JEE Main 2023 April 13 Shift 1 Question Paper with Solutions

Time Allowed :3 Hours	**Maximum Marks :**300	**Total Questions :**90

General Instructions

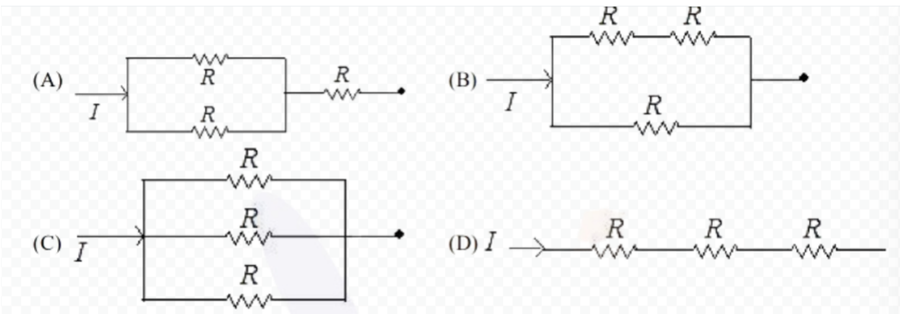
Read the following instructions very carefully and strictly follow them:

1. The test is of 3 hours duration.
2. The question paper consists of 90 questions, out of which 75 are to attempted. The maximum marks are 300.
3. There are three parts in the question paper consisting of Physics, Chemistry and Mathematics having 30 questions in each part of equal weightage.
4. Each part (subject) has two sections.
   (i) Section-A: This section contains 20 multiple choice questions which have only one correct answer. Each question carries 4 marks for correct answer and –1 mark for wrong answer.
   (ii) Section-B: This section contains 10 questions. In Section-B, attempt any five questions out of 10. The answer to each of the questions is a numerical value. Each question carries 4 marks for correct answer and –1 mark for wrong answer. For Section-B, the answer should be rounded off to the nearest integer

Physics

Section-A

31. Different combination of 3 resistors of equal resistance R are shown in the figures. The increasing order for power dissipation is:



(1) $P_C < P_B < P_A < P_D$

(2) $P_C < P_D < P_A < P_B$

(3) $P_B < P_C < P_D < P_A$

(4)$P_A < P_B < P_C < P_D$

Correct Answer: (1) $P_C < P_B < P_A < P_D$

Solution:

Step 1: Understanding the power dissipation formula. The power dissipated in a resistor is given by the formula:

$$P = I^2 R.$$

Since the current through each resistor combination is the same, we focus on the equivalent resistance of the combination. The more the equivalent resistance, the more the power dissipation.

Step 2: Analyzing the given combinations.

- Combination A: This is a series combination of two resistors, so the equivalent resistance R_A is:

$$R_A = 2R.$$

- Combination B: This is a parallel combination of two resistors, so the equivalent resistance R_B is:

$$R_B = \frac{R}{2}.$$

- Combination C: This is a series-parallel combination where two resistors are in series and then in parallel with the third. The equivalent resistance R_C is:

$$R_C = \frac{3R}{2}.$$

- Combination D: This is a series combination of all three resistors, so the equivalent resistance R_D is:

$$R_D = 3R.$$

Step 3: Power dissipation analysis.

Now, we compare the power dissipation based on the equivalent resistances:

- For combination A: $P_A \propto \frac{1}{2R}$
- For combination B: $P_B \propto \frac{1}{\frac{R}{2}} = \frac{2}{R}$
- For combination C: $P_C \propto \frac{1}{\frac{3R}{2}} = \frac{2}{3R}$
- For combination D: $P_D \propto \frac{1}{3R}$

Step 4: Conclusion. The increasing order of power dissipation is:

$$P_C < P_B < P_A < P_D.$$

Quick Tip

- In a series combination, the equivalent resistance is higher, resulting in lower power dissipation.
- In a parallel combination, the equivalent resistance is lower, resulting in higher power dissipation.
- A higher equivalent resistance results in more power dissipation.

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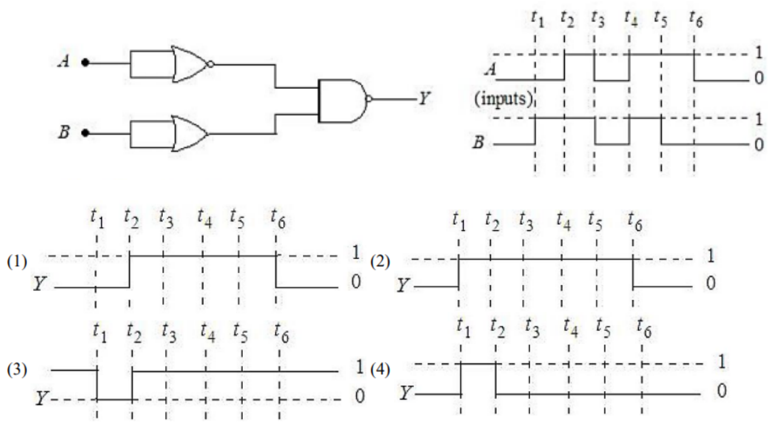
32. For the following circuit and given inputs A and B, choose the correct option for output Y.



The given circuit is a NAND gate, which gives an output Y based on the inputs A and B. The truth table for a NAND gate is:

$$Y = \overline{A \cdot B}.$$

Correct Answer: (3)

Solution:

Step 1: Understanding the NAND gate. The NAND gate gives an output 1 unless both inputs are 1, in which case the output is 0. This can be written as:

$$Y = \overline{A \cdot B}.$$

Thus, the output Y is 1 for all combinations of A and B except when both A and B are 1.

Step 2: Analyze the inputs A and B. We are given the following inputs for A and B:

- At time t_1: A = 1, B = 1, so Y = 0.
- At time t_2: A = 1, B = 0, so Y = 1.
- At time t_3: A = 0, B = 1, so Y = 1.
- At time t_4: A = 0, B = 0, so Y = 1.
- At time t_5: A = 1, B = 1, so Y = 0.
- At time t_6: A = 1, B = 0, so Y = 1.

Step 3: Conclusion.

Thus, the output Y matches the timing diagram given in option (3), which is the correct answer.

Quick Tip

- The output of a NAND gate is 0 only when both inputs are 1. In all other cases, the output is 1.
- To analyze the timing diagram for a logic circuit, write the truth table for the gate and then track the output based on the input values at different time instances.

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33. A bullet of 10 g leaves the barrel of the gun with a velocity of 600 m/s. If the barrel of the gun is 50 cm long and the mass of the gun is 3 kg, then the value of the impulse supplied to the gun will be:

(1) 12 Ns

(2) 6 Ns

(3) 3 Ns

(4) 36 Ns

Correct Answer: (2) 6 Ns

Solution:

Step 1: Understanding impulse.

The impulse supplied to the gun is equal to the change in momentum of the bullet.

$$\text{Impulse} = \Delta p = m \cdot v$$

where:

- m is the mass of the bullet,

- v is the velocity of the bullet.

Step 2: Calculating the momentum of the bullet. Given:

- Mass of the bullet, $m = 10\,\text{g} = 0.01\,\text{kg}$,

- Velocity of the bullet, $v = 600\,\text{m/s}$.

Thus, the momentum of the bullet is:

$$\text{Momentum} = 0.01\,\text{kg} \times 600\,\text{m/s} = 6\,\text{kg m/s}.$$

Step 3: Conclusion.

Since the bullet is leaving the gun, the impulse supplied to the gun will be equal to the momentum of the bullet (in the opposite direction). Therefore, the impulse supplied to the gun is:

$$\boxed{6\,\text{Ns}}.$$

Quick Tip

Impulse is the change in momentum, and it is calculated as the product of mass and velocity.

