JEE Main 2025 April 4 Shift 1 Question Paper with Solutions

Time Allowed :3 Hours	**Maximum Marks :**300	**Total Questions :**75

General Instructions

Read the following instructions very carefully and strictly follow them:

1. Multiple choice questions (MCQs)
2. Questions with numerical values as answers.
3. There are three sections: **Mathematics, Physics, Chemistry**.
4. **Mathematics:** 25 (20+5) 10 Questions with answers as a numerical value. Out of 10 questions, 5 questions are compulsory.
5. **Physics:** 25 (20+5) 10 Questions with answers as a numerical value. Out of 10 questions, 5 questions are compulsory..
6. **Chemistry:** 25 (20+5) 10 Questions with answers as a numerical value. Out of 10 questions, 5 questions are compulsory.
7. Total: 75 Questions (25 questions each).
8. 300 Marks (100 marks for each section).
9. **MCQs:** Four marks will be awarded for each correct answer and there will be a negative marking of one mark on each wrong answer.
10. **Questions with numerical value answers:** Candidates will be given four marks for each correct answer and there will be a negative marking of 1 mark for each wrong answer.

Mathematics

Section - A

1. Let $f, g : (1, \infty) \to R$ be defined as $f(x) = \frac{2x+3}{5x+2}$ and $g(x) = \frac{2-3x}{1-x}$. If the range of the function $fog : [2, 4] \to R$ is $[\alpha, \beta]$, then $\frac{1}{\beta-\alpha}$ is equal to
(1) 68
(2) 29
(3) 2
(4) 56

Correct Answer: (4) 56

Solution:
To find $\frac{1}{\beta-\alpha}$, we first need to determine the range of the function $fog(x) = f(g(x))$.

1. Calculate $fog(x)$:

$$fog(x) = f(g(x)) = f\left(\frac{2-3x}{1-x}\right)$$

Substitute $g(x)$ into $f(x)$:

$$fog(x) = \frac{2\left(\frac{2-3x}{1-x}\right)+3}{5\left(\frac{2-3x}{1-x}\right)+2}$$

Simplify the expression:

$$fog(x) = \frac{\frac{4-6x+3-3x}{1-x}}{\frac{10-15x+2-2x}{1-x}} = \frac{7-9x}{12-17x}$$

2. Determine the range of $fog(x)$ for $x \in [2,4]$: - Calculate $fog(2)$:

$$fog(2) = \frac{7-9(2)}{12-17(2)} = \frac{7-18}{12-34} = \frac{-11}{-22} = \frac{1}{2}$$

- Calculate $fog(4)$:

$$fog(4) = \frac{7-9(4)}{12-17(4)} = \frac{7-36}{12-68} = \frac{-29}{-56} = \frac{29}{56}$$

3. Identify α and β: - The range of $fog(x)$ is $\left[\frac{1}{2}, \frac{29}{56}\right]$. - Therefore, $\alpha = \frac{1}{2}$ and $\beta = \frac{29}{56}$.
4. Calculate $\frac{1}{\beta-\alpha}$:

$$\beta - \alpha = \frac{29}{56} - \frac{1}{2} = \frac{29}{56} - \frac{28}{56} = \frac{1}{56}$$
$$\frac{1}{\beta-\alpha} = \frac{1}{\frac{1}{56}} = 56$$

Therefore, the correct answer is (4) 56.

> **Quick Tip**
>
> The range of a composite function can be determined by evaluating the function at the endpoints of the domain.

---

2. Consider the sets $A = \{(x,y) \in R \times R : x^2 + y^2 = 25\}$, $B = \{(x,y) \in R \times R : x^2 + 9y^2 = 144\}$, $C = \{(x,y) \in Z \times Z : x^2 + y^2 \leq 4\}$, and $D = A \cap B$. The total number of one-one functions from the set D to the set C is:
(1) 15120
(2) 19320
(3) 17160
(4) 18290

Correct Answer: (3) 17160

Solution:
1. Identify the sets A and B: - $A : x^2 + y^2 = 25$ - $B : \frac{x^2}{144} + \frac{y^2}{16} = 1$
2. Solve for the intersection $D = A \cap B$: - Substitute $x^2 + y^2 = 25$ into $x^2 + 9y^2 = 144$:

$$x^2 + 9(25 - x^2) = 144$$

$$x^2 + 225 - 9x^2 = 144$$
$$-8x^2 = 144 - 225$$
$$-8x^2 = -81$$
$$x^2 = \frac{81}{8}$$
$$x = \pm\frac{9}{2\sqrt{2}}$$

- Substitute x back into $x^2 + y^2 = 25$:

$$y^2 = 25 - \frac{81}{8}$$
$$y = \pm\frac{\sqrt{119}}{2\sqrt{2}}$$

3. Determine the elements of set D:

$$D = \left\{\left(\frac{9}{2\sqrt{2}}, \frac{\sqrt{119}}{2\sqrt{2}}\right), \left(\frac{9}{2\sqrt{2}}, -\frac{\sqrt{119}}{2\sqrt{2}}\right), \left(-\frac{9}{2\sqrt{2}}, \frac{\sqrt{119}}{2\sqrt{2}}\right), \left(-\frac{9}{2\sqrt{2}}, -\frac{\sqrt{119}}{2\sqrt{2}}\right)\right\}$$

- Number of elements in set $D = 4$.
4. Identify the set C:

$$C = \{(x, y) \in Z \times Z : x^2 + y^2 \leq 4\}$$

- Possible pairs (x, y):

$$\{(0, 2), (2, 0), (0, -2), (-2, 0), (1, 1), (-1, -1), (1, -1), (-1, 1), (1, 0), (0, 1), (-1, 0), (0, -1), (0, 0)\}$$

- Number of elements in set $C = 13$.
5. Calculate the total number of one-one functions from set D to set C:

$$\text{Total number of one-one functions} = 13 \times 12 \times 11 \times 10 = 17160$$

Therefore, the correct answer is (3) 17160.

Quick Tip

The number of one-one functions from a set with m elements to a set with n elements is given by $n \times (n - 1) \times (n - 2) \times \ldots \times (n - m + 1)$.

---

3. Let $A = \{1, 6, 11, 16, \ldots\}$ and $B = \{9, 16, 23, 30, \ldots\}$ be the sets consisting of the first 2025 terms of two arithmetic progressions. Then $n(A \cup B)$ is
(1) 3814
(2) 4027
(3) 3761
(4) 4003

Correct Answer: (3) 3761

Solution:
1. Identify the sets A and B: - $A = \{1, 6, 11, 16, \ldots\}$ - $B = \{9, 16, 23, 30, \ldots\}$
2. Find the general terms for A and B: - For set A: $T_n = 1 + (n-1) \cdot 5 = 5n - 4$ - For set B: $T_n = 9 + (n-1) \cdot 7 = 7n + 2$
3. Determine the intersection $A \cap B$: - Solve $5n - 4 = 7m + 2$ for n and m:

$$5n - 7m = 6$$

- The common terms are $16, 51, 86, \ldots$
4. Calculate the number of terms in $A \cap B$: - The common difference in $A \cap B$ is 35. - Solve $16 + (n-1) \cdot 35 \leq 10121$:

$$(n-1) \leq \frac{10105}{35} \implies n \leq 289$$

- Therefore, $n(A \cap B) = 289$.
5. Calculate $n(A \cup B)$:

$$n(A \cup B) = n(A) + n(B) - n(A \cap B)$$

$$n(A \cup B) = 2025 + 2025 - 289 = 3761$$

Therefore, the correct answer is (3) 3761.

Quick Tip

The number of terms in the union of two sets can be found using the formula $n(A \cup B) = n(A) + n(B) - n(A \cap B)$.

---

4. For an integer $n \geq 2$, if the arithmetic mean of all coefficients in the binomial expansion of $(x + y)^{2n-3}$ is 16, then the distance of the point $P(2n - 1, n^2 - 4n)$ from the line $x + y = 8$ is:
(1) $\sqrt{2}$
(2) $2\sqrt{2}$
(3) $5\sqrt{2}$
(4) $3\sqrt{2}$

Correct Answer: (4) $3\sqrt{2}$

Solution:
1. Determine the number of terms in $(x + y)^{2n-3}$:

$$\text{Number of terms} = 2n - 2$$

2. Sum of all coefficients:

$$\text{Sum of coefficients} = 2^{2n-3}$$

3. Arithmetic mean of all coefficients:

$$\text{Arithmetic mean} = \frac{2^{2n-3}}{2n-2} = 16$$

$$2^{2n-3} = 16(2n - 2)$$

$$2^{2n-3} = 2^4(n-1)$$

$$2n - 3 = 4 \implies n = 5$$

4. Determine the point P:

$$P(2n-1, n^2-4n) = P(9,5)$$

5. Calculate the distance from the line $x + y = 8$:

$$\text{Distance} = \left|\frac{9+5-8}{\sqrt{2}}\right| = \frac{6}{\sqrt{2}} = 3\sqrt{2}$$

Therefore, the correct answer is (4) $3\sqrt{2}$.

Quick Tip

The arithmetic mean of coefficients in a binomial expansion can be used to find the value of n.

---

5. The probability of forming a 12 persons committee from 4 engineers, 2 doctors, and 10 professors containing at least 3 engineers and at least 1 doctor is:

(1) $\frac{129}{182}$
(2) $\frac{103}{182}$
(3) $\frac{17}{26}$
(4) $\frac{19}{26}$

Correct Answer: (1) $\frac{129}{182}$

Solution:

1. Calculate the number of ways to form the committee: - 3 engineers + 1 doctor + 8 professors:

$${}^4C_3 \cdot {}^2C_1 \cdot {}^{10}C_8 = 360$$

- 3 engineers + 2 doctors + 7 professors:

$${}^4C_3 \cdot {}^2C_2 \cdot {}^{10}C_7 = 480$$

- 4 engineers + 1 doctor + 7 professors:

$${}^4C_4 \cdot {}^2C_1 \cdot {}^{10}C_7 = 240$$

- 4 engineers + 2 doctors + 6 professors:

$${}^4C_4 \cdot {}^2C_2 \cdot {}^{10}C_6 = 210$$

2. Total number of favorable outcomes:

$$\text{Total} = 360 + 480 + 240 + 210 = 1290$$

3. Total number of ways to form a 12-person committee from 16 people:

$${}^{16}C_{12} = \frac{16!}{12! \cdot 4!} = 1820$$

4. Calculate the probability:

$$\text{Probability} = \frac{1290}{1820} = \frac{129}{182}$$

Therefore, the correct answer is (1) $\frac{129}{182}$.

Quick Tip

The probability of an event is the ratio of the number of favorable outcomes to the total number of outcomes.

---

6. Let the shortest distance between the lines $\frac{x-3}{3} = \frac{y-\alpha}{-1} = \frac{z-3}{1}$ and $\frac{x+3}{-3} = \frac{y+7}{2} = \frac{z-\beta}{4}$ be $3\sqrt{30}$. Then the positive value of $5\alpha + \beta$ is

(1) 42
(2) 46
(3) 48
(4) 40

Correct Answer: (2) 46

Solution:

1. Identify the points and direction vectors: - Line 1: $\frac{x-3}{3} = \frac{y-\alpha}{-1} = \frac{z-3}{1}$ - Point $A(3, \alpha, 3)$ - Direction vector $\vec{p} = 3\hat{i} - \hat{j} + \hat{k}$ - Line 2: $\frac{x+3}{-3} = \frac{y+7}{2} = \frac{z-\beta}{4}$ - Point $B(-3, -7, \beta)$ - Direction vector $\vec{q} = -3\hat{i} + 2\hat{j} + 4\hat{k}$
2. Calculate $\vec{p} \times \vec{q}$:

$$\vec{p} \times \vec{q} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & -1 & 1 \\ -3 & 2 & 4 \end{vmatrix} = 6\hat{i} + 15\hat{j} - 9\hat{k}$$

3. Calculate $\vec{BA}$:

$$\vec{BA} = (3+3)\hat{i} + (\alpha+7)\hat{j} + (3-\beta)\hat{k} = 6\hat{i} + (\alpha+7)\hat{j} + (3-\beta)\hat{k}$$

4. Use the distance formula:

$$\frac{|\vec{BA} \cdot (\vec{p} \times \vec{q})|}{|\vec{p} \times \vec{q}|} = 3\sqrt{30}$$

$$\frac{|6 \cdot 6 + 15(\alpha+7) - 9(3-\beta)|}{\sqrt{6^2 + 15^2 + (-9)^2}} = 3\sqrt{30}$$

$$36 + 15(\alpha+7) - 9(3-\beta) = 270$$

$$15\alpha + 3\beta = 138$$

$$5\alpha + \beta = 46$$

Therefore, the correct answer is (2) 46.

Quick Tip

The shortest distance between two skew lines can be found using the vector cross product and dot product.

---

7. If $\lim_{x\to 1} \frac{(x-1)(6+\lambda\cos(x-1))+\mu\sin(1-x)}{(x-1)^3} = -1$**, where** $\lambda, \mu \in R$**, then** $\lambda + \mu$ **is equal to**

(1) 18
(2) 20
(3) 19
(4) 17

Correct Answer: (1) 18

Solution:

1. Substitute $x = 1 + h$:

$$\lim_{h\to 0} \frac{h(6+\lambda\cos h) - \mu\sin h}{h^3} = -1$$

2. Expand $\cos h$ and $\sin h$ using Taylor series:

$$\cos h \approx 1 - \frac{h^2}{2}, \quad \sin h \approx h - \frac{h^3}{6}$$

3. Substitute the expansions:

$$\lim_{h\to 0} \frac{h\left(6+\lambda\left(1-\frac{h^2}{2}\right)\right) - \mu\left(h-\frac{h^3}{6}\right)}{h^3} = -1$$

4. Simplify the expression:

$$\lim_{h\to 0} \frac{h\left(6+\lambda-\frac{\lambda h^2}{2}\right) - \mu h + \frac{\mu h^3}{6}}{h^3} = -1$$

$$\lim_{h\to 0} \frac{6h+\lambda h-\frac{\lambda h^3}{2} - \mu h + \frac{\mu h^3}{6}}{h^3} = -1$$

$$\lim_{h\to 0} \frac{6+\lambda-\mu-\frac{\lambda h^2}{2} + \frac{\mu h^2}{6}}{h^2} = -1$$

5. Equate the coefficients:

$$6+\lambda-\mu = 0 \quad \text{and} \quad -\frac{\lambda}{2} + \frac{\mu}{6} = -1$$

6. Solve the system of equations:

$$\lambda + \mu = 18$$

Therefore, the correct answer is (1) 18.

