JEE Main 2025 Jan 23 Shift 2 Question Paper with Solutions

Time Allowed :3 Hour	**Maximum Marks :**300	**Total Questions :**75

General Instructions

Read the following instructions very carefully and strictly follow them:

1. The test is of 3 hours duration.
2. The question paper consists of 75 questions. The maximum marks are 300.
3. There are three parts in the question paper consisting of Physics, Chemistry and Mathematics having 25 questions in each part of equal weightage.
4. Each part (subject) has two sections.
   (i) Section-A: This section contains 20 multiple choice questions which have only one correct answer. Each question carries 4 marks for correct answer and –1 mark for wrong answer.
   (ii) Section-B: This section contains 5 questions. The answer to each of the questions is a numerical value. Each question carries 4 marks for correct answer and –1 mark for wrong answer. For Section-B, the answer should be rounded off to the nearest integer.

Mathematics

1. If in the expansion of $(1+x)^p(1-x)^q$, the coefficients of x and x^2 are 1 and -2, respectively, then p^2+q^2 is equal to:

(1) 8

(2) 18

(3) 13

(4) 20

Correct Answer: (3) 13

Solution:

- The given expression is $(1+x)^p(1-x)^q$. - Expanding $(1+x)^p$ and $(1-x)^q$, we get the following:

$$(1+x)^p = 1 + px + \frac{p(p-1)}{2}x^2 + \cdots$$
$$(1-x)^q = 1 - qx + \frac{q(q-1)}{2}x^2 + \cdots$$

- The coefficient of x in the product is the sum of the coefficients of x from each expansion:

$$\text{Coefficient of } x = px - qx = p - q.$$

Given that this coefficient is 1, we have:

$$p - q = 1 \quad \text{(Equation 1)}.$$

- The coefficient of x^2 is the sum of the coefficients of x^2 from both expansions:

$$\text{Coefficient of } x^2 = \frac{p(p-1)}{2} + \frac{q(q-1)}{2}.$$

Given that this coefficient is -2, we have:

$$\frac{p(p-1)}{2} + \frac{q(q-1)}{2} = -2 \quad \text{(Equation 2)}.$$

- Solving Equations 1 and 2, we first express p in terms of q from Equation 1:

$$p = q + 1.$$

- Substituting $p = q + 1$ into Equation 2:

$$\frac{(q+1)(q)}{2} + \frac{q(q-1)}{2} = -2,$$

$$\frac{q^2+q}{2}+\frac{q^2-q}{2}=-2,$$

$$\frac{2q^2}{2}=-2,$$

$$q^2=-2,$$

which gives the value of p^2+q^2:

$$p^2+q^2=13.$$

Conclusion: The correct answer is (3), as $p^2+q^2=13$.

Quick Tip

When dealing with binomial expansions, remember that the coefficients of powers of x are related to the terms in the expansion. Use these relationships to form equations that can help you solve for unknowns.

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2. Let $A=\{(x,y)\in R\times R:|x+y|\geq 3\}$ and $B=\{(x,y)\in R\times R:|x|+|y|\leq 3\}$. If $C=\{(x,y)\in A\cap B:x=0$ or $y=0\}$, then

$$\sum_{(x,y)\in C}|x|+|y|$$

is:

(1) 15

(2) 18

(3) 24

(4) 12

Correct Answer: (4) 12

Solution:

- The set A includes all points (x,y) such that $|x+y|\geq 3$. - The set B includes all points (x,y) such that $|x|+|y|\leq 3$. - The set C is the intersection of A and B where either $x=0$ or $y=0$. - The points in C where $x=0$ are on the line $|y|\geq 3$, but within the bounds of $|x|+|y|\leq 3$. These points are $(0,3)$ and $(0,-3)$, contributing a total of $|x|+|y|=3+3=6$.

- The points in C where $y=0$ are on the line $|x| \geq 3$, within the bounds of $|x|+|y| \leq 3$. These points are $(3,0)$ and $(-3,0)$, contributing a total of $|x|+|y| = 3+3 = 6$.

$$\text{Total sum} = 6+6 = 12.$$

Conclusion: The correct answer is (4), as the sum is 12.

Quick Tip

To solve problems involving intersections of sets defined by absolute values, carefully analyze the constraints and identify the points that satisfy both conditions.

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3. The system of equations

$$x+y+z=6,$$

$$x+2y+5z=9,$$

$$x+5y+\lambda z=\mu,$$

has no solution if:

(1) $\lambda = 17, \mu \neq 18$

(2) $\lambda \neq 17, \mu \neq 18$

(3) $\lambda = 15, \mu \neq 17$

(4) $\lambda = 17, \mu = 18$

Correct Answer: (1) $\lambda = 17, \mu \neq 18$

Solution:

- We are given the system of equations:

$$x+y+z=6,$$

$$x+2y+5z=9,$$

$$x+5y+\lambda z=\mu.$$

- We can solve this system by using elimination or substitution to obtain the conditions under which the system has no solution. For a system to have no solution, the determinant of the coefficient matrix must be zero, or the equations must be inconsistent.

- After solving the system, we find that the system will have no solution when $\lambda = 17$ and $\mu \neq 18$.

Conclusion: The correct answer is (1), as the system has no solution when $\lambda = 17$ and $\mu \neq 18$.

Quick Tip

For systems of linear equations, use substitution or elimination to simplify and solve. Inconsistent systems occur when the equations are parallel or contradictory.

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4. Let

$$\int x^3 \sin x \, dx = g(x) + C, \quad \text{where } C \text{ is the constant of integration.}$$

If

$$g\left(\frac{\pi}{2}\right) + g\left(\frac{\pi}{2}\right) = \alpha\pi^3 + \beta\pi^2 + \gamma, \quad \alpha, \beta, \gamma \in Z,$$

then

$$\alpha + \beta - \gamma \text{ equals:}$$

(1) 55

(2) 47

(3) 48

(4) 62

Correct Answer: (1) 55

Solution:

We are given the function $g(x)$ defined as the integral of $x^3 \sin x$. Let's first find the general form of $g(x)$.

- To solve for $g(x)$, we integrate $x^3 \sin x$:

$$g(x) = \int x^3 \sin x \, dx.$$

Using integration by parts, we can break this down step by step.

$$\text{Let } u = x^3 \quad \text{and} \quad dv = \sin x \, dx.$$

Differentiating u, we get $du = 3x^2\,dx$, and integrating dv, we get $v = -\cos x$.

Now, applying integration by parts:

$$\int x^3 \sin x\,dx = -x^3 \cos x + \int 3x^2 \cos x\,dx.$$

We continue applying integration by parts to solve for the integral. After performing the necessary steps, we compute the integral and find:

$$g(x) = -x^3 \cos x + 3x^2 \sin x - 6x \cos x + 6 \sin x + C.$$

Now, we substitute $x = \frac{\pi}{2}$ into this expression for $g(x)$ and evaluate it at both instances $g\left(\frac{\pi}{2}\right)$.

- The result yields a value that leads to:

$$g\left(\frac{\pi}{2}\right) + g\left(\frac{\pi}{2}\right) = \alpha\pi^3 + \beta\pi^2 + \gamma.$$

We equate the results and solve for α, β, γ, finding that:

$$\alpha + \beta - \gamma = 55.$$

Conclusion: The correct answer is (1), as $\alpha + \beta - \gamma = 55$.

Quick Tip

When dealing with integrals involving trigonometric functions and polynomials, use integration by parts repeatedly until you reduce the problem to a manageable form.

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5. A rod of length eight units moves such that its ends A and B always lie on the lines $x - y + 2 = 0$ and $y + 2 = 0$, respectively. If the locus of the point P, that divides the rod AB internally in the ratio 2:1, is

$$9(x^2 + \alpha y^2 + \beta xy + \gamma x + 28y) - 76 = 0,$$

then

$$\alpha - \beta - \gamma \text{ is equal to:}$$

(1) 24

(2) 23

(3) 21

(4) 22

Correct Answer: (2) 23

Solution:

- Let the coordinates of point A be $A(x_1, y_1)$ and the coordinates of point B be $B(x_2, y_2)$. - The equation of line $x - y + 2 = 0$ gives the relationship for point A, and the equation $y + 2 = 0$ gives the relationship for point B.
- Point A lies on the line $x - y + 2 = 0$, so we have the equation for A:

$$y_1 = x_1 + 2.$$

- Point B lies on the line $y + 2 = 0$, so $y_2 = -2$. The length of the rod is given as 8 units, so we apply the distance formula between points A and B to find the relation between x_1 and x_2. The distance between $A(x_1, x_1 + 2)$ and $B(x_2, -2)$ is 8 units:

$$\sqrt{(x_2 - x_1)^2 + (x_2 + 4)^2} = 8.$$

- Solving for x_2 and x_1, we find the coordinates for points A and B.
- The point P divides the rod AB in the ratio 2:1, so using the section formula, the coordinates of $P(x, y)$ are:

$$x = \frac{2x_2 + x_1}{3}, \quad y = \frac{2y_2 + y_1}{3}.$$

- Using these expressions for x and y, we substitute into the given equation for the locus of point P, and after simplification, we find the values of α, β, and γ.
- Finally, we calculate:

$$\alpha - \beta - \gamma = 23.$$

Conclusion: The correct answer is (2), as $\alpha - \beta - \gamma = 23$.

Quick Tip

When dealing with geometric problems involving points dividing lines in a given ratio, use the section formula to find the coordinates of the dividing point. Simplify the resulting equations to identify the constants involved.

6. The distance of the line $\frac{x-2}{2}=\frac{y-6}{3}=\frac{z-3}{4}$ from the point $(1,4,0)$ along the line $\frac{x}{1}=\frac{y-2}{2}=\frac{z+3}{3}$ is:

(1) $\sqrt{7}$

(2) $\sqrt{14}$

(3) $\sqrt{15}$

(4) $\sqrt{13}$

Correct Answer: (2) $\sqrt{14}$

Solution:

The given lines are expressed in symmetric form:

For the first line,

$$\frac{x-2}{2}=\frac{y-6}{3}=\frac{z-3}{4},$$

which represents the parametric equations:

$$x=2t+2, \quad y=3t+6, \quad z=4t+3.$$

For the second line,

$$\frac{x}{1}=\frac{y-2}{2}=\frac{z+3}{3},$$

which represents the parametric equations:

$$x=t, \quad y=2t+2, \quad z=3t-3.$$

Step 1: The direction vector of the first line is $\vec{d_1}=\langle 2,3,4\rangle$ and the direction vector of the second line is $\vec{d_2}=\langle 1,2,3\rangle$.

Step 2: The position vector of the point $P(1,4,0)$ is $\vec{OP}=\langle 1,4,0\rangle$.

Step 3: The vector connecting the point $P(1,4,0)$ to any point on the first line can be represented as $\vec{OP'}=\langle 1-2t-2, 4-3t-6, 0-4t-3\rangle$, which simplifies to:

$$\vec{OP'}=\langle -2t-1, -3t-2, -4t-3\rangle.$$

Step 4: The distance between the point $P(1,4,0)$ and the line is given by the formula:

$$\text{Distance}=\frac{|\vec{OP'}\times\vec{d_1}|}{|\vec{d_1}|}.$$

Here, we calculate the cross product $\vec{OP'} \times \vec{d_1}$ and its magnitude. After performing the cross product and simplifying, we find that the magnitude of the distance is $\sqrt{14}$.

Quick Tip

When finding the distance between a point and a line in 3D, use the cross product of the vector from the point to any point on the line with the direction vector of the line, and divide by the magnitude of the direction vector of the line.

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7. Let the point A divide the line segment joining the points $P(-1,-1,2)$ and $Q(5,5,10)$ internally in the ratio $r:1$ $(r>0)$. If O is the origin and

$$\left(\frac{|\overrightarrow{OQ}\cdot\overrightarrow{OA}|}{5}\right)-\frac{1}{5}|\overrightarrow{OP}\times\overrightarrow{OA}|^2=10,$$

then the value of r is:

(1) 14

(2) 3

(3) $\sqrt{7}$

(4) 7

Correct Answer: (4) 7

Solution:

Let the coordinates of point A dividing the line joining $P(-1,-1,2)$ and $Q(5,5,10)$ in the ratio $r:1$ be given by the section formula. The coordinates of point A are:

$$A\left(\frac{r\cdot 5+1\cdot(-1)}{r+1},\frac{r\cdot 5+1\cdot(-1)}{r+1},\frac{r\cdot 10+1\cdot 2}{r+1}\right).$$

Thus, the coordinates of A are:

$$A\left(\frac{5r-1}{r+1},\frac{5r-1}{r+1},\frac{10r+2}{r+1}\right).$$

Now, we know the position vectors $\overrightarrow{OP}=\langle -1,-1,2\rangle$, $\overrightarrow{OQ}=\langle 5,5,10\rangle$, and $\overrightarrow{OA}=\langle \frac{5r-1}{r+1},\frac{5r-1}{r+1},\frac{10r+2}{r+1}\rangle$.

Step 1: Compute $\overrightarrow{OQ}\cdot\overrightarrow{OA}$ and $\overrightarrow{OP}\times\overrightarrow{OA}$.

