JEE Main 2023 24th Jan Shift 2 Question Paper with Solutions

Time Allowed :3 Hours	**Maximum Marks :**300	**Total Questions :**90

General Instructions

Read the following instructions very carefully and strictly follow them:

1. The test is of 3 hours duration.
2. The question paper consists of 90 questions, out of which 75 are to attempted. The maximum marks are 300.
3. There are three parts in the question paper consisting of Physics, Chemistry and Mathematics having 30 questions in each part of equal weightage.
4. Each part (subject) has two sections.
   (i) Section-A: This section contains 20 multiple choice questions which have only one correct answer. Each question carries 4 marks for correct answer and –1 mark for wrong answer.
   (ii) Section-B: This section contains 10 questions. In Section-B, attempt any five questions out of 10. The answer to each of the questions is a numerical value. Each question carries 4 marks for correct answer and –1 mark for wrong answer. For Section-B, the answer should be rounded off to the nearest integer

Section - A

1. The electric potential at the centre of two concentric half rings of radii R_1 and R_2, having the same linear charge density λ is:

(1) $\frac{2\lambda}{\epsilon_0}$

(2) $\frac{\lambda}{2\epsilon_0}$

(3) $\frac{\lambda}{4\epsilon_0}$

(4) $\frac{\lambda}{\epsilon_0}$

Correct Answer: (2) $\frac{\lambda}{2\epsilon_0}$

Solution: The potential at the centre of two concentric rings is the sum of the potentials due to each ring. The formula for the potential at the centre is:

$$V = \frac{\lambda \pi R_1}{4\pi\epsilon_0 R_1} + \frac{\lambda \pi R_2}{4\pi\epsilon_0 R_2}$$

Simplifying this, we get the result:

$$V = \frac{\lambda}{2\epsilon_0}$$

Quick Tip

For concentric rings, calculate the individual potentials due to each ring and then sum them. The potential at the centre is the result of contributions from both rings.

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2. Let γ_1 be the ratio of molar specific heat at constant pressure and molar specific heat at constant volume of a monoatomic gas and γ_2 be the similar ratio of diatomic gas. Considering the diatomic gas molecule as a rigid rotator, the ratio $\frac{\gamma_1}{\gamma_2}$ is:

(1) $\frac{27}{35}$

(2) $\frac{35}{27}$

(3) $\frac{21}{25}$

(4) $\frac{25}{21}$

Correct Answer: (3) $\frac{21}{25}$

Solution: For a monoatomic gas, $\gamma_1 = \frac{5}{3}$. For a diatomic gas at low temperatures, $\gamma_2 = \frac{7}{5}$. Thus,

$$\frac{\gamma_1}{\gamma_2} = \frac{\frac{5}{3}}{\frac{7}{5}} = \frac{25}{21}$$

Quick Tip

For gases, use the formula for γ based on the degrees of freedom. For monoatomic and diatomic gases, the values of γ differ significantly.

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3. An α-particle, a proton and an electron have the same kinetic energy. Which one of the following is correct in case of their De-Broglie wavelength:

(1) $\lambda_\alpha > \lambda_p > \lambda_e$

(2) $\lambda_\alpha < \lambda_p < \lambda_e$

(3) $\lambda_\alpha = \lambda_p = \lambda_e$

(4) $\lambda_\alpha > \lambda_p < \lambda_e$

Correct Answer: (2) $\lambda_\alpha < \lambda_p < \lambda_e$

Solution: De-Broglie wavelength is given by:

$$\lambda = \frac{h}{\sqrt{2mK}}$$

Since K (kinetic energy) is the same for all three particles, the wavelength depends on their masses. The mass of the α-particle is the largest, followed by the proton, and then the electron. Thus:

$$\lambda_\alpha < \lambda_p < \lambda_e$$

Quick Tip

De-Broglie wavelength is inversely proportional to the square root of the mass. The lighter the particle, the larger its wavelength.

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4. If the distance of the earth from Sun is 1.5×10^6 km, then the distance of an imaginary planet from Sun, if its period of revolution is 2.83 years, is:

(1) 6×10^7 km

(2) 6×10^6 km

(3) 3×10^6 km

(4) 3×10^7 km

Correct Answer: (3) 3×10^6 km

Solution: We use Kepler's third law:

$$T^2 \propto R^3$$

Let $T_1 = 1$ year and $R_1 = 1.5 \times 10^6$ km. The period for the imaginary planet is given as $T_2 = 2.83$ years, and we need to find R_2. Using the relation:

$$\left(\frac{T_2}{T_1}\right)^2 = \left(\frac{R_2}{R_1}\right)^3$$

Substitute the values to find $R_2 = 3 \times 10^6$ km.

