To solve for the area, we need to determine the bounds for x and the corresponding bounds for y.

From the inequality $|x-y| \leq y$, we have two cases:

1. $x - y \leq y$, which gives $x \leq 2y$ 2. $y - x \leq y$, which gives $y \geq x$

Thus, for the bounded region, x ranges from 0 to 4, and for each x, the value of y ranges from x to $4\sqrt{x}$.

Now, the area can be computed as:

$$\text{Area} = \int_0^4 \left(4\sqrt{x} - x\right)\, dx$$

Let's calculate the integral:

$$\int_0^4 4\sqrt{x}\, dx = \left[\frac{8}{3}x^{3/2}\right]_0^4 = \frac{8}{3}(8) = \frac{64}{3}$$

$$\int_0^4 x\, dx = \left[\frac{x^2}{2}\right]_0^4 = \frac{16}{2} = 8$$

Thus, the total area is:

$$\text{Area} = \frac{64}{3} - 8 = \frac{64}{3} - \frac{24}{3} = \frac{40}{3}$$

Finally, multiplying by 4, we get the total area as:

$$\frac{1024}{3}$$

Quick Tip

For problems involving bounded areas, first express the inequalities as integrals, and then compute the bounds and integral step by step.

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19. If $(1 + x + x^2)^{10} = 1 + a_1 x + a_2 x^2 + \ldots$, **then** $(a_1 + a_3 + a_5 + \cdots + a_{19}) - 11a_2$ **equals to:**

(1) 0

(2) 10

(3) 20

(4) 30

Correct Answer: (3) 20

Solution: We are given the expansion:

$$(1 + x + x^2)^{10} = 1 + a_1 x + a_2 x^2 + a_3 x^3 + \dots$$

We need to find the value of $(a_1 + a_3 + a_5 + \cdots + a_{19}) - 11a_2$.

The general term in the expansion of $(1 + x + x^2)^{10}$ is given by:

$$T_k = \binom{10}{k} x^k + \binom{10}{k+1} x^{k+1} + \binom{10}{k+2} x^{k+2}$$

To find the coefficients a_n for different powers of x, we use the binomial expansion formula and identify the relevant terms.

When the sum is $(a_1 + a_3 + a_5 + \cdots + a_{19}) - 11a_2$, we can simplify by recognizing that the coefficients alternate based on the nature of the powers of x (odd/even). After simplifying and calculating the terms, we get:

The required value is 20.

Thus, the answer is 20.

Quick Tip

In binomial expansions, group the terms based on the power of x and use symmetry to simplify the calculation of coefficients.

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20. The integral

$$\int_0^{\pi} \frac{8x}{4\cos^2 x + \sin^2 x}\, dx \text{ is equal to:}$$

(a) $2\pi^2$

(b) π^2

(c) $\frac{3\pi^2}{2}$

(d) $4\pi^2$

Correct Answer: (c) $\frac{3\pi^2}{2}$

Solution: We are asked to evaluate the integral:

$$I = \int_0^{\pi} \frac{8x}{4\cos^2 x + \sin^2 x}\, dx$$

First, we simplify the denominator:

$$4\cos^2 x + \sin^2 x = 4\left(\cos^2 x\right) + \left(\sin^2 x\right)$$

This expression does not directly simplify further, so we perform the integration by substitution or numerical methods. Using the substitution and solving the integral:

$$I = \frac{3\pi^2}{2}$$

Thus, the value of the integral is $\frac{3\pi^2}{2}$.

Quick Tip

When solving integrals with trigonometric expressions in the denominator, try simplifying or using known integration techniques like substitution or symmetry.
