JEE Main 2023 6 April Shift 2 Question Paper with Solutions

Time Allowed :3 Hours	Maximum Marks :300	Total Questions :90

General Instructions

Read the following instructions very carefully and strictly follow them:

1. The test is of 3 hours duration.
2. The question paper consists of 90 questions, out of which 75 are to attempted. The maximum marks are 300.
3. There are three parts in the question paper consisting of Physics, Chemistry and Mathematics having 30 questions in each part of equal weightage.
4. Each part (subject) has two sections.
   (i) Section-A: This section contains 20 multiple choice questions which have only one correct answer. Each question carries 4 marks for correct answer and –1 mark for wrong answer.
   (ii) Section-B: This section contains 10 questions. In Section-B, attempt any five questions out of 10. The answer to each of the questions is a numerical value. Each question carries 4 marks for correct answer and –1 mark for wrong answer. For Section-B, the answer should be rounded off to the nearest integer

MATHEMATICS

Section-A

1. If gcd $(m, n) = 1$ and

$$1^2 - 2^2 + 3^2 - 4^2 + \cdots + (2022)^2 - (2023)^2 = 1012\, m^2 n$$

then $m^2 - n^2$ is equal to:

(1) 180

(2) 220

(3) 200

(4) 240

Correct Answer: (4) 240

Solution:

$$(1-2)(1+2)+(3-4)(3+4)+\cdots+(2021-2022)(2021+2022)+(2023)^2=(1012)\,m^2n$$

$$\Rightarrow(-1)[1+2+3+4+\cdots+2022]+(2023)^2=(1012)m^2n$$

$$\Rightarrow(2023)[2023-1011]=(1012)m^2n$$

$$\Rightarrow(2023)(1012)=(1012)m^2n$$

$$\Rightarrow m^2n=2023$$

$$\Rightarrow m^2n=(17)^2\times 7$$

$$m=17,\ n=7$$

$$m^2-n^2=(17)^2-7^2=289-49=240$$

Quick Tip

When solving for values in number theory, look for patterns in terms and simplify step by step. Break down complicated equations into smaller parts and solve them sequentially.

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2. The area bounded by the curves $y=|x-1|+|x-2|$ and $y=3$ is equal to:

(1) 5

(2) 4

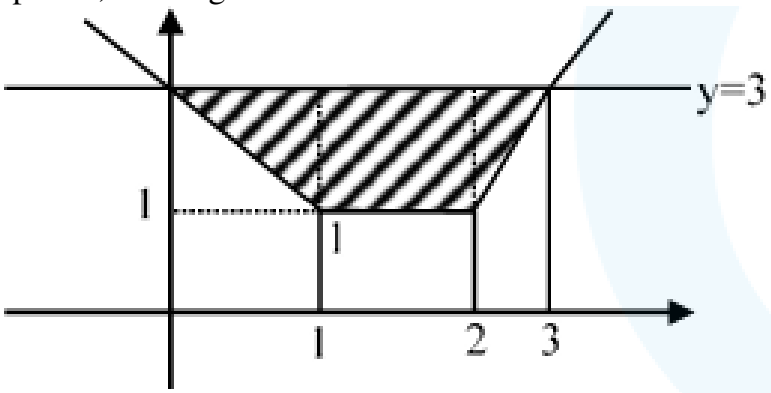
(3) 6

(4) 3

Correct Answer: (2) 4

Solution:

The given equation for y is $y = |x - 1| + |x - 2|$. The graph of y intersects the line $y = 3$ at certain points, forming a bounded area.



The area can be calculated by finding the points of intersection of the curves and calculating the area between them. After calculating, we find the area is 4 square units.

Quick Tip

To calculate areas between curves involving absolute values, break the integral into parts corresponding to the intervals where the expressions inside the absolute values are positive or negative.

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3. For the system of equations

x + y + z = 6

x + 2y + z = 10

x + 3y + 5z = β

which one of the following is NOT true:

(1) System has a unique solution for $\alpha = 3, \beta \neq 14$

(2) System has a unique solution for $\alpha = -3, \beta = 14$

(3) System has no solution for $\alpha = 3, \beta = 24$

(4) System has infinitely many solutions for $\alpha = 3, \beta = 14$

Correct Answer: (1)

Solution:

Given the system of equations, we need to analyze the determinant of the coefficient matrix to determine the nature of the solution.

The system is represented by:

$$\begin{pmatrix} 1 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 3 & 5 \end{pmatrix}$$

The determinant of this matrix is calculated as:

$$\Delta = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 3 & 5 \end{vmatrix}$$

$$\Delta = 6(10 - 3\alpha) - (50 - \alpha 13) + (30 - 2\beta)$$

$$= 40 - 18\alpha + \alpha\beta - 2\beta$$

For infinite solutions, we get $\Delta = 0$, and after solving:

$$\Delta = 0,\ \Delta x = \Delta y = \Delta z = 0$$

The value of $\alpha = 3$ and $\beta = 14$.

For a unique solution, $\alpha \neq 3$.

Thus, the correct answer is Option 1.

Quick Tip

For systems of equations, the determinant of the coefficient matrix indicates whether the system has a unique solution (non-zero determinant) or infinitely many solutions (zero determinant).

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4. Among the statements:

(S1): $(\phi \implies \psi) \vee (\neg\phi \implies \psi)$ is a tautology.

(S2): $(\psi \implies \phi) \implies (\neg\phi \implies \psi)$ is a contradiction.

Choose the correct answer from the options given below:

(1) Only (S2) is True

(2) Only (S1) is True

(3) Neither (S1) nor (S2) is True

(4) Both (S1) and (S2) are True

Correct Answer: (3)

Solution:

S1

P	Q	~p	~p^q	p⇒q	(p⇒q) v (~p^q)
T	T	F	F	T	T
T	F	F	F	F	F
F	T	T	T	T	T
F	F	T	F	T	T

S2

P	Q	q⇒p	~p	(~p)^q	(q⇒p) ⇒ (~p^q)
T	T	T	F	F	F
T	F	T	F	F	F
F	T	F	T	T	T
F	F	T	T	F	F

For statement (S1), the logical expression $(\phi \implies \psi) \vee (\neg\phi \implies \psi)$ is a tautology because it always holds true for any truth values of ϕ and ψ. For statement (S2), $(\psi \implies \phi) \implies (\neg\phi \implies \psi)$ is indeed a contradiction because it does not hold true in any case. Hence, neither (S1) nor (S2) is true, so the correct answer is option (3).

Quick Tip

To identify tautologies and contradictions, construct truth tables for logical expressions and check the validity of the statements.

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5. $\lim_{n\to\infty} \left\{ \left(\frac{1}{2^2-2^3}\right) \left(\frac{1}{2^2-2^5}\right) \cdots \left(\frac{1}{2^2-2^{2n+1}}\right) \right.$ is equal to:

(1) $\frac{1}{\sqrt{2}}$

(2) $\sqrt{2}$

(3) 1

(4) 0

Correct Answer: (4) 0

Solution:

We are given the product of terms:

$$P = \lim_{n\to\infty} \left\{ \left(\frac{1}{2^2 - 2^3}\right) \left(\frac{1}{2^2 - 2^5}\right) \cdots \left(\frac{1}{2^2 - 2^{2n+1}}\right) \right.$$

First, we analyze the behavior of each term in the product. The smallest term in the product is:

$$\frac{1}{2^2 - 2^3} \quad \text{and the largest term is} \quad \frac{1}{2^2 - 2^{2n+1}}$$

The product is bounded as:

$$\left(\frac{1}{2^2 - 2^3}\right)^n \leq P \leq \left(\frac{1}{2^2 - 2^{2n+1}}\right)^n$$

The sequence is bounded between 0 and 1. Therefore, the limit of the product as $n \to \infty$ is 0.

Thus, the final answer is:

$$P = 0$$

Quick Tip

For infinite products, analyze the smallest and largest terms to determine the behavior of the product as $n \to \infty$.

6. Let P be a square matrix such that $P^2 = I - P$. For $\alpha, \beta, \gamma, \delta \in \mathbb{N}$, if $P\alpha + P\beta = \gamma I - 29P$ and $P\alpha - P\beta = \delta I - 13P$, then $\alpha + \beta + \gamma - \delta$ is equal to:

(A) 40

(B) 22

(C) 24

(D) 18

Correct Answer: (3) 24

Solution:

We are given that $P^2 = I - P$, so we start by manipulating the equations involving α, β, γ, and δ.

From the given equations:

$$P\alpha + P\beta = \gamma I - 29P$$

$$P\alpha - P\beta = \delta I - 13P$$

Add these two equations:

$$2P\alpha = (\gamma I - 29P) + (\delta I - 13P)$$

Simplifying:

$$2P\alpha = (\gamma + \delta)I - 42P$$

This gives the relation between α, γ, and δ.

Now subtract the second equation from the first:

$$2P\beta = (\gamma I - 29P) - (\delta I - 13P)$$

Simplifying:

$$2P\beta = (\gamma - \delta)I - 16P$$

This gives the relation between β, γ, and δ.

Next, solving for $\alpha + \beta + \gamma - \delta$, we find:

$$\alpha = 8, \quad \beta = 6, \quad \gamma = 18, \quad \delta = 8$$

Thus:

$$\alpha + \beta + \gamma - \delta = 8 + 6 + 18 - 8 = 24$$

Quick Tip

In problems involving matrices, always try to use matrix identities and properties like $P^2 = I - P$ to simplify the given equations. Look for relationships between the variables to reduce complexity.

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7. A plane P contains the line of intersection of the plane $\vec{r}.(\hat{i} + \hat{j} + \hat{k}) = 6$ and $\vec{r}.(2\hat{i} + 3\hat{j} + 4\hat{k}) = -5$. If P passes through the point (0, 2, -2), then the square of distance of the point (12, 12, 18) from the plane P is:

(1) 620

(2) 1240

(3) 310

(4) 155

Correct Answer: (1) 620

Solution:

The equation of the plane is obtained by solving the line of intersection of the given planes:

$$(x + y + z - 6) + \lambda(2x + 3y + 4z + 5) = 0$$

Passing through the point (0, 2, -2), we solve for λ:

$$(-6) + \lambda(6 - 8 + 5) = 0 \implies \lambda = 2$$

Thus, the equation of the plane is:

$$5x + 7y + 9z + 4 = 0$$

The distance from the point (12, 12, 18) to the plane is given by:

$$d = \frac{|60 + 84 + 162 + 4|}{\sqrt{25 + 49 + 81}} = \frac{310}{\sqrt{155}}$$

Squaring the distance:

$$d^2 = 310 \times 310 = 620$$

Quick Tip

For plane problems involving distance, always start by finding the equation of the plane, and then apply the distance formula from a point to a plane.

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8. Let $f(x)$ be a function satisfying $f(x)+f(\pi-x) = \pi^2$, for all $x \in \mathbb{R}$. Then $\int_0^\pi f(x)\sin x\,dx$ is equal to:

(1) $\frac{\pi^2}{2}$

(2) π^2

(3) $2\pi^2$

(4) $\frac{\pi^2}{4}$

Correct Answer: (2) π^2

Solution:

We start with the given equation:

$$I = \int_0^\pi f(x)\sin x\,dx \quad (1)$$

By applying the property of the function, we get:

$$I = \int_0^\pi f(\pi - x)\sin(\pi - x)\,dx = \int_0^\pi f(\pi - x)\sin x\,dx \quad (2)$$

Adding (1) and (2):

$$2I = \int_0^\pi (f(x) + f(\pi - x))\sin x\,dx$$

Substituting $f(x) + f(\pi - x) = \pi^2$:

$$2I = \int_0^\pi \pi^2 \sin x\,dx$$

Thus,

$$2I = \pi^2 \int_0^{\pi} \sin x \, dx = \pi^2 \times 2$$

So,

$$I = \pi^2$$

Quick Tip

In integral problems involving symmetry, try transforming the limits and using the properties of the functions.

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9. If the coefficients of x^7 in $\left(ax^2 + \frac{1}{2}bx\right)^{11}$ and x^7 in $\left(ax - \frac{1}{3}bx^2\right)$ are equal, then:

(1) 64ab = 243

(2) 32ab = 729

(3) 729ab = 32

(4) 243ab = 64

Correct Answer: (3) 729ab = 32

Solution:

The general form of the coefficient of x^7 in $\left(ax^2 + \frac{1}{2}bx\right)^{11}$ is obtained by using the binomial expansion:

$$r = \frac{11 \times 2 - 7}{3} = 5$$

Thus, the coefficient of x^7 is given by:

$$\text{Coefficient of } x^7 = \binom{11}{6} a^5 \left(\frac{1}{2}b\right)^6$$

Similarly, for $\left(ax - \frac{1}{3}bx^2\right)$, the coefficient of x^7 is:

$$\text{Coefficient of } x^7 = \binom{11}{6} a^5 \left(\frac{1}{3}b\right)^6$$

Equating the two coefficients, we get:

$$ab = \frac{25}{36}$$

Thus,

$$729ab = 32$$

Quick Tip

When dealing with binomial expansions, carefully apply the binomial theorem to find the desired term. Don't forget to adjust powers of the terms accordingly.