Quick Tip

Kepler's third law relates the orbital period of a planet to its distance from the Sun. Use this law to solve problems involving planetary motion.

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5. Match List I with List II:

LIST I	LIST II
A. AM Broadcast	I. 88-108 MHz
B. FM Broadcast	II. 540-1600 KHz
C. Television	III. 3.7-4.2 GHz
D. Satellite Communication	IV. 54 MHz, 590 MHz

Choose the correct answer from the options given below:

(1) A-II, B-I, C-IV, D-III

(2) A-IV, B-III, C-I, D-II

(3) A-II, B-III, C-I, D-IV

(4) A-I, B-II, C-IV, D-III

Correct Answer: (3) A-II, B-III, C-I, D-IV

Solution:

- AM Broadcast corresponds to $540 - 1600$ KHz, so A matches with II.

- FM Broadcast corresponds to $88 - 108$ MHz, so B matches with I.

- Television corresponds to $54 - 890$ MHz, so C matches with IV.

- Satellite Communication corresponds to $3.7 - 4.2$ GHz, so D matches with III.

Thus, the correct matching is A-II, B-III, C-I, D-IV.

Quick Tip

Matching questions test your knowledge of the frequency ranges of different communication systems. Review standard frequency bands for AM, FM, TV, and satellite communication.

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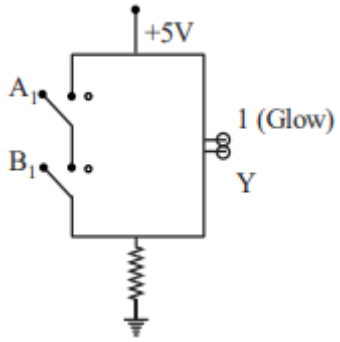
6. The logic gate equivalent to the given circuit diagram is:



(1) OR

(2) NAND

(3) NOR

(4) AND

Correct Answer: (2) NAND

Solution: The given circuit has two inputs A_1 and B_1, and its output is denoted by Y. From the truth table, we observe that the output corresponds to a NAND gate. The truth table is:

A_1	B_1	Y
0	0	1
0	1	1
1	0	1
1	1	0

The logic gate that produces this output is the NAND gate, and the equation for the output is:

$$Y = A_1 \text{ NAND } B_1$$

Quick Tip

To identify logic gates, use the truth table. For a NAND gate, the output is 1 for all combinations of inputs except when both inputs are 1.

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7. A long solenoid is formed by winding 70 turns cm^{-1}. If 2.0 A current flows, then the magnetic field produced inside the solenoid is:

(1) $1232 \times 10^{-4}\, T$

(2) $176 \times 10^{-4}\,T$

(3) $352 \times 10^{-4}\,T$

(4) $88 \times 10^{-4}\,T$

Correct Answer: (2) $176 \times 10^{-4}\,T$

Solution: The magnetic field produced inside a solenoid is given by the formula:

$$B = \mu_0 n I$$

where n is the number of turns per unit length, I is the current, and $\mu_0 = 4\pi \times 10^{-7}\,T \cdot m/A$ is the permeability of free space. Given: $n = 70$ turns/cm $= 70 \times 10^2$ turns/m $I = 2.0\,A$

Substitute the values into the formula:

$$B = 4\pi \times 10^{-7} \times 70 \times 10^2 \times 2 = 176 \times 10^{-4}\,T$$

Quick Tip

The magnetic field inside a solenoid is directly proportional to the current and the number of turns per unit length. Use this formula to quickly calculate the magnetic field.

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8. Given below are two statements:

Statement I: Acceleration due to earth's gravity decreases as you go 'up' or 'down' from earth's surface.

Statement II: Acceleration due to earth's gravity is the same at a height h and depth d from earth's surface, if $h = d$.

In the light of above statements, choose the most appropriate answer from the options given below:

(1) Statement I is incorrect but Statement II is correct

(2) Both Statement I and Statement II are incorrect

(3) Statement I is correct but Statement II is incorrect

(4) Both Statement I and Statement II are correct

Correct Answer: (3) Statement I is correct but Statement II is incorrect

Solution: Statement I: This statement is correct. As you move away from the surface or go deeper into the earth, the acceleration due to gravity decreases.