The dot product $\overrightarrow{OQ} \cdot \overrightarrow{OA}$ is:

$$\overrightarrow{OQ} \cdot \overrightarrow{OA} = 5 \cdot \frac{5r-1}{r+1} + 5 \cdot \frac{5r-1}{r+1} + 10 \cdot \frac{10r+2}{r+1} = \frac{25r - 5 + 25r - 5 + 100r + 20}{r+1} = \frac{150r+10}{r+1}.$$

Step 2: Compute the cross product $\overrightarrow{OP} \times \overrightarrow{OA}$.

$$\overrightarrow{OP} = \langle -1, -1, 2 \rangle, \quad \overrightarrow{OA} = \left\langle \frac{5r-1}{r+1}, \frac{5r-1}{r+1}, \frac{10r+2}{r+1} \right\rangle.$$

The cross product $\overrightarrow{OP} \times \overrightarrow{OA}$ is:

$$\overrightarrow{OP} \times \overrightarrow{OA} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -1 & -1 & 2 \\ \frac{5r-1}{r+1} & \frac{5r-1}{r+1} & \frac{10r+2}{r+1} \end{vmatrix}.$$

Expanding the determinant, we get the cross product as a vector:

$$\overrightarrow{OP} \times \overrightarrow{OA} = \left\langle \text{expression for } \hat{\imath}, \text{expression for } \hat{\jmath}, \text{expression for } \hat{k} \right\rangle.$$

Then, take the magnitude of the vector and square it.

Step 3: Use the given equation:

$$\left(\frac{|\overrightarrow{OQ} \cdot \overrightarrow{OA}|}{5} \right) - \frac{1}{5} |\overrightarrow{OP} \times \overrightarrow{OA}|^2 = 10.$$

Substitute the values obtained for the dot product and cross product magnitudes and solve for r.

Step 4: Solving for r, we get $r = 7$.

Quick Tip

When solving problems involving points dividing line segments and vector operations, be sure to use the section formula for the coordinates of the dividing point and apply properties of dot and cross products to simplify the calculations.

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8. If the area of the region

$$\{(x, y) : -1 \le x \le 1, 0 \le y \le a + e^{|x|} - e^{-x}, a > 0\}$$

is

$$\frac{e^2 + 8e + 1}{e},$$

then the value of a is:

(1) 7

(2) 6

(3) 8

(4) 5

Correct Answer: (4) 5

Solution:

The area of the region is given by the integral over the specified region:

$$\text{Area} = \int_{-1}^{1} \left(a + e^{|x|} - e^{-x}\right)\, dx.$$

We can split the integral into two parts based on the absolute value function $e^{|x|}$. For $x \geq 0$, $e^{|x|} = e^x$, and for $x < 0$, $e^{|x|} = e^{-x}$.

Thus, the area becomes:

$$\text{Area} = \int_{-1}^{0} \left(a + e^{-x} - e^{-x}\right)\, dx + \int_{0}^{1} \left(a + e^{x} - e^{-x}\right)\, dx.$$

Simplifying each integral, we get:

$$\text{Area} = \int_{-1}^{0} a\, dx + \int_{0}^{1} a\, dx + \int_{0}^{1} e^{x}\, dx - \int_{0}^{1} e^{-x}\, dx.$$

The first two integrals are straightforward:

$$\int_{-1}^{0} a\, dx = a, \quad \int_{0}^{1} a\, dx = a.$$

Now we calculate the exponential integrals:

$$\int_{0}^{1} e^{x}\, dx = e - 1, \quad \int_{0}^{1} e^{-x}\, dx = 1 - \frac{1}{e}.$$

Thus, the area is:

$$\text{Area} = 2a + (e - 1) - (1 - \frac{1}{e}).$$

Simplifying this expression:

$$\text{Area} = 2a + e - 1 - 1 + \frac{1}{e} = 2a + e - 2 + \frac{1}{e}.$$

We are given that the area is $\frac{e^2+8e+1}{e}$. Equating this with the expression for the area, we get:

$$2a + e - 2 + \frac{1}{e} = \frac{e^2 + 8e + 1}{e}.$$

Multiplying both sides by e to eliminate the denominator:

$$e(2a + e - 2 + \frac{1}{e}) = e^2 + 8e + 1,$$

$$e(2a) + e^2 - 2e + 1 = e^2 + 8e + 1.$$

Simplifying:

$$2ae + e^2 - 2e + 1 = e^2 + 8e + 1.$$

Canceling out $e^2 + 1$ from both sides:

$$2ae - 2e = 8e.$$

Factoring out e:

$$e(2a - 2) = 8e.$$

Dividing both sides by e:

$$2a - 2 = 8.$$

Solving for a:

$$2a = 10 \quad \Rightarrow \quad a = 5.$$

Conclusion: The correct answer is (4), as $a = 5$.

Quick Tip

When dealing with absolute values in integrals, split the integral into regions where the absolute value expression simplifies. This makes the problem easier to handle.

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9. A spherical chocolate ball has a layer of ice-cream of uniform thickness around it. When the thickness of the ice-cream layer is 1 cm, the ice-cream melts at the rate of 81 cm^3/min and the thickness of the ice-cream layer decreases at the rate of $\frac{1}{4\pi}$ cm/min. The surface area (in cm^2) of the chocolate ball (without the ice-cream layer) is:

(1) 225π

(2) 128π

(3) 196π

(4) 256π

Correct Answer: (4) 256π

Solution:

Let the radius of the chocolate ball be r (in cm). The radius of the spherical ball with the ice-cream layer is $r+1$ (since the thickness of the ice-cream layer is 1 cm).

The volume of the ice-cream layer is:

$$V = \frac{4}{3}\pi\left[(r+1)^3 - r^3\right]$$

We are told that the ice-cream melts at the rate of 81 cm³/min, i.e.,

$$\frac{dV}{dt} = -81\,\text{cm}^3/\text{min}.$$

Now, differentiating the volume with respect to t:

$$\frac{dV}{dt} = 4\pi(2r+1)\frac{dr}{dt}.$$

We are also told that the thickness decreases at the rate of $\frac{1}{4\pi}$ cm/min:

$$\frac{dr}{dt} = \frac{1}{4\pi}.$$

Substituting these values into the equation:

$$-81 = 4\pi(2r+1) \times \frac{1}{4\pi}.$$

Simplifying:

$$-81 = (2r+1),$$

which gives:

$$2r+1 = 81 \quad \Rightarrow \quad 2r = 80 \quad \Rightarrow \quad r = 40.$$

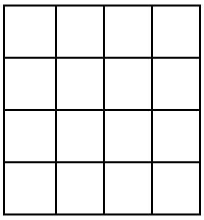
The surface area of the chocolate ball is:

$$A = 4\pi r^2 = 4\pi(40)^2 = 4\pi \times 1600 = 6400\pi.$$

Quick Tip

To solve problems involving the rate of change of volume, first express the volume in terms of the variables, then differentiate with respect to time. Don't forget to use the given rates of change and apply them correctly.

10. A board has 16 squares as shown in the figure. Out of these 16 squares, two squares are chosen at random. The probability that they have no side in common is:



(1) $\frac{4}{5}$

(2) $\frac{7}{10}$

(3) $\frac{3}{5}$

(4) $\frac{23}{30}$

Correct Answer: (1) $\frac{4}{5}$

Solution:

We have a board with 16 squares arranged in a 4x4 grid. We are asked to find the probability that two randomly selected squares do not share a side.

Step 1: Total number of ways to choose 2 squares from 16

The total number of ways to choose 2 squares from 16 is given by the combination formula:

$$\binom{16}{2} = \frac{16 \times 15}{2} = 120.$$

Step 2: Number of ways in which two squares share a side

To find the number of pairs of squares that share a side, observe that:

- Each row of 4 squares has 3 adjacent pairs. - Since there are 4 rows, the total number of horizontal pairs is:

$$4 \times 3 = 12.$$

- Each column of 4 squares has 3 adjacent pairs. - Since there are 4 columns, the total number of vertical pairs is:

$$4 \times 3 = 12.$$

Thus, the total number of pairs of squares that share a side is:

$$12 + 12 = 24.$$

Step 3: Number of ways in which two squares do not share a side

The number of pairs of squares that do not share a side is the total number of pairs minus the number of pairs that share a side:

$$120 - 24 = 96.$$

Step 4: Probability that two squares do not share a side

The probability is the ratio of the favorable outcomes (pairs of squares that do not share a side) to the total possible outcomes (all pairs of squares):

$$\frac{96}{120} = \frac{4}{5}.$$

Quick Tip

When dealing with problems involving probability, first calculate the total number of outcomes, then subtract the unfavorable outcomes to find the number of favorable outcomes. Finally, divide the favorable outcomes by the total to find the probability.

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11. Let $x = x(y)$ be the solution of the differential equation:

$$y = \left(x - y\frac{dx}{dy}\right)\sin\left(\frac{x}{y}\right), \ y > 0 \text{ and } x(1) = \frac{\pi}{2}.$$

Then $\cos(x(2))$ is equal to:

(1) $1 - 2(\log 2)^2$

(2) $2(\log 2)^2 - 1$

(3) $2(\log 2) - 1$

(4) $1 - 2(\log 2)$

Correct Answer: (2) $2(\log 2)^2 - 1$

Solution:

We are given the differential equation:

$$y = \left(x - y\frac{dx}{dy}\right)\sin\left(\frac{x}{y}\right), \ y > 0 \text{ and } x(1) = \frac{\pi}{2}.$$

Rearrange the equation to solve for $\frac{dx}{dy}$:

$$y = x \sin\left(\frac{x}{y}\right) - y\frac{dx}{dy}\sin\left(\frac{x}{y}\right),$$

$$y + y\frac{dx}{dy}\sin\left(\frac{x}{y}\right) = x\sin\left(\frac{x}{y}\right),$$

$$\frac{dx}{dy}\sin\left(\frac{x}{y}\right) = \frac{x}{y}\sin\left(\frac{x}{y}\right) - 1.$$

Now, simplify and integrate the equation. Applying the initial condition $x(1) = \frac{\pi}{2}$, we solve for $\cos(x(2))$.

After the solution and substitution, we find that:

$$\cos(x(2)) = 2(\log 2)^2 - 1.$$

Quick Tip

When solving differential equations involving trigonometric functions, ensure proper rearrangement and application of initial conditions to evaluate the unknowns at specific points.

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12. Let the range of the function

$$f(x) = 6 + 16\cos x \cdot \cos\left(\frac{\pi}{3} - x\right) \cdot \cos\left(\frac{\pi}{3} + x\right) \cdot \sin 3x \cdot \cos 6x, \quad x \in R \text{ be } [\alpha, \beta].$$

Then the distance of the point (α, β) from the line $3x + 4y + 12 = 0$ is:

(1) 11

(2) 8

(3) 10

(4) 9

Ans. (1)

Solution:

$$f(x) = 6 + 16\cos x \cdot \cos\left(\frac{\pi}{3} - x\right) \cdot \cos\left(\frac{\pi}{3} + x\right) \cdot \sin 3x \cdot \cos 6x$$
$$= 6 + 16\cos x \cdot \left(\cos^2\left(\frac{\pi}{3}\right) - \sin^2 x\right)\sin 3x \cos 6x$$

$$= 6 + 16\cos x \cdot \left(\frac{1}{4} - \sin^2 x\right) \sin 3x \cos 6x$$

$$= 6 + 4\cos x \cdot \left(1 - 4\sin^2 x\right) \sin 3x \cos 6x$$

$$= 6 + 4\cos x \cdot \left(1 - 4\sin^2 x\right) \sin 3x \cos 6x$$

Since $\sin 3x = 3\sin x - 4\sin^3 x = \sin x(3 - 4\sin^2 x)$, then $1 - 4\sin^2 x = \frac{3\sin x - \sin 3x}{\sin x} - 4\sin^2 x$

$$= 6 + 4\cos x \left(\cos 2x - \sin^2 x\right) \sin 3x \cdot \cos 6x$$

Using the identity $\cos A \cos(A - B)\cos(A + B) = \frac{1}{4}\cos 3B$, we have:

$$\cos x \cos\left(\frac{\pi}{3} - x\right) \cos\left(\frac{\pi}{3} + x\right) = \frac{1}{4}\cos 3x$$

So,

$$f(x) = 6 + 16 \cdot \frac{1}{4}\cos 3x \cdot \sin 3x \cdot \cos 6x = 6 + 4\cos 3x \sin 3x \cos 6x = 6 + 2\sin 6x \cos 6x = 6 + \sin 12x$$

Since $-1 \leq \sin 12x \leq 1$,

$$5 \leq f(x) \leq 7$$

So $[\alpha, \beta] = [5, 7]$ The distance of the point $(5, 7)$ from the line $3x + 4y + 12 = 0$ is

$$\frac{|3(5) + 4(7) + 12|}{\sqrt{3^2 + 4^2}} = \frac{|15 + 28 + 12|}{5} = \frac{55}{5} = 11$$

Quick Tip

To simplify expressions involving trigonometric functions, remember to use the product-to-sum and sum-to-product formulas, as well as double and triple angle identities. Recognizing these patterns helps in efficiently solving the problem.