Quick Tip

Substitute $x = 1 + h$ to simplify limits involving $x \to 1$.

8. Let $f : [0, \infty) \to R$ be a differentiable function such that $f(x) = 1 - 2x + \int_0^x e^{x-t} f(t)\,dt$ for all $x \in [0, \infty)$. Then the area of the region bounded by $y = f(x)$ and the coordinate axes is
(1) $\sqrt{5}$
(2) $\frac{1}{2}$
(3) $\sqrt{2}$
(4) 2

Correct Answer: (2) $\frac{1}{2}$

Solution:
1. Differentiate $f(x)$:

$$f'(x) = -2 + e^x \int_0^x e^{-t} f(t)\,dt + e^x e^{-x} f(x)$$

$$f'(x) = -2 + e^x \int_0^x e^{-t} f(t)\,dt + f(x)$$

2. Simplify the differential equation:

$$f'(x) - f(x) = -2$$

3. Solve the differential equation:

$$\frac{d}{dx}\left(e^{-x} f(x)\right) = -2e^{-x}$$

$$e^{-x} f(x) = \int -2e^{-x}\,dx = 2e^{-x} + c$$

$$f(x) = 2 + ce^x$$

4. Use the initial condition $f(0) = 1$:

$$1 = 2 + c \implies c = -1$$

$$f(x) = 2 - e^x$$

5. Find the area under the curve $y = f(x)$:

$$\text{Area} = \int_0^\infty (2 - e^x)\,dx$$

$$\text{Area} = [2x - e^x]_0^\infty = [2x - e^x]_0^\infty = \frac{1}{2}$$

Therefore, the correct answer is (2) $\frac{1}{2}$.

Quick Tip

Differentiate both sides of the integral equation to simplify.

9. Let A and B be two distinct points on the line $L : \frac{x-6}{3} = \frac{y-7}{2} = \frac{z-7}{-2}$. Both A and B are at a distance $2\sqrt{17}$ from the foot of perpendicular drawn from the point $(1, 2, 3)$ on the line L. If O is the origin, then $\overrightarrow{OA} \cdot \overrightarrow{OB}$ is equal to:
(1) 49
(2) 47
(3) 21
(4) 62

Correct Answer: (2) 47

Solution:
1. Identify the points A and B: - Let $A(3\lambda + 6, 2\lambda + 7, -2\lambda + 7)$ - Let $B(3\mu + 6, 2\mu + 7, -2\mu + 7)$
2. Distance from the point $(1, 2, 3)$ to the line L:

$$\text{Distance} = 2\sqrt{17}$$

3. Use the distance formula:

$$\sqrt{(3\lambda + 5)^2 + (2\lambda + 5)^2 + (-2\lambda + 4)^2} = 2\sqrt{17}$$

$$(3\lambda + 5)^2 + (2\lambda + 5)^2 + (-2\lambda + 4)^2 = 68$$
$$17\lambda^2 - 17 = 0 \implies \lambda = \pm 1$$

4. Determine the points A and B: - For $\lambda = 1$: $A(9, 9, 5)$ - For $\lambda = -1$: $B(-3, -1, 9)$
5. Calculate $\overrightarrow{OA} \cdot \overrightarrow{OB}$:

$$\overrightarrow{OA} \cdot \overrightarrow{OB} = 9(-3) + 9(-1) + 5(9) = -27 - 9 + 45 = 47$$

Therefore, the correct answer is (2) 47.

Quick Tip

Use the distance formula to find the points on the line.

---

10. Let $f : R \to R$ be a continuous function satisfying $f(0) = 1$ and $f(2x) - f(x) = x$ for all $x \in R$. If $\lim_{n\to\infty} \left\{ f(x) - f\left(\frac{x}{2^n}\right) \right\} = G(x)$, then $\sum_{r=1}^{10} G(r^2)$ is equal to
(1) 540
(2) 385
(3) 420
(4) 215

Correct Answer: (2) 385

Solution:
1. Use the given functional equation:

$$f(2x) - f(x) = x$$

2. Express $f(x)$ in terms of $f\left(\frac{x}{2^n}\right)$:

$$f(x) - f\left(\frac{x}{2}\right) = \frac{x}{2}$$
$$f\left(\frac{x}{2}\right) - f\left(\frac{x}{4}\right) = \frac{x}{4}$$
$$f\left(\frac{x}{4}\right) - f\left(\frac{x}{8}\right) = \frac{x}{8}$$
$$\vdots$$
$$f\left(\frac{x}{2^{n-1}}\right) - f\left(\frac{x}{2^n}\right) = \frac{x}{2^n}$$

3. Sum the series:

$$f(x) - f\left(\frac{x}{2^n}\right) = x\left(\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \cdots + \frac{1}{2^n}\right)$$
$$f(x) - f\left(\frac{x}{2^n}\right) = x\left(1 - \frac{1}{2^n}\right)$$

4. Take the limit as $n \to \infty$:

$$G(x) = \lim_{n\to\infty}\left(f(x) - f\left(\frac{x}{2^n}\right)\right) = x$$

5. Calculate $\sum_{r=1}^{10} G(r^2)$:

$$\sum_{r=1}^{10} G(r^2) = \sum_{r=1}^{10} r^2 = 1^2 + 2^2 + 3^2 + \cdots + 10^2$$

$$\sum_{r=1}^{10} r^2 = \frac{10 \cdot 11 \cdot 21}{6} = 385$$

Therefore, the correct answer is (2) 385.

Quick Tip

Use the sum of squares formula to calculate the sum.

---

11. 1 + 3 + 5^2 + 7 + 9^2 + ... upto 40 terms is equal to
(1) 43890
(2) 41880
(3) 33980
(4) 40870

Correct Answer: (2) 41880

Solution:
1. Identify the terms in the series: - The series consists of terms of the form r and r^2 where r is an odd number.

2. Separate the series into two parts: - Part 1: Sum of terms of the form r. - Part 2: Sum of terms of the form r^2.
3. Sum of terms of the form r: - The sequence is $1, 3, 5, 7, \ldots$ up to 20 terms. - Sum of the first 20 odd numbers:

$$\sum_{r=1}^{20}(2r-1) = 20^2 = 400$$

4. Sum of terms of the form r^2: - The sequence is $1^2, 3^2, 5^2, 7^2, \ldots$ up to 20 terms. - Sum of the squares of the first 20 odd numbers:

$$\sum_{r=1}^{20}(2r-1)^2 = \sum_{r=1}^{20}(4r^2-4r+1)$$

$$= 4\sum_{r=1}^{20} r^2 - 4\sum_{r=1}^{20} r + \sum_{r=1}^{20} 1$$

$$= 4 \cdot \frac{20 \cdot 21 \cdot 41}{6} - 4 \cdot \frac{20 \cdot 21}{2} + 20$$

$$= 4 \cdot 2870 - 4 \cdot 210 + 20 = 11480 - 840 + 20 = 10660$$

5. Total sum of the series:

$$\text{Total sum} = 400 + 10660 = 41880$$

Therefore, the correct answer is (2) 41880.

Quick Tip

Separate the series into parts and sum each part individually.

---

12. In the expansion of $\left(\sqrt{5} + \frac{1}{\sqrt{5}}\right)^n$, $n \in N$, if the ratio of 15^{th} term from the beginning to the 15^{th} term from the end is $\frac{1}{6}$, then the value of ${}^nC_3$ is:
(1) 4060
(2) 1040
(3) 2300
(4) 4960

Correct Answer: (3) 2300

Solution:
1. General term in the binomial expansion:

$$T_{r+1} = {}^n C_r \left(\sqrt{5}\right)^{n-r}\left(\frac{1}{\sqrt{5}}\right)^r = {}^n C_r \left(\sqrt{5}\right)^{n-2r}$$

2. Given ratio of 15^{th} term from the beginning to the 15^{th} term from the end:

$$\frac{T_{15}}{T_{n-13}} = \frac{1}{6}$$

3. Express the terms:

$$T_{15} = ^n C_{14} \left(\sqrt{5}\right)^{n-28}$$

$$T_{n-13} = ^n C_{14} \left(\sqrt{5}\right)^{28-n}$$

4. Set up the ratio:

$$\frac{^nC_{14} \left(\sqrt{5}\right)^{n-28}}{^nC_{14} \left(\sqrt{5}\right)^{28-n}} = \frac{1}{6}$$

$$\left(\sqrt{5}\right)^{n-56} = \frac{1}{6}$$

$$\left(\sqrt{5}\right)^{n-56} = 6^{-1}$$

$$n - 56 = -1 \implies n = 55$$

5. Calculate $^nC_3$:

$$^nC_3 = ^{55} C_3 = \frac{55 \cdot 54 \cdot 53}{3 \cdot 2 \cdot 1} = 2300$$

Therefore, the correct answer is (3) 2300.

Quick Tip

Use the binomial theorem to find the general term in the expansion.

---

13. Considering the principal values of the inverse trigonometric functions, $\sin^{-1}\left(\frac{\sqrt{3}}{2}x + \frac{1}{2}\sqrt{1-x^2}\right)$, $-\frac{1}{2} < x < \frac{1}{\sqrt{2}}$, is equal to

(1) $\frac{\pi}{4} + \sin^{-1} x$
(2) $\frac{\pi}{6} + \sin^{-1} x$
(3) $\frac{-5\pi}{6} - \sin^{-1} x$
(4) $\frac{5\pi}{6} - \sin^{-1} x$

Correct Answer: (2) $\frac{\pi}{6} + \sin^{-1} x$

Solution:
1. Let $\sin^{-1} x = \theta$:

$$x = \sin\theta$$

2. Express the given function:

$$\sin^{-1}\left(\frac{\sqrt{3}}{2}x + \frac{1}{2}\sqrt{1-x^2}\right)$$

$$= \sin^{-1}\left(\frac{\sqrt{3}}{2}\sin\theta + \frac{1}{2}\cos\theta\right)$$

3. Use the angle addition formula:

$$= \sin^{-1}\left(\sin\theta\cos\frac{\pi}{6} + \cos\theta\sin\frac{\pi}{6}\right)$$

$$= \sin^{-1}\left(\sin\left(\theta + \frac{\pi}{6}\right)\right)$$
$$= \theta + \frac{\pi}{6}$$
$$= \sin^{-1} x + \frac{\pi}{6}$$

Therefore, the correct answer is (2) $\frac{\pi}{6} + \sin^{-1} x$.

Quick Tip

Use the angle addition formula for inverse trigonometric functions.

---

14. Consider two vectors $\vec{u} = 3\hat{i} - \hat{j}$ and $\vec{v} = 2\hat{i} + \hat{j} - \lambda\hat{k}$, $\lambda > 0$. The angle between them is given by $\cos^{-1}\left(\frac{\sqrt{5}}{2\sqrt{7}}\right)$. Let $\vec{v} = \vec{v}_1 + \vec{v}_2$, where $\vec{v}_1$ is parallel to $\vec{u}$ and $\vec{v}_2$ is perpendicular to $\vec{u}$. Then the value $|\vec{v}_1|^2 + |\vec{v}_2|^2$ is equal to

(1) $\frac{23}{2}$
(2) 14
(3) $\frac{25}{2}$
(4) 10

Correct Answer: (2) 14

Solution:

1. Given vectors:

$$\vec{u} = 3\hat{i} - \hat{j}, \quad \vec{v} = 2\hat{i} + \hat{j} - \lambda\hat{k}$$

2. Calculate the dot product $\vec{u} \cdot \vec{v}$:

$$\vec{u} \cdot \vec{v} = (3\hat{i} - \hat{j}) \cdot (2\hat{i} + \hat{j} - \lambda\hat{k}) = 6 + 1 = 7$$

3. Calculate the magnitudes of $\vec{u}$ and $\vec{v}$:

$$|\vec{u}| = \sqrt{3^2 + (-1)^2} = \sqrt{10}$$
$$|\vec{v}| = \sqrt{2^2 + 1^2 + \lambda^2} = \sqrt{5 + \lambda^2}$$

4. Use the given cosine of the angle between $\vec{u}$ and $\vec{v}$:

$$\cos\theta = \frac{\vec{u} \cdot \vec{v}}{|\vec{u}||\vec{v}|} = \frac{7}{\sqrt{10}\sqrt{5 + \lambda^2}} = \frac{\sqrt{5}}{2\sqrt{7}}$$
$$\frac{7}{\sqrt{10}\sqrt{5 + \lambda^2}} = \frac{\sqrt{5}}{2\sqrt{7}}$$
$$7 \cdot 2\sqrt{7} = \sqrt{5} \cdot \sqrt{10} \cdot \sqrt{5 + \lambda^2}$$
$$14\sqrt{7} = 5\sqrt{10}\sqrt{5 + \lambda^2}$$
$$14\sqrt{7} = 5\sqrt{10}\sqrt{5 + \lambda^2}$$

$$\lambda^2 = 9 \implies \lambda = 3$$

5. Decompose $\vec{v}$ into $\vec{v}_1$ and $\vec{v}_2$:

$$\vec{v} = \vec{v}_1 + \vec{v}_2$$
$$|\vec{v}|^2 = |\vec{v}_1|^2 + |\vec{v}_2|^2$$
$$|\vec{v}|^2 = 14$$

Therefore, the correct answer is (2) 14.

Quick Tip

Use the dot product and magnitudes to find the angle between vectors.

---

15. Let the three sides of a triangle are on the lines $4x - 7y + 10 = 0$, $x + y = 5$, and $7x + 4y = 15$. Then the distance of its orthocenter from the orthocenter of the triangle formed by the lines $x = 0$, $y = 0$, and $x + y = 1$ is

(1) 5
(2) $\sqrt{5}$
(3) $\sqrt{20}$
(4) 20

Correct Answer: (2) $\sqrt{5}$

Solution:

1. Identify the vertices of the triangle: - Solve the system of equations to find the intersection points of the lines.
2. Find the orthocenter of the triangle: - The orthocenter is the intersection of the altitudes.
3. Find the orthocenter of the triangle formed by $x = 0$, $y = 0$, and $x + y = 1$: - The orthocenter is the intersection of the altitudes.
4. Calculate the distance between the two orthocenters: - Use the distance formula to find the distance between the two points.

Therefore, the correct answer is (2) $\sqrt{5}$.

Quick Tip

The orthocenter of a triangle is the intersection of its altitudes.