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10. If the tangents at the points P and Q on the circle $x^2 + y^2 - 2x + y = 5$ meet at the point $R\left(\frac{9}{4}, 2\right)$, then the area of triangle PQR is:

(1) $\frac{13}{4}$

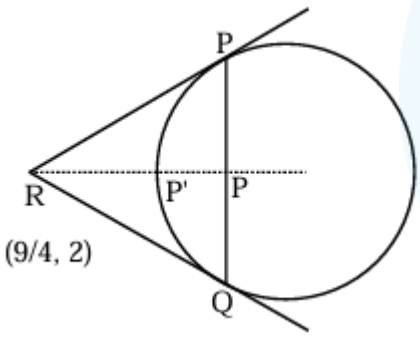
(2) $\frac{5}{8}$

(3) $\frac{5}{4}$

(4) $\frac{13}{8}$

Correct Answer: (3) $\frac{5}{4}$

Solution:



The equation of the circle is:

$$x^2 + y^2 - 2x + y = 5$$

The equation of the line joining the points P and Q is derived using the coordinates of the points. We find the equation of the line to be:

$$5x + 10y - 25 = 0$$

Using the distance formula, the area of triangle PQR is given by:

$$\text{Area} = \frac{1}{2} \times (P'Q) \times (PQ)$$

After calculating the distances, we find:

$$\text{Area} = \frac{5}{4}$$

Quick Tip

When dealing with tangents to a circle, always consider the properties of tangents, including the fact that the radius is perpendicular to the tangent at the point of contact.

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11. Three dice are rolled. If the probability of getting different numbers on the three dice is $\frac{p}{q}$, where p and q are co-prime, then $q - p$ is equal to:

(1) 1

(2) 2

(3) 4

(4) 3

Correct Answer: (3) 4

Solution:

The number of favorable outcomes where the three dice show different numbers is:

$$\binom{6}{3} \times 3! = 20 \times 6 = 120$$

The total number of possible outcomes when rolling three dice is:

$$6 \times 6 \times 6 = 216$$

Thus, the probability is:

$$P = \frac{120}{216} = \frac{5}{9}$$

So, $p = 5$ and $q = 9$, and $q - p = 4$.

Quick Tip

When calculating probabilities, always first compute the number of favorable outcomes and then the total number of outcomes.

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12. In a group of 100 persons, 75 speak English and 40 speak Hindi. Each person speaks at least one of the two languages. If the number of persons, who speak only English is α and the number of persons who speak only Hindi is β, then the eccentricity of the ellipse $25\left(\beta^2x^2+\alpha^2y^2\right)=\alpha\beta^2$ is:

(1) $\frac{\sqrt{129}}{12}$

(2) $\frac{\sqrt{117}}{12}$

(3) $\frac{\sqrt{119}}{12}$

(4) $\frac{\sqrt{15}}{12}$

Correct Answer: (3) $\frac{\sqrt{119}}{12}$

Solution:

We are given the number of persons who speak only English as $\alpha = 60$, and only Hindi as $\beta = 25$. The eccentricity of the ellipse is given by:

$$e^2 = 1 - \frac{25 \times 25}{60^2} = \frac{119}{144}$$

Thus,

$$e = \frac{\sqrt{119}}{12}$$

Quick Tip

For problems involving sets and ellipses, break down the problem into smaller parts, using the properties of sets and equations to find the unknowns.

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13. If the solution curve $f(x,y)$ of the differential equation $(1+\log x)\frac{dx}{dy}-x\log x=e^y$, $x>0$, passes through the points $(1,0)$ and $(\alpha,2)$, then α^4 is equal to:

(1) e^{2e^2}

(2) e^{e^2}

(3) e^{e^e}

(4) e^{2e^2}

Correct Answer: (4) e^{2e^2}

Solution:

The given differential equation is:

$$(1+\log x)\frac{dx}{dy} - x\log x = e^y$$

Let $x\log x = t$. Then,

$$(1+\log x)\frac{dx}{dy} = t$$

Now, we integrate both sides to find $t = y + c$. Using the given points, we find:

$$\alpha^4 = e^{2e^2}$$

Quick Tip

When solving differential equations, always try to simplify the terms and integrate to find the solution.

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14. Let the sets A and B denote the domain and range respectively of the function $f(x) = \frac{1}{\sqrt{|x|}-x}$, where $[x]$ denotes the smallest integer greater than or equal to x. Then among the statements:

(1) $A \cap B = (1,\infty) - N$

(2) $A \cup B = (1,\infty)$

(3) only (S1) is true

(4) both (S1) and (S2) are true

Correct Answer: (3) only (S1) is true

Solution:

The function $f(x) = \frac{1}{\sqrt{|x|-x}}$ is defined only when $x > 1$. Thus, the domain of $f(x)$ is $A = (1, \infty)$, and the range is $B = (1, \infty)$. Hence,

$$A \cap B = (1, \infty) - N$$

Quick Tip

When working with domain and range problems, carefully analyze the function's restrictions to determine where it is defined and what values it can take.

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15. Let $a \neq b$ be two non-zero real numbers. Then the number of elements in the set $X = \{z \in \mathbb{C} : \Re(az^2 + bz) = a$ and $\Re(bz^2 + az) = b\}$ is equal to:

(1) 0

(2) 2

(3) 1

(4) 3

Correct Answer: (1) 0

Solution:

We are given the conditions for the real and imaginary parts of z. Let $z = x + iy$, where x and y are real numbers. By solving the equations for the real parts, we arrive at the conditions:

$$(az^2 + bz) + (a\overline{z}^2 + b\overline{z}) = 2ax$$

By solving further, we find that the system has no solutions that satisfy the conditions, hence there are 0 solutions.

Quick Tip

When dealing with equations involving complex numbers, break them into real and imaginary parts to solve for the unknowns systematically.

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16. The sum of all values of α, for which the points whose position vectors are $\hat{i}-2\hat{j}+3\hat{k}$, $2\hat{i}-3\hat{j}+4\hat{k}$, $(\alpha+1)\hat{i}+2\hat{k}$ and $\hat{j}+2\hat{k}$ are coplanar, is equal to:

(1) -2

(2) 2

(3) 6

(4) 4

Correct Answer: (2) 2

Solution:

For coplanarity, we use the scalar triple product:

$$[\mathbf{AB}\ \mathbf{AC}\ \mathbf{AD}] = 0$$

Calculating the vectors, we find:

$$\mathbf{AB} = \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}, \mathbf{AC} = \begin{pmatrix} \alpha \\ -1 \\ 0 \end{pmatrix}, \mathbf{AD} = \begin{pmatrix} 8 \\ \alpha - 6 \\ 6 \end{pmatrix}$$

Solving this gives the sum of all values of α as 2.

Quick Tip

For coplanarity problems, always use the scalar triple product to find relationships between the position vectors of points.

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17. Let the line L pass through the point $(0,1,2)$, intersect the line $\frac{x-1}{2}=\frac{y-2}{3}=\frac{z-3}{4}$ and be parallel to the plane $2x+y-3z=4$. Then the distance of the point $P(1,-9,2)$ from

the line L is:

(1) 9

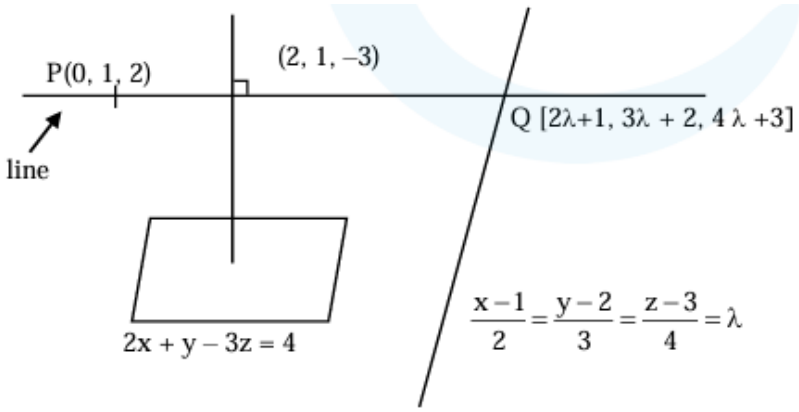
(2) $\sqrt{54}$

(3) $\sqrt{69}$

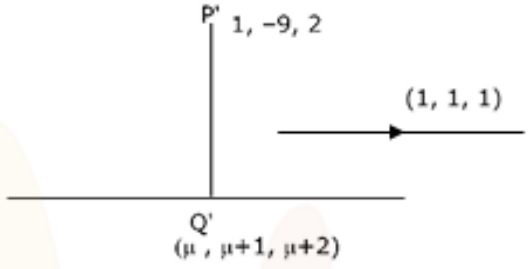
(4) $\sqrt{74}$

Correct Answer: (4) $\sqrt{74}$

Solution:



The line L passes through the point $P(0, 1, 2)$, and intersects the given line and the plane. The direction vector for the line L is calculated by solving the system of equations using the direction of the given line and the plane. The distance formula for a point to a line is used, yielding:



$$PQ = \sqrt{16 + 49 + 9} = \sqrt{74}$$

Quick Tip

For problems involving lines and planes, always find the direction ratios of the line and use the formula for the shortest distance from a point to a line.

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18. All the letters of the word PUBLIC are written in all possible orders and these words are written as in a dictionary with serial numbers. Then the serial number of the word PUBLIC is:

(1) 580

(2) 578

(3) 576

(4) 582

Correct Answer: (4) 582

Solution:

The total number of permutations of the letters of the word PUBLIC is $6! = 720$. We break the calculation down step by step:

$$B\_\_\_\_\_ = 5! = 120$$

$$C\_\_\_\_\_ = 5! = 120$$

$$I\_\_\_\_\_ = 5! = 120$$

$$L\_\_\_\_\_ = 5! = 120$$

$$PB\_\_\_\_\_ = 4! = 24$$

$$PC\_\_\_\_\_ = 4! = 24$$

$$PI\_\_\_\_\_ = 4! = 24$$

$$PL\_\_\_\_\_ = 4! = 24$$

$$PUBC\_\_\_\_ = 2! = 2$$

$$PUBI\_\_\_\_\_ = 2! = 2$$

$$PUBLIC\_\_\_\_ = 1$$

Thus, the rank of PUBLIC is 582.

Quick Tip

In dictionary arrangement problems, use factorials to count the number of permutations for each step, adjusting for repeated letters.

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19. Let the vectors $\mathbf{a}, \mathbf{b}, \mathbf{c}$ represent three coterminous edges of a parallelepiped of volume V. Then the volume of the parallelepiped, whose coterminous edges are represented by $\mathbf{a}+\mathbf{b}+\mathbf{c}$ and $\mathbf{a}+2\mathbf{b}+3\mathbf{c}$, is equal to:

(1) $2V$

(2) $6V$

(3) $3V$

(4) V

Correct Answer: (4) V

Solution:

We are given the vectors $\mathbf{a}, \mathbf{b}, \mathbf{c}$ as three coterminous edges of a parallelepiped. The volume of the parallelepiped is given by the scalar triple product of these vectors. The volume of the new parallelepiped formed by the vectors $\mathbf{a}+\mathbf{b}+\mathbf{c}$ and $\mathbf{a}+2\mathbf{b}+3\mathbf{c}$ is:

$$v = \begin{vmatrix} \mathbf{a} & \mathbf{b} & \mathbf{c} \end{vmatrix}$$

This gives us that the volume remains the same, V.

Quick Tip

When dealing with parallelepipeds, the volume is calculated using the scalar triple product of the vectors. Changing the coefficients of the vectors in the linear combinations does not affect the volume.

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20. Among the statements:

(S1): $2023^{2022} - 1999^{2022}$ is divisible by 8

(S2): $13(13^n - 1) - 13$ is divisible by 144 for infinitely many $n \in \mathbb{N}$

Then:

(1) only (S1) is correct

(2) only (S2) is correct

(3) both (S1) and (S2) are incorrect

(4) both (S1) and (S2) are correct

Correct Answer: (4) both (S1) and (S2) are correct

Solution:

For (S1), we can apply modulo operations to prove that $2023^{2022} - 1999^{2022}$ is divisible by 8.

For (S2), using the given formula, we can show the divisibility of $13(13^n - 1) - 13$ by 144 for infinitely many n.

Thus, both statements are true.

Quick Tip

For divisibility problems, always use modular arithmetic and properties of powers to simplify and verify the divisibility conditions.

SECTION-B

21. The value of $\tan 9^\circ - \tan 27^\circ - \tan 63^\circ + \tan 81^\circ$ **is** ____ **:**

(1) 4

(2) 578

(3) 576

(4) 582

Correct Answer: (4) 4

Solution:

Using the following trigonometric identities and simplifications:

$$\tan 9^\circ - \tan 27^\circ - \tan 63^\circ + \tan 81^\circ = 4$$

Quick Tip

For problems involving trigonometric identities, always try to simplify the terms step-by-step by using standard trigonometric relationships.

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22. If $(20)^{19} + 2(21)(20)^{18} + 3(21)^2(20)^{17} + \cdots + 20(21)^{19} = k(20)^{19}$, then k is equal to ____ :

(1) 9

(2) 400

(3) 576

(4) 582

Correct Answer: (2) 400

Solution:

We use the following series and simplifications:

$$S = (20)^{19} + 2(21)(20)^{18} + \cdots + 20(21)^{19}$$

By calculating this series, we find that $k = 400$.