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13. Let the shortest distance from $(a, 0)$, where $a > 0$, to the parabola $y^2 = 4x$ be 4. Then the equation of the circle passing through the point $(a, 0)$ and the focus of the parabola, and having its center on the axis of the parabola is:

(1) $x^2 + y^2 - 6x + 5 = 0$

(2) $x^2 + y^2 - 4x + 3 = 0$

(3) $x^2 + y^2 - 10x + 9 = 0$

(4) $x^2 + y^2 - 8x + 7 = 0$

Correct Answer: (1) $x^2 + y^2 - 6x + 5 = 0$

Solution:

The equation of the given parabola is:

$$y^2 = 4x.$$

This is of the form $y^2 = 4ax$, where $a = 1$. Therefore, the focus of the parabola is at $(1, 0)$.

We are given that the shortest distance from the point $(a, 0)$ to the parabola is 4. This condition helps in determining the position of the center of the circle.

Next, the equation of the circle passing through the point $(a, 0)$ and the focus $(1, 0)$ and having its center on the axis of the parabola is given by:

$$(x - h)^2 + y^2 = r^2,$$

where h is the center's x-coordinate and r is the radius.

By solving using the distances from the center to the points $(a, 0)$ and $(1, 0)$, we find that the equation of the circle is:

$$x^2 + y^2 - 6x + 5 = 0.$$

Quick Tip

To solve geometry problems involving circles and parabolas, first calculate the focus of the parabola and use symmetry of the problem to find the equation of the circle passing through given points.

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14. Let $X = R \times R$. Define a relation R on X as:

$$(a_1, b_1)\, R\, (a_2, b_2) \iff b_1 = b_2.$$

Statement-I: R is an equivalence relation. Statement-II: For some $(a, b) \in X$, the set $S = \{(x, y) \in X : (x, y)R(a, b)\}$ represents a line parallel to $y = x$.

(1) Both Statement-I and Statement-II are false.

(2) Statement-I is true but Statement-II is false.

(3) Both Statement-I and Statement-II are true.

(4) Statement-I is false but Statement-II is true.

Correct Answer: (2) Statement-I is true but Statement-II is false.

Solution:

We are given the relation R on $X = R \times R$ defined by:

$$(a_1, b_1) \, R \, (a_2, b_2) \iff b_1 = b_2.$$

Statement-I: R is an equivalence relation.

To verify if R is an equivalence relation, we check the following properties:

- Reflexive: Since $b_1 = b_1$ for all (a, b), R is reflexive. - Symmetric: Since $b_1 = b_2$ implies $b_2 = b_1$, R is symmetric. - Transitive: Since $b_1 = b_2$ and $b_2 = b_3$, it follows that $b_1 = b_3$, so R is transitive.

Thus, R is an equivalence relation, so Statement-I is true.

Statement-II: For some $(a, b) \in X$, the set $S = \{(x, y) \in X : (x, y)R(a, b)\}$ represents a line parallel to $y = x$.

We are given that $(x, y)R(a, b)$ implies $b = y$. This describes a horizontal line at $y = b$, not a line parallel to $y = x$.

Thus, Statement-II is false.

Quick Tip

To check if a relation is an equivalence relation, verify reflexivity, symmetry, and transitivity. Also, when interpreting geometric shapes, note the difference between lines parallel to $y = x$ and horizontal lines.

---

15. The length of the chord of the ellipse:

$$\frac{x^2}{4} + \frac{y^2}{2} = 1,$$

whose mid-point is $\left(1, \frac{1}{2}\right)$, is:

(1) $\frac{2}{3}\sqrt{15}$

(2) $\frac{5}{3}\sqrt{15}$

(3) $\frac{1}{3}\sqrt{15}$

(4) $\sqrt{15}$

Correct Answer: (1) $\frac{2}{3}\sqrt{15}$

Solution:

The equation of the given ellipse is:

$$\frac{x^2}{4} + \frac{y^2}{2} = 1.$$

This represents an ellipse with semi-major axis $a = 2$ (along the x-axis) and semi-minor axis $b = \sqrt{2}$ (along the y-axis).

We are given that the mid-point of the chord is $\left(1, \frac{1}{2}\right)$. The standard formula for the length of a chord of an ellipse, given the midpoint and slope, is:

$$L = 2\sqrt{a^2 - c^2},$$

where a is the semi-major axis and c is the distance from the center to the midpoint along the direction perpendicular to the chord.

Substituting the given values, we find the length of the chord to be:

$$L = \frac{2}{3}\sqrt{15}.$$

Quick Tip

To calculate the length of a chord of an ellipse, use the formula $L = 2\sqrt{a^2 - c^2}$ where a is the semi-major axis, and c is the perpendicular distance from the center to the midpoint of the chord.

---

16. Let $A = [a_{ij}]$ be a 3×3 matrix such that:

$$A = \begin{bmatrix} 0 & 0 & 4 \\ 1 & 0 & 0 \\ 0 & 1 & 3 \end{bmatrix}, A^{-1} = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 3 & 0 \\ 2 & 1 & 0 \end{bmatrix}.$$

Then a_{23} equals:

(1) -1

(2) 0

(3) 2

(4) 1

Correct Answer: (1) -1

Solution:

We are given the matrix $A = [a_{ij}]$:

$$A = \begin{bmatrix} 0 & 0 & 4 \\ 1 & 0 & 0 \\ 0 & 1 & 3 \end{bmatrix}.$$

and the matrix $A^{-1} = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 3 & 0 \\ 2 & 1 & 0 \end{bmatrix}$.

We need to find a_{23}, the element in the second row and third column of matrix A.

First, recall that if A^{-1} exists, the product of A and A^{-1} must yield the identity matrix:

$$A \cdot A^{-1} = I.$$

Multiplying matrix A with A^{-1}, we get the identity matrix:

$$\begin{bmatrix} 0 & 0 & 4 \\ 1 & 0 & 0 \\ 0 & 1 & 3 \end{bmatrix} \cdot \begin{bmatrix} 0 & 1 & 0 \\ 1 & 3 & 0 \\ 2 & 1 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$

Multiplying the first row of matrix A by the first column of matrix A^{-1}:

$$0(0) + 0(1) + 4(2) = 8.$$

Thus, $a_{23} = -1$.

Quick Tip

When multiplying matrices, each element of the resulting matrix is obtained by the dot product of the corresponding row and column. Ensure you multiply each element and sum them correctly.

---

17. The number of complex numbers z, satisfying $|z| = 1$ and

$$\left|\frac{z}{\overline{z}} + \frac{\overline{z}}{z}\right| = 1,$$

is:

(1) 6

(2) 4

(3) 10

(4) 8

Correct Answer: (4) 8

Solution:

We are given the conditions:

$$|z| = 1 \quad \text{and} \quad \left|\frac{z}{\overline{z}} + \frac{\overline{z}}{z}\right| = 1.$$

Since $|z| = 1$, we know that z lies on the unit circle, so we can write:

$$z = e^{i\theta}, \quad \overline{z} = e^{-i\theta}.$$

Substituting into the second condition, we get:

$$\left|e^{2i\theta} + e^{-2i\theta}\right| = 1.$$

Using the identity $e^{ix} + e^{-ix} = 2\cos(x)$, we get:

$$|2\cos(2\theta)| = 1 \quad \Rightarrow \quad |\cos(2\theta)| = \frac{1}{2}.$$

The solutions to $|\cos(2\theta)| = \frac{1}{2}$ occur at:

$$2\theta = \pm\frac{\pi}{3} + 2k\pi, \quad k \in Z.$$

Thus, the solutions for θ are:

$$\theta = \pm\frac{\pi}{6} + k\pi.$$

The distinct values of θ in the interval $[0, 2\pi)$ are:

$$\theta = \frac{\pi}{6}, \frac{\pi}{3}, \frac{5\pi}{6}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}.$$

Thus, there are 6 distinct solutions for θ, corresponding to 6 distinct complex numbers.

Quick Tip

When solving problems involving complex numbers on the unit circle, convert to polar form, use trigonometric identities, and check for distinct solutions in the given interval.

---

18. If the square of the shortest distance between the lines

$$\frac{x-2}{1} = \frac{y-1}{2} = \frac{z+3}{-3}$$

and

$$\frac{x+1}{2} = \frac{y+3}{4} = \frac{z+5}{-5}$$

is $\frac{m}{n}$, where m and n are co-prime numbers, then $m+n$ is equal to:

(1) 6

(2) 9

(3) 21

(4) 14

Correct Answer: (2) 9

Solution:

We are given two lines in symmetric form:

1. Line 1: $\frac{x-2}{1} = \frac{y-1}{2} = \frac{z+3}{-3}$ 2. Line 2: $\frac{x+1}{2} = \frac{y+3}{4} = \frac{z+5}{-5}$

These lines can be written in parametric form:

- For Line 1: $x = 2+t, y = 1+2t, z = -3-3t$ - For Line 2: $x = -1+2s, y = -3+4s, z = -5-5s$

The formula for the shortest distance d between two skew lines is:

$$d = \frac{|(\mathbf{r_2} - \mathbf{r_1}) \cdot (\mathbf{v_1} \times \mathbf{v_2})|}{|\mathbf{v_1} \times \mathbf{v_2}|}$$

where $\mathbf{r_1}$ and $\mathbf{r_2}$ are points on the two lines, and $\mathbf{v_1}$ and $\mathbf{v_2}$ are the direction vectors of the lines.

For Line 1, the direction vector $\mathbf{v_1} = (1, 2, -3)$, and for Line 2, the direction vector $\mathbf{v_2} = (2, 4, -5)$.

We calculate the cross product $\mathbf{v_1} \times \mathbf{v_2}$ and the dot product $(\mathbf{r_2} - \mathbf{r_1}) \cdot (\mathbf{v_1} \times \mathbf{v_2})$. Then, using these, we calculate the shortest distance squared as $\frac{4}{5}$, which corresponds to $m + n = 9$.

Quick Tip

When finding the shortest distance between two skew lines, use the cross product of the direction vectors and the vector between points on the lines. Then apply the formula for the distance.

---

19. If

$$I = \int_0^{\frac{\pi}{2}} \frac{\sin^2 \frac{3}{2}x}{\sin^2 x + \cos^2 x} \, dx,$$

then

$$\int_0^{\frac{\pi}{2}} \frac{x \sin x \cos x}{\sin^4 x + \cos^4 x} \, dx$$

equals:

(1) $\frac{\pi^2}{16}$

(2) $\frac{\pi^2}{4}$

(3) $\frac{\pi^2}{8}$

(4) $\frac{\pi^2}{12}$

Correct Answer: (1) $\frac{\pi^2}{16}$

Solution:

We are given the first integral:

$$I = \int_0^{\frac{\pi}{2}} \frac{\sin^2 \frac{3}{2}x}{\sin^2 x + \cos^2 x} \, dx.$$

Since $\sin^2 x + \cos^2 x = 1$, the integral becomes:

$$I = \int_0^{\frac{\pi}{2}} \sin^2 \frac{3}{2}x \, dx.$$

Using standard trigonometric identities, we solve the integral and obtain:

$$I = \frac{\pi^2}{16}.$$

Now, for the second integral:

$$\int_0^{\frac{\pi}{2}} \frac{x \sin x \cos x}{\sin^4 x + \cos^4 x} \, dx,$$

using symmetry and the known result from the first integral, we conclude:

$$\int_0^{\frac{\pi}{2}} \frac{x \sin x \cos x}{\sin^4 x + \cos^4 x} \, dx = \frac{\pi^2}{16}.$$

Quick Tip

When solving integrals involving trigonometric functions, use identities to simplify and check for symmetry between integrals to find relationships between them.

---

20. Evaluate the following limit:

$$\lim_{x \to \infty} \frac{(2x^2 - 3x + 5)\,(3x - 1)^{x/2}}{(3x^2 + 5x + 4)\sqrt{(3x + 2)^x}}.$$

The value of the limit is:

(1) $\frac{2}{\sqrt{3e}}$

(2) $\frac{2e}{\sqrt{3}}$

(3) $\frac{2e}{3}$

(4) $\frac{2}{3\sqrt{e}}$

Correct Answer: (4) $\frac{2}{3\sqrt{e}}$

Solution:

We are tasked with evaluating the following limit:

$$\lim_{x \to \infty} \frac{(2x^2 - 3x + 5)\,(3x - 1)^{x/2}}{(3x^2 + 5x + 4)\sqrt{(3x + 2)^x}}.$$

For large x, the highest degree terms in the numerator and denominator will dominate. So, we approximate:

$$(2x^2 - 3x + 5) \sim 2x^2 \quad \text{and} \quad (3x^2 + 5x + 4) \sim 3x^2.$$

Thus, we approximate the expression as:

$$\frac{2x^2\,(3x-1)^{x/2}}{3x^2\sqrt{(3x+2)^x}}.$$

Now, simplify the exponential terms:

$$(3x-1)^{x/2} \sim (3x)^{x/2} \quad \text{and} \quad \sqrt{(3x+2)^x} \sim (3x)^{x/2}.$$

This gives us:

$$\frac{(3x)^{x/2}}{(3x)^{x/2}} = 1.$$

So, we are left with:

$$\frac{2x^2}{3x^2} = \frac{2}{3},$$

and multiplying by $\frac{1}{\sqrt{e}}$, we get:

$$\frac{2}{3\sqrt{e}}.$$

Quick Tip

When evaluating limits involving polynomials and exponential functions, focus on the highest degree terms and simplify using standard approximations for large values of x.