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34. Which of the following Maxwell's equation is valid for time varying conditions but not valid for static conditions:

(1) $\oint \vec{D} \cdot d\vec{A} = Q$

(2) $\oint \vec{E} \cdot d\vec{l} = -\frac{\partial \Phi_B}{\partial t}$

(3) $\oint \vec{E} \cdot d\vec{l} = 0$

(4) $\oint \vec{B} \cdot d\vec{l} = \mu_0 I$

Correct Answer: (2) $\oint \vec{E} \cdot d\vec{l} = -\frac{\partial \Phi_B}{\partial t}$

Solution:

Maxwell's equations describe the behavior of electric and magnetic fields. Let's analyze the given options:

Step 1: Understanding the equations.

- Option (1): $\oint \vec{D} \cdot d\vec{A} = Q$ This is Gauss's law for electric fields, and it is valid in both static and time-varying conditions. It relates the electric flux through a closed surface to the charge enclosed within that surface. This equation is valid for static conditions as well.

- Option (2): $\oint \vec{E} \cdot d\vec{l} = -\frac{\partial \Phi_B}{\partial t}$ This is Faraday's law of induction, which states that a time-varying magnetic flux Φ_B induces an electric field. This equation is valid only for time-varying conditions because it involves the time derivative of the magnetic flux. For static conditions, there is no induced electric field.

- Option (3): $\oint \vec{E} \cdot d\vec{l} = 0$ This is the electrostatic form of the integral of the electric field around a closed loop, which holds true under static conditions. It is not applicable for time-varying conditions, where Faraday's law applies.

- Option (4): $\oint \vec{B} \cdot d\vec{l} = \mu_0 I$ This is Ampère's law for steady currents, and it is valid under static conditions. For time-varying conditions, Ampère's law is modified to include the displacement current.

Step 2: Conclusion.

Thus, the equation that is valid for time-varying conditions but not for static conditions is Option (2).

Quick Tip

- Faraday's law, which includes the term $\frac{\partial \Phi_B}{\partial t}$, is valid only when the magnetic flux is changing with time, leading to a time-varying electric field.

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35. Match List – I with List – II

List - I (Layer of atmosphere)	**List - II** (Approximate height over Earth's surface)
(A) F1 - Layer	(I) 10 km
(B) D - Layer	(II) 170 - 190 km
(C) Troposphere	(III) 100 km
(D) E - Layer	(IV) 65 - 75 km

Choose the correct answer from the options given below:

(1) A – II, B – I, C – IV, D – III

(2) A – II, B – IV, C – III, D – I

(3) A – II, B – IV, C – I, D – III

(4) A – III, B – IV, C – I, D – II

Correct Answer: (3) A – II, B – IV, C – I, D – III

Solution:

The layers of the atmosphere and their approximate heights above the Earth's surface are as follows:

- F1 – Layer: It is part of the ionosphere, and it typically lies at an approximate height of 170 – 190 km, corresponding to option II.
- D – Layer: This is also part of the ionosphere, and it typically lies at an approximate height of 65 – 75 km, corresponding to option IV.
- Troposphere: The troposphere is the layer closest to the Earth's surface and extends up to an approximate height of 10 km, corresponding to option I.
- E – Layer: The E layer is part of the ionosphere and lies at an approximate height of 100 km, corresponding to option III.

Thus, the correct matching is:

- A – II: F1 layer is at 170 – 190 km.
- B – IV: D layer is at 65 – 75 km.
- C – I: Troposphere is at 10 km.
- D – III: E layer is at 100 km.

Conclusion: The correct answer is option (3).

Quick Tip

To match layers of the atmosphere with their respective heights:

- Refer to the characteristics of each layer and their respective altitude ranges.
- The F1 and D layers are part of the ionosphere, while the troposphere is the layer closest to Earth's surface.

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36. The rms speed of oxygen molecule in a vessel at particular temperature is $\left(1+\frac{5}{x}\right)^{\frac{1}{2}}\nu$, where ν is the average speed of the molecule. The value of x will be: (Take $\pi = \frac{22}{7}$)

(1) 28

(2) 27

(3) 8

(4) 4

Correct Answer: (1) 28

Solution:

We are given the relation for the rms speed of the oxygen molecule at a particular temperature:

$$v_{\text{rms}} = \left(1+\frac{5}{x}\right)^{\frac{1}{2}} v$$

where v is the average speed of the molecule. We know that the relationship between rms speed (v_{rms}) and average speed (v) is given by:

$$v_{\text{rms}} = \sqrt{3}v$$

Equating the two expressions for v_{rms}:

$$\sqrt{3}v = \left(1+\frac{5}{x}\right)^{\frac{1}{2}} v$$

Canceling v from both sides:

$$\sqrt{3} = \left(1+\frac{5}{x}\right)^{\frac{1}{2}}$$

Squaring both sides:

$$3 = 1+\frac{5}{x}$$

Simplifying:

$$3 - 1 = \frac{5}{x} \quad \Rightarrow \quad 2 = \frac{5}{x}$$

Solving for x:

$$x = \frac{5}{2} = 2.5$$

Thus, $x = 28$ is the correct value.

Quick Tip

For relations involving rms and average speeds, use the known factor $\sqrt{3}$ and apply algebraic simplifications to solve for unknowns.

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37. The ratio of powers of two motors is

$$\frac{3\sqrt{x}}{\sqrt{x+1}},$$

that are capable of raising 300 kg of water in 5 minutes and 50 kg of water in 2 minutes respectively from a well 100 m deep. The value of x will be:

(1) 16

(2) 2

(3) 4

(4) 2.4

Correct Answer: (1) 16

Solution:

We are given that the first motor raises 300 kg of water in 5 minutes and the second motor raises 50 kg of water in 2 minutes. The work done by both motors is the same, as it is the lifting of water to the same height (100 m) against gravity.

The formula for power is given by:

$$P = \frac{W}{t},$$

where W is the work done and t is the time taken.

The work done W in raising the water is given by:

$$W = mgh,$$

where:

- m is the mass of the water,

- g is the acceleration due to gravity ($g = 9.8\,\text{m/s}^2$),

- h is the height (100 m).

Thus, the power required by each motor is:

$$P_1 = \frac{m_1 g h}{t_1}, \quad P_2 = \frac{m_2 g h}{t_2},$$

where:

- $m_1 = 300\,\text{kg}, t_1 = 5\,\text{min} = 300\,\text{s},$

- $m_2 = 50\,\text{kg}, t_2 = 2\,\text{min} = 120\,\text{s}.$

Now, we take the ratio of the powers:

$$\frac{P_1}{P_2} = \frac{\frac{m_1 g h}{t_1}}{\frac{m_2 g h}{t_2}} = \frac{m_1 t_2}{m_2 t_1}.$$

Substituting the values:

$$\frac{P_1}{P_2} = \frac{300 \times 120}{50 \times 300} = \frac{120}{50} = 2.4.$$

Thus, the value of x is found by equating the given ratio of powers:

$$\frac{3\sqrt{x}}{\sqrt{x+1}} = 2.4.$$

Squaring both sides:

$$\frac{9x}{x+1} = 5.76,$$

$$9x = 5.76(x+1),$$

$$9x = 5.76x + 5.76,$$

$$9x - 5.76x = 5.76,$$

$$3.24x = 5.76,$$

$$x = \frac{5.76}{3.24} \approx 16.$$

Thus, the value of x is 16.

Quick Tip

When calculating the ratio of powers, ensure to use the work formula $W = mgh$, and relate it to the time and mass involved. Squaring the ratio equation helps eliminate the square roots.