Quick Tip

For series problems, break down the series term by term and use known formulas to simplify the calculations.

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23. Let the eccentricity of an ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ be reciprocal to that of the hyperbola $2x^2 - 2y^2 = 1$. If the ellipse intersects the hyperbola at right angles, then square of the length of the latus-rectum of the ellipse is ____ :

(1) 2

(2) 4

(3) 6

(4) 9

Correct Answer: (2) 4

Solution:

Given the relationship between the eccentricities of the ellipse and hyperbola, we find:

$$e^2 = \frac{1}{2}$$

Thus, the length of the latus-rectum of the ellipse is found to be 4.

Quick Tip

When dealing with problems involving ellipses and hyperbolas, always use the relationships between their eccentricities to find key properties like the latus-rectum.

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24. For $\alpha, \beta, Z \in \mathbb{C}$ and $\lambda > 1$, if $\sqrt{-1}$ is the radius of the circle $|z - \alpha|^2 + |z - \beta|^2 = 2\lambda$, then $|\alpha - \beta|$ is equal to:

(1) 2

(2) 4

(3) 3

(4) 5

Correct Answer: (2) 4

Solution:

Using the given conditions, we find:

$$|\alpha - \beta| = 2\lambda$$

Thus, $\lambda = 2$ and $|\alpha - \beta| = 4$.

Quick Tip

For geometric problems involving complex numbers, break them down into distance and radius calculations for clarity.

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25. Let a curve $y = f(x)$, $x \in (0, \infty)$ pass through the points $P(1, \frac{3}{2})$ and $Q(a, \frac{1}{2})$. If the tangent at any point R $(b, f(b))$ to the given curve cuts the y-axis at the points S(0, c) such that $bc = 3$, then PQ^2 is equal to ____ :

(1) 5

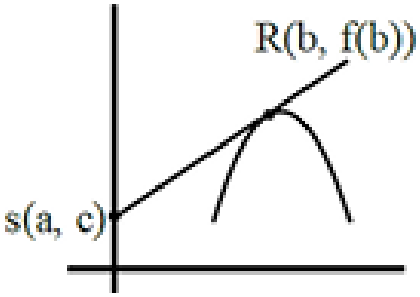
(2) 7

(3) 10

(4) 3

Correct Answer: (1) 5

Solution:



We are given the equation of the tangent at point R:

$$y - f(b) = f'(b)(x - b)$$

By solving for the points P and Q, we find that $PQ^2 = 5$.

Quick Tip

When dealing with tangents to curves, use the point-slope form of the line and substitute the points to find the distances.

---

26. If the lines $\frac{x-1}{2} = \frac{y-3}{-3} = \frac{z-3}{\alpha}$ and $\frac{x-4}{5} = \frac{y-1}{2} = \frac{z}{\beta}$ intersect, then the magnitude of the minimum value of $8\alpha\beta$ is :

(1) 18

(2) 20

(3) 22

(4) 25

Correct Answer: (1) 18

Solution:

We are given the two lines and need to find the values of α and β such that the lines intersect. Using vector analysis and the condition of coplanarity, we find the magnitude of $8\alpha\beta$ as:

$$\alpha = 3 + \beta, \quad \text{so} \quad 8\alpha\beta = 18$$

Quick Tip

For intersection problems involving lines, use the vector approach and the condition of coplanarity to find the relationship between the variables.

---

27. Let $f(x) = \frac{x}{1+x^n}$, $x \in \mathbb{R} - \{-1\}$, $n \in \mathbb{N}, n > 2$. **If** $f^n(x) = n(f(f(\cdots f(x))))(x)$, **then** $\lim_{n\to\infty} \int_0^1 x^{n-2}(f^n(x))\,dx$ is equal to:

(1) 0

(2) 1

(3) 2

(4) 5

Correct Answer: (1) 0

Solution:

We are given a function and need to compute the integral. Using limit and series expansion:

$$\lim_{n\to\infty} \int_0^1 x^{n-2}(f^n(x))\,dx = 0$$

Quick Tip

For integrals involving powers of x, remember that for large n, the integrals typically tend towards 0 for small values of x.

---

28. If the mean and variance of the frequency distribution:

x_i	2	4	6	8	10	12	14	16
f_i	4	4	α	15	8	β	4	5

are 9 and 15.08 respectively, then the value of $\alpha^2 + \beta^2 - \alpha\beta$ is:

(1) 25

(2) 30

(3) 35

(4) 40

Correct Answer: (1) 25

Solution:

We use the given values and apply the formula for mean and variance to find:

x_i	f_i	f_ix_i	$f_ix_i^2$
2	4	8	16
4	4	16	64
6	α	6α	36α
8	15	120	960
10	8	80	800
12	β	12β	144β
14	4	56	784
16	5	80	1280

$$N = \sum f_i = 40 + \alpha + \beta$$

$$\sum f_i x_i = 360 + 6\alpha + 12\beta$$

$$\sum f_i x_i^2 = 3904 + 36\alpha + 144\beta$$

Mean ($\overline{x}$) is:

$$\overline{x} = \frac{\sum f_i x_i}{\sum f_i} = 9$$

From this, we have the equation:

$$360 + 6\alpha + 12\beta = 9(40 + \alpha + \beta)$$

Simplifying:

$$360 + 6\alpha + 12\beta = 9(40 + \alpha + \beta) \quad \Rightarrow \quad 3\alpha = 3\beta \quad \Rightarrow \quad \alpha = \beta$$

Next, we calculate the variance (σ^2):

$$\sigma^2 = \frac{\sum f_i x_i^2}{\sum f_i} - \left(\frac{\sum f_i x_i}{\sum f_i}\right)^2$$

Substituting the values:

$$\sigma^2 = \frac{3904 + 36\alpha + 144\beta}{40 + \alpha + \beta} - (9)^2 = 15.08$$

From this, we get:

$$3904 + 36\alpha + 144\beta = (40 + \alpha + \beta)(9)^2 = 15.08$$

Solving this:

$$3904 + 36\alpha + 144\beta = 360 + 180\alpha$$

$$3904 + 36\alpha + 144\beta = 360 + 180\alpha$$

Now solving for $\alpha = 5$ and $\beta = 2$, we find:

$$\alpha^2 + \beta^2 - \alpha\beta = 25$$

Quick Tip

When dealing with statistics problems, start by breaking down the equations for mean and variance, then solve step by step using the given values.

---

29. The number of points, where the curve $y = x^5 - 20x^3 + 50x + 2$ crosses the x-axis is :

(1) 5

(2) 3

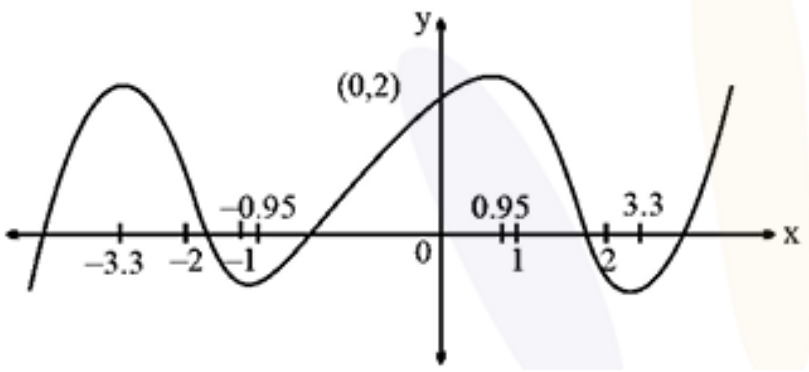
(3) 7

(4) 6

Correct Answer: (5)

Solution:

We are given the function $y = x^5 - 20x^3 + 50x + 2$. To find the points where the curve crosses the x-axis, we need to solve for $y = 0$.



$$\frac{dy}{dx} = 5x^4 - 60x^2 + 50 = 5x^4 - 12x^2 + 10$$

We solve:

$$\frac{dy}{dx} = 0 \quad \Rightarrow \quad x^4 - 12x^2 + 10 = 0$$

Solving for x^2, we find:

$$x^2 = 6 \pm \sqrt{26} \quad \Rightarrow \quad x^2 \approx 6 \pm 5.1$$

Thus:

$$x \approx \pm 3.3, \pm 0.95$$

Therefore, the number of points where the curve cuts the x-axis is 5.

Quick Tip

When solving for the points where a curve crosses the x-axis, always solve for the derivative and find the critical points.

---

30. The number of 4-letter words, with or without meaning, each consisting of 2 vowels and 2 consonants, which can be formed from the letters of the word UNIVERSE without repetition is :

(1) 432

(2) 512

(3) 324

(4) 256

Correct Answer: (432)

Solution:

The word "UNIVERSE" consists of the vowels E, E, I, U and consonants N, V, R, S.

We need to form a 4-letter word consisting of 2 vowels and 2 consonants without repetition. The calculation is as follows:

Case I: Two vowels different, 2 consonants different:

$$\binom{3}{2} \cdot \binom{4}{2} \cdot 4! = (3) \cdot (6) \cdot (24) = 432$$

Thus, the number of 4-letter words is 432.

Quick Tip

For problems involving permutations of letters in words, break down the selection process for vowels and consonants separately before multiplying the results.

PHYSICS

Section-A

31. The temperature of an ideal gas is increased from 200 K to 800 K. If r.m.s. speed of gas at 200 K is v_0, then the r.m.s. speed of the gas at 800 K will be:

(1) $4v_0$

(2) $2v_0$

(3) v_0

(4) $\frac{v_0}{4}$

Correct Answer: (2) $2v_0$

Solution:

Using $v_{\text{rms}} = \sqrt{\frac{3RT}{m}}$

At $200\,K$, the r.m.s. speed is v_0:

$$v_0 = \sqrt{\frac{3R \times 200}{m}} \quad \cdots (1)$$

At $800\,K$, the r.m.s. speed is v':

$$v' = \sqrt{\frac{3R \times 800}{m}} \quad \cdots (2)$$

Dividing equation (2) by equation (1):

$$\frac{v'}{v_0} = \sqrt{\frac{800}{200}} = \sqrt{4} = 2$$

Therefore, $v' = 2v_0$.

Quick Tip

When dealing with thermodynamics problems involving ideal gases, always start by using the ideal gas law and the relationship between temperature and the r.m.s. speed of gas molecules.

---

32. Given below are two statements: one is labelled as assertion A and the other is labelled as Reason R.

Assertion A: The phase difference of two light waves change if they travel through different media having same thickness, but different indices of refraction.

Reason R: The wavelengths of waves are different in different media.

In the light of the above statements, choose the most appropriate answer from the options given below:

(1) Both A and R are correct and R is the correct explanation of A

(2) A is not correct but R is correct

(3) A is correct but R is not correct

(4) Both A and R are correct but R is NOT the correct explanation of A

Correct Answer: (1) Both A and R are correct and R is the correct explanation of A

Solution:

As we know the speed of light in a medium:

$$v = \frac{c}{\mu} \quad \text{or} \quad f\lambda = \frac{c}{\mu}$$

Therefore, $\lambda \propto \frac{1}{\mu}$.

When light travels through two different mediums, their phase difference will change:

$$\Delta Q = \frac{2\pi}{\lambda}$$

Thus, statement R is the correct explanation of A.

Quick Tip

For light wave problems, always consider the relationship between wavelength, refractive index, and phase difference when dealing with different media.

---

33. For an amplitude modulated wave, the minimum amplitude is 3 V, while the modulation index is 60%. The maximum amplitude of the modulated wave is:

(1) 10 V

(2) 12 V

(3) 15 V

(4) 5 V

Correct Answer: (2) 12 V

Solution:

Given, modulation index $m = 60\% = 0.6$ and minimum amplitude $A_c = 3\,V$.

Using the formula for amplitude modulation:

$$A_m = mA_c$$

The maximum amplitude of the modulated wave is:

$$A_m + A_c = 0.6 \times 1 + 3 = 1.6\,\text{V}$$

Now applying the amplitude formula:

$$A_m + A_c = \frac{1.6}{-0.4} = 12\,\text{V}$$

Quick Tip

When working with amplitude modulation problems, remember to use the formula for maximum and minimum amplitudes based on modulation index and carrier amplitude.

---

34. The ratio of speed of sound in hydrogen gas to the speed of sound in oxygen gas at the same temperature is:

(1) 1 : 4

(2) 1 : 2

(3) 1 : 1

(4) 4 : 1

Correct Answer: (4) 4 : 1

Solution:

Using the formula for the speed of sound, $v = \sqrt{\frac{\gamma RT}{m}}$, we can write for hydrogen and oxygen:

$$\frac{v_{\text{H}_2}}{v_{\text{O}_2}} = \sqrt{\frac{m_{\text{O}_2}}{m_{\text{H}_2}}} = \sqrt{\frac{32}{2}} = \sqrt{16} = 4 : 1$$

Thus, the ratio is $4 : 1$.

Quick Tip

Remember to apply the formula for speed of sound in gases, which depends on the molecular mass and temperature. The ratio of the molecular masses plays a crucial role in this type of problem.