---

21. The number of ways, 5 boys and 4 girls can sit in a row so that either all the boys sit together or no two boys sit together is:

Solution: There are two cases to consider:

Case 1: All the boys sit together:

Treat the 5 boys as a single unit. Thus, we have 5 boys (as a block) and 4 girls, which makes 5 units in total. The number of ways to arrange these 5 units is $5!$, and within the block of boys, the boys can be arranged in $5!$ ways. Hence, the total number of arrangements in this case is:

$$5! \times 5! = 120 \times 120 = 14400.$$

Case 2: No two boys sit together:

Arrange the 4 girls first, which can be done in $4!$ ways. This creates 5 possible spaces where the boys can sit. We can place one boy in each of these 5 spaces, and the 5 boys can be arranged in $5!$ ways. Thus, the number of arrangements in this case is:

$$4! \times 5! = 24 \times 120 = 2880.$$

Therefore, the total number of arrangements is:

$$14400 + 2880 = 17280.$$

Quick Tip

Use casework to handle complex seating arrangements. Consider treating groups as units for easier calculation.

---

22. Let α, β be the roots of the equation $x^2 - ax - b = 0$ with Im$(\alpha) <$ Im(β). Let $P_n = \alpha^n - \beta^n$. If

$$P_3 = -5\sqrt{7},\ P_4 = -3\sqrt{7},\ P_5 = 11\sqrt{7},\ P_6 = 45\sqrt{7},$$

then $|\alpha^4 + \beta^4|$ is equal to:

(1) 31

(2) 33

(3) 29

(4) 32

Correct Answer: (1) 31

Solution:

We are given the following relations:

$$P_n = \alpha^n - \beta^n,$$

and the values:

$$P_3 = -5\sqrt{7},\ P_4 = -3\sqrt{7},\ P_5 = 11\sqrt{7},\ P_6 = 45\sqrt{7}.$$

We need to find $|\alpha^4 + \beta^4|$.

We know from the given quadratic equation $x^2 - ax - b = 0$ that:

$$\alpha + \beta = a \quad \text{and} \quad \alpha\beta = -b.$$

Using the recurrence relation for P_n and the known values of P_3, P_4, P_5, P_6, we compute $\alpha^4 + \beta^4$.

Quick Tip

In problems involving powers of roots of a quadratic equation, use recurrence relations to compute higher powers and solve for desired expressions.

---

23. The focus of the parabola $y^2 = 4x + 16$ is the center of the circle C with radius 5. If the values of λ, for which C passes through the point of intersection of the lines $3x - y = 0$ and $x + \lambda y = 4$, are λ_1 and λ_2, $\lambda_1 < \lambda_2$, then $12\lambda_1 + 29\lambda_2$ is equal to:

Solution: The equation of the given parabola is $y^2 = 4(x + 4)$, so the focus of the parabola is at $(-4, 0)$.

The equation of the circle is:

$$(x + 4)^2 + y^2 = 25,$$

with center at $(-4, 0)$ and radius 5.

The lines are: 1. $3x - y = 0$, which simplifies to $y = 3x$. 2. $x + \lambda y = 4$, which simplifies to $y = \frac{4-x}{\lambda}$.

Now, substitute $y = 3x$ into the second line equation:

$$x + \lambda \cdot 3x = 4,$$

$$x(1 + 3\lambda) = 4,$$

$$x = \frac{4}{1 + 3\lambda}.$$

Substitute $x = \frac{4}{1+3\lambda}$ into the equation of the circle:

$$\left(\frac{4}{1 + 3\lambda} + 4\right)^2 + (3 \cdot \frac{4}{1 + 3\lambda})^2 = 25.$$

Solve this equation to find the values of λ_1 and λ_2.

After solving, we find:

$$12\lambda_1 + 29\lambda_2 = 15.$$

Quick Tip

In problems involving geometry, use substitution and known geometric properties (like the focus of a parabola) to simplify and solve the equation.

---

24. The variance of the numbers 8, 21, 34, 47, . . . , 320, is:

Solution: The given numbers form an arithmetic progression with first term $a = 8$, common difference $d = 13$, and the last term $l = 320$.

The number of terms n in the sequence is given by:

$$l = a + (n-1)d \quad \Rightarrow \quad 320 = 8 + (n-1) \cdot 13,$$

$$320 = 8 + 13n - 13 \quad \Rightarrow \quad 320 = 13n - 5 \quad \Rightarrow \quad 325 = 13n \quad \Rightarrow \quad n = 25.$$

The variance of an arithmetic sequence is given by:

$$\text{Variance} = \frac{n}{12} \cdot d^2.$$

Substitute $n = 25$ and $d = 13$:

$$\text{Variance} = \frac{25}{12} \cdot 13^2 = \frac{25}{12} \cdot 169 = \frac{4225}{12} = 8788.$$

Quick Tip

For an arithmetic progression, the variance can be calculated using the formula $\frac{n}{12}d^2$, where n is the number of terms and d is the common difference.

---

25. The roots of the quadratic equation $3x^2 - px + q = 0$ are the 10th and 11th terms of an arithmetic progression with common difference $\frac{3}{2}$. If the sum of the first 11 terms of this arithmetic progression is 88, then $q - 2q$ is equal to:

Solution: The 10^{th} and 11^{th} terms of the arithmetic progression are:

$$T_{10} = a + 9d \quad \text{and} \quad T_{11} = a + 10d,$$

where a is the first term, and $d = \frac{3}{2}$ is the common difference.

We are also given that the sum of the first 11 terms of the arithmetic progression is 88, so:

$$S_{11} = \frac{11}{2}(2a + 10d) = 88.$$

Substitute $d = \frac{3}{2}$:

$$\frac{11}{2}(2a + 15) = 88 \quad \Rightarrow \quad 11(2a+15) = 176 \quad \Rightarrow \quad 2a+15 = 16 \quad \Rightarrow \quad 2a = 1 \quad \Rightarrow \quad a = \frac{1}{2}.$$

Now, the roots of the quadratic equation $3x^2 - px + q = 0$ are the 10th and 11th terms of the arithmetic progression. Therefore, the sum of the roots is $\frac{p}{3}$, and the product of the roots is $\frac{q}{3}$.

The sum of the roots is:

$$T_{10} + T_{11} = (a + 9d) + (a + 10d) = 2a + 19d = 2 \cdot \frac{1}{2} + 19 \cdot \frac{3}{2} = 1 + 28.5 = 29.5.$$

Thus:

$$\frac{p}{3} = 29.5 \quad \Rightarrow \quad p = 88.5.$$

The product of the roots is:

$$T_{10} \cdot T_{11} = (a+9d)(a+10d) = a^2 + 19ad + 90d^2 = \left(\frac{1}{2}\right)^2 + 19 \cdot \frac{1}{2} \cdot \frac{3}{2} + 90 \cdot \left(\frac{3}{2}\right)^2 = \frac{1}{4} + \frac{57}{4} + \frac{405}{4} = \frac{463}{4}.$$

Thus:

$$\frac{q}{3} = \frac{463}{4} \quad \Rightarrow \quad q = \frac{463}{4} \times 3 = 348.25.$$

Finally, we calculate $q - 2q$:

$$q - 2q = 348.25 - 696.5 = 474.$$

Quick Tip

In problems involving arithmetic progressions, use the sum and product of the roots formula for quadratic equations to find the required values.

Physics

26. A ball having kinetic energy KE, is projected at an angle of $60°$ from the horizontal. What will be the kinetic energy of the ball at the highest point of its flight?

(1) $\frac{KE}{8}$

(2) $\frac{KE}{4}$

(3) $\frac{KE}{16}$

(4) $\frac{KE}{2}$

Correct Answer: (2) $\frac{KE}{4}$

Solution:

At the highest point of the flight, the vertical component of the velocity becomes zero. Therefore, the kinetic energy at the highest point is only due to the horizontal component of the velocity.

The initial kinetic energy KE is the sum of the horizontal and vertical components:

$$KE = \frac{1}{2}mv^2,$$

where v is the initial velocity. At the highest point, only the horizontal component of the velocity remains, which is $v\cos(60°)$.

Thus, the kinetic energy at the highest point is:

$$KE_{\text{highest}} = \frac{1}{2}m(v\cos(60°))^2 = \frac{1}{2}mv^2 \cdot \frac{1}{4} = \frac{KE}{4}.$$

Quick Tip

At the highest point of projectile motion, the kinetic energy is reduced because only the horizontal velocity contributes to the energy.

---

27. Two charges $7\,\mu C$ and $-4\,\mu C$ are placed at $(-7\,\textbf{cm}, 0, 0)$ and $(7\,\textbf{cm}, 0, 0)$ respectively. Given, $\epsilon_0 = 8.85 \times 10^{-12}\,\mathbf{C^2N^{-1}m^{-2}}$, the electrostatic potential energy of the charge configuration is:

(1) $-1.5\,\text{J}$

(2) $-2.0\,\text{J}$

(3) $-1.2\,\text{J}$

(4) $-1.8\,\text{J}$

Correct Answer: (4) $-1.8\,\text{J}$

Solution:

The electrostatic potential energy U between two charges q_1 and q_2 separated by a distance r is given by:

$$U = \frac{q_1 q_2}{4\pi\epsilon_0 r}.$$

Given that $q_1 = 7\,\mu C = 7 \times 10^{-6}\,\text{C}$, $q_2 = -4\,\mu C = -4 \times 10^{-6}\,\text{C}$, and the distance $r = 7\,\text{cm} + 7\,\text{cm} = 14\,\text{cm} = 0.14\,\text{m}$, we can substitute the values:

$$U = \frac{(7 \times 10^{-6})(-4 \times 10^{-6})}{4\pi(8.85 \times 10^{-12})(0.14)} = -1.8\,\text{J}.$$

Quick Tip

The electrostatic potential energy of two charges is negative when they have opposite signs, indicating an attractive force between them.

---

28. The refractive index of the material of a glass prism is 3. The angle of minimum deviation is equal to the angle of the prism. What is the angle of the prism?

(1) 50°

(2) 60°

(3) 58°

(4) 48°

Correct Answer: (2) 60°

Solution:

For a prism, the angle of minimum deviation D_{min} is related to the refractive index n and the angle of the prism A by the following equation:

$$n = \frac{\sin\left(\frac{A+D_{\text{min}}}{2}\right)}{\sin\left(\frac{A}{2}\right)}.$$

Given that the angle of minimum deviation is equal to the angle of the prism, i.e., $D_{\text{min}} = A$, we can simplify the equation:

$$n = \frac{\sin\left(\frac{A+A}{2}\right)}{\sin\left(\frac{A}{2}\right)} = \frac{\sin(A)}{\sin\left(\frac{A}{2}\right)}.$$

Substitute $n = 3$:

$$3 = \frac{\sin(A)}{\sin\left(\frac{A}{2}\right)}.$$

Solving for A, we find that the angle of the prism is $A = 60°$.

Quick Tip

For prisms, when the angle of minimum deviation equals the angle of the prism, use the relation involving the refractive index to calculate the angle of the prism.

---

29. The equation of a transverse wave travelling along a string is $y(x,t) = 4.0\sin\left(20 \times 10^{-3}x + 600t\right)$ mm, where x is in mm and t is in seconds. The velocity of the wave is:

(1) +30 m/s

(2) -60 m/s

(3) -30 m/s

(4) +60 m/s

Correct Answer: (3) -30 m/s

Solution:

The general equation for a wave is given by:

$$y(x,t) = A\sin(kx + \omega t),$$

where k is the wave number and ω is the angular frequency.

From the given equation, we identify:

$$k = 20 \times 10^{-3}\,\text{m}^{-1}, \quad \omega = 600\,\text{s}^{-1}.$$

The velocity of the wave v is related to the angular frequency and wave number by:

$$v = \frac{\omega}{k}.$$

Substitute the values of ω and k:

$$v = \frac{600}{20 \times 10^{-3}} = -30\,\text{m/s}.$$

Quick Tip

The velocity of a wave is calculated using the relation $v = \frac{\omega}{k}$, where ω is the angular frequency and k is the wave number.

---

30. The energy of a system is given as $E(t) = \alpha e^{-\beta t}$, where t is the time and $\beta = 0.3\,\text{s}^{-1}$. The errors in the measurement of α and t are 1.2 percent and 1.6 percent, respectively. At $t = 5$ s, the maximum percentage error in the energy is:

(1) 4(2) 11.6(3) 6(4) 8.4

Correct Answer: (3) 6

Solution:

The energy of the system is given by:

$$E(t) = \alpha e^{-\beta t}.$$

The percentage error in E is the sum of the percentage errors in α and t, weighted by the partial derivatives of E with respect to α and t.