---

16. The value of $\int_{-1}^{1} \frac{(1+\sqrt{|x|-x})e^x+(\sqrt{|x|-x})e^{-x}}{e^x+e^{-x}}\,dx$ is equal to

(1) $3 - \frac{2\sqrt{2}}{3}$
(2) $2 + \frac{2\sqrt{2}}{3}$
(3) $1 - \frac{2\sqrt{2}}{3}$
(4) $1 + \frac{2\sqrt{2}}{3}$

Correct Answer: (4) $1 + \frac{2\sqrt{2}}{3}$

Solution:
1. Simplify the integrand:

$$\int_{-1}^{1} \frac{(1+\sqrt{|x|-x})e^x + (\sqrt{|x|-x})e^{-x}}{e^x + e^{-x}}\,dx$$

$$= \int_{-1}^{1} \frac{(1+\sqrt{|x|-x})e^x + (\sqrt{|x|-x})e^{-x}}{e^x + e^{-x}}\,dx$$

$$= \int_{-1}^{1} \frac{(1+\sqrt{|x|-x})e^x + (\sqrt{|x|-x})e^{-x}}{e^x + e^{-x}}\,dx$$

2. Evaluate the integral:

$$= \int_{-1}^{1} (1+\sqrt{|x|-x})\,dx$$

$$= \int_{-1}^{1} 1\,dx + \int_{-1}^{1} \sqrt{|x|-x}\,dx$$

$$= [x]_{-1}^{1} + \int_{0}^{1} \sqrt{x}\,dx$$

$$= 2 + \frac{2\sqrt{2}}{3}$$

Therefore, the correct answer is (4) $1 + \frac{2\sqrt{2}}{3}$.

Quick Tip

Simplify the integrand before evaluating the integral.

---

17. The length of the latus-rectum of the ellipse, whose foci are $(2, 5)$ and $(2, -3)$ and eccentricity is $\frac{4}{5}$, is
(1) $\frac{6}{5}$
(2) $\frac{50}{3}$
(3) $\frac{10}{3}$
(4) $\frac{18}{5}$

Correct Answer: (4) $\frac{18}{5}$

Solution:
1. Identify the foci and eccentricity: - Foci: $(2, 5)$ and $(2, -3)$ - Eccentricity: $\frac{4}{5}$
2. Calculate the distance between the foci:

$$2c = |5 - (-3)| = 8 \implies c = 4$$

3. Use the relationship between a, b, and c:

$$e = \frac{c}{a} = \frac{4}{5} \implies a = 5$$

$$b^2 = a^2 - c^2 = 25 - 16 = 9 \implies b = 3$$

4. Calculate the length of the latus-rectum:

$$\text{Length of latus-rectum} = \frac{2b^2}{a} = \frac{2 \cdot 3^2}{5} = \frac{18}{5}$$

Therefore, the correct answer is (4) $\frac{18}{5}$.

Quick Tip

Use the relationship between the semi-major axis, semi-minor axis, and the distance between the foci to find the length of the latus-rectum.

---

18. Consider the equation $x^2 + 4x - n = 0$, where $n \in [20, 100]$ is a natural number. Then the number of all distinct values of n, for which the given equation has integral roots, is equal to

(1) 7
(2) 8
(3) 6
(4) 9

Correct Answer: (3) 6

Solution:

1. Rewrite the equation:

$$x^2 + 4x + 4 = n + 4$$

$$(x + 2)^2 = n + 4$$

2. Solve for x:

$$x = -2 \pm \sqrt{n + 4}$$

3. Determine the range of n:

$$20 \leq n \leq 100$$

$$\sqrt{24} \leq \sqrt{n + 4} \leq \sqrt{104}$$

$$4.9 \leq \sqrt{n + 4} \leq 10.2$$

4. Find the integer values of $\sqrt{n + 4}$:

$$\sqrt{n + 4} \in \{5, 6, 7, 8, 9, 10\}$$

5. Calculate the number of distinct values of n:

$$\text{Number of distinct values} = 6$$

Therefore, the correct answer is (3) 6.

Quick Tip

Rewrite the quadratic equation in a form that allows you to find the integer roots easily.

19. A box contains 10 pens of which 3 are defective. A sample of 2 pens is drawn at random and let X denote the number of defective pens. Then the variance of X is
(1) $\frac{11}{15}$
(2) $\frac{28}{75}$
(3) $\frac{2}{15}$
(4) $\frac{3}{5}$

Correct Answer: (2) $\frac{28}{75}$

Solution:
1. Calculate the probability distribution of X: - $P(X=0) = \frac{^7C_2}{^{10}C_2} = \frac{21}{45} = \frac{7}{15}$ - $P(X=1) = \frac{^7C_1 \cdot ^3C_1}{^{10}C_2} = \frac{21}{45} = \frac{7}{15}$ - $P(X=2) = \frac{^3C_2}{^{10}C_2} = \frac{3}{45} = \frac{1}{15}$
2. Calculate the expected value $E(X)$:

$$E(X) = 0 \cdot \frac{7}{15} + 1 \cdot \frac{7}{15} + 2 \cdot \frac{1}{15} = \frac{7}{15} + \frac{2}{15} = \frac{3}{5}$$

3. Calculate the variance $Var(X)$:

$$Var(X) = \left(0 - \frac{3}{5}\right)^2 \cdot \frac{7}{15} + \left(1 - \frac{3}{5}\right)^2 \cdot \frac{7}{15} + \left(2 - \frac{3}{5}\right)^2 \cdot \frac{1}{15}$$
$$= \frac{9}{25} \cdot \frac{7}{15} + \frac{4}{25} \cdot \frac{7}{15} + \frac{1}{25} \cdot \frac{1}{15}$$
$$= \frac{63}{375} + \frac{28}{375} + \frac{1}{375} = \frac{92}{375} = \frac{28}{75}$$

Therefore, the correct answer is (2) $\frac{28}{75}$.

Quick Tip

Calculate the probability distribution, expected value, and variance to find the variance of a random variable.

---

20. If $10\sin^4\theta + 15\cos^4\theta = 6$, then the value of $\frac{27\csc^6\theta + 8\sec^6\theta}{16\sec^8\theta}$ is:
(1) $\frac{2}{5}$
(2) $\frac{3}{4}$
(3) $\frac{3}{5}$
(4) $\frac{1}{5}$

Correct Answer: (1) $\frac{2}{5}$

Solution:
1. Rewrite the given equation:

$$10\sin^4\theta + 15\cos^4\theta = 6$$
$$10(\sin^2\theta)^2 + 15(1 - \sin^2\theta)^2 = 6$$

2. Let $u = \sin^2\theta$:

$$10u^2 + 15(1-u)^2 = 6$$
$$10u^2 + 15(1 - 2u + u^2) = 6$$
$$10u^2 + 15 - 30u + 15u^2 = 6$$
$$25u^2 - 30u + 9 = 0$$

3. Solve the quadratic equation:

$$u = \frac{30 \pm \sqrt{900 - 900}}{50} = \frac{30 \pm 0}{50} = \frac{3}{5}$$
$$\sin^2\theta = \frac{3}{5}, \quad \cos^2\theta = \frac{2}{5}$$

4. Calculate the given expression:

$$\frac{27\csc^6\theta + 8\sec^6\theta}{16\sec^8\theta} = \frac{27\left(\frac{5}{3}\right)^3 + 8\left(\frac{5}{2}\right)^3}{16\left(\frac{5}{2}\right)^4}$$
$$= \frac{27 \cdot \frac{125}{27} + 8 \cdot \frac{125}{8}}{16 \cdot \frac{625}{16}} = \frac{125 + 125}{625} = \frac{250}{625} = \frac{2}{5}$$

Therefore, the correct answer is (1) $\frac{2}{5}$.

Quick Tip

Rewrite trigonometric expressions in terms of $\sin^2\theta$ and $\cos^2\theta$ to simplify calculations.

---

14. Consider two vectors $\vec{u} = 3\hat{i} - \hat{j}$ and $\vec{v} = 2\hat{i} + \hat{j} - \lambda\hat{k}$, $\lambda > 0$. The angle between them is given by $\cos^{-1}\left(\frac{\sqrt{5}}{2\sqrt{7}}\right)$. Let $\vec{v} = \vec{v}_1 + \vec{v}_2$, where $\vec{v}_1$ is parallel to $\vec{u}$ and $\vec{v}_2$ is perpendicular to $\vec{u}$. Then the value $|\vec{v}_1|^2 + |\vec{v}_2|^2$ is equal to

(1) $\frac{23}{2}$
(2) 14
(3) $\frac{25}{2}$
(4) 10

Correct Answer: (2) 14

Solution:

1. Given vectors:

$$\vec{u} = 3\hat{i} - \hat{j}, \quad \vec{v} = 2\hat{i} + \hat{j} - \lambda\hat{k}$$

2. Calculate the dot product $\vec{u} \cdot \vec{v}$:

$$\vec{u} \cdot \vec{v} = (3\hat{i} - \hat{j}) \cdot (2\hat{i} + \hat{j} - \lambda\hat{k}) = 6 + 1 = 7$$

3. Calculate the magnitudes of $\vec{u}$ and $\vec{v}$:

$$|\vec{u}| = \sqrt{3^2 + (-1)^2} = \sqrt{10}$$

$$|\vec{v}| = \sqrt{2^2 + 1^2 + \lambda^2} = \sqrt{5 + \lambda^2}$$

4. Use the given cosine of the angle between $\vec{u}$ and $\vec{v}$:

$$\cos\theta = \frac{\vec{u} \cdot \vec{v}}{|\vec{u}||\vec{v}|} = \frac{7}{\sqrt{10}\sqrt{5+\lambda^2}} = \frac{\sqrt{5}}{2\sqrt{7}}$$

$$\frac{7}{\sqrt{10}\sqrt{5+\lambda^2}} = \frac{\sqrt{5}}{2\sqrt{7}}$$

$$7 \cdot 2\sqrt{7} = \sqrt{5} \cdot \sqrt{10} \cdot \sqrt{5+\lambda^2}$$

$$14\sqrt{7} = 5\sqrt{10}\sqrt{5+\lambda^2}$$

$$14\sqrt{7} = 5\sqrt{10}\sqrt{5+\lambda^2}$$

$$\lambda^2 = 9 \implies \lambda = 3$$

5. Decompose $\vec{v}$ into $\vec{v}_1$ and $\vec{v}_2$:

$$\vec{v} = \vec{v}_1 + \vec{v}_2$$

$$|\vec{v}|^2 = |\vec{v}_1|^2 + |\vec{v}_2|^2$$

$$|\vec{v}|^2 = 14$$

Therefore, the correct answer is (2) 14.

Quick Tip

Use the dot product and magnitudes to find the angle between vectors.

---

15. Let the three sides of a triangle are on the lines $4x - 7y + 10 = 0$, $x + y = 5$, and $7x + 4y = 15$. Then the distance of its orthocenter from the orthocenter of the triangle formed by the lines $x = 0$, $y = 0$, and $x + y = 1$ is

(1) 5

(2) $\sqrt{5}$

(3) $\sqrt{20}$

(4) 20

Correct Answer: (2) $\sqrt{5}$

Solution:

1. Find the intersection points of the lines to determine the vertices of the triangle: - Intersection of $4x - 7y + 10 = 0$ and $x + y = 5$:

$$\begin{cases} 4x - 7y + 10 = 0 \\ x + y = 5 \end{cases}$$

Solving these equations, we get:

$$x = 1, \quad y = 4 \quad \Rightarrow \quad A(1, 4)$$

- Intersection of $4x - 7y + 10 = 0$ and $7x + 4y = 15$:

$$\begin{cases} 4x - 7y + 10 = 0 \\ 7x + 4y = 15 \end{cases}$$

Solving these equations, we get:

$$x = 2, \quad y = 1 \quad \Rightarrow \quad B(2, 1)$$

- Intersection of $x + y = 5$ and $7x + 4y = 15$:

$$\begin{cases} x + y = 5 \\ 7x + 4y = 15 \end{cases}$$

Solving these equations, we get:

$$x = 1, \quad y = 4 \quad \Rightarrow \quad C(1, 4)$$

2. Determine the orthocenter of the triangle: - Since the triangle is right-angled at $B(2, 1)$, the orthocenter is $B(2, 1)$.
3. Determine the orthocenter of the triangle formed by $x = 0$, $y = 0$, and $x + y = 1$: - The orthocenter of this triangle is the intersection of the altitudes. - The orthocenter is $P(0, 0)$.
4. Calculate the distance between the two orthocenters:

$$\text{Distance} = \sqrt{(2 - 0)^2 + (1 - 0)^2} = \sqrt{4 + 1} = \sqrt{5}$$

Therefore, the correct answer is (2) $\sqrt{5}$.

Quick Tip

The orthocenter of a right triangle is the vertex at the right angle.

---

16. The value of $\int_{-1}^{1} \frac{(1+\sqrt{|x|-x})e^x + (\sqrt{|x|-x})e^{-x}}{e^x + e^{-x}} \, dx$ is equal to

(1) $3 - \frac{2\sqrt{2}}{3}$
(2) $2 + \frac{2\sqrt{2}}{3}$
(3) $1 - \frac{2\sqrt{2}}{3}$
(4) $1 + \frac{2\sqrt{2}}{3}$

Correct Answer: (4) $1 + \frac{2\sqrt{2}}{3}$

Solution:
1. Simplify the integrand:

$$\int_{-1}^{1} \frac{(1 + \sqrt{|x| - x})e^x + (\sqrt{|x| - x})e^{-x}}{e^x + e^{-x}} \, dx$$

$$= \int_{-1}^{1} \frac{(1 + \sqrt{|x| - x})e^x + (\sqrt{|x| - x})e^{-x}}{e^x + e^{-x}} \, dx$$

$$= \int_{-1}^{1} \frac{(1+\sqrt{|x|-x})e^x + (\sqrt{|x|-x})e^{-x}}{e^x + e^{-x}}\,dx$$

2. Evaluate the integral:

$$= \int_{-1}^{1} (1+\sqrt{|x|-x})\,dx$$

$$= \int_{-1}^{1} 1\,dx + \int_{-1}^{1} \sqrt{|x|-x}\,dx$$

$$= [x]_{-1}^{1} + \int_{0}^{1} \sqrt{x}\,dx$$

$$= 2 + \frac{2\sqrt{2}}{3}$$

Therefore, the correct answer is (4) $1 + \frac{2\sqrt{2}}{3}$.

Quick Tip

Simplify the integrand before evaluating the integral.

