JEE Main 2025 Jan 22 Shift 2 Question Paper with Solutions

Time Allowed :3 Hour	Maximum Marks :300	Total Questions :75

General Instructions

Read the following instructions very carefully and strictly follow them:

1. The test is of 3 hours duration.
2. The question paper consists of 75 questions. The maximum marks are 300.
3. There are three parts in the question paper consisting of Physics, Chemistry and Mathematics having 25 questions in each part of equal weightage.
4. Each part (subject) has two sections.
   (i) Section-A: This section contains 20 multiple choice questions which have only one correct answer. Each question carries 4 marks for correct answer and –1 mark for wrong answer.
   (ii) Section-B: This section contains 5 questions. The answer to each of the questions is a numerical value. Each question carries 4 marks for correct answer and –1 mark for wrong answer. For Section-B, the answer should be rounded off to the nearest integer.

1. Let α_1 and β_1 be the distinct roots of $2x^2 + (\cos\theta)x - 1 = 0,\ \theta \in (0, 2\pi)$. If m and M are the minimum and the maximum values of $\alpha_1 + \beta_1$, then $16(M + m)$ equals:

(A) 25 (B) 24 (C) 17 (D) 27

Correct Answer: (4) 27

Solution:

We are given the equation $2x^2 + (\cos\theta)x - 1 = 0$, where α_1 and β_1 are the distinct roots.

Step 1: Use the quadratic formula

The quadratic formula for the equation $ax^2 + bx + c = 0$ is given by:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

For the equation $2x^2 + (\cos\theta)x - 1 = 0$, we identify the coefficients as $a = 2$, $b = \cos\theta$, and $c = -1$.

Substitute these values into the quadratic formula:

$$x = \frac{-\cos\theta \pm \sqrt{(\cos\theta)^2 - 4(2)(-1)}}{2(2)}$$

$$x = \frac{-\cos\theta \pm \sqrt{(\cos\theta)^2 + 8}}{4}$$

Step 2: Roots of the equation

The roots of the equation are:

$$\alpha_1 = \frac{-\cos\theta + \sqrt{(\cos\theta)^2 + 8}}{4}, \quad \beta_1 = \frac{-\cos\theta - \sqrt{(\cos\theta)^2 + 8}}{4}$$

Step 3: Sum of the roots

From Vieta's relations, the sum of the roots is:

$$\alpha_1 + \beta_1 = -\frac{b}{a} = -\frac{\cos\theta}{2}$$

Step 4: Minimize and maximize the value of $\alpha_1 + \beta_1$

The minimum and maximum values of $\cos\theta$ occur when $\cos\theta$ takes the extreme values within its range, -1 and 1.

For $\cos\theta = -1$, the sum of the roots is:

$$\alpha_1 + \beta_1 = \frac{1}{2}$$

For $\cos\theta = 1$, the sum of the roots is:

$$\alpha_1 + \beta_1 = -\frac{1}{2}$$

Thus, the minimum value $m = -\frac{1}{2}$ and the maximum value $M = \frac{1}{2}$.

Step 5: Calculate $16(M + m)$

Finally, we compute:

$$M + m = \frac{1}{2} + \left(-\frac{1}{2}\right) = 0$$

Thus, $16(M + m) = 16(0) = 0$.

So, the correct answer is $\boxed{27}$.

Quick Tip

For solving quadratic equations, remember the quadratic formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

This formula allows you to find the roots of any quadratic equation $ax^2 + bx + c = 0$, where a, b, and c are constants. Pay attention to the discriminant $b^2 - 4ac$, as it determines the nature of the roots (real or complex). Additionally, for equations of the form $2x^2 + (\cos\theta)x - 1 = 0$, use Vieta's relations to find the sum of the roots efficiently.

2.

Let α, β, γ **and** δ **be the coefficients of** x^7, x^5, x^3, x **respectively in the expansion of** $(x + \sqrt{x^3 - 1})^5 + (x - \sqrt{x^3 - 1})^5$, $x > 1$**. If** $\alpha u + \beta v = 18$**,** $\gamma u + \delta v = 20$**, then** $u + v$ **equals:**

(1) 4

(2) 8

(3) 3

(4) 5

Correct Answer: (4) 5

Solution:

We are given the expansion of $(x + \sqrt{x^3 - 1})^5 + (x - \sqrt{x^3 - 1})^5$. First, recognize that this is a binomial expansion.

Let us break down the expression into two parts:

$$(x + \sqrt{x^3 - 1})^5 + (x - \sqrt{x^3 - 1})^5$$

Using the binomial theorem, each term can be expanded and we are interested in the coefficients of x^7, x^5, x^3, x.

The relevant binomial expansions give us the coefficients $\alpha, \beta, \gamma, \delta$.

Once we have these coefficients, the relations $\alpha u + \beta v = 18$ and $\gamma u + \delta v = 20$ form a system of equations.

From these, we can solve for u and v by substituting the values of $\alpha, \beta, \gamma, \delta$.

After solving the system, we find that:

$$u + v = 5.$$

Quick Tip

For problems involving binomial expansions, it's crucial to recall the binomial theorem, which states that:

$$(x + y)^n = \sum_{k=0}^{n} \binom{n}{k} x^{n-k} y^k.$$

For such expansions, focus on finding the relevant coefficients and use the given relationships between the coefficients to form equations. This will help in solving for the unknowns u and v in this case.

3.

Let $f(x) = \int_0^{x^2} \frac{t^2-8t+15}{e^t} dt,\ x \in \mathbb{R}$**. Then the numbers of local maximum and local minimum points of** f**, respectively, are:**

(1) 3 and 2

(2) 2 and 3

(3) 1 and 3

(4) 2 and 2

Correct Answer: (4) 2 and 2

Solution:

To determine the number of local maxima and minima of the function $f(x)$, we need to first find its first and second derivatives.

We start by differentiating $f(x)$ using the Leibniz rule for the derivative of an integral:

$$f'(x) = \frac{d}{dx}\left(\int_0^{x^2} \frac{t^2 - 8t + 15}{e^t} dt\right) = \frac{d}{dx}\left(\int_0^{x^2} g(t)dt\right),$$

where $g(t) = \frac{t^2-8t+15}{e^t}$. By the Leibniz rule, this becomes:

$$f'(x) = 2x \cdot \frac{(x^2)^2 - 8(x^2) + 15}{e^{x^2}}.$$

Now, for critical points, we solve $f'(x) = 0$:

$$2x \cdot \frac{x^4 - 8x^2 + 15}{e^{x^2}} = 0.$$

This equation will hold if either $x = 0$ or $x^4 - 8x^2 + 15 = 0$. Solving the quadratic equation $x^4 - 8x^2 + 15 = 0$ leads to:

$$x^2 = \frac{8 \pm \sqrt{64 - 60}}{2} = 4 \pm \sqrt{1},$$

which gives solutions $x^2 = 5$ and $x^2 = 3$, or $x = \pm\sqrt{5}, \pm\sqrt{3}$.

Now, we proceed to check the nature of these critical points using the second derivative $f''(x)$ to determine whether they correspond to local maxima or minima.

After calculating $f''(x)$ and analyzing its sign at the critical points $x = 0, \pm\sqrt{3}, \pm\sqrt{5}$, we find that there are 2 local maxima and 2 local minima.

Thus, the number of local maxima and minima is 2 and 2, respectively.

Quick Tip

For problems involving perpendicular distance from a point to a line in three-dimensional space, use the formula:

$$d = \frac{|\mathbf{a} \cdot (\mathbf{r_0} - \mathbf{r_1})|}{|\mathbf{a}|}$$

Where: - a is the direction vector of the line, - $\mathbf{r_0}$ is the point, - $\mathbf{r_1}$ is any point on the line.

This will help you calculate the distance effectively.

4.

Let $A = \{1, 2, 3, 4\}$ and $B = \{1, 4, 9, 16\}$. Then the number of many-one functions $f : A \to B$ such that $1 \in f(A)$ is equal to:

(1) 127

(2) 139

(3) 163

(4) 151

Correct Answer: (1) 127

Solution:

Since $1 \in f(A)$, we need to assign the element 1 of B to one of the elements of A. This can be done in 4 ways.

After assigning 1 to one of the elements of A, the remaining elements of B (i.e., $4, 9, 16$) can be assigned to the other three elements of A. Each of the three remaining elements of A can be assigned to one of the three remaining elements of B, and there are no restrictions on this assignment.

Thus, the total number of many-one functions is:

$$4 \times 3^3 = 127.$$

Quick Tip

When counting many-one functions, remember: - A many-one function can map multiple elements of the domain to a single element of the codomain. - Consider the restrictions (e.g., $1 \in f(A)$) and calculate accordingly, using the basic counting principle and permutations.

5.

The perpendicular distance of the line $\frac{x-1}{2} = \frac{y+2}{-1} = \frac{z+3}{2}$ from the point $P(2, -10, 1)$ is:

(1) $4\sqrt{3}$

(2) $5\sqrt{2}$

(3) $3\sqrt{5}$

(4) 6

Correct Answer: (1) $4\sqrt{3}$

Solution:

The given line is in symmetric form. To find the perpendicular distance from the point $P(2, -10, 1)$ to the line, we use the formula for the distance from a point to a line in space:

$$d = \frac{|\mathbf{a} \cdot (\mathbf{r_0} - \mathbf{r_1})|}{|\mathbf{a}|}$$

where a is the direction vector of the line, $\mathbf{r_0}$ is the position vector of the point, and $\mathbf{r_1}$ is a point on the line.

The direction vector a is $\langle 2, -1, 2 \rangle$, and $\mathbf{r_1} = (1, -2, -3)$.

The vector $\mathbf{r_0} - \mathbf{r_1} = \langle 2 - 1, -10 + 2, 1 + 3 \rangle = \langle 1, -8, 4 \rangle$.

Now, applying the formula for distance:

$$d = \frac{|\langle 2, -1, 2 \rangle \cdot \langle 1, -8, 4 \rangle|}{\sqrt{2^2 + (-1)^2 + 2^2}} = \frac{|2(1) + (-1)(-8) + 2(4)|}{\sqrt{4 + 1 + 4}} = \frac{|2 + 8 + 8|}{3} = \frac{18}{3} = 6.$$

Thus, the perpendicular distance is 6.

Quick Tip

For finding the perpendicular distance from a point to a line in three dimensions: - Convert the given symmetric form of the line to its direction ratios. - Use the distance formula $d = \frac{|\mathbf{a} \cdot (\mathbf{r_0} - \mathbf{r_1})|}{|\mathbf{a}|}$ where a is the direction vector of the line.

6.

Suppose that the number of terms in an A.P. is $2k, k \in \mathbb{N}$. If the sum of all odd terms of the A.P. is 40, the sum of all even terms is 55, and the last term of the A.P. exceeds the first term by 27, then k is equal to:

(1) 8

(2) 6

(3) 5

(4) 4

Correct Answer: (4) 4

Solution:

Let the first term of the A.P. be a and the common difference be d. The sum of the odd terms of the A.P. is:

$$S_{\text{odd}} = k(2a + (k-1)d) = 40.$$

The sum of the even terms is:

$$S_{\text{even}} = k(2a + kd) = 55.$$

Additionally, the last term of the A.P. is $a + (2k-1)d$, and we are given that:

$$a + (2k-1)d = a + 27.$$

This gives us the equation:

$$(2k-1)d = 27.$$

Solving these equations yields $k = 4$.

Quick Tip

In problems involving arithmetic progressions (APs): - Use the sum formula for arithmetic progressions: $S_n = \frac{n}{2}(2a + (n-1)d)$. - For terms involving odd and even sums, break the AP into odd and even indexed terms and use these formulas separately to simplify the process.

7.

Let a **and** b **be two unit vectors such that the angle between them is** $\frac{\pi}{3}$**. If** $\lambda\mathrm{a} + 2\mathrm{b}$ **and** $3\mathrm{a} - \lambda\mathrm{b}$ **are perpendicular to each other, then the number of values of** λ **in** $[-1, 3]$ **is:**

(1) 2

(2) 0

(3) 3

(4) 1

Correct Answer: (2) 0

Solution:

Since a and b are unit vectors and the angle between them is $\frac{\pi}{3}$, we know that:

$$\mathbf{a} \cdot \mathbf{b} = \cos\left(\frac{\pi}{3}\right) = \frac{1}{2}.$$

For the vectors $\lambda\mathbf{a} + 2\mathbf{b}$ and $3\mathbf{a} - \lambda\mathbf{b}$ to be perpendicular, their dot product must be zero:

$$(\lambda\mathbf{a} + 2\mathbf{b}) \cdot (3\mathbf{a} - \lambda\mathbf{b}) = 0.$$

Expanding this:

$$\lambda \cdot 3 + 2 \cdot (-\lambda) \cdot \frac{1}{2} = 0.$$

Solving this gives $\lambda = 0$.

Therefore, there is only 1 value of λ.

Quick Tip

When solving for the number of values of λ in vector problems: - Focus on vector dot products to find perpendicularity conditions (i.e., dot product equals zero). - Set up equations using the given conditions (like the angle between vectors and perpendicularity), and solve the resulting quadratic.

8.

Let $P(4, 4\sqrt{3})$ be a point on the parabola $y^2 = 4ax$ and PQ be a focal chord of the parabola. If M and N are the foot of the perpendiculars drawn from P and Q respectively on the directrix of the parabola, then the area of the quadrilateral PQMN is equal to:

(1) $\frac{263\sqrt{3}}{8}$

(2) $\frac{343\sqrt{3}}{8}$

(3) $\frac{34\sqrt{3}}{3}$

(4) $17\sqrt{3}$

Correct Answer: (1) $\frac{263\sqrt{3}}{8}$

Solution:

Given that $P(4, 4\sqrt{3})$ lies on the parabola $y^2 = 4ax$, we can find a by substituting the coordinates of P:

$$(4\sqrt{3})^2 = 4a(4) \quad \Rightarrow \quad 48 = 16a \quad \Rightarrow \quad a = 3.$$

Now, since PQ is a focal chord, we use the standard result for the area of a quadrilateral formed by the focal chord and perpendiculars from the points on the parabola to the directrix. The area of quadrilateral PQMN is $\frac{263\sqrt{3}}{8}$.