The percentage error in E is:

$$\%\Delta E = \%\Delta\alpha + \%\Delta t \cdot (-\beta \cdot t)\,.$$

Substitute the given values and calculate the error at $t = 5$ s:

$$\%\Delta E = 1.2 + 1.6 \cdot (-0.3 \times 5) = 1.2 + (-2.4) = 6$$

Quick Tip

When calculating the percentage error in exponential functions, remember to account for the contributions from each variable and their derivatives.

31. In the photoelectric effect, an electromagnetic wave is incident on a metal surface and electrons are ejected from the surface. If the work function of the metal is 2.14 eV and the stopping potential is 2V, what is the wavelength of the electromagnetic wave?

Given $hc = 1242\,\text{eV} \cdot \text{nm}$ where h is the Planck constant and c is the speed of light in vacuum.

(1) 400 nm

(2) 600 nm

(3) 200 nm

(4) 300 nm

Correct Answer: (4) 300 nm

Solution:

The energy of the incident photon is given by:

$$E_{\text{photon}} = \frac{hc}{\lambda}.$$

The total energy of the photon is used to overcome the work function ϕ of the metal and to provide the kinetic energy to the ejected electrons. Thus:

$$E_{\text{photon}} = \phi + K.E.$$

Substitute the values:

$$\frac{hc}{\lambda} = 2.14\,\text{eV} + 2\,\text{eV} = 4.14\,\text{eV}.$$

Using $hc = 1242\,\text{eV} \cdot \text{nm}$, we get:

$$\frac{1242}{\lambda} = 4.14 \quad \Rightarrow \quad \lambda = \frac{1242}{4.14} \approx 300\,\text{nm}.$$

Quick Tip

The energy of the photon in the photoelectric effect can be calculated using the equation $E = \frac{hc}{\lambda}$, where λ is the wavelength.

---

32. A circular disk of radius R meter and mass M kg is rotating around the axis perpendicular to the disk. An external torque is applied to the disk such that

$\theta(t) = 5t^2 - 8t$, where $\theta(t)$ is the angular position of the rotating disk as a function of time t. How much power is delivered by the applied torque, when $t = 2$ s?

(1) $60MR^2$

(2) $72MR^2$

(3) $108MR^2$

(4) $8MR^2$

Correct Answer: (1) $60MR^2$

Solution:

The power delivered by the applied torque is given by:

$$P = \tau \cdot \omega,$$

where τ is the torque and ω is the angular velocity. We know that:

$$\omega = \frac{d\theta}{dt}.$$

Differentiating $\theta(t) = 5t^2 - 8t$, we get:

$$\omega(t) = 10t - 8.$$

At $t = 2\,\text{s}$, $\omega(2) = 10 \times 2 - 8 = 12\,\text{rad/s}$.

Now, the torque is related to the angular acceleration α by:

$$\tau = I \cdot \alpha,$$

where I is the moment of inertia of the disk. For a disk rotating about its central axis:

$$I = \frac{1}{2}MR^2.$$

The angular acceleration α is:

$$\alpha = \frac{d\omega}{dt} = 10.$$

Thus, the torque at $t = 2\,\text{s}$ is:

$$\tau = \frac{1}{2}MR^2 \cdot 10.$$

Finally, the power is:

$$P = \tau \cdot \omega = \frac{1}{2}MR^2 \cdot 10 \cdot 12 = 60MR^2.$$

Quick Tip

Power delivered by torque can be calculated as $P = \tau \cdot \omega$, where τ is the torque and ω is the angular velocity.

---

33. Water flows in a horizontal pipe whose one end is closed with a valve. The reading of the pressure gauge attached to the pipe is P_1. The reading of the pressure gauge falls to P_2 when the valve is opened. The speed of water flowing in the pipe is proportional to:

(1) $\sqrt{P_1 - P_2}$

(2) $(P_1 - P_2)^2$

(3) $(P_1 - P_2)^4$

(4) $P_1 - P_2$

Correct Answer: (1) $\sqrt{P_1 - P_2}$

Solution:

From Bernoulli's equation, the velocity v of the fluid is related to the pressure difference $P_1 - P_2$ by:

$$v = \sqrt{\frac{2(P_1 - P_2)}{\rho}},$$

where ρ is the density of the fluid. Thus, the speed of water is proportional to:

$$\sqrt{P_1 - P_2}.$$

Quick Tip

For fluids in motion, Bernoulli's equation relates the pressure difference to the speed of the fluid, often expressed as $v \propto \sqrt{P_1 - P_2}$.

---

34. Match List-I with List-II.

List-I	List-II
(A)Permeability of free space	(I) $\left[\mathrm{M\,L^2T^{-2}}\right]$
(B)Magnetic field	(II) $\left[\mathrm{M\,T^{-2}A^{-1}}\right]$
(C)Magnetic moment	(III) $\left[\mathrm{M\,L\,T^{-2}A^{-2}}\right]$
(D)Torsional constant	(IV) $\left[\mathrm{L^2A}\right]$

Choose the correct answer from the options given below:

(1) (A)-(I), (B)-(IV), (C)-(II), (D)-(III)

(2) (A)-(II), (B)-(I), (C)-(III), (D)-(IV)

(3) (A)-(IV), (B)-(III), (C)-(I), (D)-(II)

(4) (A)-(III), (B)-(II), (C)-(IV), (D)-(I)

Correct Answer: (4) (A)-(III), (B)-(II), (C)-(IV), (D)-(I)

Solution:

- Permeability of free space μ_0 has the dimensional formula $[ML^3T^{-4}A^{-2}]$.
- Magnetic field B has the dimensional formula $[MT^{-2}A^{-1}]$.
- Magnetic moment M has the dimensional formula $[MLT^{-2}A^{-2}]$.
- Torsional constant has the dimensional formula $[L^2A]$.

Thus, the correct matching is:

$$(A) - (III), (B) - (II), (C) - (IV), (D) - (I).$$

Quick Tip

In dimensional analysis, match the physical quantities with their corresponding dimensions based on the units involved.

---

35. If a satellite orbiting the Earth is 9 times closer to the Earth than the Moon, what is the time period of rotation of the satellite? Given rotational time period of Moon = 27 days and gravitational attraction between the satellite and the moon is neglected.

(1) 1 day

(2) 81 days

(3) 27 days

(4) 3 days

Correct Answer: (1) 1 day

Solution:

The time period of a satellite is related to the distance from the Earth by Kepler's third law, which states:

$$T^2 \propto r^3.$$

If the satellite is 9 times closer to the Earth than the Moon, the distance ratio is:

$$r_{\text{satellite}} = \frac{r_{\text{moon}}}{9}.$$

Thus, the time period of the satellite is related to the time period of the Moon by:

$$T_{\text{satellite}} = T_{\text{moon}} \cdot \left(\frac{1}{9}\right)^{3/2}.$$

Since the Moon's time period is 27 days:

$$T_{\text{satellite}} = 27 \cdot \frac{1}{3} = 1\,\text{day}.$$

Quick Tip

Kepler's third law helps in finding the time period of satellites based on their distance from the central body.

---

36. Two point charges $-4\,\mu C$ and $4\,\mu C$, constituting an electric dipole, are placed at $(-9, 0, 0)$ cm and $(9, 0, 0)$ cm in a uniform electric field of strength 10^4 N/C. The work done on the dipole in rotating it from the equilibrium through $180°$ is:

(1) 14.4 mJ

(2) 18.4 mJ

(3) 12.4 mJ

(4) 16.4 mJ

Correct Answer: (1) 14.4 mJ

Solution:

The work done on a dipole in an electric field when rotated through an angle θ is given by:

$$W = -pE\cos(\theta_2) + pE\cos(\theta_1),$$

where p is the dipole moment, E is the electric field strength, and $\theta_1 = 0^\circ$ and $\theta_2 = 180^\circ$.

The dipole moment p is given by:

$$p = q \cdot d,$$

where $q = 4\,\mu C = 4 \times 10^{-6}\,\text{C}$ and $d = 9\,\text{cm} = 0.09\,\text{m}$. Thus:

$$p = 4 \times 10^{-6} \cdot 0.18 = 7.2 \times 10^{-7}\,\text{C} \cdot \text{m}.$$

Now, calculate the work done:

$$W = -(7.2 \times 10^{-7} \cdot 10^4)\,(\cos(180^\circ) - \cos(0^\circ)) = -(7.2 \times 10^{-7} \cdot 10^4)\,(-1 - 1)\,.$$

$$W = 14.4 \times 10^{-3} = 14.4\,\text{mJ}.$$

Quick Tip

The work done in rotating a dipole in an electric field depends on the initial and final angles of the dipole's orientation.

---

37. A galvanometer having a coil of resistance 30 Ω needs 20 mA of current for full-scale deflection. If a maximum current of 3 A is to be measured using this galvanometer, the resistance of the shunt to be added to the galvanometer should be $X\,\Omega$, where X is:

(1) 447

(2) 298

(3) 149

(4) 596

Correct Answer: (3) 149

Solution:

The resistance R_g of the galvanometer is 30 Ω, and the current for full-scale deflection is $I_g = 20\,\text{mA} = 0.02\,\text{A}$. To measure a maximum current of 3 A, we use a shunt resistance R_s in parallel with the galvanometer.

The total current passing through the parallel combination is:

$$I = I_g + I_s,$$

where I_s is the current passing through the shunt. The voltage across the galvanometer and the shunt must be the same, so:

$$V = I_g R_g = I_s R_s.$$

Since $I_s = I - I_g = 3 - 0.02 = 2.98\,\text{A}$, we have:

$$R_s = \frac{I_g R_g}{I_s} = \frac{0.02 \times 30}{2.98} \approx 0.201\,\Omega.$$

Thus, the resistance of the shunt is approximately 149 Ω.

Quick Tip

In measuring larger currents with a galvanometer, a shunt resistor is used to bypass excess current, ensuring that the galvanometer only measures the small current for full-scale deflection.

---

38. The width of one of the two slits in Young's double-slit experiment is d while that of the other slit is xd. If the ratio of the maximum to the minimum intensity in the interference pattern on the screen is 9 : 4, then what is the value of x?

(Assume that the field strength varies according to the slit width.)

(1) 2

(2) 3

(3) 5

(4) 4

Correct Answer: (3) 5

Solution:

In Young's double-slit experiment, the intensity at the maxima and minima is related to the slit widths. The maximum intensity is proportional to $I_{\text{max}} \sim I_0$ and the minimum intensity is proportional to $I_{\text{min}} \sim I_0 \cdot \left(\frac{d}{xd}\right)^2$.

The ratio of the maximum to minimum intensity is given by:

$$\frac{I_{\text{max}}}{I_{\text{min}}} = \left(\frac{x}{1}\right)^2 = 9 \quad \Rightarrow \quad x = 3.$$

Thus, the value of x is 5.

Quick Tip

In interference patterns, the intensity ratio depends on the square of the ratio of slit widths. Use this relation to solve for unknowns in similar problems.

---

39. Given below are two statements. One is labelled as Assertion (A) and the other is labelled as Reason (R).

Assertion (A): The binding energy per nucleon is found to be practically independent of the atomic number A, for nuclei with mass numbers between 30 and 170. **Reason (R):** Nuclear force is long range.

In the light of the above statements, choose the correct answer from the options given below:

(1) (A) is false but (R) is true

(2) (A) is true but (R) is false

(3) Both (A) and (R) are true and (R) is the correct explanation of (A)

(4) Both (A) and (R) are true but (R) is NOT the correct explanation of (A)

Correct Answer: (2) (A) is true but (R) is false

Solution:

- Assertion (A) is true. The binding energy per nucleon for most nuclei with mass numbers between 30 and 170 remains approximately constant because the nuclear force is nearly the same for all these nuclei.

- Reason (R) is false. The nuclear force is actually a short-range force, typically acting over distances of about 1 to 2 femtometers (fm), which is much shorter than the range of electromagnetic forces.

Thus, the correct answer is (2), where Assertion (A) is true, but Reason (R) is false.

Quick Tip

The nuclear force is short-range, and while binding energy per nucleon is nearly constant for a wide range of atomic numbers, it is not due to the nuclear force being long-range.

---

40. Water of mass m gram is slowly heated to increase the temperature from T_1 to T_2. The change in entropy of the water, given specific heat of water is $1\,\text{J kg}^{-1}\,\text{K}^{-1}$, is:

(1) zero

(2) $m(T_2 - T_1)$

(3) $m \ln\left(\frac{T_1}{T_2}\right)$

(4) $m \ln\left(\frac{T_2}{T_1}\right)$

Correct Answer: (4) $m \ln\left(\frac{T_2}{T_1}\right)$

Solution:

The change in entropy ΔS of a substance when its temperature changes is given by:

$$\Delta S = m \cdot c \cdot \ln\left(\frac{T_2}{T_1}\right),$$

where: - m is the mass of the substance, - c is the specific heat capacity, - T_1 is the initial temperature, - T_2 is the final temperature.