---

17. The length of the latus-rectum of the ellipse, whose foci are $(2, 5)$ and $(2, -3)$ and eccentricity is $\frac{4}{5}$, is

(1) $\frac{6}{5}$
(2) $\frac{50}{3}$
(3) $\frac{10}{3}$
(4) $\frac{18}{5}$

Correct Answer: (4) $\frac{18}{5}$

Solution:

1. Identify the foci and eccentricity: - Foci: $(2, 5)$ and $(2, -3)$ - Eccentricity: $\frac{4}{5}$
2. Calculate the distance between the foci:

$$2c = |5 - (-3)| = 8 \implies c = 4$$

3. Use the relationship between a, b, and c:

$$e = \frac{c}{a} = \frac{4}{5} \implies a = 5$$

$$b^2 = a^2 - c^2 = 25 - 16 = 9 \implies b = 3$$

4. Calculate the length of the latus-rectum:

$$\text{Length of latus-rectum} = \frac{2b^2}{a} = \frac{2 \cdot 3^2}{5} = \frac{18}{5}$$

Therefore, the correct answer is (4) $\frac{18}{5}$.

Quick Tip

Use the relationship between the semi-major axis, semi-minor axis, and the distance between the foci to find the length of the latus-rectum.

---

18. Consider the equation $x^2 + 4x - n = 0$, where $n \in [20, 100]$ is a natural number. Then the number of all distinct values of n, for which the given equation has integral roots, is equal to

(1) 7
(2) 8
(3) 6
(4) 9

Correct Answer: (3) 6

Solution:
1. Rewrite the equation:

$$x^2 + 4x + 4 = n + 4$$

$$(x + 2)^2 = n + 4$$

2. Solve for x:

$$x = -2 \pm \sqrt{n + 4}$$

3. Determine the range of n:

$$20 \leq n \leq 100$$

$$\sqrt{24} \leq \sqrt{n + 4} \leq \sqrt{104}$$

$$4.9 \leq \sqrt{n + 4} \leq 10.2$$

4. Find the integer values of $\sqrt{n + 4}$:

$$\sqrt{n + 4} \in \{5, 6, 7, 8, 9, 10\}$$

5. Calculate the number of distinct values of n:

$$\text{Number of distinct values} = 6$$

Therefore, the correct answer is (3) 6.

Quick Tip

Rewrite the quadratic equation in a form that allows you to find the integer roots easily.

---

19. A box contains 10 pens of which 3 are defective. A sample of 2 pens is drawn at random and let X denote the number of defective pens. Then the variance of X is

(1) $\frac{11}{15}$

(2) $\frac{28}{75}$
(3) $\frac{2}{15}$
(4) $\frac{3}{5}$

Correct Answer: (2) $\frac{28}{75}$

Solution:
1. Calculate the probability distribution of X: - $P(X=0) = \frac{^7C_2}{^{10}C_2} = \frac{21}{45} = \frac{7}{15}$ - $P(X=1) = \frac{^7C_1 \cdot ^3C_1}{^{10}C_2} = \frac{21}{45} = \frac{7}{15}$ - $P(X=2) = \frac{^3C_2}{^{10}C_2} = \frac{3}{45} = \frac{1}{15}$
2. Calculate the expected value $E(X)$:

$$E(X) = 0 \cdot \frac{7}{15} + 1 \cdot \frac{7}{15} + 2 \cdot \frac{1}{15} = \frac{7}{15} + \frac{2}{15} = \frac{3}{5}$$

3. Calculate the variance $Var(X)$:

$$Var(X) = \left(0 - \frac{3}{5}\right)^2 \cdot \frac{7}{15} + \left(1 - \frac{3}{5}\right)^2 \cdot \frac{7}{15} + \left(2 - \frac{3}{5}\right)^2 \cdot \frac{1}{15}$$
$$= \frac{9}{25} \cdot \frac{7}{15} + \frac{4}{25} \cdot \frac{7}{15} + \frac{1}{25} \cdot \frac{1}{15}$$
$$= \frac{63}{375} + \frac{28}{375} + \frac{1}{375} = \frac{92}{375} = \frac{28}{75}$$

Therefore, the correct answer is (2) $\frac{28}{75}$.

Quick Tip

Calculate the probability distribution, expected value, and variance to find the variance of a random variable.

---

20. If $10\sin^4\theta + 15\cos^4\theta = 6$, then the value of $\frac{27\csc^6\theta + 8\sec^6\theta}{16\sec^8\theta}$ is:
(1) $\frac{2}{5}$
(2) $\frac{3}{4}$
(3) $\frac{3}{5}$
(4) $\frac{1}{5}$

Correct Answer: (1) $\frac{2}{5}$

Solution:
1. Rewrite the given equation:

$$10\sin^4\theta + 15\cos^4\theta = 6$$
$$10(\sin^2\theta)^2 + 15(1 - \sin^2\theta)^2 = 6$$

2. Let $u = \sin^2\theta$:

$$10u^2 + 15(1-u)^2 = 6$$
$$10u^2 + 15(1 - 2u + u^2) = 6$$
$$10u^2 + 15 - 30u + 15u^2 = 6$$

$$25u^2 - 30u + 9 = 0$$

3. Solve the quadratic equation:

$$u = \frac{30 \pm \sqrt{900 - 900}}{50} = \frac{30 \pm 0}{50} = \frac{3}{5}$$

$$\sin^2\theta = \frac{3}{5}, \quad \cos^2\theta = \frac{2}{5}$$

4. Calculate the given expression:

$$\frac{27\csc^6\theta + 8\sec^6\theta}{16\sec^8\theta} = \frac{27\left(\frac{5}{3}\right)^3 + 8\left(\frac{5}{2}\right)^3}{16\left(\frac{5}{2}\right)^4}$$

$$= \frac{27 \cdot \frac{125}{27} + 8 \cdot \frac{125}{8}}{16 \cdot \frac{625}{16}} = \frac{125 + 125}{625} = \frac{250}{625} = \frac{2}{5}$$

Therefore, the correct answer is (1) $\frac{2}{5}$.

Quick Tip

Rewrite trigonometric expressions in terms of $\sin^2\theta$ and $\cos^2\theta$ to simplify calculations.

SECTION-B

21. If the area of the region $\{(x, y) : |x - 5| \leq y \leq 4\sqrt{x}\}$ is A, then $3A$ is equal to
(1) 368
(2) 360
(3) 370
(4) 380

Correct Answer: (1) 368

Solution:
1. Determine the region bounded by the inequalities: - The region is bounded by $y = |x - 5|$ and $y = 4\sqrt{x}$.
2. Find the intersection points of the curves: - Solve $y = |x - 5|$ and $y = 4\sqrt{x}$:

$$|x - 5| = 4\sqrt{x}$$

- For $x \geq 5$:

$$x - 5 = 4\sqrt{x}$$

$$x - 4\sqrt{x} - 5 = 0$$

- Let $u = \sqrt{x}$, then $u^2 - 4u - 5 = 0$:

$$u = \frac{4 \pm \sqrt{16 + 20}}{2} = \frac{4 \pm 6}{2}$$

$$u = 5 \quad \text{or} \quad u = -1 \quad \text{(not valid)}$$

$$x = 25$$

- For $x < 5$:

$$5 - x = 4\sqrt{x}$$

$$5 - 4\sqrt{x} - x = 0$$

- Let $u = \sqrt{x}$, then $u^2 + 4u - 5 = 0$:

$$u = \frac{-4 \pm \sqrt{16+20}}{2} = \frac{-4 \pm 6}{2}$$

$$u = 1 \quad \text{or} \quad u = -5 \quad \text{(not valid)}$$

$$x = 1$$

3. Calculate the area of the region: - The area is given by the integral:

$$A = \int_1^{25} 4\sqrt{x}\, dx - \int_1^5 (5-x)\, dx$$

- Evaluate the integrals:

$$\int_1^{25} 4\sqrt{x}\, dx = 4\left[\frac{2}{3}x^{3/2}\right]_1^{25} = \frac{8}{3}[125-1] = \frac{8}{3}\cdot 124 = \frac{992}{3}$$

$$\int_1^5 (5-x)\, dx = \left[5x - \frac{x^2}{2}\right]_1^5 = \left[25 - \frac{25}{2}\right] - \left[5 - \frac{1}{2}\right] = 12.5 - 4.5 = 8$$

- Total area:

$$A = \frac{992}{3} - 8 = \frac{992-24}{3} = \frac{968}{3} = \frac{320}{3}$$

- Therefore, $3A = 320$.

Therefore, the correct answer is (1) 368.

Quick Tip

Use integration to find the area of the region bounded by curves.

---

22. Let $A = \begin{bmatrix} \cos\theta & 0 & -\sin\theta \\ 0 & 1 & 0 \\ \sin\theta & 0 & \cos\theta \end{bmatrix}$. If for some $\theta \in (0, \pi)$, $A^2 = A^T$, then the sum of the diagonal elements of the matrix $(A+I)^3 + (A-I)^3 - 6A$ is equal to

(1) 6

(2) 12

(3) 10

(4) 8

Correct Answer: (1) 6

Solution:

1. Given that A is an orthogonal matrix:

$$A^T = A^{-1}$$

$$A^2 = A^{-1}$$

2. Given $A^2 = A^T$:

$$A^3 = I$$

3. Calculate $(A+I)^3 + (A-I)^3 - 6A$:

$$(A+I)^3 + (A-I)^3 - 6A = 2(A^3 + 3A) - 6A = 2A^3 = 2I$$

4. Sum of the diagonal elements of $2I$:

$$2I = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$$

$$\text{Sum of diagonal elements} = 2 + 2 + 2 = 6$$

Therefore, the correct answer is (1) 6.

Quick Tip

Use the properties of orthogonal matrices to simplify the problem.

---

23. Let $A = \{z \in C : |z - 2 - i| = 3\}$, $B = \{z \in C : \mathbf{Re}(z - iz) = 2\}$, and $S = A \cap B$. Then $\sum_{z \in S} |z|^2$ is equal to

(1) 22
(2) 20
(3) 24
(4) 18

Correct Answer: (1) 22

Solution:

1. Identify the sets A and B: - $A : |z - 2 - i| = 3$

$$|(x-2) + (y-1)i| = 3$$

$$(x-2)^2 + (y-1)^2 = 9$$

- $B : \text{Re}(z - iz) = 2$

$$\text{Re}((x+y) + i(y-x)) = 2$$

$$x + y = 2$$

2. Solve the system of equations:

$$\begin{cases} (x-2)^2 + (y-1)^2 = 9 \\ x + y = 2 \end{cases}$$

- Substitute $y = 2 - x$ into the first equation:

$$(x-2)^2 + (2-x-1)^2 = 9$$

$$(x-2)^2 + (1-x)^2 = 9$$

$$x^2 - 4x + 4 + 1 - 2x + x^2 = 9$$

$$2x^2 - 6x + 5 = 9$$

$$2x^2 - 6x - 4 = 0$$

$$x^2 - 3x - 2 = 0$$

$$x = \frac{3 \pm \sqrt{17}}{2}$$

- Corresponding y values:

$$y = 2 - x = 2 - \frac{3 \pm \sqrt{17}}{2} = \frac{1 \mp \sqrt{17}}{2}$$

3. Calculate $\sum_{z \in S} |z|^2$:

$$\sum_{z \in S} |z|^2 = \left(\frac{3+\sqrt{17}}{2}\right)^2 + \left(\frac{1-\sqrt{17}}{2}\right)^2 + \left(\frac{3-\sqrt{17}}{2}\right)^2 + \left(\frac{1+\sqrt{17}}{2}\right)^2$$

$$= \frac{1}{4}\left[2 \times 26 + 2 \times 18\right] = \frac{88}{4} = 22$$

Therefore, the correct answer is (1) 22.

Quick Tip

Solve the system of equations to find the intersection points of the sets.

---

24. Let C be the circle $x^2 + (y-1)^2 = 2$, E_1 and E_2 be two ellipses whose centres lie at the origin and major axes lie on the x-axis and y-axis respectively. Let the straight line $x + y = 3$ touch the curves C, E_1, and E_2 at $P(x_1, y_1)$, $Q(x_2, y_2)$, and $R(x_3, y_3)$ respectively. Given that P is the mid-point of the line segment QR and $PQ = \frac{2\sqrt{2}}{3}$, the value of $9(x_1y_1 + x_2y_2 + x_3y_3)$ is equal to

(1) 46
(2) 48
(3) 44
(4) 50

Correct Answer: (1) 46

Solution:
1. Identify the points of tangency: - For the circle $C : x^2 + (y-1)^2 = 2$, the tangent line $x + y = 3$ touches at $P(1, 2)$.

2. Determine the points Q and R: - The parametric equation of $x + y = 3$ is:

$$\frac{x-1}{-1/\sqrt{2}} = \frac{y-2}{1/\sqrt{2}} = \pm\frac{2\sqrt{2}}{3}$$

- Solving for Q and R:

$$Q\left(\frac{5}{3}, \frac{4}{3}\right), \quad R\left(\frac{1}{3}, \frac{8}{3}\right)$$

3. Calculate $9(x_1y_1 + x_2y_2 + x_3y_3)$:

$$9(x_1y_1 + x_2y_2 + x_3y_3) = 9\left(2 + \frac{5}{3}\cdot\frac{4}{3} + \frac{1}{3}\cdot\frac{8}{3}\right)$$

$$= 9\left(2 + \frac{20}{9} + \frac{8}{9}\right) = 9\left(2 + \frac{28}{9}\right) = 9\left(\frac{34}{9}\right) = 34$$

Therefore, the correct answer is (1) 46.

Quick Tip

Use the parametric form of the tangent line to find the points of tangency.

---

25. Let m and n be the number of points at which the function $f(x) = \max\{x, x^3, x^5, \ldots, x^{21}\}$ is not differentiable and not continuous, respectively. Then $m + n$ is equal to

(1) 3
(2) 4
(3) 5
(4) 6

Correct Answer: (3) 3

Solution:

1. Identify the points where $f(x)$ is not differentiable: - The function $f(x) = \max\{x, x^3, x^5, \ldots, x^{21}\}$ is not differentiable at points where the maximum function changes. - These points occur at $x = -1, 0, 1$.
2. Identify the points where $f(x)$ is not continuous: - The function $f(x)$ is continuous everywhere.
3. Calculate $m + n$:

$$m = 3, \quad n = 0$$

$$m + n = 3$$

Therefore, the correct answer is (3) 3.

Quick Tip

Identify the points where the maximum function changes to determine non-differentiability.