Statement II: This statement is incorrect. The acceleration due to gravity is the same at a height h and depth d from the earth's surface only if the height h is equal to the depth d, which is not generally true.

The formula for acceleration due to gravity at a height or depth is:

$$g' = g\left(\frac{1 + h/R}{1 + d/R}\right)$$

where g is the acceleration due to gravity at the surface, R is the radius of the earth, and h and d are the height and depth, respectively.

Thus, the correct answer is that Statement I is correct, but Statement II is incorrect.

Quick Tip

Gravity decreases with height and depth, but the exact relationship involves both the height above and the depth below the surface.

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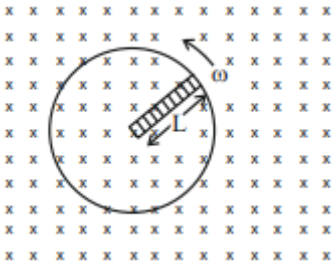
9. A metallic rod of length L is rotated with an angular speed of ω normal to a uniform magnetic field B about an axis passing through one end of the rod as shown in the figure. The induced emf will be:



(1) $\frac{1}{4}B^2L\omega$

(2) $\frac{1}{4}BL\omega$

(3) $\frac{1}{2}BL^2\omega$

(4) $\frac{1}{2}B^2L^2\omega$

Correct Answer: (3) $\frac{1}{2}BL^2\omega$

Solution: The induced emf in a rotating rod in a magnetic field can be determined using the following relationship for the induced emf:

$$\text{emf} = \int_0^L B\, v\, dx$$

where $v = \omega x$ is the linear velocity at a distance x from the axis of rotation, and L is the length of the rod.

Substitute $v = \omega x$ into the equation:

$$\text{emf} = \int_0^L B\,(\omega x)\,dx = B\omega \int_0^L x\,dx$$

$$\text{emf} = B\omega \left[\frac{x^2}{2}\right]_0^L = \frac{1}{2}BL^2\omega$$

Thus, the induced emf is $\frac{1}{2}BL^2\omega$.

Quick Tip

To calculate the induced emf in a rotating rod, integrate the velocity over the length of the rod. The linear velocity depends on the distance from the axis of rotation.

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10. When a beam of white light is allowed to pass through a convex lens parallel to the principal axis, the different colours of light converge at different points on the principal axis after refraction. This is called:

(1) Scattering

(2) Chromatic aberration

(3) Spherical aberration

(4) Polarisation

Correct Answer: (2) Chromatic aberration

Solution: When a beam of white light passes through a convex lens, different wavelengths (colours) of light refract by different amounts due to their varying refractive indices. This phenomenon, where different colours converge at different points along the principal axis, is known as chromatic aberration.

Quick Tip

Chromatic aberration occurs because the lens refracts different colours of light at different angles. This can be minimized using achromatic lenses.

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11. The frequency (v) of an oscillating liquid drop may depend upon radius (r) of the drop, density (ρ) of liquid and the surface tension (s) of the liquid as:

$$v = r^a \rho^b s^c$$

The values of a, b, and c respectively are:

(1) $\left(\frac{3}{2}, -\frac{1}{2}, \frac{1}{2}\right)$

(2) $\left(\frac{3}{2}, \frac{1}{2}, -\frac{1}{2}\right)$

(3) $\left(\frac{3}{2}, \frac{1}{2}, \frac{1}{2}\right)$

(4) $\left(\frac{3}{2}, -\frac{1}{2}, -\frac{1}{2}\right)$

Correct Answer: (1) $\left(\frac{3}{2}, -\frac{1}{2}, \frac{1}{2}\right)$

Solution: We start with the dimensional analysis. The frequency of oscillation is given as:

$$[v] = [L^0 T^{-1}]$$

Now, the units of r, ρ, and s are:

$$[r] = L, \quad [\rho] = ML^{-3}, \quad [s] = MT^{-2}$$

Using the dimensional formula $T^1 = M^a L^b T^c$, we get the following system of equations for the exponents:

$$c = \frac{1}{2}, \quad b = -\frac{1}{2}, \quad a = \frac{3}{2}$$

Thus, $a = \frac{3}{2}, b = -\frac{1}{2}, c = \frac{1}{2}$.

Quick Tip

Dimensional analysis helps in determining the relation between physical quantities. Ensure the dimensions on both sides of the equation are balanced for accurate results.