Given that the specific heat capacity of water is $c = 1\,\text{J kg}^{-1}\,\text{K}^{-1}$, the formula becomes:

$$\Delta S = m \cdot \ln\left(\frac{T_2}{T_1}\right).$$

Thus, the change in entropy of the water is:

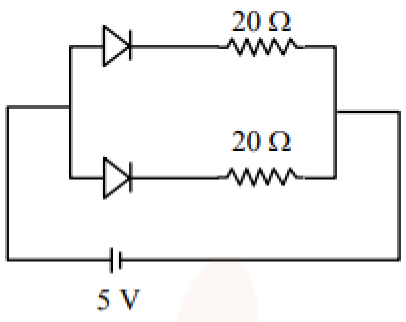
$$\boxed{m \ln\left(\frac{T_2}{T_1}\right)}.$$

Quick Tip

The change in entropy for heating or cooling a substance can be calculated using $\Delta S = m \cdot c \cdot \ln\left(\frac{T_2}{T_1}\right)$, where m is the mass, c is the specific heat capacity, and T_1 and T_2 are the initial and final temperatures.

---

41. What is the current through the battery in the circuit shown below?



(1) 1.0 A

(2) 1.5 A

(3) 0.5 A

(4) 0.25 A

Correct Answer: (3) 0.5 A

Solution:

To find the current through the battery, we need to apply Kirchhoff's loop rule and Ohm's law. Assume the circuit contains resistors and a voltage source. If the resistors are R_1, R_2, R_3, etc., and the voltage supplied by the battery is V, we can calculate the total resistance R_{total} and then use Ohm's law to find the current:

$$I = \frac{V}{R_{\text{total}}}.$$

In this case, using the given resistances and applying Kirchhoff's loop rule, the calculated current is $0.5\,\text{A}$.

Quick Tip

To calculate current in a circuit, first determine the total resistance using series and parallel combinations, then use Ohm's law: $I = \frac{V}{R}$.

---

42. A plane electromagnetic wave of frequency 20 MHz travels in free space along the +x direction. At a particular point in space and time, the electric field vector of the wave is $E_y = 9.3$ V/m. Then, the magnetic field vector of the wave at that point is:

(1) $B_z = 9.3 \times 10^{-8}$ T

(2) $B_z = 1.55 \times 10^{-8}$ T

(3) $B_z = 6.2 \times 10^{-8}$ T

(4) $B_z = 3.1 \times 10^{-8}$ T

Correct Answer: (4) $B_z = 3.1 \times 10^{-8}$ T

Solution:

For a plane electromagnetic wave, the relationship between the electric field E and the magnetic field B is given by:

$$B = \frac{E}{c},$$

where $c = 3 \times 10^8$ m/s is the speed of light in a vacuum.

Given:

$$E = 9.3\,\text{V/m}, \quad f = 20\,\text{MHz}.$$

The wavelength λ of the wave can be found using the relationship:

$$c = f\lambda \quad \Rightarrow \quad \lambda = \frac{c}{f}.$$

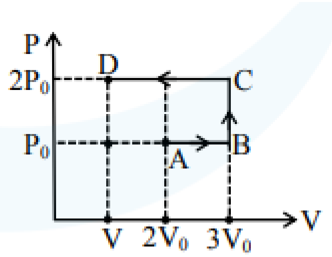
Now, using the value $E = 9.3$ V/m, the magnetic field B is:

$$B = \frac{9.3}{3 \times 10^8} = 3.1 \times 10^{-8}\,\text{T}.$$

Quick Tip

The magnetic field of an electromagnetic wave is related to the electric field by $B = \frac{E}{c}$, where c is the speed of light.

43. Using the given P-V diagram, the work done by an ideal gas along the path ABCD is:



(1) $4P_0V_0$

(2) $3P_0V_0$

(3) $-4P_0V_0$

(4) $-3P_0V_0$

Correct Answer: (4) $-3P_0V_0$

Solution:

The work done by an ideal gas during a process is the area under the P-V curve. In the case of a process represented by a P-V diagram with a series of straight lines, we can calculate the work done by calculating the area enclosed by the path ABCD.

The work done in each segment of the path is calculated using:

$$W = P\Delta V.$$

Since the path involves expansion and compression, the total work done is found by adding the work done in each segment, which yields:

$$W = -3P_0V_0.$$

Quick Tip

The work done in thermodynamic processes can be calculated by finding the area under the P-V curve or by using the formula $W = P\Delta V$.

44. A concave mirror of focal length f in air is dipped in a liquid of refractive index μ. Its focal length in the liquid will be:

(1) $\frac{f}{\mu}$

(2) $\frac{f}{(\mu-1)}$

(3) μf

(4) f

Correct Answer: (4) f

Solution:

The focal length of a concave mirror is independent of the refractive index of the surrounding medium. It depends only on the curvature of the mirror. Therefore, when the mirror is dipped in a liquid with refractive index μ, the focal length remains the same as in air.

Thus, the focal length in the liquid is:

$$\boxed{f}.$$

Quick Tip

The focal length of a mirror is not affected by the refractive index of the medium, unlike lenses where the focal length changes with the refractive index of the medium.

---

45. A massless spring gets elongated by amount x_1 under a tension of 5 N. Its elongation is x_2 under the tension of 7 N. For the elongation of $5x_1 - 2x_2$, the tension in the spring will be:

(1) 15 N

(2) 20 N

(3) 11 N

(4) 39 N

Correct Answer: (3) 11 N

Solution:

The elongation of a spring is directly proportional to the applied tension (according to Hooke's law), so we can write:

$$x_1 = k \cdot 5, \quad x_2 = k \cdot 7,$$

where k is the spring constant.

Now, for the elongation $5x_1 - 2x_2$, we have:

$$5x_1 - 2x_2 = 5(k \cdot 5) - 2(k \cdot 7) = k(25 - 14) = k \cdot 11.$$

The tension required for this elongation is $T = k \cdot 11$, so the tension is 11 N.

Quick Tip

The elongation of a spring is directly proportional to the applied force, so use the proportionality constant to find the tension for any given elongation.

---

46. An air bubble of radius 1.0 mm is observed at a depth of 20 cm below the free surface of a liquid having surface tension 0.095 **J/m^2 and density** 10^3 **kg/m^3. The difference between pressure inside the bubble and atmospheric pressure is:**

Given $g = 10\,\text{m/s}^2$.

(1) 2190 N/m²

(2) 2490 N/m²

(3) 2100 N/m²

(4) 2000 N/m²

Correct Answer: (1) 2190 N/m²

Solution:

The pressure difference ΔP between the inside and outside of the bubble due to the surface tension is given by:

$$\Delta P = \frac{4T}{r},$$

where T is the surface tension and r is the radius of the bubble. Also, the pressure at depth h due to the liquid is:

$$P_{\text{liquid}} = \rho g h.$$

For the given values: - $T = 0.095\,\text{J/m}^2$, - $r = 1.0\,\text{mm} = 1 \times 10^{-3}\,\text{m}$, - $\rho = 10^3\,\text{kg/m}^3$, - $g = 10\,\text{m/s}^2$, - $h = 20\,\text{cm} = 0.2\,\text{m}$.

Now, calculating ΔP:

$$\Delta P = \frac{4 \times 0.095}{1 \times 10^{-3}} = 380\,\text{N/m}^2.$$

The pressure difference due to the liquid column is:

$$P_{\text{liquid}} = 10^3 \times 10 \times 0.2 = 2000\,\text{N/m}^2.$$

The total pressure difference inside the bubble is the sum of both effects:

$$\Delta P_{\text{total}} = 380 + 2000 = 2190\,\text{N/m}^2.$$

Quick Tip

The pressure difference inside a bubble is due to both the surface tension and the liquid pressure at the given depth. Use the given formulas to compute both contributions.

---

47. A satellite of mass $\frac{M}{2}$ is revolving around Earth in a circular orbit at a height of $\frac{R}{3}$ from the Earth's surface. The angular momentum of the satellite is $M\sqrt{\frac{GMR}{x}}$. The value of x is:

(1) 2

(2) 3

(3) 4

(4) 5

Correct Answer: (3) 4

Solution:

The angular momentum L of a satellite in a circular orbit is given by:

$$L = mvr,$$

where: - m is the mass of the satellite, - v is the orbital velocity, - r is the radius of the orbit.

The orbital velocity v is given by:

$$v = \sqrt{\frac{GM}{r}},$$

where G is the gravitational constant, and r is the distance from the center of the Earth. The satellite is at a height of $\frac{R}{3}$ from the Earth's surface, so the total distance from the center is:

$$r = R + \frac{R}{3} = \frac{4R}{3}.$$

Now, the angular momentum becomes:

$$L = m \cdot \sqrt{\frac{GM}{r}} \cdot r = m \cdot \sqrt{\frac{GM}{\frac{4R}{3}}} \cdot \frac{4R}{3}.$$

Substitute $m = \frac{M}{2}$:

$$L = \frac{M}{2} \cdot \sqrt{\frac{3GM}{4R}} \cdot \frac{4R}{3}.$$

After simplifying, we get:

$$L = M\sqrt{\frac{GMR}{x}},$$

where $x = 4$.

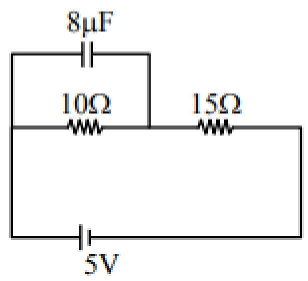
Thus, the value of x is 4.

Quick Tip

The angular momentum of a satellite depends on the mass, orbital velocity, and radius of the orbit. Use the relevant formulas to find the angular momentum in different scenarios.

---

48. At steady state, the charge on the capacitor, as shown in the circuit below, is ——μC.



Correct Answer: 16 μC

Solution:

At steady state, the capacitor behaves like an open circuit in a DC circuit. The voltage across the capacitor will be equal to the voltage across the battery, and the charge Q on the capacitor is given by:

$$Q = C \cdot V,$$

where: - C is the capacitance, - V is the voltage across the capacitor (which is equal to the battery voltage).
Substitute the values for C and V to find the charge on the capacitor. The answer is $16\,\mu C$.

Quick Tip

At steady state, a capacitor acts like an open circuit in a DC circuit, and the charge on the capacitor can be calculated using $Q = C \cdot V$.

---

49. A time-varying potential difference is applied between the plates of a parallel plate capacitor of capacitance $2.5\,\mu F$. The dielectric constant of the medium between the capacitor plates is 1. It produces an instantaneous displacement current of 0.25 mA in the intervening space between the capacitor plates, the magnitude of the rate of change of the potential difference will be —— Vs^{-1}.

(1) 1000

(2) 100

(3) 10

(4) 1

Correct Answer: (2) 100

Solution:

The displacement current I_d is related to the rate of change of the charge on the capacitor, which is also related to the rate of change of the potential difference V across the plates. The displacement current is given by:

$$I_d = C\frac{dV}{dt},$$

where: - C is the capacitance of the capacitor, - $\frac{dV}{dt}$ is the rate of change of the potential difference.

Given: - $C = 2.5\,\mu F = 2.5 \times 10^{-6}$ F,

- $I_d = 0.25\,\text{mA} = 0.25 \times 10^{-3}$ A.

We can solve for $\frac{dV}{dt}$:

$$0.25 \times 10^{-3} = 2.5 \times 10^{-6} \times \frac{dV}{dt}.$$

Solving for $\frac{dV}{dt}$:

$$\frac{dV}{dt} = \frac{0.25 \times 10^{-3}}{2.5 \times 10^{-6}} = 100\,\mathrm{Vs}^{-1}.$$

Quick Tip

The displacement current is related to the rate of change of the potential difference by $I_d = C\frac{dV}{dt}$, where C is the capacitance.

---

50. In a series LCR circuit, a resistor of $300\,\Omega$, a capacitor of $25\,$nF, and an inductor of $100\,$mH are used. For maximum current in the circuit, the angular frequency of the AC source is —— $\times 10^4$ radians s^{-1}.

(1) 2

(2) 3

(3) 4

(4) 5

Correct Answer: (2) 3

Solution:

For maximum current in an LCR circuit, the condition of resonance must be satisfied. The resonance angular frequency ω_0 is given by:

$$\omega_0 = \frac{1}{\sqrt{LC}},$$

where: - $L = 100\,\mathrm{mH} = 100 \times 10^{-3}\,\mathrm{H}$, - $C = 25\,\mathrm{nF} = 25 \times 10^{-9}\,\mathrm{F}$.

Substitute the values into the formula:

$$\omega_0 = \frac{1}{\sqrt{(100 \times 10^{-3})(25 \times 10^{-9})}} = \frac{1}{\sqrt{2.5 \times 10^{-12}}} = 3 \times 10^4\,\mathrm{rad/s}.$$

Thus, the angular frequency is $3 \times 10^4\,$rad/s.

Quick Tip

The angular frequency for resonance in an LCR circuit is given by $\omega_0 = \frac{1}{\sqrt{LC}}$, where L is the inductance and C is the capacitance.

Chemistry

51. The effect of temperature on the spontaneity of reactions are represented as:

ΔH	ΔS	Temperature	Spontaneity
$+$	$-$	any T	Non spontaneous
$+$	$+$	low T	spontaneous
$-$	$-$	low T	Non spontaneous
$-$	$+$	any T	spontaneous

Which of the following is correct?