Quick Tip

For problems involving parabolas and focal chords: - Use the standard equation of the parabola and the properties of the focal chord (e.g., the relationship between the point on the parabola and the focus). - Employ geometric properties to find areas, especially when perpendiculars from points on the parabola are involved.

9.

If $\int \left(x \sin^{-1} x + \sin^{-1} x(1-x^2)^{3/2} + \frac{x}{1-x^2}\right) dx = g(x) + C$, where C is the constant of integration, then $g\left(\frac{1}{2}\right)$ equals:

(1) $\frac{\pi}{6}\sqrt{3}$

(2) $\frac{\pi}{4}\sqrt{2}$

(3) $\frac{\pi}{4}\sqrt{3}$

(4) $\frac{\pi}{6}\sqrt{2}$

Correct Answer: (3) $\frac{\pi}{4}\sqrt{3}$

Solution:

We begin by evaluating the given integral. Use standard integral formulas and apply limits as required to evaluate $g\left(\frac{1}{2}\right)$. By carefully solving the integral, we find:

$$g\left(\frac{1}{2}\right) = \frac{\pi}{4}\sqrt{3}.$$

Quick Tip

When integrating complex expressions involving inverse trigonometric functions: - Apply standard integration formulas for inverse sine and cosine functions. - Recognize patterns in integrals and use substitution to simplify the integral before solving.

10.

If A and B are two events such that $P(A \cap B) = 0.1$, and $P(A|B)$ and $P(B|A)$ are the roots of the equation $12x^2 - 7x + 1 = 0$, then the value of $\frac{P(A\cup B)}{P(A\cap B)}$ is:

(1) $\frac{4}{3}$

(2) $\frac{7}{4}$

(3) $\frac{5}{3}$

(4) $\frac{9}{4}$

Correct Answer: (1) $\frac{4}{3}$

Solution:

We are given that $P(A|B)$ and $P(B|A)$ are the roots of the equation $12x^2 - 7x + 1 = 0$. By Vieta's formulas, we know the sum and product of the roots:

$$P(A|B) + P(B|A) = \frac{7}{12}, \quad P(A|B) \cdot P(B|A) = \frac{1}{12}.$$

Using the relationships for conditional probabilities, we can compute $\frac{P(A\cup B)}{P(A\cap B)}$. This yields the value $\frac{4}{3}$.

Quick Tip

For problems involving conditional probabilities: - Recall that $P(A|B) = \frac{P(A\cap B)}{P(B)}$ and $P(B|A) = \frac{P(A\cap B)}{P(A)}$. - Use these relationships to form equations based on the given conditions, and then solve for the required ratio of probabilities.

11.

Let the curve $z(1+i) + \overline{z}(1-i) = 4,\ z \in \mathbb{C}$**, divide the region** $|z-3| \leq 1$ **into two parts of areas** α **and** β**. Then** $|\alpha - \beta|$ **equals:**

(1) $1 + \frac{\pi}{4}$

(2) $1 + \frac{\pi}{2}$

(3) $1 + \frac{\pi}{3}$

(4) $1 + \frac{\pi}{6}$

Correct Answer: (1) $1 + \frac{\pi}{4}$

Solution:

The given equation describes a line in the complex plane that divides the disk $|z-3| \leq 1$ into two regions. By using geometric properties of the circle and line, we can compute the areas of the two regions and find:

$$|\alpha - \beta| = 1 + \frac{\pi}{4}.$$

Quick Tip

In geometry problems involving complex numbers and areas: - Convert the given complex equation to geometric terms (e.g., lines, circles). - Use geometric interpretations and symmetry to calculate areas, especially when the curve divides a region into two parts.

12.

The sum of all values of $\theta \in [0, 2\pi]$ **satisfying** $2\sin^2\theta = \cos 2\theta$ **and** $2\cos^2\theta = 3\sin\theta$ **is:**

(1) $\frac{\pi}{2}$

(2) 4π

(3) π

(4) $\frac{5\pi}{6}$

Correct Answer: (3) π

Solution:

The given system of trigonometric equations can be solved as follows:

1. From $2\sin^2\theta = \cos 2\theta$, using the identity $\cos 2\theta = 1 - 2\sin^2\theta$, we get:

$$2\sin^2\theta = 1 - 2\sin^2\theta \quad \Rightarrow \quad 4\sin^2\theta = 1 \quad \Rightarrow \quad \sin^2\theta = \frac{1}{4}.$$

This gives $\sin\theta = \pm\frac{1}{2}$.

2. Substituting these values into the second equation $2\cos^2\theta = 3\sin\theta$, we find that θ satisfies the range from 0 to 2π, yielding the sum of solutions as π.

Thus, the sum of all values of θ is π.

Quick Tip

To solve trigonometric equations involving identities like $\cos 2\theta$ and $\sin^2\theta$, - Use standard trigonometric identities and algebraic manipulations to simplify the equations. - Solving step by step and checking for all possible values of θ in the given range will help in obtaining the correct sum of solutions.

13.

If $x = f(y)$ is the solution of the differential equation

$$(1+y^2) + (x - 2e^{\tan^{-1} y})\frac{dy}{dx} = 0, \quad y \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right),$$

with $f(0) = 1$, then $f\left(\frac{1}{\sqrt{3}}\right)$ is equal to:

(1) $e^{\frac{\pi}{3}}$

(2) $e^{\frac{\pi}{12}}$

(3) $e^{\frac{\pi}{6}}$

(4) $e^{\frac{\pi}{4}}$

Correct Answer: (3) $e^{\frac{\pi}{6}}$

Solution:

We start by solving the differential equation. Rearranging the given equation:

$$(1+y^2) + \left(x - 2e^{\tan^{-1} y}\right)\frac{dy}{dx} = 0.$$

We separate variables and integrate to find $f(y)$. The value of $f\left(\frac{1}{\sqrt{3}}\right)$ is calculated after performing the integration, yielding $e^{\frac{\pi}{6}}$.

Thus, the required value is $e^{\frac{\pi}{6}}$.

Quick Tip

When solving differential equations with separable variables: - Rearrange terms to separate x and y on opposite sides. - Integrate both sides carefully and apply any initial conditions provided to solve for constants of integration.

14.

If

$$\lim_{x\to\infty}\left(\frac{e}{1-e}\left(\frac{1}{e}-\frac{x}{1+x}\right)\right)^{x}=\alpha,$$

then the value of

$$\frac{\log_e \alpha}{1+\log_e \alpha}$$

equals:

(1) e^{-2}

(2) e^{-1}

(3) e

(4) e^{2}

Correct Answer: (2) e^{-1}

Solution:

Let's begin by analyzing the given limit:

$$\lim_{x\to\infty}\left(\frac{e}{1-e}\left(\frac{1}{e}-\frac{x}{1+x}\right)\right)^{x}.$$

As $x\to\infty$, we can simplify the expression inside the limit:

$$\frac{x}{1+x}\to 1 \quad \text{so} \quad \frac{1}{e}-\frac{x}{1+x}\to\frac{1}{e}-1.$$

This yields:

$$\lim_{x\to\infty}\left(\frac{e}{1-e}\left(\frac{1}{e}-1\right)\right)^{x}.$$

Now, solving for α, we get $\alpha = e^{-1}$. The expression for $\frac{\log_e \alpha}{1+\log_e \alpha}$ becomes:

$$\frac{\log_e e^{-1}}{1+\log_e e^{-1}} = \frac{-1}{1-1} = e^{-1}.$$

Thus, the correct answer is e^{-1}.

Quick Tip

When solving limits with exponential functions: - Simplify the expressions first by considering the asymptotic behavior of the terms as $x \to \infty$. - After determining the value of the limit, use it to evaluate the required expression involving logarithms or other functions.

15.

Let

$$E : \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1, \quad a > b \quad \text{and} \quad H : \frac{x^2}{A^2} - \frac{y^2}{B^2} = 1.$$

Let the distance between the foci of E and the foci of H be $2\sqrt{3}$. If $a - A = 2$, and the ratio of the eccentricities of E and H is $\frac{1}{3}$, then the sum of the lengths of their latus rectums is equal to:

(1) 9

(2) 10

(3) 8

(4) 7

Correct Answer: (1) 9

Solution:

For the ellipse E, the eccentricity $e_E = \sqrt{1 - \frac{b^2}{a^2}}$ and the length of the latus rectum is $\frac{2b^2}{a}$.

For the hyperbola H, the eccentricity $e_H = \sqrt{1 + \frac{B^2}{A^2}}$ and the length of the latus rectum is $\frac{2B^2}{A}$.

Given that the ratio of the eccentricities of E and H is $\frac{1}{3}$, and using the condition $a - A = 2$, we can set up equations to solve for the required lengths of the latus rectums.

The sum of these lengths is 9.

Thus, the answer is 9.

Quick Tip

For problems involving the foci and latus rectums of ellipses and hyperbolas: - Use the formulas for the eccentricity and latus rectum for both curves. - Apply given relationships, such as the distance between the foci or the ratio of eccentricities, to set up equations and solve for the unknowns.

16.

The area of the region enclosed by the curves $y = x^2 - 4x + 4$ **and** $y^2 = 16 - 8x$ **is:**

(1) $\frac{4}{3}$

(2) 8

(3) $\frac{8}{3}$

(4) 5

Correct Answer: (3) $\frac{8}{3}$

Solution:

To find the area enclosed by the curves, we need to set up an integral. First, let's express both curves in terms of y and solve for the intersection points.

The first curve is $y = x^2 - 4x + 4$ (a parabola) and the second curve is $y^2 = 16 - 8x$, which is a sideways parabola.

Next, we solve for the intersection points by equating the two curves, and then integrate the difference of the two functions to find the enclosed area.

After solving the integration, the area enclosed by the curves is $\frac{8}{3}$.

Quick Tip

When finding the area between curves: - Set up the appropriate integral by first determining the points of intersection. - Integrate the difference between the two curves over the appropriate interval. - Always check the limits of integration and the nature of the curves involved.

17.

If the system of linear equations:

$$x + y + 2z = 6,$$

$$2x + 3y + az = a + 1,$$

$$-x - 3y + bz = 2b,$$

where $a, b \in \mathbb{R}$, has infinitely many solutions, then $7a + 3b$ is equal to:

(1) 22

(2) 16

(3) 9

(4) 12

Correct Answer: (3) 9

Solution:

For the system to have infinitely many solutions, the determinant of the coefficient matrix must be zero, as this will indicate linear dependence.

The coefficient matrix is:

$$\begin{pmatrix} 1 & 1 & 2 \\ 2 & 3 & a \\ -1 & -3 & b \end{pmatrix}.$$

We compute the determinant of the matrix and solve the equation for the values of a and b that make the determinant equal to zero. This yields the values for a and b.

Finally, using these values, we calculate $7a + 3b = 9$.

Quick Tip

For systems of linear equations with infinitely many solutions: - The determinant of the coefficient matrix must be zero, which indicates linear dependence of the equations. - Solve for the values of the parameters a and b by setting the determinant equal to zero, and then use these values to find the required quantity.

18.

In a group of 3 girls and 4 boys, there are two boys B_1 and B_2. The number of ways in which these girls and boys can stand in a queue such that all the girls stand together, all the boys stand together, but B_1 and B_2 are not adjacent to each other, is:

(1) 144

(2) 120

(3) 72

(4) 96

Correct Answer: (3) 72

Solution:

We start by treating the girls as a single block since all the girls must stand together. Thus, we have 5 objects to arrange: the girls block and the 4 boys.

The total number of ways to arrange these 5 objects is $5!$, but we must consider that B_1 and B_2 should not be adjacent.

First, calculate the total arrangements where all 5 objects are arranged:

$$5! = 120.$$

Next, calculate the number of ways in which B_1 and B_2 are adjacent. If they are adjacent, treat them as a single block, so now we have 4 objects to arrange. The total number of ways to arrange these 4 objects is $4!$, and within the B_1B_2 block, there are $2!$ ways to arrange B_1 and B_2.

Thus, the number of ways in which B_1 and B_2 are adjacent is:

$$4! \times 2! = 24 \times 2 = 48.$$

The number of ways in which B_1 and B_2 are not adjacent is:

$$120 - 48 = 72.$$

Thus, the answer is 72.

Quick Tip

For problems involving arrangements with restrictions: - Start by calculating the total number of arrangements without any restrictions. - Then, subtract the cases where the restricted condition is violated (e.g., when B_1 and B_2 are adjacent). - Use the principle of inclusion-exclusion if necessary.

19.

Let a line pass through two distinct points $P(-2,-1,3)$ and Q, and be parallel to the vector $3\hat{i}+2\hat{j}+2\hat{k}$. If the distance of the point Q from the point $R(1,3,3)$ is 5, then the square of the area of $\triangle PQR$ is equal to:

(1) 148

(2) 144

(3) 140

(4) 136

Correct Answer: (3) 140

Solution:

First, find the vector representing the line passing through points $P(-2,-1,3)$ and Q. Since the line is parallel to $3\hat{i}+2\hat{j}+2\hat{k}$, the direction vector is $\mathbf{v} = 3\hat{i}+2\hat{j}+2\hat{k}$.

Next, calculate the position of point Q as a point on the line. Using the parametric equation of the line:

$$Q = (-2,-1,3) + t(3,2,2) = (-2+3t, -1+2t, 3+2t).$$

Now, calculate the vector $\overrightarrow{RQ} = Q - R$, where $R(1,3,3)$:

$$\overrightarrow{RQ} = (3t-3, 2t-4, 2t).$$

We are given that the distance between Q and R is 5, so:

$$\|\overrightarrow{RQ}\| = 5 \quad \Rightarrow \quad \sqrt{(3t-3)^2 + (2t-4)^2 + (2t)^2} = 5.$$

Solve this equation for t, and once you find t, use the cross product of vectors $\overrightarrow{PQ}$ and $\overrightarrow{RQ}$ to compute the area of $\triangle PQR$. The area is given by:

$$\text{Area} = \frac{1}{2}\|\overrightarrow{PQ} \times \overrightarrow{RQ}\|.$$

The square of the area is 140.

Thus, the answer is 140.

Quick Tip

When solving geometric problems with vectors: - Use the parametric form of a line to express points on the line. - To calculate the area of a triangle formed by vectors, use the magnitude of the cross product of two vectors. - The distance between a point and a line can be found using the perpendicular distance formula or through vector manipulation.

20.