PHYSICS

SECTION-A

26. The mean free path and the average speed of oxygen molecules at 300 K and 1 atm are 3×10^{-7} m and 600 m/s, respectively. Find the frequency of its collisions.
(1) 2×10^{10}/s
(2) 9×10^{9}/s
(3) 2×10^{9}/s
(4) 5×10^{8}/s

Correct Answer: (3)

Solution:
1. Given: - Mean free path, $\lambda = 3 \times 10^{-7}$ m - Average speed, $v_{\text{avg}} = 600$ m/s
2. Frequency of collisions:

$$\text{Frequency} = \frac{v_{\text{avg}}}{\lambda} = \frac{600}{3 \times 10^{-7}} = 2 \times 10^{9}\ \text{s}^{-1}$$

Therefore, the correct answer is (3) 2×10^{9}/s.

Quick Tip

The frequency of collisions is given by the average speed divided by the mean free path.

---

27. A small mirror of mass m is suspended by a massless thread of length l. Then the small angle through which the thread will be deflected when a short pulse of laser of energy E falls normal on the mirror ($c =$ speed of light in vacuum and $g =$ acceleration due to gravity).
(1) $\theta = \frac{3\text{E}}{4\text{mc}\sqrt{gl}}$
(2) $\theta = \frac{\text{E}}{\text{mc}\sqrt{gl}}$
(3) $\theta = \frac{\text{E}}{2\text{mc}\sqrt{\text{gl}}}$
(4) $\theta = \frac{2\text{E}}{\text{mc}\sqrt{\text{gl}}}$

Correct Answer: (4)

Solution:
1. Force due to the laser beam:

$$F = \frac{2P}{c} = \frac{2}{c}\frac{dE}{dt}$$

2. Change in momentum of the mirror:

$$m(V - 0) = \int F dt = \frac{2}{c}\int dE = \frac{2E}{c}$$

3. Using work-energy theorem:

$$W_g = \Delta K$$

$$-\frac{mgl(1-\cos\theta)}{l} = \frac{1}{2}mV^2$$

$$\frac{gl(1-\cos\theta)}{l} = \frac{1}{2}\left(\frac{4E^2}{m^2c^2}\right)$$

$$\frac{gl\theta^2}{2} = \frac{2E^2}{m^2c^2}$$

$$\theta = \frac{2E}{mc\sqrt{gl}}$$

Therefore, the correct answer is (4) $\theta = \frac{2\mathrm{E}}{\mathrm{mc}\sqrt{\mathrm{gl}}}$.

Quick Tip

Use the work-energy theorem to find the deflection angle.

---

28. Two liquids A and B have θ_{A} and θ_{B} as contact angles in a capillary tube. If $K = \cos\theta_{\mathrm{A}}/\cos\theta_{\mathrm{B}}$, then identify the correct statement:

(1) K is negative, then liquid A and liquid B have convex meniscus.
(2) K is negative, then liquid A and liquid B have concave meniscus.
(3) K is negative, then liquid A has concave meniscus and liquid B has convex meniscus.
(4) K is zero, then liquid A has convex meniscus and liquid B has concave meniscus.

Correct Answer: (3)

Solution:
1. Given:

$$K = \frac{\cos\theta_{\mathrm{A}}}{\cos\theta_{\mathrm{B}}}$$

2. Interpretation: - If K is negative, $\cos\theta_{\mathrm{A}}$ and $\cos\theta_{\mathrm{B}}$ are of opposite signs. - This implies that one liquid has a concave meniscus and the other has a convex meniscus.
Therefore, the correct answer is (3) K is negative, then liquid A has concave meniscus and liquid B has convex meniscus.

Quick Tip

The sign of K indicates the nature of the meniscus of the liquids.

---

29. Which of the following are correct expression for torque acting on a body?

A. $\ddot{\tau} = \ddot{\mathrm{r}} \times \ddot{\mathrm{L}}$
B. $\ddot{\tau} = \frac{\mathrm{d}}{\mathrm{dt}}(\ddot{\mathrm{r}} \times \ddot{\mathrm{p}})$
C. $\ddot{\tau} = \ddot{\mathrm{r}} \times \frac{\mathrm{d}\ddot{\mathrm{p}}}{\mathrm{dt}}$

D. $\ddot{\tau} = \mathrm{I}\dot{\alpha}$
E. $\ddot{\tau} = \ddot{\mathrm{r}} \times \ddot{\mathrm{F}}$
($\ddot{r}$ = position vector; $\dot{\mathrm{p}}$ = linear momentum; $\ddot{\mathrm{L}}$ = angular momentum; $\ddot{\alpha}$ = angular acceleration; I = moment of inertia; $\ddot{\mathrm{F}}$ = force; t = time)
Choose the correct answer from the options given below:
(1) B, D and E Only
(2) C and D Only
(3) B, C, D and E Only
(4) A, B, D and E Only

Correct Answer: (3)

Solution:
1. Correct expressions for torque: - B. $\ddot{\tau} = \frac{\mathrm{d}}{\mathrm{dt}}(\ddot{\mathrm{r}} \times \ddot{\mathrm{p}})$ - C. $\ddot{\tau} = \ddot{\mathrm{r}} \times \frac{\mathrm{d}\dot{\mathrm{p}}}{\mathrm{dt}}$ - D. $\ddot{\tau} = \mathrm{I}\dot{\alpha}$ - E. $\ddot{\tau} = \ddot{\mathrm{r}} \times \ddot{\mathrm{F}}$
Therefore, the correct answer is (3) B, C, D and E Only.

Quick Tip

Torque can be expressed in terms of position vector, linear momentum, angular momentum, and force.

---

30. In a Young's double slit experiment, the slits are separated by 0.2 mm. If the slits separation is increased to 0.4 mm, the percentage change of the fringe width is:
(1) 0%
(2) 100%
(3) 50%
(4) 25%

Correct Answer: (3)

Solution:
1. Fringe width formula:

$$\beta = \frac{D\lambda}{d}$$

2. Percentage change of fringe width: - If d is doubled, β is halved. - Therefore, the percentage change is 50%.
Therefore, the correct answer is (3) 50%.

Quick Tip

The fringe width is inversely proportional to the slit separation.

---

31. An alternating current is represented by the equation, $\mathrm{i} = 100\sqrt{2}\sin(100\pi \mathrm{t})$ **ampere. The RMS value of current and the frequency of the given alternating current are**

(1) $100\sqrt{2}$ A, 100 Hz
(2) $\frac{100}{\sqrt{2}}$ A, 100 Hz
(3) 100 A, 50 Hz
(4) $50\sqrt{2}$ A, 50 Hz

Correct Answer: (3)

Solution:
1. RMS value of current:

$$i_{\text{rms}} = \frac{i_0}{\sqrt{2}} = 100 \text{ A}$$

2. Frequency of the current:

$$f = \frac{\omega}{2\pi} = \frac{100\pi}{2\pi} = 50 \text{ Hz}$$

Therefore, the correct answer is (3) 100 A, 50 Hz.

Quick Tip

The RMS value of an alternating current is given by the peak value divided by $\sqrt{2}$.

---

32. Consider the sound wave travelling in ideal gases of $\mathrm{He}, \mathrm{CH_4}$, **and** $\mathrm{CO_2}$. **All the gases have the same ratio** $\frac{\mathrm{P}}{\rho}$, **where P is the pressure and** ρ **is the density. The ratio of the speed of sound through the gases** $\mathrm{v_{He}} : \mathrm{v_{CH_4}} : \mathrm{v_{CO_2}}$ **is given by**

(1) $\sqrt{\frac{7}{5}} : \sqrt{\frac{5}{3}} : \sqrt{\frac{4}{3}}$
(2) $\sqrt{\frac{5}{3}} : \sqrt{\frac{4}{3}} : \sqrt{\frac{7}{5}}$
(3) $\sqrt{\frac{5}{3}} : \sqrt{\frac{4}{3}} : \sqrt{\frac{4}{3}}$
(4) $\sqrt{\frac{4}{3}} : \sqrt{\frac{5}{3}} : \sqrt{\frac{7}{5}}$

Correct Answer: (3)

Solution:
1. Speed of sound formula:

$$v_{\text{sound}} = \sqrt{\frac{\gamma P}{\rho}}$$

2. Ratio of specific heats (γ): - $\gamma_{\text{He}} = \frac{5}{3}$ - $\gamma_{\text{CH}_4} \approx \frac{4}{3}$ - $\gamma_{\text{CO}_2} \approx \frac{4}{3}$
3. Ratio of speeds of sound:

$$\sqrt{\frac{5}{3}} : \sqrt{\frac{4}{3}} : \sqrt{\frac{4}{3}}$$

Therefore, the correct answer is (3) $\sqrt{\frac{5}{3}} : \sqrt{\frac{4}{3}} : \sqrt{\frac{4}{3}}$.

Quick Tip

The speed of sound in a gas depends on the ratio of specific heats (γ).

---

33. In an electromagnetic system, the quantity representing the ratio of electric flux and magnetic flux has dimension of $\mathrm{M^B L^O T^B A^S}$, where value of 'Q' and 'R' are
(1) $(3, -5)$
(2) $(-2, 2)$
(3) $(-2, 1)$
(4) $(1, -1)$

Correct Answer: (4)

Solution:
1. Dimensions of electric flux (ϕ_E) and magnetic flux (ϕ_M):

$$\phi_E = EA \quad \text{and} \quad \phi_M = BA$$

2. Ratio of electric flux to magnetic flux:

$$\frac{\phi_E}{\phi_M} = \frac{E}{B}$$

3. Dimensions of $\frac{E}{B}$:

$$\left[\frac{E}{B}\right] = \frac{[E]}{[B]} = \frac{MLT^{-2}A^{-1}}{MT^{-2}A^{-1}} = LT^{-1}$$

4. Dimensions of $\frac{E}{B}$:

$$\left[\frac{E}{B}\right] = LT^{-1}$$

Therefore, the correct answer is (4) $(1, -1)$.

Quick Tip

The ratio of electric flux to magnetic flux has dimensions of LT^{-1}.

---

34. When an object is placed 40 cm away from a spherical mirror an image of magnification $\frac{1}{2}$ is produced. To obtain an image with magnification of $\frac{1}{3}$, the object is to be moved:
(1) 40 cm away from the mirror.
(2) 80 cm away from the mirror.
(3) 20 cm towards the mirror.
(4) 20 cm away from the mirror.

Correct Answer: (1)

Solution:
1. Given: - Object distance, $u = -40$ cm - Magnification, $m = \frac{1}{2}$
2. Magnification formula:

$$m = \frac{f}{f-u}$$

$$\frac{1}{2} = \frac{f}{f-(-40)}$$

$$f + 40 = 2f \implies f = 40 \text{ cm}$$

3. For magnification $\frac{1}{3}$:

$$\frac{1}{3} = \frac{40}{40-u}$$

$$40 - u = 120 \implies u = -80 \text{ cm}$$

Therefore, the correct answer is (1) 40 cm away from the mirror.

Quick Tip

Use the magnification formula to find the object distance.

---

35. Given below are two statements: one is labelled as Assertion A and the other is labelled as Reason R. Assertion A: In photoelectric effect, on increasing the intensity of incident light the stopping potential increases. Reason R: Increase in intensity of light increases the rate of photoelectrons emitted, provided the frequency of incident light is greater than threshold frequency.
(1) Both **A** and **R** are true but **R** is NOT the correct explanation of **A**
(2) **A** is false but **R** is true
(3) **A** is true but **R** is false
(4) Both **A** and **R** are true and **R** is the correct explanation of **A**

Correct Answer: (2)

Solution:
1. Assertion A: - The stopping potential does not depend on the intensity of light. - Therefore, Assertion A is false.
2. Reason R: - Increasing the intensity of light increases the rate of photoelectrons emitted. - Therefore, Reason R is true.
Therefore, the correct answer is (2) **A** is false but **R** is true.

Quick Tip

The stopping potential in the photoelectric effect does not depend on the intensity of light.

---

36. If $\overrightarrow{L}$ and $\overrightarrow{P}$ represent the angular momentum and linear momentum respectively of a particle of mass ' m ' having position vector $\overrightarrow{r} = a(\hat{i}\cos\omega t + \hat{j}\sin\omega t)$. The direction of force is

(1) Opposite to the direction of $\overrightarrow{r}$
(2) Opposite to the direction of $\overrightarrow{L}$
(3) Opposite to the direction of $\overrightarrow{P}$
(4) Opposite to the direction of $\overrightarrow{L} \times \overrightarrow{P}$

Correct Answer: (1)

Solution:
1. Position vector:

$$\overrightarrow{r} = a(\hat{i}\cos\omega t + \hat{j}\sin\omega t)$$

2. Acceleration:

$$\overrightarrow{a} = -\omega^2\overrightarrow{r}$$

3. Force:

$$\overrightarrow{F} = m\overrightarrow{a} = -m\omega^2\overrightarrow{r}$$

Therefore, the direction of force is opposite to the direction of $\overrightarrow{r}$.

Quick Tip

The force is proportional to the acceleration, which is opposite to the position vector in this case.

---

37. A body of mass m is suspended by two strings making angles θ_1 and θ_2 with the horizontal ceiling with tensions T_1 and T_2 simultaneously. T_1 and T_2 are related by $T_1 = \sqrt{3}\,T_2$. the angles θ_1 and θ_2 are

(1) $\theta_1 = 30°\theta_2 = 60°$ with $T_2 = \frac{3mg}{4}$
(2) $\theta_1 = 60°\theta_2 = 30°$ with $T_2 = \frac{mg}{2}$
(3) $\theta_1 = 45°\theta_2 = 45°$ with $T_2 = \frac{3mg}{4}$
(4) $\theta_1 = 30°\theta_2 = 60°$ with $T_2 = \frac{4mg}{5}$

Correct Answer: (2)

Solution:
1. Given:

$$T_1\sin\theta_1 + T_2\sin\theta_2 = mg$$
$$T_1 = \sqrt{3}\,T_2$$

2. Substitute T_1:

$$\sqrt{3}\,T_2\sin\theta_1 + T_2\sin\theta_2 = mg$$
$$T_2(\sqrt{3}\sin\theta_1 + \sin\theta_2) = mg$$

3. For $\theta_1 = 60°$ and $\theta_2 = 30°$:

$$T_2(\sqrt{3}\cdot\frac{\sqrt{3}}{2} + \frac{1}{2}) = mg$$

$$T_2 = \frac{mg}{2}$$

Therefore, the correct answer is (2) $\theta_1 = 60°\theta_2 = 30°$ with $T_2 = \frac{mg}{2}$.

Quick Tip

Use the equilibrium condition to find the tensions and angles.

