JEE Main 2023 Jan 30 Shift 2 Maths Question Paper

Time Allowed :3 Hours	**Maximum Marks :**300	**Total Questions :**90

General Instructions

Read the following instructions very carefully and strictly follow them:

1. The test is of 3 hours duration.
2. The question paper consists of 90 questions, out of which 75 are to attempted. The maximum marks are 300.
3. There are three parts in the question paper consisting of Physics, Chemistry and Mathematics having 30 questions in each part of equal weightage.
4. Each part (subject) has two sections.
   (i) Section-A: This section contains 20 multiple choice questions which have only one correct answer. Each question carries 4 marks for correct answer and –1 mark for wrong answer.
   (ii) Section-B: This section contains 10 questions. In Section-B, attempt any five questions out of 10. The answer to each of the questions is a numerical value. Each question carries 4 marks for correct answer and –1 mark for wrong answer. For Section-B, the answer should be rounded off to the nearest integer

Maths

Section-A

61. Consider the following statements:

P : I have fever

Q : I will not take medicine

R : I will take rest

The statement ”If I have fever, then I will take medicine and I will take rest” is equivalent to:

(1) $(\ P) \vee (\ Q) \wedge ((\ P) \vee\ \ R)$

(2) $(\ P) \vee (\ Q) \wedge ((\ P) \vee R)$

(3) $(P \vee Q) \wedge (\ P) \vee R$

(4) $(P \vee\ \ Q) \wedge (P \vee\ \ R)$

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62. Let A be a point on the x-axis. Common tangents are drawn from A to the curves $x^2 + y^2 = 8$ and $y^2 = 16x$. If one of these tangents touches the two curves at Q and R, then $(QR)^2$ is equal to:

(1) 64

(2) 76

(3) 81

(4) 72

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63. Let q be the maximum integral value of p in $[0, 10]$ for which the roots of the equation $x^2 - px + \frac{5p}{4} = 0$ are rational. Then the area of the region $\{(x, y) : 0 \leq y \leq (x-q)^2, 0 \leq x \leq q\}$ is:

(1) 243

(2) 25

(3) $\frac{125}{3}$

(4) 164

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64. If the functions $f(x) = \frac{x^3}{3} + 2bx + \frac{ax}{2}$ and $g(x) = \frac{x^3}{3} + ax + bx^2, a \neq 2b$ have a common extreme point, then $a + 2b + 7$ is equal to:

(1) 4

(2) $\frac{3}{2}$

(3) 3

(4) 6

65. The range of the function $f(x) = \sqrt{3 - x + \sqrt{2 + x}}$ **is:**

(1) $[\sqrt{5}, \sqrt{10}]$

(2) $[2\sqrt{2}, \sqrt{11}]$

(3) $[\sqrt{5}, \sqrt{13}]$

(4) $[\sqrt{2}, \sqrt{7}]$

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66. The solution of the differential equation **The solution of the differential equation**

$$\frac{dy}{dx} = \frac{x^2 + 3y^2}{3x^2 + y^2},\ y(1) = 0 \text{ is:}$$

(1) $\log_e |x + y| - \frac{xy}{(x+y)^2} = 0$

(2) $\log_e |x + y| + \frac{xy}{(x+y)^2} = 0$

(3) $\log_e |x + y| + \frac{2xy}{(x+y)^2} = 0$

(4) $\log_e |x + y| - \frac{2xy}{(x+y)^3} = 0$

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67. Let x = $\left(8\sqrt{3} + 13\right)^{13} andy = \left(7\sqrt{2} + 9\right)^{9}. If [t] denotesthegreatestinteger \leq t$, then **Let** $x = \left(8\sqrt{3} + 13\right)^{13}$ **and** $y = \left(7\sqrt{2} + 9\right)^{9}$**. If** $[t]$ **denotes the greatest integer** $\leq t$**, then**