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12. A body of mass 200g is tied to a spring of spring constant 12.5 N/m, while the other end of the spring is fixed at point O. If the body moves about O in a circular path on a smooth horizontal surface with constant angular speed 5 rad/s, then the ratio of extension in the spring to its natural length will be:

(1) 1:2

(2) 1:1

(3) 2:3

(4) 2:5

Correct Answer: (3) 2:3

Solution: Let the natural length of the spring be L_0 and the extension be x. For the circular motion of the body, the centripetal force is provided by the restoring force of the spring:

$$F = m\omega^2 r = kx$$

where $\omega = 5\,\text{rad/s}$, $m = 200\,\text{g} = 0.2\,\text{kg}$, and $k = 12.5\,\text{N/m}$. The total length of the spring is $L_0 + x$, and the centripetal force is given by:

$$F = 0.2 \times (5)^2 = 12.5x$$

Simplifying this, we get:

$$12.5 = 12.5x \quad \Rightarrow \quad x = \frac{2}{3}L_0$$

Thus, the ratio of the extension to the natural length is $\frac{2}{3}$, or 2:3.

Quick Tip

For problems involving circular motion, the restoring force from the spring provides the necessary centripetal force. Use this relation to calculate the extension of the spring.

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13. A cell of emf 90 V is connected across a series combination of two resistors each of 100 Ω resistance. A voltmeter of resistance 400 Ω is used to measure the potential difference across each resistor. The reading of the voltmeter will be:

(1) 40 V

(2) 45 V

(3) 80 V

(4) 90 V

Correct Answer: (1) 40 V

Solution: The resistors are connected in series, so the equivalent resistance R_{eq} is:

$$R_{\text{eq}} = 100\,\Omega + 100\,\Omega = 200\,\Omega$$

The total current i in the circuit is:

$$i = \frac{90\,\text{V}}{200\,\Omega} = \frac{1}{2}\,\text{A}$$

Now, the voltage across each resistor is given by:

$$V = i \times R = \frac{1}{2} \times 100 = 50\,\text{V}$$

However, the voltmeter has a resistance of 400 Ω, and it is connected across one of the resistors. The effective resistance of the combination is:

$$R_{\text{eq}} = \frac{100 \times 400}{100 + 400} = 80\,\Omega$$

So, the total current in the circuit becomes:

$$i = \frac{90\,\text{V}}{180\,\Omega} = \frac{1}{2}\,\text{A}$$

Finally, the reading of the voltmeter is:

$$V_{\text{reading}} = \frac{1}{2} \times 400 \times \frac{100}{500} = 40\,\text{V}$$

Quick Tip

When measuring potential difference across a resistor with a voltmeter, account for the internal resistance of the voltmeter, which forms a parallel combination with the resistor.

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14. The electric field and magnetic field components of an electromagnetic wave going through vacuum is described by:

$$E_x = E_0 \sin(kz - \omega t)$$

$$B_y = B_0 \sin(kz - \omega t)$$

Then the correct relation between E_0 and B_0 is given by:

(1) $kE_0 = B_0$

(2) $E_0 B_0 = ck$

(3) $\omega E_0 = kB_0$

(4) $E_0 = kB_0$

Correct Answer: (1) $kE_0 = B_0$

Solution: For an electromagnetic wave in a vacuum, the electric and magnetic fields are related by the following equation:

$$E_0 = cB_0$$

where c is the speed of light. Additionally, the wave number k and the angular frequency ω are related to the speed of light as:

$$c = \frac{\omega}{k}$$

Thus, combining these relations, we obtain:

$$kE_0 = B_0$$

Quick Tip

For electromagnetic waves, the electric and magnetic fields are perpendicular to each other and the direction of wave propagation. Use the relationship $E_0 = cB_0$ to find the relationship between field components.

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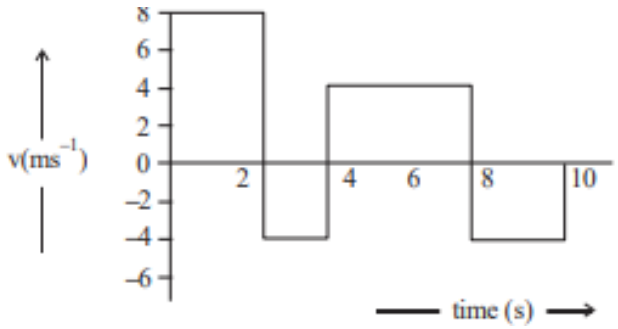
15. The velocity-time graph of a body moving in a straight line is shown in the figure.