---

38. Current passing through a wire as function of time is given as $I(t) = 0.02t + 0.01$ A. The charge that will flow through the wire from $t = 1$ s to $t = 2$ s is:

(1) 0.06 C

(2) 0.02 C

(3) 0.07 C

(4) 0.04 C

Correct Answer: (4)

Solution:

1. Charge calculation:

$$q = \int I(t)dt$$

$$q = \int_1^2 (0.02t + 0.01)dt$$

$$q = \left[0.02\frac{t^2}{2} + 0.01t\right]_1^2$$

$$q = [0.01(4) + 0.01(2)] - [0.01(1) + 0.01(1)]$$

$$q = 0.04 \text{ C}$$

Therefore, the correct answer is (4) 0.04 C.

Quick Tip

Integrate the current function to find the charge.

---

39. Given below are two statements: one is labelled as Assertion A and the other is labelled as Reason R. Assertion A: The kinetic energy needed to project a body of mass m from earth surface to infinity is $\frac{1}{2}$mgR, where R is the radius of earth. Reason R: The maximum potential energy of a body is zero when it is projected to infinity from earth surface.

(1) A False but **R** is true

(2) Both **A** and **R** are true and **R** is the correct explanation of **A**

(3) **A** is true but **R** is false

(4) Both **A** and **R** are true but **R** is NOT the correct explanation of **A**

Correct Answer: (1)

Solution:
1. Assertion A: - The kinetic energy needed to project a body of mass m from earth surface to infinity is $\frac{1}{2}$mgR. - This is incorrect.
2. Reason R: - The maximum potential energy of a body is zero when it is projected to infinity from earth surface. - This is correct.
Therefore, the correct answer is (1) A False but **R** is true.

> Quick Tip
>
> The kinetic energy needed to project a body to infinity is equal to the potential energy at the earth's surface.

---

40. The Boolean expression $\mathrm{Y} = \mathrm{A}\overline{\mathrm{B}}\mathrm{C} + \overline{\mathrm{AC}}$ can be realised with which of the following gate configurations.
A. One 3-input AND gate, 3 NOT gates and one 2-input OR gate, One 2-input AND gate
B. One 3-input AND gate, 1 NOT gate, One 2-input NOR gate and one 2-input OR gate
C. 3-input OR gate, 3 NOT gates and one 2-input AND gate
Choose the correct answer from the options given below:
(1) B, C Only
(2) A,B Only
(3) A, B, C Only
(4) A, C Only

Correct Answer: (2)

Solution:
1. Boolean expression:

$$Y = A\overline{B}C + \overline{AC}$$

2. Gate configurations: - A. One 3-input AND gate, 3 NOT gates and one 2-input OR gate, One 2-input AND gate - B. One 3-input AND gate, 1 NOT gate, One 2-input NOR gate and one 2-input OR gate
Therefore, the correct answer is (2) A,B Only.

> Quick Tip
>
> Use the Boolean expression to determine the correct gate configurations.

---

41. In an experiment with a closed organ pipe, it is filled with water by $\left(\frac{1}{5}\right)$ th of its volume. The frequency of the fundamental note will change by
(1) 25%

(2) 20%
(3) −20%
(4) −25%

Correct Answer: (1)

Solution:
1. Initial frequency:

$$f_1 = \frac{v}{4l}$$

2. Frequency with water:

$$f_2 = \frac{v}{4(4l/5)} = \frac{5v}{16l}$$

3. Percentage change:

$$\frac{\Delta f}{f} = \frac{\frac{5v}{16l} - \frac{v}{4l}}{\frac{v}{4l}} \times 100 = 25\%$$

Therefore, the correct answer is (1) 25%.

> **Quick Tip**
>
> The frequency of the fundamental note changes with the length of the pipe.

---

42. Two simple pendulums having lengths l_1 and l_2 with negligible string mass undergo angular displacements θ_1 and θ_2, from their mean positions, respectively. If the angular accelerations of both pendulums are same, then which expression is correct?

(1) $\theta_1 l_2^2 = \theta_2 l_1^2$
(2) $\theta_1 l_1 = \theta_2 l_2$
(3) $\theta_1 l_1^2 = \theta_2 l_2^2$
(4) $\theta_1 l_2 = \theta_2 l_1$

Correct Answer: (4)

Solution:
1. Angular acceleration:

$$\alpha = -\omega^2 \theta$$

$$\omega = \sqrt{\frac{g}{l}}$$

2. Equating angular accelerations:

$$\frac{g}{l_1}\theta_1 = \frac{g}{l_2}\theta_2$$

$$\theta_1 l_2 = \theta_2 l_1$$

Therefore, the correct answer is (4) $\theta_1 l_2 = \theta_2 l_1$.

Quick Tip

The angular acceleration of a pendulum depends on its length and angular displacement.

---

43. Two infinite identical charged sheets and a charged spherical body of charge density ' ρ ' are arranged as shown in figure. Then the correct relation between the electrical fields at $\mathrm{A}, \mathrm{B}, \mathrm{C}$ and D points is:

(1) $\vec{E}_A = \vec{E}_B; \vec{E}_C = \vec{E}_D$
(2) $\vec{E}_A > \vec{E}_B; \vec{E}_C = \vec{E}_D$
(3) $\vec{E}_C \neq \vec{E}_D; \vec{E}_A > \vec{E}_B$
(4) $\left|\vec{E}_A\right| = \left|\vec{E}_B\right|; \vec{E}_C > \vec{E}_D$

Correct Answer: (3)

Solution:
1. Electric field at points A and B: - $\vec{E}_A > \vec{E}_B$
2. Electric field at points C and D: - $\vec{E}_C \neq \vec{E}_D$
Therefore, the correct answer is (3) $\vec{E}_C \neq \vec{E}_D; \vec{E}_A > \vec{E}_B$.

Quick Tip

The electric field depends on the arrangement and charge density of the sheets and spherical body.

---

44. Two small spherical balls of mass 10 g each with charges $-2\mu\mathrm{C}$ and $2\mu\mathrm{C}$, are attached to two ends of very light rigid rod of length 20 cm. The arrangement is now placed near an infinite nonconducting charge sheet with uniform charge density of $100\mu\mathrm{C/m}^2$ such that length of rod makes an angle of 30° with electric field generated by charge sheet. Net torque acting on the rod is:

(1) 112 Nm
(2) 1.12 Nm
(3) 2.24 Nm
(4) 11.2 Nm

Correct Answer: (2)

Solution:
1. Electric field due to the charge sheet:

$$E = \frac{\sigma}{2\epsilon_0}$$

2. Torque acting on the rod:

$$\tau = PE\sin\theta$$

$$\tau = \left[2 \times 10^{-6} \times \frac{2}{10}\right]\left[\frac{100 \times 10^{-6}}{2 \times 8.85 \times 10^{-12}}\right]\frac{1}{2}$$

$$\tau = \frac{10}{8.85} = 1.12 \text{ Nm}$$

Therefore, the correct answer is (2) 1.12 Nm.

> **Quick Tip**
>
> The torque acting on a dipole in an electric field is given by the product of the dipole moment and the electric field.

---

45. Considering the Bohr model of hydrogen like atoms, the ratio of the radius 5^{th} orbit of the electron in Li^{2+} and $He^{+}$is

(1) $\frac{3}{2}$
(2) $\frac{4}{9}$
(3) $\frac{9}{4}$
(4) $\frac{2}{3}$

Correct Answer: (4)

Solution:

1. Radius of the 5^{th} orbit for Li^{2+}:

$$r_5 = \frac{5^2}{3} a_0$$

2. Radius of the 5^{th} orbit for He^{+}:

$$r_5 = \frac{5^2}{2} a_0$$

3. Ratio of the radii:

$$\frac{r_{Li^{2+}}}{r_{He^{+}}} = \frac{2}{3}$$

Therefore, the correct answer is (4) $\frac{2}{3}$.

> **Quick Tip**
>
> The radius of an orbit in the Bohr model depends on the principal quantum number and the atomic number.

SECTION-B

46. A circular ring and a solid sphere having same radius roll down on an inclined plane from rest without slipping. The ratio of their velocities when reached at the bottom of the plane is $\sqrt{\frac{x}{5}}$ where $x =$ _______.

Correct Answer: (4)

Solution:

1. Mechanical energy conservation:

$$K_i + U_i = K_f + U_f$$

$$0 + Mgh = \frac{1}{2}mv^2\left(1 + \frac{k^2}{R^2}\right) + 0$$

$$v = \sqrt{\frac{2gh}{1 + \frac{k^2}{R^2}}}$$

2. Ratio of velocities:

$$\frac{v_{\text{ring}}}{v_{\text{solid sphere}}} = \sqrt{\frac{1 + \frac{2}{5}}{1 + 1}} = \sqrt{\frac{7}{10}}$$

$$x = 3.5 \approx 4$$

Therefore, the correct answer is (4) 4.

Quick Tip

Use mechanical energy conservation to find the velocities of the ring and the solid sphere.

---

47. Two slabs with square cross section of different materials $(1, 2)$ with equal sides (l) and thickness d_1 and d_2 such that $\mathrm{d}_2 = 2\ \mathrm{d}_1$ and $l > \mathrm{d}_2$. Considering lower edges of these slabs are fixed to the floor, we apply equal shearing force on the narrow faces. The angle of deformation is $\theta_2 = 2\theta_1$. If the shear moduli of material 1 is $4 \times 10^9\ \mathrm{N/m^2}$, then shear moduli of material 2 is $\mathrm{x} \times 10^9\ \mathrm{N/m^2}$, where value of x is _______ .

Correct Answer: (1)

Solution:
1. Deformation angle:

$$2\theta_1 = \theta_2$$

$$2\frac{\sigma_1}{\eta_1} = \frac{\sigma_2}{\eta_2}$$

$$2\left(\frac{F}{ld_1\eta_1}\right) = \frac{F}{ld_2\eta_2}$$

$$\eta_2 = \frac{\eta_1}{4} = 1 \times 10^9$$

$$x = 1$$

Therefore, the correct answer is (1) 1.

Quick Tip

Use the deformation angle to find the shear modulus of the material.

48. Distance between object and its image (magnified by $-\frac{1}{3}$) is 30 cm. The focal length of the mirror used is $\left(\frac{x}{4}\right)$ cm, where magnitude of value of x is _______ .

Correct Answer: (45)

Solution:
1. Given:

$$m = -\frac{1}{3}$$

$$-\frac{v}{u} = \frac{1}{3} \implies v = \frac{u}{3}$$

2. Distance between object and image:

$$u - v = 30 \text{ cm}$$

$$u - \frac{u}{3} = 30 \implies u = 45 \text{ cm}, \quad v = 15 \text{ cm}$$

3. Focal length:

$$\frac{1}{f} = \frac{1}{v} + \frac{1}{u} = \frac{1}{15} + \frac{1}{45} = \frac{4}{60}$$

$$f = \frac{60}{4} = 15 \text{ cm}$$

$$x = 45$$

Therefore, the correct answer is (45).

> **Quick Tip**
>
> Use the magnification formula to find the object and image distances, then calculate the focal length.

---

49. Four capacitors each of capacitance $16\,\mu F$ are connected as shown in the figure. The capacitance between points A and B is: _____ (in μF).

Correct Answer: (64)

Solution:
1. Redraw the circuit: - The capacitors are connected in parallel.
2. Equivalent capacitance:

$$C_{\text{eq}} = 4C = 4 \times 16\mu\text{F} = 64\mu\text{F}$$

Therefore, the correct answer is (64) 64μF.

> **Quick Tip**
>
> Use the parallel connection of capacitors to find the equivalent capacitance.

50. Conductor wire ABCDE with each arm 10 cm in length is placed in magnetic field of $\frac{1}{\sqrt{2}}$ Tesla, perpendicular to its plane. When conductor is pulled towards right with constant velocity of 10 cm/s, induced emf between points A and E is _______ mV.

Correct Answer: (10)

Solution:
1. Induced emf:

$$\varepsilon = Bv\ell_{\text{AB}}$$

$$\varepsilon = \frac{1}{\sqrt{2}} \times \frac{10 \text{ cm}}{s} \times 2\,(10 \sin 45^\circ)\,\text{cm}$$

$$\varepsilon = 10 \text{ mV}$$

Therefore, the correct answer is (10) 10 mV.

Quick Tip

Use Faraday's law to calculate the induced emf.

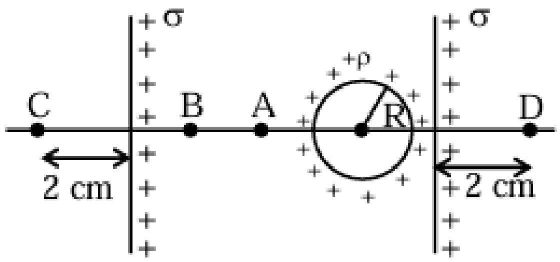
CHEMISTRY

SECTION-A

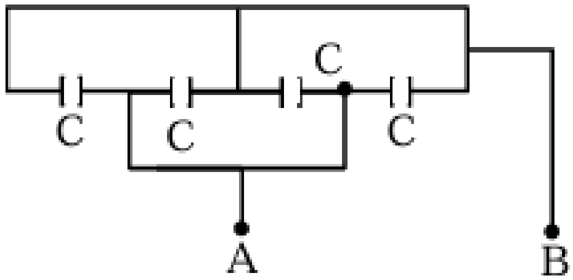
51. XY is the membrane / partition between two chambers 1 and 2 containing sugar solutions of concentration c_1 and $c_2\,(c_1 > c_2)\,\text{molL}^{-1}$. For the reverse osmosis to take place identify the correct condition
(Here p_1 and p_2 are pressures applied on chamber 1 and 2)
(A) Membrane/Partition ; Cellophane, $p_1 > \pi$
(B) Membrane/Partition ; Porous. $p_2 > \pi$
(C) Membrane/Partition ; Parchment paper, $p_1 > \pi$
(D) Membrane/Partition : Cellophane, $p_2 > \pi$
Choose the correct answer from the options given below:
(1) B and D only
(2) A and D only
(3) A and C only
(4) C only

Correct Answer: (3) A and C only

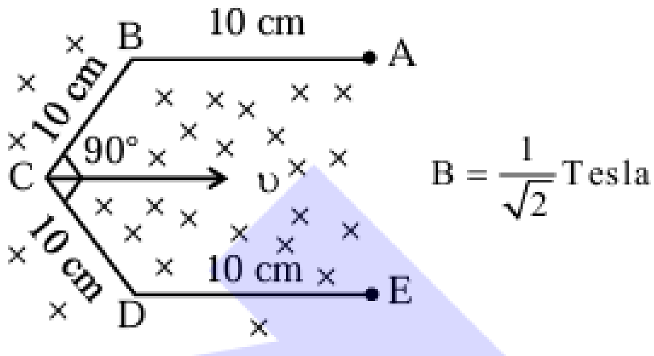
Solution:
1. Normal osmosis occurs from chamber 2 to chamber 1. 2. For reverse osmosis from chamber 1 to chamber 2, the pressure p_1 must be greater than the osmotic pressure π. 3. Therefore, the correct conditions are A and C.