(1) $[x] + [y]$ is even

(2) $[x]$ is odd but $[y]$ is even

(3) $[x]$ is even but $[y]$ is odd

(4) Both $[x]$ and $[y]$ are both odd

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68. A vector v in the first octant is inclined to the x-axis at 60°, to the y-axis at 45° and to the z-axis at an acute angle. If a plane passing through the points $\left(\sqrt{2}, -1, 1\right)$ and (a, b, c), is normal to v, then **A vector** v **in the first octant is inclined to the x-axis at 60°, to the y-axis at 45° and to the z-axis at an acute angle. If a plane passing through the points**

$\left(\sqrt{2}, -1, 1\right)$ **and** (a, b, c)**, is normal to v, then**

(1) $\sqrt{2}a + b + c = 1$

(2) $a + b + \sqrt{2}c = 1$

(3) $a + \sqrt{2}b + c = 1$

(4) $\sqrt{2}a - b + c = 1$

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69. Let f, g and h be the real valued functions defined on $\mathbb{R}$ as $f(x) = \left\{ \frac{x}{|x|}, \quad x \neq 0 \right.$

(1) f is continuous at $x = 0$

(2) g is continuous at $x = 0$

(3) h is continuous at $x = 0$

(4) f, g, h are continuous at $x = 0$

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70. The number of ways of selecting two numbers a and b, $a \in \{2, 4, 6, \ldots, 100\}$ and $b \in \{1, 3, 5, 7, \ldots, 99\}$ such that 2 is the remainder when $a + b$ is divided by 23 is:

(1) 186

(2) 54

(3) 108

(4) 268

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71. If P is a 3x3 real matrix such that $P^T = aP + (a - 1)I$, where $a > 1$, then:

(1) P is a singular matrix

(2) $|\text{Adj}P| > 1$

(3) $|\text{Adj}P| = \frac{1}{2}$

(4) $|\text{Adj}P| = 1$

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72. Let $\lambda \in \mathbb{R}$, $\mathbf{a} = \lambda i + 2j - 3k$, $\mathbf{b} = i - \lambda j + 2k$. If $((\mathbf{a} + \mathbf{b}) \times (\mathbf{a} \times \mathbf{b})) \times (\mathbf{a} - \mathbf{b}) = 8i - 40j - 24k$, then $|\lambda(\mathbf{a} + \mathbf{b}) \times (\mathbf{a} - \mathbf{b})|^2$ is equal to ......

(1) 140

(2) 132

(3) 144

(4) 136

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73. Let $\mathbf{a}$ and $\mathbf{b}$ be two vectors. Let $|\mathbf{a}| = 1, |\mathbf{b}| = 4$ and $\mathbf{a} \cdot \mathbf{b} = 2$. If $\mathbf{c} = (2\mathbf{a} \times \mathbf{b}) - 3\mathbf{b}$, then the value of $\mathbf{b} \cdot \mathbf{c}$ is

(1) -24

(2) -48

(3) -84

(4) -60

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74. Let $a_1 = 1, a_2, a_3, a_4, \ldots$ be consecutive natural numbers. Then

$$\tan^{-1}\left(\frac{1}{1+a_1a_2}\right) + \tan^{-1}\left(\frac{1}{1+a_2a_3}\right) + \cdots + \tan^{-1}\left(\frac{1}{1+a_{2021}a_{2022}}\right)$$

is equal to:

(1) $\frac{\pi}{4} - \cot^{-1}(2022)$

(2) $\cot^{-1}(2022) - \frac{\pi}{4}$

(3) $\tan^{-1}(2022) - \frac{\pi}{4}$

(4) $\frac{\pi}{4} - \tan^{-1}(2022)$

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75. The parabolas: $ax^2 + 2bx + cy = 0$ and $dx^2 + 2ex + fy = 0$ intersect on the line $y = 1$. If a, b, c, d, e, f are positive real numbers and a, b, c, d, e, f are in G.P., then

(1) d, e, f are in A.P.

(2) $\frac{d}{a}, \frac{e}{b}, \frac{f}{c}$ are in G.P.

(3) $\frac{d}{a}, \frac{e}{b}, \frac{f}{c}$ are in A.P.

(4) d, e, f are in G.P.