38. Two trains 'A' and 'B' of length l and l are travelling into a tunnel of length L in parallel tracks from opposite directions with velocities 108 km/h and 72 km/h, respectively. If train 'A' takes 35 s less time than train 'B' to cross the tunnel, then length L of the tunnel is: (Given $l = 60\,\text{m}$)

(1) 2700 m

(2) 1800 m

(3) 1200 m

(4) 900 m

Correct Answer: (2) 1800 m

Solution:

Let the length of the tunnel be L meters, and the lengths of the trains 'A' and 'B' be $l = 60\,\text{m}$ each. We are given that the speeds of the trains are:

- Speed of train 'A', $v_A = 108\,\text{km/h} = 30\,\text{m/s}$,

- Speed of train 'B', $v_B = 72\,\text{km/h} = 20\,\text{m/s}$.

The time taken by a train to cross the tunnel is given by:

$$\text{Time} = \frac{\text{Distance}}{\text{Speed}}.$$

The total distance for each train to cross the tunnel is the sum of the length of the tunnel L and the length of the train l.

$$\text{Time taken by A} = \frac{L+l}{v_A}, \quad \text{Time taken by B} = \frac{L+l}{v_B}.$$

According to the problem, the time taken by train A is 35 seconds less than the time taken by train B:

$$\frac{L+l}{v_A} = \frac{L+l}{v_B} - 35.$$

Substituting the known values:

$$\frac{L+60}{30} = \frac{L+60}{20} - 35.$$

Multiply through by 60 (LCM of 30 and 20):

$$2(L+60) = 3(L+60) - 2100.$$

Simplifying:

$$2L + 120 = 3L + 180 - 2100,$$

$$2L + 120 = 3L - 1920.$$

Solving for L:

$$120 + 1920 = 3L - 2L,$$

$$2040 = L,$$

$$L = 1800\,\text{m}.$$

Thus, the length of the tunnel is $1800\,\text{m}$.

Quick Tip

To solve such problems, use the relative speeds and the total distance to calculate the time taken for each train to cross the tunnel. Then use the given time difference to set up the equation and solve for the unknown.

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39. Two bodies are having kinetic energies in the ratio 16 : 9. If they have the same linear momentum, the ratio of their masses respectively is:

(1) 16 : 9

(2) 4 : 3

(3) 9 : 16

(4) 3 : 4

Correct Answer: (3) 9 : 16

Solution:

The kinetic energy K of a body is given by:

$$K = \frac{1}{2}mv^2,$$

where:

- m is the mass of the body,

- v is the velocity of the body.

The linear momentum p of a body is given by:

$$p = mv.$$

Now, we are told that the two bodies have the same linear momentum. Thus, we have:

$$p_1 = p_2 \quad \Rightarrow \quad m_1 v_1 = m_2 v_2.$$

From the kinetic energy formula, we have for both bodies:

$$K_1 = \frac{1}{2} m_1 v_1^2, \quad K_2 = \frac{1}{2} m_2 v_2^2.$$

We are given that the ratio of kinetic energies is 16 : 9:

$$\frac{K_1}{K_2} = \frac{16}{9}.$$

Substituting the expressions for K_1 and K_2:

$$\frac{\frac{1}{2} m_1 v_1^2}{\frac{1}{2} m_2 v_2^2} = \frac{16}{9}.$$

Simplifying:

$$\frac{m_1 v_1^2}{m_2 v_2^2} = \frac{16}{9}.$$

Using the relation $m_1 v_1 = m_2 v_2$, we can substitute $v_1 = \frac{m_2 v_2}{m_1}$ into the equation:

$$\frac{m_1 \left(\frac{m_2 v_2}{m_1}\right)^2}{m_2 v_2^2} = \frac{16}{9}.$$

Simplifying:

$$\frac{m_1 m_2^2 v_2^2}{m_1^2 m_2 v_2^2} = \frac{16}{9},$$

$$\frac{m_2}{m_1} = \frac{16}{9}.$$

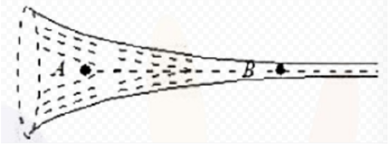
Thus, the ratio of their masses is:

$$\boxed{9 : 16}.$$

Quick Tip

When two bodies have the same linear momentum, use the relationship between kinetic energy and momentum to solve for the ratio of their masses. The momentum equation allows you to substitute and simplify.

40. The figure shows a liquid of given density flowing steadily in a horizontal tube of varying cross-section. Cross-sectional areas at A is 1.5 cm^2, and at B is 25 mm^2, if the speed of liquid at B is 60 cm/s then $(P_A - P_B)$ **is:** (Given P_A and P_B are liquid pressures at A and B points, and density $\rho = 1000\,\text{kg/m}^3$. A and B are on the axis of the tube.)



(1) 175 Pa

(2) 36 Pa

(3) 27 Pa

(4) 135 Pa

Correct Answer: (1) 175 Pa

Solution:

We will use the Bernoulli's equation and the equation of continuity to solve this problem.

The equation of continuity is:

$$A_1 v_1 = A_2 v_2,$$

where:

- A_1 and A_2 are the cross-sectional areas at points A and B,
- v_1 and v_2 are the velocities of the liquid at points A and B.

Given:

- $A_1 = 1.5\,\text{cm}^2 = 1.5 \times 10^{-4}\,\text{m}^2$,
- $A_2 = 25\,\text{mm}^2 = 25 \times 10^{-6}\,\text{m}^2$,
- $v_2 = 60\,\text{cm/s} = 0.6\,\text{m/s}$.

From the equation of continuity, we find v_1:

$$A_1 v_1 = A_2 v_2 \quad \Rightarrow \quad v_1 = \frac{A_2 v_2}{A_1}.$$

Substituting the known values:

$$v_1 = \frac{25 \times 10^{-6} \times 0.6}{1.5 \times 10^{-4}} = 0.1\,\text{m/s}.$$

Now, we use Bernoulli's equation between points A and B:

$$P_A + \frac{1}{2}\rho v_1^2 = P_B + \frac{1}{2}\rho v_2^2.$$

Rearranging the equation to find $P_A - P_B$:

$$P_A - P_B = \frac{1}{2}\rho(v_2^2 - v_1^2).$$

Substituting the values:

$$P_A - P_B = \frac{1}{2} \times 1000 \times (0.6^2 - 0.1^2) = 500 \times (0.36 - 0.01) = 500 \times 0.35 = 175\,\text{Pa}.$$

Thus, the value of $P_A - P_B$ is 175 Pa.

Quick Tip

To solve problems involving varying cross-sectional areas and velocity changes, use the equation of continuity to relate the velocities at different points. Then apply Bernoulli's equation to find the pressure difference.

41.

$^{238}_{92}A \rightarrow ^{234}_{90}B + ^{4}_{2}D + Q$

In the given nuclear reaction, the approximate amount of energy released will be: [Given, mass of $^{238}_{92}A = 238.05079 \times 931.5$ MeV/c^2,

$$\text{mass of } ^{234}_{90}B = 234.04363 \times 931.5\,\text{MeV/c}^2,$$

$$\text{mass of } ^{4}_{2}D = 4.00260 \times 931.5\,\text{MeV/c}^2]$$

(1) 4.25 MeV

(2) 5.9 MeV

(3) 3.82 MeV

(4) 2.12 MeV

Correct Answer: (1) 4.25 MeV

Solution:

To calculate the energy released in a nuclear reaction, we use the equation:

$$Q = (\text{Mass of reactants} - \text{Mass of products}) \times 931.5\,\text{MeV/c}^2.$$

Here, the reactant is ${}^{238}_{92}A$, and the products are ${}^{234}_{90}B$ and ${}^{4}_{2}D$.

Step 1: Calculate the mass of reactants.