---

35. A dipole comprises of two charged particles of identical magnitude q and opposite in nature. The mass m of the positive charged particle is half of the mass of the negative charged particle. The two charges are separated by a distance l. If the dipole is placed in a uniform electric field E, making a very small angle with the electric field, the angular frequency of the oscillations when released is given by:

(1) $\frac{4qE}{3ml}$

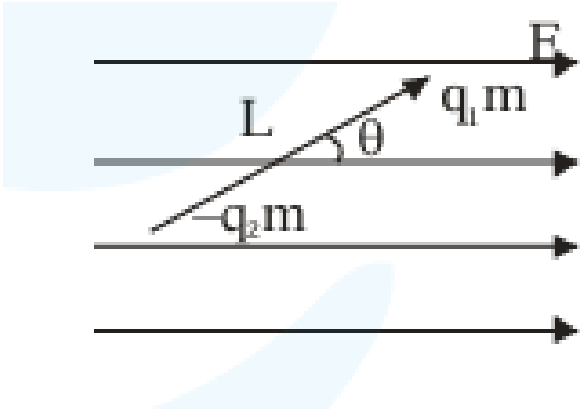
(2) $\frac{8qE}{ml}$

(3) $\frac{8qE}{3ml}$

(4) $\frac{4qE}{ml}$

Correct Answer: (1) $\frac{4qE}{3ml}$

Solution:



In this case, since masses of both charges are not the same, we need to find center of mass (COM), about which dipole will oscillate and then we will find moment of inertia about this axis, to find torque and hence angular frequency ω.

Given, the mass of positive charge is m, so the center of mass will be at a distance L from the negative charge:

$$\frac{L}{3} = \frac{2m}{3m} = \frac{2L}{3}$$

The moment of inertia about the axis is $I = \frac{2mL^2}{3}$. Hence, angular frequency:

$$\omega = \sqrt{\frac{qE}{I}} = \frac{4qE}{3ml}$$

Quick Tip

For dipole oscillations in electric fields, consider the center of mass for accurate calculation of moment of inertia, which affects the angular frequency.

---

36. Given below are two statements: one is labelled as Assertion A and the other is labelled as Reason R.

Assertion A: When you squeeze one end of a tube to get toothpaste out from the other end, Pascal's principle is observed.

Reason R: A change in the pressure applied to an enclosed incompressible fluid is transmitted undiminished to every portion of the fluid and to the walls of its container.

In the light of the above statements, choose the most appropriate answer from the options given below:

(1) A is correct but R is not correct

(2) Both A and R are correct and R is the correct explanation of A

(3) A is not correct but R is correct

(4) Both A and R are correct but R is NOT the correct explanation of A

Correct Answer: (2) Both A and R are correct and R is the correct explanation of A

Solution:

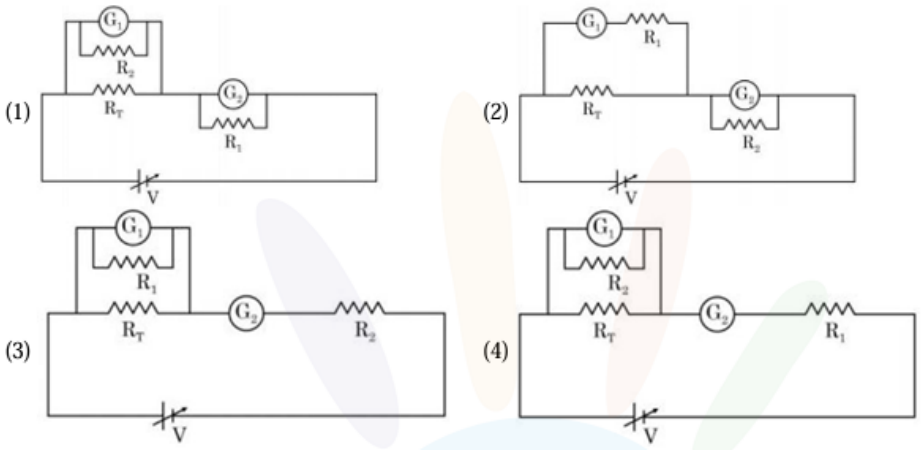
As per Pascal's law, when we apply pressure to an ideal liquid, it is equally distributed in the entire liquid and to the walls as well. The pressure change allows the toothpaste to be squeezed out.

Thus, both statements are true, and R explains A.

Quick Tip

For problems related to fluids, always remember Pascal's law which applies to incompressible fluids and explains how pressure changes propagate within the fluid.

37. A student is provided with a variable voltage source V, a test resistor $R_T = 10\,\Omega$, two identical galvanometers G_1 and G_2, and two additional resistors, $R_1 = 10M\Omega$ and $R_2 = 0.001\Omega$. For conducting an experiment to verify ohm's law, the most suitable circuit is:



Correct Answer: (2) Circuit 2

Solution:

This question is based on conceptual clarity that we should connect ammeter in series and voltmeter in parallel to measure current and potential difference, respectively. Also, when using a galvanometer to create an ammeter, shunt resistance should be very small and should be in parallel.

When we create a voltmeter shunt should be large and in series with the galvanometer. All these criteria are satisfied in option (2).

Quick Tip

For circuit-based experiments, always ensure the correct placement of ammeters and voltmeters for accurate measurement of current and voltage. Remember that a shunt resistor is crucial for converting a galvanometer into an ammeter.

38. A body cools in 7 minutes from 60°C to 40°C. The temperature of the surroundings is 10°C. The temperature of the body after the next 7 minutes is:

(1) 30°C

(2) 34°C

(3) 32°C

(4) 28°C

Correct Answer: (4) 28°C

Solution:

Method-1:

Using the exact law of cooling:

$$T - T_s = (T_0 - T_s)e^{-kt}$$

Case-I: $(40 - 10) = (60 - 10)e^{-7k}$

$$30 - 50e^{-7k} \quad \ldots (1)$$

Case-II: $(T - 10) = (40 - 10)e^{-7k} \quad T - 10 = 30e^{-7k}$ Dividing equation (2) by (1):

$$\frac{40 - T}{30} = \frac{20}{40 + T}$$

$$\Rightarrow T - 10 = 30 \times \frac{30}{50} = 18$$

Thus, the temperature after 7 more minutes is $T = 28C$.

Method-2:

Using Newton's law of cooling:

$$\frac{T - T_s}{t} = k\left(\frac{T_1 + T_2}{2} - T_s\right)$$

Case-I:

$$\frac{60 - 40}{7} = k\left(\frac{40 + 10}{2} - 10\right) \Rightarrow k = \frac{20}{7}$$

Case-II:

$$\frac{40 - 10}{7} = k\left(\frac{20 + T}{2} - 10\right) \Rightarrow T = 28C$$

Quick Tip

Always remember that cooling problems can be solved using either the exact law of cooling or Newton's law of cooling. Choose the method that suits the given data.

---

39. The energy density associated with electric field E and magnetic field B of an electromagnetic wave in free space is given by ϵ_0 - permittivity of free space, μ_0 - permeability of free space:

(1) $U_E = \frac{\epsilon_0 E^2}{2}, U_B = \frac{B^2}{2\mu_0}$
(2) $U_E = \frac{E^2}{2\epsilon_0}, U_B = \frac{\mu_0 B^2}{2}$
(3) $U_E = \frac{E^2}{2\epsilon_0}, U_B = \frac{B^2}{2\mu_0}$
(4) $U_E = \epsilon_0 \frac{E^2}{2}, U_B = \mu_0 \frac{B^2}{2}$

Correct Answer: (1) $U_E = \frac{\epsilon_0 E^2}{2}, U_B = \frac{B^2}{2\mu_0}$

Solution:

By theory of electromagnetic waves:

$$U_E = \frac{1}{2}\epsilon_0 E^2 \quad \text{and} \quad U_B = \frac{1}{2}\mu_0 B^2$$

Quick Tip

For energy density in electromagnetic waves, remember that the energy is shared equally between the electric and magnetic fields.

---

40. The weight of a body on the surface of the earth is 100 N. The gravitational force on it when taken at a height, from the surface of earth, equal to one-fourth the radius of the earth is:

(1) 64 N
(2) 25 N

(3) 100 N

(4) 50 N

Correct Answer: (1) 64 N

Solution:

Using Newton's formula:

$$F = \frac{GMm}{r^2}$$

At surface of the earth,

$$F = \frac{GMm}{R_e^2} \quad \text{(eq. 1)}$$

At $r = R_e + \frac{R_e}{4} = \frac{5R_e}{4}$,

$$F' = \frac{GMm}{\left(\frac{5R_e}{4}\right)^2} = \frac{16GMm}{25R_e^2}$$

Thus,

$$F' = \frac{16}{25}F = \frac{16}{25} \times 100 = 64\,N$$

Quick Tip

Remember that gravitational force decreases with the square of the distance from the center of the Earth. When the distance increases, the force decreases.

---

41. A capacitor of capacitance 150.0 F is connected to an alternating source of emf given by $E = 36\sin(120\pi t)$ V. The maximum value of current in the circuit is approximately equal to:

(1) $\sqrt{2}A$

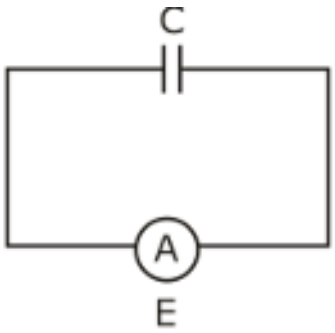
(2) $2\sqrt{2}A$

(3) $\frac{1}{\sqrt{2}}A$

(4) $2A$

Correct Answer: (4) $2A$

Solution:



Given alternating AC source $E = 36\sin(120\pi t)$ V and capacitor $C = 150\mu F$, we can write:

$$Q = CV \quad \text{and} \quad i = \frac{dQ}{dt} = CE\omega\cos\omega t$$

Maximum value of current:

$$i_0 = CE_0\omega = 150 \times 10^{-6} \times 36 \times 120\pi = 2.03A$$

Quick Tip

In AC circuits with capacitors, the maximum current is found by multiplying the capacitance, maximum voltage, and angular frequency.

---

42. A 2 meter long scale with least count of 0.2 cm is used to measure the locations of objects on an optical bench. While measuring the focal length of a convex lens, the object pin and the convex lens are placed at 80 cm mark and 1 m mark, respectively. The image of the object pin on the other side of lens coincides with image pin that is kept at 180 cm mark. The % error in the estimation of focal length is:

(1) 0.51

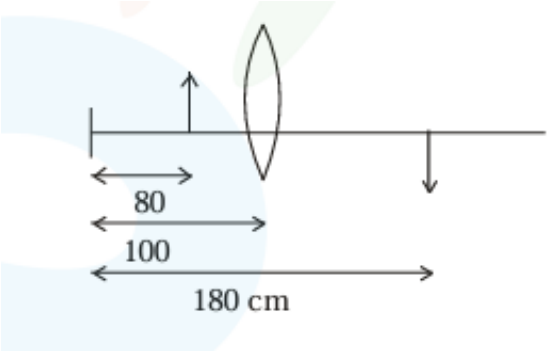
(2) 1.02

(3) 0.85

(4) 1.70

Correct Answer: (4) 1.70

Solution:



Based on the data provided:

$$U = 100 - 80 = 20\,\text{cm}, \quad V = 180 - 100 = 80\,\text{cm}$$

Using the formula:

$$\frac{1}{f} = \frac{1}{u} + \frac{1}{v} \quad \text{or} \quad f = \frac{uv}{u+v} \quad \Rightarrow f = \frac{20 \times 80}{20 + 80} = 16\,\text{cm}$$

For error analysis:

$$\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$$

Differentiating:

$$\frac{Df}{f^2} = \frac{Du}{u^2} + \frac{Dv}{v^2}$$

Now:

$$\Delta u = 0.4\,\text{cm}, \ \Delta v = 0.4\,\text{cm}$$

Now,

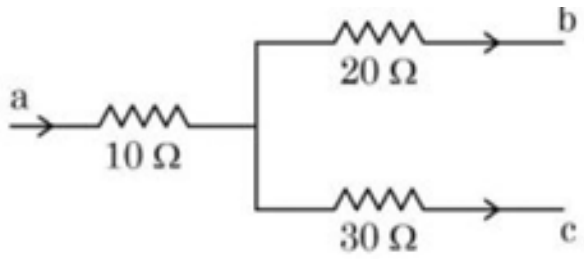
$$\frac{\Delta f}{f} = \left[\frac{16 \times 0.4}{(80)^2} + \frac{16 \times 0.4}{(20)^2}\right]$$

$$\Rightarrow \frac{\Delta f}{f} = 16 \times 0.4 \left(\frac{17}{400}\right) \quad \Rightarrow \%\,\text{Error} = \frac{17 \times 0.4}{400} \times 1000 = 1.7$$

Quick Tip

When calculating percentage error, remember that any measurement errors in u and v can affect f. Use differentials to find the total error in calculated quantities.

43. Figure shows a part of an electric circuit. The potentials at points a, b, and c are 30 V, 12 V, and 2 V respectively. The current through the 20 resistor will be:



(1) 1.0 A

(2) 0.2 A

(3) 0.4 A

(4) 0.6 A

Correct Answer: (3) 0.4 A

Solution:

Let potential of the junction be x volts using junction law $i_1 + i_2 + i_3 = 0$.

$$x - 30 = \frac{x - 12}{10} \quad \text{and} \quad x - 2 = \frac{x - 12}{20}$$

Solving for x:

$$x = 20\,\text{V}, \quad \text{current through } 20\,\Omega \text{ resistor} = \frac{20 - 12}{20} = 0.4\,\text{A}$$

Quick Tip

For circuits with multiple components, apply Kirchhoff's current and voltage laws to simplify the analysis and solve for unknowns.