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76. If a plane passes through the points $(-1, k, 0), (2, k, -1), (1, 1, 2)$ and is parallel to the line

$\frac{x-1}{1} = \frac{2y+1}{2} = \frac{z+1}{-1}$, then the value of $\frac{k^2+1}{(k-1)(k-2)}$ is:

(1) $\frac{17}{5}$

(2) $\frac{5}{17}$

(3) $\frac{6}{13}$

(4) $\frac{13}{6}$

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77. Let a, b, c ¿ 1, a^3, b^3 and c^3 be in A.P., and $\log_b a, \log_a c$ and $\log_c b$ be in G.P. If the sum of the first 20 terms of an A.P., whose first term is $\frac{a+4b+c}{3}$ and the common difference is $\frac{a-8b+c}{10} = -444$, then abc is equal to:

(1) 343

(2) 216

(3) 343

(4) $\frac{125}{8}$

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78. Let S be the set of all values of a_1 for which the mean deviation about the mean of 100 consecutive positive integers $a_1, a_2, a_3, \ldots, a_{100}$ is 25. Then S is:

(1) $\emptyset$

(2) $\{99\}$

(3) $\mathbb{N}$

(4) {}

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79. $\lim_{n\to\infty} \left(3n \left[4 + \left(2 + \frac{1}{n}\right)^2 + \left(2 + \frac{2}{n}\right)^2 + \cdots + \left(3 - \frac{1}{n}\right)^2\right]\right)$

(1) 12

(2) $\frac{19}{3}$

(3) 0

(4) 19

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80. For $\alpha, \beta \in \mathbb{R}$, suppose the system of linear equations $x - y + z = 5$ $2x + 2y + \alpha z = 8$

$3x - y + 4z = \beta$

(1) $x^2 - 10x + 16 = 0$

(2) $x^2 + 18x + 56 = 0$

(3) $x^2 - 18x + 56 = 0$

(4) $x^2 + 14x + 24 = 0$

SECTION-B

81. 50th root of a number x is 12 and 50th root of another number y is 18. Then the remainder obtained on dividing $(x + y)$ by 25 is ....

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82. Let $A = \{1, 2, 3, 5, 8, 9\}$. Then the number of possible functions $f : A \to A$ such that $f(m \cdot n) = f(m) \cdot f(n)$ for every $m, n \in A$ with $m \cdot n \in A$ is equal to ....

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83. Let $P(a_1, b_1)$ and $Q(a_2, b_2)$ be two distinct points on a circle with center $C(\sqrt{2}, \sqrt{3})$. Let O be the origin and OC be perpendicular to both CP and CQ. If the area of the triangle OPC is $\frac{\sqrt{35}}{2}$, then $a_1^2 + a_2^2 + b_1^2 + b_2^2$ is equal to ....

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84. The 8th common term of the series $S_1 = 3+7+11+15+19+\ldots$, $S_2 = 1+6+11+16+21+\ldots$, is ....

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85. Let a line L pass through the point $P(2, 3, 1)$ and be parallel to the line $x + 3y - 2z - 2 = 0$ i.e. $x - y + 2z = 0$. If the distance of L from the point $(5, 3, 8)$ is α, then $3\alpha^2$ is equal to ....

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86. If $\int \sqrt{\sec 2x - 1}\, dx = \alpha \log_e (\cos 2x + \beta) + \sqrt{\cos 2x\,(1 + \cos 2x)\left(\frac{1}{\beta}\right)}$ + constant, then $\beta - \alpha$ is equal to ....

87. If the value of real number $a > 0$ for which $x^2 - 5ax + 1 = 0$ and $x^2 - ax - 5 = 0$ have a common real root is $\frac{3}{\sqrt{2}}$, then β is equal to ....

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88. The number of seven-digit odd numbers that can be formed using all the seven digits 1, 2, 2, 2, 3, 3, 5 is ....

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89. A bag contains six balls of different colours. Two balls are drawn in succession with replacement. The probability that both the balls are of the same colour is p. Next four balls are drawn in succession with replacement and the probability that exactly three balls are of the same colours is q. If $p : q = m : n$, where m and n are coprime, then $m+n$ is equal to ....

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90. Let A be the area of the region $\{(x, y) : y \geq x^2, y \geq (1-x)^2, y \leq 2x(1-x)\}$.Then 540A is equal to ....
