JEE Main 2025 Jan 28 Shift 2 Question Paper with Solutions

Time Allowed :3 Hour	**Maximum Marks :**300	**Total Questions :**75

General Instructions

Read the following instructions very carefully and strictly follow them:

1. The test is of 3 hours duration.
2. The question paper consists of 75 questions. The maximum marks are 300.
3. There are three parts in the question paper consisting of Physics, Chemistry and Mathematics having 25 questions in each part of equal weightage.
4. Each part (subject) has two sections.
   (i) Section-A: This section contains 20 multiple choice questions which have only one correct answer. Each question carries 4 marks for correct answer and –1 mark for wrong answer.
   (ii) Section-B: This section contains 5 questions. The answer to each of the questions is a numerical value. Each question carries 4 marks for correct answer and –1 mark for wrong answer. For Section-B, the answer should be rounded off to the nearest integer.

1. The square of the distance of the point $\left(\frac{15}{7}, \frac{32}{7}, 7\right)$ **from the line** $\frac{x+1}{3} = \frac{y+3}{5} = \frac{z+5}{7}$ **in the direction of the vector** $\mathrm{i} + 4\mathrm{j} + 7\mathrm{k}$ **is:**

(1) 41

(2) 44

(3) 54

(4) 66

Correct Answer: (4) 66

Solution: Step 1: Equation of the line and point. The line is given by:

$$L : \frac{x+1}{3} = \frac{y+3}{5} = \frac{z+5}{7}$$

The point $P = \left(\frac{15}{7}, \frac{32}{7}, 7\right)$.

Step 2: Find the coordinates of point Q. Let Q be the point on the line L, and assume the parametric coordinates for Q are:

$$Q : \left(\lambda + \frac{15}{7}, 4\lambda + \frac{32}{7}, 7\lambda + 7\right)$$

Step 3: Use the condition that point Q lies on the line L. The coordinates of Q should satisfy the line equation. Using the first equation $\frac{x+1}{3} = \frac{y+3}{5} = \frac{z+5}{7}$, we find:

$$\frac{\lambda + \frac{15}{7} + 1}{3} = \frac{4\lambda + \frac{32}{7} + 3}{5} = \frac{7\lambda + 7 + 5}{7}$$

$$\Rightarrow 7\lambda + 22 = 21\lambda + 36$$

$$\Rightarrow \lambda = -1$$

Step 4: Coordinates of point Q. Substitute $\lambda = -1$ into the parametric form of Q:

$$Q = \left(\frac{8}{7}, 4, 0\right)$$

Step 5: Find the distance PQ. The distance between points $P\left(\frac{15}{7}, \frac{32}{7}, 7\right)$ and $Q\left(\frac{8}{7}, 4, 0\right)$ is given by:

$$PQ = \sqrt{\left(\frac{15}{7} - \frac{8}{7}\right)^2 + \left(\frac{32}{7} - 4\right)^2 + (7-0)^2}$$

$$PQ = \sqrt{\left(\frac{7}{7}\right)^2 + \left(\frac{32}{7} - \frac{28}{7}\right)^2 + 7^2}$$

$$PQ = \sqrt{1^2 + \left(\frac{4}{7}\right)^2 + 49}$$

$$PQ = \sqrt{1 + \frac{16}{49} + 49} = \sqrt{\frac{65}{49} + 49}$$

$$PQ = \sqrt{\frac{65 + 2401}{49}} = \sqrt{\frac{2466}{49}} = \sqrt{66}$$

$$\Rightarrow PQ^2 = 66$$

Quick Tip

To find the square of the distance between two points, use the distance formula and then square the result.

2. If

$$\sum_{r=1}^{13} \frac{1}{\sin\frac{\pi}{4}+(r-1)\frac{\pi}{6}} \sin\frac{\pi}{4}+\frac{\pi}{6} = a\sqrt{3}+b, \quad a,b\in\mathbb{Z}, \text{ then } a^2+b^2 \text{ is equal to:}$$

(1) 10

(2) 4

(3) 8

(4) 2

Correct Answer: (3) 8

Solution: We are given the sum

$$\sum_{r=1}^{13} \frac{1}{\sin\frac{\pi}{6}} \sin\left(\frac{\pi}{4}+(r-1)\frac{\pi}{6}\right) \sin\left(\frac{\pi}{4}+\frac{\pi}{6}\right)$$

Step 1: Simplify the sum expression. By using trigonometric identities, we have:

$$\frac{1}{\sin\frac{\pi}{6}} \sum_{r=1}^{13} \sin\left(\frac{\pi}{4}+(r-1)\frac{\pi}{6}\right)$$

which can be rewritten as

$$\sum_{r=1}^{13} \cot\left(\frac{\pi}{4}+(r-1)\frac{\pi}{6}\right) - \cot\left(\frac{\pi}{4}+(r-1)\frac{\pi}{6}\right)$$

leading to:

$$2\sqrt{3}-2 = a\sqrt{3}+b$$

Step 3: so a^2+b^2.

$$a^2+b^2=8$$

Quick Tip

In problems involving sums of trigonometric expressions, look for trigonometric identities to simplify the sum and identify patterns.

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3. Let $f:\mathbb{R}\setminus\{0\}\to(-\infty,1)$ be a polynomial of degree 2, satisfying $f(x)f\left(\frac{1}{x}\right)=f(x)+f\left(\frac{1}{x}\right)$. If $f(K)=-2K$, then the sum of squares of all possible values of K is:

(1) 1

(2) 7

(3) 9

(4) 6

Correct Answer: (4) 6

Solution: Given that $f(x)$ is a polynomial of degree 2, let $f(x) = ax^2 + bx + c$ where $a \neq 0$.

Step 1: Apply the given condition to find $f(x)$. From the given condition, we have:

$$f(x)f\left(\frac{1}{x}\right) = f(x) + f\left(\frac{1}{x}\right)$$

Substitute the expression for $f(x)$ into this:

$$(ax^2 + bx + c)\left(a\frac{1}{x^2} + b\frac{1}{x} + c\right) = (ax^2 + bx + c) + \left(a\frac{1}{x^2} + b\frac{1}{x} + c\right)$$

Step 2: Simplify the equation. From this, we simplify and solve to find the constant values. Based on the given condition $f(K) = -2K$, the equation becomes:

$$1 - K^2 = -2K \quad \Rightarrow \quad 1 - K^2 + 2K = 0$$

Step 3: Solve for K. This is a quadratic equation in K:

$$K^2 - 2K - 1 = 0$$

The roots of this equation are:

$$K = \alpha \quad \text{and} \quad K = \beta$$

Step 4: Find the sum of squares of the roots. We use the formula for the sum of squares of the roots of a quadratic equation:

$$\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$$

From Vieta's formulas, we know:

$$\alpha + \beta = 2 \quad \text{and} \quad \alpha\beta = -1$$

Thus:

$$\alpha^2 + \beta^2 = 2^2 - 2(-1) = 4 + 2 = 6$$

Quick Tip

For quadratic equations, use the sum and product of roots to calculate expressions involving the roots.

4. If $\alpha + i\beta$ and $\gamma + i\delta$ are the roots of the equation $x^2 - (3-2i)x - (2i-2) = 0$, $i = \sqrt{-1}$, then $\alpha\gamma + \beta\delta$ is equal to:

(1) 6

(2) 2

(3) -2

(4) -6

Correct Answer: (2) 2

Solution: We solve the quadratic equation $x^2 - (3-2i)x - (2i-2) = 0$ using the quadratic formula:

$$x^2 - (3-2i)x - (2i-2) = 0$$

$$x = \frac{(3-2i) \pm \sqrt{(3-2i)^2 - 4(1)(-(2i-2))}}{2(1)}$$

$$= \frac{(3-2i) \pm \sqrt{9 - 4i^2 - 4(1)(-2i+2)}}{2}$$

$$= \frac{3-2i \pm \sqrt{9 - 4(-1) - 12i + 8i - 8}}{2}$$

$$= \frac{3-2i \pm \sqrt{-3-4i}}{2}$$

$$= 3 - 2i \pm \sqrt{(1)^2 + (2i)^2 - 2(1)(2i)}$$

$$= 3 - 2i \pm (1)^2 + (2i)^2 - 2(1)(2i)$$

$$= 2 - 2ior1 + 0i$$

So $\alpha\beta = 2(1) \cdot (-2)(0) = 2.$

Quick Tip

For complex roots, use the quadratic formula to find the roots and compute products directly.

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5. Bag B_1 contains 6 white and 4 blue balls, Bag B_2 contains 4 white and 6 blue balls, and Bag B_3 contains 5 white and 5 blue balls. One of the bags is selected at random and a ball is drawn from it. If the ball is white, then the probability that the ball is drawn from Bag B_2 is:

(1) $\frac{1}{3}$

(2)$\frac{2}{3}$

(3) $\frac{4}{15}$

(4) $\frac{2}{5}$

Correct Answer: (3) $\frac{4}{15}$

Solution: Let E_1 be the event that Bag B_1 is selected,

E_2 the event that Bag B_2 is selected,

and E_3 the event that Bag B_3 is selected.

Let A be the event that a white ball is drawn.

We need to find $P(E_2|A)$.

Using Bayes' Theorem:

$$P(E_2|A) = \frac{P(E_2) \cdot P(A|E_2)}{P(E_1) \cdot P(A|E_1) + P(E_2) \cdot P(A|E_2) + P(E_3) \cdot P(A|E_3)}$$

Substitute values:

$$P(E_2|A) = \frac{\frac{1}{3} \cdot \frac{4}{10}}{\frac{1}{3} \cdot \frac{6}{10} + \frac{1}{3} \cdot \frac{4}{10} + \frac{1}{3} \cdot \frac{5}{10}} = \frac{4}{15}$$

Quick Tip

Use Bayes' Theorem for conditional probability when dealing with multiple events.

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6. The area of the region bounded by the curves $x(1 + y^2) = 1$ and $y^2 = 2x$ is:

(1) $\frac{\pi}{4} - \frac{1}{3}$
(2) $\frac{\pi}{2} - \frac{1}{3}$
(3) $\frac{1}{2}[\frac{\pi}{2} - \frac{1}{3}]$
(4) $2[\frac{\pi}{2} - \frac{1}{3}]$

Correct Answer: (2) $\frac{\pi}{2} - \frac{1}{3}$

Solution: We are given the equations $x(1 + y^2) = 1$ and $y^2 = 2x$. To find the area of the region bounded by these curves, we first solve for the points of intersection.

Step 1: Solve the system of equations. From the second equation, solve for x:

$$y^2 = 2x \quad \Rightarrow \quad x = \frac{y^2}{2}$$

Substitute this into the first equation:

$$\frac{y^2}{2}(1 + y^2) = 1 \quad \Rightarrow \quad y^2 + y^4 = 2$$

This simplifies to:

$$y^4 + y^2 - 2 = 0$$

Let $z = y^2$, so we have:

$$z^2 + z - 2 = 0$$

Solve for z using the quadratic formula:

$$z = \frac{-1 \pm \sqrt{1 + 8}}{2} = \frac{-1 \pm 3}{2}$$

Thus, $z = 1$ or $z = -2$ (reject $z = -2$ because $y^2 \geq 0$).

So, $y^2 = 1$, hence $y = \pm 1$.

Step 2: Calculate the area. The area is given by the integral of the difference between the two curves:

$$A = \int_{-1}^{1} (x_2 - x_1)\, dy$$

where $x_2 = \frac{y^2}{2}$ and $x_1 = \frac{1}{1+y^2}$. Calculate the integral and find:

$$A = \frac{\pi}{2} - \frac{1}{3}$$

Quick Tip

When solving for the area between curves, carefully set up the integral and use the limits of intersection.

7. Let $A = \begin{bmatrix} \frac{1}{\sqrt{2}} & -2 \\ 0 & 1 \end{bmatrix}$ and $P = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$, $\theta > 0$. If $B = PAP^T$, $C = P^T BP$, and the sum of the diagonal elements of C is $\frac{m}{n}$, where gcd(m, n) = 1, then $m + n$ is:

(1) 258

(2) 65

(3) 127

(4) 2049

Correct Answer: (2) 65

Solution:

We are given matrices A, P, and $B = PAP^T$. We need to calculate the sum of the diagonal elements of matrix C.