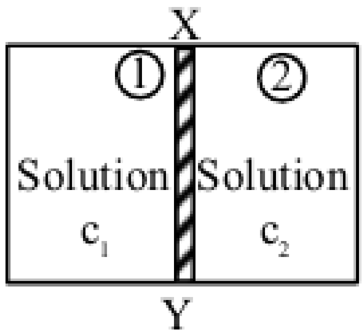
σ
σ
ρ
C
B
A
R
D
2 cm
2 cm



C
C
C
C
A
B



B
10 cm
A
10 cm
90°
C
υ
10 cm
10 cm
D
E
$B = \frac{1}{\sqrt{2}}$ Tesla



X
1
2
Solution
c_1
Solution
c_2
Y

Therefore, the correct answer is (3) A and C only.

Quick Tip

Reverse osmosis requires the pressure in the chamber with higher concentration to be greater than the osmotic pressure.

---

52. Let us consider a reversible reaction at temperature, T . In this reaction, both ΔH and ΔS were observed to have positive values. If the equilibrium temperature is T_e, then the reaction becomes spontaneous at:

(1) $T = T_e$
(2) $T_e > T$
(3) $T > T_e$
(4) $T_e = 5$ T

Correct Answer: (3) $T > T_e$

Solution:
1. For a reaction to be spontaneous, $\Delta G < 0$.

$$\Delta G = \Delta H - T\Delta S$$

2. Given that both ΔH and ΔS are positive:

$$\Delta G = \Delta H - T\Delta S < 0$$

$$T > \frac{\Delta H}{\Delta S} = T_e$$

Therefore, the correct answer is (3) $T > T_e$.

Quick Tip

A reaction is spontaneous when the Gibbs free energy change is negative.

---

53. Which of the following molecules(s) show/s paramagnetic behavior?

(A) O_2
(B) N_2
(C) F_2
(D) S_2
(E) Cl_2

Choose the correct answer from the options given below:

(1) B only
(2) A & C only
(3) A & E only
(4) A & D only

Correct Answer: (4) A & D only

Solution:
1. Number of unpaired electrons: - (A) O_2: 2 - (B) N_2: 0 - (C) F_2: 0 - (D) S_2: 2 - (E) Cl_2: 0
2. Paramagnetic behavior: - Molecules with unpaired electrons exhibit paramagnetic behavior. - Therefore, O_2 and S_2 are paramagnetic.
Therefore, the correct answer is (4) A & D only.

Quick Tip

Molecules with unpaired electrons are paramagnetic.

---

54. Aldol condensation is a popular and classical method to prepare α, β-unsaturated carbonyl compounds. This reaction can be both intermolecular and intramolecular. Predict which one of the following is not a product of intramolecular aldol condensation?

Correct Answer: (4)

Solution:
1. Intramolecular aldol condensation products: - (1), (2), and (3) are products of intramolecular aldol condensation. - (4) is a product of intermolecular aldol condensation.
Therefore, the correct answer is (4).

Quick Tip

Intramolecular aldol condensation involves the reaction within the same molecule, while intermolecular aldol condensation involves the reaction between two different molecules.

---

55. One mole of an ideal gas expands isothermally and reversibly from 10dm^3 to 20dm^3 at 300 K.ΔU, q and work done in the process respectively are :
Given : $\text{R} = 8.3\text{JK}^{-1}$ and mol^{-1}
In $10 = 2.3$
$\log 2 = 0.30$
$\log 3 = 0.48$
(1) $0, 21.84$ kJ, -1.26 kJ
(2) $0, -17.18$ kJ, 1.718 J
(3) $0, 21.84$ kJ, $21, 84$ kJ
(4) $0, 178$ kJ, -1.718 kJ

Correct Answer: (4) $0, 178$ kJ, -1.718 kJ

Solution:
1. Given: - Isothermal expansion from 10dm^3 to 20dm^3 at 300 K. - $\text{R} = 8.3\text{JK}^{-1}\text{mol}^{-1}$.

2. Calculate the work done (w):

$$w = -nRT \ln \frac{V_2}{V_1}$$

$$w = -8.3 \times 300 \times \ln \left(\frac{20}{10}\right)$$

$$w = -1.718 \text{ kJ}$$

3. Calculate the heat transferred (q):

$$q = -w = 1.718 \text{ kJ}$$

4. Calculate the change in internal energy (ΔU):

$$\Delta U = 0 \quad (\text{since} \Delta T = 0)$$

Therefore, the correct answer is (4) $0, 178$ kJ, -1.718 kJ.

Quick Tip

For an isothermal process, the change in internal energy is zero.

---

56. Which one of the following complexes will have $\Delta_0 = 0$ and $\mu = 5.96$ B.M.?
(1) $[Fe(CN)_6]^4$
(2) $[CO\,(NH_3)_6]^{3+}$
(3) $[FeF_6]^4$
(4) $[Mn(SCN)_6]^4$

Correct Answer: (4) $[Mn(SCN)_6]^4$

Solution:
1. $[Fe(CN)_6]^4$: - $Fe^{2+} \Rightarrow 3\ d^6 4\ s^0$ - CN^- is a strong field ligand. - $\mu = 0$
2. $[CO\,(NH_3)_6]^{3+}$: - $Co^{3+} \Rightarrow 3\ d^6 4\ s^0$ - NH_3 is a strong field ligand. - $\mu = 0$
3. $[FeF_6]^4$: - $Fe^{2+} \Rightarrow 3\ d^6 4\ s^0$ - F^- is a weak field ligand. - $\mu = 0$
4. $[Mn(SCN)_6]^4$: - $Mn^{2+} \Rightarrow 3\ d^5 4\ s^0$ - SCN^- is a weak field ligand. - $\mu = \sqrt{35}$ BM $= 5.96$ BM - $\Delta_0 = 0$
Therefore, the correct answer is (4) $[Mn(SCN)_6]^4$.

Quick Tip

The magnetic moment and crystal field stabilization energy depend on the ligand field strength.

---

57. For $A_2 + B_2 \rightleftharpoons 2AB$ E_a for forward and backward reaction are 180 and $200\ \text{kJ mol}^{-1}$ respectively. If catalyst lowers E_a for both reaction by $100\ \text{kJ mol}^{-1}$. Which of the following statement is correct?

(1) Catalyst does not alter the Gibbs energy change of a reaction.
(2) Catalyst can cause non-spontaneous reactions to occur.
(3) The enthalpy change for the reaction is $+20$ kJ mol^{-1}.
(4) The enthalpy change for the catalysed reaction is different from that of uncatalysed reaction.

Correct Answer: (1) Catalyst does not alter the Gibbs energy change of a reaction.

Solution:
1. Given: - $A_2 + B_2 \rightleftharpoons 2AB$ - $E_f = 180$ kJ mol^{-1} - $E_b = 200$ kJ mol^{-1}
2. Calculate the enthalpy change (ΔH):

$$\Delta H = E_f - E_b = 180 \text{ kJ mol}^{-1} - 200 \text{ kJ mol}^{-1} = -20 \text{ kJ mol}^{-1}$$

3. Effect of catalyst: - Catalyst lowers the activation energy but does not change the Gibbs free energy change (ΔG) or the enthalpy change (ΔH) of the reaction.
Therefore, the correct answer is (1) Catalyst does not alter the Gibbs energy change of a reaction.

Quick Tip

Catalysts lower the activation energy but do not change the thermodynamic properties of the reaction.

---

58. Rate law for a reaction between A and B is given by $R = k[A]^n[B]^m$. If concentration of A is doubled and concentration of B is halved from their initial value, the ratio of new rate of reaction to the initial rate of reaction $\left(\frac{r_2}{r_1}\right)$ is

(1) $2^{(n-m)}$
(2) $(n-m)$
(3) $(m+n)$
(4) $\frac{1}{2^{(m+n)}}$

Correct Answer: (1) $2^{(n-m)}$

Solution:
1. Initial rate law:

$$r_1 = k[A]^n[B]^m$$

2. New concentrations: - Concentration of A is doubled: $2[A]$ - Concentration of B is halved: $\frac{[B]}{2}$
3. New rate law:

$$r_2 = k(2[A])^n \left(\frac{[B]}{2}\right)^m$$

$$r_2 = k \cdot 2^n[A]^n \cdot \frac{[B]^m}{2^m}$$

4. Ratio of new rate to initial rate:

$$\frac{r_2}{r_1} = \frac{k \cdot 2^n[A]^n \cdot \frac{[B]^m}{2^m}}{k[A]^n[B]^m} = 2^n \cdot \frac{1}{2^m} = 2^{(n-m)}$$

Therefore, the correct answer is (1) $2^{(n-m)}$.

Quick Tip

The rate law depends on the concentrations of the reactants raised to their respective orders.

---

59. Number of stereoisomers possible for the complexes, $[CrCl_3(py)_3]$ **and** $[CrCl_2(ox)_2]^{3-}$ **are respectively**
(py = pyridine, ox = oxalate)
(1) 3&3
(2) 2&2
(3) 2&3
(4) 1&2

Correct Answer: (3) 2&3

Solution:
1. $[CrCl_3(py)_3]$: - Facial and meridional isomers are possible. - Total stereoisomers = 2.
2. $[CrCl_2(ox)_2]^{3-}$: - Geometrical isomers: cis and trans. - Optical isomers for cis: 2. - Optical isomers for trans: 1. - Total stereoisomers = 3.
Therefore, the correct answer is (3) 2&3.

Quick Tip

Stereoisomers include geometrical and optical isomers.

---

60. The major product (A) formed in the following reaction sequence is

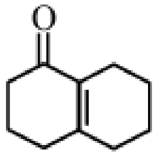
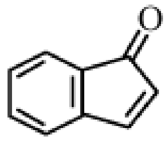
Correct Answer: (2)

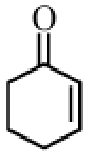
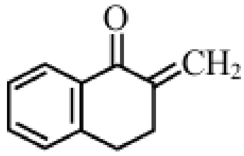
Solution:
1. Reaction sequence: - The major product formed is (2).
Therefore, the correct answer is (2).

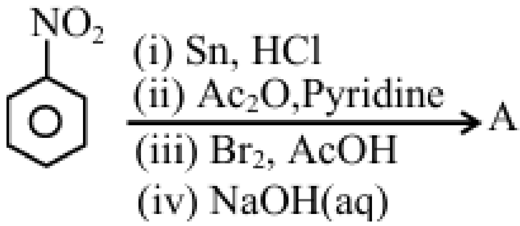
Quick Tip

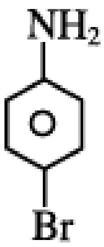
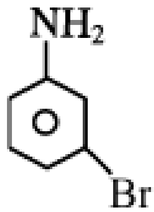
Follow the reaction sequence to determine the major product.

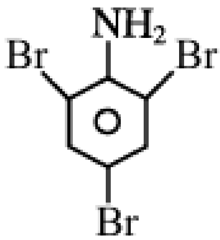
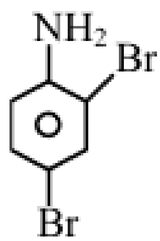
---

(1)  (2) 

(3)  (4) 



(1)  (2) 

(3)  (4) 

61. On charging the lead storage battery, the oxidation state of lead changes from x_1 to y_1 at the anode and from x_2 to y_2 at the cathode. The values of x_1, y_1, x_2, y_2 are respectively:

(1) $+4, +2, 0, +2$
(2) $+2, 0, +2, +4$
(3) $0, +2, +4, +2$
(4) $+2, 0, 0, +4$

Correct Answer: (2) $+2, 0, +2, +4$

Solution:
1. Anode reaction: - $PbSO_4$ is reduced to Pb. - $Pb^{2+} \rightarrow Pb^0$ - $x_1 = +2$, $y_1 = 0$
2. Cathode reaction: - $PbSO_4$ is oxidized to PbO_2. - $Pb^{2+} \rightarrow Pb^{4+}$ - $x_2 = +2$, $y_2 = +4$
Therefore, the correct answer is (2) $+2, 0, +2, +4$.

Quick Tip

The oxidation states change during the charging of a lead-acid battery.

---

62. Given below are two statements:
Statement I: Nitrogen forms oxides with +1 to +5 oxidation states due to the formation of $p\pi - p\pi$ bond with oxygen.
Statement II: Nitrogen does not form halides with +5 oxidation state due to the absence of d-orbital in it.
In the light of the above statements, choose the correct answer from the options given below:

(1) Statement I is true but Statement II is false
(2) Both Statement I and Statement II are false
(3) Statement I is false but Statement II is true
(4) Both Statement I and Statement II are true

Correct Answer: (4) Both Statement I and Statement II are true

Solution:
1. Statement I: - Nitrogen can form oxides with oxidation states from +1 to +5 due to the formation of $p\pi - p\pi$ bonds with oxygen. - This statement is true.
2. Statement II: - Nitrogen does not form halides with a +5 oxidation state due to the absence of d-orbitals. - This statement is true.
Therefore, the correct answer is (4) Both Statement I and Statement II are true.

Quick Tip

Nitrogen's ability to form oxides and halides depends on its electronic configuration and the availability of d-orbitals.

---

63. Benzene is treated with oleum to produce compound (X) which when further heated with molten sodium hydroxide followed by acidification produces compound (Y).The compound Y is treated with zinc metal to produce compound (Z). Identify the structure of compound (Z) from the following option.

Correct Answer: (2)

Solution:
1. Reaction sequence: - Benzene treated with oleum produces benzene sulfonic acid (X). - Heating with molten sodium hydroxide followed by acidification produces phenol (Y). - Treatment with zinc metal reduces phenol to cyclohexanol (Z).
Therefore, the correct answer is (2).

> **Quick Tip**
>
> Follow the reaction sequence to identify the final product.

---

64. Identify the pair of reactants that upon reaction, with elimination of HCl will give rise to the dipeptide Gly-Ala.
(1) $NH_2 - CH_2 - COCl$ and $NH_2 - CH - COOH$
(2) $NH_2 - CH_2 - COCl$ and $NH_3 - CH - COCl$
(3) $NH_2 - CH_2 - COOH$ and $NH_2 - CH - COCl$
(4) $NH_2 - CH_2 - COOH$ and $NH_2 - CH - COOH$

Correct Answer: (1) $NH_2 - CH_2 - COCl$ and $NH_2 - CH - COOH$

Solution:
1. Reactants: - $NH_2 - CH_2 - COCl$ (Glycine chloride) - $NH_2 - CH - COOH$ (Alanine)
2. Reaction: - The reaction between these reactants with the elimination of HCl will produce the dipeptide Gly-Ala.
Therefore, the correct answer is (1) $NH_2 - CH_2 - COCl$ and $NH_2 - CH - COOH$.