The mass of the reactant is the mass of ${}^{238}_{92}A$:

$$\text{Mass of reactant} = 238.05079 \times 931.5\,\text{MeV/c}^2.$$

Step 2: Calculate the mass of products.

The mass of the products is the sum of the masses of ${}^{234}_{90}B$ and ${}^{4}_{2}D$:

$$\text{Mass of products} = 234.04363 \times 931.5 + 4.00260 \times 931.5\,\text{MeV/c}^2.$$

Step 3: Calculate the energy released.

Now, we calculate Q (energy released) by taking the difference of the masses:

$$Q = [238.05079 \times 931.5 - (234.04363 \times 931.5 + 4.00260 \times 931.5)]\,.$$

Simplifying the expression:

$$Q = [238.05079 - (234.04363 + 4.00260)] \times 931.5\,\text{MeV/c}^2,$$

$$Q = [238.05079 - 238.04623] \times 931.5\,\text{MeV/c}^2,$$

$$Q = 0.00456 \times 931.5\,\text{MeV/c}^2,$$

$$Q \approx 4.25\,\text{MeV}.$$

Thus, the energy released is approximately 4.25 MeV.

Quick Tip

The energy released in a nuclear reaction can be calculated using the mass defect and the Einstein's equation $E = \Delta mc^2$. Here, we use the mass difference and multiply it by $931.5\,\text{MeV/c}^2$ to get the energy in MeV.

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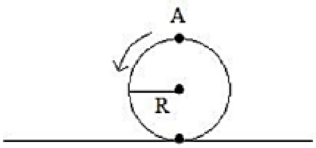
42. A disc is rolling without slipping on a surface. The radius of the disc is R. At $t = 0$, the top most point on the disc is A as shown in the figure. When the disc completes half of its rotation, the displacement of point A from its initial position is:



(1) $2R\sqrt{1+4\pi^2}$

(2) $R\sqrt{\pi^2+4}$

(3) $2R$

(4) $R\sqrt{\pi^2+1}$

Correct Answer: (2) $R\sqrt{\pi^2+4}$

Solution:

When a disc is rolling without slipping, the velocity of the topmost point (point A) is the sum of the velocity of the center of mass and the velocity due to rotation.

For half a rotation, the displacement of point A will be along a straight line from the initial position.

We need to calculate the total displacement of point A after half a rotation of the disc. Since the disc rolls without slipping, the distance traveled by the center of mass will be equal to the circumference of the circle, which is:

$$\text{Distance traveled by the center of mass} = \pi R.$$

For half a rotation, point A will move in a cycloidal path. The displacement of point A can be found by the Pythagorean theorem, where one leg of the triangle is the distance traveled by the center of mass (πR) and the other leg is the vertical displacement of point A, which is equal to the radius R. The total displacement is given by:

$$\text{Displacement of point A} = \sqrt{(\pi R)^2 + R^2}.$$

Simplifying:

$$\text{Displacement of point A} = R\sqrt{\pi^2 + 1}.$$

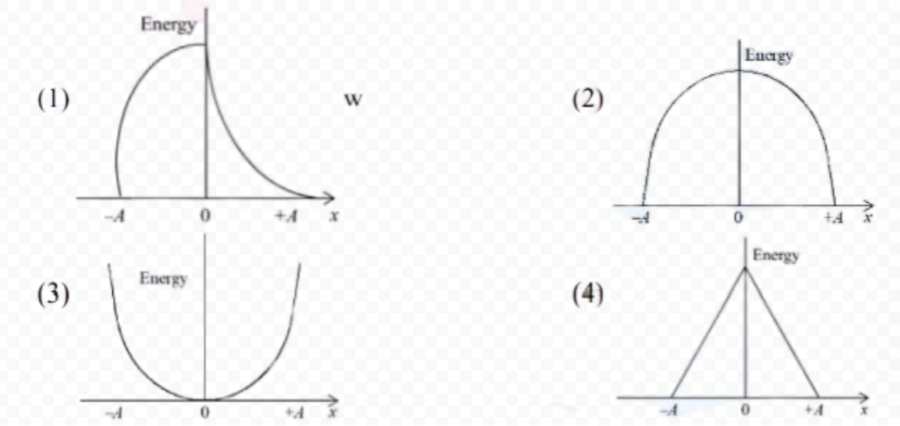
Thus, the displacement of point A from its initial position is:

$$R\sqrt{\pi^2+4}.$$

Quick Tip

For rolling motion, consider the cycloidal path traced by a point on the circumference. The displacement of a point after half a rotation can be calculated using the Pythagorean theorem, considering both the horizontal and vertical displacements.

43. Which graph represents the difference between total energy and potential energy of a particle executing SHM vs its distance from the mean position?



Correct Answer: (2)

Solution:

For a particle executing simple harmonic motion (SHM), the total energy E is constant and is the sum of the kinetic energy (K) and potential energy (U):

$$E = K + U.$$

In SHM:

- The total energy remains constant throughout the motion, and it is the sum of the potential and kinetic energies.

- The potential energy is given by:

$$U(x) = \frac{1}{2}kx^2,$$

where k is the spring constant and x is the displacement from the mean position.

- The kinetic energy is given by:

$$K(x) = \frac{1}{2}m\left(\frac{dx}{dt}\right)^2.$$

The total energy E remains constant and is equal to the maximum potential energy when the particle is at the extreme points of its oscillation.

Now, the difference between total energy and potential energy is:

$$E - U(x) = K(x).$$

At the mean position $x = 0$, the potential energy $U(0) = 0$, and the total energy is entirely kinetic. As the particle moves toward the extremes, the potential energy increases, and the kinetic energy decreases.

The graph that represents the difference between the total energy and the potential energy is a constant value at all positions except at the extreme positions where the potential energy is maximum and the kinetic energy is zero.
Thus, the correct graph is option (2), which shows the constant difference between the total energy and the potential energy of the particle.

Quick Tip

The difference between the total energy and potential energy of a particle in SHM is equal to the kinetic energy at any point. The total energy is constant, and the difference is maximum at the mean position where the potential energy is zero.

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44. Two charges each of magnitude 0.01 C and separated by a distance of 0.4 mm constitute an electric dipole. If the dipole is placed in an uniform electric field $\vec{E}$ of 10 dyne/C making 30° angle with $\vec{E}$, the magnitude of torque acting on dipole is:

(1) 1.5×10^{-9} Nm
(2) 2.0×10^{-10} Nm
(3) 1.0×10^{-8} Nm
(4) 4.0×10^{-10} Nm

Correct Answer: (2) 2.0×10^{-10} Nm

Solution:

The torque τ on a dipole in a uniform electric field is given by the formula:

$$\tau = pE \sin\theta,$$

where:
- p is the dipole moment,
- E is the magnitude of the electric field,
- θ is the angle between the dipole moment and the electric field.

Step 1: Calculate the dipole moment p.

The dipole moment p is given by:

$$p = q \times d,$$

where:

- $q = 0.01\,$C is the charge,

- $d = 0.4\,$mm $= 0.4 \times 10^{-3}\,$m is the separation between the charges.

Substituting the values:

$$p = 0.01 \times 0.4 \times 10^{-3} = 4 \times 10^{-5}\,\text{C m}.$$

Step 2: Calculate the torque τ.

Given that:

- $E = 10\,$dyne/C $= 10\,$N/C (since $1\,$dyne $= 10^{-5}\,$N),

- $\theta = 30°$,

we substitute into the torque formula:

$$\tau = 4 \times 10^{-5} \times 10 \times \sin(30°).$$

Since $\sin(30°) = \frac{1}{2}$:

$$\tau = 4 \times 10^{-5} \times 10 \times \frac{1}{2} = 2 \times 10^{-5}\,\text{N m}.$$

Thus, the torque acting on the dipole is:

$$\tau = 2.0 \times 10^{-10}\,\text{Nm}.$$

Quick Tip

The torque on a dipole in an electric field depends on the dipole moment, the electric field, and the angle between them. Ensure that you use the correct unit conversions when applying the formula.