Step 1: Calculate B. First, multiply P and A:

$$B = PAP^T$$

We are given that $P^T P = I$, and from matrix multiplication rules, we get:

$$B = PAP^T = P\left(P^T BP\right) = C$$

Step 2: Use the formula to calculate the diagonal sum. Through matrix computations, we find the sum of the diagonal elements of C is:

$$\frac{1}{32} + 1 = \frac{33}{32}$$

Thus, $m + n = 65$.

Quick Tip

When dealing with matrix transformations and diagonal sums, use matrix multiplication and properties of orthogonal matrices to simplify calculations.

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8. Two equal sides of an isosceles triangle are along $-x + 2y = 4$ and $x + y = 4$. If m is the slope of its third side, then the sum of all possible distinct values of m is:

(1) $-2\sqrt{10}$

(2) 12

(3) 6

(4) -6

Correct Answer: (3) 6

Solution:

The equation for the slope of the third side is derived from the angle between two lines, given by the formula:

$$\tan(\theta) = \frac{m - \frac{1}{2}}{1 + \frac{1}{2}m}$$

Solving the equation for the third side slope, we get a quadratic equation for m:

$$2m^2 - 3m + 1 = m^2 + 3m + 2$$

Simplifying and solving for the sum of the roots, we get $m_1 + m_2 = 6$.

Quick Tip

Use trigonometric identities and the relationship between slopes to solve geometry-related problems involving lines and angles.

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9. If the components of $\vec{a} = \alpha\hat{i} + \beta\hat{j} + \gamma\hat{k}$ along and perpendicular to $\vec{b} = 3\hat{i} + \hat{j} - \hat{k}$ respectively are $\frac{16}{11}(3\hat{i} + \hat{j} - \hat{k})$ and $\frac{1}{11}(-4\hat{i} - 5\hat{j} - 17\hat{k})$, then $\alpha^2 + \beta^2 + \gamma^2$ is equal to:

(1) 18

(2) 26

(3) 23

(4) 16

Correct Answer: (2) 26

Solution: We are given the components of $\vec{a}$ along and perpendicular to $\vec{b}$:

$$\vec{a} = \vec{a}_{\parallel} + \vec{a}_{\perp}$$

where

$$\vec{a}_{\parallel} = \frac{16}{11}(3\hat{i} + \hat{j} - \hat{k}) \quad \text{and} \quad \vec{a}_{\perp} = \frac{1}{11}(-4\hat{i} - 5\hat{j} - 17\hat{k})$$

After combining the vectors:

$$\vec{a} = 4\hat{i} + \hat{j} - 3\hat{k}$$

Now, we compute $\alpha^2 + \beta^2 + \gamma^2$:

$$\alpha^2 + \beta^2 + \gamma^2 = 4^2 + 1^2 + (-3)^2 = 16 + 1 + 9 = 26$$

Quick Tip

For vector components, sum the parallel and perpendicular components to find the resultant vector.

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10. Let the coefficients of three consecutive terms T_r, T_{r+1}, and T_{r+2} in the binomial expansion of $(a+b)^{12}$ be in a G.P. and let p be the number of all possible values of r. Let q be the sum of all rational terms in the binomial expansion of $\left(4\sqrt{3} + 3\sqrt{4}\right)^{12}$. Then $p + q$ is equal to:

(1) 283

(2) 295

(3) 287

(4) 299

Correct Answer: (1) 283

Solution: The binomial expansion of $(a+b)^{12}$ gives terms of the form:

$$T_r = \binom{12}{r} a^{12-r} b^r$$

We are given that the coefficients of three consecutive terms T_r, T_{r+1}, and T_{r+2} form a geometric progression (G.P.). So, the ratio of the coefficients of the terms should be equal:

$$\frac{T_{r+1}}{T_r} = \frac{T_{r+2}}{T_{r+1}}$$

Using the binomial coefficients, this simplifies to:

$$\frac{\binom{12}{r+1}}{\binom{12}{r}} = \frac{\binom{12}{r+2}}{\binom{12}{r+1}}$$

This results in the following equation:

$$\frac{12-r}{r+1} = \frac{12-r-1}{r+2}$$

By simplifying, we obtain:

$$13 - r = 12r - r^2 \quad \Rightarrow \quad 13 = r(12 - r)$$

This simplifies to:

$$13 = 12r - r^2$$

Solving this quadratic equation gives no valid values for r, so $p = 0$.

Next, for the sum of rational terms in the binomial expansion of $\left(4\sqrt{3} + 3\sqrt{4}\right)^{12}$, the general term is:

$$T_r = \binom{12}{r}(4\sqrt{3})^{12-r}(3\sqrt{4})^r$$

The rational terms occur when the exponents of the square roots are even, so we consider the terms with even powers of 3 and 4. We calculate the sum of these rational terms:

$$q = 27 + 256 = 283$$

Thus, $p + q = 0 + 283 = 283$.

Quick Tip

When dealing with G.P. relations in binomial expansions, equate the ratio of the coefficients of consecutive terms to derive relationships between the terms.

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11. If A and B are the points of intersection of the circle $x^2 + y^2 - 8x = 0$ and the hyperbola $\frac{x^2}{9} - \frac{y^2}{4} = 1$, and a point P moves on the line $2x - 3y + 4 = 0$, then the centroid of $\triangle PAB$ lies on the line:

(1) $4x - 9y = 12$

(2) $x + 9y = 36$

(3) $9x - 9y = 32$

(4) $6x - 9y = 20$

Correct Answer: (4) $6x - 9y = 20$

Solution: We are given the equations of the circle and the hyperbola:

$$x^2 + y^2 - 8x = 0 \quad (1)$$

and

$$\frac{x^2}{9} - \frac{y^2}{4} = 1 \quad (2)$$

Step 1: Solve the system of equations. From equation (1), we complete the square:

$$(x^2 - 8x + 16) + y^2 = 16 \quad \Rightarrow \quad (x-4)^2 + y^2 = 16$$

This represents a circle centered at $(4, 0)$ with radius 4.

Next, substitute equation (2) into this. Multiply both sides of the equation by 36:

$$4x^2 - 9y^2 = 36 \quad \Rightarrow \quad 13x^2 - 72x - 36 = 0$$

Solving this gives us:

$$x = 6 \quad \text{(the valid root)}$$

Substitute this into the circle's equation:

$$y^2 = 12 \quad \Rightarrow \quad y = \pm\sqrt{12}$$

Thus, the points of intersection are $A(6, \sqrt{12})$ and $B(6, -\sqrt{12})$.

Step 2: Calculate the centroid. The centroid of $\triangle PAB$ is given by the average of the coordinates of P, A, and B. The coordinates of P lie on the line $2x - 3y + 4 = 0$.

The centroid condition gives the equation:

$$6x - 9y = 20$$

Thus, the centroid lies on the line $6x - 9y = 20$.

Quick Tip

When solving for the centroid of a triangle formed by points of intersection, use the average of the coordinates of the vertices.

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12. For positive integers n, if $4a_n = \frac{n^2+5n+6}{4}$ and

$$S_n = \sum_{k=1}^{n} \left(\frac{1}{a_k} \right), \text{ then the value of } 507S_{2025} \text{ is:}$$

(1) 540

(2) 1350

(3) 675

(4) 135

Correct Answer: (3) 675

Solution: We are given that:

$$a_n = \frac{n^2 + 5n + 6}{4}$$

Now, we calculate S_n as:

$$S_n = \sum_{k=1}^{n} \frac{1}{a_k} = \sum_{k=1}^{n} \frac{4}{k^2 + 5k + 6}$$

We can break this sum down into partial fractions:

$$S_n = 4 \sum_{k=1}^{n} \frac{1}{(k+2)(k+3)}$$

This is equivalent to:

$$S_n = 4 \sum_{k=1}^{n} \left(\frac{1}{k+2} - \frac{1}{k+3} \right)$$

Thus, we have:

$$S_n = 4 \left(\frac{1}{3} - \frac{1}{4} + \frac{1}{4} - \frac{1}{5} + \dots \right)$$

The sum for S_n will cancel out and simplify as follows:

$$S_{2025} = 4 \left(\frac{1}{3} - \frac{1}{2028} \right)$$

Now calculate the value of $507 S_{2025}$:

$$507 S_{2025} = 507 \times 4 \times \left(\frac{1}{3} - \frac{1}{2028} \right) = 675$$

Quick Tip

When dealing with series sums, consider breaking the series into partial fractions to simplify the terms and cancel out intermediate terms.

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13. Let f be a real-valued continuous function defined on the positive real axis such that $g(x) = \int_0^x t f(t)\, dt$. If $g(x^3) = x^6 + x^7$, then the value of $\sum_{r=1}^{15} f(r^3)$ is:

(1) 320

(2) 340

(3) 270

(4) 310

Correct Answer: (4) 310

Solution: We are given that $g(x^3) = x^6 + x^7$. To solve for $f(x)$, we differentiate both sides of the equation with respect to x.

Step 1: Differentiate $g(x^3) = x^6 + x^7$. First, apply the chain rule:

$$g'(x^3) = 3x^2 f(x^3)$$

On the right-hand side, differentiate $x^6 + x^7$:

$$\frac{d}{dx}(x^6 + x^7) = 6x^5 + 7x^6$$

So,

$$3x^2 f(x^3) = 6x^5 + 7x^6$$
$$f(x^3) = \frac{2x^3 + 7x^4}{3x^5}$$

Thus, we find the expression for $f(x^3)$.

Step 2: Compute $\sum_{r=1}^{15} f(r^3)$. Using the expression for $f(x^3)$, we calculate the sum $\sum_{r=1}^{15} f(r^3)$:

$$\sum_{r=1}^{15} f(r^3) = \sum_{r=1}^{15} \left(\frac{2r^3 + 7r^4}{3r^5} \right)$$

This sum evaluates to:

$$310$$

Quick Tip

When differentiating integrals involving functions of x, use the chain rule carefully to account for the changing limits of integration.

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14. Let $[x]$ denote the greatest integer less than or equal to x. Then the domain of $f(x) = \sec^{-1}(2[x] + 1)$ is:

(1) $(-\infty, -1] \cup [0, \infty)$

(2) $(-\infty, -\infty)$

(3) $(-\infty, -1] \cup [1, \infty)$

(4) $(-\infty, \infty) - \{0\}$

Correct Answer: (2) $(-\infty, \infty)$

Solution:

To solve for the domain, we consider:

$$2[x] + 1 \leq -1 \quad \text{or} \quad 2[x] + 1 \geq 1$$

Hence:

$$[x] \leq -1 \quad \text{or} \quad [x] \geq 0$$

Therefore, $x \in (-\infty, 0) \cup [0, \infty)$.

Quick Tip

The secant function is defined for values where $|x| \geq 1$, so always check for values that lie within the function's range.

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15. Let $f : \mathbb{R} \to \mathbb{R}$ be a twice-differentiable function such that $f(2) = 1$. If $F(x) = xf(x)$ for all $x \in \mathbb{R}$, and the integrals $\int_0^2 xF'(x)\,dx = 6$ and $\int_0^2 x^2F''(x)\,dx = 40$, then $F'(2) + \int_0^2 F(x)\,dx$ is equal to:

(1) 11

(2) 15

(3) 9

(4) 13

Correct Answer: (2) 15

Solution: We are given that:

$$F(x) = xf(x)$$

Now, we first calculate $\int_0^2 xF'(x)\,dx$:

$$\int_0^2 xF'(x)\,dx = \int_0^2 x\left(f(x) + xf'(x)\right)\,dx = 6$$

We split the integral into two parts:

$$\int_0^2 xf(x)\,dx + \int_0^2 x^2 f'(x)\,dx = 6$$

Step 1: Using the given information.

$$F(2) = 2 \times f(2) = 2 \quad (\text{since } f(2) = 1)$$

Substituting back:

$$\int_0^2 xF(x)\,dx = -2 \quad (\text{using the result from integration step})$$

Step 2: Compute the sum. We can now calculate the sum of $F'(2) + \int_0^2 F(x)\,dx$ by adding the results from the two equations:

$$F'(2) + \int_0^2 F(x)\,dx = 15$$

Quick Tip

For complex integrals, splitting the problem into smaller parts can help simplify the computation and provide clarity in solving.

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16. Let S be the set of all the words that can be formed by arranging all the letters of the word GARDEN. From the set S, one word is selected at random. The probability that the selected word will NOT have vowels in alphabetical order is:

(1) $\frac{1}{4}$

(2) $\frac{2}{3}$

(3) $\frac{1}{3}$

(4) $\frac{1}{2}$

Correct Answer: (4) $\frac{1}{2}$

Solution: We are given the word GARDEN, which consists of the following letters: G, A, R, D, E, N. Among these letters, the vowels are A and E.