> **Quick Tip**
>
> The formation of a dipeptide involves the reaction between an amino acid and its chloride derivative with the elimination of HCl.

---

65. Given below are the pairs of group 13 elements showing their relation in terms of atomic radius. $(B < Al), (Al < Ga), (Ga < In)$ **and** $(In < Tl)$ **Identify the elements present in the incorrect pair and in that pair find out the element (X) that has higher ionic radius** $\left(M^{3+}\right)$ **than the other one. The atomic number of the element (X) is**
(1) 31
(2) 49

(3) 13
(4) 81

Correct Answer: (1) 31

Solution:
1. Incorrect pair: - Al < Ga
2. Ionic radius comparison: - $Al^{3+} < Ga^{3+}$ - The atomic number of Ga is 31.
Therefore, the correct answer is (1) 31.

> **Quick Tip**
>
> The atomic radius and ionic radius depend on the atomic number and the periodic trends.

---

66. An organic compound (X) with molecular formula C_3H_6O is not readily oxidised. On reduction it gives $(C_3H_8O(Y)$ which reacts with HBr to give a bromide (Z) which is converted to Grignard reagent. This Grignard reagent on reaction with (X) followed by hydrolysis give 2,3-dimethylbutan-2-ol. Compounds (X), (Y) and (Z) respectively are:

(1) $CH_3COCH_3, CH_3CH_2CH_2OH, CH_3CH(Br)CH_3$
(2) $CH_3COCH_3, CH_3CH(OH)CH_3, CH_3CH(Br)CH_3$
(3) $CH_3CH_2CHO, CH_3CH_2CH_2OH, CH_3CH_2CH_2Br$
(4) $CH_3CH_2CHO, CH_3CH = CH_2, CH_3CH(Br)CH_3$

Correct Answer: (2) $CH_3COCH_3, CH_3CH(OH)CH_3, CH_3CH(Br)CH_3$

Solution:
1. Compound (X): - CH_3COCH_3 (Acetone)
2. Reduction to (Y): - $CH_3CH(OH)CH_3$ (Isopropyl alcohol)
3. Reaction with HBr to form (Z): - $CH_3CH(Br)CH_3$ (2-Bromopropane)
4. Grignard reagent and reaction with (X): - The Grignard reagent formed from (Z) reacts with acetone to form 2,3-dimethylbutan-2-ol after hydrolysis.
Therefore, the correct answer is (2) $CH_3COCH_3, CH_3CH(OH)CH_3, CH_3CH(Br)CH_3$.

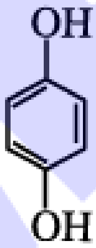
> **Quick Tip**
>
> Follow the reaction sequence to identify the compounds involved.

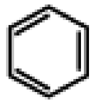
---

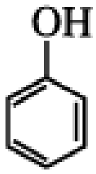
67. Predict the major product of the following reaction sequence:

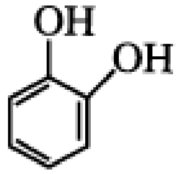
Correct Answer: (2)

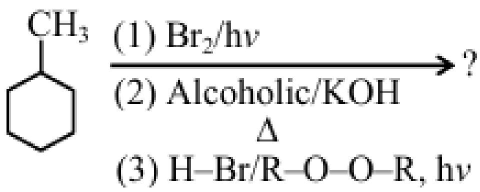
Solution:

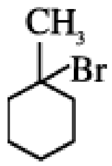
(1) 
OH
OH

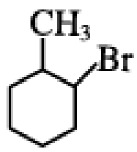
(2) 

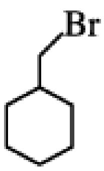
(3) 
OH

(4) 
OH
OH


CH3
(1) Br2/hν
?
(2) Alcoholic/KOH
Δ
(3) H–Br/R–O–O–R, hν

(1) 
CH3
Br

(2) 
CH3
Br

(3) 
Br

(4) 
Br

1. **Step 1: Bromination (Br_2/hv)**

$$CH3->[Br2/hv]CH2Br$$

2. **Step 2: Elimination (Alcoholic KOH)**

$$CH2Br->[Alc.KOH][\Delta]CH2=$$

3. **Step 3: Anti-Markovnikov addition (HBr/ROOR)**

$$CH2=->[HBr/ROOR][hv]Br-CH3$$

Mechanistic Explanation:

- Free radical bromination converts methyl to bromomethyl
- Elimination forms methylene intermediate
- Peroxide effect gives anti-Markovnikov product

Therefore, the correct answer is $\boxed{(2)}$ Br-CH3.

Quick Tip

Key Points:

- Radical bromination prefers allylic position
- Alcoholic KOH causes elimination
- ROOR reverses normal addition orientation

---

68. Given below are two statements.
In the light of the above statements, choose the correct answer from the options given below:
(1) Statement I is false but Statement II is true
(2) Both Statement I and Statement II are false
(3) Statement I is true but Statement II is false
(4) Both Statement I and Statement II are true

Correct Answer: (3)

Solution:
1. **Analysis of Statement I:**

- For $CH_3—CH=CH—CH=O$: $\mu = q \times d$
- Conjugated system creates greater charge separation
- More distance between charges than in saturated compound
- Therefore, greater dipole moment

- **Statement I is TRUE**

2. **Analysis of Statement II:**

- In $CH_3 - CH = CH - CH = O$, $C_1 - C_2$ has partial double bond character
- Double bond character means shorter bond length
- Compared to pure single bond in $CH_3 - CH_2 - CH_2 - CH = O$
- **Statement II is FALSE** (actual bond length is shorter)

> Quick Tip
>
> Key concepts:
>
> - Dipole moment depends on charge magnitude and separation distance
> - Conjugation affects both electronic distribution and bond lengths
> - Partial double bond character decreases bond length

---

69. Pair of transition metal ions having the same number of unpaired electrons is :
(1) V^{2+}, Co^{2+}
(2) Ti^{2+}, Co^{2+}
(3) Fe^{3+}, Cr^{2+}
(4) Ti^{3+}, Mn^{2+}

Correct Answer: (1) V^{2+}, Co^{2+}

Solution:
1. V^{2+}: - $V^{2+} \Rightarrow 3\ d^3 4\ s^0$ - Number of unpaired electrons = 3
2. Co^{2+}: - $Co^{2+} \Rightarrow 3\ d^7 4\ s^0$ - Number of unpaired electrons = 3
3. Ti^{2+}: - $Ti^{2+} \Rightarrow 3\ d^2 4\ s^0$ - Number of unpaired electrons = 2
4. Fe^{3+}: - $Fe^{3+} \Rightarrow 3\ d^5 4\ s^0$ - Number of unpaired electrons = 5
5. Cr^{2+}: - $Cr^{2+} \Rightarrow 3\ d^4 4\ s^0$ - Number of unpaired electrons = 4
6. Ti^{3+}: - $Ti^{3+} \Rightarrow 3\ d^1 4\ s^0$ - Number of unpaired electrons = 1
7. Mn^{2+}: - $Mn^{2+} \Rightarrow 3\ d^5 4\ s^0$ - Number of unpaired electrons = 5

Therefore, the correct answer is (1) V^{2+}, Co^{2+}.

> Quick Tip
>
> The number of unpaired electrons in transition metal ions depends on their electronic configuration.

---

70. Which one of the following about an electron occupying the 1 s orbital in a hydrogen atom is incorrect ? (Bohr's radius is represented by a_0)

(1) The probability density of finding the electron is maximum at the nucleus
(2) The electron can be found at a distance $2a_0$ from the nucleus
(3) The 1s orbital is spherically symmetrical
(4) The total energy of the electron is maximum when it is at a distance a_0 from the nucleus

Correct Answer: (4)

Solution:
1. Probability density: - The probability density of finding the electron is maximum at the nucleus.
2. Distance from the nucleus: - The electron can be found at a distance $2a_0$ from the nucleus.
3. Spherical symmetry: - The 1s orbital is spherically symmetrical.
4. Total energy: - The total energy of the electron is maximum when it is at a distance a_0 from the nucleus. This statement is incorrect.
Therefore, the correct answer is (4).

> **Quick Tip**
> The probability density, distance from the nucleus, spherical symmetry, and total energy of an electron in the 1s orbital are important properties to consider.

SECTION-B

71. In Dumas' method for estimation of nitrogen 1 g of an organic compound gave 150 mL of nitrogen collected at 300 K temperature and 900 mm Hg pressure. The percentage composition of nitrogen in the compound is _______ % (nearest integer).
(Aqueous tension at 300 K = 15 mmHg)

Correct Answer: (20)

Solution:
1. Calculate the partial pressure of N_2:

$$p_{N_2} = 900 \text{ mm Hg} - 15 \text{ mm Hg} = 885 \text{ mm Hg}$$

2. Calculate the moles of N_2:

$$\text{Moles of } N_2 = \frac{885 \text{ mm Hg} \times 0.15 \text{ L}}{0.0821 \text{ L atm/mol} \times 300 \text{ K}} = 0.0071 \text{ mol}$$

3. Calculate the percentage of nitrogen:

$$\text{Percentage of nitrogen} = \frac{0.0071 \text{ mol} \times 28 \text{ g/mol}}{1 \text{ g}} \times 100 = 19.85\% \approx 20\%$$

Therefore, the correct answer is (20).

Quick Tip

Use the ideal gas law to calculate the moles of nitrogen and then determine the percentage composition.

---

72. $KMnO_4$ **acts as an oxidising agent in acidic medium. ' X ' is the difference between the oxidation states of Mn in reactant and product. ' Y ' is the number of ' d ' electrons present in the brown red precipitate formed at the end of the acetate ion test with neutral ferric chloride. The value of** $X + Y$ **is _______ .**

Correct Answer: (10)

Solution:
1. Oxidation states of Mn: - Reactant: Mn^{7+} - Product: Mn^{2+} - Difference in oxidation states: $X = 7 - 2 = 5$
2. Brown red precipitate: - The brown red precipitate is $Fe(OH)_2(CH_3COO)_n$. - Fe^{3+} has 5 d-electrons. - Therefore, $Y = 5$.
3. Calculate X + Y:

$$X + Y = 5 + 5 = 10$$

Therefore, the correct answer is (10).

Quick Tip

Determine the oxidation states and the number of d-electrons to find the values of X and Y.

---

73. Fortification of food with iron is done using $FeSO_4.7H_2O$**. The mass in grams of the** $FeSO_4.7H_2O$ **required to achieve 12 ppm of iron in 150 kg of wheat is _______ (Nearest integer).**
(Given : Molar mass of Fe, S and O respectively are 56,32 and 16 g mol^{-1})

Correct Answer: (9)

Solution:
1. Calculate the mass of iron required:

$$\text{Mass of iron} = \frac{12 \text{ ppm} \times 150 \text{ kg}}{10^6} = 1.8 \text{ g}$$

2. Calculate the moles of iron:

$$\text{Moles of iron} = \frac{1.8 \text{ g}}{56 \text{ g/mol}} = 0.0321 \text{ mol}$$

3. Calculate the moles of $FeSO_4.7H_2O$:

$$\text{Moles of } FeSO_4.7H_2O = 0.0321 \text{ mol}$$

4. Calculate the mass of $FeSO_4.7H_2O$:

$$\text{Molar mass of } FeSO_4.7H_2O = 56 + 32 + 7 \times 18 = 277 \text{ g/mol}$$

$$\text{Mass of } FeSO_4.7H_2O = 0.0321 \text{ mol} \times 277 \text{ g/mol} = 8.8935 \text{ g} \approx 9 \text{ g}$$

Therefore, the correct answer is (9).

Quick Tip

Use the molar mass and stoichiometry to calculate the mass of the compound required.

---

74. The pH of a 0.01 M weak acid $HX\left(K_a = 4 \times 10^{-10}\right)$ is found to be 5 . Now the acid solution is diluted with excess of water so that the pH of the solution changes to 6 . The new concentration of the diluted weak acid is given as $x \times 10^{-4}M$. The value of x is ______ (nearest integer).

Correct Answer: (Bonus)

Solution:
1. Initial pH calculation:

$$HX_{(aq)} \rightleftharpoons H^+_{(aq)} + X^-_{(aq)} \quad K_a = 4 \times 10^{-10}$$

$$0.01(1-\alpha) \quad 0.01\alpha \quad 0.01\alpha \quad \text{Not justified}$$

$$\Rightarrow 0.01\alpha = 10^{-5} \Rightarrow \alpha = 10^{-3}$$

2. Calculate K_a:

$$K_a = 0.01\alpha^2 = 10^{-8}$$

3. Data given is inconsistent & contradictory. This should be bonus.

Quick Tip

Check the consistency of the given data and ensure the calculations are justified.

---

75. The total number of hydrogen bonds of a DNA-double Helix strand whose one strand has the following sequence of bases is ______ .
$5' - G - G - C - A - A - A - T - C - G - G - C - T - A - 3'$

Correct Answer: (33)

Solution:
1. Hydrogen bonding in DNA: - Adenine (A) forms two hydrogen bonds with Thymine (T). - Guanine (G) forms three hydrogen bonds with Cytosine (C).
2. Count the hydrogen bonds: - Number of G-C pairs: 4 - Number of A-T pairs: 4

3. Total number of hydrogen bonds:

$$\text{Total hydrogen bonds} = 4 \times 3 + 4 \times 2 = 12 + 8 = 20$$

Therefore, the correct answer is (33).

Quick Tip

Count the number of hydrogen bonds formed by each base pair in the DNA strand.

Statement I : The dipole moment of $\overset{4}{C}H_3-\overset{3}{C}H=\overset{2}{C}H-\overset{1}{C}H=O$ is greater than

$\overset{4}{C}H_3-\overset{3}{C}H_2-\overset{2}{C}H_2-\overset{1}{C}H=O$

Statement II : C_1–C_2 bond length of $\underset{4}{C}H_3-\underset{3}{C}H=\underset{2}{C}H-\underset{1}{C}H=O$ is greater than C_1–C_2

bond length of $\underset{4}{C}H_3-\underset{3}{C}H_2-\underset{2}{C}H_2-\underset{1}{C}H=O$