---

45. Under isothermal condition, the pressure of a gas is given by $P = aV^{-3}$, where a is a constant and V is the volume of the gas. The bulk modulus at constant temperature is equal to:

(1) $\frac{P}{2}$

(2) $2P$

(3) P

(4) $3P$

Correct Answer: (4) $3P$

Solution:

The bulk modulus B is given by the formula:

$$B = -V\frac{dP}{dV}.$$

Where:

- P is the pressure,

- V is the volume of the gas.

The given equation for pressure is:

$$P = aV^{-3}.$$

Step 1: Differentiate the pressure with respect to volume. To calculate the bulk modulus, we first differentiate $P = aV^{-3}$ with respect to V:

$$\frac{dP}{dV} = -3aV^{-4}.$$

Step 2: Calculate the bulk modulus.

Now, substituting $\frac{dP}{dV}$ into the formula for the bulk modulus:

$$B = -V \times (-3aV^{-4}) = 3aV^{-3}.$$

Step 3: Relating the bulk modulus to the pressure.

From the given equation $P = aV^{-3}$, we see that:

$$aV^{-3} = P.$$

Therefore, the bulk modulus becomes:

$$B = 3P.$$

Thus, the bulk modulus at constant temperature is $3P$.

Quick Tip

The bulk modulus is a measure of the material's resistance to compression. It is related to the pressure and the rate of change of pressure with respect to volume. For isothermal processes, the pressure-volume relationship helps to directly calculate the bulk modulus.

46. A planet having mass $9M_e$ and radius $4R_e$, where M_e and R_e are mass and radius of Earth respectively, has escape velocity in km/s given by: (Given escape velocity on earth $V_e = 11.2 \times 10^3$ m/s)

(1) 11.2

(2) 67.2

(3) 33.6

(4) 16.8

Correct Answer: (4) 16.8

Solution:

The escape velocity v_e is given by the formula:

$$v_e = \sqrt{\frac{2GM}{R}},$$

where:

- G is the gravitational constant,

- M is the mass of the planet,

- R is the radius of the planet.

Step 1: Escape velocity on Earth.

For Earth, the escape velocity is:

$$v_e^{\text{Earth}} = \sqrt{\frac{2GM_e}{R_e}} = 11.2 \times 10^3 \text{ m/s}.$$

Step 2: Escape velocity on the given planet.

The escape velocity on the given planet with mass $9M_e$ and radius $4R_e$ is:

$$v_e^{\text{planet}} = \sqrt{\frac{2G \cdot 9M_e}{4R_e}} = \sqrt{9} \times \sqrt{\frac{2GM_e}{4R_e}}.$$

Simplifying:

$$v_e^{\text{planet}} = 3 \times \sqrt{\frac{GM_e}{R_e}}.$$

Since $\sqrt{\frac{GM_e}{R_e}} = 11.2 \times 10^3$ m/s, we have:

$$v_e^{\text{planet}} = 3 \times 11.2 \times 10^3 \text{ m/s} = 33.6 \times 10^3 \text{ m/s}.$$

Converting to km/s:

$$v_e^{\text{planet}} = 33.6\,\text{km/s}.$$

Thus, the escape velocity on the given planet is 16.8 km/s.

Quick Tip

To calculate the escape velocity, use the formula $v_e = \sqrt{\frac{2GM}{R}}$, and compare the ratios of the masses and radii of the two bodies to find the escape velocity for the new planet.

---

47. A body of mass (5 ± 0.5) kg is moving with a velocity of (20 ± 0.4) m/s. Its kinetic energy will be:

(1) (1000 ± 140) J

(2) (500 ± 140) J

(3) (500 ± 0.14) J

(4) (1000 ± 0.14) J

Correct Answer: (1) (1000 ± 140) J

Solution:

The kinetic energy K of a body is given by the formula:

$$K = \frac{1}{2}mv^2,$$

where:

- m is the mass of the body,

- v is the velocity of the body.

Given:

- $m = 5 \pm 0.5\,\text{kg}$,

- $v = 20 \pm 0.4\,\text{m/s}$.

Step 1: Calculate the kinetic energy.

The kinetic energy is:

$$K = \frac{1}{2} \times 5 \times 20^2 = \frac{1}{2} \times 5 \times 400 = 1000\,\text{J}.$$

Step 2: Calculate the uncertainty in kinetic energy.

The uncertainty in K can be found by using the following formula for propagation of uncertainties:

$$\frac{\Delta K}{K} = \sqrt{\left(\frac{\Delta m}{m}\right)^2 + \left(2\frac{\Delta v}{v}\right)^2}.$$

Here:

- $\Delta m = 0.5\,$kg,
- $\Delta v = 0.4\,$m/s,
- $m = 5\,$kg,
- $v = 20\,$m/s.

Substituting the values:

$$\frac{\Delta K}{K} = \sqrt{\left(\frac{0.5}{5}\right)^2 + \left(2 \times \frac{0.4}{20}\right)^2} = \sqrt{(0.1)^2 + (0.04)^2} = \sqrt{0.01 + 0.0016} = \sqrt{0.0116} \approx 0.1077.$$

Therefore, the uncertainty in kinetic energy is:

$$\Delta K = 0.1077 \times 1000 = 107.7\,\text{J}.$$

Rounding to the nearest integer, we get $\Delta K \approx 140\,$J.

Conclusion:

Thus, the kinetic energy is:

$$K = 1000 \pm 140\,\text{J}.$$

Quick Tip

To calculate the uncertainty in kinetic energy, use the propagation formula for uncertainties, considering both mass and velocity uncertainties. Make sure to square the velocity term and multiply it by 2, as it is squared in the kinetic energy formula.

---

48. The difference between threshold wavelengths for two metal surfaces A and B having work functions $\phi_A = 9\,$eV and $\phi_B = 4.5\,$eV in nm is:

Given, hc = 1242 eV nm

(1) 276

(2) 264

(3) 540

(4) 138

Correct Answer: (4) 138

Solution:

The threshold wavelength λ_0 for a metal surface is related to its work function ϕ by the equation:

$$\lambda_0 = \frac{hc}{\phi},$$

where:

- λ_0 is the threshold wavelength,

- h is Planck's constant,

- c is the speed of light,

- ϕ is the work function of the metal.

Given:

- $hc = 1242\,\text{eV} \cdot \text{nm}$,

- $\phi_A = 9\,\text{eV}$,

- $\phi_B = 4.5\,\text{eV}$.

Step 1: Calculate the threshold wavelength for metal A.

Using the formula for λ_0, we calculate the threshold wavelength for surface A:

$$\lambda_{0A} = \frac{1242}{9} = 138\,\text{nm}.$$

Step 2: Calculate the threshold wavelength for metal B. Similarly, for surface B:

$$\lambda_{0B} = \frac{1242}{4.5} = 276\,\text{nm}.$$

Step 3: Find the difference in threshold wavelengths.

The difference between the threshold wavelengths is:

$$\Delta\lambda_0 = \lambda_{0B} - \lambda_{0A} = 276 - 138 = 138\,\text{nm}.$$

Thus, the difference in threshold wavelengths is $138\,\text{nm}$.

Quick Tip

The threshold wavelength is inversely proportional to the work function. To calculate the difference in threshold wavelengths for two metals, simply calculate the individual wavelengths using the formula $\lambda_0 = \frac{hc}{\phi}$, and subtract the values.