The ratio of displacement to distance travelled by the body in time 0 to 10s is:

(1) 1:1

(2) 1:4

(3) 1:2

(4) 1:3

Correct Answer: (4) 1:3

Solution:

The displacement is the area under the velocity-time graph.

The displacement is calculated as:

$$\text{Displacement} = \text{Area} = 16 - 8 + 16 - 8 = 16\,\text{m}$$

The total distance travelled is the sum of the areas, which is:

$$\text{Distance} = \sum (\text{area}) = 48\,\text{m}$$

Thus, the ratio of displacement to distance is:

$$\frac{\text{Displacement}}{\text{Distance}} = \frac{16}{48} = \frac{1}{3}$$

Quick Tip

When dealing with velocity-time graphs, the area under the graph represents displacement, and the total distance is the sum of the absolute areas.

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16. Given below are two statements: one is labelled as Assertion A and the other is labelled as Reason R

Assertion A: Steel is used in the construction of buildings and bridges.

Reason R: Steel is more elastic and its elastic limit is high.

In the light of above statements, choose the most appropriate answer from the options given below:

(1) Both A and R are correct but R is NOT the correct explanation of A

(2) A is not correct but R is correct

(3) Both A and R are correct and R is the correct explanation of A

(4) A is correct but R is not correct

Correct Answer: (3) Both A and R are correct and R is the correct explanation of A

Solution: Assertion A is correct because steel is indeed used in the construction of buildings and bridges due to its strength. Reason R is also correct because steel is more elastic and has a high elastic limit, which makes it ideal for construction purposes where high strength and flexibility are needed.

Since Reason R provides the correct explanation for Assertion A, the correct answer is option (3).

Quick Tip

In assertion-reasoning questions, always check whether the reason correctly explains the assertion.

17. Given below are two statements: one is labelled as Assertion A and the other is labelled as Reason R

Assertion A: A pendulum clock when taken to Mount Everest becomes fast.

Reason R: The value of g (acceleration due to gravity) is less at Mount Everest than its value on the surface of the Earth.

In the light of the above statements, choose the most appropriate answer from the options given below:

(1) Both A and R are correct but R is NOT the correct explanation of A

(2) Both A and R are correct and R is the correct explanation of A

(3) A is not correct but R is correct

(4) A is correct but R is not correct

Correct Answer: (3) A is not correct but R is correct

Solution: Assertion A is incorrect because a pendulum clock becomes slower at higher altitudes due to the lower value of g. Reason R is correct as the value of gravity decreases with altitude, which is why the pendulum clock runs slower on Mount Everest. Thus, A is incorrect, but R is correct.

Quick Tip

In such problems, always check the correctness of both the assertion and the reason before selecting the appropriate answer.

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18. A photon is emitted in transition from $n = 4$ to $n = 1$ level in hydrogen atom. The corresponding wavelength for this transition is (given, $h = 4 \times 10^{-15}$ eV $\cdot$ s):

(1) 94.1 nm

(2) 941 nm

(3) 97.4 nm

(4) 99.3 nm

Correct Answer: (1) 94.1 nm

Solution: The energy of the photon is given by the difference in energy levels:

$$E = h\nu = hc\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$$

Substitute the values $n_1 = 1$, $n_2 = 4$, and $h = 4 \times 10^{-15}\,\text{eV} \cdot \text{s}$, $c = 3 \times 10^8\,\text{m/s}$:

$$E = 4 \times 10^{-15} \times 3 \times 10^8 \times \left(\frac{1}{1^2} - \frac{1}{4^2}\right) = 13.6\,\text{eV}$$

The corresponding wavelength λ is given by:

$$\lambda = \frac{hc}{E} = \frac{4 \times 10^{-15} \times 3 \times 10^8}{13.6} = 94.1\,\text{nm}$$

Quick Tip

For transitions between energy levels in hydrogen, use the Rydberg formula to find the energy difference and subsequently the wavelength.