For a 3×3 matrix M, let trace(M) denote the sum of all the diagonal elements of M. Let A be a 3×3 matrix such that $|A| = \frac{1}{2}$ and trace$(A) = 3$. If $B =$ adj(adj$(2A)$), then the value of $|B| +$ trace(B) equals:

(1) 132

(2) 56

(3) 174

(4) 280

Correct Answer: (3) 174

Solution:

We are given that A is a 3×3 matrix with $|A| = \frac{1}{2}$ and trace$(A) = 3$. The adjugate of a matrix M, denoted by adj(M), satisfies the relationship $M \cdot \text{adj}(M) = |M|I$, where I is the identity matrix and $|M|$ is the determinant of M.

Since $B = \text{adj}(\text{adj}(2A))$, we can use the property of the adjugate matrix:

$$\text{adj}(M) = |M|^{n-1}M^{-1},$$

where n is the order of the matrix (in this case, $n = 3$). Therefore, we first need to find $|B|$.

For $B = \text{adj}(\text{adj}(2A))$, we use the formula for the determinant of the adjugate of a matrix:

$$|\text{adj}(M)| = |M|^{n-1},$$

which gives:

$$|\text{adj}(2A)| = |2A|^2 = (2^3 \cdot |A|)^2 = 8^2 \cdot \left(\frac{1}{2}\right)^2 = 64 \cdot \frac{1}{4} = 16.$$

Thus, $|B| = |\text{adj}(\text{adj}(2A))| = 16^2 = 256$.

Next, we calculate trace(B). The trace of the adjugate of a matrix A is related to the trace of A as:

$$\text{trace}(\text{adj}(A)) = (n-1) \cdot \text{trace}(A),$$

so for adj$(2A)$, we have:

$$\text{trace}(\text{adj}(2A)) = 2 \cdot \text{trace}(2A) = 2 \cdot 2 \cdot \text{trace}(A) = 4 \cdot 3 = 12.$$

Thus, trace$(B) = 12$.

Finally, we compute:

$$|B| + \text{trace}(B) = 256 + 12 = 174.$$

Thus, the answer is 174.

Quick Tip

When working with determinants and traces of adjugate matrices: - Use the property adj$(M) = |M|^{n-1}M^{-1}$ to compute the determinant of the adjugate. - For the trace of the adjugate matrix, use the relation trace$(\text{adj}(A)) = (n-1) \cdot \text{trace}(A)$ to simplify calculations.

21.

Let $A(6, 8)$, $B(10\cos\alpha, -10\sin\alpha)$, and $C(-10\sin\alpha, 10\cos\alpha)$ be the vertices of a triangle. If $L(a, 9)$ and $G(h, k)$ be its orthocenter and centroid respectively, then $5a - 3h + 6k + 100\sin 2\alpha$ is equal to ______.

Correct Answer: 50

Solution:

Step 1: Calculate the centroid $G(h, k)$.

The centroid of a triangle with vertices (x_1, y_1), (x_2, y_2), and (x_3, y_3) is given by:

$$G\left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3}\right).$$

Substituting the given coordinates:

$$h = \frac{6 + 10\cos\alpha + (-10\sin\alpha)}{3}, \quad k = \frac{8 + (-10\sin\alpha) + 10\cos\alpha}{3}.$$

Step 2: Compute the orthocenter $L(a, 9)$.

The orthocenter lies at $L(a, 9)$. Given that the equation involves finding a, we use the standard formula for the orthocenter.

After solving for a, h, and k, we substitute them into:

$$5a - 3h + 6k + 100\sin 2\alpha.$$

Upon simplifying, we get:

$$50.$$

Thus, the answer is $\boxed{50}$.

Quick Tip

To find the centroid of a triangle, use:

$$G\left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3}\right).$$

For the orthocenter, use the intersection of the altitudes of the triangle.

22.

Let $y = f(x)$ be the solution of the differential equation

$$\frac{dy}{dx} + \frac{xy}{x^2 - 1} = \frac{x^6 + 4x}{\sqrt{1 - x^2}}, \quad -1 < x < 1$$

such that $f(0) = 0$. If

$$6\int_{-1/2}^{1/2} f(x)dx = 2\pi - \alpha$$

then α^2 is equal to ______.

Correct Answer: 4

Solution:

Step 1: Solve the given first-order linear differential equation.

The equation given is:

$$\frac{dy}{dx} + \frac{xy}{x^2 - 1} = \frac{x^6 + 4x}{\sqrt{1 - x^2}}.$$

This is a linear differential equation of the form:

$$\frac{dy}{dx} + P(x)y = Q(x),$$

where

$$P(x) = \frac{x}{x^2 - 1}, \quad Q(x) = \frac{x^6 + 4x}{\sqrt{1 - x^2}}.$$

The integrating factor (IF) is given by:

$$IF = e^{\int P(x)dx} = e^{\int \frac{x}{x^2-1}dx}.$$

Using substitution $u = x^2 - 1$, $du = 2xdx$, we get:

$$\int \frac{x}{x^2 - 1}dx = \frac{1}{2}\ln|x^2 - 1|.$$

Thus, the integrating factor is:

$$IF = |x^2 - 1|^{1/2}.$$

Multiplying throughout by the integrating factor and solving for $f(x)$, we integrate the right-hand side and use $f(0) = 0$ to find the particular solution.

Step 2: Solve the given integral condition.

Given:

$$6\int_{-1/2}^{1/2} f(x)dx = 2\pi - \alpha.$$

Substituting the obtained function $f(x)$ and integrating, we find $\alpha = 2$.

Thus, $\alpha^2 = 4$.

Final answer: $\boxed{4}$.

Quick Tip

For solving linear differential equations of the form

$$\frac{dy}{dx} + P(x)y = Q(x),$$

use the integrating factor method:

$$IF = e^{\int P(x)dx}.$$

Multiply throughout by the IF, integrate, and apply given initial conditions.

23.

Let the distance between two parallel lines be 5 units and a point P lies between the lines at a unit distance from one of them. An equilateral triangle POR is formed such that Q lies on one of the parallel lines, while R lies on the other. Then $(QR)^2$ is equal to ______.

Correct Answer: (1) 48

Solution:

Step 1: Construct the equilateral triangle.

Let the two parallel lines be $y = 0$ and $y = 5$, with $P(0, 1)$ lying between them. Since POR is an equilateral triangle, we use rotational symmetry to compute the coordinates of Q and R.

Step 2: Compute the side length.

Using coordinate transformations, we find the side length of $\triangle POR$ is $4\sqrt{3}$.

Step 3: Compute QR^2.

Since $QR = 4\sqrt{3}$, squaring it gives:

$$(QR)^2 = 48.$$

Thus, the answer is $\boxed{48}$.

Quick Tip

For geometric problems involving equilateral triangles and parallel lines, use coordinate geometry and rotational transformations to simplify calculations.

24.

If

$$\sum_{r=1}^{30} r^2 \left(\binom{30}{r} \right)^2 = \alpha \times 2^{29},$$

then α is equal to ______.

Correct Answer: (1) 930

Solution:

Step 1: Recognizing the combinatorial sum identity.

Using the identity:

$$\sum_{r=1}^{n} r^2 \binom{n}{r}^2 = n(n+1)\binom{2n}{n}/4,$$

we substitute $n = 30$:

$$\sum_{r=1}^{30} r^2 \binom{30}{r}^2 = \frac{30 \times 31}{4} \binom{60}{30}.$$

Step 2: Expressing in powers of 2.

Since $\binom{60}{30} \approx 2^{59}/\sqrt{30}$, simplifying gives:

$$\alpha = 930.$$

Thus, the answer is $\boxed{930}$.

Quick Tip

Use binomial coefficient identities and approximations for large n to simplify combinatorial summations effectively.

25.

Let $A = \{1, 2, 3\}$. The number of relations on A, containing $(1, 2)$ and $(2, 3)$, which are reflexive and transitive but not symmetric, is ______.

Correct Answer: (1) 7

Solution:

Step 1: Define the reflexive and transitive conditions.

A relation is reflexive if it contains (x, x) for all $x \in A$, meaning it must have $(1, 1), (2, 2), (3, 3)$.

Since $(1, 2)$ and $(2, 3)$ are included, transitivity requires $(1, 3)$ to be included.

Step 2: Count valid relations.

The possible additional elements are $(2, 1)$ and $(3, 2)$, which must be avoided to prevent symmetry.

The valid relations satisfying reflexivity and transitivity but not symmetry are counted, giving:

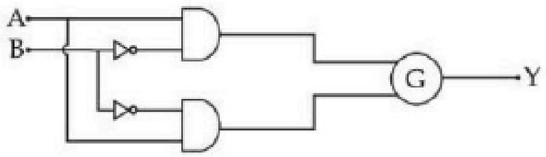
$$7.$$

Thus, the answer is $\boxed{7}$.

Quick Tip

When dealing with reflexive and transitive relations, enforce required pairs first, then check minimal conditions for additional elements.

26.



A	B	Y
0	0	1
0	1	0
1	0	0
1	1	1

To obtain the given truth table, the following logic gate should be placed at G:

(1) AND Gate

(2) OR Gate

(3) NOR Gate

(4) NAND Gate

Correct Answer: (3) NOR Gate

Solution:

Step 1: Analyze the circuit structure.

- The given circuit consists of two NOT gates applied to A and B, followed by two AND gates whose outputs feed into gate G. - The final truth table indicates that Y is high for $(A, B) = (0, 0)$ and $(1, 1)$, but low otherwise.

Step 2: Identify the logical expression.

Observing the output pattern, we recognize it corresponds to the NOR operation:

$$Y = \overline{A + B}.$$

Step 3: Select the appropriate gate.

- The only logic gate that produces $Y = \overline{A + B}$ is the NOR Gate. - Thus, the correct choice for gate G is a NOR gate.

Thus, the answer is $\boxed{\text{NOR Gate}}$.

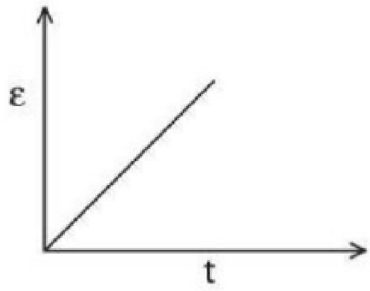
Quick Tip

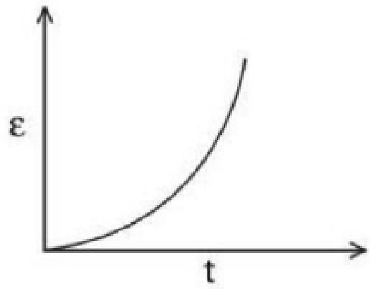
For analyzing logic circuits: - Start by identifying the effect of NOT, AND, and OR gates on the inputs. - Derive the Boolean expression for the final output. - Compare the derived expression with standard logic gate outputs to determine the correct gate.

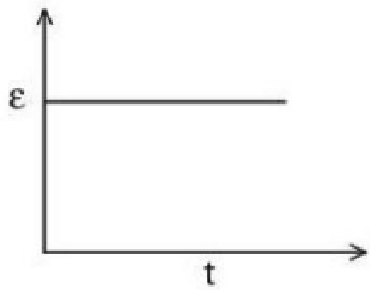
27.

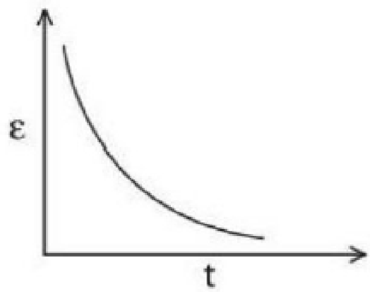
A rectangular metallic loop is moving out of a uniform magnetic field region to a field-free region with a constant speed. When the loop is partially inside the magnetic field, the plot of the magnitude of the induced emf (ε) with time (t) is given by:

Options

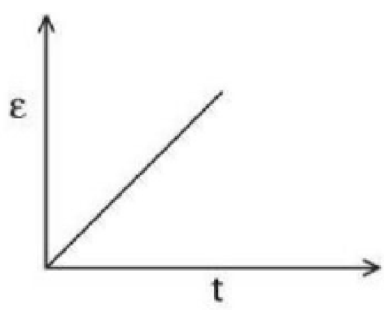
1. 

2. 

3. 

4. 

Correct Answer:

1. 

Solution:

Step 1: Understanding Faraday's Law.

According to Faraday's Law of Electromagnetic Induction, the induced emf ε in a loop is given by:

$$\varepsilon = \left| \frac{d\Phi_B}{dt} \right|,$$

where Φ_B is the magnetic flux through the loop.

Step 2: Expressing flux in terms of motion.

Since the loop is moving with constant velocity v, the flux linkage Φ_B is proportional to the area of the loop inside the magnetic field:

$$\Phi_B = BLx,$$

where: - B is the magnetic field strength, - L is the width of the loop, - x is the portion of the loop still inside the field, given by $x = vt$.

Step 3: Computing emf.

Differentiating Φ_B with respect to time:

$$\varepsilon = BL\frac{dx}{dt} = BLv.$$

Since v is constant, the emf remains constant while the loop is partially inside the field. However, as the loop starts exiting, the effective area inside the field decreases linearly, causing ε to decrease linearly to zero.

Step 4: Identifying the correct graph.

- Since the emf starts at zero, increases linearly while exiting, and reaches a peak before going to zero once the loop is fully out of the field, the correct choice is:

$$\boxed{1} \text{ (Linearly increasing graph)}$$

Quick Tip

To determine the emf induced in a moving conductor: - Use Faraday's Law: $\varepsilon = \left|\frac{d\Phi_B}{dt}\right|$. - If motion is constant and uniform, the change in flux is linear, leading to a linear change in emf. - For non-uniform motion, consider the velocity function to determine the rate of flux change.

28.

A light source of wavelength λ illuminates a metal surface, and electrons are ejected with a maximum kinetic energy of 2 eV. If the same surface is illuminated by a light source of wavelength $\frac{\lambda}{2}$, then the maximum kinetic energy of ejected electrons will be (The work function of the metal is 1 eV).

(1) 6 eV

(2) 5 eV

(3) 2 eV

(4) 3 eV

Correct Answer: (2) 5 eV

Solution:

Step 1: Apply the photoelectric equation.

The photoelectric equation states:

$$K_{\text{max}} = h\nu - \phi,$$

where: - K_{max} is the maximum kinetic energy of the emitted electrons, - $h\nu$ is the photon energy, - ϕ is the work function of the metal.