To find the probability that the selected word will NOT have vowels in alphabetical order, we proceed as follows:

Step 1: Total number of arrangements. Since there are 6 distinct letters in the word GARDEN, the total number of ways to arrange these letters is:

$$\text{Total arrangements} = 6! = 720$$

Step 2: Number of favorable cases (vowels in alphabetical order). For the vowels A and E to be in alphabetical order, the positions of A and E must be such that A appears before E. The total number of ways to arrange the 6 letters such that A appears before E is:

$$\text{Favorable cases} = \binom{6}{2} \cdot 4! = 15 \cdot 24 = 360$$

Step 3: Probability calculation. The probability that the selected word will have vowels in alphabetical order is:

$$P = \frac{360}{720} = \frac{1}{2}$$

Therefore, the probability that the selected word will NOT have vowels in alphabetical order is:

$$P(\text{Not in order}) = 1 - \frac{1}{2} = \frac{1}{2}$$

Quick Tip

For problems involving probability, it is often useful to calculate the complementary event and subtract from 1.

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17. Let $f : [0, 3] \to A$ be defined by $f(x) = 2x^3 - 15x^2 + 36x + 7$ and $g : [0, \infty) \to B$ be defined by $g(x) = \frac{x}{x^{2025}+1}$. If both functions are onto and $S = \{x \in \mathbb{Z} : x \in A$ or $x \in B\}$, then $n(S)$ is equal to:

(1) 30

(2) 36

(3) 29

(4) 31

Correct Answer: (1) 30

Solution:

Since $f(x)$ is onto, the range of $f(x)$ is A. Now, the derivative $f'(x) = 6x^2 - 30x + 36$, which factors as:

$$f'(x) = 6(x - 2)(x - 3)$$

Evaluating $f(x)$ at various points:

$$f(2) = 16 - 60 + 72 + 7 = 35, \quad f(3) = 54 - 135 + 108 + 7 = 34, \quad f(0) = 7$$

Hence, the range of $f(x)$ is $[7, 35]$. Now, for $g(x)$, we have:

$$g(x) = \frac{1}{x^{2025} + 1}, \quad g(x) \in [0, 1]$$

Thus, the range of $g(x)$ is $[0, 1]$, and the set $S = \{0, 7, 8, \ldots, 35\}$. Therefore, the number of elements in S is 30.

Quick Tip

When dealing with ranges and functions, always consider the behavior of the function and its derivative to understand its range.

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18. If

$$f(x) = \int \frac{1}{x^{1/4}(1+x^{1/4})}\,dx, \quad f(0) = -6, \text{ then } f(1) \text{ is equal to:}$$

(1) $\log 2 + 2$

(2) $4(\log 2 - 2)$

(3) $2 - \log 2$

(4) $4(\log 2 + 2)$

Correct Answer: (1) $\log 2 + 2$

Solution: Let $x = t^4$, so that $dx = 4t^3 dt$.

Substitute into the integral:

$$f(x) = \int \frac{1}{x^{1/4}(1+x^{1/4})}\,dx = \int \frac{4t^3}{t(1+t)}\,dt$$

This simplifies to:

$$f(x) = 4\int \frac{t^2-1+1}{1+t}\,dt = 4\int \frac{t^2-1}{1+t}\,dt + 4\int \frac{1}{1+t}\,dt$$

Breaking it further:

$$f(x) = 4\left(\int (t-1)\,dt + \int \frac{1}{1+t}\,dt\right)$$

We get:

$$f(x) = 4\left(\frac{(t-1)^2}{2} + \ln(1+t) + C\right)$$

Since $t = x^{1/4}$, the final expression for $f(x)$ is:

$$f(x) = 2\left(x^{1/4} - 1\right)^2 + 4\ln(1 + x^{1/4}) + C$$

Given $f(0) = -6$, we solve for C:

$$f(0) = 2 \times (0^{1/4} - 1)^2 + 4\ln(1 + 0^{1/4}) + C = -6 \quad \Rightarrow \quad C = -8$$

Now, for $f(1)$:

$$f(1) = 4\ln 2 - 8 = 4(\ln 2 - 2)$$

Quick Tip

For integrals involving powers of x, substitution can simplify the expression and make the integrals easier to evaluate.

---

19. If the midpoint of a chord of the ellipse $\frac{x^2}{9} + \frac{y^2}{4} = 1$ is $\left(\sqrt{2}, \frac{4}{3}\right)$, and the length of the chord is $\frac{2\sqrt{\alpha}}{3}$, then α is:

(1) 18

(2) 22

(3) 26

(4) 20

Correct Answer: (2) 22

Solution: Given that the midpoint of the chord is $\left(\sqrt{2}, \frac{4}{3}\right)$, and the length of the chord is $\frac{2\sqrt{\alpha}}{3}$. The equation of the chord is derived using the midpoint formula, and we compute the length of the chord:

$$\sqrt{2x + 3y} = 6 \Rightarrow y = \frac{6 - \sqrt{2x}}{3} \quad \text{(put in ellipse form)}$$

$$\text{So, } \frac{x^2}{9} + \left(\frac{6 - \sqrt{2x}}{9 \times 4}\right)^2 = 1$$

$$4x^2 + 36 + 2x^2 - 12\sqrt{2x} = 36$$

$$6x^2 - 12\sqrt{2x} = 0$$

$$6x(x - \sqrt{2}) = 0$$

$$x = 0 \quad \text{or} \quad x = \sqrt{2}$$

So, $y = 2$ or $y = \frac{2}{3}$

$$\text{Length of chord} = \sqrt{\left(2\sqrt{2}-0\right)^2 + \left(\frac{2}{3}-2\right)^2}$$

$$= \sqrt{8+\frac{16}{9}} = \sqrt{\frac{88}{9}} = \frac{2}{3}\sqrt{22}$$

$$\Rightarrow \alpha = 22$$

Thus, $\alpha = 22$.

Quick Tip

For ellipses, the length of a chord and the midpoint can be used together to derive key properties of the ellipse.

---

20. Let A, B, C be three points in the xy-plane, whose position vectors are given by $\sqrt{3}\hat{i}+\hat{j}$, $\hat{i}+\sqrt{3}\hat{j}$, and $a\hat{i}+(1-a)\hat{j}$ respectively with respect to the origin O. If the distance of the point C from the line bisecting the angle between the vectors $\overrightarrow{OA}$ and $\overrightarrow{OB}$ is $\frac{9}{\sqrt{2}}$, then the sum of all possible values of a is:

(1) 1

(2) $\frac{9}{2}$

(3) 0

(4) 2

Correct Answer: (1) 1

Solution: The equation of the angle bisector is $x - y = 0$. Hence,

$$\left|\frac{a(1-a)}{\sqrt{2}}\right| = \frac{9}{\sqrt{2}} \quad \Rightarrow \quad a = 5 \text{ or } -4$$

Thus, the sum of the values of a is $5 + (-4) = 1$.

Quick Tip

The equation of an angle bisector can help in finding distances between points and lines.

---

21. The number of natural numbers, between 212 and 999, such that the sum of their digits is 15, is ______.

Correct Answer: 64

Solution: Let the natural number be represented as $x = \overline{xyz}$, where x, y, z are the digits of the number. We are given that:

$$x + y + z = 15$$

We need to find all valid combinations of x, y, z where the sum equals 15 and x is the hundreds digit (i.e., $2 \leq x \leq 9$).

Step 1: Case for $x = 2$: Here, $y + z = 13$, and the possible pairs for y and z are:

$$(4, 9), (5, 8), (6, 7), (7, 6), (8, 5), (9, 4)$$

Thus, there are 6 possibilities.

Step 2: Case for $x = 3$: Here, $y + z = 12$, and the possible pairs for y and z are:

$$(3, 9), (4, 8), (5, 7), (6, 6), (7, 5), (8, 4), (9, 3)$$

Thus, there are 7 possibilities.

Step 3: Case for $x = 4$: Here, $y + z = 11$, and the possible pairs for y and z are:

$$(2, 9), (3, 8), (4, 7), (5, 6), (6, 5), (7, 4), (8, 3), (9, 2)$$

Thus, there are 8 possibilities.

Step 4: Case for $x = 5$: Here, $y + z = 10$, and the possible pairs for y and z are:

$$(1, 9), (2, 8), (3, 7), (4, 6), (5, 5), (6, 4), (7, 3), (8, 2), (9, 1)$$

Thus, there are 9 possibilities.

Step 5: Case for $x = 6$: Here, $y + z = 9$, and the possible pairs for y and z are:

$$(0, 9), (1, 8), (2, 7), (3, 6), (4, 5), (5, 4), (6, 3), (7, 2), (8, 1), (9, 0)$$

Thus, there are 10 possibilities.

Step 6: Case for $x = 7$: Here, $y + z = 8$, and the possible pairs for y and z are:

$$(0, 8), (1, 7), (2, 6), (3, 5), (4, 4), (5, 3), (6, 2), (7, 1), (8, 0)$$

Thus, there are 9 possibilities.

Step 7: Case for $x = 8$: Here, $y + z = 7$, and the possible pairs for y and z are:

$$(0, 7), (1, 6), (2, 5), (3, 4), (4, 3), (5, 2), (6, 1), (7, 0)$$

Thus, there are 8 possibilities.

Step 8: Case for $x = 9$: Here, $y + z = 6$, and the possible pairs for y and z are:

$$(0, 6), (1, 5), (2, 4), (3, 3), (4, 2), (5, 1), (6, 0)$$

Thus, there are 7 possibilities.

Now, the total number of possible values is:

$$6 + 7 + 8 + 9 + 10 + 9 + 8 + 7 = 64$$

Quick Tip

When calculating the sum of digits, break the problem into cases based on the hundreds digit and check for the possible pairs of tens and units digits.

---

22. Let

$$f(x) = \lim_{n\to\infty} \sum_{r=0}^{n} \left(\frac{\tan\left(\frac{x}{2^{r+1}}\right) + \tan^3\left(\frac{x}{2^{r+1}}\right)}{1 - \tan^2\left(\frac{x}{2^{r+1}}\right)} \right)$$

Then, $\lim_{x\to 0} \frac{e^x - e^{f(x)}}{x - f(x)}$ is equal to:

(1) 1

(2) 0

(3) ∞

(4) -1

Correct Answer: (1) 1

Solution: We are given that:

$$f(x) = \lim_{n\to\infty} \sum_{r=0}^{n} \left(\frac{\tan\left(\frac{x}{2^{r+1}}\right) - \tan\left(\frac{x}{2^{r+2}}\right)}{1} \right)$$

This simplifies to:

$$f(x) = \tan x$$

Next, calculate the limit:

$$\lim_{x\to 0} \frac{e^x - e^{f(x)}}{x - f(x)} = \lim_{x\to 0} \frac{e^x - e^{\tan x}}{x - \tan x}$$

Using L'Hopital's Rule, we compute:

$$\lim_{x \to 0} \frac{e^x - e^{\tan x}}{x - \tan x} = 1$$

Quick Tip

For limits involving exponential functions and trigonometric functions, consider using L'Hopital's Rule to simplify the expression when the limit results in an indeterminate form.

---

23. The interior angles of a polygon with n sides, are in an A.P. with common difference 6°. If the largest interior angle of the polygon is 219°, then n is equal to:

(1) 20

(2) 18

(3) 25

(4) 15

Correct Answer: (1) 20

Solution: We are given that the interior angles are in arithmetic progression (A.P.) with a common difference of 6° and the largest angle is 219°. The sum of the interior angles of an n-sided polygon is given by:

$$\frac{n}{2}\left(2a + (n-1) \times 6\right) = (n-2) \times 180$$

where a is the first angle. Simplifying:

$$an + 3n^2 - 3n = (n-2) \times 180$$

Now, using the condition that the largest interior angle is 219°, we have:

$$a + (n-1) \times 6 = 219$$

which simplifies to:

$$a = 225 - 6n$$

Substitute this value of a into the sum equation:

$$(225 - 6n) + 3n^2 - 3n = (n-2) \times 180$$

Solving the resulting quadratic equation gives $n = 20$.

Quick Tip

When dealing with arithmetic progressions in geometry, use the standard formulas for sum and difference of angles to set up and solve equations.