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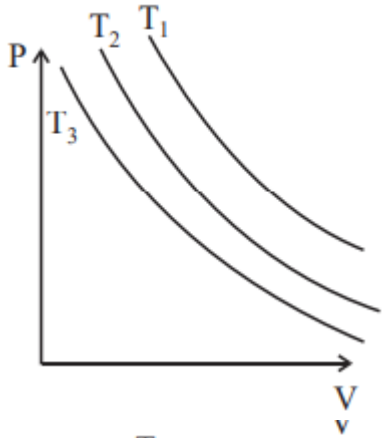
19. In an Isothermal change, the change in pressure and volume of a gas can be represented for three different temperatures; $T_3 > T_2 > T_1$ as:

(1)



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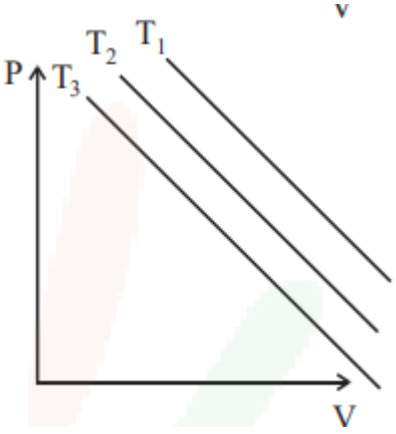

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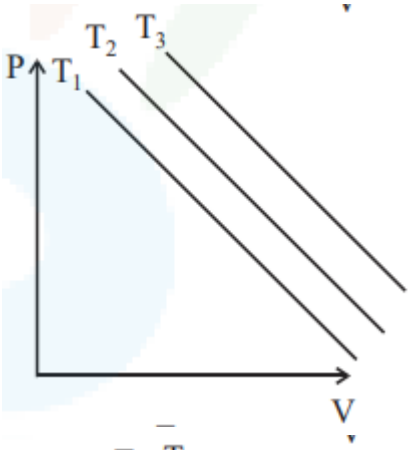
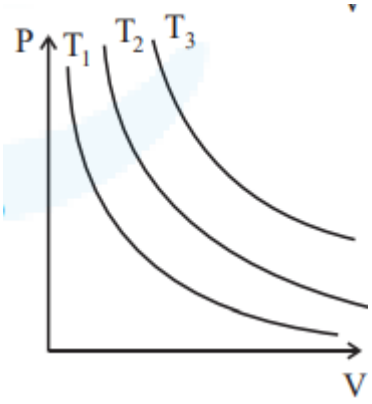
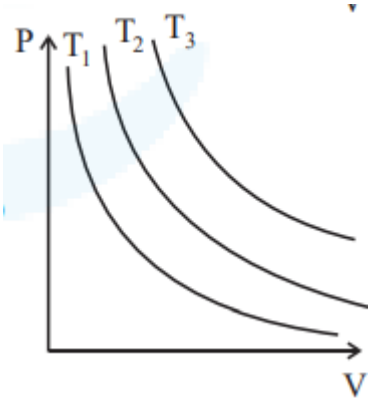


(4)





Correct Answer: (4)

Solution: For an isothermal process, the $P - V$ graph is represented as a rectangular hyperbola, with the pressure inversely proportional to volume. The higher the temperature, the greater the volume for the same pressure, which is why $T_3 > T_2 > T_1$ leads to the following graph. The dotted line in the graph represents an isobaric line.

Quick Tip

In an isothermal process, temperature remains constant, so the product of pressure and volume is constant. This relationship is represented as a hyperbolic curve in the $P - V$ graph.

20. If two vectors $\mathbf{P} = \hat{i} + 2\hat{j} + m\hat{k}$ and $\mathbf{Q} = 4\hat{i} - 2\hat{j} + \hat{k}$ are perpendicular to each other, then the value of m will be:

(1) 1

(2) -1

(3) 3

(4) 2

Correct Answer: (4) 2

Solution: For two vectors to be perpendicular, their dot product must be zero. So,

$$\mathbf{P} \cdot \mathbf{Q} = 0$$

$$(i + 2j + mk) \cdot (4i - 2j + k) = 0$$

$$4 - 4 + m = 0$$

$$m = 2$$

Quick Tip

To check if two vectors are perpendicular, calculate their dot product. If it equals zero, the vectors are perpendicular.