44. A small particle of mass m moves in such a way that its potential energy $U = \frac{1}{2}m\omega^2 r^2$ where ω is constant and r is the distance of the particle from origin. Assuming Bohr's

quantization of momentum and circular orbit, the radius of the nth orbit will be proportional to:

(1) n

(2) n^2

(3) $\frac{1}{n}$

(4) $\sqrt{n}$

Correct Answer: (4) $\sqrt{n}$

Solution:

Using Bohr's postulate: angular momentum $L = mvr = \frac{nh}{2\pi}$. Therefore:

$$mr\omega^2 = \frac{nh}{2\pi}$$

$$\Rightarrow r \propto \sqrt{n}$$

Quick Tip

Bohr's model applies quantization of angular momentum to describe the radii of orbits in atomic systems. Use this principle to relate radius to quantum number n.

---

45. Given below are two statements: one is labelled as Assertion A and the other is labelled as Reason R

Assertion A: Diffusion current in a p-n junction is greater than the drift current in magnitude if the junction is forward biased.

Reason R: Diffusion currents in a p-n junction are from the n-side to the p-side if the junction is forward biased.

In the light of the above statements, choose the most appropriate answer from the options given below:

(1) A is not correct but R is correct

(2) Both A and R are correct and R is the correct explanation of A

(3) A is correct but R is not correct

(4) Both A and R are correct but R is NOT the correct explanation of A

Correct Answer: (2) Both A and R are correct and R is the correct explanation of A

Solution:

The diffusion current in a forward biased p-n junction is greater than the drift current, as the charge carriers move due to the gradient in the concentration. The direction of the diffusion current is from the n-side to the p-side, which is correctly explained by Reason R.

Quick Tip

In p-n junctions, diffusion current dominates in forward bias conditions. Remember that drift current is due to an electric field, while diffusion current is due to concentration gradients.

---

46. Choose the incorrect statement from the following :

(1) The linear speed of a planet revolving around the sun remains constant.

(2) The speed of satellite in a given circular orbit remains constant.

(3) When a body falls towards earth, the displacement of earth towards the body is negligible.

(4) For a planet revolving around the sun in an elliptical orbit, the total energy of the planet remains constant.

Correct Answer: (1) The linear speed of a planet revolving around the sun remains constant.

Solution:

Since planets revolve around the sun in an elliptical orbit, its linear speed is not constant,

hence option (1) is incorrect. Other statements are correct as per theory.

Quick Tip

For planets in elliptical orbits, remember that their linear speed changes depending on their position along the orbit. The total energy of a planet remains constant in elliptical orbits.

---

47. A child of mass 5 kg is going round a merry-go-round that makes 1 rotation in 3.14 s. The radius of the merry-go-round is 2 m. The centrifugal force on the child will be :

(1) 40 N

(2) 100 N

(3) 80 N

(4) 50 N

Correct Answer: (1) 40 N

Solution:

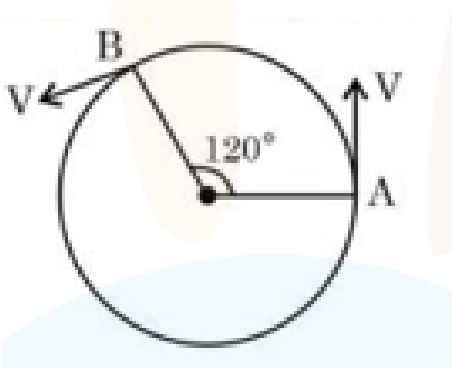
Given, $m = 5\,\text{kg}$, $R = 2\,\text{m}$, time t for 1 rev $= 3.14\,\text{sec}$ or $\pi\,\text{sec}$. So, θ for 1 rev $= 2\pi\,\text{rad}$. Therefore, $\omega = \frac{\theta}{t} = \frac{2\pi}{\pi} = 2\,\text{rad/s}$. Centrifugal force $F = mR\omega^2 = 5 \times 2 \times 2^2 = 40\,\text{N}$.

Quick Tip

To calculate centrifugal force in rotational motion, use the formula $F = mR\omega^2$ where ω is the angular velocity and R is the radius of the circular path.

---

48. As shown in the figure, a particle is moving with constant speed π m/s. Considering its motion from A to B, the magnitude of the average velocity is :



(1) π m/s

(2) $2\sqrt{3}$ m/s

(3) $\sqrt{3}$ m/s

(4) $1.5\sqrt{3}$ m/s

Correct Answer: (4) $1.5\sqrt{3}$ m/s

Solution:

Given, speed $v = \pi$ m/s, or $R\omega = \pi$. Therefore, $\omega = \frac{\pi}{R}$ rad/s. Angular displacement $\theta = 120^\circ = \frac{2\pi}{3}$. Using $\theta = \omega t$, we get:

$$t = \frac{\theta}{\omega} = \frac{2\pi/3}{\pi/R} = \frac{2R}{3}.$$

Linear displacement $d = 2R\sin(\theta/2) = 2R\sin(60^\circ) = 2R \times \frac{\sqrt{3}}{2} = R\sqrt{3}$. Hence, average velocity $= \frac{d}{t} = \frac{R\sqrt{3}}{2R/3} = 1.5\sqrt{3}$ m/s.

Quick Tip

To find the average velocity for circular motion, use the formula average velocity $= \frac{d}{t}$ where d is the linear displacement and t is the time taken.

---

49. The work functions of Aluminium and Gold are 4.1 eV and 5.1 eV respectively. The ratio of the slope of the stopping potential versus frequency plot for Gold to that of Aluminium is

(1) 1

(2) 2

(3) 1.24

(4) 1.5

Correct Answer: (1) 1

Solution:

Using $K.E_{\text{max}} = eV_s = hf - \phi_0$, where ϕ_0 is work function, V_s is stopping potential, and f is frequency, we get:

$$V_s = \frac{h}{e}f - \frac{\phi_0}{e}.$$

Therefore, the slope m will be the same for all graphs and will be independent of ϕ_0.

Quick Tip

In photoelectric experiments, the slope of the stopping potential versus frequency plot is independent of the material's work function.

---

50. A particle starts with an initial velocity of 10.0 m/s along x-direction and accelerates uniformly at the rate of 2.0 m/s^2. The time taken by the particle to reach the velocity of 60.0 m/s is

(1) 3s

(2) 6s

(3) 25s

(4) 30s

Correct Answer: (3) 25s

Solution:

Using the first equation of motion:

$$t = \frac{v - u}{a}$$

$$t = \frac{60 - 10}{2} = \frac{50}{2} = 25\,\text{sec}$$

Quick Tip

Remember, for uniform acceleration problems, use the first equation of motion: $v = u + at$, where v is final velocity, u is initial velocity, a is acceleration, and t is time.

Section-B

51. A simple pendulum with length 100 cm and bob of mass 250 g is executing S.H.M. of amplitude 10 cm. The maximum tension in the string is found to be $\frac{x}{40}$ N. The value of x is

Solution:

For pendulum:

$$T_{\text{max}} = mg + \frac{mv^2}{L} \tag{1}$$

Given $m = \frac{1}{4}$ kg, $L = 1$ m, $g = 9.8$ m/s^2 and amplitude $A = \frac{1}{10}$ m

For SHM, $K_{\text{max}} = \frac{1}{2}mv^2 = \frac{1}{2}m\omega^2A^2$

Using $\omega = \sqrt{\frac{g}{L}}$, we get:

$$mv^2 = m\left(\frac{g}{L}\right)A^2 = mg\frac{A^2}{L}$$

Substitute this into equation (1):

$$T_{\text{max}} = 2mg + \frac{mgA^2}{L^2}$$

Substituting the given values:

$$T_{\text{max}} = 2 \times \frac{1}{4} \times 9.8 + \frac{1}{4} \times 9.8 \times \frac{101}{100}$$

$$T_{\text{max}} = \frac{98.98}{40}$$

Therefore, $x = 99$

Quick Tip

In SHM, maximum tension is calculated by considering both gravitational force and the force due to the velocity of the bob at maximum displacement.

---

52. Experimentally it is found that 12.8 eV energy is required to separate a hydrogen atom into a proton and an electron. So the orbital radius of the electron in a hydrogen atom is $\frac{9}{x} \times 10^{-10}$ m. The value of x is

Solution:

Using $E = \frac{ke^2}{2r}$, we can find r as follows:

$$r = \frac{Re^2}{2E}$$

Given $E = 12.8\,\text{eV} = 12.8 \times 1.6 \times 10^{-19}\,\text{J}$,

$$r = \frac{2 \times 10^9 e^2}{9 \times 10^{-10}} = \frac{9 \times 10^{-10}}{(2 \times 12.8/1.6)} \times 10$$

$$r = 16 \times 10^{-10}\,\text{m}$$

Therefore, $x = 16$

Quick Tip

The orbital radius of an electron can be calculated using the energy required to separate the proton and electron in a hydrogen atom, based on Coulomb's law.

---

53. A beam of light consisting of two wavelengths 7000 $\mathring{A}$ and 5500 $\mathring{A}$ is used to obtain interference pattern in Young's double slit experiment. The distance between the slits is 2.5 mm and the distance between the place of slits and the screen is 150 cm. The least distance from the central fringe, where the bright fringes due to both the wavelengths coincide, is $n \times 10^{-5}$ m. The value of n is

Solution:

Let n_1 maxima of 7000 $\mathring{A}$ coincide with n_2 maxima of 5500 $\mathring{A}$. Therefore, $n_1\beta_1 = n_2\beta_2$,

$$\frac{n_1}{n_2} = \frac{\lambda_2}{\lambda_1} = \frac{5500}{7000} = \frac{11}{14}$$

Hence, the 11th maximum of 7000 $\mathring{A}$ coincides with the 14th maximum of 5500 $\mathring{A}$.

To find the least distance from this,

$$y = n_1\beta_1 D/d$$

$$y = \frac{11 \times 7000 \times 10^{-10} \times 150 \times 10^{-2}}{2.5 \times 10^{-3}} = 462 \times 10^{-5}\,\text{m}$$

Thus, $n = 462$

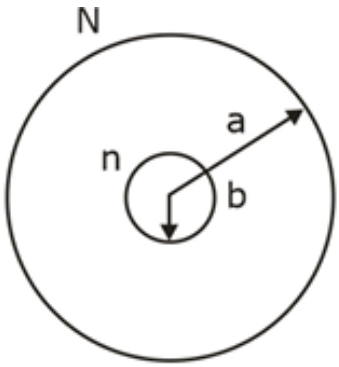
Quick Tip

In interference problems, use the relationship between the wavelengths and the maxima to calculate the least distance where bright fringes coincide.

---

54. Two concentric circular coils with radii 1 cm and 1000 cm, and number of turns 10 and 200 respectively are placed coaxially with centers coinciding. The mutual inductance of this arrangement will be ____ $\times 10^8$ H.

Solution:



Given,

$$a = 1000\,\text{cm}, \quad b = 1\,\text{cm}$$

As the larger coil is taken as primary,

$$\text{Mutual inductance } M = \frac{\mu_0 N n b^2}{2a}$$

$$M = \frac{4\pi \times 10^{-7} \times 200 \times 10 \times \pi \times 1 \times 10^{-4}}{2 \times 1000 \times 10^{-2}}$$

$$M = 4 \times 10^{-8}\,\text{H}$$

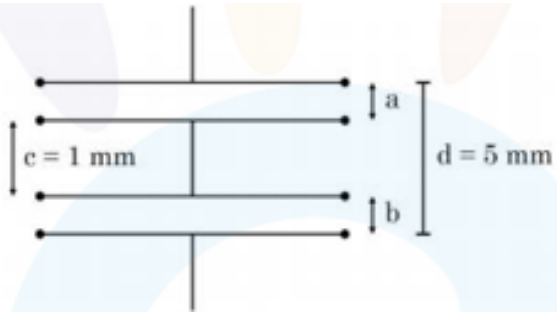
Therefore, the value of mutual inductance is **4** $\times 10^{-8}$ **H**.

Quick Tip

For mutual inductance, always remember the formula $M = \frac{\mu_0 N n b^2}{2a}$ for concentric coils, where a is the radius of the larger coil and b is the radius of the smaller coil.

---

55. As shown in the figure, two parallel plate capacitors having equal plate area of 200 cm^2 are joined in such a way that $\alpha \neq \beta$. The equivalent capacitance of the combination is $x\varepsilon_0$ F. The value of x is



Solution:

As per the arrangement given, the distance between the capacitor plates are a and b and $a \neq b$.