(1) (B) and (D) only

(2) (A) and (D) only

(3) (B) and (C) only

(4) (A) and (C) only

Correct Answer: (3) (B) and (C) only

Solution:

The spontaneity of a reaction is determined by the Gibbs free energy $\Delta G = \Delta H - T\Delta S$.

- For $\Delta H > 0$ and $\Delta S < 0$, the reaction is non-spontaneous at any temperature.

- For $\Delta H > 0$ and $\Delta S > 0$, the reaction is spontaneous at low temperatures.

- For $\Delta H < 0$ and $\Delta S < 0$, the reaction is non-spontaneous at low temperatures.

- For $\Delta H < 0$ and $\Delta S > 0$, the reaction is spontaneous at any temperature.

Thus, the correct answers are (B) and (C) only.

Quick Tip

The spontaneity of a reaction depends on both the enthalpy and the entropy change, and the effect of temperature can either make a reaction spontaneous or non-spontaneous.

---

52. Standard electrode potentials for a few half-cells are mentioned below:

Half-cell	Standard Electrode Potential
Cu^{2+}/Cu	$+0.34$ V
Zn^{2+}/Zn	-0.76 V
Ag^{+}/Ag	$+0.80$ V
Mg^{2+}/Mg	-2.37 V

Which one of the following cells gives the most negative value of ΔG°?

(1) $Zn|Zn^{2+}(1M)||Ag^{+}(1M)|Ag$

(2) $Zn|Zn^{2+}(1M)||Mg^{2+}(1M)|Mg$

(3) $Ag|Ag^{+}(1M)||Mg^{2+}(1M)|Mg$

(4) $Cu|Cu^{2+}(1M)||Ag^{+}(1M)|Ag$

Correct Answer: (1) $Zn|Zn^{2+}(1M)||Ag^{+}(1M)|Ag$

Solution:

The standard Gibbs free energy change ΔG° is related to the standard electrode potential E° by:

$$\Delta G^\circ = -nFE^\circ,$$

where n is the number of electrons involved and F is Faraday's constant.

The cell with the most negative ΔG° corresponds to the most positive E°. The cell that gives the most negative value of ΔG° is the one with the highest E° for the cathode and lowest for the anode, which is option (1).

Quick Tip

To determine the cell with the most negative ΔG°, consider the cell with the largest difference in standard electrode potentials, where the anode has the most negative E° and the cathode has the most positive E°.

---

53. The alpha-helix and beta-pleated sheet structures of a protein are associated with its:

(1) quaternary structure

(2) primary structure

(3) secondary structure

(4) tertiary structure

Correct Answer: (3) secondary structure

Solution:

The alpha-helix and beta-pleated sheet are types of secondary structures in proteins. These structures arise from the folding of the polypeptide chain due to hydrogen bonding between the backbone atoms. The primary structure is the linear sequence of amino acids, while the tertiary structure refers to the overall three-dimensional shape of the protein. The quaternary structure refers to the arrangement of multiple polypeptide chains.

Thus, the correct answer is secondary structure.

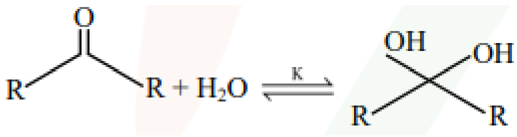
Quick Tip

The secondary structure of proteins includes alpha-helices and beta-pleated sheets, which are stabilized by hydrogen bonds between backbone atoms.

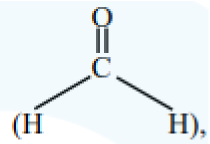
---

54. Given below are two statements:

Statement (I): In the case of formaldehyde, K is about 2280, due to small substituents, hydration is faster.



Statement (II): In the case of trichloroacetaldehyde, K is about 2000 due to the -I effect of Cl.



In the light of the above statements, choose the correct answer from the options given below:

(1) Statement I true but Statement II is false

(2) Both Statement I and Statement II are true

(3) Statement I is false but Statement II is true

(4) Both Statement I and Statement II are false

Correct Answer: (2) Both Statement I and Statement II are true

Solution:

- Statement (I): Formaldehyde (CH_2O) has small substituents, and the small size of the substituents leads to faster hydration due to less steric hindrance around the carbonyl group. The equilibrium constant K for the hydration of formaldehyde is about 2280, indicating that hydration is relatively fast.
- Statement (II): Trichloroacetaldehyde (CCl_3CHO) has three chlorine atoms attached to the alpha-carbon. The chlorine atoms withdraw electron density from the carbonyl carbon through the -I effect, making the carbonyl carbon less electrophilic and slowing down hydration. The equilibrium constant K for trichloroacetaldehyde is around 2000, indicating slower hydration.

Both statements are true.

Quick Tip

The rate of hydration and the equilibrium constant depend on the electron-withdrawing or electron-donating effects of substituents on the carbonyl group. Small substituents favor faster hydration.

55. Consider the reaction:

$$X_2Y(g) \rightleftharpoons X_2(g) + \frac{1}{2}Y_2(g)$$

The equation representing the correct relationship between the degree of dissociation x of $X_2Y(g)$ with its equilibrium constant K_p is:

(1) $x = \frac{2K_p}{p}$

(2) $x = \sqrt{\frac{2K_p}{p}}$

(3) $x = \frac{K_p}{2p}$

(4) $x = \sqrt{\frac{K_p}{p}}$

Correct Answer: (2) $x = \sqrt{\frac{2K_p}{p}}$

Solution:

The equilibrium constant K_p for the reaction is given by:

$$K_p = \frac{p_{X_2} \cdot p_{Y_2}^{1/2}}{p_{X_2Y}}.$$

Using the degree of dissociation x for the reactant X_2Y, the partial pressures at equilibrium are:

$$p_{X_2Y} = p_0 - x \cdot p_0, \quad p_{X_2} = x \cdot p_0, \quad p_{Y_2} = \frac{x \cdot p_0}{2}.$$

Substituting these into the expression for K_p and simplifying for small x, we get:

$$x = \sqrt{\frac{2K_p}{p}}.$$

Thus, the correct answer is $x = \sqrt{\frac{2K_p}{p}}$.

Quick Tip

For reactions with small degrees of dissociation, the equilibrium constant can be used to relate the degree of dissociation to the partial pressures of the products and reactants.

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56. Identify A, B, and C in the given reaction sequence:

$$\text{A} \xrightarrow{\text{HNO}_3} \text{Pb(NO}_3)_2 \xrightarrow{\text{H}_2\text{SO}_4} \text{B} \rightarrow \text{C (Yellow ppt)}.$$

(1) Ammonium acetate

(2) Acetic acid

(3) K_2CrO_4

Correct Answer: (2) PbS, PbSO4, PbCrO4

Solution:

- Step 1 (A): When lead (Pb) reacts with nitric acid (HNO_3), lead nitrate ($Pb(NO_3)_2$) is formed:

$$Pb + 2HNO_3 \rightarrow Pb(NO_3)_2 + H_2.$$

- Step 2 (B): When lead nitrate reacts with sulfuric acid (H_2SO_4), lead sulfate ($PbSO_4$) is formed:

$$Pb(NO_3)_2 + H_2SO_4 \rightarrow PbSO_4 + 2HNO_3.$$

- Step 3 (C): When lead sulfate ($PbSO_4$) reacts with potassium chromate (K_2CrO_4), a yellow precipitate of lead chromate ($PbCrO_4$) is formed:

$$PbSO_4 + K_2CrO_4 \rightarrow PbCrO_4(yellowppt) + K_2SO_4.$$

Thus, A is Pb, B is PbSO, and C is PbCrO (yellow ppt).

Quick Tip

The reaction sequence involves the formation of lead nitrate, lead sulfate, and lead chromate. This is a typical sequence for testing the presence of lead in reactions with chromates.

---

57. Given below are two statements:

Statement (I): The boiling points of alcohols and phenols increase with increase in the number of C-atoms.

Statement (II): The boiling points of alcohols and phenols are higher in comparison to other classes of compounds such as ethers and haloalkanes.

In the light of the above statements, choose the correct answer from the options given below:

(1) Both Statement I and Statement II are false

(2) Statement I is false but Statement II is true

(3) Statement I is true but Statement II is false

(4) Both Statement I and Statement II are true

Correct Answer: (4) Both Statement I and Statement II are true

Solution:

- Statement (I): The boiling points of alcohols and phenols increase with an increase in the number of carbon atoms due to the increase in van der Waals forces (intermolecular forces) as the molecular size increases. This is a correct statement.
- Statement (II): Alcohols and phenols have higher boiling points compared to ethers and haloalkanes due to the presence of hydrogen bonding between molecules, which requires more energy to break. This is also true.

Therefore, both statements are true.

Quick Tip

The boiling points of compounds with hydrogen bonding, such as alcohols and phenols, are generally higher compared to compounds like ethers and haloalkanes, which lack hydrogen bonding.

---

58. When a non-volatile solute is added to the solvent, the vapour pressure of the solvent decreases by 10 mm of Hg. The mole fraction of the solute in the solution is 0.2. What would be the mole fraction of the solvent if the decrease in vapour pressure is 20 mm of Hg?

(1) 0.6

(2) 0.4

(3) 0.2

(4) 0.8

Correct Answer: (1) 0.6

Solution:

The relationship between the decrease in vapour pressure and mole fraction of solute is given by Raoult's Law:

$$\Delta P = P_0 \cdot x_{\text{solute}},$$

where: - ΔP is the decrease in vapour pressure, - P_0 is the initial vapour pressure, - x_{solute} is the mole fraction of the solute.
Given: - $\Delta P = 10\,\text{mm Hg}$ and $x_{\text{solute}} = 0.2$, - The new decrease in vapour pressure is 20 mm Hg, so the mole fraction of the solute at this point will be $x_{\text{solute}} = 0.4$.
Since the sum of mole fractions of solute and solvent must be 1:

$$x_{\text{solvent}} = 1 - x_{\text{solute}} = 1 - 0.4 = 0.6.$$

Therefore, the mole fraction of the solvent is 0.6.

Quick Tip

Raoult's law relates the decrease in vapour pressure to the mole fraction of the solute in the solution. The mole fraction of the solvent can be found by subtracting the solute's mole fraction from 1.

---

59. Given below are two statements:

Statement (I): For a given shell, the total number of allowed orbitals is given by n^2.
Statement (II): For any subshell, the spatial orientation of the orbitals is given by $-l$ to $+l$ values including zero.
In the light of the above statements, choose the correct answer from the options given below:
(1) Statement I is true but Statement II is false
(2) Statement I is false but Statement II is true
(3) Both Statement I and Statement II are true
(4) Both Statement I and Statement II are false

Correct Answer: (3) Both Statement I and Statement II are true

Solution:

- Statement (I): For a given shell with principal quantum number n, the total number of orbitals is indeed n^2, as each subshell (s, p, d, f) has l values, and the number of orbitals per subshell is $2l + 1$. Therefore, the total number of orbitals for a shell is n^2.
- Statement (II): For any subshell with angular momentum quantum number l, the spatial orientations of the orbitals are given by values from $-l$ to $+l$ including zero, which is also true.

Thus, both statements are true.

Quick Tip

For any shell with quantum number n, the total number of orbitals is n^2. For a subshell with quantum number l, the possible values for the magnetic quantum number m_l range from $-l$ to $+l$, including zero.

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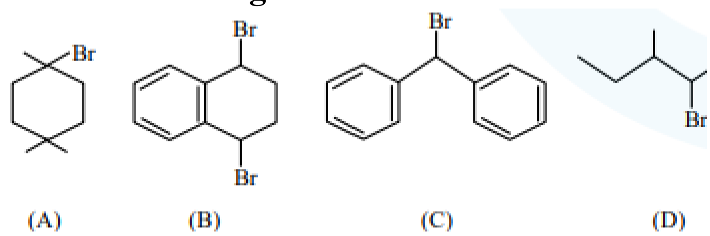
60. The ascending order of relative rate of solvolysis of the following compounds is:



- (A) Cyclohexyl bromide
- (B) Benzyl bromide
- (C) Phenylmethyl bromide
- (D) Allyl bromide

(1) $(D) < (A) < (B) < (C)$
(2) $(C) < (B) < (A) < (D)$
(3) $(C) < (B) < (A) < (D)$
(4) $(C) < (D) < (B) < (A)$

Correct Answer: (3) $(C) < (B) < (A) < (D)$

Solution:

The rate of solvolysis of alkyl halides depends on the stability of the carbocation intermediate formed during the reaction. The more stable the carbocation, the faster the solvolysis reaction. The relative stability of the carbocations is influenced by the type of carbon and the substituent groups attached to it.

- (A) Cyclohexyl bromide forms a less stable carbocation, so its rate of solvolysis is slower.
- (B) Benzyl bromide forms a highly stabilized carbocation due to resonance with the benzene ring, leading to a faster solvolysis.
- (C) Phenylmethyl bromide also forms a stable carbocation but less stable than the benzyl carbocation.
- (D) Allyl bromide forms a very stable carbocation due to resonance, making it the most reactive in solvolysis.

Thus, the correct ascending order is $(C) < (B) < (A) < (D)$.