Section - B

21. A uniform solid cylinder with radius R and length L has moment of inertia I_1, about the axis of the cylinder. A concentric solid cylinder of radius $R' = \frac{R}{2}$ and length $L' = \frac{L}{2}$ is carved out of the original cylinder. If I_2 is the moment of inertia of the carved-out portion of the cylinder, then $\frac{I_1}{I_2}$ is:

Solution: The moment of inertia of a solid cylinder about its axis is given by:

$$I = \frac{1}{2}mR^2$$

where m is the mass and R is the radius. For the full cylinder:

$$I_1 = \frac{1}{2}mR^2$$

For the carved-out cylinder, the radius is $R' = \frac{R}{2}$ and the length is $L' = \frac{L}{2}$, so:

$$I_2 = \frac{1}{2}m'R'^2 = \frac{1}{2}\left(\frac{m}{4}\right)\left(\frac{R}{2}\right)^2 = \frac{1}{2} \times \frac{m}{4} \times \frac{R^2}{4} = \frac{mR^2}{32}$$

Thus,

$$\frac{I_1}{I_2} = \frac{\frac{1}{2}mR^2}{\frac{mR^2}{32}} = 32$$

Quick Tip

For concentric cylinders, calculate the moment of inertia using the formula $I = \frac{1}{2}mR^2$, and apply the ratio of masses and radii to determine the moment of inertia of the carved-out portion.

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22. A mass m attached to the free end of a spring executes SHM with a period of 1s. If the mass is increased by 3 kg, the period of oscillation increases by one second, the value of mass m is:

Solution: The formula for the period T of a mass-spring system is given by:

$$T = 2\pi\sqrt{\frac{m}{k}}$$

where m is the mass and k is the spring constant. Given that the period increases by 1 second when the mass increases by 3 kg, we can set up the equation for the new and original periods:

$$T' = T + 1 = 2\pi\sqrt{\frac{m+3}{k}}$$

Substituting for the initial period $T = 1$ second:

$$1 + 1 = 2\pi\sqrt{\frac{m+3}{k}} \quad \Rightarrow \quad m = 1\,\text{kg}$$

Quick Tip

For SHM, the period is proportional to the square root of the mass. Use this relationship to set up equations for the initial and final periods and solve for the unknown mass.

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23. The energy released per fission of nucleus of ^{240}X is 200 MeV. The energy released if all the atoms in 120g of pure ^{240}X undergo fission is:

Solution: Given that the energy released per fission of a nucleus is 200 MeV, and the number of moles is:

$$\text{No. of moles} = \frac{120}{240} = \frac{1}{2}\,\text{moles}$$

The number of molecules is:

$$\text{No. of molecules} = \frac{1}{2}N_A$$

where $N_A = 6 \times 10^{23}$ is Avogadro's number. The total energy released is:

$$E = \frac{1}{2} \times 6 \times 10^{23} \times 200 = 6 \times 10^{25}\,\text{MeV}$$

Quick Tip

Use Avogadro's number to find the total number of molecules, and then multiply by the energy released per fission event to find the total energy released.

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24. A parallel plate capacitor with air between the plates has a capacitance of 15 pF. The separation between the plates becomes twice and the space between them is filled with a material of dielectric constant 3.5. Then the capacitance becomes:

Solution: The capacitance of a parallel plate capacitor is given by:

$$C = \frac{\varepsilon_0 A}{d}$$

where A is the area of the plates, ε_0 is the permittivity of free space, and d is the distance between the plates. When the dielectric constant is added, the capacitance becomes:

$$C' = \kappa C$$

Given that $\kappa = 3.5$, and the separation between the plates becomes twice, the capacitance becomes:

$$C' = \frac{15}{2} \times 3.5 = 4\,\text{pF}$$

Quick Tip

When the distance between plates of a capacitor is doubled, the capacitance is halved. Additionally, the dielectric constant increases the capacitance.

25. A body of mass 1 kg begins to move under the action of a time dependent force $\mathbf{F} = \hat{i} + 3t\hat{j}$ N. where $\hat{i}$ and $\hat{j}$ are the unit vectors along x and y axis. The power developed by above force, at the time $t = 2$ s will be:

Solution: The power developed is given by the dot product of force and velocity:

$$P = \mathbf{F} \cdot \mathbf{v}$$

We are given that the force $\mathbf{F} = \hat{i} + 3t\hat{j}$, so the acceleration is:

$$\frac{d\mathbf{v}}{dt} = \hat{i} + 3t\hat{j}$$

Integrating with respect to time:

$$\mathbf{v} = \int \left(\hat{i} + 3t\hat{j}\right) dt = t\hat{i} + \frac{3t^2}{2}\hat{j}$$

At $t = 2$ s, the velocity is:

$$\mathbf{v} = 2\hat{i} + 6\hat{j}$$

The power is:

$$P = \mathbf{F} \cdot \mathbf{v} = (1\hat{i} + 6\hat{j}) \cdot (2\hat{i} + 6\hat{j}) = 2 + 6 \times 6 = 100\,\text{W}$$

Quick Tip

When calculating power, use the dot product of the force and velocity vectors. The units of power are watts (W), which are joules per second (J/s).