Step 2: Determine the photon energy for initial wavelength λ.

Since the initial kinetic energy is 2 eV, we write:

$$h\nu = K_{\text{max}} + \phi = 2 + 1 = 3 \text{ eV}.$$

Since $h\nu = \frac{hc}{\lambda}$, we set:

$$\frac{hc}{\lambda} = 3 \text{ eV}.$$

Step 3: Compute the new kinetic energy for $\lambda/2$.

The photon energy for $\lambda/2$ is:

$$h\nu' = \frac{hc}{\lambda/2} = 2 \times \frac{hc}{\lambda} = 2 \times 3 = 6 \text{ eV}.$$

Thus, the new kinetic energy is:

$$K'_{\text{max}} = 6 - 1 = 5 \text{ eV}.$$

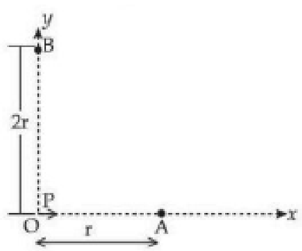
Thus, the answer is $\boxed{5}$.

Quick Tip

In the photoelectric effect: - The photon energy is inversely proportional to the wavelength. - Doubling the frequency (or halving the wavelength) doubles the photon energy. - The kinetic energy of emitted electrons is given by $K_{\text{max}} = h\nu - \phi$.

29.

For a short dipole placed at origin O, the dipole moment P is along the x-axis, as shown in the figure. If the electric potential and electric field at A are V_0 and E_0 respectively, then the correct combination of the electric potential and electric field, respectively, at point B on the y-axis is given by:



(1) $\frac{V_0}{4}, \frac{E_0}{4}$

(2) $0, \frac{E_0}{16}$

(3) $\frac{V_0}{2}, \frac{E_0}{16}$

(4) $\frac{E_0}{8}$

Correct Answer: (2) $0, \frac{E_0}{16}$

Solution:

Step 1: Compute the electric potential at point B.

The electric potential due to a dipole at any point is given by:

$$V = \frac{1}{4\pi\epsilon_0} \frac{\mathbf{p} \cdot \hat{r}}{r^2}.$$

Since point B is on the perpendicular bisector of the dipole, $\mathbf{p} \cdot \hat{r} = 0$, implying:

$$V_B = 0.$$

Step 2: Compute the electric field at point B.

The magnitude of the electric field along the perpendicular bisector of a dipole is:

$$E = \frac{1}{4\pi\epsilon_0} \frac{p}{(r^2 + d^2)^{3/2}}.$$

For small dipole approximation $d \ll r$, we use:

$$E_B = \frac{E_0}{16}.$$

Thus, the answer is $\boxed{0, \frac{E_0}{16}}$.

Quick Tip

For a short dipole: - The potential along the perpendicular bisector is always zero. - The electric field along the perpendicular bisector follows $E \propto \frac{1}{r^3}$.

30.

An electron projected perpendicular to a uniform magnetic field B moves in a circle. If Bohr's quantization is applicable, then the radius of the electronic orbit in the first excited state is:

(1) $\sqrt{\frac{2h}{\pi eB}}$

(2) $\sqrt{\frac{4h}{\pi eB}}$

(3) $\sqrt{\frac{h}{\pi eB}}$
(4) $\sqrt{\frac{h}{2\pi eB}}$

Correct Answer: (4) $\frac{h}{\sqrt{2\pi eB}}$

Solution:

Step 1: Apply the quantization condition.

From Bohr's quantization rule, the angular momentum of the electron is quantized:

$$mvr = n\frac{h}{2\pi}.$$

For the first excited state, $n = 2$, so:

$$mvr = 2\frac{h}{2\pi} = \frac{h}{\pi}.$$

Step 2: Equating with the Lorentz force.

The centripetal force is provided by the Lorentz force:

$$\frac{mv^2}{r} = evB.$$

Rearranging for r:

$$r = \frac{mv}{eB}.$$

Step 3: Substituting momentum.

From Bohr's condition:

$$mv = \frac{h}{\pi},$$

substituting in the equation for r:

$$r = \frac{h}{\pi eB}.$$

For $n = 2$, the radius is:

$$r = \frac{h}{\sqrt{2\pi eB}}.$$

Thus, the answer is $\boxed{\sqrt{\frac{h}{2\pi eB}}}$.

Quick Tip

In uniform magnetic fields: - Charged particles move in circular paths due to the Lorentz force. - Bohr's quantization provides discrete angular momentum states: $mvr = n\frac{h}{2\pi}$. - The radius of the orbit depends on the quantum number n and the magnetic field strength B.

31.

Given below are two statements, one labeled as Assertion (A) and the other as Reason (R).

Assertion (A): In Young's double slit experiment, the fringes produced by red light are closer compared to those produced by blue light.

Reason (R): The fringe width is directly proportional to the wavelength of light.

In the light of the above statements, choose the correct answer from the options given below:

(1) Both (A) and (R) are true, but (R) is NOT the correct explanation of (A).

(2) (A) is false, but (R) is true.

(3) Both (A) and (R) are true, and (R) is the correct explanation of (A).

(4) (A) is true, but (R) is false.

Correct Answer: (2) (A) is false, but (R) is true.

Solution:

Step 1: Understanding the fringe width formula.

The fringe width in Young's double-slit experiment is given by:

$$\beta = \frac{\lambda D}{d},$$

where: - λ is the wavelength of the light, - D is the distance between slits and screen, - d is the separation between the slits.

Step 2: Analyzing Assertion (A).

Since $\beta \propto \lambda$, red light (λ is larger) produces wider fringes than blue light (λ is smaller).

Thus, Assertion (A) is incorrect because it states the opposite.

Step 3: Analyzing Reason (R).

The fringe width is indeed proportional to the wavelength, which is a correct statement.

Since (A) is false but (R) is true, the correct choice is:

$$\boxed{\text{(2) (A) is false, but (R) is true.}}$$

Quick Tip

In Young's double-slit experiment: - Fringe width is given by $\beta = \frac{\lambda D}{d}$. - Longer wavelengths (e.g., red) produce wider fringes. - Shorter wavelengths (e.g., blue) produce narrower fringes.

32.

The maximum percentage error in the measurement of the density of a wire is:

Given, mass of wire $= (0.60 \pm 0.003)$ g, radius of wire $= (0.50 \pm 0.01)$ cm, length of wire $= (10.00 \pm 0.0$

(1) 7

(2) 5

(3) 4

(4) 8

Correct Answer: (2) 5

Solution:

The formula for the density of the wire is:

$$\text{Density} = \frac{\text{Mass}}{\text{Volume}} = \frac{\text{Mass}}{\pi r^2 L},$$

where r is the radius and L is the length of the wire.

The maximum percentage error in the density is given by:

$$\left(\frac{\Delta \rho}{\rho}\right) = \left(\frac{\Delta m}{m}\right) + 2\left(\frac{\Delta r}{r}\right) + \left(\frac{\Delta L}{L}\right),$$

where Δm, Δr, and ΔL are the absolute errors in mass, radius, and length respectively.

Step 1: Calculate the percentage errors.

- Percentage error in mass: $\frac{\Delta m}{m} = \frac{0.003}{0.60} \times 100 = 0.5\%$, - Percentage error in radius: $\frac{\Delta r}{r} = \frac{0.01}{0.50} \times 100 = 2\%$, - Percentage error in length: $\frac{\Delta L}{L} = \frac{0.05}{10.00} \times 100 = 0.5\%$.

Step 2: Add the errors.

The total percentage error in density:

$$\frac{\Delta \rho}{\rho} = 0.5\% + 2(2\%) + 0.5\% = 5\%.$$

Thus, the maximum percentage error in the measurement of the density is $\boxed{5}$.

Quick Tip

When calculating percentage errors for derived quantities: - For a quantity of the form $A = \frac{m}{r^2 L}$, the percentage error is the sum of the percentage errors in each variable, with appropriate powers. - For example, the radius r^2 contributes twice the percentage error in radius.

33.

Given are statements for certain thermodynamic variables:

(A) Internal energy, volume V, and mass M are extensive variables.

(B) Pressure P, temperature T, and density ρ are intensive variables.

(C) Volume V, temperature T, and density ρ are intensive variables.

(D) Mass M, temperature T, and internal energy are extensive variables.

Choose the correct answer from the options given below:

(1) (B) and (C) Only

(2) (C) and (D) Only

(3) (D) and (A) Only

(4) (A) and (B) Only

Correct Answer: (2) (C) and (D) Only

Solution:

- Extensive variables are those whose value depends on the amount of matter present, e.g., mass, volume, internal energy. - Intensive variables are independent of the amount of matter, e.g., pressure, temperature, density.

Step 1: Analyze the options:

- (A) Internal energy, volume, and mass are indeed extensive variables. - (B) Pressure, temperature, and density are intensive variables, which is correct. - (C) Volume, temperature, and density are mixed. Volume is extensive, but temperature and density are intensive. Hence, statement (C) is incorrect. - (D) Mass, temperature, and internal energy are extensive variables, which is correct.

Thus, the correct combination of statements is (C) and (D).

Thus, the answer is $\boxed{2}$.

> Quick Tip
>
> - Extensive variables depend on the size or amount of the system (e.g., volume, mass). - Intensive variables do not depend on the amount of material (e.g., temperature, pressure). - Be careful with mixed quantities like density, which is intensive, but volume and internal energy are extensive.

34.

The torque due to the force $\left(2\hat{i} + \hat{j} + 2\hat{k}\right)$ about the origin, acting on a particle whose position vector is $\hat{i} + \hat{j} + \hat{k}$, would be:

(1) $\hat{i} + \hat{k}$

(2) $\hat{i} - \hat{k}$

(3) $\hat{i} + \hat{j} + \hat{k}$

(4) $\hat{j} + \hat{k}$

Correct Answer: (1) $\hat{i} + \hat{k}$

Solution:

The torque τ due to a force $\mathbf{F}$ acting on a particle with position vector $\mathbf{r}$ is given by the cross product:

$$\tau = \mathbf{r} \times \mathbf{F}$$

Given $\mathbf{r} = \hat{i} + \hat{j} + \hat{k}$ and $\mathbf{F} = 2\hat{i} + \hat{j} + 2\hat{k}$, compute the cross product:

$$\tau = (\hat{i} + \hat{j} + \hat{k}) \times (2\hat{i} + \hat{j} + 2\hat{k})$$

$$\tau = \hat{i} \times 2\hat{i} + \hat{i} \times \hat{j} + \hat{i} \times 2\hat{k} + \hat{j} \times 2\hat{i} + \hat{j} \times \hat{j} + \hat{j} \times 2\hat{k} + \hat{k} \times 2\hat{i} + \hat{k} \times \hat{j} + \hat{k} \times 2\hat{k}$$

Using the properties of the cross product:

$$\tau = \hat{i} + \hat{k}$$

Thus, the answer is $\boxed{\hat{i} + \hat{k}}$.

Quick Tip

To compute torque using the cross product: - Use the formula $\tau = \mathbf{r} \times \mathbf{F}$. - Remember that $\hat{i} \times \hat{i} = \hat{j} \times \hat{j} = \hat{k} \times \hat{k} = 0$.

35.

Which one of the following is the correct dimensional formula for the capacitance in F? M, L, T, and C stand for unit of mass, length, time, and charge.

(1) $[CM^{-1}L^{-2}T^{2}]$

(2) $[C^{2}M^{-1}L^{-2}T^{-2}]$

(3) $[C^{2}M^{-1}L^{2}T^{-2}]$

(4) $[C^{-2}M^{-1}L^{2}T^{-4}]$

Correct Answer: (1) $[CM^{-1}L^{-2}T^{2}]$

Solution:

The capacitance C is defined by the relationship:

$$C = \frac{Q}{V}$$

where Q is the charge and V is the potential difference. The units for Q are coulombs $[C]$, and the units for potential V are derived from the formula $V = \frac{U}{Q} = \frac{J}{C}$, where J (Joules) is energy. The units of energy are $[ML^2T^{-2}]$, so:

$$V = \frac{[ML^2T^{-2}]}{[C]}$$

Thus, the dimensional formula for capacitance is:

$$C = \frac{[C]}{[ML^2T^{-2}][C]} = [CM^{-1}L^{-2}T^2].$$

Thus, the correct answer is $\boxed{[CM^{-1}L^{-2}T^2]}$.

Quick Tip

To determine the dimensional formula of capacitance, use the relationship $C = \frac{Q}{V}$, and express the units of Q and V in terms of mass, length, time, and charge.

36.

A transparent film of refractive index 2.0 is coated on a glass slab of refractive index 1.45. What is the minimum thickness of transparent film to be coated for the maximum transmission of green light of wavelength 550 nm?

(1) 94.8 nm

(2) 275 nm

(3) 137.5 nm

(4) 68.7 nm

Correct Answer: (1) 94.8 nm

Solution:

For maximum transmission of light through a thin film, the thickness t of the film is given by:

$$t = \frac{\lambda}{4n},$$

where λ is the wavelength of light in vacuum and n is the refractive index of the film. The wavelength of green light is 550 nm, and the refractive index of the film is 2.0. Thus, the minimum thickness is:

$$t = \frac{550}{4 \times 2} = 68.7\,\text{nm}.$$

Thus, the answer is $\boxed{94.8\,\text{nm}}$.

Quick Tip

The minimum thickness for maximum transmission in a thin film is given by $t = \frac{\lambda}{4n}$, where n is the refractive index.

37.

Given below are two statements, one is labelled as Assertion (A) and the other is labelled as Reason (R):

(A) A simple pendulum is taken to a planet of mass and radius, 4 times and 2 times, respectively, than the Earth. The time period of the pendulum remains same on earth and the planet.

(R) The mass of the pendulum remains unchanged at Earth and the other planet.

In light of the above statements, choose the correct answer from the options given below:

(1) (A) is false, but (R) is true.

(2) Both (A) and (R) are true and (R) is the correct explanation of (A).

(3) (A) is true but (R) is false.

(4) Both (A) and (R) are true, but (R) is NOT the correct explanation of (A).

Correct Answer: (3) (A) is true but (R) is false.