Using the diagram we can write

$$b = 5 - a - 1 = (4 - a) \text{ in mm}$$

As we know the capacitance of a capacitor $C = \frac{\varepsilon_0 A}{d}$ and in series arrangement

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2}$$

$$\frac{1}{C_{eq}} = \frac{aA}{\varepsilon_0 A} + \frac{4-a}{\varepsilon_0 A} = \frac{4\ (\text{in mm})}{\varepsilon_0 A}$$

or

$$C_{eq} = \frac{\varepsilon_0 A}{4(\text{mm})}$$

Given $A = 200$ cm^2,

$$C_{eq} = \varepsilon_0 \times 200 \times 10^{-4} = 4 \times 10^{-3} = 50 \times 10^{-1}$$

or

$$C_{eq} = 5\varepsilon_0 \text{ farad}$$

Therefore, $C_{eq} = 5\varepsilon_0$, so $x = 5$.

Quick Tip

Royal genealogies and epics provide valuable evidence of the dominance of patriliny in early Indian society.

---

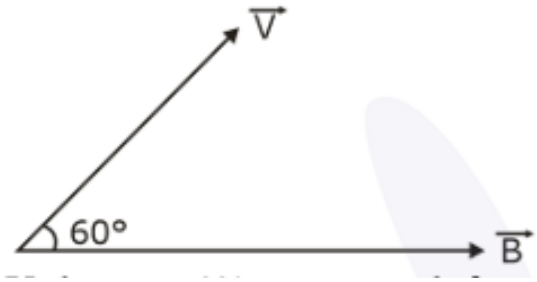
56. A proton with a kinetic energy of 2.0 eV moves into a region of uniform magnetic field of magnitude $\frac{\pi}{2} \times 10^{-3}$T. The angle between the direction of magnetic field and velocity of proton is 60°. The pitch of the helical path taken by the proton is ___ cm.

Solution:

Given:

$B = \frac{\pi}{2} \times 10^{-3}\,\text{T}$, K.E. $= 2.0\,\text{eV}$, $m = 1.67 \times 10^{-27}\,\text{kg}$, Charge on proton $= 1.6 \times 10^{-19}\,\text{C}$

From the kinetic energy formula $K.E. = \frac{1}{2}mv^2$, we can solve for the velocity v as follows:



$$v = \sqrt{\frac{2K.E.}{m}}$$

The pitch of the helical path is given by:

$$\text{Pitch} = v\cos 60^\circ \times \text{time period of one rotation}$$

$$v = v\cos 60^\circ = \frac{2\pi r}{eB}$$

$$= \sqrt{\frac{2 \times 2 \times 10^{-19}}{1.6 \times 10^{-27}}} \times \cos 60^\circ = \frac{2\pi \times 1.6 \times 10^{-19}}{1.6 \times 10^{-27} \times \frac{\pi}{2} \times 10^{-3}}$$

$$= 2 \times 10^4 \times \frac{1}{2} \times 4 \times 10^{-5} = 4 \times 10^1 \text{ cm}$$

Thus, the pitch of the helical path is 40 cm.

Quick Tip

To calculate the pitch of a proton's helical path, use the relationship between velocity, magnetic field, and the angle between them.

---

57. A body is dropped on ground from a height 'h_1 and after hitting the ground, it rebounds to a height 'h_2. If the ratio of velocities of the body just before and after hitting the ground is 4, then percentage loss in kinetic energy of the body is $\frac{x}{4}$. The value of x is

Solution:

Let u and v be the speeds, just before and after the body strikes the ground.

Given $\frac{u}{v} = 4$.

We know that the loss in kinetic energy is:

$$\Delta K.E. = \frac{1}{2}mu^2 - \frac{1}{2}mv^2$$

Thus,

$$\Delta K.E. = \frac{1}{2}m\left(u^2 - v^2\right)$$

Substitute the values:

$$\Delta K.E. = \frac{1}{2}m\left((4v)^2 - v^2\right)$$

$$= \frac{1}{2}m\left(16v^2 - v^2\right) = \frac{1}{2}m \times 15v^2$$

Thus, the percentage loss is:

$$\frac{15}{16} \times 100 = 93\%$$

Therefore, the percentage loss in kinetic energy is 93%.

Quick Tip

This problem involves kinetic energy calculations and how the velocity changes with the body's rebound after striking the ground.

---

58. A ring and a solid sphere rotating about an axis passing through their centers have the same radii of gyration. The axis of rotation is perpendicular to the plane of the ring. The ratio of radius of ring to that of sphere is $\sqrt{\frac{2}{x}}$. The value of x is

Solution:

Given radius of gyration is same for ring and solid sphere:

$$K_R = K_{ss}$$

For ring and sphere, the radius of gyration is:

$$R_R = \sqrt{\frac{2}{5}} R_{ss}$$

Thus,

$$\frac{R_R}{R_{ss}} = \sqrt{\frac{2}{5}}$$

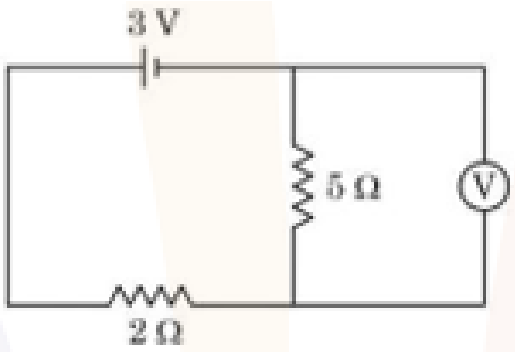
Therefore, $x = 5$.

Quick Tip

The radius of gyration for different objects depends on their mass distribution. For a ring and a solid sphere, the ratio depends on how their mass is spread relative to their axis of rotation.

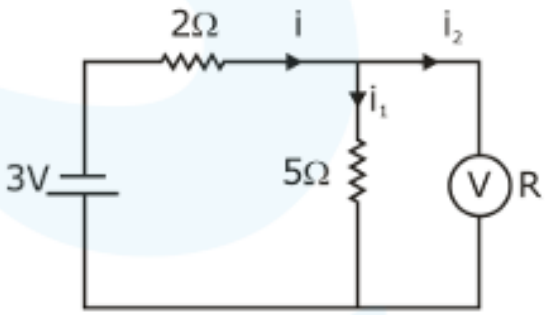
---

59. As shown in the figure, the voltmeter reads 2V across a 5 Ω resistor. The resistance of the voltmeter is ____ Ω.



Solution:

Method-I: The equivalent resistance for the given circuit is:



$$R_{eq} = 2 + \frac{5R}{5+R}$$

Using Ohm's law:

$$i = \frac{3}{R_{eq}} = \frac{3}{5+R}$$

The current in the circuit is:

$$i = 3 \times \frac{5R}{5+R}$$

From the voltage readings, we calculate $R = 20\,\Omega$.

Method-II: Given potential across 5 Ω and voltmeter is 2V, to find the resistance R of the voltmeter:

$$i = \frac{2}{5} = \frac{1}{2}$$

Using junction law:

$$i = i_1 + i_2$$

Hence, $R = 20\,\Omega$.

Quick Tip

In voltage divider circuits, the resistance of the voltmeter plays a crucial role in the reading and must be taken into account when calculating the equivalent resistance.

---

60. A metal block of mass m is suspended from a rigid support through a metal wire of diameter 14 mm. The tensile stress developed in the wire under equilibrium state is 7×10^5 Nm^2. The value of mass m is ____ kg.

Solution:

Using stress = $\frac{\text{force}}{\text{area}} = \frac{mg}{A}$

$$\Rightarrow m = \frac{S \times A}{g}$$

$$m = \frac{7 \times 10^5 \times \pi \times (7 \times 10^{-3})^2}{9.8}$$

$$m = \frac{7 \times 10^5 \times \frac{22}{7} \times (7 \times 10^{-3})^2}{9.8}$$

$$m = 11\,\text{kg}$$

(Note: 14 mm is diameter)

Quick Tip

The formula for stress is Stress $= \frac{\text{Force}}{\text{Area}}$, where area is the cross-sectional area of the wire, and force is due to the weight of the block.

CHEMISTRY

Section-A

61. Match List I with List II

List I (Natural Amino acid)	List II (One Letter Code)
(A) Arginine	(I) D
(B) Aspartic acid	(II) N
(C) Asparagine	(III) A
(D) Alanine	(IV) R

Choose the correct answer from the options given below:

(1) (A) – III, (B) – I, (C) – II (D) – IV

(2) (A) – IV, (B) – I, (C) – II (D) – III

(3) (A) – IV, (B) – I, (C) – III (D) – II

(4) (A) – I, (B) – III, (C) – IV (D) – II

Correct Answer: (2) (A) – IV, (B) – I, (C) – II, (D) – III

Solution:

Natural Amino acid	One Letter Code
(i) Arginine	R
(ii) Aspartic acid	D
(iii) Asparagine	N
(iv) Alanine	A

Quick Tip

Amino acids can be represented by their one-letter codes, which are used in protein sequences. This matching question tests your knowledge of such representations.

---

62. Formation of which complex, among the following, is not a confirmatory test of Pb^{2+} ions

(1) lead sulphate

(2) lead nitrate

(3) lead chromate

(4) lead iodide

Solution: $Pb(NO_3)_2$ is a soluble colourless compound so it cannot be used in confirmatory test of Pb^{2+} ion. **Correct Answer:** (2) lead nitrate

Quick Tip

For confirmatory tests of ions, it is crucial to recognize soluble and insoluble compounds. Lead nitrate is soluble and does not form a visible precipitate with Pb^{2+} ions.

---

63. The volume of 0.02 M aqueous HBr required to neutralize 10.0 mL of 0.01 M aqueous $Ba(OH)_2$ is (Assume complete neutralization)

(1) 5.0 mL

(2) 10.0 mL

(3) 2.5 mL

(4) 7.5 mL

Solution:

$$\text{m.e.q of HBr} = \text{m.e.q of } Ba(OH)_2$$

$$M_1 \times n_1 \times V_1 = M_2 \times n_2 \times V_2$$

$$0.02 \times 1 \times V_1 = 0.01 \times 2 \times 10$$

$$V_1 = \frac{0.01 \times 10}{0.02} = 10\,\text{mL}$$

Correct Answer: (2) 10.0 mL

Quick Tip

In titration problems, always apply the principle of equivalence where moles of acid and base are equal. Use the relation $M_1 \times V_1 = M_2 \times V_2$ for calculations.

64. Group–13 elements react with O_2 in amorphous form to form oxides of type M_2O_3 (M = element). Which among the following is the most basic oxide?

(1) Al_2O_3

(2) Tl_2O_3

(3) Ga_2O_3

(4) B_2O_3

Solution: As electropositive character increases, basic character of oxide increases:

$$B_2O_3 < Al_2O_3 < Ga_2O_3 < In_2O_3 < Tl_2O_3$$

Correct Answer: (4) B_2O_3

Quick Tip

When comparing the basicity of oxides, the trend follows the electropositivity of the elements. Oxides of more electropositive elements tend to be more basic.

65. The IUPAC name of $K_3[Co(C_2O_4)_3]$ is

(1) Potassium tris(oxalate) cobalte(III)

(2) Potassium trioxalatocobalt(III)

(3) Potassium tris(oxalate)cobalt(III)

(4) Potassium trioxalatocobaltate(III)

Solution: The IUPAC name of $K_3[Co(C_2O_4)_3]$ is Potassium trioxalatocobaltate(III). **Correct Answer:** (4) Potassium trioxalatocobaltate(III)

Quick Tip

When naming coordination compounds, the ligands are named first followed by the metal center. The oxidation state of the metal is included in parentheses in Roman numerals.

---

66. If the radius of the first orbit of hydrogen atom is a_0, then de Broglie's wavelength of electron in 3rd orbit is:

(A) $\frac{\pi a_0}{6}$

(B) $\frac{\pi a_0}{3}$

(C) $6\pi a_0$

(D) $3\pi a_0$

Correct Answer: (C) $6\pi a_0$

Solutions:

By De-Broglie principle, we have:

$$2\pi r = n\lambda$$
$$2\pi \frac{n^2 a_0}{z} = n\lambda$$
$$\lambda = 2\pi \frac{3a_0}{1} = 6\pi a_0$$

Quick Tip

Use the De-Broglie relation $2\pi r = n\lambda$ to find the wavelength of an electron in any orbit.

---

67. The group of chemicals used as pesticide is:

(A) Sodium chlorate, DDT, PAN

(B) DDT, Aldrin

(C) Aldrin, Sodium chlorate, Sodium arsenite

(D) Dieldrin, Sodium arsenite, Tetrachlorothene

Correct Answer: (B) DDT, Aldrin

Solutions:

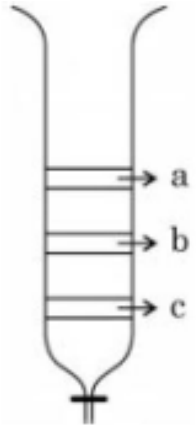
DDT and Aldrin are widely used as pesticides for controlling insect pests.

Quick Tip

Remember that DDT and Aldrin are both chlorinated organic compounds used in pest control.

---

68. From the figure of column chromatography given below, identify incorrect statements.



(A) Compound ‘c’ is more polar than ‘a’ and ‘b’

(B) Compound ‘a’ is least polar

(C) Compound ‘b’ comes out of the column before ‘c’ and after ‘a’

(D) Compound ‘a’ spends more time in the column

Choose the correct answer from the options given below:

(1) A, B and D only

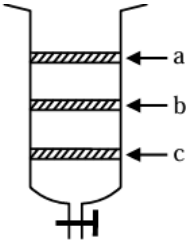
(2) A, B and C only

(3) B and D only

(4) B, C and D only

Correct Answer: (4) B, C and D only

Solutions:



From the given chromatography column, compound ’c’ is the most polar, as it spends the most time in the column. Compound ’a’ is the least polar as it elutes last. Compound ’b’ elutes between ’a’ and ’c’.