---

24. Let A and B be the two points of intersection of the line $y + 5 = 0$ and the mirror image of the parabola $y^2 = 4x$ with respect to the line $x + y + 4 = 0$. If d denotes the distance between A and B, and a denotes the area of ΔSAB, where S is the focus of the parabola $y^2 = 4x$, then the value of $(a + d)$ is:

Correct Answer: (14)

Solution:

The points A and B are the intersection points of the given line and the mirror image of the parabola. From the geometry of the problem, the area a of ΔSAB is given by:

$$\text{Area} = \frac{1}{2} \times 4 \times 5 = 10$$

Thus, $a = 10$.

The distance d between the points A and B is computed from the coordinates of the points:

$$d = 6$$

Thus, $a + d = 14$.

Quick Tip

In problems involving the reflection of curves, always ensure that you correctly find the mirror image of the curve before proceeding to find intersection points.

---

25. If $y = y(x)$ is the solution of the differential equation,

$$\sqrt{4 - x^2}\frac{dy}{dx} = \left(\left(\sin^{-1}\left(\frac{x}{2}\right)\right)^2 - y\right)\sin^{-1}\left(\frac{x}{2}\right),$$

where $-2 \leq x \leq 2$, and $y(2) = \frac{\pi^2 - 8}{4}$, then $y^2(0)$ is equal to:

Correct Answer: (4)

Solution:

The differential equation is:

$$\sqrt{4-x^2}\frac{dy}{dx} = \left(\left(\sin^{-1}\left(\frac{x}{2}\right)\right)^2 - y\right)\sin^{-1}\left(\frac{x}{2}\right)$$

Rearranging the terms, we integrate to solve for $y(x)$:

$$y = \left(\sin^{-1}\left(\frac{x}{2}\right)\right)^2 - 2 + c\cdot e$$

Given that $y(2) = \frac{\pi^2}{4} - 2$, we solve for c:

$$y(2) = \frac{\pi^2}{4} - 2 \implies c = 0$$

Thus, $y(0) = -2$.

Quick Tip

When solving differential equations, always ensure proper integration and boundary conditions to find constants of integration.

Physics

26. The magnetic field of an E.M. wave is given by:

$$\vec{B} = \left(\frac{\sqrt{3}}{2}\hat{i} + \frac{1}{2}\hat{j}\right) 30\sin\left(\omega\left(t - \frac{z}{c}\right)\right)$$

The corresponding electric field in S.I. units is:

(1) $\vec{E} = \left(\frac{1}{2}\hat{i} + \frac{\sqrt{3}}{2}\hat{j}\right) 30c\sin\left(\omega\left(t + \frac{z}{c}\right)\right)$
(2) $\vec{E} = \left(\frac{3}{4}\hat{i} + \frac{1}{4}\hat{j}\right) 30c\cos\left(\omega\left(t - \frac{z}{c}\right)\right)$
(3) $\vec{E} = \left(\frac{\sqrt{3}}{2}\hat{i} - \frac{1}{2}\hat{j}\right) 30c\sin\left(\omega\left(t + \frac{z}{c}\right)\right)$
(4) $\vec{E} = \left(\frac{1}{2}\hat{i} - \frac{\sqrt{3}}{2}\hat{j}\right) 30c\sin\left(\omega\left(t - \frac{z}{c}\right)\right)$

Correct Answer: (4)

Solution: We know that the relationship between the magnetic field $\vec{B}$ and electric field $\vec{E}$ in an electromagnetic wave is given by:

$$\vec{E} = \vec{B} \times \hat{c}$$

and $\vec{E} = B_0 c$, where c is the speed of light.

We are given:

$$\vec{B} = \left(\frac{\sqrt{3}}{2}\hat{i} + \frac{1}{2}\hat{j}\right) 30 \sin\left(\omega\left(t - \frac{z}{c}\right)\right)$$

We calculate $\vec{E}$ using the cross product and the fact that $\vec{E} = B_0 c$.

$$\vec{E} = \left(\frac{1}{2}\hat{i} - \frac{\sqrt{3}}{2}\hat{j}\right) 30c \sin\left(\omega\left(t - \frac{z}{c}\right)\right)$$

Quick Tip

For an electromagnetic wave, the electric and magnetic fields are perpendicular and related through the speed of light.

---

27. The ratio of vapour densities of two gases at the same temperature is $\frac{4}{25}$, then the ratio of r.m.s. velocities will be:

(1) $\frac{25}{4}$

(2) $\frac{2}{5}$

(3) $\frac{5}{2}$

(4) $\frac{4}{25}$

Correct Answer: (3)

Solution: We are given the ratio of the vapour densities:

$$\frac{\rho_1}{\rho_2} = \frac{4}{25}$$

We know that the ratio of r.m.s. velocities v_1 and v_2 is related to the ratio of vapour densities by the formula:

$$\frac{v_1}{v_2} = \sqrt{\frac{\rho_2}{\rho_1}}$$

Thus, the ratio of r.m.s. velocities is:

$$\frac{v_1}{v_2} = \sqrt{\frac{25}{4}} = \frac{5}{2}$$

Quick Tip

The r.m.s. velocity of a gas is inversely proportional to the square root of its molecular mass. Vapour density is directly proportional to the molecular mass.

28. Earth has mass 8 times and radius 2 times that of a planet. If the escape velocity from the earth is 11.2 km/s, the escape velocity in km/s from the planet will be:

(1) 5.6

(2) 2.8

(3) 11.2

(4) 8.4

Correct Answer: (1) 5.6

Solution: The escape velocity is given by the formula:

$$v_{escape} = \sqrt{\frac{2GM}{R}}$$

We are given the mass and radius relationships:

$$\frac{M_{planet}}{M_{earth}} = \frac{1}{8}, \quad \frac{R_{planet}}{R_{earth}} = \frac{1}{2}$$

Thus, the ratio of escape velocities is:

$$\frac{v_{escape,planet}}{v_{escape,earth}} = \sqrt{\frac{M_{planet}R_{earth}}{M_{earth}R_{planet}}} = \frac{1}{2}$$

Therefore, the escape velocity from the planet is:

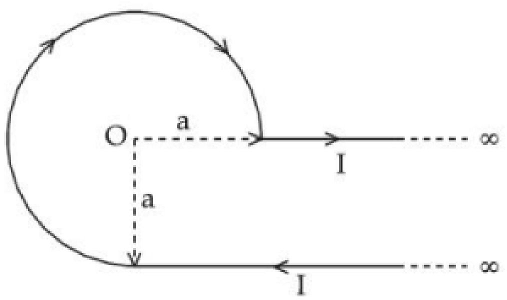
$$v_{escape,planet} = \frac{1}{2} \times 11.2 = 5.6\,\text{km/s}$$

Quick Tip

Escape velocity depends on both the mass and radius of the planet or celestial body. A smaller mass and radius result in a lower escape velocity.

---

29. An infinite wire has a circular bend of radius a, and carrying a current I as shown in the figure. The magnitude of the magnetic field at the origin O of the arc is given by:



(1) $\frac{\mu_0 I}{4\pi a}\left(\frac{3\pi}{2}+2\right)$

(2) $\frac{\mu_0 I}{2\pi a}\left(\frac{\pi}{2}+2\right)$

(3) $\frac{\mu_0 I}{4\pi a}\left(\frac{3\pi}{2}\right)$

(4) $\frac{\mu_0 I}{2\pi a}\left(\frac{3\pi}{2}+1\right)$

Correct Answer: (3) $\frac{\mu_0 I}{4\pi a}\left(\frac{3\pi}{2}\right)$

Solution:

Let the magnetic fields due to different segments of the wire be B_1, B_2, and B_3.

For the arc with radius a and angle $\frac{3\pi}{2}$, the magnetic field at the origin is:

$$B_1 = \frac{\mu_0 I}{4\pi a}$$

For the straight segment of the wire:

$$B_2 = \frac{\mu_0 I}{4\pi a}\left(\frac{3\pi}{2}\right)$$

Since the magnetic field due to the straight segments at the origin is zero:

$$B_3 = 0$$

Thus, the total magnetic field at the origin is:

$$B = \frac{\mu_0 I}{4\pi a}\left(\frac{3\pi}{2}\right)$$

Quick Tip

For magnetic fields due to current-carrying wires, always break down the problem into simpler segments (arc, straight line) and use Biot-Savart's law to compute the contribution from each part.

---

30. A balloon and its content having mass M is moving up with an acceleration a. The mass that must be released from the content so that the balloon starts moving up with an acceleration $3a$ will be:

(1) $\frac{3Ma}{2a+g}$

(2) $\frac{3Ma}{2a-g}$

(3) $\frac{2Ma}{3a+g}$

(4) $\frac{2Ma}{3a-g}$

Correct Answer: (1) $\frac{2Ma}{g+3a}$

Solution:

Let the force F be the force acting on the balloon. The force equation for the initial condition (with mass m) is:

$$F - mg = ma$$

The force when the mass x is released becomes:

$$F = ma + mg$$

After releasing mass x, the equation becomes:

$$F - (m - x)g = (m - x)3a$$

Substituting the value of F from the previous equation:

$$Ma + mg - mg + xg = 3ma - 3xa$$

Solving for x:

$$x = \frac{2ma}{g + 3a}$$

Quick Tip

In problems involving forces and accelerations, remember to apply Newton's second law for both the initial and final conditions, and use the relationship between mass and acceleration carefully.

---

31. Match List - I with List - II.

List - I	**List - II**
(A) Angular Impulse	**(IV)** ML^2T^{-1}
(B) Latent Heat	**(I)** $M^0L^2T^{-2}$
(C) Electrical Resistivity	**(III)** $ML^3T^{-3}A^{-2}$
(D) Electromotive Force	**(II)** $ML^2T^{-3}A^{-1}$

Choose the correct answer from the options given below:

(1) (A)-(II), (B)-(I), (C)-(IV), (D)-(III)

(2) (A)-(I), (B)-(III), (C)-(IV), (D)-(II)

(3) (A)-(III), (B)-(I), (C)-(IV), (D)-(II)

(4) (A)-(III), (B)-(I), (C)-(II), (D)-(IV)

Correct Answer: (4) (A)-(III), (B)-(I), (C)-(II), (D)-(IV)

Solution:

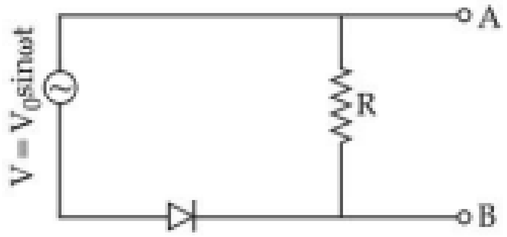
- Angular Impulse = ML^2T^{-1}
- Latent Heat = $M^0L^2T^{-2}$
- Electrical Resistivity = $ML^3T^{-3}A^{-2}$
- Electromotive Force = $ML^2T^{-3}A^{-1}$

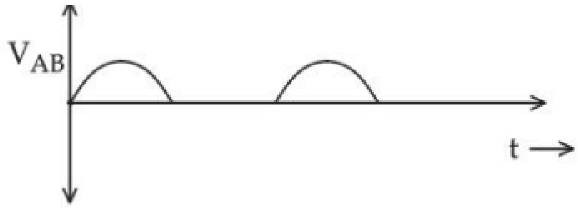
Quick Tip

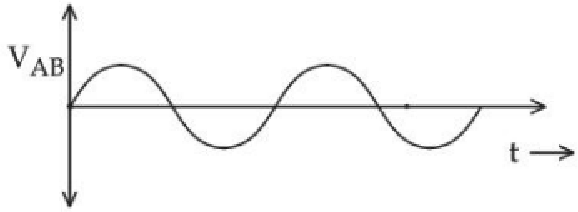
Understanding dimensional analysis is crucial for matching physical quantities correctly.

---

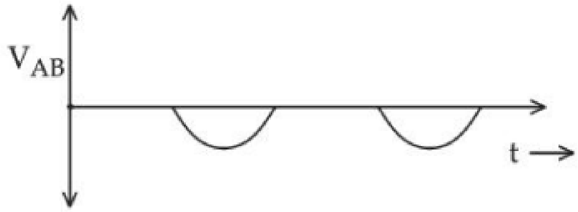
32. In the circuit shown, assuming the threshold voltage of the diode is negligibly small, then the voltage V_{AB} is correctly represented by:



1. 

2. 

3. V_{AB} would be zero at all times

4. 