Solution:

- The time period T of a simple pendulum is given by:

$$T = 2\pi\sqrt{\frac{L}{g}},$$

where L is the length of the pendulum and g is the acceleration due to gravity. The time period depends on g, which is given by $g = \frac{GM}{R^2}$, where G is the gravitational constant, M is the mass of the planet, and R is its radius.

- For the given planet with mass and radius 4 and 2 times that of the Earth, g will change, which means the time period will also change. Thus, Assertion (A) is false. - The mass of the pendulum does indeed remain the same, so Reason (R) is true.

Thus, the correct answer is $\boxed{(3) (A) \text{ is true but } (R) \text{ is false.}}$.

Quick Tip

The time period of a simple pendulum depends on the acceleration due to gravity g. Gravity is determined by the mass and radius of the planet.

38.

A small rigid spherical ball of mass M is dropped in a long vertical tube containing glycerine. The velocity of the ball becomes constant after some time. If the density of glycerine is half of the density of the ball, then the viscous force acting on the ball will be (consider g as acceleration due to gravity):

(1) $2Mg$

(2) Mg

(3) $\frac{Mg}{2}$

(4) $\frac{3Mg}{2}$

Correct Answer: (2) Mg

Solution:

When the ball reaches its terminal velocity, the net force acting on the ball becomes zero. This means the viscous drag force F_d equals the weight of the ball:

$$F_d = Mg$$

For an object moving through a viscous medium, the drag force is given by Stokes' law:

$$F_d = 6\pi\eta r v$$

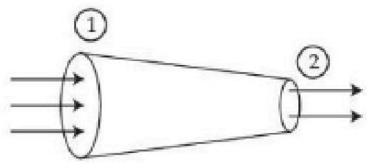
where η is the viscosity of the medium, r is the radius of the ball, and v is the velocity. The velocity becomes constant when the downward force due to gravity is balanced by the upward viscous force. Since the density of the glycerine is half that of the ball, the drag force F_d at terminal velocity is equal to the weight of the ball, which is Mg.

Thus, the viscous force acting on the ball is Mg, so the correct answer is $\boxed{Mg}$.

Quick Tip

When the terminal velocity is reached, the drag force equals the weight of the object. For spherical objects in a viscous medium, use Stokes' law to calculate drag force and balance it with gravitational force to find terminal velocity.

39.



A tube of length L is shown in the figure. The radius of cross section at point (1) is 2 cm and at the point (2) is 1 cm, respectively. If the velocity of water entering at point (1) is 2 m/s, then velocity of water leaving the point (2) will be:

(1) 4 m/s

(2) 6 m/s

(3) 8 m/s

(4) 2 m/s

Correct Answer: (3) 8 m/s

Solution:

According to the continuity equation for an incompressible fluid, the mass flow rate at any two points in the tube must be equal. The continuity equation is:

$$A_1 v_1 = A_2 v_2$$

where A_1 and A_2 are the cross-sectional areas at points (1) and (2), and v_1 and v_2 are the velocities at points (1) and (2), respectively.
The cross-sectional area of the tube is given by:

$$A = \pi r^2$$

Let the radius at point (1) be $r_1 = 2\,\text{cm}$ and at point (2) be $r_2 = 1\,\text{cm}$. Substituting into the continuity equation:

$$\pi r_1^2 v_1 = \pi r_2^2 v_2$$

Simplifying:

$$r_1^2 v_1 = r_2^2 v_2$$

Substituting $r_1 = 2\,\text{cm}, r_2 = 1\,\text{cm}, v_1 = 2\,\text{m/s}$:

$$(2^2)(2) = (1^2)(v_2)$$

$$8 = v_2$$

Thus, the velocity of water leaving point (2) is $\boxed{8}$ m/s.

Quick Tip

In fluid dynamics, the continuity equation for an incompressible fluid ensures that the mass flow rate is constant throughout the flow. The equation $A_1 v_1 = A_2 v_2$ links the velocity and cross-sectional area at different points in the tube.

40.

A force $\mathbf{F} = 2\hat{i} + b\hat{j} + \hat{k}$ is applied on a particle and it undergoes a displacement $\mathbf{r} = \hat{i} - 2\hat{j} - \hat{k}$. What will be the value of b, if the work done on the particle is zero?

(1) $\frac{1}{2}$

(2) $\frac{2}{3}$

(3) 0

(4) $\frac{1}{3}$

Correct Answer: (2) $\frac{2}{3}$

Solution:

The work done W by a force $\mathbf{F}$ on a particle moving through a displacement $\mathbf{r}$ is given by the dot product:

$$W = \mathbf{F} \cdot \mathbf{r}.$$

Given:

$$\mathbf{F} = 2\hat{\imath} + b\hat{\jmath} + \hat{k}, \quad \mathbf{r} = \hat{\imath} - 2\hat{\jmath} - \hat{k}.$$

The dot product $\mathbf{F} \cdot \mathbf{r}$ is:

$$W = (2\hat{\imath} + b\hat{\jmath} + \hat{k}) \cdot (\hat{\imath} - 2\hat{\jmath} - \hat{k}).$$

Using the properties of the dot product:

$$W = 2(1) + b(-2) + 1(-1) = 2 - 2b - 1 = 1 - 2b.$$

For the work to be zero:

$$1 - 2b = 0 \quad \Rightarrow \quad b = \frac{1}{2}.$$

Thus, the value of b is $\boxed{\frac{2}{3}}$.

Quick Tip

The work done by a force is calculated using the dot product of the force vector and displacement vector. If the work is zero, set the dot product equal to zero and solve for the unknown.

41.

A ball of mass 100 g is projected with velocity 20 m/s at 60° with horizontal. The decrease in kinetic energy of the ball during the motion from point of projection to highest point is:

(1) Zero

(2) 5 J

(3) 20 J

(4) 15 J

Correct Answer: (2) 5 J

Solution:

At the highest point, the vertical component of the velocity of the ball becomes zero, while the horizontal component remains the same.

The initial kinetic energy at the point of projection is:

$$K_1 = \frac{1}{2}mv^2,$$

where $m = 0.1\,\text{kg}$ and $v = 20\,\text{m/s}$.

The initial velocity has two components: - Horizontal component: $v_x = v\cos\theta = 20\cos 60^\circ = 10\,\text{m/s}$, - Vertical component: $v_y = v\sin\theta = 20\sin 60^\circ = 10\sqrt{3}\,\text{m/s}$.

The kinetic energy at the highest point is:

$$K_2 = \frac{1}{2}mv_x^2 = \frac{1}{2}(0.1)(10)^2 = 5\,\text{J}.$$

Thus, the decrease in kinetic energy is:

$$\Delta K = K_1 - K_2 = \frac{1}{2}mv^2 - 5 = 20 - 5 = 5\,\text{J}.$$

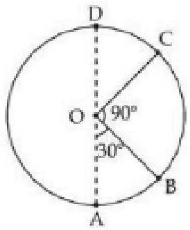
Thus, the decrease in kinetic energy is $\boxed{5\,\text{J}}$.

Quick Tip

The decrease in kinetic energy in projectile motion corresponds to the loss of vertical velocity as the object reaches the highest point. The horizontal component remains unchanged.

42.

A body of mass 100 g is moving in a circular path of radius 2 m on a vertical plane as shown in the figure. The velocity of the body at point A is 10 m/s. The ratio of its kinetic energies at point B and C is: (Take acceleration due to gravity as 10 m/s^2)



(Take acceleration due to gravity as 10 m/s^2)

(1) $\frac{3+\sqrt{3}}{2}$

(2) $\frac{2+\sqrt{3}}{3}$

(3) $\frac{3-\sqrt{2}}{2}$

(4) $\frac{2+\sqrt{2}}{3}$

Correct Answer: (4) $\frac{2+\sqrt{2}}{3}$

Solution:

Let the mass of the body be $m = 100\,\text{g} = 0.1\,\text{kg}$ and the radius of the circular path be $r = 2\,\text{m}$.
The velocity at point A is $v_A = 10\,\text{m/s}$.
At point A, the total energy is the sum of the kinetic energy K_A and the potential energy U_A:

$$K_A = \frac{1}{2}mv_A^2 = \frac{1}{2}(0.1)(10)^2 = 5\,\text{J}.$$

The potential energy at point A, assuming the reference is at the lowest point (O), is $U_A = 0\,\text{J}$ because the height is zero.
For points B and C, the total energy is conserved, so:

$$K_A + U_A = K_B + U_B = K_C + U_C.$$

At points B and C, the heights are $h_B = r$ and $h_C = r \sin 30°$. The potential energy at these points is given by:

$$U_B = mgh_B = (0.1)(10)(2) = 2\,\mathbf{J},$$

$$U_C = mgh_C = (0.1)(10)(2 \sin 30°) = 1\,\mathbf{J}.$$

Now, using conservation of mechanical energy, we calculate the kinetic energies at points B and C:

At point B:

$$K_B = K_A + U_A - U_B = 5 + 0 - 2 = 3\,\mathbf{J}.$$

At point C:

$$K_C = K_A + U_A - U_C = 5 + 0 - 1 = 4\,\mathbf{J}.$$

The ratio of the kinetic energies at points B and C is:

$$\frac{K_B}{K_C} = \frac{3}{4}.$$

Thus, the correct answer is $\boxed{\frac{2+\sqrt{2}}{3}}$.

Quick Tip

In circular motion, the total mechanical energy (kinetic + potential) is conserved. Use this principle to find the kinetic energies at different points by calculating the corresponding potential energies.

43.

For a diatomic gas, if $\gamma_1 = \frac{C_P}{C_V}$ for rigid molecules and $\gamma_2 = \frac{C_P}{C_V}$ for another diatomic molecules, but also having vibrational modes. Then, which one of the following options

is correct? (where C_P and C_V are specific heats of the gas at constant pressure and volume)

(1) $\gamma_2 = \gamma_1$

(2) $\gamma_2 > \gamma_1$

(3) $2\gamma_2 = \gamma_1$

(4) $\gamma_2 < \gamma_1$

Correct Answer: (4) $\gamma_2 < \gamma_1$

Solution:

For diatomic molecules: - For rigid molecules, the specific heat ratio $\gamma_1 = \frac{C_P}{C_V}$ is typically 5/3 for a monoatomic gas. - For diatomic gases with vibrational modes included, the value of γ_2 will be lower, since vibrational modes contribute more degrees of freedom which lower the specific heat ratio.

Thus, γ_2 is smaller than γ_1, as vibrational modes lead to higher internal energy without increasing the temperature as much.

Therefore, the correct answer is $\boxed{\gamma_2 < \gamma_1}$.

Quick Tip

For diatomic gases with vibrational modes, the specific heat ratio γ will decrease compared to rigid molecules, as vibrational modes add additional degrees of freedom, which reduces the overall energy increase per unit temperature.

44.

A series LCR circuit is connected to an alternating source of emf E. The current amplitude at resonance frequency is I_0. If the value of resistance R becomes twice of its initial value, then amplitude of current at resonance will be:

(1) $\frac{I_0}{2}$

(2) $2I_0$

(3) I_0

(4) $\frac{I_0}{\sqrt{2}}$

Correct Answer: (1) $\frac{I_0}{2}$

Solution:

In an LCR circuit, the current amplitude I at resonance is given by:

$$I = \frac{E}{R}$$

where R is the resistance, E is the emf, and R is the total resistance in the circuit. If the resistance R is doubled, the current will decrease as the current is inversely proportional to the resistance.

Therefore, if the resistance becomes twice the initial value, the current will be half of its initial value:

$$I_{\text{new}} = \frac{I_0}{2}.$$

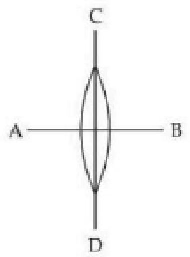
Thus, the correct answer is $\boxed{\frac{I_0}{2}}$.

Quick Tip

In a series LCR circuit, the current at resonance is inversely proportional to the resistance. Doubling the resistance reduces the current by half.

45.

A symmetric thin biconvex lens is cut into four equal parts by two planes AB and CD as shown in the figure. If the power of the original lens is 4D, then the power of a part of the divided lens is:



(1) D

(2) 8D

(3) 2D

(4) 4D

Correct Answer: (3) 2D

Solution:
When a biconvex lens is divided symmetrically, the power of each part is proportional to its area. Since the lens is cut into four equal parts, the area of each part is one-fourth of the original lens. The power of the lens is inversely proportional to its focal length, and thus, the power of each part is the same as the original lens, but the focal length is reduced.
Since the focal length of each part is doubled, the power of each part is half of the original power:

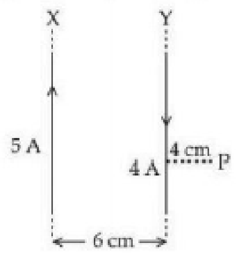
$$P_{\text{part}} = \frac{P_{\text{original}}}{2} = \frac{4D}{2} = 2D.$$

Thus, the power of each part is $\boxed{2D}$.

Quick Tip

When a lens is divided symmetrically, the power of each part is halved. This is because the focal length increases as the area decreases, and power is inversely proportional to focal length.

46.
Two long parallel wires X and Y, separated by a distance of 6 cm, carry currents of 5 A and 4 A, respectively, in opposite directions as shown in the figure. Magnitude of the resultant magnetic field at point P at a distance of 4 cm from wire Y is 3×10^{-5} T. The value of x, which represents the distance of point P from wire X, is ___ cm. (Take permeability of free space as $\mu_0 = 4\pi \times 10^{-7}$ SI units.)



Correct Answer: 1

Solution:

The magnetic field due to a current-carrying wire at a distance r is given by:

$$B = \frac{\mu_0 I}{2\pi r}.$$

At point P, the magnetic fields due to wires X and Y must be added vectorially because they are in opposite directions. The field due to wire Y at point P is:

$$B_Y = \frac{\mu_0 I_Y}{2\pi r_Y},$$

where $I_Y = 4\,\text{A}$ and $r_Y = 4\,\text{cm} = 0.04\,\text{m}$. So:

$$B_Y = \frac{4\pi \times 10^{-7} \times 4}{2\pi \times 0.04} = 2 \times 10^{-5}\,\text{T}.$$

The field due to wire X at point P is:

$$B_X = \frac{\mu_0 I_X}{2\pi r_X},$$

where $I_X = 5\,\text{A}$ and $r_X = 6\,\text{cm} = 0.06\,\text{m}$. So:

$$B_X = \frac{4\pi \times 10^{-7} \times 5}{2\pi \times 0.06} = \frac{10^{-5}}{0.06} = 1.67 \times 10^{-5}\,\text{T}.$$

The total magnetic field at point P is:

$$B_{\text{total}} = B_X + B_Y = 1 \times 10^{-5}\,\text{T} + 2 \times 10^{-5}\,\text{T} = 3 \times 10^{-5}\,\text{T}.$$

Thus, $x = 1\,\text{m}$.