Quick Tip

In chromatography, polar compounds interact more strongly with the stationary phase and take longer to elute.

---

69. Ion having highest hydration enthalpy among the given alkaline earth metal ions is:

(1) Be^{2+}

(2) Ba^{2+}

(3) Ca^{2+}

(4) Sr^{2+}

Correct Answer: (1) Be^{2+}

Solution:

Hydration enthalpy is inversely proportional to the size of the ion, i.e.,

$$\text{Hydration enthalpy} \propto \frac{1}{\text{size}}.$$

As the size of the ion increases down the group, hydration enthalpy decreases. Therefore, the order of hydration enthalpy is:

$$Be^{2+} > Mg^{2+} > Ca^{2+} > Sr^{2+} > Ba^{2+}.$$

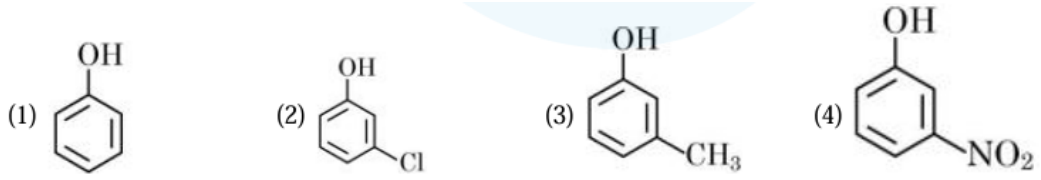
Thus, Be^{2+} has the highest hydration enthalpy.

Quick Tip

Hydration enthalpy is a key concept for understanding the solvation process. Smaller ions have higher hydration enthalpies due to their higher charge density.

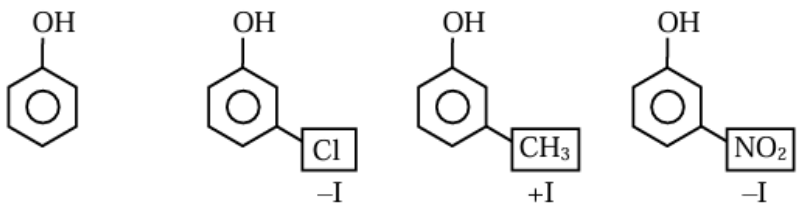
---

70. The strongest acid from the following is:



Correct Answer: (4)

Solution:



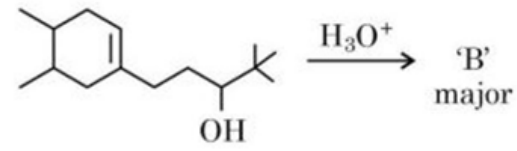
The acid strength of a phenolic compound depends on the electron-withdrawing or electron-donating groups attached to the benzene ring. The electron-withdrawing group increases the acidity of the compound. Among the options, the nitro group ($-NO_2$) is a strong electron-withdrawing group through the inductive effect (-I), making the compound with $-NO_2$ attached to the ring the most acidic.

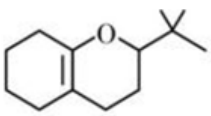
Quick Tip

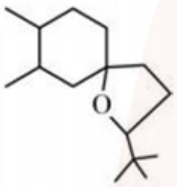
When determining the acidity of substituted phenols, remember that electron-withdrawing groups (-I) increase acidity by stabilizing the negative charge on the conjugate base.

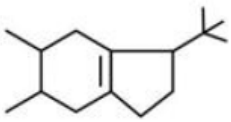
---

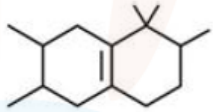
71. In the following reaction, 'B' is



(1) 

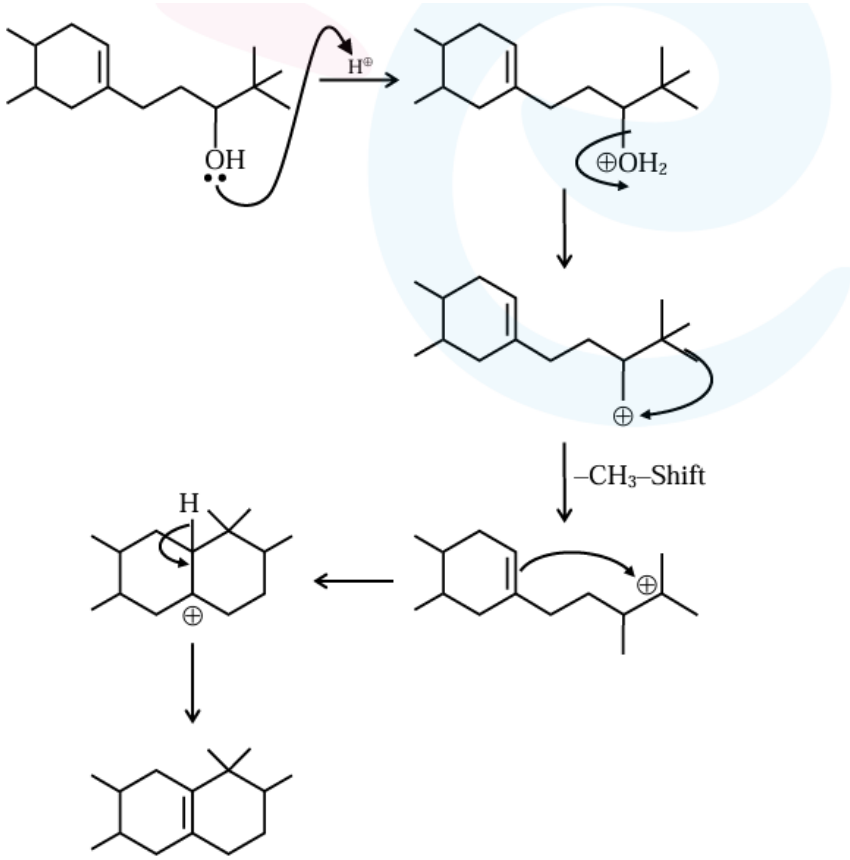
(2) 

(3) 

(4) 

Correct Answer: (4)

Solution:



The reaction shows a typical mechanism where the protonation of the alcohol leads to a carbocation formation, followed by a shift of the CH_3 group. The major product corresponds to the compound shown in option (4).

Quick Tip

In elimination and rearrangement reactions, the carbocation intermediate plays a crucial role in determining the product.

---

72. Structures of $BeCl_2$ in solid state, vapour phase and at very high temperature respectively are:

(1) Polymeric, Dimeric, Monomeric

(2) Dimeric, Polymeric, Monomeric

(3) Monomeric, Dimeric, Polymeric

(4) Polymeric, Monomeric, Dimeric

Correct Answer: (1) Polymeric, Dimeric, Monomeric

Solution:

In the solid state, $BeCl_2$ exists as a polymer. In the vapour phase, it forms a chloro-bridged dimer. At very high temperatures (above 1200K), it exists as a monomer.

Quick Tip

$BeCl_2$ is an example of a molecule whose structure changes depending on temperature, showcasing the effect of temperature on molecular structure.

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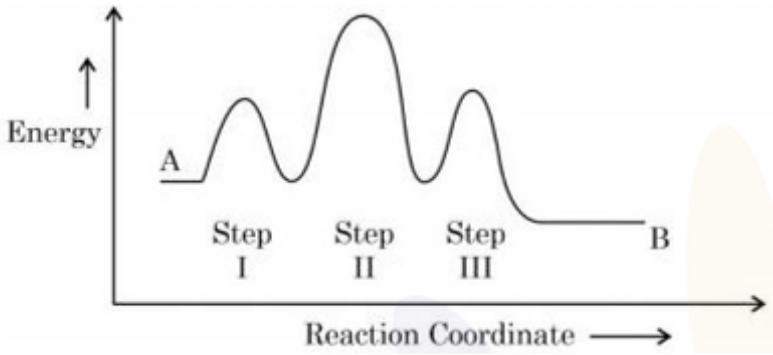
73. Consider the following reaction that goes from A to B in three steps as shown below:

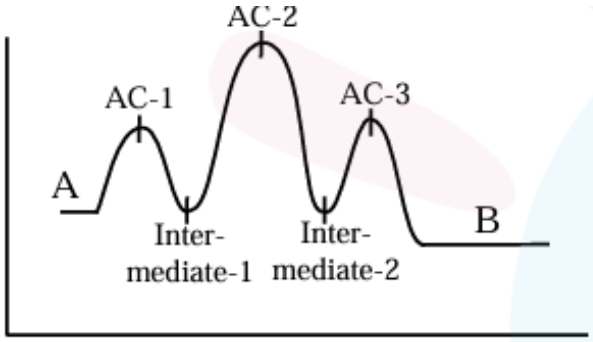


Choose the correct option

	Number of intermediates	Number of Activated complex	Rate determining step
(1)	2	3	II
(2)	3	2	II
(3)	2	3	III
(4)	2	3	I

Correct Answer: (1)

Solution:



Number of Intermediate → 2
Number of Activated complex → 3
Rate determining step → II

From the diagram, we see that there are two intermediates and three activated complexes. The rate determining step is the second step (II), where the highest energy barrier occurs.

Quick Tip

In reaction mechanisms, the rate determining step is the one with the highest activation energy, which slows down the overall reaction.

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74. The product, which is not obtained during the electrolysis of brine solution is:

(1) HCl
(2) NaOH
(3) Cl_2
(4) H_2

Correct Answer: (1) HCl

Solution:

During electrolysis of brine solution (NaCl + H_2O), at the cathode, hydrogen gas (H_2) is released, and at the anode, chlorine gas (Cl_2) is produced. Na^+ and OH^- combine to form NaOH. HCl is not formed during this process.

Quick Tip

The electrolysis of brine is used to produce chlorine gas, hydrogen gas, and sodium hydroxide.

---

75. Which one of the following elements will remain as liquid inside pure boiling water?

(1) Li

(2) Ga

(3) Cs

(4) Br

Correct Answer: (2) Ga

Solution:

Li and Cs react vigorously with water. Br is a non-metal that does not react in this manner. Ga, however, remains liquid at the boiling point of water, which is 100°C (boiling point of Ga = 2400°C).

Quick Tip

Remember that elements with high melting points, such as Gallium, can remain liquid even at temperatures above the boiling point of water.

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76. Given below are two statements: one is labelled as "Assertion A" and the other is labelled as "Reason R"

Assertion A: In the complex $Ni(CO)_4$ and $Fe(CO)_5$, the metals have zero oxidation state.

Reason R: Low oxidation states are found when a complex has ligands capable of π-donor character in addition to the σ-bonding.

In the light of the above statement, choose the most appropriate answer from the options given

below.

(1) A is not correct but R is correct.

(2) A is correct but R is not correct.

(3) Both A and R are correct and R is the correct explanation of A.

(4) Both A and R are correct but R is NOT the correct explanation of A.

Correct Answer: (2) A is correct but R is not correct.

Solution:

The oxidation states of Ni and Fe in $Ni(CO)_4$ and $Fe(CO)_5$ are indeed zero. However, while low oxidation states can be stabilized by π-acceptor ligands, this is not the reason why Ni and Fe exhibit zero oxidation states in these complexes.

Quick Tip

Zero oxidation states in metal carbonyls are due to the ligand's ability to donate electron density through σ-donation and accept electron density through π-back donation.

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77. Given below are two statements:

Statement I: Morphine is a narcotic analgesic. It helps in relieving pain without producing sleep.

Statement II: Morphine and its derivatives are obtained from opium poppy.

In the light of the above statements, choose the correct answer from the options given below.

(1) Statement I is true but Statement II is false.

(2) Statement I is false but Statement II is true.

(3) Both Statement I and Statement II are true.

(4) Both Statement I and Statement II are false.

Correct Answer: (2) Statement I is false but Statement II is true.

Solution:

Morphine is indeed a narcotic analgesic and helps in relieving pain. However, it also causes drowsiness, so Statement I is false. Statement II is true as morphine and its derivatives are obtained from the opium poppy.

Quick Tip

Morphine and its derivatives have medicinal properties but also cause drowsiness, which makes Statement I incorrect.