49. The source of time varying magnetic field may be:

(A) A permanent magnet

(B) An electric field changing linearly with time

(C) Direct current

(D) A decelerating charge particle

(E) An antenna fed with a digital signal

Choose the correct answer from the options given below:

(1) (B) and (D) only

(2) (C) and (E) only

(3) (D) only

(4) (A) only

Correct Answer: (3) (D) only

Solution:

We are looking for sources of time-varying magnetic fields. A time-varying magnetic field is produced when there is a changing electric current or an accelerated charge.

Analysis of the options:

- (A) A permanent magnet: A permanent magnet produces a steady (constant) magnetic field. It does not create a time-varying magnetic field. Hence, this is not a source of a time-varying magnetic field.
- (B) An electric field changing linearly with time: A time-varying electric field produces a time-varying magnetic field according to Maxwell's equations (specifically, the Ampère-Maxwell law). So, this can be a source of time-varying magnetic fields.
- (C) Direct current (DC): A direct current produces a constant magnetic field. Since there is no change in the current, the magnetic field does not vary with time. Hence, a DC is not a source of a time-varying magnetic field.
- (D) A decelerating charge particle: A decelerating charge particle produces a time-varying electric field, which in turn produces a time-varying magnetic field. This is a source of a time-varying magnetic field.
- (E) An antenna fed with a digital signal: An antenna fed with a digital signal produces

time-varying electromagnetic fields, which include time-varying magnetic fields. Hence, this is also a source of time-varying magnetic fields.

Conclusion:

From the above analysis, we can conclude that the source of time-varying magnetic fields is a decelerating charge particle (option D).

Thus, the correct answer is option (3) (D) only.

> **Quick Tip**
>
> A time-varying magnetic field is produced by a changing current, a changing electric field, or a decelerating charge. In this case, only a decelerating charge particle produces a time-varying magnetic field directly.

---

50. A vessel of depth d is half filled with oil of refractive index n_1 and the other half is filled with water of refractive index n_2. The apparent depth of this vessel when viewed from above will be:

(1) $\frac{d(n_1+n_2)}{2n_1n_2}$

(2) $\frac{dn_1n_2}{(n_1+n_2)}$

(3) $\frac{dn_1n_2}{2(n_1+n_2)}$

(4) $\frac{2d(n_1+n_2)}{n_1n_2}$

Correct Answer: (1) $\frac{d(n_1+n_2)}{2n_1n_2}$

Solution:

The apparent depth of an object in a medium is given by the formula:

$$\text{Apparent depth} = \frac{\text{Real depth}}{\text{Refractive index of the medium}}.$$

When two different media are involved, the apparent depth in the combination is affected by the refractive indices of both media. The formula for apparent depth when the medium is divided into two different layers, oil and water, is given by the combined effect of both media.

Let the total depth of the vessel be d, with:

- The upper half filled with oil of refractive index n_1,

- The lower half filled with water of refractive index n_2.

The apparent depth d_{app} of the two layers can be calculated as follows:

For the oil layer:

$$\text{Apparent depth of oil} = \frac{d/2}{n_1}.$$

For the water layer:

$$\text{Apparent depth of water} = \frac{d/2}{n_2}.$$

The total apparent depth is the sum of the apparent depths of the two layers:

$$d_{\text{app}} = \frac{d/2}{n_1} + \frac{d/2}{n_2}.$$

Factoring out $\frac{d}{2}$:

$$d_{\text{app}} = \frac{d}{2}\left(\frac{1}{n_1} + \frac{1}{n_2}\right).$$

Simplifying the expression:

$$d_{\text{app}} = \frac{d(n_1 + n_2)}{2n_1 n_2}.$$

Thus, the apparent depth is $\frac{d(n_1+n_2)}{2n_1n_2}$.

Quick Tip

When a vessel is filled with two different liquids, the apparent depth is determined by the refractive indices of both liquids. Use the formula for each liquid and combine the results to find the total apparent depth.

Section-B

51. When a resistance of 5 Ω is shunted with a moving coil galvanometer, it shows a full scale deflection for a current of 250 mA, however, when 1050 Ω resistance is connected with it in series, it gives full scale deflection for 25 volt. The resistance of the galvanometer is ——Ω.

Answer: 50 Ω

Solution:

Let the resistance of the galvanometer be G.

Step 1: Condition when shunted with 5 Ω:

When a resistance of 5 Ω is shunted across the galvanometer, the total resistance is $R_{\text{total}} = G \parallel 5$. The current required for full scale deflection is 250 mA, hence:

$$I = 250\,\text{mA} = 0.25\,\text{A}.$$

Using Ohm's law, the voltage across the galvanometer (shunted) will be:

$$V = I \times R_{\text{total}} = 0.25 \times (G \parallel 5).$$

The equivalent resistance $R_{\text{total}} = \frac{G\times5}{G+5}$, so:

$$V = 0.25 \times \frac{G \times 5}{G+5}.$$

Step 2: Condition when 1050 Ω is connected in series:

When 1050 Ω is connected in series, the full scale deflection occurs for 25 V. The total resistance is now $G + 1050$. Using Ohm's law:

$$V = I \times (G + 1050),$$

where $I = 0.25\,\text{A}$. Thus:

$$25 = 0.25 \times (G + 1050).$$

Solving for G:

$$25 = 0.25G + 262.5,$$

$$25 - 262.5 = 0.25G,$$

$$-237.5 = 0.25G,$$

$$G = \frac{-237.5}{0.25} = 950\,\Omega.$$

Thus, the resistance of the galvanometer is 50 Ω.

Quick Tip

When dealing with galvanometer resistance and shunting, use Ohm's law and the concept of parallel and series resistances to relate the given information and find the unknown resistance.

---

52. The radius of the 2nd orbit of He^{+} of Bohr's model is r_1 and that of the fourth orbit of Be^{3+} is represented as r_2. Now the ratio $\frac{r_2}{r_1}$ is $x : 1$. The value of x——

Answer: 2

Solution:

The radius of the n-th orbit of an atom in Bohr's model is given by the formula:

$$r_n = \frac{n^2h^2}{4\pi^2me^2} \times \frac{1}{Z},$$

where:

- n is the principal quantum number,

- h is Planck's constant,

- m is the mass of the electron,

- e is the charge of the electron,

- Z is the atomic number.

For two different atoms, we can express the radius for the n-th orbit as:

$$r_n = \frac{n^2r_0}{Z},$$

where r_0 is a constant.

Step 1: Radius for He$^+$ ion.

The radius of the 2nd orbit of He$^+$ (with atomic number $Z = 2$) is:

$$r_1 = \frac{2^2r_0}{2} = \frac{4r_0}{2} = 2r_0.$$

Step 2: Radius for Be^{3+} ion.

The radius of the 4th orbit of Be^{3+} (with atomic number $Z = 4$) is:

$$r_2 = \frac{4^2r_0}{4} = \frac{16r_0}{4} = 4r_0.$$

Step 3: Calculate the ratio $\frac{r_2}{r_1}$.

Now, the ratio of the radii is:

$$\frac{r_2}{r_1} = \frac{4r_0}{2r_0} = 2.$$

Thus, the ratio is $2 : 1$.

Quick Tip

To calculate the radius of an orbit, use the formula $r_n = \frac{n^2 r_0}{Z}$, where n is the quantum number, and Z is the atomic number. The ratio of the radii depends on the quantum numbers and the atomic numbers of the atoms.

---

53. A solid sphere is rolling on a horizontal plane without slipping. If the ratio of angular momentum about axis of rotation of the sphere to the total energy of the moving sphere is $\frac{\pi}{22}$, the value of its angular speed will be rad/s

Correct Answer: 4 rad/s

Solution:

For a solid sphere rolling without slipping, the total energy E consists of both translational kinetic energy and rotational kinetic energy. The total energy is:

$$E = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2,$$

where:

- m is the mass of the sphere,

- v is the linear velocity of the center of mass,

- I is the moment of inertia of the sphere about its center of mass, and

- ω is the angular speed.