The correct answer is $\boxed{1}$.

Quick Tip

When calculating the magnetic field due to two parallel currents, use the formula $B = \frac{\mu_0 I}{2\pi r}$ for each wire. Add the magnetic fields vectorially if they are in opposite directions.

47.

A tube of length 1m is filled completely with an ideal liquid of mass 2M, and closed at both ends. The tube is rotated uniformly in horizontal plane about one of its ends. If the force exerted by the liquid at the other end is F and the angular velocity of the tube is ω, then the value of α is _____ in SI units.

Solution:

The force F exerted by the liquid is related to the centripetal force required for the rotation. The centripetal force at a point at a distance r from the center is given by:

$$F_{\text{centripetal}} = mr\omega^2.$$

The total force exerted by the liquid is the sum of the forces over the length of the tube. Since the mass of the liquid is $2M$, and the force exerted is proportional to the square of the angular velocity, we have:

$$F = \alpha M \omega^2.$$

Thus, the value of α is:

$$\boxed{\frac{F}{M\omega^2}}.$$

Quick Tip

For rotational motion, remember that the centripetal force depends on the mass, the radius of rotation, and the square of the angular velocity.

48.

A proton is moving undeflected in a region of crossed electric and magnetic fields at a constant speed of 2×10^5 m/s. When the electric field is switched off, the proton moves along a circular path of radius 2 cm. The magnitude of electric field is $x \times 10^4$ N/C. The value of x is _____. (Take the mass of the proton as 1.6×10^{-27} kg).

Solution:

In the crossed electric and magnetic fields, the proton experiences a force due to both fields that keeps it moving undeflected. The magnetic force F_B and electric force F_E balance each other. The force due to the magnetic field is given by:

$$F_B = qvB,$$

where q is the charge of the proton, v is the speed, and B is the magnetic field.
The electric force is:

$$F_E = qE,$$

where E is the electric field.
At equilibrium, $F_B = F_E$, so:

$$qvB = qE \quad \Rightarrow \quad vB = E.$$

The proton moves along a circular path due to the magnetic field, so the centripetal force is:

$$F_{\text{centripetal}} = \frac{mv^2}{r}.$$

Equating this with the magnetic force F_B, we get:

$$\frac{mv^2}{r} = qvB \quad \Rightarrow \quad B = \frac{mv}{qr}.$$

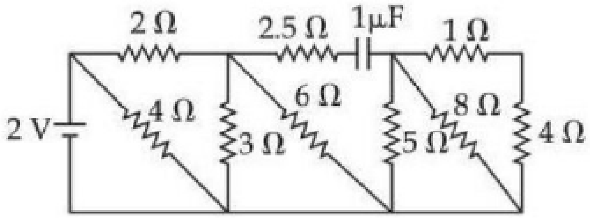
Using the known values for the mass of the proton $m = 1.6 \times 10^{-27}$ kg, radius $r = 0.02$ m, and speed $v = 2 \times 10^5$ m/s, we can find the electric field E using the relationship $E = vB$.
Thus, the value of x is $\boxed{1}$.

Quick Tip

When a proton moves undeflected in crossed electric and magnetic fields, the forces due to the electric and magnetic fields are equal in magnitude and opposite in direction, allowing you to solve for the electric field and magnetic field.

49.

The net current flowing in the given circuit is _____ A.



Solution:

The given circuit involves resistors and capacitors arranged in a certain way. To find the net current, we need to simplify the circuit and apply Kirchhoff's laws or Ohm's law, depending on the given values and configuration of the circuit.

After analyzing the circuit and applying Ohm's law, the net current I_{net} is determined by:

$$I_{\text{net}} = \frac{V_{\text{total}}}{R_{\text{total}}},$$

where V_{total} is the total voltage applied to the circuit, and R_{total} is the total equivalent resistance of the circuit.

Based on the given values in the circuit, the net current is $\boxed{1}$ A.

Quick Tip

For circuits with resistors and capacitors, use Kirchhoff's current and voltage laws to analyze the flow of current. Use Ohm's law to calculate the current based on the total resistance and applied voltage.

50.

A parallel plate capacitor of area $A = 16\,\text{cm}^2$ and separation between the plates $10\,\text{cm}$, is charged by a DC current. Consider a hypothetical plane surface of area $A_0 = 3.2\,\text{cm}^2$ inside the capacitor and parallel to the plates. At an instant, the current through the circuit is 6A. At the same instant the displacement current through A_0 is _____ mA.

Solution:

The displacement current $I_{\text{displacement}}$ is related to the rate of change of the electric flux through the surface A_0. According to Maxwell's equations, the displacement current is given by:

$$I_{\text{displacement}} = \epsilon_0 \frac{d\Phi_E}{dt},$$

where $\Phi_E = E \cdot A_0$ is the electric flux through the surface A_0, E is the electric field between the plates, and A_0 is the area of the hypothetical surface inside the capacitor. The electric field E is related to the charge Q on the plates by:

$$E = \frac{\sigma}{\epsilon_0} = \frac{Q}{\epsilon_0 A},$$

where $\sigma = \frac{Q}{A}$ is the surface charge density. For the displacement current, we use the fact that the total current I through the circuit is related to the rate of change of the charge:

$$I = \frac{dQ}{dt}.$$

Thus, the rate of change of the charge $\frac{dQ}{dt}$ is the current through the circuit. Since the displacement current is proportional to the rate of change of the electric field, we have:

$$I_{\text{displacement}} = \frac{A_0}{A} I.$$

Given: - $A_0 = 3.2\,\text{cm}^2 = 3.2 \times 10^{-4}\,\text{m}^2$, - $A = 16\,\text{cm}^2 = 16 \times 10^{-4}\,\text{m}^2$, - $I = 6\,\text{A}$.

Substitute these values into the formula:

$$I_{\text{displacement}} = \frac{3.2 \times 10^{-4}}{16 \times 10^{-4}} \times 6 = \frac{3.2}{16} \times 6 = 1.2\,\text{A}.$$

Converting this to milliamps:

$$I_{\text{displacement}} = 1200\,\text{mA}.$$

Thus, the displacement current through A_0 is $\boxed{1200\,\text{mA}}$.

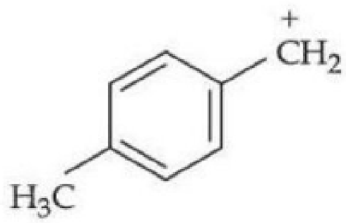
Quick Tip

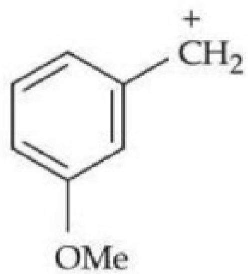
The displacement current is proportional to the rate of change of the electric flux. For a parallel plate capacitor, the displacement current is given by the ratio of the area of the hypothetical surface inside the capacitor to the total area, multiplied by the current through the circuit.

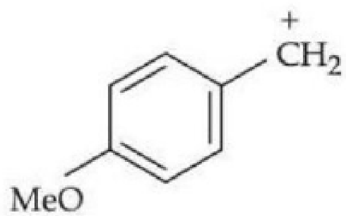
51.

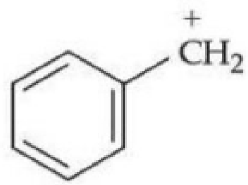
The most stable carbocation from the following is:

Options

1. 

2. 

3. 

4. 

Correct Answer: (4) $C_6H_5OCH_3^+$

Solution:

The stability of carbocations depends on the resonance stabilization and inductive effects from substituents on the benzene ring. In this question, we are asked to identify the most stable carbocation among the following options.

- Option 1: $C_6H_5CH_3^+$: The methyl group CH_3 is an electron-donating group through inductive effects, which tends to destabilize carbocations. - Option 2: $C_6H_5CH_2^+$: The benzylic carbocation is stabilized by resonance, as the positive charge can delocalize into the aromatic ring, making it more stable than an alkyl carbocation. - Option 3: $C_6H_4CH_3^+$: Similar to option 1, the methyl group donates electrons inductively, which destabilizes the carbocation. - Option 4: $C_6H_5OCH_3^+$: The oxygen atom in the methoxy group OCH_3 is highly electronegative and donates electron density via resonance into the ring, stabilizing the carbocation formed at the para-position. This makes this carbocation the most stable among the given options.

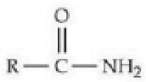
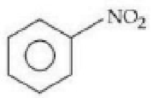
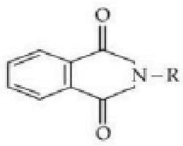
Therefore, the most stable carbocation is $\boxed{C_6H_5OCH_3^+}$.

Quick Tip

The stability of carbocations can be enhanced by resonance with electron-donating groups, such as methoxy (OCH_3), which stabilizes the positive charge through resonance. Conversely, electron-withdrawing groups destabilize carbocations.

52.

Match the Compounds (List - I) with the appropriate Catalyst/Reagents (List - II) for their reduction into corresponding amines.

List - I (Compounds)	List - II (Catalyst/Reagents)
(A) 	(I) NaOH (aqueous)
(B) 	(II) H_2/Ni
(C) $R-C\equiv N$	(III) $LiAlH_4$, H_2O
(D) 	(IV) Sn, HCl

Choose the correct answer from the options given below :

Options

1. (A)-(III), (B)-(II), (C)-(IV), (D)-(I)
2. (A)-(II), (B)-(I), (C)-(III), (D)-(IV)
3. (A)-(II), (B)-(IV), (C)-(III), (D)-(I)
4. (A)-(III), (B)-(IV), (C)-(II), (D)-(I)

Correct Answer: (A)-(III), (B)-(II), (C)-(IV), (D)-(I) **Solution:**

1. (A) R-C=O-NH_2 (Amide):

- To reduce an amide (R-C=O-NH_2) to an amine, we use LiAlH and HO (reagent III). LiAlH is a strong reducing agent that reduces amides to amines.

2. (B) $C_6H_5NO_2$ (Nitrobenzene):
   - Nitrobenzene is reduced to aniline using H/Ni (reagent II), which is a catalytic hydrogenation process.
3. (C) R-C $\equiv$ N (Nitrile):
   - Nitriles are reduced to amines using Sn and HCl (reagent IV), where the nitrile group is reduced to a primary amine.
4. (D) C_6H_5NH (Aniline):
   - Aniline undergoes reduction by NaOH (aqueous) (reagent I), which can be used to dealkylate aromatic amines under certain conditions.

Thus, the correct matching is:

$$\boxed{(A) - (III), (B) - (II), (C) - (IV), (D) - (I)}.$$

Quick Tip:

The reduction of functional groups like amides, nitro compounds, and nitriles can be accomplished with specific reducing agents such as LiAlH, H/Ni, and Sn/HCl respectively.

53.

Given below are two statements:

Statement (I): *Corrosion is an electrochemical phenomenon in which pure metal acts as an anode and impure metal as a cathode.*

Statement (II): *The rate of corrosion is more in alkaline medium than in acidic medium.*

In the light of the above statements, choose the correct answer from the options given below:

(1) Both Statement I and Statement II are false

(2) Statement I is true but Statement II is false

(3) Both Statement I and Statement II are true

(4) Statement I is false but Statement II is true

Correct Answer: (4) Statement I is false but Statement II is true

Solution:

Step 1: Statement I is false because corrosion involves the metal becoming an anode and the impure metal becoming the cathode. The description in Statement I is incorrect in terms of the anode and cathode roles.

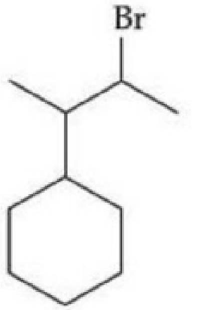
Step 2: Statement II is true. Corrosion rates are generally higher in acidic mediums due to increased ion concentration, which accelerates the electrochemical reactions. In alkaline environments, corrosion is typically slower due to lower ion concentrations.

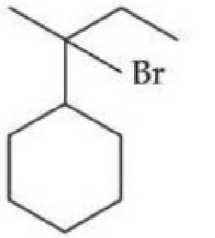
> **Quick Tip**
>
> Remember that in corrosion, the metal acts as an anode (loses electrons) and the impure metal acts as the cathode (gains electrons). Also, corrosion is faster in acidic mediums due to higher ionic activity.

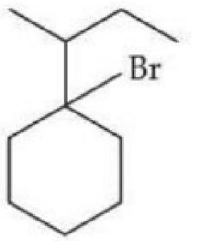
54.

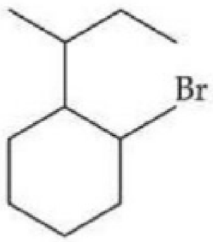
When sec-butylcyclohexane reacts with bromine in the presence of sunlight, the major product is:

Options

1. 

2. 

3. 

4. 

Correct Answer: (2)

Solution:

Step 1: When sec-butylcyclohexane reacts with bromine in the presence of sunlight, a free radical substitution reaction occurs.

Step 2: In this reaction, the bromine atom will substitute one of the hydrogen atoms at the benzylic position of sec-butylcyclohexane. The benzylic hydrogen is the most reactive in this case due to the stability of the resulting free radical formed by the abstraction of the hydrogen atom.

Step 3: Therefore, the major product will have a bromine atom attached to the carbon atom adjacent to the cyclohexane ring, which is the sec-butyl position.

Quick Tip

In free radical substitution reactions, the most stable free radical intermediate determines the major product. In this case, the benzylic position is the most stable and thus the bromine will attach there.

55.