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26. If a copper wire is stretched to increase its length by 20%. The percentage increase in resistance of the wire is:

Solution: The resistance R of a wire is proportional to its length L, so:

$$R \propto L$$

Thus, if the length increases by 20

$$\Delta R = 24 + 20 = 44\% \text{ increase in resistance.}$$

Quick Tip

Resistance of a wire is directly proportional to its length and inversely proportional to its cross-sectional area. If the wire is stretched, the resistance increases.

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27. A single turn current loop in the shape of a right angle triangle with sides 5 cm, 12 cm, 13 cm is carrying a current of 2A. The loop is in a uniform magnetic field of magnitude 0.75 T whose direction is parallel to the current in the 13 cm side of the loop. The magnitude of the magnetic force on the 5 cm side will be:

Solution: The magnetic force on a current-carrying conductor is given by:

$$F = ILB \sin\theta$$

where $I = 2\,\text{A}$, $L = 5\,\text{cm} = 0.05\,\text{m}$, $B = 0.75\,\text{T}$, and $\theta = 90^\circ$. Thus, the force is:

$$F = (2)(0.05)(0.75) = 9\,\text{N}$$

Quick Tip

For calculating the magnetic force on a current-carrying wire, ensure you know the current, length of the wire in the field, magnetic field strength, and the angle between the wire and field.

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28. Three identical resistors with resistance $R = 12\,\Omega$ and two identical inductors with self inductance $L = 5$ mH are connected to an ideal battery with emf of 12 V as shown in figure. The current through the battery long after the switch has been closed will be:

Solution: After a long time, the inductor behaves as a resistance-less path, so the current through the resistors is governed by the total resistance. The equivalent resistance is:

$$R_{\text{eq}} = \frac{R}{3} = 4\,\Omega$$

Thus, the current through the battery is:

$$I = \frac{12}{R_{\text{eq}}} = \frac{12}{4} = 3\,\text{A}$$

Quick Tip

When inductors are connected in a DC circuit long after the switch is closed, they act like short circuits because the current through an inductor stabilizes.

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29. A convex lens of refractive index 1.5 and focal length 18 cm in air is immersed in water. The change in focal length of the lens will be: (Given refractive index of water $= \frac{4}{3}$)

Solution: The formula for the focal length of a lens is:

$$\frac{1}{f_{\text{air}}} = \left(\frac{\mu_l}{\mu_{\text{air}}} - 1\right)\left(\frac{2}{R}\right)$$

where μ_l is the refractive index of the lens and $\mu_{\text{air}} = 1$. For the lens immersed in water, the refractive index of water is $\frac{4}{3}$, so:

$$\frac{1}{f_{\text{water}}} = \left(\frac{\mu_l}{\mu_{\text{water}}} - 1\right)\left(\frac{2}{R}\right)$$

Thus,

$$f_{\text{water}} = 72\,\text{cm}$$

The change in focal length is:

$$72 - 54 = 18\,\text{cm}$$

Quick Tip

When a lens is immersed in a medium, the refractive index of the medium affects the focal length. In water, the focal length increases as the refractive index of water is less than that of air.

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30. A spherical ball of radius 1mm and density 10.5 g/cc is dropped in glycerine of coefficient of viscosity 9.8 poise and density 1.5 g/cc. Viscous force on the ball when it attains constant velocity is 3696 $\times 10^{-7}$ N. The value of x is:

Solution: When the ball attains terminal velocity, the force balance is given by:

$$F = mg - F_b$$

where F_b is the buoyant force. The expression for the viscous force is:

$$F = V\rho_g g$$

The volume of the ball is $V = \frac{4}{3}\pi r^3$. Substituting the values:

$$F = \left(\frac{4}{3}\pi(10^{-3})^3 \times 9.8 \times (10.5 - 1.5) \times 10^3\right) = 3696 \times 10^{-7}\,\text{N}$$

Thus, $x = 7$.

Quick Tip

For objects moving through a fluid, use the viscous force equation for terminal velocity. Remember to use the volume of the sphere to calculate the force and density values.