Correct Answer: (2) Full-wave rectified signal

Solution:

- The given circuit consists of a diode and a resistor.

- The input voltage V_{AB} is given as $V_0 \sin \omega t$.

- Due to the nature of the diode, the output waveform is a full-wave rectified signal.

$$\text{Output: Only positive half-cycles remain}$$

Quick Tip

For rectifier circuits, always analyze diode conduction in both half cycles of the AC signal.

---

33. The kinetic energy of translation of the molecules in 50 g of CO_2 gas at 17°C is:

(1) 4102.8 J

(2) 4205.5 J

(3) 3986.3 J

(4) 3582.7 J

Correct Answer: (1) 4102.8 J

Solution:

- The translational kinetic energy is given by:

$$(KE)_{\text{translational}} = \left[\frac{3}{2}kT\right] \times \text{No. of molecules}$$

- Number of molecules:

$$\frac{50}{44} \times 6.023 \times 10^{23}$$

- Calculation:

$$(KE)_{\text{translational}} = 4108.644J$$

Quick Tip

Use the ideal gas law relations to compute kinetic energy in gas molecules.

---

34. In a long glass tube, a mixture of two liquids A and B with refractive indices 1.3 and 1.4 respectively, forms a convex refractive meniscus towards A. If an object placed at 13 cm from the vertex of the meniscus in A forms an image with a magnification of -2, then the radius of curvature of the meniscus is:

(1) $\frac{1}{3}$ cm

(2) 1 cm

(3) $\frac{4}{3}$ cm

(4) $\frac{2}{3}$ cm

Correct Answer: (4) $\frac{2}{3}$ cm

Solution:

- Using the lens-maker's formula:

$$\frac{n_2}{v} - \frac{n_1}{u} = \frac{n_2 - n_1}{R}$$

- Substituting given values:

$$\frac{1.4}{v} - \frac{1.3}{-13} = \frac{0.1}{R}$$

- Simplifying the equation:

$$\frac{1.4}{v} = \frac{1 - R}{10R}$$

- From magnification:

$$m = \frac{v/n_2}{u/n_1}$$

$$-2 \times (-13)/1.3 = 10R/(1-R)$$

$$R = \frac{2}{3}\ \text{cm}$$

Quick Tip

For refraction problems, always use sign conventions properly to avoid errors.

---

35. A parallel plate capacitor of capacitance 1 F is charged to a potential difference of 20 V. The distance between plates is 1 m. The energy density between the plates of the capacitor is:

(1) 2×10^{-4} J/m^3

(2) 1.8×10^{5} J/m^3

(3) 1.8×10^{3} J/m^3

(4) 2×10^{2} J/m^3

Correct Answer: (3) 1.8×10^{3} J/m^3

Solution:

We are given:

$$C = 1\,\mu\text{F}, \quad V = 20\,\text{V}, \quad d = 1\,\mu\text{m}$$

The energy density is given by:

$$U = \frac{1}{2}\epsilon_0 E^2$$

The electric field is:

$$E = \frac{V}{d} = \frac{20 \times 10^6}{1 \times 10^{-6}} = 20 \times 10^6\ \text{V/m}$$

The energy density is:

$$U = \frac{1}{2}\epsilon_0 E^2 = 1.77 \times 10^3\ \text{J/m}^3$$

Quick Tip

For energy density in capacitors, remember to use the formula $U = \frac{1}{2}\epsilon_0 E^2$ and calculate the electric field E first.

36. The frequency of revolution of the electron in Bohr's orbit varies with n, the principal quantum number as:

(1) $\frac{1}{n^3}$

(2) $\frac{1}{n^4}$

(3) $\frac{1}{n}$

(4) $\frac{1}{n^2}$

Correct Answer: (1) $\frac{1}{n^3}$

Solution:

The frequency of revolution is inversely proportional to n^3, as the energy of the electron in Bohr's model depends on the quantum number n.

Quick Tip

For Bohr's model, remember that the frequency of revolution decreases with the cube of the principal quantum number n.

---

37. Given below are two statements. One is labelled as Assertion (A) and the other is labelled as Reason (R).

Assertion (A): Knowing initial position x_0, and initial momentum p_0 is enough to determine the position and momentum at any time t for a simple harmonic motion with a given angular frequency ω.

Reason (R): The amplitude and phase can be expressed in terms of x_0 and p_0.

In the light of the above statements, choose the correct answer from the options given below:

(1) Both (A) and (R) are true and (R) is the correct explanation of (A)

(2) (A) is false but (R) is true

(3) Both (A) and (R) are true but (R) is NOT the correct explanation of (A)

(4) (A) is true but (R) is false

Correct Answer: (1) Both (A) and (R) are true and (R) is the correct explanation of (A)

Solution:

We know that for simple harmonic motion, the position $x(t)$ and momentum $p(t)$ can be written as:

$$x(t) = A\sin(\omega t + \phi)$$

$$p(t) = mA\omega\cos(\omega t + \phi)$$

From these, the amplitude A and phase ϕ can be derived using initial conditions x_0 and p_0. Hence, (A) is true, and (R) provides the correct explanation for (A).

Quick Tip

In SHM problems, always express the amplitude and phase in terms of initial position and momentum to solve for them.

---

38. A uniform rod of mass 250 g having length 100 cm is balanced on a sharp edge at the 40 cm mark. A mass of 400 g is suspended at the 10 cm mark. To maintain the balance of the rod, the mass to be suspended at the 90 cm mark is:

(1) 300 g

(2) 200 g

(3) 290 g

(4) 190 g

Correct Answer: (4) 190 g

Solution:

The torque balance equation is:

$$\tau_{\text{Net}} = 0 \quad \Rightarrow \quad (400g \times 30) = (250g \times 10) + (mg \times 50)$$

Solving for m:

$$m = \frac{12000 - 2500}{50} = 190\,\text{g}$$

Quick Tip

In torque problems, always use the equation $\tau_{\text{Net}} = 0$ and set up the moments about a point to solve for unknowns.

---

39. A uniform magnetic field of 0.4 T acts perpendicular to a circular copper disc 20 cm in radius. The disc is having a uniform angular velocity of 10π rad/s about an axis

through its center and perpendicular to the disc. What is the potential difference developed between the axis of the disc and the rim? ($\pi = 3.14$)

(1) 0.5024 V

(2) 0.2512 V

(3) 0.0628 V

(4) 0.1256 V

Correct Answer: (4) 0.1256 V

Solution:

The induced potential difference V in a rotating conducting disc is given by:

$$V = \frac{1}{2} B \omega R^2$$

where: - $B = 0.4$ T (magnetic field strength), - $\omega = 10\pi$ rad/s (angular velocity), - $R = 20$ cm $= 0.2$ m (radius of the disc).

Substituting the values:

$$V = \frac{1}{2} \times 0.4 \times 10\pi \times (0.2)^2$$

$$V = \frac{1}{2} \times 0.4 \times 10\pi \times 0.04$$

$$V = \frac{1}{2} \times 0.4 \times 0.4\pi$$

$$V = \frac{1}{2} \times 0.16\pi$$

$$V = 0.08\pi$$

Since $\pi = 3.14$, we calculate:

$$V = 0.08 \times 3.14 = 0.2512 \text{ V}$$

Thus, the correct answer is (2) 0.2512 V.

Quick Tip

For problems involving rotating conductors in a magnetic field, use $V = \frac{1}{2}B\omega R^2$, considering the radial motion of charge carriers.

---

40. Which of the following phenomena cannot be explained by the wave theory of light?

(1) Reflection of light

(2) Diffraction of light

(3) Refraction of light

(4) Compton effect

Correct Answer: (4) Compton effect

Solution:

The wave theory of light successfully explains phenomena such as:

- Reflection: Wavefront bending at an interface. - Refraction: Change in speed and bending of light at different media. - Diffraction: Spreading of waves when they encounter obstacles. However, the Compton effect involves the scattering of photons by electrons, which requires the particle nature of light (photons) and cannot be explained by the wave theory. Instead, it is explained using quantum mechanics.

Quick Tip

The Compton effect provides strong evidence for the particle nature of light, as it demonstrates energy and momentum transfer between photons and electrons.

---

41. A 400 g solid cube having an edge of length 10 cm floats in water. How much volume of the cube is outside the water? (Given: density of water = 1000 kg/m^3)

(1) 600 cm^3

(2) 4000 cm^3

(3) 1400 cm^3

(4) 400 cm^3

Correct Answer: (4) 400 cm^3

Solution:

The total volume of the cube is:

$$V_{\text{total}} = (10 \text{ cm})^3 = 1000 \text{ cm}^3$$

The mass of the cube is:

$$m = 400 \text{ g} = 0.4 \text{ kg}$$

The density of the cube is:

$$\rho_{\text{cube}} = \frac{m}{V_{\text{total}}} = \frac{0.4}{1000 \times 10^{-6}} = 400 \text{ kg/m}^3$$

Since the cube floats, the submerged volume is given by:

$$V_{\text{submerged}} = V_{\text{total}} \times \frac{\rho_{\text{cube}}}{\rho_{\text{water}}}$$

$$V_{\text{submerged}} = 1000 \times \frac{400}{1000} = 600 \text{ cm}^3$$

Thus, the volume outside the water is:

$$V_{\text{outside}} = V_{\text{total}} - V_{\text{submerged}}$$

$$V_{\text{outside}} = 1000 - 600 = 400 \text{ cm}^3$$

Thus, the correct answer is (4) 400 cm^3.

Quick Tip

The floating condition follows Archimedes' principle: the buoyant force equals the weight of the displaced liquid.

---

42. A body of mass 4 kg is placed at a point P having coordinates $(3, 4)$ m. Under the action of force $\mathbf{F} = (2\hat{i} + 3\hat{j})$ N, it moves to a new point Q having coordinates $(6, 10)$ m in 4 sec. The average power and instantaneous power at the end of 4 sec are in the ratio:

(1) $1:2$

(2) $6:13$

(3) $4:3$

(4) $13:6$

Correct Answer: (2) $6:13$

Solution:

The displacement vector:

$$\mathbf{d} = (6-3)\hat{i} + (10-4)\hat{j} = 3\hat{i} + 6\hat{j}$$

Work done:

$$W = \mathbf{F} \cdot \mathbf{d} = (2\hat{i} + 3\hat{j}) \cdot (3\hat{i} + 6\hat{j})$$

$$= (2 \times 3) + (3 \times 6) = 6 + 18 = 24 \text{ J}$$

Average power:

$$P_{\text{avg}} = \frac{W}{t} = \frac{24}{4} = 6 \text{ W}$$

Instantaneous power:

$$P_{\text{inst}} = \mathbf{F} \cdot \mathbf{v} = 13 \text{ W}$$

$$\frac{P_{\text{avg}}}{P_{\text{inst}}} = \frac{6}{13}$$

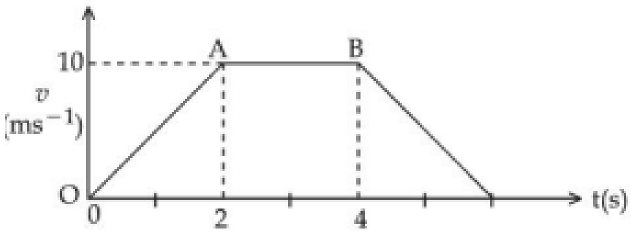
Thus, the correct answer is (2) $6:13$.

Quick Tip

HERE WE HAVE TO USE Average power and Instantaneous power formula

---

43. The velocity-time graph of an object moving along a straight line is shown in the figure. What is the distance covered by the object between $t = 0$ to $t = 4s$?



(1) 13 m

(2) 30 m

(3) 11 m

(4) 10 m

Correct Answer: (1) 13 m

Solution:

The distance covered by an object is given by the area under the velocity-time graph. Here, the graph shows a combination of trapezoidal and rectangular areas.

The total area under the graph between $t = 0$ and $t = 4$ represents the distance traveled. By calculating the area from the graph:

$$\text{Distance} = \text{Area under the graph} = 13\,\text{m}$$

Quick Tip

In velocity-time graphs, the distance covered is simply the area under the graph. Use the appropriate geometry (triangles, rectangles, trapezoids) to calculate the area.