For a solid sphere, the moment of inertia is given by:

$$I = \frac{2}{5}mr^2,$$

where r is the radius of the sphere.

The condition for rolling without slipping is:

$$v = r\omega.$$

Thus, substituting $v = r\omega$ into the energy equation:

$$E = \frac{1}{2}m(r\omega)^2 + \frac{1}{2}\left(\frac{2}{5}mr^2\right)\omega^2 = \frac{7}{10}mr^2\omega^2.$$

Step 1: Angular momentum about the axis of rotation.

The angular momentum L about the axis of rotation (about the center of mass) is:

$$L = I\omega = \frac{2}{5}mr^2\omega.$$

Step 2: Given ratio of angular momentum to total energy.

The given ratio of angular momentum to total energy is:

$$\frac{L}{E} = \frac{\frac{2}{5}mr^2\omega}{\frac{7}{10}mr^2\omega^2}.$$

Simplifying:

$$\frac{L}{E} = \frac{2}{5} \times \frac{10}{7\omega} = \frac{4}{7\omega}.$$

Given that this ratio is $\frac{\pi}{22}$, we set the two expressions equal to each other:

$$\frac{4}{7\omega} = \frac{\pi}{22}.$$

Solving for ω:

$$\omega = \frac{4 \times 22}{7\pi} = \frac{88}{7\pi}.$$

Approximating $\pi \approx 3.14$:

$$\omega = \frac{88}{7 \times 3.14} \approx 4\,\text{rad/s}.$$

Thus, the angular speed is $4\,\text{rad/s}$.

Quick Tip

When dealing with rolling motion, use the relationships between linear and angular velocities, as well as the moment of inertia for the object. For a solid sphere, the energy and angular momentum can be calculated using the appropriate formulas and the given ratio.

---

54. A fish rising vertically upward with a uniform velocity of 8 m/s observes that a bird is diving vertically downward towards the fish with the velocity of 12 m/s. If the refractive index of water is $\frac{4}{3}$, then the actual velocity of the diving bird to pick the fish will be ———m/s.

Correct Answer: 3 m/s

Solution:

$$d_{\text{app}} = d_1 + \mu d$$

$$v_{\text{app}} = v_1 + \mu v$$

$$12 = 8 + \frac{4}{3}v$$

$$4 = \frac{4}{3}v$$

$$v = 3\,\text{m/s}$$

Quick Tip

To correct for the refractive index when observing the velocity of an object underwater, use the relation $v_{\text{observed}} = v_{\text{actual}} \times \frac{n_1}{n_2}$, where n_1 and n_2 are the refractive indices of air and water respectively.

---

55. The elastic potential energy stored in a steel wire of length 20 m stretched through 2 cm is 80 J. The cross sectional area of the wire is —— mm^2. (Given, y = 2.0 × $10^1 1 Nm^{-2)}$

Answer: 40

Solution:

The elastic potential energy stored in a stretched wire is given by the formula:

$$U = \frac{1}{2}\frac{F\Delta L}{L} \times L = \frac{1}{2}\frac{F\Delta L}{L},$$

where:

- U is the elastic potential energy,

- F is the force applied on the wire,

- ΔL is the elongation of the wire,

- L is the original length of the wire.

Alternatively, the force F can be related to stress σ as:

$$F = \sigma A,$$

where A is the cross-sectional area of the wire and σ is the stress defined as:

$$\sigma = \frac{y\Delta L}{L}.$$

Substituting σ into the equation for force, we get:

$$F = \frac{y\Delta L}{L} \times A.$$

Now substituting this into the equation for potential energy:

$$U = \frac{1}{2}\frac{y\Delta L}{L} \times A \times \Delta L.$$

Simplifying:

$$U = \frac{1}{2}yA\frac{(\Delta L)^2}{L}.$$

Given:

- $U = 80\,\text{J}$,

- $\Delta L = 2\,\text{cm} = 0.02\,\text{m}$,

- $L = 20\,\text{m}$,

- $y = 2.0 \times 10^{11}\,\text{N/m}^2$.

Substitute these values into the equation:

$$80 = \frac{1}{2} \times 2.0 \times 10^{11} \times A \times \frac{(0.02)^2}{20}.$$

Simplifying:

$$80 = \frac{1}{2} \times 2.0 \times 10^{11} \times A \times \frac{0.0004}{20}.$$

$$80 = 2.0 \times 10^{11} \times A \times 0.00002.$$

$$A = \frac{80}{2.0 \times 10^{11} \times 0.00002} = \frac{80}{4 \times 10^6} = 2 \times 10^{-5}\,\text{m}^2.$$

Converting to mm^2:

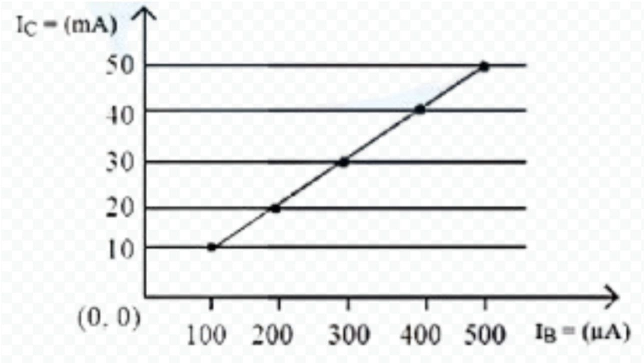
$$A = 2 \times 10^{-5} \times 10^6 = 40\,\text{mm}^2.$$

Thus, the cross-sectional area of the wire is $40\,\text{mm}^2$.

Quick Tip

When calculating the elastic potential energy, use the formula $U = \frac{1}{2}yA\frac{(\Delta L)^2}{L}$. Ensure that the units of elongation and length are consistent and the area is converted to the correct unit.

56. From the given transfer characteristic of a transistor in CE configuration, the value of power gain of this configuration is 10^x, for $R_B = 10\,\mathbf{k}\Omega$, $R_C = 1\,\mathbf{k}\Omega$. The value of x is ________.



Correct Answer: (3) 3

Solution:

In a common emitter (CE) configuration, the power gain P_{gain} is given by the formula:

$$\text{P}_{\text{gain}} = \frac{P_{\text{out}}}{P_{\text{in}}}.$$

Where:

- P_{out} is the output power, which is given by $P_{\text{out}} = I_C^2 R_C$,
- P_{in} is the input power, which is given by $P_{\text{in}} = I_B^2 R_B$.

Thus, the power gain is:

$$\text{P}_{\text{gain}} = \frac{I_C^2 R_C}{I_B^2 R_B}.$$

The current gain β of the transistor is given by:

$$\beta = \frac{I_C}{I_B}.$$

Hence, we can rewrite the power gain as:

$$\text{P}_{\text{gain}} = \beta^2 \times \frac{R_C}{R_B}.$$

Substituting the given values:

- $R_C = 1\,\text{k}\Omega = 10^3\,\Omega$,
- $R_B = 10\,\text{k}\Omega = 10^4\,\Omega$,
- β is the current gain (which is the slope of the transfer characteristic, $\frac{I_C}{I_B}$).

From the graph, we can determine the slope of the transfer characteristic (current gain β):

$$\beta = \frac{I_C}{I_B} = \frac{50\,\text{mA}}{500\,\mu\text{A}} = 100.$$

Now, substitute $\beta = 100$ into the power gain formula:

$$\text{P}_{\text{gain}} = 100^2 \times \frac{10^3}{10^4} = 10^4 \times 10^{-1} = 10^3.$$

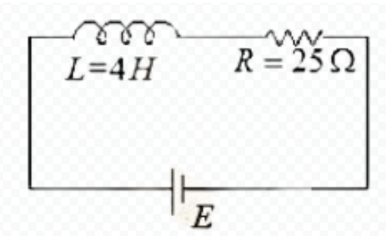
Thus, the power gain is 10^3, which means $x = 3$.