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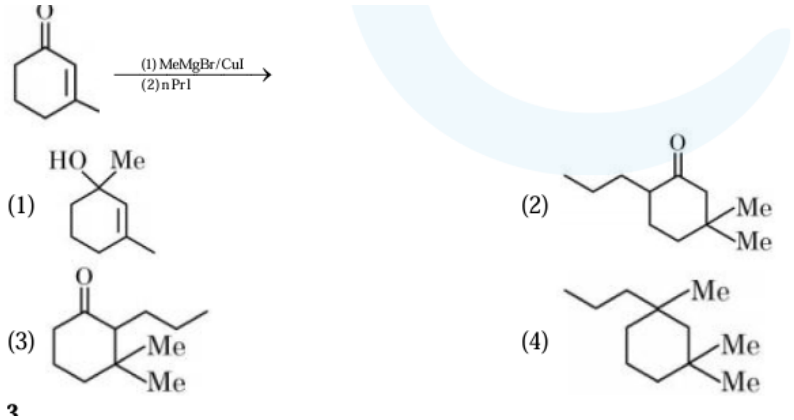
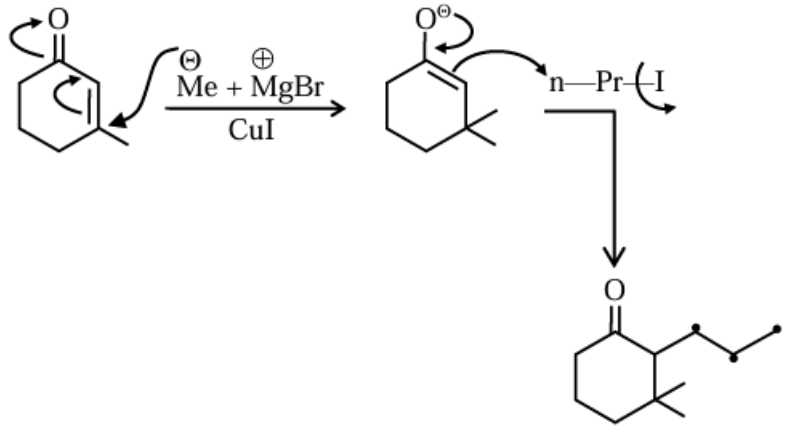
78. Find out the major product from the following reaction.



Correct Answer: (3) **Solution:**



The reaction is a Grignard reagent reaction with the carbonyl group, followed by reaction with n-PrI. The major product is the one shown in option (3), as a new carbon-carbon bond is formed.

Quick Tip

In Grignard reactions, a nucleophilic attack on the carbonyl group leads to the formation of an alcohol after the addition of a halide.

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79. During the reaction of permanganate with thiosulphate, the change in oxidation of manganese occurs by value of 3. Identify which of the below medium will favour the reaction.

(1) aqueous neutral
(2) aqueous acidic
(3) both aqueous acidic and neutral
(4) both aqueous acidic and faintly alkaline

Correct Answer: (1) aqueous neutral

Solution:

In neutral or weakly alkaline solution, the oxidation state of Mn changes by 3 units: MnO_4^- $\rightarrow$ MnO_2. Therefore, the reaction favours the neutral medium.

Quick Tip

The oxidation state change of Mn in reactions involving permanganate depends on the solution's pH. Neutral or slightly alkaline solutions are best for this reaction.

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80. Element not present in Nessler's reagent is

(1) K

(2) N

(3) I

(4) Hg

Correct Answer: (2) N

Solution:

Nessler reagent is $K_2[HgI_4]$, which contains K, Hg, and I. Nitrogen (N) is not present in the reagent.

Quick Tip

Nessler's reagent is used for the detection of ammonia and is made of mercury and iodine, but does not contain nitrogen.

Section-B

81. The standard reduction potentials at 298 K for the following half cells are given below:

NO_3^- + 4H$^+$ + 3e$^-$ $\rightarrow$ NO(g) + 2H_2O $\quad E^0 = 0.97V$

V^{2+} + 2e$^-$ $\rightarrow$ V $\quad E^0 = -1.19V$

Fe^{3+} + 3e$^-$ $\rightarrow$ Fe $\quad E^0 = -0.04V$

Ag^+ + e$^-$ $\rightarrow$ Ag(s) $\quad E^0 = 0.80V$

Au^{3+} + 3e$^-$ $\rightarrow$ Au(s) $\quad E^0 = 1.40V$

The number of metal(s) which will be oxidized by NO_3^- in aqueous solution is

Solution:

In the given half reactions, metals V, Fe, and Ag will be oxidized by NO_3^- because their reduction potentials are less than 0.97 V. Thus, three metals will be oxidized by NO_3^-.

Quick Tip

When comparing reduction potentials, metals with lower reduction potentials are more easily oxidized.

---

82. Number of crystal system from the following where body centred unit cell can be found is ____

Cubic, tetragonal, orthorhombic, hexagonal, rhombohedral, monoclinic, triclinic

Solution:

Body-centered cubic (BCC) crystal structure is found in cubic, tetragonal, and orthorhombic systems. Thus, the number of crystal systems where BCC can be found is 3.

Quick Tip

BCC structure is commonly found in metals like iron and chromium.

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83. Among the following the number of compounds which will give positive iodoform reaction is ___

(a) 1-Phenylbutan-2-one

(b) 2-Methylbutan-2-ol

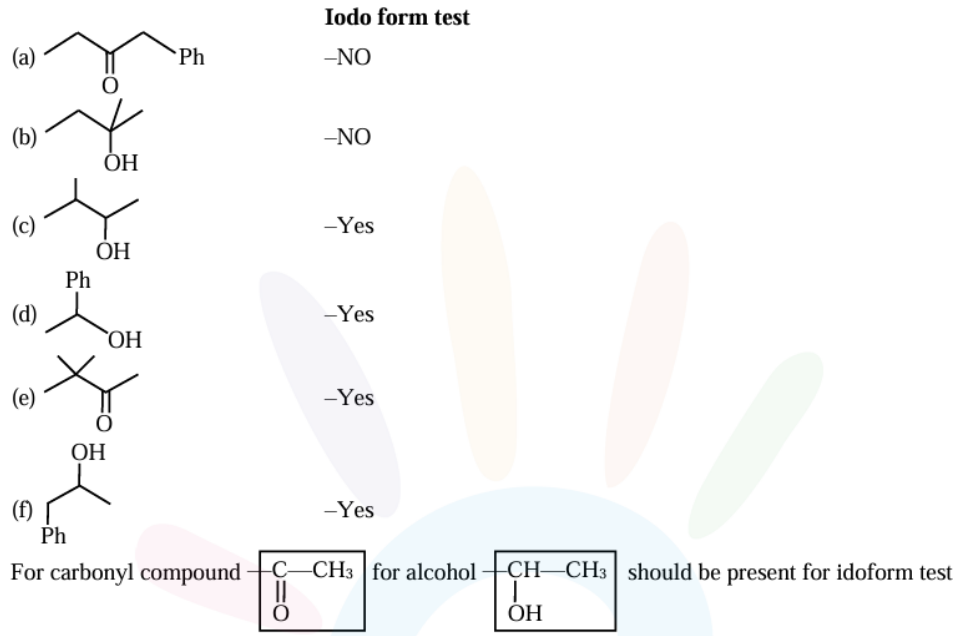
(c) 3-Methylbutan-2-ol

(d) 1-Phenylethanol

(e) 3,3-dimethylbutan-2-one

(f) 1-Phenylpropan-2-ol

Solution:



The iodoform test is positive for compounds that contain a methyl group ($-CH_3$) adjacent to a carbonyl group ($C = O$) or for alcohols that can be oxidized to acetone. Based on the structure and functional groups, compounds (c), (d), (e), and (f) will give a positive iodoform test. Thus, the answer is 4 compounds.

Quick Tip

For iodoform test to be positive, the structure must have either a $-COCH_3$ group or a secondary alcohol adjacent to a methyl group.

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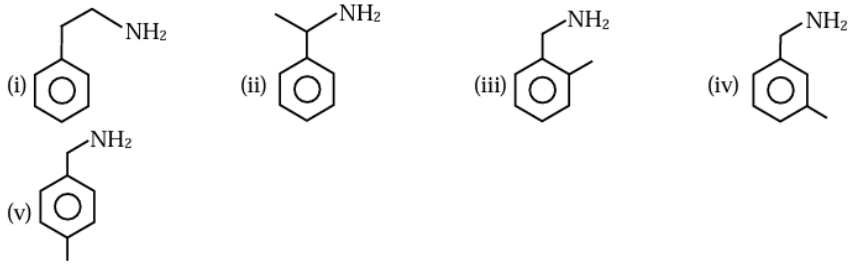
84. Number of isomeric aromatic amines with molecular formula C_8H_11N, which can be synthesized by Gabriel Phthalimide synthesis is_____

Solution:

By Gabriel phthalimide synthesis, i-amine is prepared, which should be aromatic and a-amine.

The degree of unsaturation (Du) helps in determining the number of possible isomers.

Du = C + 1 - H - N/2, which equals 4, indicating the presence of a benzene ring. Thus, the number of isomeric aromatic amines that can be synthesized is 5.



Quick Tip

Gabriel Phthalimide synthesis is a key reaction for preparing aromatic amines from halides, especially when the structure is aromatic.

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85. Consider the following pairs of solution which will be isotonic at the same temperature. The number of pairs of solutions is/are_____

A. 1 M aq. NaCl and 2 M aq. Urea

B. 1 M aq. $CaCl_2$ and 1.5 M aq. KCl

C. 1.5 M aq. $AlCl_3$ and 2 M aq. Na_2SO_4

D. 2.5 M aq. KCl and 1 M aq. $Al_2(SO_4)_3$

Solution:

For two solutions to be isotonic, the total number of ions should be the same. Checking the pairs for ion concentration:

- A: 1 M NaCl gives 2 ions, and 2 M Urea gives 2 ions — isotonic.

- B: 1 M $CaCl_2$ gives 3 ions, and 1.5 M KCl gives 3 ions — isotonic.

- C: 1.5 M $AlCl_3$ gives 6 ions, and 2 M Na_2SO_4 gives 6 ions — isotonic.

- D: 2.5 M KCl gives 5 ions, and 1 M $Al_2(SO_4)_3$ gives 5 ions — isotonic.

Thus, all 4 pairs are isotonic.

Quick Tip

Isotonic solutions have the same osmotic pressure, which is determined by the concentration of dissolved particles.

86. The number of colloidal systems from the following, which will have 'liquid' as the dispersion medium, is_____

Gem stones, paints, smoke, cheese, milk, hair cream, milk, insecticide sprays, froth, soap lather

Solution:

Liquid dispersion medium colloidal systems include paints, milk, hair cream, froth, soap lather. Thus, there are 5 such systems.

Quick Tip

Liquid colloidal systems are common in everyday life, especially in products like paints and creams where the dispersion medium is liquid.

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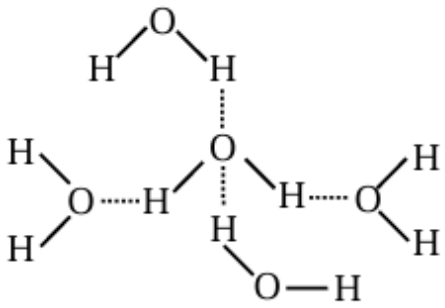
87. In an ice crystal, each water molecule is hydrogen bonded to_____ neighbouring molecules.

Solution:



In an ice crystal, each water molecule is hydrogen bonded to four neighbouring molecules, forming a tetrahedral arrangement.

Quick Tip

Hydrogen bonding in ice leads to the characteristic structure that makes ice less dense than liquid water.

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88. Consider the following data:

Heat of combustion of H_2 (g) = -241.8 kJ mol^{-1}

Heat of combustion of C(s) = -393.5 kJ mol^{-1}

Heat of combustion of C_2H_5OH(l) = -1234.7 kJ mol^{-1}

The heat of formation of C_2H_5OH(l) is (-) _____ kJ mol^{-1} (Nearest integer).

Solution:

We can use Hess's Law and combine the equations to calculate the heat of formation of C_2H_5OH(l):

$\Delta H_f = -393.5 \times 2$ - $241.5 \times 8 \times 3$ + 1234.7 = -277.7 kJ mol^{-1}

Quick Tip

Hess's Law states that the heat of a reaction is the sum of the heats of the reactions into which it can be divided.

---

89. The equilibrium composition for the reaction PCl_3 + Cl_2 PCl_5 at 298 K is given below:

l_3

= 0.2 mol L^{-1}, [Cl_2] = 0.1 mol L^{-1}, [PCl_5] = 0.40 mol L^{-1}

If 0.2 mol of Cl_2 is added at the same temperature, the equilibrium concentrations of PCl_5 is _____ $\times 10^{-2}$ mol L^{-1}.

Solution:

We are given the equilibrium constant Kc for the reaction at 298 K as 20.

$K_c = \frac{[PCl_5]}{[PCl_3][Cl_2]} = \frac{0.40}{0.20 \times 0.10} = 20$

$After adding 0.2 mol of Cl_2$, the new concentrations become:

PCl_3 = 0.2 - x, Cl_2 = 0.2 + x, PCl_5 = 0.4 + x.

Solving for x, we find that x = 0.084. Thus, the equilibrium concentration of PCl_5 is 0.484 mol L^{-1}.

Quick Tip

For equilibrium reactions, the change in concentrations due to shifting of equilibrium can be calculated using the reaction's equilibrium constant.

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90. The number of species having a square planar shape from the following is_____

XeF_4, SF_4, SiF_4, BF_4, BrF_4, $[Cu(NH_3)_4]^{2+}$, $[FeCl_4]^{2-}$, $[PtCl_4]^{2-}$

Solution:

The species with square planar geometry include XeF_4, $[Cu(NH_3)_4]^{2+}$, $[PtCl_4]^{2-}$. Thus, there are 4 species with a square planar shape.

Quick Tip

Square planar geometry is common in transition metal complexes, especially for d^8 configuration metals.