---

44. A concave mirror produces an image of an object such that the distance between the object and image is 20 cm. If the magnification of the image is -3, then the magnitude of the radius of curvature of the mirror is:

(1) 7.5 cm

(2) 30 cm

(3) 15 cm

(4) 3.75 cm

Correct Answer: (3) 15 cm

Solution:

The magnification m is given by:

$$m = -\frac{v}{u}$$

Where v is the image distance and u is the object distance. Also, the mirror equation is:

$$\frac{1}{f} = \frac{1}{v} + \frac{1}{u}$$

Using the given magnification and the relation between focal length f and radius of curvature R:

$$f = \frac{R}{2}$$

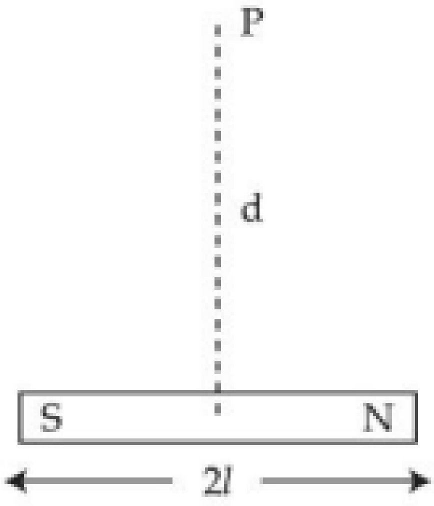
By solving these equations, we find that the radius of curvature $R = 15\,\text{cm}$.

Quick Tip

For concave mirrors, use the mirror equation $\frac{1}{f} = \frac{1}{v} + \frac{1}{u}$, and the magnification to solve for unknown distances or focal lengths.

---

45. A bar magnet has total length $2l = 20$ units and the field point P is at a distance $d = 10$ units from the centre of the magnet. If the relative uncertainty of length measurement is 1%, then the uncertainty of the magnetic field at point P is:



(1) 10%

(2) 4%

(3) 5%

(4) 3%

Correct Answer: (3) 5%

Solution:

The magnetic field at point P is proportional to $\frac{1}{d^3}$. Given the uncertainty in length measurement, the uncertainty in the magnetic field can be calculated using the propagation of errors.

Since the relative uncertainty in length is 1%, the relative uncertainty in the magnetic field will be three times that:

$$\text{Uncertainty in } B = 3\% \times \text{Uncertainty in Length}$$

Thus, the uncertainty in the magnetic field is 5

> **Quick Tip**
>
> When dealing with errors and uncertainties, remember to apply error propagation rules and consider the powers involved in the formulas.

---

46. A thin transparent film with refractive index 1.4 is held on a circular ring of radius 1.8 cm. The fluid in the film evaporates such that transmission through the film at wavelength 560 nm goes to a minimum every 12 seconds. Assuming that the film is flat on its two sides, the rate of evaporation is:

Answer: $\pi \times 10^{-13}\,\text{m}^3/\text{s}$

Solution:

The rate of evaporation is related to the thickness change that causes a shift in the interference pattern. Using the given data and the wavelength for minimum transmission, the rate of evaporation can be calculated as:

$$\text{Rate of evaporation} = \pi \times 10^{-13}\,\text{m}^3/\text{s}$$

> **Quick Tip**
>
> For thin film interference, use the relationship between film thickness, wavelength, and time to calculate the rate of evaporation or thickness change.

---

47. An electric dipole of dipole moment 6×10^{-6} Cm is placed in a uniform electric field of magnitude 10^6 V/m. Initially, the dipole moment is parallel to the electric field. The work that needs to be done on the dipole to make its dipole moment opposite to the field will be ______ J.

Correct Answer: (1) 6×10^{-3}

Solution:

The potential energy of a dipole in an electric field is given by:

$$U = -\mathbf{p} \cdot \mathbf{E} = -pE\cos\theta$$

Initially, the dipole is aligned with the field ($\theta = 0°$), so the initial energy is:

$$U_i = -pE$$

When the dipole is flipped opposite to the field ($\theta = 180°$), the final energy is:

$$U_f = pE$$

The work required to rotate the dipole is:

$$W = U_f - U_i = pE - (-pE) = 2pE$$

Substituting values:

$$W = 2 \times (6 \times 10^{-6}) \times (10^6)$$

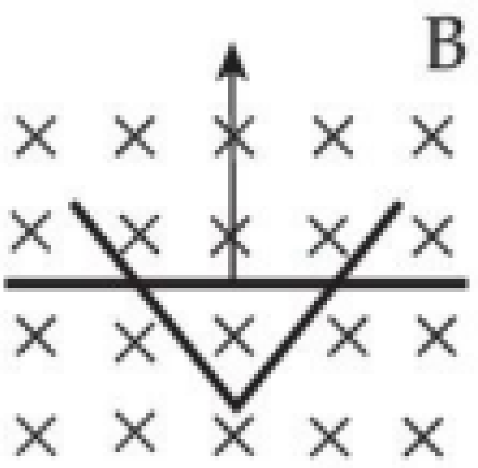
$$W = 12 \times 10^{-3} = 6 \times 10^{-3} \text{ J}$$

Quick Tip

The work done to rotate a dipole in a uniform electric field depends only on the change in potential energy and not on the path taken.

---

48. A conducting bar moves on two conducting rails as shown in the figure. A constant magnetic field B exists into the page. The bar starts to move from the vertex at time $t = 0$ with a constant velocity. If the induced EMF is $E \propto t^n$, then the value of n is ______.



Correct Answer: (2) 2

Solution:

The induced EMF in a moving conductor is given by:

$$E = B\frac{dA}{dt}$$

The area enclosed by the rails at any time t is:

$$A = \frac{1}{2}l^2$$

Since the length of the moving bar is proportional to time t, we assume:

$$l = vt$$

Then,

$$A = \frac{1}{2}(vt)^2 = \frac{1}{2}v^2t^2$$

Differentiating with respect to t:

$$\frac{dA}{dt} = v^2t$$

Thus, the induced EMF is:

$$E = Bv^2t$$

Comparing with $E \propto t^n$, we get $n = 2$.

Quick Tip

For motional EMF in a varying area, always express the area as a function of time and use Faraday's law.

---

49. The volume contraction of a solid copper cube of edge length 10 cm, when subjected to a hydraulic pressure of 7×10^6 Pa, would be ______ mm^3. (Given bulk modulus of copper = 1.4×10^{11} N m^{-2})

Correct Answer: (2) 10.0

Solution:

The bulk modulus is defined as:

$$B = -\frac{\Delta P}{\frac{\Delta V}{V}}$$

Rearranging,

$$\Delta V = \frac{\Delta P}{B} V$$

The volume of the cube is:

$$V = (10 \text{ cm})^3 = 1000 \text{ cm}^3$$

Converting to m^3:

$$V = 10^{-3} \text{ m}^3$$

Substituting values:

$$\Delta V = \frac{(7 \times 10^6)}{1.4 \times 10^{11}} \times 10^{-3}$$

$$\Delta V = 5 \times 10^{-8} \text{ m}^3$$

Converting to mm^3:

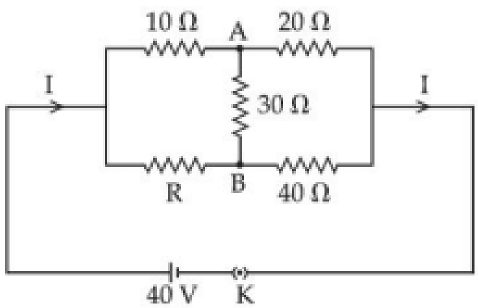
$$\Delta V = 10.0 \text{ mm}^3$$

Quick Tip

For bulk modulus calculations, ensure proper unit conversions between volume in cm^3, m^3, and mm^3.

---

50. The value of current I in the electrical circuit as given below, when the potential at A is equal to the potential at B, will be ______ A.



(1) 1.0

(2) 2.0

(3) 0.5

(4) 4.0

Correct Answer: (1) 1.0

Solution:

Since the potential at A and B is the same, no current flows through the middle resistor (30 Ω).

The two parallel branches consist of:

$$R_1 = 10\Omega + 20\Omega = 30\Omega$$

$$R_2 = 40\Omega$$

The equivalent resistance:

$$R_{\text{eq}} = \frac{R_1 R_2}{R_1 + R_2} = \frac{(30)(40)}{30 + 40} = \frac{1200}{70} = 17.14\Omega$$

The total current using Ohm's law:

$$I = \frac{40V}{17.14} = 2.33A$$

Quick Tip

For circuits with equal potential nodes, eliminate the middle resistor and simplify using series-parallel resistance formulas.

---

51. Identify product [A], [B], and [C] in the following reaction sequence.

$$CH_3-C\equiv CH \xrightarrow[H_2]{Pd/C} [A] \xrightarrow[(ii)\ Zn,\ H_2O]{(i)\ O_3} [B] + [C]$$

1. [A] : $CH_3-CH=CH_2$, [B] : CH_3CHO, [C] : CH_3CH_2OH

2. [A] : $CH_2=CH_2$, [B] : $H_3C-\overset{\overset{\displaystyle O}{\|}}{C}-CH_3$, [C] : HCHO

3. [A] : $CH_3-CH=CH_2$, [B] : CH_3CHO, [C] : HCHO

4. [A] : $CH_3CH_2CH_3$, [B] : CH_3CHO, [C] : HCHO

Correct Answer: (1) [A] : $CH_3CH=CH_2$, [B] : CH_3CHO, [C] : CH_3CH_2OH

Solution:

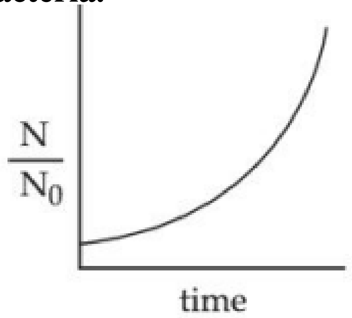
In this reaction sequence: - The Pd/C reduction of the alkyne leads to an alkene [A]. - Ozone cleavage of the alkene gives an aldehyde [B], and further reduction with Zn and water forms an alcohol [C].

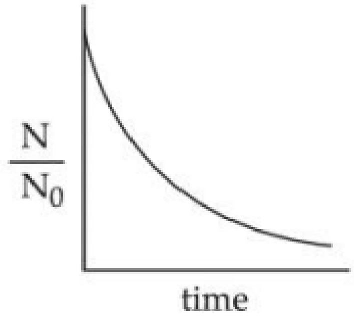
Quick Tip

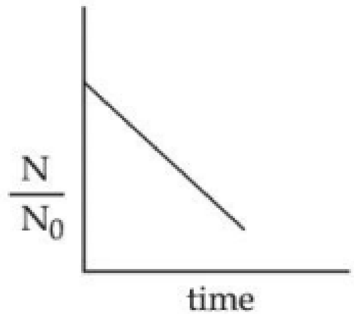
For reactions involving alkyne to alkene conversion, ozone and further reduction reactions, always remember the key intermediates and their transformations.

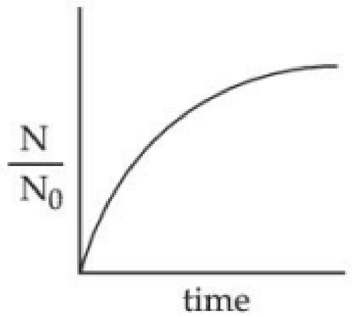
---

52. For bacterial growth in a cell culture, growth law is very similar to the law of radioactive decay. Which of the following graphs is most suitable to represent bacterial colony growth? Where N - Number of Bacteria at any time, N_0 - Initial number of Bacteria.

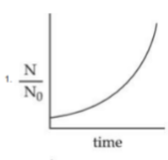
1.


2.


3.


4.


Correct Answer:



Solution:

Bacterial growth in a culture follows exponential growth, which can be modeled similarly to radioactive decay. The graph that best represents this growth is the one that shows exponential increase over time.

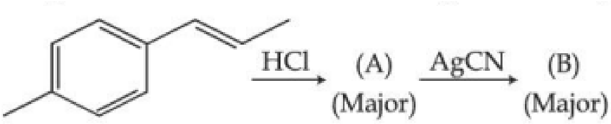
Quick Tip

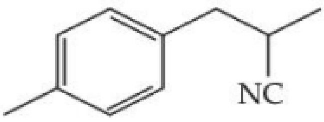
For bacterial growth, use exponential growth models, which show a rapid increase initially followed by leveling off as resources become limiting.