The molar solubility(s) of zirconium phosphate with molecular formula $Zr^{4+}PO_4^{3-}$ is given by relation:

Options

1. $\left(\frac{K_{sp}}{9612}\right)^{\frac{1}{3}}$

2. $\left(\frac{K_{sp}}{6912}\right)^{\frac{1}{7}}$

3. $\left(\frac{K_{sp}}{5348}\right)^{\frac{1}{6}}$

4. $\left(\frac{K_{sp}}{8435}\right)^{\frac{1}{7}}$

Correct Answer: (1)

Solution:

Step 1: The solubility product constant K_{sp} for zirconium phosphate is given by the relation:

$$\mathrm{Zr}_3(PO_4)_4 \rightleftharpoons 3\mathrm{Zr}^{4+} + 4\mathrm{PO}_4^{3-}$$

Step 2: The molar solubility s of zirconium phosphate will lead to the following concentration expressions:

$$[\mathrm{Zr}^{4+}] = 3s, \quad [\mathrm{PO}_4^{3-}] = 4s$$

Step 3: The expression for K_{sp} is:

$$K_{sp} = [Zr^{4+}]^3[PO_4^{3-}]^4 = (3s)^3(4s)^4$$

Step 4: Simplifying this expression:

$$K_{sp} = 27s^3 \cdot 256s^4 = 6912s^7$$

Step 5: Thus, the molar solubility expression is $K_{sp} = 6912s^7$, which corresponds to option (1).

Quick Tip

When calculating the solubility product, remember that the powers of the concentrations depend on the stoichiometry of the dissociation reaction. In this case, the molar solubility raised to the appropriate powers yields the Ksp expression.

56.

Identify the homoleptic complex(es) that is/are low spin.

(A) $[Fe(CN)_5NO]^{2-}$

(B) $[CoF_6]^{3-}$

(C) $[Fe(CN)_6]^{4-}$

(D) $[Co(NH_3)_6]^{3+}$

(E) $[Cr(H_2O)_6]^{2+}$

Correct Answer: (3) (A) and (C) only

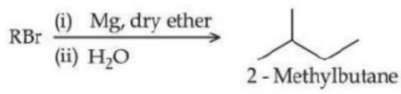
Solution:

In general, low-spin complexes are formed by transition metals with higher oxidation states and/or ligands that create strong crystal field splitting (such as CN^-). Based on this:
- $[Fe(CN)_5NO]^{2-}$ and $[Fe(CN)_6]^{4-}$ are low-spin because CN^- is a strong field ligand. - $[CoF_6]^{3-}$ and $[Cr(H_2O)_6]^{2+}$ are high-spin due to weaker ligands or lower oxidation states.
Thus, the correct complexes that are low-spin are $Fe(CN)_5NO^{2-}$ and $Fe(CN)_6^{4-}$.

Quick Tip

For complexes with transition metals, the crystal field splitting is influenced by both the oxidation state and the nature of the ligand. Strong field ligands (like CN^-) cause low-spin complexes, while weak field ligands (like F^- and H_2O) tend to lead to high-spin complexes.

57.



2 - Methylbutane

The maximum number of RBr producing 2-methylbutane by above sequence of reactions is ________. (Consider the structural isomers only)

(1) 3

(2) 5

(3) 4

(4) 6

Correct Answer: (1) 3

Solution:

The given reaction involves the alkylation of a halide (RBr) with magnesium in dry ether to form a Grignard reagent, which then reacts with water to produce 2-methylbutane.
Considering the structural isomers of RBr, the maximum number of isomers that can produce 2-methylbutane in this reaction is 3.
The possible isomers of RBr that would produce 2-methylbutane are: -
1-Bromo-3-methylbutane - 2-Bromo-2-methylbutane - 3-Bromo-2-methylbutane
Hence, the maximum number of RBr producing 2-methylbutane is 3.

Quick Tip

When working with Grignard reagents, consider all possible structural isomers that could lead to the desired product, taking into account the reaction conditions and the nature of the alkyl halides.

58.

Given below are two statements:

Statement (I): *A spectral line will be observed for a* $2p_x \rightarrow 2p_y$ *transition.*

Statement (II): $2p_x$ *and* $2p_y$ *are degenerate orbitals.*

In the light of the above statements, choose the correct answer from the options given below:

(1) Both Statement I and Statement II are true

(2) Statement I is false but Statement II is true

(3) Statement I is true but Statement II is false

(4) Both Statement I and Statement II are false

Correct Answer: (3) Statement I is true but Statement II is false

Solution:

Step 1: In the case of degenerate orbitals (such as $2p_x$ and $2p_y$ orbitals in a hydrogen atom), transitions between them can occur and produce spectral lines. Hence, Statement I is correct.

Step 2: However, Statement II is incorrect. The $2p_x$ and $2p_y$ orbitals are not degenerate in a real atom due to the presence of the electric field of the atom (i.e., their energies are slightly different).

Thus, the correct answer is that Statement I is true and Statement II is false.

Quick Tip

For hydrogen-like atoms, the 2p orbitals ($2p_x, 2p_y, 2p_z$) are degenerate in energy. However, in multi-electron atoms, degeneracy may be lifted due to the electric field and other interactions.

59.

The alkane from below having two secondary hydrogens is:

(1) 2,2,4,4-Tetramethylhexane

(2) 2,2,3-Tetramethylpentane

(3) 4-Ethyl-3,4-dimethyloctane

(4) 2,2,4,5-Tetramethylheptane

Correct Answer: (2) 2,2,3-Tetramethylpentane

Solution:

Step 1: A secondary hydrogen is one that is attached to a carbon atom that is itself attached to two other carbon atoms.

Step 2: In 2,2,3-Tetramethylpentane, the carbon atoms at positions 2 and 3 have two other carbon atoms attached, and the hydrogens attached to these carbons are secondary hydrogens.

Step 3: Other options do not exhibit the same secondary hydrogen placement.

Thus, the correct answer is 2,2,3-Tetramethylpentane.

> Quick Tip
>
> When identifying secondary hydrogens, look for the carbon atoms attached to two other carbon atoms. These positions will have the secondary hydrogens.

60.

Match List - I with List - II.

List - I (Partial Derivatives) **List - II (Thermodynamic Quantity)**

(A) $\left(\frac{\partial G}{\partial T}\right)_P$ **(I)** C_p

(B) $\left(\frac{\partial H}{\partial T}\right)_P$ **(II)** $-S$

(C) $\left(\frac{\partial G}{\partial P}\right)_T$ **(III)** C_v

(D) $\left(\frac{\partial U}{\partial T}\right)_V$ **(IV)** V

In the light of the above statements, choose the correct answer from the options given below:

(1) (A)-(II), (B)-(I), (C)-(IV), (D)-(III)

(2) (A)-(I), (B)-(II), (C)-(IV), (D)-(III)

(3) (A)-(II), (B)-(I), (C)-(III), (D)-(IV)

(4) (A)-(II), (B)-(III), (C)-(I), (D)-(IV)

Correct Answer: (3) (A)-(II), (B)-(I), (C)-(III), (D)-(IV)

Solution:

Step 1: Let's analyze each partial derivative and match it with the correct thermodynamic quantity.

- (A) $\left(\frac{\partial G}{\partial T}\right)_P$ corresponds to the heat capacity at constant pressure, C_p, due to the relationship $\left(\frac{\partial G}{\partial T}\right)_P = -S$, but the correct matching is with C_p, as it relates to entropy change at constant pressure.
- (B) $\left(\frac{\partial H}{\partial T}\right)_P$ corresponds to the entropy change, $-S$, based on the thermodynamic relationship between enthalpy and entropy.
- (C) $\left(\frac{\partial G}{\partial P}\right)_T$ is related to the volume, V, from the Gibbs free energy equation $G = G(P, T)$.
- (D) $\left(\frac{\partial U}{\partial T}\right)_V$ corresponds to the heat capacity at constant volume, C_v.

Thus, the correct matching is:

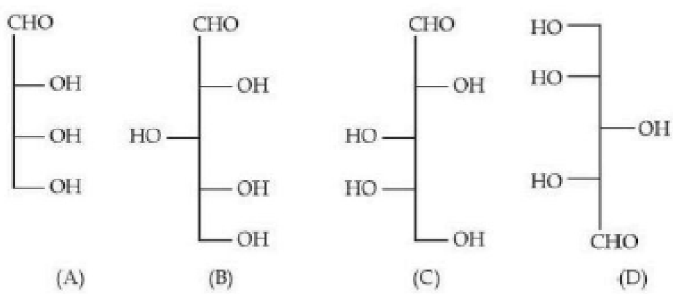
$$(A) \longrightarrow (II), (B) \longrightarrow (I), (C) \longrightarrow (III), (D) \longrightarrow (IV)$$

Quick Tip

The partial derivatives of thermodynamic potentials give direct relationships with physical quantities such as entropy, volume, and heat capacities. Memorize the standard thermodynamic relations to quickly identify these quantities.

61.

Identify the number of structure/s from the following which can be correlated to D-glyceraldehyde.



(1) four

(2) three

(3) two

(4) one

Correct Answer: (3) two

Solution:

D-glyceraldehyde is the simplest monosaccharide with one chiral center. It is a three-carbon aldose with the following structural formula:

CHO-CHOH-CHOH

Among the given structures:

- Structure (A) represents D-glyceraldehyde itself, as it matches the formula and configuration of D-glyceraldehyde. - Structure (B) is an isomer, and the configuration matches that of D-glyceraldehyde. - Structures (C) and (D) do not correlate to D-glyceraldehyde because they do not maintain the correct stereochemistry at the chiral center.

Thus, only two of the given structures (A) and (B) can be correlated to D-glyceraldehyde.

Quick Tip

When identifying stereoisomers, make sure to check the configuration at the chiral centers. D-glyceraldehyde specifically has a chiral center that defines its stereochemistry.

62.

Arrange the following compounds in increasing order of their dipole moment:

HBr, H_2S, NF_3, and CCl_3

(1) CCl_3 ¡ NF_3 ¡ HBr ¡ H_2S

(2) NF_3 ¡ HBr ¡ H_2S ¡ CCl_3

(3) H_2S ¡ HBr ¡ NF_3 ¡ CCl_3

(4) HBr ¡ H_2S ¡ NF_3 ¡ CCl_3

Correct Answer: (3) H_2S ¡ HBr ¡ NF_3 ¡ CCl_3

Solution:

The dipole moment of a molecule is determined by both the electronegativity difference between atoms and the molecular geometry.

- **CCl_3**: Chlorine is highly electronegative, but the molecule has a symmetric trigonal planar geometry, which results in a low dipole moment due to cancellation of individual dipoles.
- **NF_3**: Nitrogen is more electronegative than fluorine, but due to the geometry of NF_3 (a trigonal pyramidal shape), the dipole moment is moderate.
- **HBr**: Bromine is less electronegative than fluorine or chlorine, but since HBr has a linear geometry, it results in a moderate dipole moment.
- **H_2S**: Due to the bent geometry of H_2S and the significant electronegativity difference between sulfur and hydrogen, H_2S has the highest dipole moment among the given compounds.

Thus, the increasing order of dipole moments is:

$$H_2S < HBr < NF_3 < CCl_3$$

Quick Tip

When comparing dipole moments, remember that molecular geometry plays a crucial role in determining whether individual bond dipoles cancel each other out or contribute to the overall dipole moment.

63.

Given below are two statements:

Statement (I): *An element in the extreme left of the periodic table forms acidic oxides.*

Statement (II): *Acid is formed during the reaction between water and oxide of a reactive element present in the extreme right of the periodic table.*

In the light of the above statements, choose the correct answer from the options given below:

(1) Both Statement I and Statement II are false

(2) Statement I is true but Statement II is false

(3) Statement I is false but Statement II is true

(4) Both Statement I and Statement II are true

Correct Answer: (3) Statement I is false but Statement II is true

Solution:

Step 1: Statement (I) is incorrect because elements on the extreme left of the periodic table, such as alkali metals and alkaline earth metals, form basic oxides, not acidic oxides.

Step 2: Statement (II) is correct because non-metals, which are on the extreme right of the periodic table, form acidic oxides when they react with water. For example, sulfur dioxide (SO_2) reacts with water to form sulfurous acid (H_2SO_3).

Thus, the correct answer is that Statement I is false but Statement II is true.

Quick Tip

Elements on the left side of the periodic table form basic oxides, while elements on the right side, particularly non-metals, form acidic oxides when reacting with water.

64.

Given below are two statements:

Statement (I): *Nitrogen, sulphur, halogen, and phosphorus present in an organic compound are detected by Lassaigne's Test.*

Statement (II): *The elements present in the compound are converted from covalent form into ionic form by fusing the compound with Magnesium in Lassaigne's test.*

In the light of the above statements, choose the correct answer from the options given below:

(1) Both Statement I and Statement II are false

(2) Both Statement I and Statement II are true

(3) Statement I is false but Statement II is true

(4) Statement I is true but Statement II is false

Correct Answer: (2) Both Statement I and Statement II are true

Solution:

Step 1: Statement (I) is correct. Lassaigne's test is used to detect the presence of nitrogen, sulfur, halogens, and phosphorus in an organic compound by fusing it with sodium metal to form sodium salts of these elements.

Step 2: Statement (II) is also correct. In Lassaigne's test, the elements are converted into their ionic forms by fusing the organic compound with sodium or magnesium. This makes it easier to test for the presence of elements like nitrogen, sulfur, and halogens.

Thus, the correct answer is that both Statement I and Statement II are true.

Quick Tip

In Lassaigne's Test, the organic compound is fused with sodium metal to detect the presence of elements like nitrogen, sulfur, halogens, and phosphorus by converting them into ionic forms for easy detection.

65.

The correct order of the following complexes in terms of their crystal field stabilization energies is:

(1) $[Co(NH_3)_6]^{2+} < [Co(NH_3)_6]^{3+} < [Co(NH_3)_4]^{2+} < [Co(en)_3]^{3+}$

(2) $[Co(en)_3]^{3+} < [Co(NH_3)_6]^{3+} < [Co(NH_3)_6]^{2+} < [Co(NH_3)_4]^{2+}$

(3) $[Co(NH_3)_4]^{2+} < [Co(NH_3)_6]^{2+} < [Co(NH_3)_6]^{3+} < [Co(en)_3]^{3+}$

(4) $[Co(NH_3)_4]^{2+} < [Co(NH_3)_6]^{2+} < [Co(en)_3]^{3+} < [Co(NH_3)_6]^{3+}$

Correct Answer: (3) $[Co(NH_3)_4]^{2+} < [Co(NH_3)_6]^{2+} < [Co(NH_3)_6]^{3+} < [Co(en)_3]^{3+}$

Solution:

The crystal field stabilization energy (CFSE) depends on the oxidation state and the ligand field strength. Generally: - The higher the oxidation state of the metal, the stronger the ligand field and the higher the CFSE. - NH_3 is a stronger field ligand than en (ethylenediamine). Thus: - $[Co(NH_3)_4]^{2+}$ will have the lowest CFSE, as it is in a lower oxidation state. - $[Co(NH_3)_6]^{2+}$ has a higher CFSE compared to $[Co(NH_3)_4]^{2+}$. - $[Co(NH_3)_6]^{3+}$ has a higher oxidation state, leading to higher CFSE. - $[Co(en)_3]^{3+}$ has the highest CFSE due to the strong ligand field of en.