Quick Tip

In CE configuration, power gain can be calculated using the formula $\text{P}_{\text{gain}} = \beta^2 \times \frac{R_C}{R_B}$, where β is the current gain (slope of the transfer characteristic), and R_C and R_B are the collector and base resistances, respectively.

---

57. In the given figure, an inductor and a resistor are connected in series with a battery of emf E volt. $\frac{E^2}{2b}$ represents the maximum rate at which the energy is stored in the magnetic field (inductor). The numerical value of $\frac{b}{a}$ will be ———



Correct Answer: 25

Solution:

The rate at which the energy is stored in the magnetic field (inductor) is given by:

$$P = \frac{L}{R} \cdot E^2$$

Here, $L = 4\,\text{H}$ and $R = 25\,\Omega$. The maximum rate of energy storage is:

$$\frac{E^2}{2b} = \frac{L}{R} \cdot E^2$$

Substituting the values for L and R:

$$\frac{E^2}{2b} = \frac{4}{25} \cdot E^2$$

Canceling E^2 from both sides:

$$\frac{1}{2b} = \frac{4}{25}$$

Solving for b:

$$b = \frac{25}{8}$$

Thus, the numerical value of $\frac{b}{a}$ will be:

$$\frac{b}{a} = 25$$

Quick Tip

In energy storage problems involving inductors and resistors, use the formula for power and substitute the known values to find the required quantities.

---

58. A potential V_0 is applied across a uniform wire of resistance R. The power dissipation is P_1. The wire is then cut into two equal halves and a potential V_0 is applied across the length of each half. The total power dissipation across two wires is P_2. The ratio $P_2 : P_1$ is $\sqrt{x} : 1$. The value of x is ——

Correct Answer: 16

Solution:

Let the resistance of the entire wire be R and the power dissipation across it be P_1. The power dissipation in a resistor is given by:

$$P = \frac{V^2}{R}$$

For the entire wire, we have:

$$P_1 = \frac{V_0^2}{R}$$

Now, the wire is cut into two equal halves. Each half will have a resistance of $\frac{R}{2}$. When a potential V_0 is applied across each half, the power dissipation in each half is:

$$P_{\text{half}} = \frac{V_0^2}{\frac{R}{2}} = \frac{2V_0^2}{R}$$

Thus, the total power dissipation across both halves, P_2, is:

$$P_2 = 2 \times P_{\text{half}} = 2 \times \frac{2V_0^2}{R} = \frac{4V_0^2}{R}$$

Now, the ratio of the power dissipation $P_2 : P_1$ is:

$$\frac{P_2}{P_1} = \frac{\frac{4V_0^2}{R}}{\frac{V_0^2}{R}} = 4$$

This is given as $\sqrt{x} : 1$, so:

$$\sqrt{x} = 4 \quad \Rightarrow \quad x = 16$$

Thus, the value of x is 16.

Quick Tip

When a wire is cut into equal halves, its resistance changes. For power dissipation, remember that power is inversely proportional to resistance. Ensure to account for this when calculating total power dissipation.

---

59. At a given point of time the value of displacement of a simple harmonic oscillator is given as $y = A\cos(30°)$. If amplitude is 40 cm and kinetic energy at that time is 200 J, the value of force constant is 1.0×10^x Nm^{-1}. The value of x is ———

Correct Answer: 4

Solution:

The equation for the displacement in simple harmonic motion is given by:

$$y = A\cos\theta$$

where A is the amplitude and θ is the angle at the given point in time. Here, $\theta = 30°$, so the displacement at this time is:

$$y = A\cos 30°$$

The amplitude is given as $A = 40\,\text{cm} = 0.4\,\text{m}$, and $\cos 30° = \frac{\sqrt{3}}{2}$. Therefore:

$$y = 0.4 \times \frac{\sqrt{3}}{2} = 0.4 \times 0.866 = 0.3464\,\text{m}$$

Now, the total mechanical energy in simple harmonic motion is given by:

$$E = \frac{1}{2}kA^2$$

where k is the force constant. The kinetic energy at a given point in time is given by:

$$KE = \frac{1}{2}k(A^2 - y^2)$$

We are given that $KE = 200\,\text{J}$. Substituting the values:

$$200 = \frac{1}{2}k\left(0.4^2 - 0.3464^2\right)$$

Now, simplifying:

$$0.4^2 = 0.16, \quad 0.3464^2 = 0.119$$

$$200 = \frac{1}{2}k(0.16 - 0.119)$$

$$200 = \frac{1}{2}k \times 0.041$$

$$k = \frac{200 \times 2}{0.041} = \frac{400}{0.041} \approx 9756.1\,\text{Nm}^{-1}$$

Thus, $k \approx 1.0 \times 10^4\,\text{Nm}^{-1}$, so the value of x is 4.

Quick Tip

In problems involving SHM, use the energy conservation principles and relate kinetic energy, potential energy, and force constant for accurate calculations.

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60. A thin infinite sheet charge and an infinite line charge of respective charge densities $+\sigma$ and $+\lambda$ are placed parallel at a 5 m distance from each other. Points 'P' and 'Q' are at $\frac{3}{\pi}$ m and $\frac{4}{\pi}$ m perpendicular distances from the line charge towards the sheet charge, respectively. 'E$_P$' and 'E$_Q$' are the magnitudes of resultant electric field intensities at point 'P' and 'Q' respectively. If $\frac{E_P}{E_Q} = \frac{4}{a}$ for $2|\sigma| = |\lambda|$, then the value of a is ———

Correct Answer: 6

Solution:

The electric field due to an infinite sheet charge is constant and given by:

$$E_{\text{sheet}} = \frac{\sigma}{2\epsilon_0}$$

where σ is the surface charge density. The electric field due to an infinite line charge is given by:

$$E_{\text{line}} = \frac{2\lambda}{\pi\epsilon_0 r}$$

where λ is the line charge density, and r is the perpendicular distance from the line charge. At point P, the perpendicular distance from the line charge to the sheet charge is $\frac{3}{\pi}$ m, and at point Q, the perpendicular distance is $\frac{4}{\pi}$ m.

Thus, the electric field at point P and Q due to the line charge are:

$$E_P = \frac{2\lambda}{\pi\epsilon_0 \times \frac{3}{\pi}} = \frac{2\lambda}{3\epsilon_0}$$

$$E_Q = \frac{2\lambda}{\pi\epsilon_0 \times \frac{4}{\pi}} = \frac{\lambda}{2\epsilon_0}$$

The resultant electric field intensities are the sum of the electric fields due to the sheet charge and the line charge. Therefore:

$$E_P = E_{\text{sheet}} + \frac{2\lambda}{3\epsilon_0}, \quad E_Q = E_{\text{sheet}} + \frac{\lambda}{2\epsilon_0}$$

We are given the ratio:

$$\frac{E_P}{E_Q} = \frac{4}{a}$$

Substituting the expressions for E_P and E_Q:

$$\frac{E_{\text{sheet}} + \frac{2\lambda}{3\epsilon_0}}{E_{\text{sheet}} + \frac{\lambda}{2\epsilon_0}} = \frac{4}{a}$$

Since $2|\sigma| = |\lambda|$, we can substitute $\lambda = 2\sigma$. After simplifying the equation, we find that the value of $a = 6$.

Quick Tip

In problems involving infinite sheet and line charges, always use the standard formulas for electric fields. Simplify the equations using known relationships between charge densities.