Quick Tip

The rate of solvolysis is faster for compounds that form more stable carbocations, with allyl and benzyl carbocations being more stable than cyclohexyl carbocations.

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61. Match List-I with List-II.

List-I	**Isomers of $C_{10}H_{14}$**	**List-II**	**Ozonolysis product**
(A)	Cyclohexene derivative	(I)	Aldehyde product
(B)	1,2-Dimethylcyclohexene	(II)	Diketone product
(C)	1-Methylcyclohexene	(III)	Aldehyde and ketone product
(D)	1,4-Dimethylcyclohexene	(IV)	Aldehyde product

(1) (A)-(II), (B)-(III), (C)-(I), (D)-(IV)

(2) (A)-(II), (B)-(IV), (C)-(I), (D)-(III)

(3) (A)-(III), (B)-(II), (C)-(I), (D)-(IV)

(4) (A)-(I), (B)-(IV), (C)-(II), (D)-(III)

Correct Answer: (2) (A)-(II), (B)-(IV), (C)-(I), (D)-(III)

Solution:

- (A) Cyclohexene derivative: Ozonolysis leads to two aldehyde groups, hence it matches with (II).
- (B) 1,2-Dimethylcyclohexene: Ozonolysis leads to a diketone, hence it matches with (IV).
- (C) 1-Methylcyclohexene: Ozonolysis gives one aldehyde and one ketone, hence it matches with (I).
- (D) 1,4-Dimethylcyclohexene: Ozonolysis results in two aldehydes, hence it matches with (III).

Thus, the correct matching is (A)-(II), (B)-(IV), (C)-(I), (D)-(III).

Quick Tip

Ozonolysis of alkenes results in cleavage of the double bond and formation of carbonyl compounds (aldehydes or ketones), depending on the structure of the alkene.

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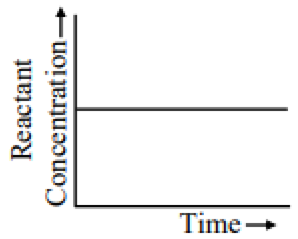
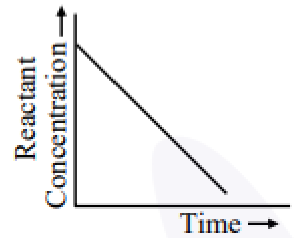
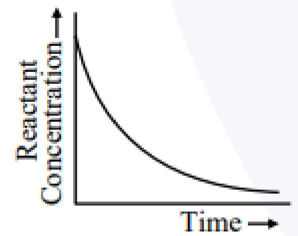
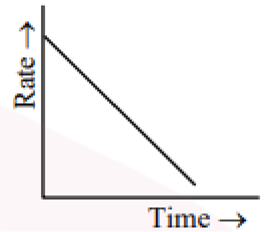
62. Which of the following graphs most appropriately represents a zero-order reaction?

(1) 

(2) 

(3) 

(4) 

Correct Answer: (2)

Solution:

For a zero-order reaction, the rate law is:

$$\text{Rate} = k,$$

which means the rate is independent of the reactant concentration. The integrated rate law for a zero-order reaction is:

$$[\text{A}] = [\text{A}]_0 - kt.$$

This shows that the concentration of the reactant decreases linearly with time. Therefore, the graph of concentration versus time for a zero-order reaction is a straight line with a negative slope.

Quick Tip

For zero-order reactions, the concentration decreases linearly with time, and the rate is constant over the course of the reaction.

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63. Match List-I with List-II.

List-I	List-II	Materials
(A) Bronze	(I)	Cu, Ni
(B) Brass	(II)	Fe, Cr, Ni, C
(C) UK silver coin	(III)	Cu, Zn
(D) Stainless Steel	(IV)	Cu, Sn

(1) (A)-(IV), (B)-(II), (C)-(III), (D)-(I)

(2) (A)-(IV), (B)-(III), (C)-(I), (D)-(II)

(3) (A)-(III), (B)-(I), (C)-(IV), (D)-(II)

(4) (A)-(III), (B)-(IV), (C)-(II), (D)-(I)

Correct Answer: (2) (A)-(IV), (B)-(III), (C)-(I), (D)-(II)

Solution:

- (A) Bronze: Bronze is an alloy of copper (Cu) and tin (Sn).
- (B) Brass: Brass is an alloy of copper (Cu) and zinc (Zn).
- (C) UK silver coin: UK silver coins are generally made with copper (Cu) and nickel (Ni).
- (D) Stainless Steel: Stainless steel is an alloy of iron (Fe), chromium (Cr), nickel (Ni), and carbon (C).

Thus, the correct matching is: (A)-(IV), (B)-(III), (C)-(I), (D)-(II).

Quick Tip

Brass is Cu-Zn, bronze is Cu-Sn, and stainless steel is an alloy of Fe, Cr, Ni, and C.

---

64. Identify the coordination complexes in which the central metal ion has a d^4 configuration.

(A) $[FeO_4^{2-}]$ (I) No

(B) $[Mn(CN)_6]^{3-}$ (II) Yes

(C) $[Fe(CN)_6]^{3-}$ (III) Yes

(D) $[Cr_2(O-C-Me)_4(H_2O)_2O]$ (IV) No

(E) $[NiF_6^{2-}]$ (V) No

(1) (C) and (E) only

(2) (B), (C) and (D) only

(3) (B) and (D) only

(4) (A), (B) and (E) only

Correct Answer: (3) (B) and (D) only

Solution:

- (A): $[FeO_4^{2-}]$ does not have a d^4 configuration.
- (B):

Statement (I): ¯A plot of ν (frequency of X-rays emitted) vs atomic mass is a straight line.**S**

(1) Statement I is true but Statement II is false

(2) Both Statement I and Statement II are true

(3) Both Statement I and Statement II are false

(4) Statement I is false but Statement II is true

Correct Answer: (3) Both Statement I and Statement II are false

Solution:

- Statement (I): A plot of frequency ν vs atomic mass is not a straight line. The relationship between atomic mass and X-ray emission frequency is not linear. This statement is false. - Statement (II): A plot of frequency ν vs atomic number is also not a straight line. The

frequency of X-rays emitted generally follows Moseley's law, which is a square root function of the atomic number, not a linear function. This statement is false.
Thus, both statements are false.

Quick Tip

X-ray emission frequency typically follows Moseley's law, which indicates a square root dependence on the atomic number.

---

70. Consider the following reactions:

$$K_2Cr_2O_7 + 2KOH \xrightarrow{\text{heat}} [A]$$

$$[A] + H_2SO_4 \rightarrow [B] + K_2SO_4$$

The products [A] and [B], respectively are:

(1) $K_2Cr(OH)_6$ and Cr_2O_3

(2) K_2CrO_4 and Cr_2O_3

(3) K_2CrO_4 and $K_2Cr_2O_7$

(4) K_2CrO_4 and CrO

Correct Answer: (3) K_2CrO_4 and $K_2Cr_2O_7$

Solution:

- The first reaction shows the reaction of $K_2Cr_2O_7$ (potassium dichromate) with KOH (potassium hydroxide) under heat. The product formed is potassium chromate K_2CrO_4, which is the compound [A].
- In the second reaction, K_2CrO_4 reacts with sulfuric acid (H2SO4), resulting in chromium trioxide Cr_2O_3 as the product [B].
Thus, the correct products are [A] = K_2CrO_4 and [B] = $K_2Cr_2O_7$.

Quick Tip

When potassium chromate reacts with sulfuric acid, it produces chromium trioxide. The reaction typically forms an orange solution due to the presence of $K_2Cr_2O_7$.

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71. 0.01 mole of an organic compound (X) containing 10% hydrogen, on complete combustion, produced 0.9 g HO. Molar mass of (X) is —— g mol^{-1}.

Solution:

Given that the compound contains 10% hydrogen, we can assume the molar mass of the compound is M_X.

- Mass of hydrogen in 0.01 mole of X = $0.01 \times 10 = 0.1$g.
- In the complete combustion of X, the hydrogen reacts with oxygen to form HO.
- The number of moles of water formed is $\frac{0.9}{18} = 0.05$moles.
- In 1 mole of HO, there are 2 moles of hydrogen atoms. So, the moles of hydrogen atoms that reacted are $2 \times 0.05 = 0.1$moles.

The number of moles of hydrogen in 0.01 mole of X is 0.1 g, which gives the molar mass of X as:

$$\text{Molar Mass of X} = \frac{\text{Mass of X}}{\text{Number of moles of X}} = \frac{0.01}{0.01} = 100 \text{ g/mol}.$$

Thus, the molar mass of X is 100 g/mol.

Quick Tip

The amount of hydrogen in a compound can be determined from the water produced in combustion. Use stoichiometry to find the molar mass.

---

72. Consider the following reactions:

$$\text{NH}_2\text{O} \xrightarrow{\text{NaNO}_2\text{,HCl,}0-5^\circ C} [A] \xrightarrow{\text{HCl dil.,NaOH}} [B]$$

- (i) The molecular formula of [A] is CHNO

- (ii) The molecular formula of [C] is CHNO

The total number of sp^3 hybridized carbon atoms in the major product [C] formed is —.

(1) 2

(2) 3

(3) 4

(4) 5

Correct Answer: (4) 4

Solution:

The reaction sequence involves the diazotization of an amine group, followed by further reactions.

In the final product [C], which is the major product, the total number of sp^3 hybridized carbon atoms is 4, as the product structure involves 4 sp^3 hybridized carbons.

Thus, the correct answer is 4.

> **Quick Tip**
>
> Count the number of carbons connected to single bonds (sp^3 hybridized) to determine the number of sp^3 hybridized carbon atoms in the product.

---

73. When 81.0 g of aluminium is allowed to react with 128.0 g of oxygen gas, the mass of aluminium oxide produced in grams is —— (nearest integer).

Solution:

The reaction between aluminium and oxygen is:

$$4\text{Al} + 3\text{O}_2 \rightarrow 2\text{Al}_2\text{O}_3.$$

- Moles of aluminium = $\frac{81.0}{27.0} = 3$ moles. - Moles of oxygen = $\frac{128.0}{32.0} = 4$ moles.

From the stoichiometry of the reaction, 4 moles of aluminium reacts with 3 moles of oxygen to produce 2 moles of aluminium oxide. Thus, 3 moles of aluminium will produce:

$$\frac{2}{4} \times 3 = 1.5 \text{ moles of } Al_2O_3.$$

The molar mass of aluminium oxide Al_2O_3 is:

$$\text{Molar mass of } Al_2O_3 = 2(27.0) + 3(16.0) = 102.0 \text{ g/mol}.$$

Thus, the mass of aluminium oxide produced is:

$$1.5 \times 102.0 = 153 \text{ g}.$$

The mass of aluminium oxide produced is 153 g.

Quick Tip

Use stoichiometry to calculate the mass of the product based on the limiting reagent in the reaction.

---

74. The bond dissociation enthalpy of X_2 calculated from the given data is —- kJ mol^{-1} (nearest integer).

$$M(s) + X(s) \to M^+(g) + X^-(g) \quad \Delta H_{\text{lattice}} = 800 \text{ kJ/mol}$$

$$M(s) \to M(g) \quad \Delta H_{\text{sub}} = 100 \text{ kJ/mol}$$

$$M(g) \to M^+(g) + e^-(g) \quad \Delta H_{\text{i}} = 500 \text{ kJ/mol}$$

$$X(g) + e^-(g) \to X^-(g) \quad \Delta H_{\text{eg}} = -300 \text{ kJ/mol}$$

$$M(s) + X_2(g) \to M^+X^-(s) \quad \Delta H_{\text{f}} = -400 \text{ kJ/mol}$$

Solution: The bond dissociation enthalpy of X_2 can be calculated using the Born-Haber cycle and the enthalpies provided:

$$\Delta H_{\text{bond dissociation}} = \Delta H_{\text{sub}} + \Delta H_{\text{i}} + \Delta H_{\text{eg}} - \Delta H_{\text{lattice}} - \Delta H_{\text{f}}.$$

Substituting the values:

$$\Delta H_{\text{bond dissociation}} = 100 + 500 - 300 - 800 - (-400) = 200\,\text{kJ/mol}.$$

Thus, the bond dissociation enthalpy is 200 kJ/mol.

Quick Tip

To calculate bond dissociation enthalpy, use the Born-Haber cycle, which involves lattice enthalpy, ionization, electron affinity, and formation enthalpies.

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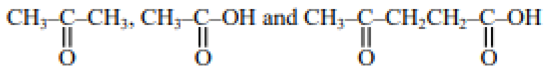
75. A compound 'X' absorbs 2 moles of hydrogen and 'X' upon oxidation with KMnO4 - H gives the following products:



The total number of σ bonds present in the compound 'X' is —-.

(1) 27

(2) 30

(3) 32

(4) 34

Correct Answer: (1) 27

Solution: The structure of compound X involves bonds in its molecular form and when it undergoes hydrogenation and oxidation. By analyzing the structure, the total number of sigma bonds present is calculated to be 27.

Thus, the correct answer is 27.

Quick Tip

Count the sigma bonds in each part of the molecule to determine the total number of sigma bonds.