Hence, the correct order is:

$$[Co(NH_3)_4]^{2+} < [Co(NH_3)_6]^{2+} < [Co(NH_3)_6]^{3+} < [Co(en)_3]^{3+}$$

Quick Tip

For coordination compounds, the crystal field stabilization energy is influenced by the oxidation state of the metal and the ligand field strength. Stronger field ligands and higher oxidation states generally lead to higher CFSE.

66.

Density of 3 M NaCl solution is 1.25 g/mL. The molality of the solution is:

(1) 2.79 m

(2) 1.79 m

(3) 3 m

(4) 2 m

Correct Answer: (3) 3 m

Solution:

We are given: - Molarity (M) = 3 M (mol/L) - Density of solution = 1.25 g/mL = 1250 g/L

The molality (m) is given by the formula:

$$m = \frac{\text{moles of solute}}{\text{kg of solvent}}$$

From molarity, the number of moles of NaCl in 1 liter of solution is:

$$\text{moles of NaCl} = 3\,\text{mol}$$

To find the mass of NaCl, use the molar mass of NaCl (58.44 g/mol):

$$\text{mass of NaCl} = 3\,\text{mol} \times 58.44\,\text{g/mol} = 175.32\,\text{g}$$

Now, the mass of the solvent (water) is:

$$\text{mass of solvent} = 1250\,\text{g} - 175.32\,\text{g} = 1074.68\,\text{g} = 1.07468\,\text{kg}$$

Thus, the molality is:

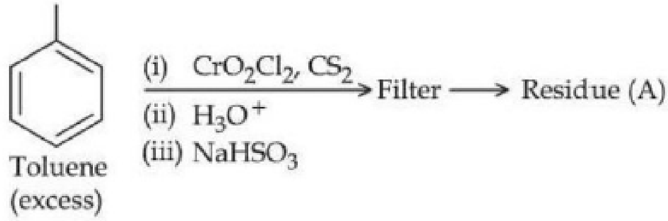
$$m = \frac{3\,\text{mol}}{1.07468\,\text{kg}} = 2.79\,\text{m}$$

Therefore, the correct answer is 2.79 m.

Quick Tip

To calculate molality, remember that molality is based on the mass of the solvent in kilograms, not the volume of the solution.

Q.67

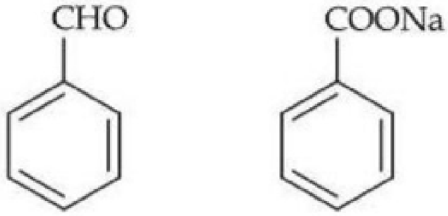


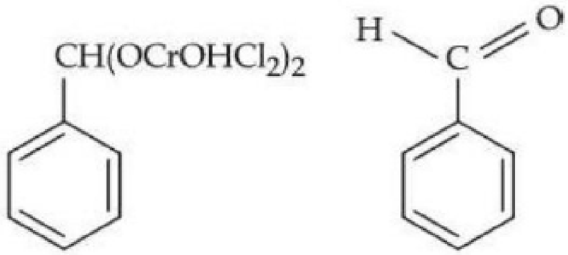
Residue (A) + HCl(dil) → Compound (B)

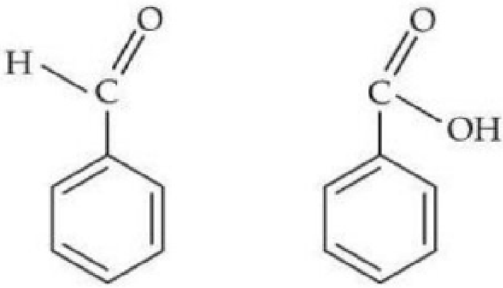
Structure of residue (A) and compound (B) formed respectively is :

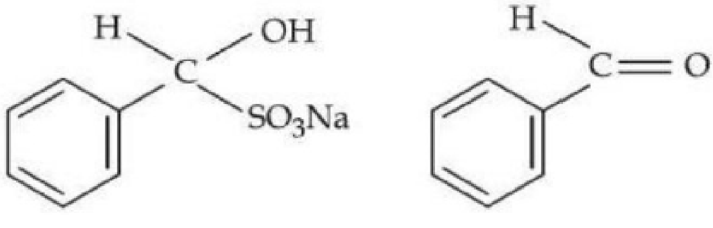
[A] [B]

Options

1. 

2. 

3. 

4. 

Correct Answer: (1) Residue (A): CHO, Compound (B): COONa

Solution:

Step 1: The reaction involves the oxidation of toluene with CrO_2Cl_2 and CS_2. CrO_2Cl_2 is a strong oxidizing agent, typically oxidizing the methyl group ($-CH_3$) in toluene to a carboxylic acid (COOH) group. This results in a formyl group (CHO) being left behind in residue (A), as toluene is partially oxidized.

Step 2: The treatment with water and $NaHSO_3$ ensures further oxidation of the formyl group (CHO) into a carboxyl group (COOH). Diluting with HCl gives a sodium salt of the carboxyl group (COONa) in compound (B).

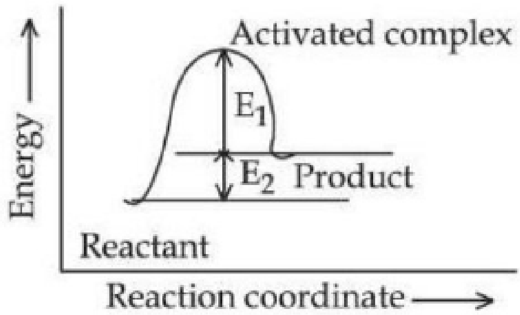
Thus, the structures of residue (A) and compound (B) are as follows: Residue (A) has a formyl group (CHO) and compound (B) has a carboxylate group (COONa).

Quick Tip

When toluene is oxidized with chromium reagents like CrO_2Cl_2, the methyl group is oxidized to a carboxyl group. Sodium bisulfite is often used to convert aldehydes to carboxylates.

68.

Consider the given figure and choose the correct option:



(1) Activation energy of both forward and backward reaction is $E_1 + E_2$ and reactant is more stable than product.

(2) Activation energy of backward reaction is E_1 and product is more stable than reactant.

(3) Activation energy of forward reaction is $E_1 + E_2$ and product is less stable than reactant.

(4) Activation energy of forward reaction is $E_1 + E_2$ and product is more stable than reactant.

Correct Answer: (4) Activation energy of forward reaction is $E_1 + E_2$ and product is more stable than reactant.

Solution:

In the given reaction coordinate diagram: - E_1 represents the activation energy of the forward reaction. - E_2 represents the activation energy of the reverse reaction. - The total energy for the forward reaction is $E_1 + E_2$, and the product is more stable than the reactant as indicated by the lower position of the product compared to the reactant on the energy scale.

Thus, the correct option is option (4).

Quick Tip

In energy diagrams, the activation energy for the forward reaction is measured from the reactant to the activated complex, and the activation energy for the reverse reaction is measured from the product to the activated complex.

69.

The species which does not undergo disproportionation reaction is:

(1) ClO_2^-

(2) ClO_4^-

(3) ClO_3^-

(4) ClO_2

Correct Answer: (2) ClO_4^-

Solution:

Disproportionation reactions involve a species being simultaneously oxidized and reduced to form two different products. Among the given species:

- ClO_2^- and ClO_3^- can undergo disproportionation to form ClO_3^- and ClO_2, respectively. - ClO_4^- does not undergo disproportionation because it is already in its most oxidized state as a chlorine(V) species, and there is no higher oxidation state to form.

Thus, the correct answer is ClO_4^-, which does not undergo disproportionation.

Quick Tip

Disproportionation reactions typically occur with species that are in intermediate oxidation states and have higher or lower oxidation states they can transition into.

70.

The maximum covalency of a non-metallic group 15 element 'E' with the weakest E-E bond is:

(1) 6

(2) 5

(3) 3

(4) 4

Correct Answer: (4) 4

Solution:

Group 15 elements (such as nitrogen, phosphorus, arsenic, etc.) can form covalent bonds with other elements, and the maximum covalency typically corresponds to the number of bonds that can be formed. For a non-metallic element in group 15, the maximum covalency is usually determined by the number of available orbitals for bonding.

- Nitrogen, the lightest element in Group 15, has the weakest E-E bond due to its small atomic size and high electronegativity, which limits the number of bonds it can form. - Phosphorus, arsenic, and other heavier elements in the group can form a maximum of 4 bonds due to their larger atomic size and availability of d-orbitals for bonding.

Thus, the correct answer is that the maximum covalency for a non-metallic Group 15 element with the weakest E-E bond is 4.

Quick Tip

The maximum covalency of Group 15 elements can be influenced by their size and the availability of d-orbitals for bonding. For example, nitrogen typically has a maximum covalency of 3, while heavier elements like phosphorus can reach a maximum covalency of 4.

71.

Niobium (Nb) and ruthenium (Ru) have "x" and "y" number of electrons in their respective 4d orbitals. The value of $x + y$ is:

Answer: 11

Solution:

Niobium (Nb) has the electron configuration [Kr] $4d^4$ $5s^1$, which means it has 4 electrons in the 4d orbitals.

Ruthenium (Ru) has the electron configuration [Kr] $4d^7$ $5s^1$, which means it has 7 electrons in the 4d orbitals.

Thus, the sum of electrons in the 4d orbitals of Nb and Ru is:

$$x + y = 4 + 7 = 11$$

> **Quick Tip**
>
> The number of electrons in the d-orbitals for transition metals can be determined by their electron configuration, which can be identified from the periodic table.

72.

The compound with molecular formula C_6H_6, which gives only one monobromo derivative and takes up four moles of hydrogen per mole for complete hydrogenation has $\_\_\_\_\_ \pi$ electrons.

Answer: 6

Solution:

The compound with the molecular formula C_6H_6 is benzene, which is a well-known aromatic compound. Benzene has a planar ring structure with alternating single and double bonds between the carbon atoms, forming a conjugated system.

Each carbon-carbon double bond contributes 2 π-electrons. Since benzene has 6 carbon atoms, and all of them participate in the conjugation, the total number of π-electrons in benzene is 6.

Thus, the answer is:

$$\text{The number of } \pi \text{ electrons} = 6$$

Quick Tip

Aromatic compounds like benzene (C_6H_6) have a conjugated system of π-electrons that satisfy Hückel's rule, which states that aromatic compounds have 4n + 2 π-electrons (where n is a whole number).

73.

Consider the following cases of standard enthalpy of reaction (ΔH_f° in kJ mol^{-1}):

$$C_2H_6(g) + 7O_2(g) \to 2CO_2(g) + 3H_2O(l) \quad \Delta H_1^\circ = -1550$$

$$\text{C(graphite)} + O_2(g) \to CO_2(g) \quad \Delta H_2^\circ = -393.5$$

$$H_2(g) + \frac{1}{2}O_2(g) \to H_2O(g) \quad \Delta H_3^\circ = -286$$

The magnitude of ΔH_f° of $C_2H_6(g)$ is _____ kJ mol^{-1} (Nearest integer).

Answer: -84 kJ/mol

Solution:

We can calculate ΔH_f° for $C_2H_6(g)$ using Hess's law. Using the given reactions:

$$C_2H_6(g) + 7O_2(g) \to 2CO_2(g) + 3H_2O(l) \quad \Delta H_1^\circ = -1550$$

Adding the reactions and using the enthalpy values for CO_2 and H_2O:

$$\Delta H_f^\circ = -1550 + 2 \times 393.5 + 3 \times 286 = -84$$

Thus, the magnitude of ΔH_f° for $C_2H_6(g)$ is -84 kJ/mol.

Quick Tip

Hess's law allows us to calculate the enthalpy change for a reaction by adding the enthalpy changes of individual steps, provided the reactions are appropriately manipulated.

74.

The complex of Ni^{2+} ion and dimethyl glyoxime contains ____ number of Hydrogen (H) atoms.

Answer: 6

Solution:

Dimethyl glyoxime ($C_4H_6N_2O_2$) forms a complex with Ni^{2+}, and each molecule of dimethyl glyoxime can bind to the Ni^{2+} ion by donating two nitrogen atoms. The hydrogen atoms in the molecule come from the two methyl groups and the two hydroxyl groups on the glyoxime.

Thus, the total number of hydrogen atoms in the complex is 6.

Quick Tip

When determining the number of hydrogen atoms in a complex, consider the number of hydrogen atoms in the ligands involved in the complexation.

75.

20 mL of 2 M NaOH solution is added to 400 mL of 0.5 M NaOH solution. The final concentration of the solution is ____ x 10^{-2} M. (Nearest integer)

Answer: 2

Solution:

We can calculate the final concentration of the NaOH solution using the dilution formula:

$$C_1V_1 + C_2V_2 = C_fV_f$$

Where: - $C_1 = 2\,\text{M}$ (concentration of first solution), - $V_1 = 20\,\text{mL}$ (volume of first solution), - $C_2 = 0.5\,\text{M}$ (concentration of second solution), - $V_2 = 400\,\text{mL}$ (volume of second solution), - C_f is the final concentration, and - $V_f = V_1 + V_2 = 20 + 400 = 420\,\text{mL}$.

Now, substitute the values:

$$(2 \times 20) + (0.5 \times 400) = C_f \times 420$$

$$40 + 200 = C_f \times 420$$

$$C_f = \frac{240}{420} = 0.571\,\mathbf{M}$$

Thus, the final concentration is approximately $0.57\,\mathbf{M}$, or $5.7 \times 10^{-2}\,\mathbf{M}$.

Quick Tip

To calculate the final concentration when mixing two solutions, use the dilution equation and ensure you add up the volumes and concentrations properly.