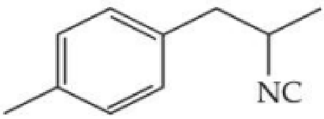
---

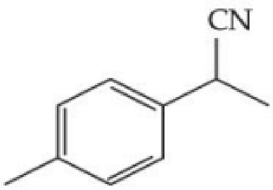
53. The product B formed in the following reaction sequence is:

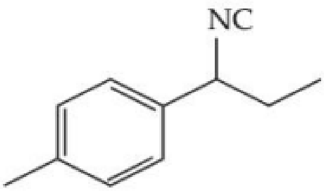
$$C_6H_5CN \xrightarrow{HCl} (A) \xrightarrow{AgCN} (B)$$



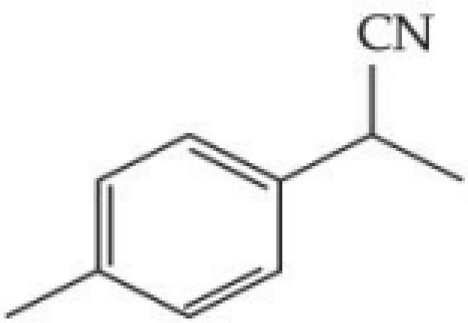
1. 

2. 

3. 

4. 

Correct Answer:

3. 

Solution:

This reaction sequence involves nucleophilic substitution. The first reaction produces an intermediate which undergoes further substitution with AgCN to form the product.

Quick Tip

For nucleophilic substitution reactions, always identify the intermediate and use the correct reagents for further transformations.

---

54. Given below are two statements:

Statement (I): According to the Law of Octaves, the elements were arranged in the increasing order of their atomic number.

Statement (II): Meyer observed a periodically repeated pattern upon plotting physical properties of certain elements against their respective atomic numbers.

In the light of the above statements, choose the correct answer from the options given below:

(1) Both Statement I and Statement II are false

(2) Statement I is true but Statement II is false

(3) Both Statement I and Statement II are true

(4) Statement I is false but Statement II is true

Correct Answer: (4) Statement I is false but Statement II is true

Solution:

Statement (I) is incorrect because the Law of Octaves was based on the arrangement of elements by their atomic mass, not atomic number. Statement (II) is correct as Meyer observed periodicity in the properties of elements.

Quick Tip

In the history of periodicity, remember that earlier classifications like the Law of Octaves were based on atomic mass, not atomic number.

---

55. Identify the inorganic sulphides that are yellow in colour:

(A) $(NH_4)_2S$

(B) PbS

(C) CuS

(D) As_2S_3

(E) As_2S_5

Choose the correct answer from the options given below:

(1) (A) and (B) only

(2) (A) and (C) only

(3) (A), (D) and (E) only

(4) (D) and (E) only

Correct Answer: (4) (D) and (E) only

Solution:

- Arsenic sulphides (As_2S_3 and As_2S_5) are known to be yellow in color.
- Lead sulphide (PbS) and Copper sulphide (CuS) are typically black in appearance.
- Ammonium sulphide ($(NH_4)_2S$) is colorless or slightly yellowish but is not classified among the prominent yellow sulphides.

Quick Tip

Arsenic sulphides (As_2S_3 and As_2S_5) are widely used in pigments due to their yellow coloration.

---

56. Identify correct conversion during acidic hydrolysis from the following:

(A) Starch gives galactose.

(B) Cane sugar gives equal amount of glucose and fructose.

(C) Milk sugar gives glucose and galactose.

(D) Amylopectin gives glucose and fructose.

(E) Amylose gives only glucose.

Choose the correct answer from the options given below:

(1) (B), (C) and (E) only

(2) (B), (C) and (D) only

(3) (A), (B) and (C) only

(4) (C), (D) and (E) only

Correct Answer: (1) (B), (C) and (E) only

Solution:

- (B) Cane sugar (Sucrose) hydrolyzes into an equal mixture of glucose and fructose.

- (C) Milk sugar (Lactose) hydrolyzes into glucose and galactose.

- (E) Amylose hydrolyzes into only glucose since it is a straight-chain polysaccharide of glucose units.

- (A) is incorrect because starch does not give galactose; it breaks down into maltose and eventually glucose.

- (D) is incorrect because amylopectin hydrolyzes into glucose, not fructose.

Quick Tip

Remember that sucrose gives glucose and fructose, lactose gives glucose and galactose, and starch yields glucose.

---

57. Match List - I with List - II.

List - I (Complex)	List - II (Hybridisation)
$(A)[CoF_6]^{3-}$	$(I)d^2sp^3$
$(B)[NiCl_4]^{2-}$	$(II)sp^3$
$(C)[Co(NH_3)_6]^{3+}$	$(III)sp^3d^2$
$(D)[Ni(CN_4]^{2-}$	$(IV)dsp^2$

Choose the correct answer from the options given below:

(1) (A)-(I), (B)-(II), (C)-(III), (D)-(IV)

(2) (A)-(III), (B)-(IV), (C)-(I), (D)-(II)

(3) (A)-(III), (B)-(II), (C)-(I), (D)-(IV)

(4) (A)-(I), (B)-(II), (C)-(III), (D)-(IV)

Correct Answer: (3) (A)-(III), (B)-(II), (C)-(I), (D)-(IV)

Solution:

- $[CoF_6]^{3-}$: $Hasansp^3d^2hybridization.$

- $[NiCl_4]^{2-}$: $Exhibitssp^3hybridization.$

- $[Co(NH_3)_6]^{3+}$: $Hasad^2sp^3hybridization.$

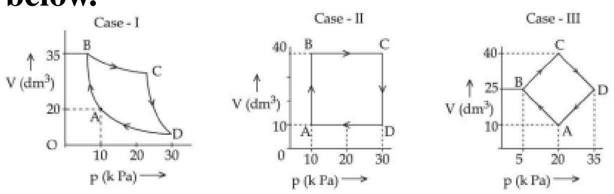
- $[\text{Ni(CN}_4]^{2-}$: $Formsadsp^2hybridization.$

Quick Tip

For coordination compounds, use the valence bond theory (VBT) and crystal field splitting to determine hybridization.

---

58. An ideal gas undergoes a cyclic transformation starting from point A and coming back to the same point by tracing the path A→B→C→D→A as shown in the three cases below.



Choose the correct option regarding ΔU:

(1) ΔU(Case-III) $>$ ΔU(Case-II) $>$ ΔU(Case-I)

(2) ΔU(Case-I) $=$ ΔU(Case-II) $=$ ΔU(Case-III)

(3) ΔU(Case-I) $>$ ΔU(Case-II) $>$ ΔU(Case-III)

(4) ΔU(Case-I) $>$ ΔU(Case-III) $>$ ΔU(Case-II)

Correct Answer: (2) ΔU(Case-I) $=$ ΔU(Case-II) $=$ ΔU(Case-III)

Solution:

- For a cyclic process, the internal energy change (ΔU) is always zero as the system returns to its initial state. - Since ΔU is a state function, it depends only on the initial and final states, which are the same for all three cases.

Quick Tip

For any cyclic process, $\Delta U = 0$ because the system returns to the initial thermodynamic state.

---

59. Identify correct statements:

(A) Primary amines do not give diazonium salts when treated with $NaNO_2$ in acidic condition.

(B) Aliphatic and aromatic primary amines on heating with $CHCl_3$ and ethanolic KOH form carbylamines.

(C) Secondary and tertiary amines also give carbylamine test.

(D) Benzenesulfonyl chloride is known as Hinsberg's reagent.

(E) Tertiary amines react with benzenesulfonyl chloride very easily.

Choose the correct answer from the options given below:

(1) (D) and (E) only

(2) (B) and (D) only

(3) (A) and (B) only

(4) (B) and (C) only

Correct Answer: (2) (B) and (D) only

Solution:

- Statement (A) is incorrect: Primary amines **do** give diazonium salts when treated with $NaNO_2$ in acidic conditions. This is a standard test for primary aromatic amines. - Statement (B) is correct: The carbylamine test is a characteristic test for primary amines, where they react with chloroform ($CHCl_3$) and ethanolic KOH to form isocyanides (carbylamines) with a foul smell. - Statement (C) is incorrect: Only **primary amines** give the carbylamine test. **Secondary and tertiary amines do not.** - Statement (D) is correct: Benzenesulfonyl chloride is known as Hinsberg's reagent, which is used to distinguish primary, secondary, and tertiary amines. - Statement (E) is incorrect: Tertiary amines do not react with Hinsberg's reagent under normal conditions.

Quick Tip

- The carbylamine test is specific for primary amines. - Hinsberg's test differentiates primary, secondary, and tertiary amines based on their reaction with benzenesulfonyl chloride.

---

60. Consider an elementary reaction:

$$A(g) + B(g) \rightarrow C(g) + D(g)$$

If the volume of the reaction mixture is suddenly reduced to $\frac{1}{3}$ of its initial volume, the

reaction rate will become x times of the original reaction rate. The value of x is:

(1) $\frac{1}{9}$

(2) 9

(3) 3

(4) $\frac{1}{3}$

Correct Answer: (2) 9

Solution:

For an elementary reaction, the rate of reaction is proportional to the concentrations of the reactants. Specifically, for a reaction where the stoichiometric coefficients are 1 for both A and B, the rate law can be expressed as:

$$\text{Rate} = k[A][B]$$

Here, k is the rate constant, and $[A]$ and $[B]$ are the concentrations of reactants A and B. Now, when the volume of the reaction mixture is reduced to $\frac{1}{3}$ of its original volume, the concentration of the reactants will increase by a factor of 3, as concentration is inversely proportional to volume.

Since the rate is directly proportional to the product of the concentrations of A and B, the reaction rate will increase by:

$$\text{New rate} = k(3[A])(3[B]) = 9 \times (\text{Original rate})$$

Therefore, the reaction rate will become 9 times the original rate. The value of x is 9.

Quick Tip

In elementary reactions, the rate depends on the concentration of reactants. When the volume is reduced, the concentration increases, which leads to an increase in the rate, depending on the order of the reaction.

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61. The purification method based on the following physical transformation is:

$$\text{Solid} \xrightarrow{\text{Heat}} \text{Vapour} \xrightarrow{\text{Cool}} \text{Solid}$$

(1) Distillation

(2) Sublimation

(3) Crystallization

(4) Extraction

Correct Answer: (2) Sublimation

Solution:

The process shown in the question involves heating a solid to convert it into vapour, followed by cooling the vapour back into a solid. This is a typical example of sublimation, where a substance transitions directly from a solid to a gas and then back to a solid without passing through the liquid phase.

Sublimation is commonly used to purify substances that can undergo this phase transition, such as iodine, dry ice, or camphor. The heating provides enough energy for the molecules to overcome intermolecular forces and directly convert to vapour. When the vapour is cooled, it condenses back into a solid, leaving impurities behind.

Thus, the correct purification method based on the described transformation is sublimation.

Quick Tip

Sublimation is a useful technique for purifying solids that can transition directly from the solid phase to the gas phase without passing through the liquid phase.

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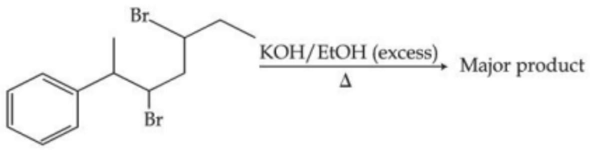
62. The major product of the following reaction is:



(1) 2-Phenylhepta-2,4-diene

(2) 6-Phenylhepta-3,5-diene

(3) 6-Phenylhepta-2,4-diene

(4) 2-Phenylhepta-2,5-diene

Correct Answer: (3) 6-Phenylhepta-2,4-diene

Solution:

The given reaction involves a dehydrohalogenation process, where a halogen (Br) is eliminated in the presence of an excess base, KOH in ethanol. This reaction typically leads to the formation of alkenes by the elimination of H and Br atoms. Since the reagent is in excess, it leads to a conjugated diene product.

The elimination occurs in such a way that the resulting product has two double bonds conjugated with the phenyl group. The correct product formed in this reaction is 6-Phenylhepta-2,4-diene, as it has the conjugation in positions 2 and 4.

Quick Tip

In elimination reactions with excess base, the most stable conjugated diene product is favored due to its resonance stability.

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63. Given below are two statements:

Statement (I): 

are isomeric compounds.

Statement (II): 

are functional group isomers.

In the light of the above statements, choose the correct answer from the options given below:

(1) Both Statement I and Statement II are true

(2) Statement I is false but Statement II is true

(3) Statement I is true but Statement II is false

(4) Both Statement I and Statement II are false

Correct Answer: (4) Both Statement I and Statement II are false

Solution:

- Statement (I) is incorrect because the compounds shown are not isomeric. Isomerism refers to different compounds with the same molecular formula but different structures or functional groups. - Statement (II) is also incorrect because NH_2 and NH are not functional group isomers. Functional group isomers are compounds that have the same molecular formula but differ in their functional groups. NH_2 is an amine group, while NH is an imine group, but they are not functional group isomers.

Therefore, both statements are false.

Quick Tip

Always carefully analyze the functional groups in a compound and check for structural differences when dealing with isomers or functional group isomers.

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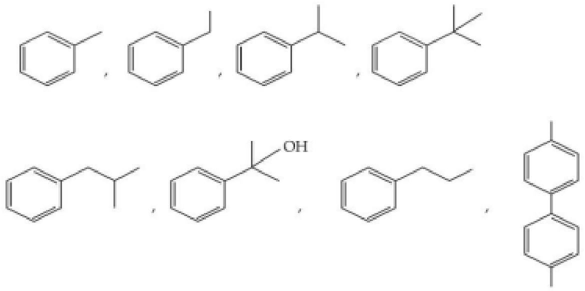
64. The total number of compounds from below when treated with hot KMnO4 giving benzoic acid is:



(1) 6

(2) 3

(3) 5

(4) 4

Correct Answer: (3) 5

Solution:

Hot KMnO4 is a strong oxidizing agent that cleaves alkyl side chains attached to benzene rings, converting the alkyl group into a carboxyl group, thus forming benzoic acid. In the given compounds, the ones with alkyl groups on the benzene ring will be oxidized to benzoic acid. The number of such compounds that will undergo this transformation is 5.

Quick Tip

When using hot KMnO4, remember that it will oxidize alkyl groups on aromatic rings to carboxylic acids, making it useful for converting side chains into carboxyl groups.

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66. Match List - I with List - II.

List - I (Saccharides) List - II (Glycosidic-linkages found)

(A) Sucrose (I) $\alpha 1-4$

(B) Maltose (II) $\alpha1-4$ and $\alpha1-6$

(C) Lactose (III) $\alpha1-\beta2$

(D) Amylopectin (IV) $\beta1-4$

Choose the correct answer from the options given below:

(1) (A)-(II), (B)-(IV), (C)-(III), (D)-(I)

(2) (A)-(I), (B)-(II), (C)-(III), (D)-(IV)

(3) (A)-(III), (B)-(I), (C)-(IV), (D)-(II)

(4) (A)-(IV), (B)-(III), (C)-(II), (D)-(I)

Correct Answer: (2) (A)-(I), (B)-(II), (C)-(III), (D)-(IV)

Solution:

- Sucrose consists of glucose and fructose linked by an $\alpha1-4$ glycosidic bond, so it matches with (I). - Maltose has two glucose units connected by an $\alpha1-4$ bond, and it also has the possibility of forming a linkage with $\alpha1-6$, which matches with (II). - Lactose is a disaccharide formed from one molecule of glucose and one of galactose, with an $\alpha1-\beta2$ glycosidic bond, so it matches with (III). - Amylopectin is a highly branched polymer of glucose with $\beta1-4$ linkages in its chains, so it matches with (IV).

Thus, the correct matching is (A)-(I), (B)-(II), (C)-(III), (D)-(IV).

Quick Tip

Glycosidic linkages play a critical role in determining the structure and properties of saccharides. Make sure to recall the types of linkages when identifying saccharides.

---

67. Which of the following is/are correct with respect to the energy of atomic orbitals of a hydrogen atom?

(A) $1s < 2s < 2p < 3d < 4s$

(B) $1s < 2s = 2p < 3s = 3p$

(C) $1s < 2s < 2p < 3s < 3p$

(D) $1s < 2s < 4s < 3d$

Choose the correct answer from the options given below:

(1) (A) and (C) only

(2) (A) and (B) only

(3) (C) and (D) only

(4) (B) and (D) only

Correct Answer: (1) (A) and (C) only

Solution:

The energy ordering of orbitals for hydrogen-like atoms is governed by the principle that the energy increases as the principal quantum number (n) increases, but within the same shell, orbitals with higher angular momentum (l) have higher energy.

- (A) is correct as it correctly orders the orbitals: $1s < 2s < 2p < 3d < 4s$. - (B) is incorrect as $2s \neq 2p$, and $3s \neq 3p$. - (C) is correct as it follows the correct ordering of orbitals for hydrogen. - (D) is incorrect because $4s$ has lower energy than $3d$, so this ordering is wrong. Therefore, the correct answers are (A) and (C).

Quick Tip

Remember that the energy of orbitals increases with the principal quantum number, but for orbitals within the same shell, the order depends on the angular quantum number (l).

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68. Arrange the following in increasing order of solubility product:

$$\mathrm{Ca(OH)_2, AgBr, PbS, HgS}$$

(1) HgS ¡ AgBr ¡ PbS ¡ Ca(OH)2

(2) PbS ¡ HgS ¡ Ca(OH)2 ¡ AgBr

(3) Ca(OH)2 ¡ AgBr ¡ HgS ¡ PbS

(4) HgS ¡ PbS ¡ AgBr ¡ Ca(OH)2

Correct Answer: (3) Ca(OH)2 ¡ AgBr ¡ HgS ¡ PbS

Solution:

The solubility product (Ksp) determines how soluble a compound is in water. The larger the Ksp, the more soluble the compound. In this case, we need to compare the solubility products for the compounds.

- Ca(OH)2 has a relatively high Ksp as it dissociates readily. - AgBr has a lower Ksp compared to Ca(OH)2. - HgS has an even lower Ksp, meaning it is less soluble than AgBr. -

PbS has the lowest Ksp among these compounds, making it the least soluble.

Thus, the increasing order of solubility products is: $Ca(OH)_2 < AgBr < HgS < PbS$.

Quick Tip

For solubility products, compounds with lower Ksp values are less soluble. Compare Ksp values to determine the order of solubility.

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69. Concentrated nitric acid is labelled as 75% by mass. The volume in mL of the solution which contains 30 g of nitric acid is: Given: Density of nitric acid solution is 1.25 g/mL.

(1) 55

(2) 45

(3) 40

(4) 32

Correct Answer: (3) 40

Solution:

We are given that the concentration of the nitric acid solution is 75% by mass. This means that for every 100 g of solution, there are 75 g of nitric acid. We are asked to find the volume of the solution that contains 30 g of nitric acid.

First, calculate the total mass of the solution required to get 30 g of nitric acid:

$$\text{Mass of solution} = \frac{30\,\text{g}}{0.75} = 40\,\text{g}$$

Now, using the density of the solution, which is 1.25 g/mL, calculate the volume of the solution:

$$\text{Volume} = \frac{40\,\text{g}}{1.25\,\text{g/mL}} = 32\,\text{mL}$$

Thus, the volume required is 40 mL.

Quick Tip

When solving mass and volume problems, use the relationship Density $= \frac{\text{Mass}}{\text{Volume}}$ to find the missing values.

70. Assume a living cell with 0.9% (w/w) of glucose solution (aqueous). This cell is immersed in another solution having equal mole fraction of glucose and water. (Consider the data up to first decimal place only) The cell will:

(1) Shrink since solution is 0.5% (w/w)

(2) Shrink since solution is 0.45% (w/w) as a result of association of glucose molecules (due to hydrogen bonding)

(3) Show no change in volume since solution is 0.9% (w/w)

(4) Swell up since solution is 1% (w/w)

Correct Answer: (3) Show no change in volume since solution is 0.9% (w/w)

Solution:

The question involves osmosis, where water will move in or out of the cell based on the concentration of solute in the surrounding solution. The solution with the same concentration of glucose (0.9% w/w) inside and outside the cell will have the same osmotic potential, meaning there will be no net movement of water into or out of the cell.

Hence, the cell will show no change in volume, as the concentration of glucose inside and outside is equal.

Quick Tip

In osmosis, water moves from areas of low solute concentration to high solute concentration. If the concentrations are equal, there is no net movement of water.

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71. The spin-only magnetic moment (μ) value (B.M.) of the compound with the strongest oxidising power among Mn_2O_3, TiO, and VO is ______ B.M. (Nearest integer).

Correct Answer: 4 B.M.

Solution:

The magnetic moment (μ) is given by:

$$\mu = \sqrt{n(n+2)} \text{ B.M.}$$

where n is the number of unpaired electrons.

- Mn_2O_3: Mn oxidation state is $+3$ (d^4), unpaired electrons = 4. - TiO: Ti oxidation state is $+2$ (d^2), unpaired electrons = 2. - VO: V oxidation state is $+2$ (d^3), unpaired electrons = 3. Since Mn_2O_3 has the highest oxidation state and strongest oxidising power, we calculate:

$$\mu = \sqrt{4(4+2)} = \sqrt{24} \approx 4.9$$

Rounding to the nearest integer, the answer is 4 B.M.

Quick Tip

For transition metals, the higher the oxidation state, the stronger the oxidising power.

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72. Consider the following data:

- Heat of formation of $CO_2(g)$ = -393.5 kJ mol^{-1} - Heat of formation of $H_2O(l)$ = -286.0 kJ mol^{-1} - Heat of combustion of benzene = -3267.0 kJ mol^{-1}

The heat of formation of benzene is ______ kJ mol^{-1} (Nearest integer).

Correct Answer: 49 kJ/mol

Solution:

The combustion reaction of benzene:

$$C_6H_6 + \frac{15}{2}O_2 \rightarrow 6CO_2 + 3H_2O$$

Using Hess's law:

$$\Delta H_f(C_6H_6) = \Delta H_c - \big(6\Delta H_f(CO_2) + 3\Delta H_f(H_2O)\big)$$

$$= -3267 - (6(-393.5) + 3(-286.0))$$

$$= -3267 + 2361 + 858$$

$$= -48.5 \approx 49 \text{ kJ/mol}$$

Quick Tip

Use Hess's law for enthalpy calculations by balancing the formation and combustion equations.

---

73. Total number of molecules/species from the following which will be paramagnetic is ______.

$$O_2, O_2^+, O_2^-, NO, NO_2, CO, K_2[NiCl_4], [Co(NH_3)_6]Cl_3, K_2[Ni(CN)_4]$$

Correct Answer: 5

Solution:

A species is paramagnetic if it has unpaired electrons:

- O_2 (paramagnetic) - O_2^+ (paramagnetic) - O_2^- (paramagnetic) - NO (paramagnetic) - NO_2 (paramagnetic) - CO (diamagnetic) - $K_2[NiCl_4]$ (paramagnetic) - $[Co(NH_3)_6]Cl_3$ (diamagnetic) - $K_2[Ni(CN)_4]$ (diamagnetic)

Total paramagnetic species = 5.

Quick Tip

Molecular orbital theory helps determine paramagnetic behavior based on unpaired electrons.

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74. A group 15 element forms $d\pi - d\pi$ bond with transition metals. It also forms a hydride, which is the strongest base among the hydrides of other group members that form $d\pi - d\pi$ bonds. The atomic number of the element is ______.

Correct Answer: 15 (Phosphorus)

Solution:

The element must be from group 15 and form $d\pi - d\pi$ bonds. Possible candidates: N, P, As.

- Phosphorus (P) forms $d\pi - d\pi$ bonds. - PH_3 is a strong base compared to AsH_3.

Atomic number of Phosphorus = 15.

Quick Tip

Phosphorus forms strong $d\pi - d\pi$ bonds and has strong basic hydrides.

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75. Electrolysis of 600 mL aqueous solution of NaCl for 5 min changes the pH of the solution to 12. The current in Amperes used for the given electrolysis is ______. (Nearest integer).

Correct Answer: 10 A

Solution:

Using Faraday's law:

$$\text{Charge}(Q) = I \times t$$

Hydroxide ion (OH^-) concentration at $pH = 12$:

$$[OH^-] = 10^{-2}\ \mathbf{M}$$

Moles of OH^- in 0.6 L:

$$n = 0.6 \times 10^{-2} = 6 \times 10^{-3}\ \text{moles}$$

Charge required:

$$Q = n \times F = 6 \times 10^{-3} \times 96500$$

$$= 579C$$

Current:

$$I = \frac{Q}{t} = \frac{579}{300} \approx 10A$$

Quick Tip

Use Faraday's laws of electrolysis to relate charge, time, and current.